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Odderon in baryon–baryon scattering from the AdS/CFT correspondence

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ABSTRACT: Based on the AdS/CFT correspondence, we present a holographic description of various C–odd exchanges in high energy baryon–(anti)baryon scattering, and calculate their respective contributions to the difference in the total cross sections. We show that, due to the warp factor of AdS_5, the single Odderon exchange gives a larger total cross section in baryon–baryon collisions than in baryon–antibaryon collisions at asymptotically high energies.

Contents

1. Introduction

The Odderon is a Regge exchange carrying odd charge parity, $C = -1$ [1, 2], as compared to its more well–known $C = +1$ partner, the Pomeron. It is a hypothetical excitation lying on the C–odd glueball Regge trajectory (meaning that the relevant amplitudes go like $s^{\alpha(t)}$ in the Regge limit $s \gg |t|$) with a putative intercept $\alpha(0) \sim 1$, and is usually considered as a separate entity from mesonic C–odd states like the vector meson ('Reggeon') trajectory. While the Pomeron is responsible for the observed rise of the pp total cross section, the Odderon should manifest itself in the differences between pp and $p\bar{p}$ cross sections, as well as in other exclusive reactions which involve C–odd exchanges [3–12]. However, it has turned out be rather difficult to prove the existence of the Odderon unambiguously, partly because the existing experimental data are not sufficient or accurate enough. The bulk of the total cross section difference in pp and $p\bar{p}$ collisions

$$\Delta\sigma(s) \equiv \sigma^{p\bar{p}}(s) - \sigma^{pp}(s)\,, \tag{1.1}$$

appears to be well described by the Reggeon exchange with an intercept $\alpha_R(0) \sim 0.5$ (namely, $\Delta\sigma \sim s^{\alpha_R(0)-1}$), leaving little room for the Odderon contribution. Other signatures are inconclusive as well, except perhaps some positive indication from the differential elastic cross section $d\sigma/dt$ for pp and $p\bar{p}$ collisions performed at the CERN ISR. [See, [2] for a comprehensive review.]

Describing the observed cross sections for pp and $p\bar{p}$ collisions is not feasible using perturbative QCD (pQCD) alone, as the dominant, soft processes involve large values of the coupling constant. One is therefore led to use models in order to fit the data, and indeed, for the Pomeron there exists a very rich and broad phenomenological analysis [13]. On the other hand, the Odderon accounts for a small fraction of the total cross section, and therefore it is extremely sensitive to the details of various assumptions in the fitting parameterizations. Clearly, guidance from first principle calculations is highly desirable in order to better constrain and discriminate different models.

One possible way to approach the nonperturbative, strong coupling regime of gauge theories is the AdS/CFT correspondence which relates weakly coupled type IIB superstring theory on $AdS_5 \times S^5$ to strongly coupled $\mathcal{N} = 4$ supersymmetric Yang–Mills (SYM) theory. Of course, the SYM theory, being maximally supersymmetric, conformal and having non-chiral particle multiplets, is in many aspects quite different from QCD. However, given the fact that we almost completely lack any insight into the strongly coupled dynamics of QCD from first principles, one can study the problem in this different setting and try to extract universal features that QCD might possibly share.

In this paper, based on the AdS/CFT correspondence, we propose a coherent description of various C–odd exchanges in $\mathcal{N} = 4$ SYM and calculate their respective contributions to high energy baryon–baryon and baryon–antibaryon scattering. Previously, in [14] the Odderon was identified on the string theory side with the fluctuations of the antisymmetric Kalb–Ramond two–form, B, and its Regge intercept at strong coupling was derived. Here we extend the work of [14] in several important directions. First, we point out that the vector meson (Reggeon) exchange can be naturally accommodated in the strong coupling formalism as different tensor components of the B–field which were not considered in [14], thereby suggesting that the Odderon and the Reggeon are unified in ten–dimensions. This interpretation naturally comes from an inspection of the corresponding $\mathcal{N} = 4$ SYM operator, and we shall determine its anomalous dimension as well as the Reggeon intercept. Next we study the coupling of the different components of the B–field to "baryons" which are on the string theory side represented by wrapped D–branes. As we shall emphasize throughout this paper, the discussion of the Odderon would never be complete without knowing how it couples to external objects. This is in fact a subtle but crucial problem already in weak coupling perturbation theory, and even more so in the AdS/CFT context.

