A mesoscale approach to simulate residual deformations in complex laser welding processes

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Abstract

Laser welding can be characterized by very small radii of beam, in the order of tenths of a millimeter, and very short high power inputs (more kW in few ms) and thus it can be certainly classified as a microscale process with a high level of physical complexity. This is clearly incompatible, due to the high computational costs, with the analysis of macroscale processes related to large geometries and non-uniform welding patterns. In order to overcome this issue, a simplified Finite Element Method (FEM) - based thermo-elastoplastic model is presented to simulate heat transfer and residual deformations due to thermal expansion and material plasticity. The idea is to substitute the microscale analysis with a mesoscale approach that renounces to describe in detail all the physical phenomena occurring in the heated zone and focuses attention on the correct prediction of the keyhole depth and weld pool size, that are the most important parameters to describe the mechanical characteristics of the welded joint. The concept of passive element, based on the numerical adjustment of the material properties in order to take into account the orthotropic behavior during the keyhole formation, is introduced. In particular the new approach has been tested on the pulsed laser welding process of two overlapping DC04 steel plates with thickness of 0.5 mm (so called sandwich) and validated through experimental tests involving different input parameters, such as power, pulse duration and frequency, speed and geometrical pattern.

Keywords: laser welding simulation, mesoscale keyhole approximation, FEM element passivation

1. Introduction

In laser welding the power intensity [W/mm^2] can be so high that the vaporization temperature is reached. Part of the vaporized metal leaves the system, representing a power loss, but allows a much higher heat penetration compared to a low intensity process. This because the combined effects of the material removal and the vapour pressure on the liquid sidewalls create the so called keyhole, a microstructural hole in the metal layer in which the laser beam can penetrate directly and indirectly through multiple reflections. For this reason laser welding has a high energetic efficiency (figure 1).

From a physical point of view this process is very complex, not only because it involves all three phases, solid, liquid and vapour, but also for the complexity of the thermal and mass transfer dynamics and for the interacting forces as well described by Fabbro [1] who analysed experimentally the keyhole structure at different speed regimes emphasizing the effects of ablation pressure, vapour plume, surface tension, gravity and induced electromagnetic forces.

Several models have been proposed to analyse numerically all these phenomena: Courtois [2,3] defined a 3D model to describe both heat and fluid flow characteristics, taking into considerations also the multi-reflections of the laser beam in the keyhole, treated under its wave form by solving Maxwell's equations. Zhang [4] worked with the Volume Of Fluid (VOF) method and analysed the multi-reflections of the laser beam using the ray-tracing algorithm. Both methods have in common high complexity and computational time. In alternative to the so called self-consistent methods like the two mentioned above, characterized by a very high level of detail, simplified methods are available. Typical simplifications apply to the heat source and, instead of modelling the multi-reflections of the laser beam, the power is defined as function of the penetration: Hozoorbakhsh [5] proposes a heat source that is the combination of a surface and an adaptive volumetric component, Shanmugam [6] uses the double-ellipsoidal heat source concept based on Goldak studies [7], while Artinov introduced a 2-step thermal analysis where the moving heat flux is the result of a moving temperature distribution based on a fixed laser beam thermal analysis.

If the study is focusing on the keyhole depth only, evaluating the size of the melted area, and thus of the welded joint, is a fundamental parameter and other simplifications can be adopted. Fabbro [8] proposed for example a model that describes the laser beam penetration as function only of a normalized aspect ratio R_0 and of a characteristic speed V_0 related to the Peclet number.

After solving the thermal fluid dynamical equations, in order to evaluate the residual stresses and deformation after welding, a mechanical simulation has to be added based on the temperature distribution calculated with the first model. Huang [9] suggested to use a local solid model to evaluate temperature distribution and plastic strains and a global shell model to evaluate the total deformation from the local plastic strains. With his model Xu [10] was able to capture the change in convexity of ultra-thin 316 stainless steel plates welded with a pulsed laser process depending on the input power and the related angular deformation and longitudinal shrinkage force.

