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A fast design technique for robust industrial controllers

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Abstract

This paper provides a new fast design method for robust industrial controllers via majorant systems in the frequency domain. The proposed methodology allows to establish several fast design techniques for a broad class of industrial controllers of plants with internal and/or external delays, parametric and/or structural uncertainties, and subject to disturbances, when an analytical model of the plant or data acquired from simple experimental tests are available. The provided design and control techniques are more general with respect to the Ziegler-Nichols ones and their numerous variants, which, in some cases, do not guarantee the control system stability.

The used key idea consists in increasing the frequency response of the process to be controlled with the frequency response of a simpler system, also of order greater than one, with external delay, which allows designing, using simple formulas, controllers of PI, PID, PIDR, PI2, PI2D, PI2DR, PI2D2, and PI2D2R types. The designed controllers always guarantee stability margins larger than those of appropriate reference systems. Therefore, good performance of robustness of the stability and tracking precision of smooth references, with respect to parametric and/or structural uncertainties and/or smooth disturbances, are always guaranteed.

The stated general methodology and various performance comparisons, also about the tracking precision of references with bounded first or second derivative, are illustrated and validated in several case studies, experimentally too.

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1. Introduction

The notable diffusion of industrial proportional-integral-derivative (PID) controllers is due to their low cost, good performance which they are able to guarantee for many plants, and also their ease of design.

Indeed, it is well-known that in a large number of process control applications the most used controllers are of PID type, and, in several industrial cases, a large percentage is of proportional-integral (PI) type [1,2,4-59].

The above considerations are the main reasons why, for almost a century, there have been great attention and considerable efforts to extend and improve the most commonly used tuning methods, which are mainly the Ziegler-Nichols (ZN) tuning rules [1], Cohen-Coon methods [2], the approach based on internal model control (IMC) [5,48], and Tyreus and Luyben technique [11], with the aim to overcome some criticisms, above all in cases where the controlled system has not negligible delays [48,58,59], uncertainties [29,38,40,46,50,51,54,56], nonlinearities [29,38,40,46,50,51,54,56], and disturbances [49,51,54,56], is multi-input multi-output (MIMO) [13,29,36,44,54,56], or unstable [17,26,41,44].

Moreover, some tuning techniques based on fuzzy approaches [16,20,22,28,36,52], and the use of artificial neural networks have been proposed [29,57], too.

Recently, in [54] it has been proposed a unified approach via majorant systems in the time domain (*TMSs*), which allows one to easily design a family of robust, smooth and effective control laws, also simple to implement, of proportional - *h* order integral - *k* order derivative (PIhDk) - type for broad classes of uncertain nonlinear MIMO systems, including mechatronic and transportation processes with ideal or real actuators and amplifies, subject to bounded disturbances and measurement errors.

In this paper, a new fast design method for robust industrial controllers via majorant systems in the frequency domain (*MSs*) is proposed. The provided methodology allows to establish several fast design techniques for a broad class of industrial controllers of plants with internal and/or external delays, parametric and/or structural uncertainties, and subject to disturbances, when an analytical model of the plant or data acquired from simple experimental tests are available.

The proposed design and control techniques are more general with respect to the ZN ones and their numerous variants, which, in some cases, do not guarantee the control system stability (see Subsection 4.1), and consider as control system design specifications also the ones related to the tracking of sufficiently smooth references.

The used key idea consists in increasing the frequency response of the process to be controlled, supposed asymptotically stable and with positive static gain, with the frequency response of a simpler system (of order greater than one, too) with an external delay. This allows designing, using simple formulas, controllers with a proportional action, a single or double integral action, and a single or double derivative action (ideal or real), i.e., of PI, PID, PIDR, PI2, PI2D, PI2DR, PI2D2, and PI2D2R types, which allow gradually to improve the performance of the control system.

The designed controllers always guarantee stability margins larger than those of appropriate reference systems. Therefore, good performance of robustness of the stability and tracking precision of sufficiently smooth references, with respect to parametric and/or structural uncertainties and/or sufficiently smooth disturbances, are always guaranteed.

If the considered process is not asymptotically stable, or the goal is to further improve the dynamic performance of the control system, the design of the industrial controller can be made jointly with the one of a dynamic compensator with two degrees of freedom (see [3] and Remark 16).

This paper also provides some methods to easily determine, theoretically or experimentally, good *MSs*, and approaches to estimate the maximum tracking error of a generic reference with bounded first or second derivative, in presence of disturbances with bounded first or second derivative, too.

It is explicitly highlighted that, in the literature, there are no fast design techniques for controllers of PIDR, PI2, PI2D, PI2DR, PI2D2, PI2D2R types, and the proposed control method can be used also to easily design other controllers able to guarantee more stringent performance requirements.

Finally, it is worth noting that the easiness of design, applying the provided techniques, is within the reach of any engineer or technician in the information and industrial fields.

The paper is organized as follows. In Section 2, they are given the problem statement and useful preliminary results about the generalized gains to estimate the tracking error, the determination of appropriate *MSs*, the experimental determination of the frequency response of a process, and the proposal of some reference control systems.

In Section 3, the main theorems, using the Nyquist criterion, are established about the proposed fast design techniques for the robust controllers PI, PID, PIDR, PI2, PI2D, PI2DR, PI2D2, PI2D2R, providing some guidelines to easily design the given controllers and comparisons with the most commonly used tuning methods in the literature.

Section 4 includes three groups of examples, which show the limitations of the most used design methods for industrial controllers available in the literature, their overcoming by the proposed design method, and the effectiveness of the numerous industrial controllers that can easily be designed with it.

Finally, Section 5 outlines the main peculiarities and advantages of the provided results and presents the ongoing research.

2. Problem statement and preliminaries

2.1. Problem statement

Consider a plant described by the following equations:

$$\begin{aligned}\dot{x}(t) &= A_1(p)x(t) + A_2(p)x(t-\tau_i) + B(p)u(t-\tau_u) + E(p)d(t)\\ y(t) &= C(p)x(t-\tau_y) + D(p)d(t)\ ,\end{aligned} \tag{1}$$

where $x \in R^n$ is the state, $u, y, d \in R$ are the input, the output, and a disturbance, respectively, $\tau_i, \tau_u, \tau_y \geq 0$ are the possible internal, input and output delays, respectively, $p \in \wp \subset R^\mu$ is the vector of uncertain parameters, and A_1, A_2, B, E, C, D are matrices of appropriate dimensions.

Remark 1. It is worth explicitly noting that the considered plants (1) are linear, time-invariant, and uncertain. Hence, the uncertain parameters p and also the delays τ_i, τ_u, τ_y are time-invariant. The class of the above systems is significant not only from a theoretical point of view but also from an engineering one. Indeed, many mechanical, electrical, electro-mechanical, thermal, fluid dynamical, and medical systems can be modelled with systems of the type

$$L(p)\dot{x} + M(p)x + N(p)u = 0 \Rightarrow \dot{x} = -L^{-1}(p)M(p)x - L^{-1}(p)N(p)u = A(p)x + B(p)u, \tag{2}$$

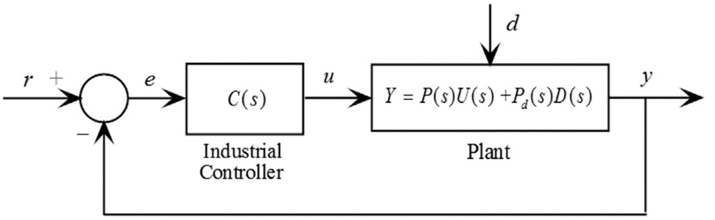

Fig. 1. Control scheme of a process controlled with an industrial controller.

with $L(p), M(p), N(p)$ matrices linear with respect to some physical parameters p of its components, which always present measurement errors and/or can change in the case of their replacement during a maintenance process. Moreover, the above mentioned systems often have constant delays, not always perfectly known, due to transportation phenomena and/or remote monitoring and control devices.

From (1), using the Laplace transform, with null initial conditions, it is

$$\begin{aligned} &Y(s) = P(s)U(s)+P_d(s)D(s) \\ &P(s) = C\left(sI - A_1 - A_2 e^{-s\tau_i}\right)^{-1} B e^{-s\tau_e},\ \tau_e = \tau_u + \tau_y \\ &P_d(\mathrm{s}) = C\left(sI - A_1 - A_2 e^{-s\tau_i}\right)^{-1} E e^{-s\tau_y} + D. \end{aligned} \tag{3}$$

The aim of this paper is to provide a general design method for the industrial controllers $C(s)$ described by

$$\begin{aligned} &K_p + \frac{K_i}{s} = \frac{K_p s + K_i}{s} \quad (PI) \\ &K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s} \quad (PID) \\ &K_p + \frac{K_i}{s} + K_d \frac{s}{s/N+1} = \frac{k_d s^2 + k_p s + k_i}{s(s/N+1)} \quad (PIDR) \\ &K_p + \frac{K_{i1}}{s} + \frac{K_{i2}}{s^2} = \frac{K_p s^2 + K_{i1} s + K_{i2}}{s^2} \quad (PI2) \\ &K_p + \frac{K_{i1}}{s} + \frac{K_{i2}}{s^2} + K_d s = \frac{K_d s^3 + K_p s^2 + K_{i1} s + K_{i2}}{s^2} \quad (PI2D) \\ &K_p + \frac{K_{i1}}{s} + \frac{K_{i2}}{s^2} + K_d \frac{s}{s/N+1} = \frac{k_d s^3 + k_p s^2 + k_{i1} s + k_{i2}}{s^2(s/N+1)} \quad (PI2DR) \\ &K_p + \frac{K_{i1}}{s} + \frac{K_{i2}}{s^2} + K_{d1} s + K_{d2} s^2 = \frac{K_{d2} s^4 + K_{d1} s^3 + K_p s^2 + K_{i1} s + K_{i2}}{s^2} \quad (PI2D2) \\ &K_p + \frac{K_{i1}}{s} + \frac{K_{i2}}{s^2} + K_{d1} \frac{s}{s/N+1} + K_{d2} \frac{s^2}{(s/N+1)^2} \\ &\quad = \frac{k_{d2} s^4 + k_{d1} s^3 + k_p s^2 + k_{i1} s + k_{i2}}{s^2(s/N+1)^2} \quad (PI2D2R)\ , \end{aligned} \tag{4}$$

to control the plant Eq. (3), using the control scheme in Fig. 1, so as to guarantee sufficient stability margins and track, with an acceptable precision, standard references or generic ones with a bounded first or second derivative.

Remark 2. Note that, nowadays, since with the help of the new technologies is easy to implement also complex output feedback controllers, a main issue is their design, not their complexity. Hence, other more effective controllers can be considered, e.g., inserting in cascade to the above considered controllers appropriate phase-lead and/or phase-lag compensators. The design of such more complex controllers, as it can be shown in Section 3, can easily be made using the proposed control method.

Remark 3. It is worth noting that sufficiently large stability margins are needed since the asymptotically stability of the controlled system has to be guaranteed also in disturbed conditions, due to unmodelled dynamics and/or unexpected parametric variations. Concerning this, take into account that several electro-mechanical systems are modelled neglecting the electric dynamics, many flexible structures are modelled neglecting the high-frequency modes, numerous plants have components whose parameters vary with the environmental conditions of their working areas (which are unforeseeable), etc. Moreover, as it is well-known (see Tables 1-4 reported in Subsection 2.5, the switching technique from open to closed loop using the Nichols chart, etc.), high values of the stability margins (which can easily be determined by the open loop frequency response) lead to small resonance peaks of the closed-loop control system and, hence, small oscillations or overshoots during the transient phases, and to not excessive control signals.

Remark 4. The proposed controllers are all output feedback controllers; hence, they can easily be implemented. If the output measurement is affected by not negligible noises, the ideal derivative actions can be replaced with real derivative actions, with an appropriate bandwidth, using the controllers PIDR, PI2DR, or PI2D2R. These controllers, using the proposed method, are designed taking into account *a priori* the bandwidth of the real derivative actions, not choosing it after, as the most control methods in the literature commonly made, which can be reason of instability.

2.2. Generalized gains to estimate the tracking error

Concerning the tracking error of a reference having a generic waveform but with bounded first or second derivative, the following theorem is useful.

Theorem 1. *Using the PI, PID, PIDR (PI2, PI2D, PI2DR, PI2D2, PI2D2R) controllers, the maximum absolute value of the tracking error of a generic reference $r(t)$ with bounded first derivative (bounded second derivative), in the presence of a generic disturbance $d(t)$ with bounded first derivative (bounded second derivative) satisfies the following inequality:*

$$\begin{aligned}&|e(t)| \le H_1 \max|\dot{r}(t)| + H_{1d} \max\left|\dot{d}(t)\right| \\ &\left(|e(t)| \le H_2 \max|\ddot{r}(t)| + H_{2d} \max\left|\ddot{d}(t)\right|\right),\end{aligned} \tag{5}$$

where H_1, H_{1d}, (H_2, H_{2d}) are appropriate constants (see Eq. (9)).