Our main calculations are performed in Section 3. We evaluate the forward baryon–(anti)baryon scattering amplitude from the single Odderon exchange contribution using the exact B–field propagator, and extract the leading contribution to the imaginary part which is related to the total cross section difference. Interestingly, we shall find that the Odderon exchange gives a *larger* cross section in baryon–baryon scattering than in baryon–antibaryon scattering, namely, $\Delta\sigma < 0$ at very high energies. This looks counterintuitive based on our experience with ordinary flat space calculations—the exchange of fields described by differential forms such as the B–field would imply $\Delta\sigma > 0$, and this is indeed the case for the Reggeon exchange. As we shall see, the extra overall minus sign in the Odderon case is essentially due to the warp factor of AdS_5, and therefore represents one of the hallmarks of gauge/gravity duality.

The emerging picture from our analysis is that the total cross section difference (1.1) has two components; the Odderon which tends to give $\Delta\sigma < 0$, and the Reggeon which tends to give $\Delta\sigma > 0$. In the final section, we shall interpret the existing experimental data in view of our findings and discuss the possibility to observe a negative cross section difference $\Delta\sigma < 0$ in QCD at very large s. Actually, the possibility of having $\Delta\sigma < 0$ at large energies was discussed already in the very first paper on the Odderon [1], and more recently in [9] on the basis of an extrapolation of phenomenological fits to the LHC energy scale.

2. Odderon at weak and strong coupling

We will start this section by giving a short overview of the status of the Odderon within pQCD. As mentioned already, one of the important aspects of the Odderon is its coupling to external objects, so we recall how this issue arises in the weakly coupled problem. Then in section 2.2 we discuss the Odderon in strongly coupled $\mathcal{N} = 4$ SYM. We first describe the argument behind the identification of the B–field with the Odderon, and then list the Regge intercepts for all the relevant components of the ten–dimensional B–field. Then in section 2.3 we discuss their physical interpretations in terms of the gauge invariant dual operators in the $\mathcal{N} = 4$ theory.

2.1 Odderon in weakly coupled QCD

The existence of the Odderon is predicted by pQCD. To lowest order, it is given by a symmetric color singlet C–odd combination of three gluons in the t–channel. Higher order corrections containing logarithms of energy $(\ln s)^n$ can be resummed by the Bartels–Kwiecinski–Praszalowicz (BKP) equation [15, 16] which describes the pairwise interaction of three Reggeized gluons.

The BKP equation has attracted much attention because of its connection to integrability. The equation was originally derived in momentum space, and it was later understood that its Fourier transformed formulation in coordinate (impact parameter) space is, under certain assumptions, identical to the eigenvalue problem of an exactly solvable spin–chain model (namely the XXX Heisenberg spin $s = 0$ model) [17, 18]. So far, two exact solutions of the BKP equation have been found; the Janik–Wosiek (JW) solution [19] (see, also, [20]) with a Regge intercept[1] slightly less than 1, $j_O(0) \approx 1 - 0.2472\frac{\alpha_s N_c}{\pi}$, and the Bartels–Lipatov–Vacca (BLV) solution [21] with $j_O = 1$.

Apart from the difference in the intercept, these two solutions have markedly different coordinate dependences which crucially determine their relevance in physical amplitudes. The JW solution is constrained such that it vanishes when two of the three gluons sit at the same point. This means that it does not couple to $q\bar{q}$ states including a virtual photon

[1] For a reason to become clear later, henceforth we distinguish the intercept associated with the spin of gauge theory operators j_O from the one associated with the total cross section $\alpha_O(0)$.

in DIS. Though it in principle couples to three–quark states (like a proton), the constraint of vanishing at equal points is incompatible with gauge invariant initial conditions [22].

On the other hand, the BLV solution does not have this constraint, and hence it naturally couples to both $q\bar{q}$ and qqq states in a gauge invariant way [22, 23]. Nevertheless, this solution is sometimes deemed 'exceptional' because its description in the integrability framework turned out to be tricky: One has to extend the Hilbert space of eigenstates of the spin–chain Hamiltonian so that it includes solutions which do not vanish at equal points. However, physically there is no reason to expect that the Odderon amplitude vanishes at equal points, and indeed the BKP equation does not *a priori* imply such a property. In fact, this constraint was artificially imposed by hand by neglecting certain terms in the full BKP Hamiltonian in order to establish the equivalence with the spin–chain Hamiltonian. Therefore, the seemingly unusual status of the BLV solution is illusory, and one can fairly conclude that it is *the* Odderon solution in perturbative QCD in the leading logarithmic approximation. Note that, as was originally done in [21], the BLV solution can be explicitly constructed in momentum space without any reference to integrability.