In this paper, a thermo-elastoplastic FEM model is proposed to simulate pulsed laser welding processes of sandwich layers, in particular the residual deformations (figures 2a and 2b). This model is intended for industrial purposes and for this reason, in order to be compatible with the industrial needs, it has to be not only accurate but also fast from a computational time and set up point of view and suitable for complex geometries and welding patterns. The microscale of the keyhole and the macroscale of the real parts to be welded cannot cohabit in the same model without increasing exponentially the computational time. For this reason the keyhole description is simplified at the mesoscale level introducing the concept of passive elements, that allows to predict accurately the size of the keyhole acting just on the material properties of the components. Extending Tirand's proposal to increase the thermal conductivity to simulate the Marangoni effect [11], in this case the thermal conductivity is modified to simulate also the metal vaporization and thus the air inside the keyhole that add almost zero thermal resistance.

The paper is organized as follows: the next paragraph describes the project methodology pointing out the main aspects of the FEM model including the adopted assumptions. In the successive paragraph the FEM model is then validated through experimental data coming from literature and original tests and the results are finally summarized and discussed in the last paragraph.

Figure 1 Schematic representation of low and high intensity laser beam welding regimes

Figure 2 Problem identification: case of two metal sheets welded together that lose planarity (a) and detail of the complex welding pattern (b)

2. Methodology description

2.1 Simulation procedure

The proposed FEM model is a 3D model calculated in two phases: a transient thermal simulation followed by a transient quasi-static mechanical one (figure 3). In the first simulation, the input is the laser power source, function of position and time, and the output is the temperature field, function of time. The output of the thermal simulation is the input for the mechanical one and the final output are the residual deformations, defined in this case as out-of-plane distortion.

Figure 3 Schematic representation of the simulation flow and main characteristics and innovations of the model

2.2 Heat source

Regarding the laser beam, as mentioned before, the heat source is function of position and time. With position is meant only the in-plane position and not the coordinate along the layer thickness. This means that the defocusing of the laser beam and the laser reflections along the keyhole do not modify neither the total power nor its distribution. However more complex approximations like the 3D conical heat source or the 3D conical and cylindrical combination heat source described by Fabbro [12] and in general the definition of the heat source in direction of the penetration are in this case not required because, using the passive elements that allow heat penetration, the heat source is 2D and applied only to the upper face of the plate.

A generic heat source is numerically described by a heat flux distribution, function of space (x,y) and time. Dimensionally this quantity is a power per unit area. Every welding technology has a specific power distribution that can be approximated by a mathematical function or a combination of functions. Bradáč [13] sustains for example that if the 3D Gaussian distribution suits better for laser and electron beam welding simulation, arc welding is better described by a hemispherical surface or by a double-ellipsoid source.

For a laser beam a good fit is represented by the Gaussian distribution, characterized by a peak in the center and exponentially decreasing values with the distance.

The Gaussian distribution on a plane (figure 4) has following general formulation:

$$f_{G\_plane} = \frac{\mathrm{P}}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad (1)$$

Figure 4 Plot of the 1D Gaussian distribution defined in (1) with σ=1, μ=5 and P=1

where σ is the standard deviation, μ the mean and P the total power. The standard deviation indicates the distance from the mean that includes the 34.15% of the total power and it is commonly associated with half of the radius of the beam r [14]. This means that the radius of the beam is equal to 2 standard deviations. At this distance the power is 13.5% of the peak.

In the 3D space (figure 5) the Gaussian function assumes this general form,

$$f_{G\_volume} = Ae^{-\left(\frac{(x-\mu)^2}{2{\sigma_x}^2}+\frac{(y-\mu)^2}{2{\sigma_y}^2}\right)} \qquad (2)$$

Figure 5 Plot of the 2D Gaussian distribution defined in (2) with σ=1, μ=0 and P=1

If $\sigma_x = \sigma_y$ and $\mu = 0$, then

$$f_{G\_volume} = Ae^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)} \qquad (3)$$

Integrating in the domain]-inf,inf[it can be demonstrated that

$$\iint Ae^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)} dxdy = 2\pi A\sigma^2 \qquad (4)$$