Proof. Setting

$$C_1(s) = \begin{cases} K_p s + K_i \quad (PI \text{ case}) \\ K_d s^2 + K_p s + K_i \quad (PID \text{ case}) \\ (k_d s^2 + k_p s + k_i)/(s/N + 1) \quad (PIDR \text{ case}) \end{cases}$$

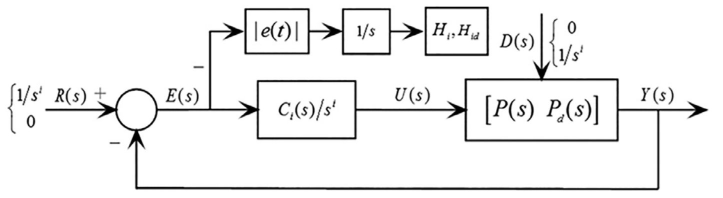

Fig. 2. Scheme to compute the generalized gains H_i, H_{id}.

$$C_2(s) = \begin{cases} K_p s^2 + K_{i1} s + K_{i2} \quad (PI2 \text{ case}) \\ K_d s^3 + K_p s^2 + K_{i1} s + K_{i2} \quad (PI2D \text{ case}) \\ (k_d s^3 + k_p s^2 + k_{i1} s + k_{i2})/(s/N+1) \quad (PI2DR \text{ case}) \\ K_{d2} s^3 + K_{d1} s^3 + K_p s^2 + K_{i1} s + K_{i2} \quad (PI2D2 \text{ case}) \\ (k_{d2} s^4 + k_{d1} s^3 + k_p s^2 + k_{i1} s + k_{i2})/(s/N+1)^2 \quad (PI2D2R \text{ case}) , \end{cases} \tag{6}$$

from the scheme in Fig. 1 it is

$$\begin{aligned} &E(s) = R(s) - Y(s) = S_i(s) s^i R(S) + S_{id}(s) s^i D(S) \\ &S_i(s) = \frac{1}{s^i + P(s) C_i(s)}, S_{id}(s) = -\frac{P_d(s)}{s^i + P(s) C_i(s)}, i = 1, 2, \end{aligned} \tag{7}$$

from which, if the control system is asymptotically stable, yields

$$|e(t)| \le H_i \max \left| d^i r(t)/dt^i \right| + H_{id} \max \left| d^i d(t)/dt^i \right|, i = 1, 2, \tag{8}$$

where

$$H_i = \int_0^\infty |w_i(\tau)| d\tau, H_{id} = \int_0^\infty |w_{id}(\tau)| d\tau, w_i(\tau) = L^{-1}(S_i(s)), w_{id}(\tau) = L^{-1}(S_{id}(s)). \tag{9}$$

Remark 5. From Eq. (7) it easily follows that the *generalized gains* $H_i, H_{id}, i = 1, 2$, can be computed with the scheme in Fig. 2.

Remark 6. If $w_i(t) \ge 0$ it is easy to prove that $H_i = 1/K_i$, where $K_i = C_i(0)P(0), i = 1, 2$; i.e., H_1 is equal to the inverse of the velocity constant K_v, and H_2 is equal to the inverse of the acceleration constant K_a.

The condition $w_i(t) = L^{-1}(W_i(s)) \ge 0$ is said to be *external positivity condition* of the system with transfer function (*t.f.*) $W_i(s)$. In [31], some results are reported, which easily allow to establish the external positivity condition of $W_i(s)$.

Remark 7. Given a reference signal $r(t)$, the change of variable $t = c\tau$ yields $dr/d\tau = c dr/dt$, and $d^2 r/d\tau^2 = c^2 d^2 r/dt^2$. Hence, by reducing the "velocity $\dot{r}(t)$" of $r(t)$ the tracking error decreases. E.g., halving the "velocity" of $r(t)$ (i.e., assuming $c = 0.5$), with the PI, PID, PIDR controllers the maximum absolute value of the tracking error, if $d(t) = 0$, halves, while, with the PI2, PI2D, PI2DR, PI2D2, PI2D2R controllers the maximum absolute value of the tracking error, if $d(t) = 0$, is the fourth part.

Therefore, varying the "velocity $\dot{r}(t)$" of $r(t)$ by an appropriate factor c, it is possible to keep the tracking error within an acceptable tolerance.

Remark 8. A reference signal $r(t)$ with a bounded ith derivative, $i = 1, 2$, can be obtained by interpolating a set of given points $(t_k,\ r_k), k = 0,\ 1,\ \ldots,\ n_r$, with appropriate splines or by filtering any piecewise constant or piecewise linear signal $\tilde{r}(t)$ with a second (third)-order filter

$$\dot{\zeta} = \begin{bmatrix} 0 & 1 \\ -f_2 & -f_1 \end{bmatrix}\zeta + \begin{bmatrix} 0 \\ f_2 \end{bmatrix}\tilde{r},\quad \begin{bmatrix} r \\ \dot{r} \end{bmatrix} = \zeta$$
$$\left(\dot{\zeta} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -f_3 & -f_2 & -f_1 \end{bmatrix}\zeta + \begin{bmatrix} 0 \\ 0 \\ f_3 \end{bmatrix}\tilde{r},\quad \begin{bmatrix} r \\ \dot{r} \\ \ddot{r} \end{bmatrix} = \zeta\right). \tag{10}$$

In the last case, it is also possible to reduce the control action, in particular during the transient phase by suitably choosing the initial conditions of the filter and its cutoff frequency.

Note that if the filter is a Bessel one with cutoff angular frequency ω_c, larger or equal to the angular frequency of $\tilde{r}(t)$, then $r(t) \cong \tilde{r}(t - t_r)$, where $t_r = \pi/(2\omega_c)$ for a second-order filter, $t_r = 3\pi/(4\omega_c)$ for a third-order one.

Remark 9. Approximating the control system with a time delay of duration $\tau_\delta \geq \tau_e$ the tracking error can be more effectively defined as follows:

$$E(s) = R(s)e^{-s\tau_\delta} - Y(s), \tag{11}$$

where the time delay τ_δ can be computed such to minimize $pH_r + (1-p)H_d,\ p \in [0, 1]$. To solve the above stated problems, the following preliminaries are given.

2.3. Majorant systems

Definition 1. Consider an asymptotically stable system with a positive static gain

$$P(s)|_{s=j\omega} = G(s, e^{-s\tau_i})e^{-s\tau_e}|_{s=j\omega} = M(\omega)e^{j\varphi(\omega)}, \tag{12}$$

said to be of *class SPG*.

A system

$$\hat{P}(s)|_{s=j\omega} = \hat{G}(s)e^{-sT}|_{s=j\omega} = \hat{M}(\omega)e^{j\hat{\varphi}(\omega)}, T \geq 0, \tag{13}$$

with $\hat{G}(s)$ without zeros or with negative real part zeros such that (see Fig. 3)

$$\hat{M}(\omega) \geq M(\omega),\quad \hat{\varphi}(\omega) \leq \varphi(\omega),\quad \forall\omega \geq 0, \tag{14}$$

is said to be majorant system in the frequency domain (*MS*) of the system $P(s)$, and the notation $\hat{P}(s) \geq P(s)$ will be used.

Remark 10. As it will be shown in the following of the treatment, the condition that the possible zeros of $\hat{G}(s)$ have always negative real parts is needed to apply the Nyquist criterion, with the aim to always guarantee acceptable stability margins.

The following Lemmas 1-4 hold, whose proofs are easy to be derived.

Lemma 1. *A system $P(s)$ of class SPG without zeros or with negative real part zeros is a MS of itself. More in general, $\hat{P}(s) = KP(s)e^{-sT} \geq P(s), \forall K \geq 1, \forall T \geq 0$.*

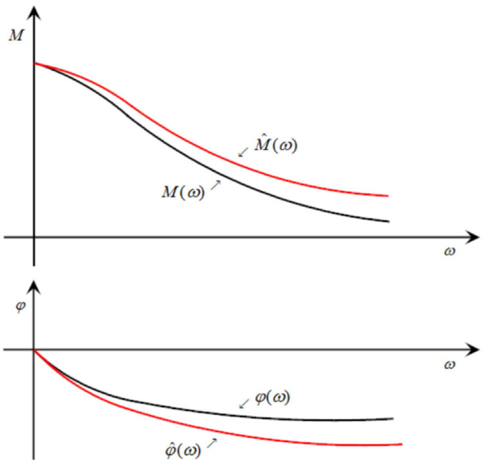

Fig. 3. Frequency responses of the system $P(s)$ (black) and its majorant $\hat{P}(s)$ (red). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Lemma 2. *Let $P_1(s)$, $P_2(s)$ be two systems of class SPG and $\hat{P}_1(s) \geq P_1(s)$, $\hat{P}_2(s) \geq P_2(s)$ their MSs, respectively. Then, $\hat{P}_1(s)\hat{P}_2(s) \geq P_1(s)P_2(s)$.*

Lemma 3. *Let $P(s)|_{s=j\omega} = M(\omega)e^{j\varphi(\omega)}$ be a system of class SPG and $\tilde{P}(s)|_{s=j\omega} = \tilde{M}(\omega)e^{j\tilde{\varphi}(\omega)}$ a system without zeros or with negative real part zeros. Then,*

$$\hat{P}(s) = K\tilde{P}(s)e^{-sT} \geq P(s), \tag{15}$$

where

$$K \geq \sup_{\omega\geq 0} \frac{M(\omega)}{\tilde{M}(\omega)}, T \geq \max\left\{0, \sup_{\omega>0} \frac{\tilde{\varphi}(\omega) - \varphi(\omega)}{\omega}\right\}. \tag{16}$$

Lemma 4. *Consider a system $P(s)$ of class SPG. Setting $P(s) = P_1(s)P_2(s)$, with $P_1(s)$ without zeros or with negative real part zeros, then*

$$\hat{P}(s) = KP_1(s)e^{-sT} \geq P(s), \tag{17}$$

where

$$K \geq \sup_{\omega\geq 0} M_2(\omega), T \geq \max\left\{0, \sup_{\omega>0} \frac{-\varphi_2(\omega)}{\omega}\right\}, P_2(s)|_{s=j\omega} = M_2(\omega)e^{j\varphi_2(\omega)} . \tag{18}$$

Lemma 5. *Consider a system of class SPG*

$$P(s) = \frac{G}{(1+s\tau_1)(1+s\tau_2)\cdots(1+s\tau_n)}, \quad \tau_1 \geq \tau_2 \geq \cdots \geq \tau_n > 0. \tag{19}$$

The following systems are *MSs* of the system (19):

$$\hat{P}_0(s) = Ge^{-(\tau_1+\tau_1+\cdots+\tau_n)s}, \quad \hat{P}_1(s) = \frac{G}{1+s\tau_1}e^{-(\tau_2+\tau_3+\cdots+\tau_n)s},$$

$$\hat{P}_2(s) = \frac{G}{(1+s\tau_1)(1+s\tau_2)} e^{-(\tau_3+\tau_4+\cdots+\tau_n)s}. \tag{20}$$

Proof. The proof easily follows noting that $\mathrm{mod}(1/(1+j\omega\tau)) \le 1$, $\arg(1/(1+j\omega\tau)) \ge -\omega\tau, \, if \;\tau > 0$.

Lemma 6. *Consider the system of class SPG*

$$P(s) = \frac{1+s\tau'}{1+s\tau}, \tau > \left|\tau'\right| \ge 0. \tag{21}$$

The following system is a *MS* of the system Eq. (21):

$$\hat{P}(s) = 1e^{-(\tau-\tau')s}. \tag{22}$$

Proof. The proof is quite simple in the case where $\tau' \le 0$ since it is easy to prove that $\mathrm{mod}((1+j\omega\tau')/(1+j\omega\tau)) \le 1$ and $\arg((1+j\omega\tau')/(1+j\omega\tau)) \ge -(\tau-\tau')\omega$.

The proof in the case where $\tau' > 0$ follows taking into account that $\mathrm{mod}((1+j\omega\tau')/(1+j\omega\tau)) \le 1$ and $\varphi(\omega) = \mathrm{atan}(\omega\tau') - \mathrm{atan}(\omega\tau) + (\tau-\tau')\omega \ge 0$ since $\varphi(0) = 0$ and

$$\begin{aligned} \frac{d\varphi(\omega)}{d\omega} &= \frac{\tau'}{1+(\tau'\omega)^2} - \frac{\tau}{1+(\tau\omega)^2} + \tau - \tau' = \\ &= (\tau-\tau')\left(1 - \frac{\tau}{1+(\tau\omega)^2}\right) + \tau'\left(\frac{1}{1+(\tau'\omega)^2} - \frac{1}{1+(\tau\omega)^2}\right) > 0, \forall\omega > 0. \end{aligned} \tag{23}$$

Lemma 7. *Consider the system of class SPG*

$$P(s) = \frac{G}{1+2\zeta/\omega_n s + s^2/\omega_n^2}, \zeta \in [0.1, 1]. \tag{24}$$

The following systems are *MSs* of the system (24):

$$\hat{P}_0(s) = G_0 e^{-T_0 s}, \quad \hat{P}_1(s) = \frac{G_1}{1+s/\omega_n} e^{-T_1 s}, \tag{25}$$

where G_0, T_0 and G_1, T_1 can be derived from Figs. 4 and 5, respectively.