The above situation in perturbative QCD illustrates an important point when studying the Odderon: It is one thing to derive the Odderon intercept, but it is quite another to discuss *how* a given Odderon solution actually couples to external states. Whether it has nonvanishing coupling depends on the quantum numbers and the internal structure of the colliding hadrons. As we shall see, this problem appears also in the approach based on the AdS/CFT correspondence.

Another point worth noting is that the BLV odderon amplitude is purely real. This is most readily seen by the identification of the odderon amplitude as the imaginary part of the eikonal amplitude made up of lightlike Wilson lines [22]

$$\mathcal{A}_{odderon} \sim \mathrm{Im}\,\mathrm{tr}\, W\,. \tag{2.1}$$

This implies, in particular, that in perturbation theory one cannot really address the issue of the total cross section difference which, via the optical theorem, is related to the imaginary part of the forward amplitude $\mathrm{Im}\,\mathcal{A}(t=0)$. In order to generate a nonzero imaginary part, one has to have some nonperturbative inputs, and this is what we shall explore in the following.

Before leaving this subsection, we should like to mention that there have been a lot of activities in phenomenological applications [4–10] as well as theoretical extensions of the BLV odderon treatment. The latter includes the corrections of next–to–leading logarithmic terms and beyond [24, 25], suggesting that the Odderon intercept will remain at $j_O(0) = 1$ to all orders in (leading–twist) perturbation theory, and also non–linear effects from gluon saturation [22, 23, 26–29] which tend to suppress the odderon amplitude.

2.2 Odderon in strongly coupled $\mathcal{N} = 4$ SYM

In this and the next subsections we describe the nature of the Odderon in $\mathcal{N} = 4$ supersymmetric Yang–Mills (SYM) theory at strong coupling. Via the AdS/CFT correspondence, the problem can be equivalently formulated in type–IIB superstring theory on the background of $AdS_5 \times S^5$ with the metric

$$\begin{aligned} ds^2 &= G_{mn} dX^m dX^n + G_{\alpha\beta} d\theta^\alpha d\theta^\beta \\ &= R^2 \frac{-2dx^+ dx^- + d\vec{x}_\perp^2 + dz^2}{z^2} + R^2 (d\theta^2 + \sin^2\theta d\Omega_4^2) \,, \end{aligned} \tag{2.2}$$

where $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$, $\vec{x}_\perp = (x^1, x^2)$ and R is the common radius of AdS_5 and S^5. We use the notation $X^m = (x^\mu, z)$ for the AdS_5 coordinates and $\theta^\alpha = (\theta \equiv \theta^1, \theta^2, \theta^3, \theta^4, \theta^5)$ for the S^5 coordinates.

In $\mathcal{N} = 4$ SYM where the fermions belong to a real (namely, the adjoint) representation of the gauge group, C–conjugation is a rather uninteresting operation. However, if one introduces charges ('quarks') in the fundamental representation living on some 'flavor' branes, one can define the C–conjugation with respect to these charges. Specifically, we shall later introduce baryons, or D–branes which carry the NS-NS charges on their worldvolume. The NS-NS antisymmetric B–field couples to the fundamental and anti-fundamental charges with opposite signs, thus it can be identified as the Odderon in this context.[2]

More properly, the Odderon is the *Reggeized* B–field,[3] which roughly means that it is a coherent superposition of string excited states lying on a Regge trajectory starting from the massless B–field. In flat ten–dimensional spacetime, this can be seen by inspection of the Shapiro–Virasoro amplitude and takes the form

$$f(\alpha' t)(1 - e^{-i\pi\alpha(t)}) s^{\alpha(t)} \,, \tag{2.3}$$

where $\alpha(t) = 1 + \frac{\alpha' t}{2}$. One recognizes the usual Regge behavior with the negative signature factor, but importantly the prefactor $f(\alpha' t)$ has no pole at $t = 0$. This means that the massless B–field decouples from the s–channel closed strings such as the dilaton ('glueball'). The fact that one nevertheless gets a nonzero amplitude with an imaginary part (2.3) is due to the Reggeization of the B–field.