Considering that the integral of the heat flux is the total heat flow, this means

$$2\pi A\sigma^2 = P\ [W] \qquad (5)$$

And thus,

$$A = \frac{P}{2\pi\sigma^2}\ [W/mm^2] \qquad (6)$$

Substituting (6) in (3),

$$f_{G_{volume}}(x,y) = \frac{P}{2\pi\sigma^2} e^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)} \qquad (7)$$

valid for a steady laser beam with center in the origin and no losses [15].
For a moving heat flux with a certain efficiency described by a parameter η, (7) becomes,

$$f_{G_{volume}}(x,y,t) = \frac{\eta P}{2\pi\sigma^2} e^{-\left(\frac{(x-x_0(t))^2+(y-y_0(t))^2}{2\sigma^2}\right)} \qquad (8)$$

where x_0 and y_0 is the moving position of the laser beam center, function of time, and η an overall efficiency that takes into account all losses and has been evaluated experimentally during the calibration process.

2.3 Passive elements

In order to simulate the material vaporization at high temperature and the keyhole formation the concept of passive element is introduced. It is a simplified method to enhance the heat penetration acting at mathematical level on the thermal material properties of the welded part, instead of physically modelling the keyhole, e.g. using the ekill command in Ansys to deactivate the elements above the vaporization temperature as suggested by Zhang [16].
This method consists in modifying the thermal conductivity, increasing it strongly in the direction of penetration and at the same time setting the in-plane values to 0. In this way the heat is almost instantaneously transferred to the bottom of the keyhole and the penetration continues. The thermal conductivity above the vaporization temperature changes from isotropic to orthotropic. The thermal conductivity is set to 10^11 W/mK, imposing that in stationary conditions, with a power of 3.5 kW applied on a radius of 0.2 mm, the temperature difference per unit length is less than 1 °C (figure 6).

The elements become thermally passive and this explains the definition. To be even more precise they are passive for the energy above the vaporization temperature but they still store the energy of the solid and liquid phase. This energy is in reality partially lost with the released vapour and has to be taken into account in the overall process efficiency defined in (8) and evaluated experimentally.

Figure 6 Influence of the thermal conductivity k on the stationary temperature distribution of an infinite thick DC04 steel plate with a heat source of 3.5 kW even distributed on a circular surface of radius 0.2 mm. This temperature difference introduces a numerical error due to the non-infinite k

2.4 Critical bonding temperature (TBND)

The melting temperature is considered the critical bonding temperature [17]: at this temperature a bonded contact at the layers interface is established and it is maintained also once the temperature drops.

2.5 Contact step control

During the welding process a magnetic field is generated to keep the parts in position and guarantee a good welding accuracy. At the end the magnetic field is deactivated, the parts are released and the residual deformation becomes visible.
To simulate this condition a contact is defined at the interface with frictional no separation properties during welding and simple frictional properties during cooling down with a frictional coefficient in both phases of 0.2.

3. Validation and calibration

The approach presented in this paper to simulate the residual deformation after welding has been validated through a two-step process: the first step was based on experimental data from literature in order to check the validity of the passive element concept. The second step, based on original experimental data, helped to validate and calibrate the full model. In total 7 different experiments have been conducted. Part of the data, 3 experiments, has been used to calibrate the model and 4 to validate it. During the calibration process the overall efficiency of the welding process, indicated by η, has been evaluated.

3.1 Passive element validation

The first validation is based on experimental data found in literature [18]. The problem is axisymmetric and can be simulated with a 2D model. The experimental data describe the keyhole penetration depth on a steel plate for 3 different laser powers (1000, 1250 and 1500 W) in a 25 ms transient. The power source is steady. Assuming an overall process efficiency of 65% (included the power loss due to metal evaporation) there is a very good accordance in all 3 cases (figures 7a, 7b, 7c and 8a, 8b, 8c). In conclusion the passive elements, known the overall efficiency of the welding process, can well describe the keyhole formation and, considering the simple and ductile definition and their timesaving nature, are a valid concept to analyze welding processes within industrial activities.