Lemma 8. *Consider the system of class SPG*

$$P(s) = \frac{G(1-s\tau)}{1+2\zeta/\omega_n s + s^2/\omega_n^2}, \tau > 0. \tag{26}$$

The following system is a *MS* of the system (26):

$$\hat{P}(s) = \frac{G(1+s\tau)}{1+2\zeta/\omega_n s + s^2/\omega_n^2} e^{-s2\tau}. \tag{27}$$

Proof. The proof easily follows taking into account that $\mathrm{mod}(P(j\omega)) = \mathrm{mod}(\hat{P}(j\omega))$ and $-\mathrm{atan}(\omega\tau) \ge \mathrm{atan}(\omega\tau) - 2\omega\tau$.

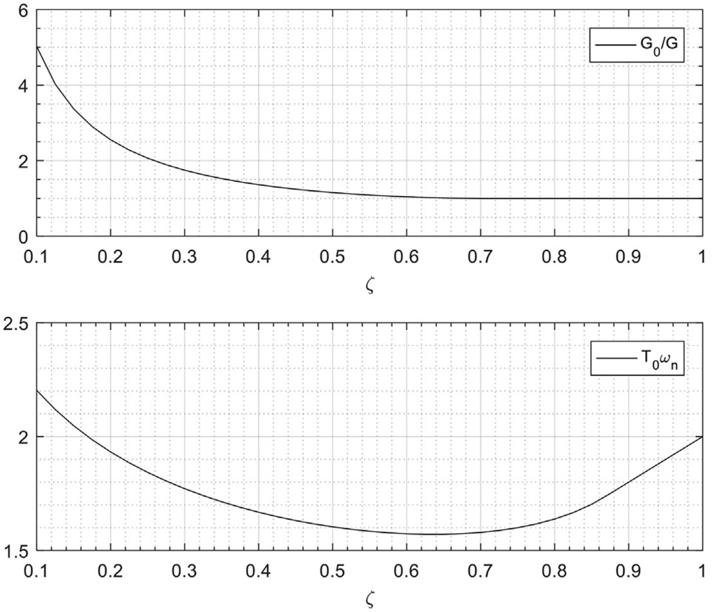

Fig. 4. Parameters of the *MS* $\hat{P}_0(s)$ of the system Eq. (24).

Lemma 9. *Let* $P(s,p)|_{s=j\omega} = M(\omega,p)e^{j\varphi(\omega,p)}$ *be a system with parametric uncertainties* $p \in \wp \subset R^{\mu}$, *of class SPG* $\forall p \in \wp$. *Clearly, a MS of* $\overset{\frown}{M}(\omega)e^{j\breve{\varphi}(\omega)}$, $\overset{\frown}{M}(\omega) \geq M(\omega,p)$, $\breve{\varphi}(\omega) \leq \varphi(\omega,p), \forall p \in \wp$, *is a MS of* $P(s,p), \forall p \in \wp$.

Remark 11. It is worth noting that, in the practice, there exist several systems with structural uncertainties, e.g., of multiplicative and/or additive type. For such systems it is easy to provide efficient numerical methods to determine some *MSs*.

Remark 12. Note that other techniques to determine a *MS* can be stated. E.g., some techniques based on the theory of dominant poles and zeros, and the ones which approximate the frequency response of the process, also the experimental one, using the Matlab command invfreqs or fmincon (see Algorithm 2 in the Appendix).

Finally, the following theorem holds, which is a basic result for the fast design of several industrial controllers.

Theorem 2. *Let* $P(s)$ *be a system of class SPG and* $\hat{P}(s) = \hat{G}(s)e^{-sT}$ *be its MS. Then,*

$$1e^{-sT} \geq \hat{G}^{-1}(s)P(s). \tag{28}$$

Proof. If it is posed $P(s)|_{s=j\omega} = G(s, e^{-sT_i})e^{-sT_e}|_{s=j\omega} = M(\omega)e^{j\varphi(\omega)}$ and $\hat{P}(s)|_{s=j\omega} = \hat{G}(s)e^{-sT}|_{s=j\omega} = \hat{M}(\omega)e^{j\hat{\varphi}(\omega)}e^{-j\omega T}$, from the relation $\hat{P}(s) \geq P(s)$ it is

$$\frac{M(\omega)}{\hat{M}(\omega)} \leq 1, \quad \varphi(\omega) - \left(\hat{\varphi}(\omega) - \omega T\right) \geq 0, \tag{29}$$

from which Eq. (28) follows.

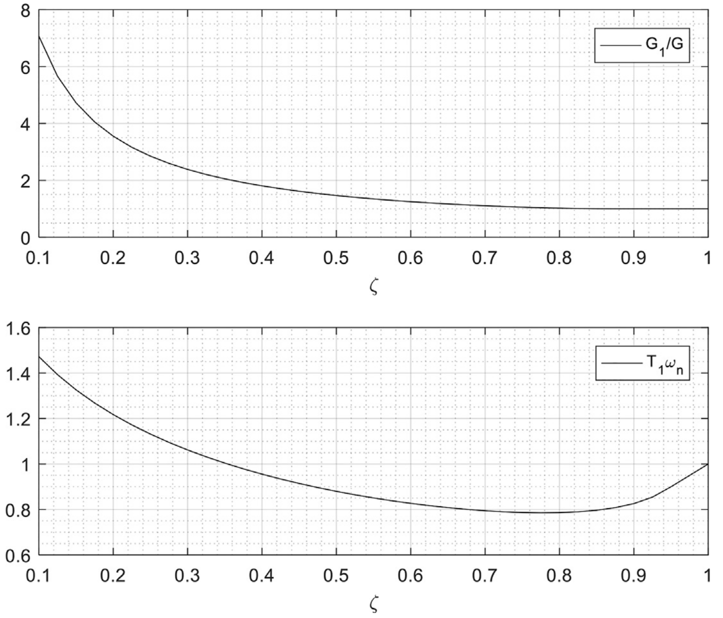

Fig. 5. Parameters of the *MS* $\hat{P}_1(s)$ of the system Eq. (24).

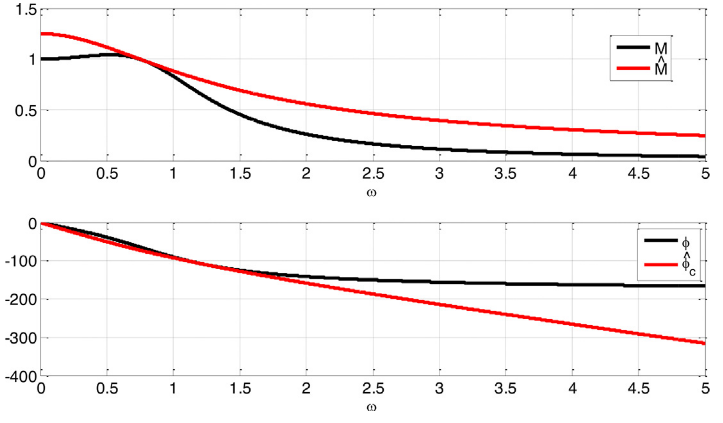

Fig. 6. M, φ of $P(s)$ Eq. (30) and $\hat{M}$, $\hat{\varphi}$ of the *MS* $\hat{P}(s)$ Eq. (31).

Example 1. Consider the system

$$P(s) = \frac{1}{s^2 + 1.2s + 1}. \tag{30}$$

Using Lemma 7, a *MS* of Eq. (30) is (see also Fig. 6)

$$\hat{P}(s) = \frac{1.25}{1 + s} e^{-0.827s}. \tag{31}$$

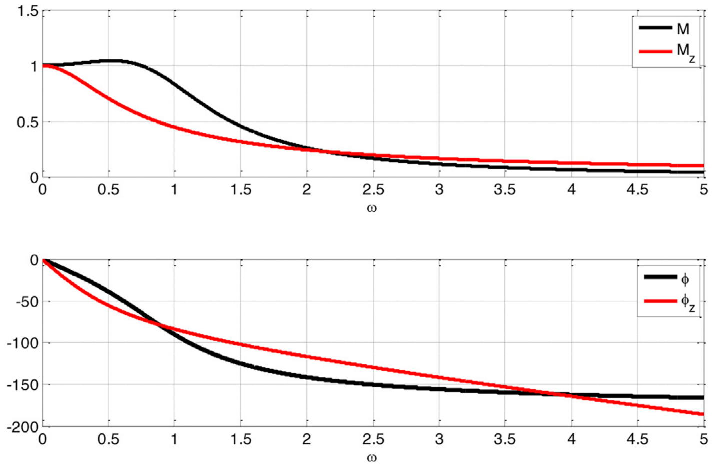

Fig. 7. M, φ of $P(s)$ Eq. (30); M_z, φ_z of the not *MS* $P_z(s)$ Eq. (32), obtained with the ZN's reaction curve method.

It is easy to verify that the approximation of the system $P(s)$ Eq. (30) with the first-order plus dead time (FOPDT) system

$$P_z(s) = \frac{1}{1+\tau_z s} e^{-T_z s} = \frac{1}{1+2.00s} e^{-0.355s}, \tag{32}$$

obtained with the ZN's reaction curve method (e.g., [7,9,13], and Fig. 17) is not a *MS* of $P(s)$ Eq. (30) (see Fig. 7).

In the same way, it is readily verify that the approximation of the system $P(s)$ Eq. (30) with the FOPDT system

$$P_a(s) = \frac{1}{1+\tau_a s} e^{-T_a s} = \frac{1}{1+0.669s} e^{-0.533s}, \tag{33}$$

obtained with the "areas method" (as in Fig. 19), is not a *MS* of the system $P(s)$ (see Fig. 8).

Example 2. Consider the system

$$P(s) = \frac{(s+1)(s+50)}{(s^2+4s+8)(s+20)} e^{-0.5s}. \tag{34}$$

Using Lemma 6, a second-order *MS* of Eq. (34) with a zero is

$$P_{2z}(s) = \frac{2.5(s+1)}{s^2+4s+8} e^{-(0.5+1/20-1/50)s} = \frac{2.5(s+1)}{s^2+4s+8} e^{-0.530s}. \tag{35}$$

Remark 13. Other *MSs*, which maximize also the velocity constant $K_v = \lim_{s\to 0} sP(s)C(s)$ or the acceleration constant $K_a = \lim_{s\to 0} s^2 P(s)C(s)$, can be obtained using Algorithm 2 reported in the Appendix or a similar one to maximize K_a.

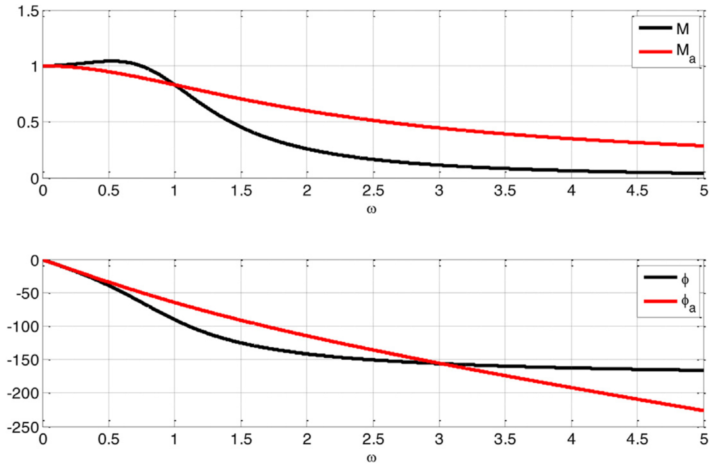

Fig. 8. M, φ of $P(s)$ Eq. (30); M_a, φ_a of the not *MS* $P_a(s)$ Eq. (33), obtained with the “areas method”.

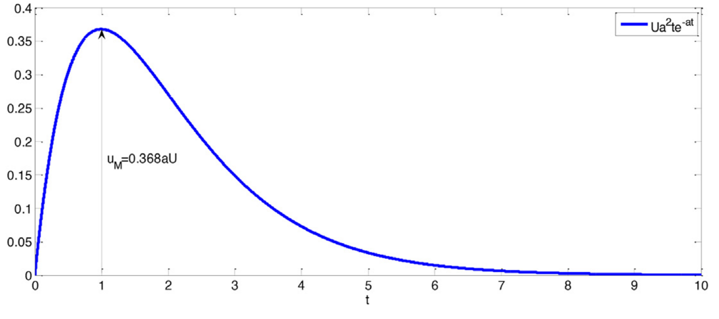

Fig. 9. Impulse Eq. (36) of area $U = 1$ and cutoff angular frequency $\omega_{u6dB} = 1$.

2.4. Experimental determination of the frequency response of a process

If no analytical model of the process to be controlled is available, a *MS* can be obtained experimentally determining the frequency response $M_s(\omega)e^{j\varphi_s(\omega)}$ of the process with the following algorithm.