The analogous problem in the background of $AdS_5 \times S^5$ is considerably more complicated. Upon compactification on S^5, the ten–dimensional B–field splits into the diagonal modes

$$B_{mn} \,, \tag{2.4}$$

with both indices along the AdS_5 direction, and the off–diagonal modes

$$B_{m\alpha} \,, \tag{2.5}$$

with one of the indices along the S^5 direction.[4] Furthermore, each mode undergoes the

[2]If one is interested in colliding objects which carry the R-R charges, such as D1–branes and monopoles, the R-R two–form would serve as the Odderon.

[3]The exchange of the bare (i.e., not Reggeized) B–field in high energy scattering was previously considered in [30].

[4]We ignore the modes $B_{\alpha\beta}$ with both indices on S^5 because they are irrelevant at high energy.

Kaluza–Klein (KK) decomposition [31]

$$
\begin{aligned}
B_{mn} &= \sum_{k=0}^{\infty} B_{mn}^{(k)}(X) Y^{(k)}(\Omega_5)\,, \\
B_{m\alpha} &= \sum_{k=1}^{\infty} \left(b_m^{(k)}(X) Y_\alpha^{(k)}(\Omega_5) + d_m^{(k)}(X) \nabla_\alpha Y^{(k)}(\Omega_5) \right)\,,
\end{aligned} \tag{2.6}
$$

where $Y^{(k)}$ and $Y_\alpha^{(k)}$ are the rank–k scalar and vector spherical harmonics on S^5, respectively. These harmonics are reviewed in Appendix A. Summation over different harmonics with the same value of k is understood. By choosing the gauge $\nabla^\alpha B_{m\alpha} = 0$, one can always set $d_m^{(k)} = 0$ (see (A.14)) which we shall subsequently do.

The Reggeization of the B–field has to be done for each KK mode by analyzing the supergravity equation of motion [32]. For the diagonal modes (2.4), there are two equations of motions for each value of k. Accordingly, there are two branches of the Regge intercepts. They have been worked out in [14] with the results

$$
j_O = 1 - \frac{M_I^2}{2\sqrt{\lambda}}\,, \qquad (I = 1, 2) \tag{2.7}
$$

where λ is the 't Hooft coupling and the respective masses M_I are given by

$$
M_1 = k\,, \qquad M_2 = k + 4\,. \qquad (k = 0, 1, 2. \cdots) \tag{2.8}
$$

Note that the lowest KK state $M_1 = k = 0$ has $j_O = 1$. In view of the finding of [25], it is tempting to regard this mode as the fate of the BLV odderon at strong coupling. However, in the gravity description this mode is pure gauge, and decouples from physical amplitudes. We shall have more to say about this below.

The intercept for the off–diagonal mode $B_{m\alpha} \sim b_m$ can be determined as follows. The relevant equation of motion is the massive Maxwell equation [31]

$$
\nabla^n f_{nm} - \frac{M^2}{R^2} b_m = 0\,, \tag{2.9}
$$

where $f_{mn} \equiv \partial_m b_n - \partial_n b_m$ and

$$
M^2 = (k+1)(k+3)\,. \qquad (k = 1, 2, 3, \cdots) \tag{2.10}
$$

Without the mass term, this is identical to the equation for the gauge boson (dual to the $\mathcal{R}$–current operator) considered in [33]. Thus the intercept can be immediately obtained by making the following change in the result of [33]

$$
j = 1 - \frac{1}{2\sqrt{\lambda}} \quad \rightarrow \quad j_R = 1 - \frac{1+M^2}{2\sqrt{\lambda}} = 1 - \frac{(k+2)^2}{2\sqrt{\lambda}}\,, \tag{2.11}
$$

where the meaning of the subscript R will be explained shortly. Interestingly, despite the difference in the equations of motion in AdS_5, the spectrums of j in (2.7) and (2.11) are overlapping except for the first few KK states. This might be due to the fact that these modes originally come from a single field, namely the B–field, in ten dimensions.

2.3 Correspondence with operators in gauge theory

The AdS/CFT correspondence identifies fields on the string theory side with local gauge invariant operators on the field theory side. Now that we know the intercepts of various KK excitations of the B–field, let us discuss to which operators in $\mathcal{N}=4$ SYM these states correspond.