Figure 7 Comparison of experimental and numerical results with P=1500 W at 2 ms (a), 8 ms (b) and 18 ms (c)

Figure 8 Simulated and experimental keyhole depth at 1000 W (a), 1250 W (b) and 1500 W (c) function of time

3.2 Full thermomechanical model validation

The test model used for the validation of the full thermomechanical model consists in 2 squared plates with edge 100 mm and 0.5 mm thickness made of DC04 steel. Please note that the 2 welded plates are represented as single layer in the thermal simulation. In addition, only for the thermal analysis, a 10 mm thick steel plate is bonded at the bottom to simulate the conductive thermal dissipation of the support. On the other side, a convection heat transfer with coefficient 10 W/m^{2}K is applied. At the end of the welding process the coefficient is increased of a factor 4 to cool down the parts to room temperature. On the thin plates the mesh is locally refined where the pulsing power is applied (figures 9a and 9b).

Figure 9 Geometry (a) and mesh (b) of the 3D test model

An energy balance check is performed to verify if the geometry and mesh independent input power is correctly applied to the model. In order to do that the full model is used to calculate the temperature distribution caused by one single welding spot and the simulation ends when the temperature is uniform on the plate. For this particular case there are no power losses and the material properties are constant and evaluated at room temperature.

The input energy has to match with the increment of internal energy of the plate.

After 1000 s the average temperature is 22.794 °C. The total energy increment in the plate can be calculated as following:

$$\Delta E = \rho V c \Delta T = 7850 \cdot 0.1 \cdot 0.1 \cdot 0.001 \cdot 434 \cdot (22.794 - 22) = 27.050 \frac{J}{spot} \qquad (9)$$

with ρ the density [kg/m^3], V the total volume of the plate [m^3], c the specific heat [J/kgK] and ΔT the temperature difference between initial and final conditions.

The total input energy, considering for this case a power (P) of 3.6 kW, a pulse duration (t) of 9 ms and an efficiency (η) of 83.5% is equal to:

$$Q = P \cdot t \cdot \eta = 3600 \cdot 0.009 \cdot 0.835 = 27.054 \frac{J}{spot} \qquad (10)$$

The energy balance is perfectly satisfied.

With the described model several simulations have been conducted and repeated experimentally to validate the results. The experiments have been performed on a Trumpf Trulaser Cell 3000 laser machine: its peculiarity is the dual phase laser beam, this means that the total power can be distributed on 2 concentric areas, the external one with radius 0.2 mm and the inner one with radius 0.05 mm (figure 10). For this project a power ratio 70% outside and 30% inside has been chosen and the Gaussian profile has been accordingly adapted to take into consideration both sources (figure 11). Excessive power concentrations in small areas could in fact lead to instability in the keyhole formation and unexpected welding performances [19].

Figure 10 Concept of dual phase laser beam: power can be distributed between two concentric areas to optimize power intensity and welding area. For the current case the power ratio between outside and inside is 2.33

Figure 11 Comparison between ratio 70-30 and ratio 0-100 (entire energy concentrated in the inner circle) with P=3800 W. The two distribution have the same underlying volume in the 3D space

After welding the residual deformations have been measured accurately with a 3D scanner and compared with the simulations.

The variables are frequency, power, duration of the pulse, speed of the laser arm and welding pattern. Part of these data has been used to calibrate the model and, in particular, to estimate the overall efficiency of the process. The rest has been used to validate the model. From the calibration process an average efficiency of 83.5% has been calculated and the final validation has been conducted simulating the welding process with 4 different powers (3800, 3600, 3400 and 3200 W). For every configuration the final deformation at the 4 corners referred to the center has been evaluated and compared to the experimental data. The other parameters are defined as following: pulse 9 ms, frequency 33.15 Hz and speed 0.25 m/s. The welding pattern consists in 5 horizontal lines with 7 mm distance from the left and right edges, 10 mm from top and bottom edges and 20 mm equal spaced.