Algorithm 1. (Identification of the Frequency Response)

Step 1. Force the system with the real impulse (always applicable since it does not have an oscillatory behavior)

$$u = Ua^2te^{-at} \tag{36}$$

of area U and amplitude $u_M = 0.368aU$ (see Fig. 9), with a such that its “cutoff angular frequency $\omega_{u6dB} = a$” is about equal to the one ω_c of the system.

Step 2. Compute the magnitude spectrum M_u and phase one F_u of u, and the ones M_y, F_y of the corresponding output y with the Matlab command fft.

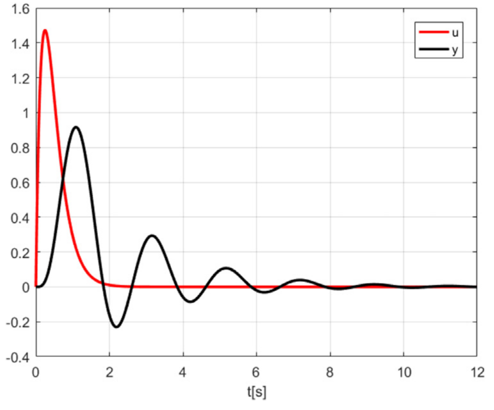

Fig. 10. The real impulse $u(t) = 4^2te^{-4t}$ and the response $y(t)$ of the system Eq. (37) to $u(t)$.

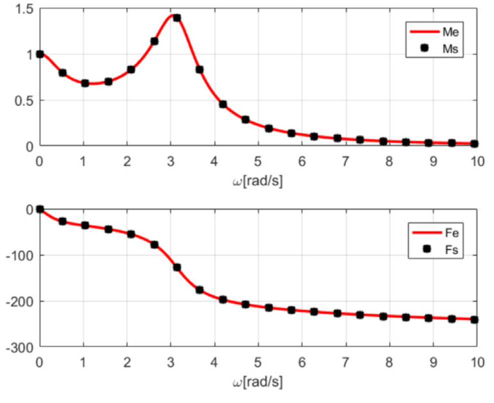

Fig. 11. Actual (-) and estimated (.) frequency responses of the system Eq. (37).

Step 3. Compute the experimental gain Ms and experimental phase Fs using the following Matlab commands:

```
Ms=My./Mu;
Fs=Fy-Fu;Fs=unwrap(Fs*pi/180)*180/pi;.
```

Example 3. In the following, the proposed experimental identification algorithm is illustrated, first considering a process without delays, and then a system with internal and external delays.

(a) By soliciting the system

$$P(s) = \frac{25(s+1)}{(s^2+s+10)(s^2+5.5s+2.5)} \tag{37}$$

with the real impulse $u(t) = 4^2te^{-4t}$ the response in Fig. 10 is obtained. Using the fast Fourier transform (FFT) with a sampling frequency $f_c = 100Hz$, in Fig. 11, the estimated frequency response and the actual one are shown.

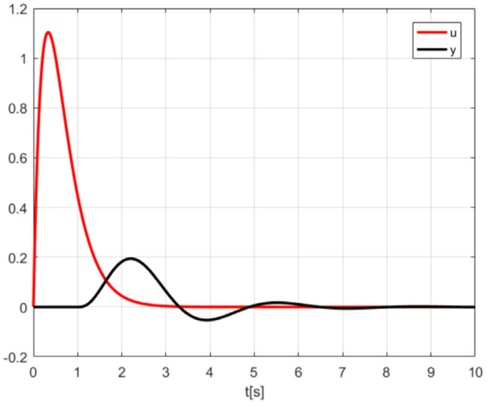

Fig. 12. The real impulse $u(t) = 3^2te^{-3t}$ and the response $y(t)$ of the system Eq. (38) to $u(t)$.

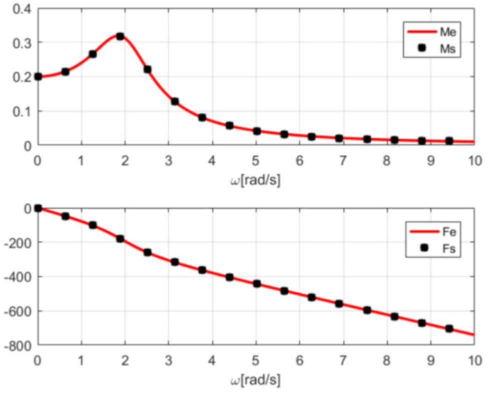

Fig. 13. Actual (-) and estimated (.) frequency responses of the system Eq. (38).

(b) Soliciting the system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} x(t-0.5) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t-1), \quad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \tag{38}$$

with the real impulse $u(t) = 3^2te^{-3t}$ the response in Fig. 12 is obtained. Using the FFT with a sampling frequency $f_c = 100Hz$, in Fig. 13, the estimated frequency response and the actual one are shown.

2.5. Reference majorant control systems

In the following, two reference type 1 control systems and two reference type 2 ones are proposed, which can be used to quickly design several industrial controllers. For the above mentioned systems, the main parameters concerning the stability robustness and the tracking precision are provided.

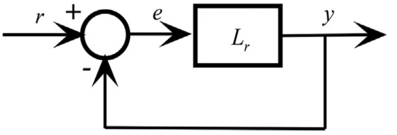

Fig. 14. Reference control system.

Table 1
Values of $m_g, M_\varphi, M_r,\ H_1$ for $KT = \pi/6, \pi/4, \pi/3$ if $L_r = Ke^{-sT}/s$.

KT	$\pi/3$ =1.047	$\pi/4$ =0.785	$\pi/6$ =0.524
m_g[dB]	3.52	6.02	9.54
M_φ[deg]	30	45	60
M_r	2.62	1.47	1.01
H_1	3.12T	2.28T	2.14T

Table 2
Values of $m_g, M_\varphi, M_r,\ H_1$ for $KT = 1.304,\ 1.006,\ 0.686$ if $L_r = K(1 + 0.25Ts)e^{-sT}/s$.

KT	1.304	1.006 ≅ 1	0.686
m_g[dB]	2.89	5.15	8.47
M_φ[deg]	30	45	60
M_r	2.97	1.57	1.00
H_1	2.61T	1.81T	1.60T

Lemma 10. *Consider the closed-loop control system in Fig. 14, where*

$$L_r(s) = K\frac{e^{-sT}}{s} = KT\frac{e^{-sT}}{sT}. \tag{39}$$

The gain margins m_g, the phase ones M_φ, the resonance peaks M_r, and the generalized gains H_1 for $KT = \pi/6, \pi/4, \pi/3$ are reported in Table 1.

Lemma 11. *Consider the closed-loop control system in Fig. 14, where*

$$L_r(s) = K(1 + 0.25Ts)\frac{e^{-sT}}{s} = KT(1 + 0.25Ts)\frac{e^{-sT}}{sT}. \tag{40}$$

The gain margins m_g, the phase ones M_φ, the resonance peaks M_r, and the generalized gains H_1 for $KT = 1.304, 1.006, 0.686$ are reported in Table 2.

Lemma 12. *Consider the feedback control system in Fig. 14, where*

$$L_r(s) = K(1 + 7Ts)\frac{e^{-sT}}{s^2} = KT^2(1 + 7Ts)\frac{e^{-sT}}{(Ts)^2}. \tag{41}$$

The gain margins m_g, the phase ones M_φ, the resonance peaks M_r, and the generalized gains H_2 for $KT^2 = 0.13, 0.10, 0.07$ are provided in Table 3.

Lemma 13. *Consider the feedback control system in Fig. 14, where*

$$L_r(s) = K(1 + 4sT)(1 + 0.4sT)\frac{e^{-sT}}{s^2} = KT^2\left(1 + 4.4Ts + 1.6T^2s^2\right)\frac{e^{-sT}}{(Ts)^2}. \tag{42}$$

Table 3
Values of $m_g, M_\varphi, M_r,\ H_2$ for $KT^2 = 0.13,\ 0.10,\ 0.07$ if $L_r = K(1+7Ts)e^{-sT}/s^2$.

KT^2	0.13	0.10	0.07
m_g[dB]	5.15	7.43	10.53
M_φ[deg]	28.42	37.78	45.16
M_r	2.21	1.59	1.35
H_2	$7.69T^2$	$10.00T^2$	$14.29T^2$

Table 4
Values of $m_g, M_\varphi, M_r,\ H_2$ for $KT^2 = 0.30,\ 0.20,\ 0.10$ if $L_r = K(1+4.4Ts+1.6T^2s^2)e^{-sT}/s^2$.

KT^2	0.30	0.20	0.10
m_g[dB]	2.70	6.22	12.24
M_φ[deg]	29.03	43.08	45.59
M_r	3.11	1.38	1.50
H_2	$3.33T^2$	$5.00T^2$	$11.95T^2$

The gain margins m_g, the phase ones M_φ, the resonance peaks M_r, and the generalized gains H_2 for $KT^2 = 0.30, 0.20, 0.10$ are reported in Table 4.

Proofs of Lemmas 10-13. Parameters $m_g, M_\varphi,\ M_r$ are of easy derivation, while the parameters H_1, H_2 can easily be computed using the simulation scheme in Fig. 2.

3. Main results

In the present section, some main theorems are established about fast design techniques for the robust controllers PI, PID, PIDR, PI2, PI2D, PI2DR, PI2D2, PI2D2R, providing also some guidelines to easily design the proposed controllers.

To this aim, in the following, only some *MSs* $\hat{P}(s) = \hat{G}(s)e^{-sT}$ with $T > 0$ are considered.

3.1. Fast design techniques for PI, PID and PIDR controllers

Theorem 3. *Let $P(s)$ be a system of class SPG and $\hat{P}(s) = \hat{G}(s)e^{-sT}$ be its MS. Then, the control system in Fig. 1 with*

$$C(s) = K\frac{\hat{G}^{-1}(s)}{s} = \frac{\pi}{4T}\frac{\hat{G}^{-1}(s)}{s} \tag{43}$$

is type 1 with $M_\varphi \geq 45^\circ, m_g \geq 6dB$, and a velocity constant $K_v = \frac{\pi}{4T}\frac{P(0)}{\hat{G}(0)}$.

Proof. Multiplying both members of Eq. (28) by K/s it turns out to be

$$L_r(s) = K\frac{e^{-sT}}{s} \geq K\frac{\hat{G}^{-1}(s)P(s)}{s} = C(s)P(s) = L(s). \tag{44}$$

The proof follows taking into account that $\hat{G}^{-1}(s)P(s)$ have all negative real part poles, from the Nyquist criterion, Eq. (44), the third column of Table 1, and Fig. 15, where $M_r(\omega)e^{j\varphi_r(\omega)} = L_r(s)|_{s=j\omega}$ and $M(\omega)e^{j\varphi(\omega)} = L(s)|_{s=j\omega}$.

Analogous results are obtained if $K \in [\pi/6,\pi/3]/T$.

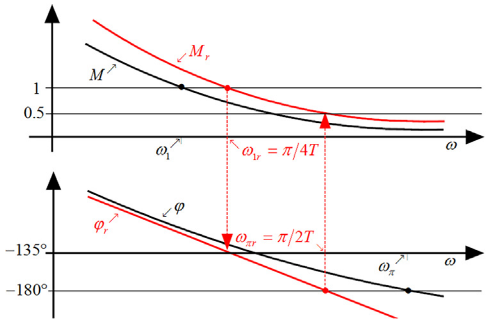

Fig. 15. Margins of $L_r(s)$ and $L(s)$.

Theorem 4. *Let $P(s)$ be a system of class SPG and $\hat{P}(s) = \hat{G}(s)e^{-sT}$ be its MS. Then, the control system in Fig. 1 with*

$$C(s) = K(1 + 0.25Ts)\frac{\hat{G}^{-1}(s)}{s} = \frac{1}{T}(1 + 0.25Ts)\frac{\hat{G}^{-1}(s)}{s} \tag{45}$$

is type 1 with $M_\varphi \geq 45^\circ$, $m_g \geq 5.14$, and $K_v = \frac{1}{T}\frac{P(0)}{\hat{G}(0)}$.

Proof. Multiplying both members of Eq. (28) by $K(1 + 0.25Ts)/s$ it turns out to be

$$L_r(s) = K(1 + 0.25Ts)\frac{e^{-sT}}{s} \geq K\frac{(1 + 0.25Ts)\hat{G}^{-1}(s)P(s)}{s} = C(s)P(s) = L(s). \tag{46}$$

The proof follows taking into account that $\hat{G}^{-1}(s)P(s)$ has all negative real part poles, from the Nyquist criterion, Eq. (46), and the third column of Table 2.
Analogous results are obtained if $K \in [0.686, 1.304]/T$.
From Theorems 3 and 4 several design techniques of industrial controllers derive.
In the following, some of the above mentioned techniques are reported.