Mathematically, the Reggeization means that the spin of the B–field is analytically continued to $j \neq 1$. Accordingly, the spin of the corresponding operator should also be continued.[5] In the case of the diagonal modes (2.4), the relation between the dimension of the operator with spin j and the mass of the KK state was derived in [14]

$$\Delta(j) = 2 + 2\sqrt{\frac{\sqrt{\lambda}}{2}(j - j_O)} = 2 + 2\sqrt{\frac{\sqrt{\lambda}}{2}\left(j - 1 + \frac{M_I^2}{2\sqrt{\lambda}}\right)}\,. \tag{2.12}$$

Consider first the branch $M_1 = k$ which has a larger intercept and should therefore dominate the cross section. One has

$$\Delta(1) = 2 + k\,. \tag{2.13}$$

As already mentioned, the $k = 0$ mode is pure gauge and therefore does not correspond to propagating physical degrees of freedom. Indeed there is no gauge–invariant, spin–1, dimension–2 operator in $\mathcal{N}=4$ SYM. For $k = 1$, the corresponding gauge theory operator is

$$\mathrm{tr}\big(\psi^A \sigma_{\mu\nu} \psi^B + 2i\phi^{AB} F^+_{\mu\nu}\big)\,, \quad (1 \leq A, B \leq 4) \tag{2.14}$$

which belongs to the **6** representation of $SU(4)$ and the (1,0) of the Lorentz $SO(3,1)$ group. [$F^{\pm}$ denotes the self–dual/anti self–dual part of the field strength, belonging to $(1,0)$ and $(0,1)$, and the trace is in the fundamental representation.] The operators with $k \geq 2$ are multiplied by higher powers of scalars with appropriate $SU(4)$ representations. These operators are denoted as $\mathcal{O}_k^{(4)}$ in Table 7 of [34].

Next, let us turn to the $M_2 = k + 4$ branch. In this case $\Delta(1) = k + 6$, and the corresponding operators are given by

$$\mathrm{tr}\left(F^+_{\mu\nu} F^2_- \phi^k\right)\,. \tag{2.15}$$

These operators are denoted as $\mathcal{O}_k^{(16)}$ in Table 7 of [34]. Note that for $k = 0$ this is a gluonic operator which, at weak coupling, starts out with three gluons. Based on this analogy, and also on the fact that the fields on the two branches $I = 1, 2$ can be treated on the same footing in actual calculations, we shall collectively call the diagonal modes $B^{(k)}_{mn}$ the Odderon as in [14].

Finally, for the off–diagonal modes, one has instead [33]

$$\Delta(j) = 2 + 2\sqrt{\frac{\sqrt{\lambda}}{2}(j - j_R)} = 2 + 2\sqrt{\frac{\sqrt{\lambda}}{2}\left(j - 1 + \frac{1 + M^2}{2\sqrt{\lambda}}\right)}\,. \tag{2.16}$$

[5] This implies that the operators become nonlocal.

Using $M^2 = (k+1)(k+3)$, one finds $\Delta(1) = k+4$. The corresponding operators are denoted as $\mathcal{O}^{(10)}_{k-1}$ in Table 7 of [34] and read

$$\mathrm{tr}\left(F^{+}_{\mu\nu}\bar{\psi}_A\bar{\sigma}^{\nu}\psi_B\phi^{k-1}\right) . \tag{2.17}$$

The lowest $k = 1$ mode is the **15** of $SU(4)$ and the $(\frac{1}{2}, \frac{1}{2})$ of $SO(3,1)$. It looks like an interpolating operator of vector mesons. Therefore we will refer to the off–diagonal modes $b^{(k)}_m$ as the Reggeon, whence the subscript in j_R.

Concluding this section, we have identified all the possible C–odd exchanges originating from the ten–dimensional B–field and listed their respective Regge intercepts and the corresponding operators in SYM. Anticipating the later developments, here we note the following two caveats when applying the formal results of this section to actual scattering processes: (i) Our experience with pQCD tells us that it may very well be that not all of the KK modes actually couple to external hadrons. (ii) The formulas presented above for the Regge "intercept" lead to the following behavior

$$\mathcal{A}(s,b) \sim s^{j_O - 1} f(s,b) , \tag{2.18}$$

of the scattering amplitude at *fixed* impact parameter b, with f being some function of b and the energy s. In order to obtain the total cross section difference, one has to integrate $\mathrm{Im}\mathcal{A}(b)$ over b. If the function f had no energy dependence, then the cross section would have the dependence $s^{\alpha_O(0)-1}$ with $\alpha_O(0) = j_O$. However, in certain cases the b–integration can modify the s–dependence so that $\alpha_O(0) \neq j_O$. In the next section, we shall see that both of these caveats are indeed relevant.