For the experimental tests plates made of steel DC04 have been used. In the FEM model the thermal and mechanical material properties have been defined as follows (table 1 and figures 12a, 12b and 12c):

Table 1 Thermal properties of steel DC04. k_1 is the in-plane and k_2 the out of plane value of thermal conductivity

	solid	**liquid**	**vapour**
T [K]	*<1798*		*>3134*
k_1 [W/mK]	60.5	33	*0*
k_2 [W/mK]	60.5	33	*10^11*
cp [J/kgK]	434	573	573
ρ [kg/m3]	7850	7287	7287
ΔH_melt [J/m3]	1.85E+09		-
ΔH_vap [J/m3]	-	4.44E+10	

Figure 12 Mechanical properties of steel DC04 function of temperature: isotropic instantaneous coefficient of thermal expansion (a), isotropic elasticity (b) and bilinear isotropic hardening (c)

The results of the validation are visualized in figures 13a1, 13a2, 13b1, 13b2, 13c1, 13c2, 13d1 and 13d2 and summarized in table 2:

Figure 13 Comparison between simulation (1) and experimental (2) results at different powers: 3800 W (a), 3600 W (b), 3400 W (c) and 3200 W (d). Soll is the theoretical value, Ist the measured one and Abw. is the abbreviation for Abweichung and indicates the deviation

Table 2 Validation results: for every configuration (power) is reported the simulated, the measured and the differential deformation in z direction (out of plane) using the plate center as reference

		corner											
		1			2			3			4		
conf.	*power [W]*	sim [mm]	exp [mm]	Δd [mm]	sim [mm]	exp [mm]	Δd [mm]	sim [mm]	exp [mm]	Δd [mm]	sim [mm]	exp [mm]	Δd [mm]
V1M	*3800*	0.75	0.31	0.44	0.39	0.39	0.00	0.63	0.55	0.08	0.24	0.24	0.00
V9M	*3600*	0.71	0.33	0.38	0.37	0.42	-0.05	0.54	0.49	0.05	0.22	0.28	-0.06
V10M	*3400*	0.68	0.23	0.45	0.37	0.4	-0.03	0.6	0.62	-0.02	0.33	0.28	0.05
V11M	*3200*	0.62	0.33	0.29	0.42	0.52	-0.10	0.57	0.84	-0.27	0.37	0.36	0.01

The average difference between simulated and experimental results is 0.14 mm, 0.08 mm if considering also the positive or negative sign. Except for one corner, corresponding to the initial spot, the simulation deliveries good results. Further investigations to explain this issue are actually on going. Figure 14 shows three snapshots of the thermal simulation.

Figure 14 Temperature distribution in °C at different time steps and corresponding position and direction of laser beam, indicated by a black spot and an arrow respectively. Horizontal lines are welding lines, along the oblique lines the laser beam moves from the end of a welding line to the next

4. Conclusions

The fundamental aspect of the presented model is the concept of passive element, based on the idea that the keyhole depth can be accurately predicted treating the material in the welded area as an orthotropic material whose thermal conductivity in the direction of the keyhole is much higher than the in-plane values. In addition, an overall process efficiency is defined in order to take into account the power losses due to metal vaporization.

The model has been subjected to a double validation: the first one verifies the validity of the passive element concept and the second one considers the whole 3D thermo-elastoplastic model.

The first validation, based on literature data, confirms the validity of the approach: depth is accurately predicted both in size and time at different powers and considering a constant efficiency, valid for all cases.

The second validation, based on direct experimental data, is successful both qualitatively and quantitatively: deformation convexity is continuously well captured and the model quite accurately predicts the residual deformation after welding. Deformation has been evaluated at the four corners of the plates and in most cases the discrepancy from the experimental data is within 0.1 mm. The average is 0.14 mm (0.08 mm considering also the positive and negative sign).

Laser welding processes are characterized by many factors namely frequency, pulse duration, power, speed and welding pattern. The discussed model is a tentative approach to give a quick and accurate instrument to whomever is interested in evaluating and minimizing the residual deformation after welding and does not have the time or the resources to run multiple experimental tests in order to optimize all the welding parameters. Results are available within two hours for the simulated case (1.5 hours for the thermal analysis and 0.5 hour for the mechanical analysis) and for this reason it can be stated that the model is suitable not only for the simulation of a few cases but also for multi-parameter optimizations with a medium to large number of Design Of Experiments (DOEs).

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Declaration of conflicting interests

The authors declare that there is no conflict of interest.

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