(a) **Design of two PI controllers.**

If

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = \frac{b}{s + a}, \tag{47}$$

using Theorem 3, a first PI controller turns out to be

$$C(s) = K_p + \frac{K_i}{s}, \quad K_p = \frac{\pi}{4Tb}, K_i = \frac{\pi a}{4Tb} = K_p a, K_v{=}K_i P(0). \tag{48}$$

If, instead,

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = b, \tag{49}$$

using Theorem 4, a second PI controller is

$$C(s) = K_p + \frac{K_i}{s}, \quad K_p = \frac{1}{4b}, K_i = \frac{1}{Tb}, K_v{=}K_i P(0). \tag{50}$$

(b) **Design of two PID controllers.**

If

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = \frac{b}{s^2 + a_1 s + a_2}, \tag{51}$$

using Theorem 3, a first PID controller is

$$C(s) = K_p + \frac{K_i}{s} + K_d s, \quad K_p = \frac{\pi a_1}{4Tb}, K_i = \frac{\pi a_2}{4Tb}, K_d = \frac{\pi}{4Tb}, K_v{=}K_i P(0). \tag{52}$$

If, instead,

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = \frac{b}{s + a}, \tag{53}$$

using Theorem 4, a second PID controller is

$$C(s) = K_p + \frac{K_i}{s} + K_d s, \quad K_p = \frac{aT + 4}{4Tb}, K_i = \frac{a}{Tb}, K_d = \frac{1}{4b}, K_v{=}K_i P(0). \tag{54}$$

(c) **Design of two PIDR controllers.**

If

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = \frac{b(s/N + 1)}{s^2 + a_1 s + a_2}, \tag{55}$$

using Theorem 3, a first PIDR controller turns out to be

$$\begin{aligned} C(s) &= \frac{k_d s^2 + k_p s + k_i}{s(1 + s/N)}, \quad k_p = \frac{\pi a_1}{4Tb}, k_i = \frac{\pi a_2}{4Tb}, k_d = \frac{\pi}{4Tb}, K_v{=}\frac{\pi a_2}{4Tb}P(0), \\ &= K_p + \frac{K_i}{s} + K_d \frac{s}{1 + s/N}, \quad K_p = k_p - k_i/N, K_i = k_i, K_d = k_d - K_p/N \ . \end{aligned} \tag{56}$$

If, instead,

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = \frac{b(s/N + 1)}{s + a}, \tag{57}$$

using Theorem 4, a second PIDR controller is

$$\begin{aligned} C(s) &= \frac{k_d s^2 + k_p s + k_i}{s(1 + s/N)}, \quad k_p = \frac{aT + 4}{4Tb}, k_i = \frac{a}{Tb}, k_d = \frac{1}{4b}, K_v{=}k_i P(0), \\ &= K_p + \frac{K_i}{s} + K_d \frac{s}{1 + s/N}, \quad K_p = k_p - k_i/N, K_i = k_i, K_d = k_d - K_p/N. \end{aligned} \tag{58}$$

3.2. Fast Design Techniques for PI2, PI2D, PI2DR, PI2D2, and PI2D2R Controllers

Theorem 5. *Let $P(s)$ be a system of class SPG and $\hat{P}(s) = \hat{G}(s)e^{-sT}$ be its MS. Then, the control system in Fig. 1 with*

$$C(s) = K\frac{(1 + 7Ts)\hat{G}^{-1}(s)}{s^2} = \frac{0.10}{T^2}\frac{(1 + 7Ts)\hat{G}^{-1}(s)}{s^2} \tag{59}$$

is type 2 with $M_\varphi \geq 37.78^\circ, m_g \geq 7.43dB$, and acceleration constant $K_a = \frac{0.10}{T^2}\frac{P(0)}{\hat{G}(0)}$.

Proof. Multiplying both members of Eq. (28) by $K(1+7Ts)/s^2$ it is

$$L_r(s) = K(1+7Ts)\frac{e^{-sT}}{s^2} \geq K\frac{(1+7Ts)\hat{G}^{-1}(s)P(s)}{s^2} = C(s)P(s) = L(s). \tag{60}$$

The proof follows taking into account that $\hat{G}^{-1}(s)P(s)$ has all negative real part poles, from the Nyquist criterion, Eq. (60), and the third column of Table 3.

Analogous results are obtained if $K \in [0.07, 0.13]/T^2$.

Theorem 6. *Let $P(s)$ be a system of class SPG and $\hat{P}(s) = \hat{G}(s)e^{-sT}$ be its MS. Then, the control system in Fig. 1 with*

$$C(s) = K\frac{\left(1+4.4Ts+1.6T^2s^2\right)\hat{G}^{-1}(s)}{s^2} = \frac{0.2}{T^2}\frac{\left(1+4.4Ts+1.6T^2s^2\right)\hat{G}^{-1}(s)}{s^2} \tag{61}$$

is type 2 with $M_\varphi \geq 43.08^\circ, m_g \geq 6.22dB$, and $K_a = \frac{0.2}{T^2}\frac{P(0)}{\hat{G}(0)}$.

Proof. Multiplying both members of Eq. (28) by $K(1+4.4Ts+1.6T^2s)/s^2$ it is

$$L_r(s) = K(1+7Ts)\frac{e^{-sT}}{s^2} \geq K\frac{(1+7Ts)\hat{G}^{-1}(s)P(s)}{s^2} = C(s)P(s) = L(s). \tag{62}$$

The proof follows taking into account that $\hat{G}^{-1}(s)P(s)$ has all negative real part poles, from the Nyquist criterion, Eq. (62), and the third column of Table 4.
Analogous results are obtained if $K \in [0.10, 0.30]/T^2$.
From Theorems 5 and 6 other design techniques of industrial controllers derive.
In the following, some of the above mentioned techniques are reported.

(a) **Design of two PI2 controllers.**

If

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = \frac{b}{s+a}, \tag{63}$$

using Theorem 5, a first PI2 controller turns out to be

$$C(s) = \frac{K_p s^2 + K_{i1}s + K_{i2}}{s^2}, \quad K_p = \frac{0.7}{Tb}, K_{i1} = \frac{0.7Ta+0.1}{T^2b}, K_{i2} = \frac{0.1a}{T^2b}, K_a = K_{i2}P(0). \tag{64}$$

If, instead,

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = b, \tag{65}$$

using Theorem 6, a second PI2 controller is

$$C(s) = \frac{K_p s^2 + K_{i1}s + K_{i2}}{s^2}, \quad K_p = \frac{0.32}{b}, K_{i1} = \frac{0.88}{Tb}, K_{i2} = \frac{0.2}{T^2b}, K_a = K_{i2}P(0). \tag{66}$$

(b) **Design of two PI2D controllers.**

If

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = \frac{b}{s^2 + a_1 s + a_2}, \tag{67}$$

using Theorem 5, a first PI2D controller is

$$C(s) = \frac{K_d s^3 + K_p s^2 + K_{i1} s + K_{i2}}{s^2}$$
$$K_d = \frac{0.7}{bT}, K_p = \frac{0.7Ta_1 + 0.1}{bT^2}, K_{i1} = \frac{0.7Ta_2 + 0.1a_1}{bT^2}, K_{i2} = \frac{0.1a_2}{bT^2}, K_a = K_{i2}P(0). \tag{68}$$

If, instead,

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = \frac{b}{s + a}, \tag{69}$$

using Theorem 6, a second PI2D controller is

$$C(s) = \frac{K_d s^3 + K_p s^2 + K_{i1} s + K_{i2}}{s^2}$$
$$K_d = \frac{0.32}{b}, K_p = \frac{0.32Ta + 0.88}{bT}, K_{i1} = \frac{0.88Ta + 0.2}{bT^2}, K_{i2} = \frac{0.2a}{bT^2}, K_a = K_{i2}P(0). \tag{70}$$

(c) **Design of two PI2DR controllers.**

If

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = \frac{b(s/N + 1)}{s^2 + a_1 s + a_2}, \tag{71}$$

using Theorem 5, a first PI2DR controller turns out to be

$$C(s) = \frac{k_d s^3 + k_p s^2 + k_{i1} s + k_{i2}}{s^2(s/N + 1)}$$
$$k_d = \frac{0.7}{bT}, k_p = \frac{0.7Ta_1 + 0.1}{bT^2}, k_{i1} = \frac{0.7Ta_2 + 0.1a_1}{bT^2}, k_{i2} = \frac{0.1a_2}{bT^2}, K_a = k_{i2}P(0). \tag{72}$$

If, instead,

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = \frac{b(s/N + 1)}{s + a}, \tag{73}$$

using Theorem 6, a second PI2DR controller is

$$C(s) = \frac{k_d s^3 + k_p s^2 + k_{i1} s + k_{i2}}{s^2(s/N + 1)}$$
$$k_d = \frac{0.32}{b}, k_p = \frac{0.32Ta + 0.88}{bT}, k_{i1} = \frac{0.88Ta + 0.2}{bT^2}, k_{i2} = \frac{0.2a}{bT^2}, K_a = k_{i2}P(0). \tag{74}$$

(d) **Design of two PI2D2 controllers.**

If

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = \frac{b}{s^3 + a_1 s^2 + a_2 s + a_3}, \tag{75}$$

using Theorem 5, a PI2D2 controller is

$$C(s) = \frac{(7s+1)(s^3 + a_1 s^2 + a_2 s + a_3)/(10bT^2)}{s^2} = \frac{K_{d2}s^4 + K_{d1}s^3 + K_p s^2 + K_{i1}s + K_{i2}}{s^2}$$
$$= K_p + \frac{K_{i1}}{s} + \frac{K_{i2}}{s^2} + K_{d1}s + K_{d2}s^2,\ K_a = K_{i2}P(0). \tag{76}$$

If, instead,

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = \frac{b}{s^2 + as_1 + a_2}, \tag{77}$$

using Theorem 6, a second PI2D$_2$ controller is

$$C(s) = \frac{(1 + 4.4Ts + 1.6T^2 s^2)(s^2 + a_1 s + a_2)/(5bT^2)}{s^2} = \frac{K_{d2}s^4 + K_{d1}s^3 + K_p s^2 + K_{i1}s + K_{i2}}{s^2}$$
$$= K_p + \frac{K_{i1}}{s} + \frac{K_{i2}}{s^2} + K_{d1}s + K_{d2}s^2,\ K_a = K_{i2}P(0). \tag{78}$$

(e) **Design of two PI2D2R controllers.**

If

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = \frac{b(s/N+1)^2}{s^3 + a_1 s^2 + a_2 s + a_3}, \tag{79}$$

using Theorem 5, a PI2D2R controller is

$$C(s) = \frac{(7s+1)(s^3 + a_1 s^2 + a_2 s + a_3)/(10bT^2)}{s^2(s/N+1)^2} = \frac{k_{d2}s^4 + k_{d1}s^3 + k_p s^2 + k_{i1}s + k_{i2}}{s^2(s/N+1)^2}$$
$$= K_p + \frac{K_{i1}}{s} + \frac{K_{i2}}{s^2} + K_{d1}\frac{s}{s/N+1} + K_{d2}\frac{s^2}{(s/N+1)^2},\ K_a = k_{i2}P(0). \tag{80}$$

If, instead,

$$\hat{P}(s) = \hat{G}(s)e^{-sT}, \quad \hat{G}(s) = \frac{b(s/N+1)^2}{s^2 + as_1 + a_2}, \tag{81}$$

using Theorem 6, a second PI2D2 controller is

$$C(s) = \frac{(1 + 4.4Ts + 1.6T^2 s^2)(s^2 + a_1 s + a_2)/(5bT^2)}{s^2(s/N+1)^2} = \frac{k_{d2}s^4 + k_{d1}s^3 + k_p s^2 + k_{i1}s + k_{i2}}{s^2(s/N+1)^2}$$
$$= K_p + \frac{K_{i1}}{s} + \frac{K_{i2}}{s^2} + K_{d1}s + K_{d2}s^2,\ K_a = k_{i2}P(0). \tag{82}$$

Remark 14. Realization of PID, PI2D (PI2D2) controllers requires the measurements of y and $\dot{y}$ (y, $\dot{y}$ and $\ddot{y}$). If the measurement of y is affected by a large bandwidth error, especially

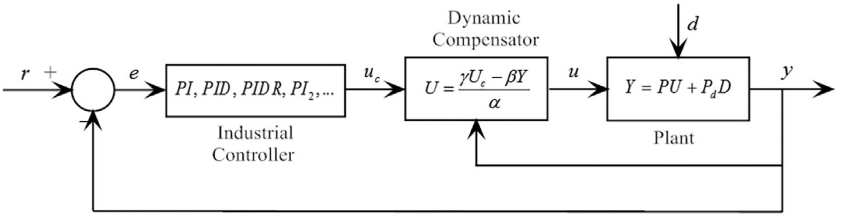

Fig. 16. Control of a plant with a dynamic compensator and an industrial controller.

at medium frequency, instead of using a real derivative action to obtain $\dot{y}$ ($\ddot{y}$), it can directly be measured $\dot{y}$ or, alternately, measured $\ddot{y}$ and estimated $\dot{y}$ using an optimal estimator (see e.g., [55]). In this regard, note that nowadays, in many cases, the direct measurement of speed, and even acceleration, can easily be achieved with accurate economic sensors.