3. Odderon exchange in baryon–baryon scattering at strong coupling

In this main section we calculate the Odderon exchange contribution to the baryon–(anti)baryon scattering amplitude by representing baryons as D–branes. Ideally, the D–brane amplitude should be exactly calculated (as can be done in flat space [35]) so that various string exchanges are automatically included. However, in a curved background like $AdS_5 \times S^5$, such exact results are unavailable, and we must content ourselves with the single Odderon exchange. Actually, in $\mathcal{N} = 4$ SYM it is probably possible to go beyond the single Odderon approximation [14] by including unitarity corrections in the form of the graviton (Pomeron) eikonalization [36, 37]. We will comment on the issue of unitarity corrections along with their relevance to QCD in the discussion section.

We first describe the D–brane solution of [38, 39] and investigate its coupling to the various KK excitations of the B–field. It will turn out that the coupling to the Reggeon $b^{(k)}_m$ vanishes identically. We therefore focus only on the Odderon $B^{(k)}_{mn}$ contribution, calculating first the bare propagator in section 3.2 from which the Reggeized propagator is immediately obtained. The latter is then used in section 3.3 to calculate the Reggeized amplitude whose imaginary part determines the cross section difference. Our main result is displayed in equation (3.33) which is an explicit analytical formula for the total cross section difference.

3.1 Baryon–Odderon coupling

In the context of gauge/string duality, baryons can be realized as D–branes wrapping on some compact manifold [40]. In the canonical example of the $\mathcal{N} = 4$ SYM/type IIB correspondence, they are D5–branes wrapping on S^5. Among the different versions of baryon configurations proposed in the literature, we employ here a particular BPS solution constructed in [38, 39].

Consider the embedding of a D5–brane in $AdS_5 \times S^5$ using the coordinate mapping $\beta(\xi) = (x^\mu(\xi), z(\xi), \theta^\alpha(\xi))$ where $\xi^a = (x^0, \theta^\alpha)$ are the coordinates parameterizing the worldvolume of the D5–brane. The dynamics of the D5–brane is governed by the Born–Infeld action plus the Chern–Simons term

$$S = T_5 \int d^6\xi \left\{ -\sqrt{-\det(\tilde{G} + \tilde{B} + 2\pi\alpha' F)} + 2\pi\alpha' F \wedge c_{(4)} \right\} , \tag{3.1}$$

where $\tilde{G} \equiv \beta^* G$, $\tilde{B} \equiv \beta^* B$ and $c_{(4)} \equiv \beta^* C_{(4)}$ are the pullbacks of the graviton, the B–field and the R–R 4–form on the brane, respectively, and F is the two–form $U(1)$ field strength on the brane. The solution [38,39] which respects the BPS condition has $S^4 \subset S^5$ symmetry and is extended in the z–direction along which there is an electric flux $F_{0z} = (\partial\theta/\partial z) F_{0\theta}$. [Remember that $\theta \equiv \theta^1$.] The embedding function $z = z(\theta)$ is given by

$$z(\theta) = \frac{r_h \sin\theta}{\left[\frac{3}{2}(\theta - \sin\theta\cos\theta)\right]^{\frac{1}{3}}} , \tag{3.2}$$

where $r_h = z(\theta = 0)$ is an arbitrary length scale which we identify with the radius of the baryon. As θ goes to π, $z(\theta)$ monotonously decreases to zero where the boundary of AdS_5 is located. In order to obtain a finite baryon mass, one has to put a UV cutoff at small z, or equivalently, in θ near π.