It is also worth noting that, if a controller is designed using the ideal derivative action, as in ZN-like methods and its numerous variants, and the ideal derivative action is replaced by a real derivative one, the control system can be unstable.

Remark 15. To design a controller, for a process $P(s)$ of class *SMG*, with a real derivative action, useful to reduce the effect of the measurement error and/or moderate the control action, it can be determined a *MS* $\hat{P}_N(s)$ of $P(s)$ with a numerator factor equal to $(s/N+1)^i$, $i=1,2$, using one of the following methods:

Method 1. If $\hat{P}(s) \geq P(s)$ it is easy to verify that $\hat{P}_N(s) = \hat{P}(s)e^{-is/N}(s/N+1)^i \geq P(s)$, $i=1,2$.

Method 2. If $\hat{P}(s) \geq P(s)/(s/N+1)^i$ it is easy to verify that $\hat{P}_N(s) = \hat{P}(s)(s/N+1)^i \geq P(s), i=1,2$.

A *MS* $\hat{P}(s)$ of $P(s)$, if Method 1 is used, or of $P(s)/(s/N+1)^i$, if Method 2 is used, of types

$$\frac{b}{s+a}e^{-sT}, \frac{b}{s^2+a_1s+a_2}e^{-sT}, \frac{b}{s^3+a_1s^2+a_2s+a_3}e^{-sT}, \tag{83}$$

such to maximize one of the following parameters $\frac{a}{bT} \equiv K_{\mathrm{v}}, \frac{a_2}{bT} \equiv K_{\mathrm{v}}$, $\frac{a}{bT^2} \equiv K_{\mathrm{a}}, \frac{a_2}{bT^2} \equiv K_a, \frac{a_3}{bT^2} \equiv K_a$ depending on the controller to design, can easily be determined from the analytical or experimental frequency response of $P(s)$ (see Algorithm 2 in the Appendix, and Remark 21).

Remark 16. If the plant is not asymptotically stable or the goal is to more improve the control system performance, the control can be made with an industrial controller and a dynamic compensator with two degrees of freedom as shown in the scheme in Fig. 16.

Concerning the above matter, note that if $P(s) = b(s)/a(s)$, for $d=0$ it is

$$P_{DC}(s) = Y(s)/U_c(s) = b(s)\gamma(s)/(a(s)\alpha(s) + b(s)\beta(s)). \tag{84}$$

Hence, if $b(s)$ and $a(s)$ are coprime polynomials with a compensator of order $\nu = \deg(a) - 1$ it is possible to assign at will all the poles and ν zeros of $P_{DC}(s)$. Clearly, using a compensator of order smaller than $\deg(a) - 1$, in many cases, it is possible to transform the plant in a system of *SPG* class, which can effectively be controlled with an industrial controller designed with one of the proposed methods.

In both cases, the parameters of the compensator can be chosen such to optimize the parameters of the *MS*, e.g., minimizing $(K_v - \hat{K}_v)^2$ or $(K_a - \hat{K}_a)^2$, where $\hat{K}_v$ $(\hat{K}_a)$ is the desired value of K_v (K_a) of the open loop control system.

In both cases, the parameters of the compensator can be chosen such to optimize the parameters of the *MS*, e.g., maximizing K_v or K_a of the open loop control system.

3.3. Guidelines for a fast design for industrial controllers and comparisons

3.3.1. Design of industrial controllers with the proposed method

The proposed method, always applicable to the broad class of systems Eq. (1), can be articulated as follows.

Step 1. Starting from the frequency response of the system to be controlled $P(s)$, which is determined analytically, or experimentally applying to $P(s)$ a real impulse (always applicable since it does not have an oscillatory behavior), $P(s)$ is approximated with one of the following *MSs*:

$$\hat{P}_0 = be^{-sT}, \hat{P}_1 = \frac{b}{s+a}e^{-sT}, \hat{P}_{20} = \frac{b}{s^2+a_1s+a_2}e^{-sT}, \hat{P}_{21} = \frac{b_1s+b_2}{s^2+a_1s+a_2}e^{-sT},$$
$$\hat{P}_{30} = \frac{b}{s^3+a_1s^2+a_2s+a_3}e^{-sT}, \hat{P}_{31} = \frac{b_1s+b_2}{s^3+a_1s^2+a_2s+a_3}e^{-sT}, \text{etc.},$$

or with the systems having an additional numerator factor of the type $(1+s/N)^i, i = 1, 2$. This can be made using Lemmas 1-9, or Algorithm 2 in the Appendix which maximizes K_v, or a similar one such to maximize K_a.

Step 2. With the parameters of one of the determined *MSs*, using the provided simple formulas, the corresponding controllers are designed, which can be not only of the type PI, PID, but also of PIDR, PI2, PI2D, PI2DR, PI2D2, and PI2D2R types, depending on the determined *MS*.

Remark 17. It is worth noting that

- with the proposed controllers the gain and phase margins are always greater than or equal to the ones deduced from Tables 1-4 in Subsection 2.5, which are a function of the chosen value of KT or KT^2.

If, for very complex systems, the stability margins should be too high then they can be reduced by choosing greater values of KT or KT^2, obtaining smaller tracking errors, at least at steady-state.

- Control system performance, also concerning the maximum tracking error of a generic reference, but with first or second bounded derivative, improve by increasing the order of the *MS*.

- If the generalized gains H_i, H_{id} are computed with the scheme in Fig. 2 then it is possible to estimate the tracking errors of references with bounded first or second derivative and keep them within acceptable values by varying the reference velocities (see Remark 7).

3.3.2. Design of industrial controllers with the most used methods available in the literature

As highlighted in the Introduction, in the literature there exist numerous fast design control methods for industrial controllers, mostly of PI and PID types.

Most of them mainly work according to the following two procedures.

Procedure 1 (Open Loop)

Step 1. The step response $W_{-1}(t)$ of the process to be controlled $P(s)$ is analytically or experimentally determined, and $P(s)$ is approximated, with various methods which use only $W_{-1}(t)$, with the *t.f.* $P_a(s) = \frac{G}{1+s\tau}e^{-sT}$ of a FOPDT system.

Step 2. The parameters of the controllers PI and/or PID are computed, from parameters G, τ, T, with empirical formulas or optimizing specific quality indices.

Procedure 2 (Closed Loop)

Step 1. It is determined the gain K_{pc} of a proportional controller which causes a persistent oscillation of period T_o in the closed loop.

Step 2. The parameters of the controllers PI and/or PID are computed, from parameters K_{pc}, T_o, with empirical formulas or optimizing specific quality indices.

Remark 18. As regards, it is worth noting that, at the best of the author's knowledge, in the literature it is not available any theoretical result which guarantees that the closed loop control system, with the controllers designed using one of the above methods, have always acceptable performance, e.g., the basic one of the asymptotic stability.

Moreover, in some cases, the design formulas fail (e.g., if $\tau = 0$ the value of K_p, with the ZN open loop tuning rules and PID tuning rule using ITAE criterion - ITAE: Integral of the Time-weighted Absolute Error -, is equal to zero, etc.). In addition, Procedure 2 is not always applicable since some systems are asymptotically stable for all the values of $K_p > 0$ and, hence, do not have a persistent oscillation. Furthermore, there exist also systems that, by increasing K_p, have a divergent aperiodic mode and not a persistent oscillation.

4. Examples

This section presents three groups of examples, which illustrate the proposed control design techniques, and show their superiority over the most know methods available in the literature, in terms of applicability, performance, and numerousness of the industrial controllers which can be designed. Moreover, some experimental validations are given.

4.1. First group

In this subsection, some examples are provided, which show that the PI and PID controllers designed with the most known methods available in the literature, in some cases, make the control system unstable.

Example 4. Consider the process

$$P(s) = \frac{24(s+4)}{(s+2)(s+6)(s^2+4s+16)}e^{-sT_p} \,. \tag{85}$$

The approximation obtained with the ZN' reaction curve method in the hypothesis that $T_p = 0$ is shown in Fig. 17.

The respective PI and PID controllers, designed with the ZN formulas, and the corresponding stability margins are

$C_{PI}(s) = 5.7401 + 8.0589/s, m_g{=}-1.181\text{dB}, M_\varphi = -5.625\,\text{deg}$

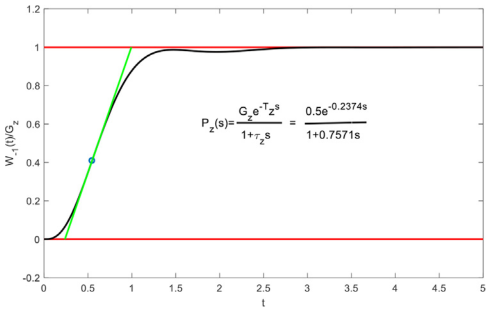

Fig. 17. Approximate model of the process Eq. (85), case $T_p = 0$, obtained with the ZN' reaction curve method.

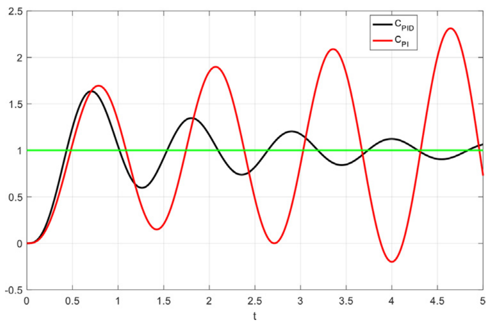

Fig. 18. Step responses of the control system of the plant Eq. (85), case $T_p = 0$, using the PI and PID controllers (86) designed with the ZN' reaction curve method.

$$C_{PID}(s) = 7.6535 + 16.1178/s + 0.9085s,\ m_g = 13.071\text{dB},\ M_\varphi = 13.040\,\text{deg}. \tag{86}$$

As it can be noted, using the PI controller in Eq. (86), the control system is unstable, while with the PID controller the phase margin of the control system is very small (see also Fig. 18).

It is easy to verify that if $T_p = 0.5$, using the ZN method, the designed PI and PID controllers, and the corresponding stability margins are

$$C_{PI}(s) = 1.8481 + 0.8354/s,\ m_g = 2.977\text{dB}, M_\varphi = 96.182\,\text{deg}$$
$$C_{PID}(s) = 2.4641 + 1.6708/s + 0.9085s,\ m_g = -1.088\text{dB}, M_\varphi = -36.975\,\text{deg}. \tag{87}$$

Hence, using the PID controller in Eq. (87) the corresponding control system is unstable.

Example 5. There exist other plants which cannot be controlled using a controller designed with the ZN method. E.g., the control systems of the following processes:

$$P(s) = \frac{1}{s^2 + 2\zeta s + 1} e^{-s},\ \zeta \le 0.5, P(s) = \frac{4}{s^3 + 3s^2 + 6s + 4} e^{-sT_p},\ T_p \in [0.5, 2.5] \tag{88}$$

controlled with a PID designed with the ZN tuning rules are unstable.

In addition, it is easy to verify that the control systems of the process

$$P(s) = \frac{4}{s^3 + 4s^2 + 7s + 4} e^{-6s} \tag{89}$$

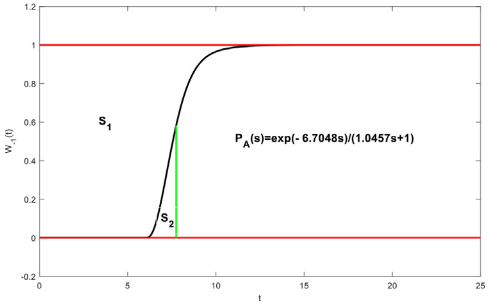

Fig. 19. Approximate model of the process Eq. (89) with the “areas method”.

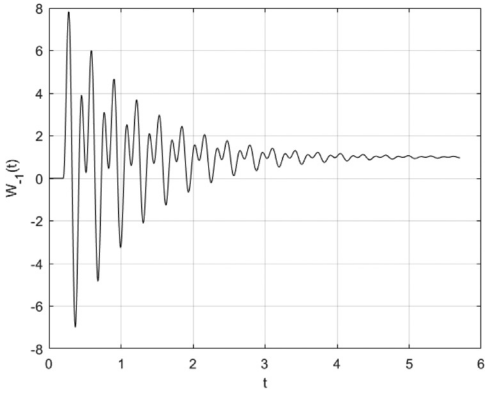

Fig. 20. Step response of the system Eq. (90).

approximated with the “areas method” (see Fig. 19) and controlled with the respective controllers PI_{ITAE} and PID_{ITAE} [23] are unstable.

Furthermore, the control systems of the plant

$$P(s) = \frac{6400(s^2 + 10s + 100)}{(s^2 + 2s + 20^2)(s^2 + 2s + 40^2)} e^{-0.2s}, \tag{90}$$

whose step response is shown in Fig. 20, controlled with the PID controllers designed with the ZN open loop tuning rules, Cohen-Coon rules, PID tuning rule using ITAE criterion, and Procedure 2 are all unstable.

4.2. Second group

Now, some examples are given, which show the better performance of the PI and PID controllers designed with the proposed method with respect to the ones of the PI and PID controllers designed using other well-known methods.

Example 6. Consider the simple process described by

$$P(s) = \frac{1}{s^2 + 2s + 1}. \tag{91}$$

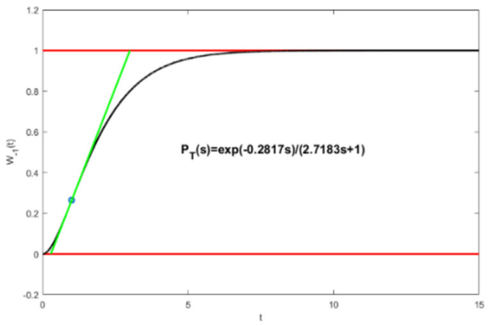

Fig. 21. Approximate model of the process Eq. (91) with the ZN' reaction curve method.

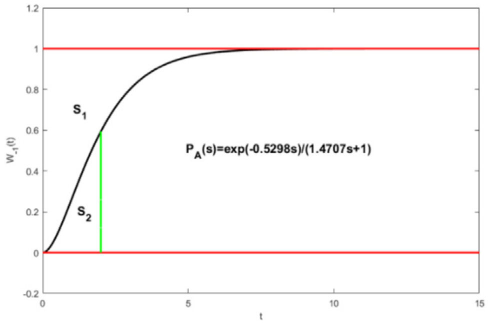

Fig. 22. Approximate model of the process Eq. (91) with the "areas method".

The approximate models obtained with the ZN' reaction curve method and the "areas one" are shown in Figs. 21 and 22, respectively.

A *MS* $\hat{P}_N(s)$ of the process Eq. (91), taking into account Remark 9, is

$$\hat{P}_N(s) = \frac{s/N + 1}{s^2 + 2s + 1} e^{-s/N}, N > 0. \tag{92}$$

The PID controllers related to $P_Z(s)$ and $P_A(s)$, designed with the ZN formulas, respectively, and the PIDR controllers related to $\hat{P}_N(s)$ with $N = 20,\ 50$, designed using Eq. (56), the corresponding stability margins and the constants H_1 are

$$\begin{aligned}
&C_{P_Z}(s) = 11.5787 + 20.5502/s + 1.6310s,\, m_g = \text{inf},\, M_\varphi = 29.252\,\text{deg},\, H_1 = 1.036\\
&C_{P_A}(s) = 1.4930 + 0.9853/s + 0.4053s,\, m_g = \text{inf},\, M_\varphi = 68.295\,\text{deg},\, H_1 = 1.409\\
&C_{\hat{P}20}(s) = 30.6305 + 15.7080/s + 14.1764^{S}/_{(s/20+1)},\\
&m_g = 86.1242\text{dB},\, M_\varphi = 56.712\,\text{deg},\, H_1 = 0.09266\\
&C_{\hat{P}50}(s) = 77.7544 + 39.2699/s + 37.7148^{S}/_{(s/50+1)},\\
&m_g = 71.4407\text{dB},\, M_\varphi = 56.711\,\text{deg},\, H_1 = 0.03706.
\end{aligned} \tag{93}$$

In Fig. 23, the step responses of the control systems using the controllers Eq. (93) are shown.

Example 7. Consider again the system Eq. (85) with $T_p = 0$. A first-order *MS* which maximizes K_v, the corresponding PI and PID controllers, stability margins and generalized gain

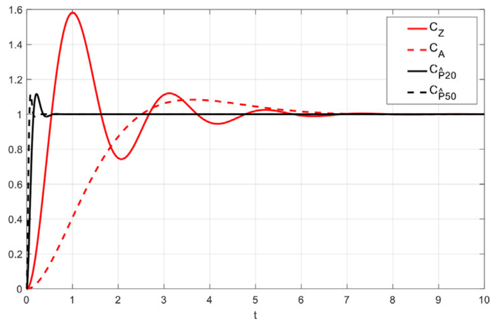

Fig. 23. Step responses of the control systems with the controllers Eq. (93).

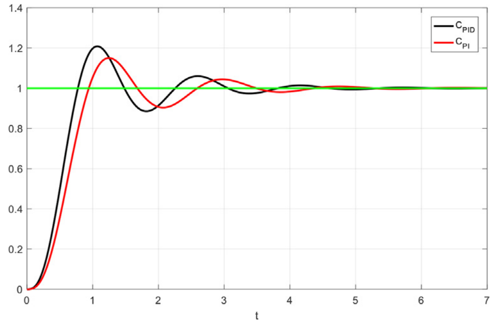

Fig. 24. Step responses of the control system of the plant Eq. (85), case $T_p = 0$, with the PI and PID controllers (95) designed with the proposed method.

are

$$\hat{P}_1(s) = \frac{1.363}{s + 2.359} e^{-0.3715s} \tag{94}$$

$$C_{PI}(s) = 1.5513 + 3.6592/s,\ m_g = 7.756\text{dB}, M_\varphi = 59.601\ \text{deg},\ H_1 = 0.749$$
$$C_{PID}(s) = 2.4075 + 4.6596/s + 0.1834s,\ m_g = 11.040\text{dB}, M_\varphi = 56.646\ \text{deg},\ H_1 = 0.694. \tag{95}$$

In Fig. 24, the step responses of the control systems obtained with the PI and PID controllers (95) designed with the proposed method are shown.

A PI2D2R controller, with $N = 30\omega_c \cong 90$, of the system Eq. (85) with $T_p = 0$, the corresponding stability margins and generalized gain are

$$C_{PI2D2R}(s) = \frac{1.58s^4 + 21.2s^3 + 112.4s^2 + 361.7s + 441.5}{(s/90 + 1/s)^2 s^2}$$
$$m_g = 12.44\text{dB}, M_\varphi = 39.76\ \text{deg},\ H_2 = 0.00470. \tag{96}$$

In Fig. 25, the control system response to a ramp signal with the PI2D2R controller Eq. (96) is shown. From the above mentioned figure, the very small value of H_2, and good stability margins can be deduced that the control system performance with the designed PI2D2R controller are optimum.

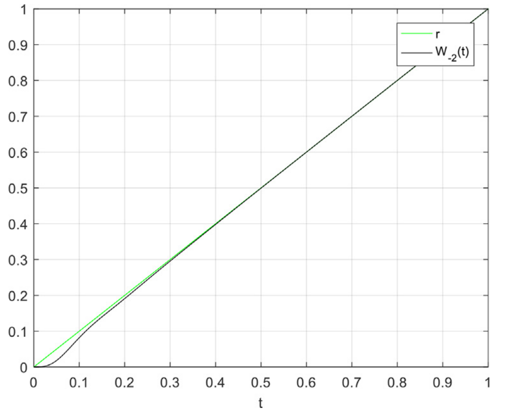

Fig. 25. Response of the control system of the plant Eq. (85), case $T_p = 0$, to a ramp with the PI2D2R controller Eq. (96).

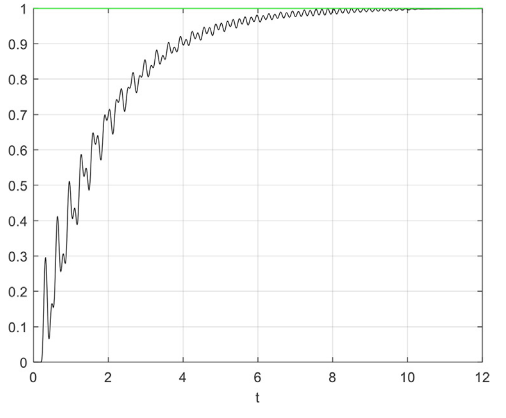

Fig. 26. Step response of the control system of the plant Eq. (90) with the PID controller designed with the proposed method.

Remark 19. Note that the control system of the plant Eq. (85) with $T_p = 0.5$ with the PID controllers designed both using a first-order *MS* and a second-order one has good performance.

Example 8. Consider again the system Eq. (90). It is easy to verify that the step response of this controlled system with the PID $C(s) = 0.0001662 + 0.56443/s + 0.00042438s$, designed using the proposed method, is shown in Fig. 26. Moreover, the corresponding stability margins and generalized gain are m_g= 8.17dB,$M_\varphi = 86.57$ deg, $H_1 = 1.77$.

4.3. Third group

Some significant examples are provided to show that, using the proposed method, numerous controllers can easily be designed, also when an analytical model of the plant is not available, with stability margins of the control system always larger than prefixed values and acceptable tracking errors of a broad class of references.

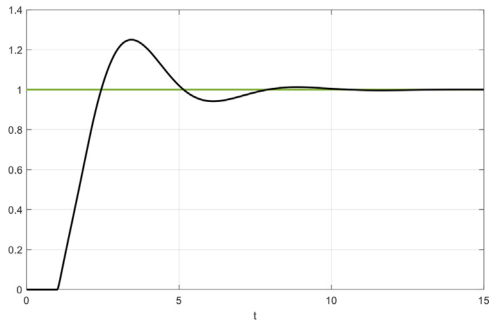

Fig. 27. Step response of the control system with the PIDR controller Eq. (99).

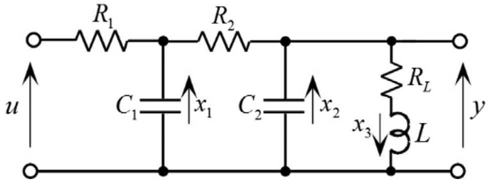

Fig. 28. Electric analog model of a process.

Example 9. Consider the process with internal and external delays

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} x(t-0.5) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t-1), \quad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \tag{97}$$

A second-order *MS*, with a zero in $N = 10\omega_c \cong 30$, which maximizes K_v, obtained with the Matlab command invfreqs, the corresponding PIDR controller, stability margins and generalized gain are

$$\hat{P}(s) = \frac{0.94563(s/30+1)}{s^2 + 1.4858s + 4.536} e^{-1.0411s}. \tag{98}$$

$$\begin{aligned} &C_{PIDR}(s) = 1.0648 + 3.6188/s + 0.7623s/(1 + 1/30s) \\ &m_g = 6.243\text{dB},\ M_\varphi = 47.9274\text{deg},\ H_1 = 2.243. \end{aligned} \tag{99}$$

In Fig. 27, the step response of the closed-loop control system using the PIDR controller Eq. (99) is shown.

Example 10. Consider the model of a process whose analog electric circuit is shown in Fig. 28.

In the hypothesis that $R_1 = R_2 = 10\Omega, R_L = 15\Omega, C_1 = C_2 = 10m\ F, L = 2H$, it is

$$\dot{x} = \begin{bmatrix} -20 & 10 & 0 \\ 10 & -10 & -100 \\ 0 & 0.50 & -7.50 \end{bmatrix} x + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u, y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x \Rightarrow$$

$$P(s) = \frac{100s + 750}{s^3 + 37.5s^2 + 375s + 1750}. \tag{100}$$

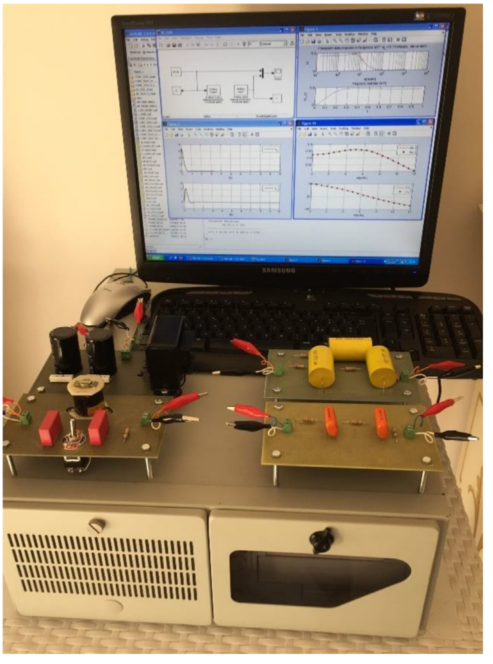

Fig. 29. Experimental prototype of the control system.

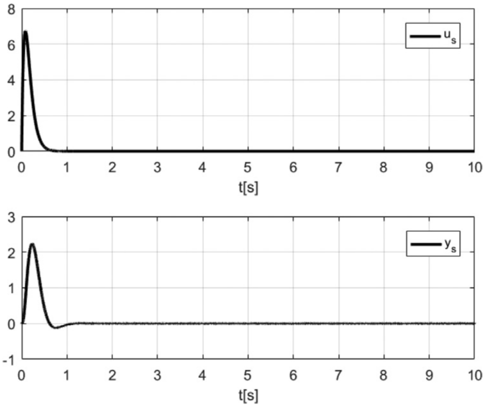

Fig. 30. Experimental response $y_s(t)$ to the real impulse $u_s(t)$.

Suppose that the model Eq. (100) is unknown, but only the degrees m and n of the numerator and denominator of $P(s)$ are known.

Hence, to design robust and effective industrial controllers, the first step is the experimental identification of $P(s)$. To this aim, by using an industrial PC equipped with a 12 bits input/output data acquisition board by National Instruments and Matlab Real-Time Windows Target with a 1 kHz sampling frequency (see Fig. 29), the real impulse $u_s = 1.5 \times 13^2 te^{-13t}$ is applied to the circuit in Fig. 28, obtaining the response y_s of Fig. 30.

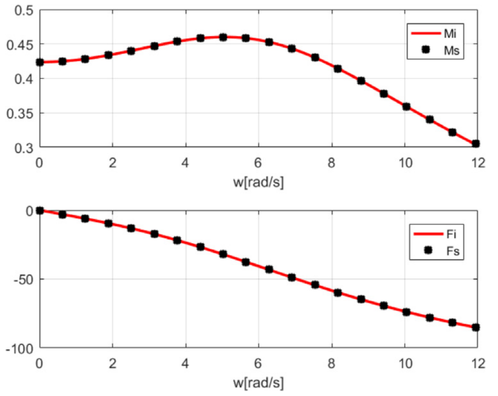

Fig. 31. Experimental frequency response M_s, F_s. .

By compensating the delay due to the digital hardware and using the FFT, the experimental frequency response M_s, F_s of the circuit is obtained and shown in Fig. 31.
Finally, using the Matlab code
Hs=Ms.*exp(j*Fs*pi/180);[nums dens]=invfreqs(Hs,ws,1,3,[]);Ps=tf(nums, dens), the estimated model turns out to be

$$P_s(s) = \frac{96.15s + 726}{s^3 + 36.96s^2 + 369s + 1713}, \tag{101}$$

which is reliable, taking into account the measurement errors of the parameters $R_1, R_2, R_L, C_1, C_2, L$, and of y_s.

To realize digital PIDR and PI2DR controllers or to make more robust the corresponding control systems, to the aim of the controllers design, it can be considered the model

$$P(s) = \frac{96.15s + 726}{s^3 + 36.96s^2 + 369s + 1713} e^{-sT_p}, T_p = 1ms. \tag{102}$$

A second-order MS, with a zero in $N = 10\omega_c \cong 120$, of the system Eq. (102), which maximizes K_v, obtained with the Matlab command *invfreqs*, the corresponding PIDR controller, stability margins and generalized gain are

$$\hat{P}_2(s) = \frac{80.634(s/120 + 1)}{s^2 + 17.694s + 148.038} e^{-0.007617s}, K_v = 80.226 \tag{103}$$

$$C_{PIDR}(s) = 21.050 + 189.310/s + 1.103s/(1 + 1/120s) = \frac{\text{numc}(s)}{\text{denc}(s)}$$
$$m_g = 19.21\text{dB}, M_\varphi = 53.35\deg, H_1 = 0.0168 > 1/K_v = 0.0125. \tag{104}$$

Instead, the PI2DR controller of $\hat{P}(s)$, the corresponding stability margins and generalized gain are

$$C_{PI2DR}(s) = \frac{1.139s^3 + 41.53s^2 + 546.7s + 3162}{(s/120 + 1)s^2} = \frac{\text{numc}(s)}{\text{denc}(s)}$$
$$m_g = 18.84\text{dB}, M_\varphi = 43.99\deg, H_2 = 0.000783 > 1/K_a = 0.000746\,. \tag{105}$$

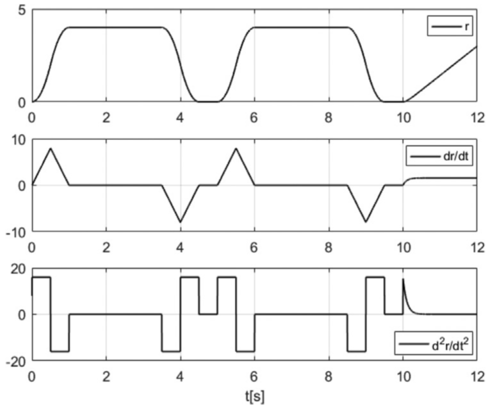

Fig. 32. Time histories of $r(ct), \dot{r}(ct), \ddot{r}(ct)$ with $c = 1$.

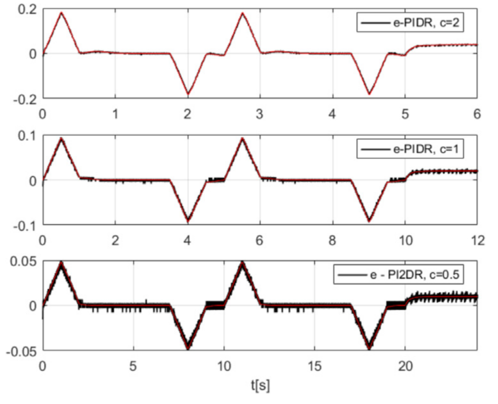

Fig. 33. Time histories of the experimental errors $e(ct)$ for $c = 0.5, 1, 2$ with the designed PIDR controller.

The designed analog controllers can be digitalized with the following Matlab code:
[Ac Bc Cc Dc]=tf2ss(numc,denc);[Ac Bc]=c2d(Ac,Bc,1e-3); .

By controlling the system in Fig. 28 with the designed digital controllers, using the industrial PC in Fig. 29, the tracking errors of the references $r(ct), c = 0.5, 1, 2$, (see Fig. 32) are reported in Figs. 33 and 34.

Since $\max(|\dot{r}(ct)|) = 8c$, $\max(|\ddot{r}(t)|) = 16c^2$, theoretically, it is

$$|e(ct)| \leq H_1 \max(|e(ct)| = 0.1344c\,, \quad with\ \ PIDR\ \ controller \tag{106}$$

$$|e(ct)| \leq H_2 \max(|e(ct)| = 0.0125c^2\,, \quad with\ \ PI2DR\ \ controller. \tag{107}$$

As it can easily be verified, the actual tracking errors, unless the quantization errors and noises, satisfy relations Eqs. (106),(107).

Finally, it is worth noting that with the proposed PIDR controller the obtained tracking error is almost null in correspondence of the time intervals in which the reference signal is

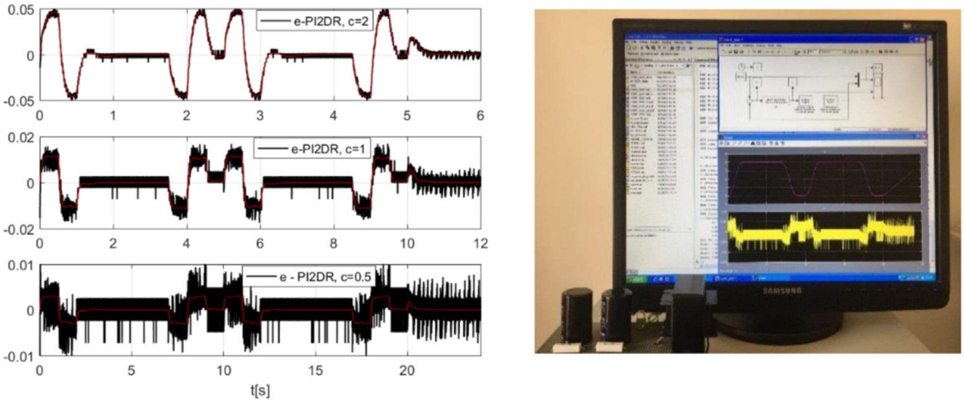

Fig. 34. Time histories of the experimental errors $e(ct)$ for $c = 0.5, 1, 2$ with the designed PI2DR controller.

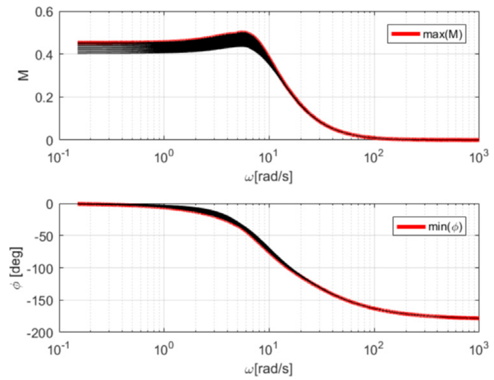

Fig. 35. Frequency responses $M(\omega, R_L, L)$, $\phi(\omega, R_L, L)$ (black), and behaviors of $\overset{\frown}{M}(\omega)$ and $\overset{\smile}{\phi}(\omega)$ (red). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

almost constant, while, using the designed PI2DR controller, the tracking error is almost null in correspondence of the time intervals in which the reference signal is almost linear.

Remark 20. Suppose to replace the components R_L and L with other ones affected by errors of ±10% and ±20% with respect to their nominal values, respectively. In Fig. 35, the behaviors of $\overset{\frown}{M}(\omega) \geq M(\omega, R_L, L)$ and $\overset{\smile}{\phi}(\omega) \leq \phi(\omega, R_L, L)$ are reported in red, which can be used to redesign the above controllers. Since $\max_{R_L,L}(\delta M(\omega, R_L, L)_{|dB}) = 1.23dB$ and $\max_{R_L,L}(\delta\phi(\omega, R_L, L)) = 7.76\,\deg$ it is that the stability margins of the control system with the controllers designed with the nominal values of R_L and L are reduced a little and, hence, the control system performance using these controllers continue to be good.

5. Conclusions

In conclusion, the main peculiarities and advantages of the obtained results can be summarized as follows.

(1) A new fast, general and systematic design control method via *MS* is provided. It allows one to design industrial controllers with a proportional, integral or double integral, derivative or double derivative (ideal or real) actions of PI, PID, PIDR, PI2, PI2D, PI2DR, PI2D2, PI2D2R types, which allow gradually to improve the performance of the control system.
(2) The proposed method can easily be applied to control systems with internal and/or external delays and with parametric and/or structural uncertainties and disturbances, also when an analytical model of the plant is not available, but data acquired from simple experimental tests are available.
(3) Some methods to easily determine, theoretically or experimentally, good *MSs* (of order greater than one, too) are provided; moreover, some methods to estimate the maximum tracking error of a generic reference with bounded first or second derivative, also in the presence of a generic disturbance with bounded first or second derivative, are given.
(4) The proposed control methodology can be used also to easily design other controllers able to guarantee more stringent performance requirements.
(5) The simplicity of design is within the reach of any engineer or technician in the information and industrial areas.

The ongoing research aims at extending the proposed results to MIMO uncertain plants with internal and/or external delays, parametric and/or structural uncertainties, and subject to disturbances.

Declaration of Competing Interest

The author declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

In this appendix, an algorithm to compute a *MS* of the type $\hat{P}(s) = b(s/N+1)^i e^{-isT}/(s^2+a_1 s+a_2), i=1,$ which maximizes K_v, using Method 1, is provided.

Algorithm 2

Step 1. Choose the cutoff angular frequency N of the real derivative action on the basis of the cutoff angular frequency ω_c of the process $P(s)$, the bandwidth, and the amplitude of the derivative of the measurement error $e(t) = r(t) - y(t)$.

Step 2. Compute the static gain G, the cutoff angular frequency ω_c, the resonance angular frequency ω_r, the resonance peak M_r of $P(s)|_{s=j\omega} = M(\omega)e^{jF(\omega)}$, and the maximum delay $T_M = \sup_{\omega>0}(-F(\omega)/\omega)$.

Step 3. Compute the frequency response $H(j\omega_s) = P(j\omega_s)$ in the points

$$\omega_s = \begin{cases} [\omega_c/1000, \omega_c/2, \omega_c, 2\omega_c], \text{ if } M_r < 1.1G \\ [\omega_r/1000, \ \omega_r/2, \omega_r, 2\omega_r, 4\omega_r], \text{ if } M_r \geq 1.1G \end{cases}.$$

Step 4. For $\tau \in [0, T_M]$ compute, using the Matlab command *invfreqs* or *fmincon*, the optimal interpolation $W_\tau(s) = \beta_\tau/(s^2 + a_{1\tau}s + a_{2\tau})$ of $H_\tau = H(j\omega_s)e^{j\omega_s\tau}$ with the conditions $a_{1\tau} > 0, \ a_{2\tau} > 0, \ \beta_\tau > 0.$

Step 5. Maximize $K_v \equiv a_{2\tau}/(b_\tau T_\tau)$ with respect to τ, where $b_\tau = \sup_{\omega>0}(M(\omega)/M_\tau(\omega))\beta_\tau$; $T_\tau = \sup_\omega((F_\tau(\omega) - F(\omega))/\omega)$; $M_\tau(\omega)e^{jF_\tau(\omega)} = W_\tau(s)|_{s=j\omega}$.

Step 6. The *MS* turns out to be $\hat{P}(s) = b(s/N+1)^i e^{-isT}/(s^2 + a_1 s + a_2)$, $T = T_o + i/N$, where b, a_1, a_2, T_o are the values of $b_\tau, a_{1\tau}, a_{2\tau}, T_\tau$ in correspondence of the optimal value τ_o of τ computed at step 5.

Remark 21. Similar algorithms can be established to determine other *MSs* (e.g., with the goal to maximize K_a).

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