We shall be interested in the high energy collision of a baryon and a (anti-)baryon in this D–brane representation, exchanging the B–field in the t–channel. Since the mass of a baryon is of order N_c, high energy means that the center–of–mass energy $\sqrt{s}$ is parametrically of order N_c up to a boost factor. For a baryon moving in the $\pm x^3$ direction near the speed of light $v \approx 1$, it is convenient to take $x^\pm$ (instead of x^0) as the 'time' coordinate on the brane with the constraint $x^\mp \approx 0$. The coupling between the D–brane moving in the $+x^3$ direction and the B–field is

$$\begin{aligned} S_{int} &= \frac{1}{\mathrm{Vol}_{S^4}} \int dx^+ d\theta d\Omega_4 \frac{\partial \mathcal{L}}{\partial F_{+\theta}} \frac{\tilde{B}_{+\theta}}{2\pi\alpha'} \\ &= \frac{n}{2\pi\alpha' \mathrm{Vol}_{S^4}} \int dx^+ d\theta d\Omega_4 \left(B_{+\theta} + \frac{\partial z}{\partial\theta} B_{+z} \right) , \end{aligned} \tag{3.3}$$

where $\mathrm{Vol}_{S^4} = \frac{8\pi^2}{3}$ and $n = \frac{\partial\mathcal{L}}{\partial F_{+\theta}}$ is an integer which measures the string charge on the D–brane. For a baryon (anti–baryon) we take $n = N_c$ ($n = -N_c$).

The B–fields in (3.3) are decomposed into the KK modes as in (2.6). Then the issue arises as to whether the $d\Omega_4$ integral of the spherical harmonics is nonvanishing. In Appendix A, we show that the integral of the θ–component of the vector spherical harmonics over S^4 vanishes identically for all k,

$$\int d\Omega_4 \, Y_\theta^{(k)}(\Omega_5) = 0 \,. \tag{3.4}$$

This means that the off–diagonal modes $B_{+\theta}^{(k)}$ (Reggeon) decouple from the the baryon under consideration. On the other hand, for any $k \geq 0$ at least one component of the scalar harmonics $Y^{(k)}$ gives a nonvanishing value after the integration over S^4, so the diagonal modes $B_{+z}^{(k)}$ (Odderon) do couple to our baryon. Therefore in the remainder of this section we only consider the diagonal mode B_{+z}. We shall return to the relevance of the off–diagonal modes in the discussion section.

The full amplitude in impact parameter space for the exchange of the B–field is

$$\begin{aligned} i\mathcal{A}^{\pm}(s,b) = \pm i^2 \left(\frac{N_c}{2\pi\alpha' \mathrm{Vol}_{S^4}} \right)^2 \sum_k \int dx^+ dz d\Omega_4 Y^{(k)}(\Omega) \\ \times \int dx'^- dz' d\Omega_4' Y^{(k)}(\Omega') \langle B_{+z}^{(k)}(x^+, 0, x_\perp, z) B_{-'z'}^{(k)}(0, x'^-, x'_\perp, z') \rangle \,, \end{aligned} \tag{3.5}$$

where the plus (minus) sign corresponds to baryon–baryon (baryon–antibaryon) scattering. Our next task is now to calculate the propagator $\langle B_{+z} B_{-'z'} \rangle$.

3.2 Bare B–field propagator

The equation of motion for the components $B_{mn}^{(k)}$ has been derived in [31] by dimensionally reducing the type IIB supergravity equation of motion. Alternatively, one may perform the dimensional reduction directly on the supergravity action taking into account the mixing with the Ramond–Ramond (R-R) two–form. This yields, for a given value of k, [41]

$$S = -\frac{R^5\pi^3}{2\kappa^2} \sum_{I=1,2} \int d^5X \sqrt{-G} \Big(\frac{i}{2} \epsilon^{mnlpq} (a_{mn}^I)^* \partial_l a_{pq}^I + M_I (a_{mn}^I)^* a_I^{mn} \Big) \,, \tag{3.6}$$

where we have normalized the spherical harmonics as $\int d\Omega_5 |Y^{(k)}|^2 = \pi^3 = \mathrm{Vol}_{S^5}$. The complex fields $a^{1,2}$ are certain linear combinations of the B–field and the R-R two–form [41], such that the B–field can be written[6]

$$B_{mn}^{(k)} = \frac{1}{\sqrt{2(k+2)}} \left(a_{mn}^1 + a_{mn}^{1*} + a_{mn}^2 + a_{mn}^{2*} \right) \,. \tag{3.7}$$

The Kaluza–Klein masses M_I are as in (2.8).

The propagator of the $a^{(1,2)}$ field satisfies the following equation (up to the prefactor $\kappa^2/R^5\pi^3$)

$$\begin{aligned} \frac{i}{2} \epsilon_{mn}{}^{lpq} \partial_l D_{pq,m'n'}(X,X') + M D_{mn,m'n'}(X,X') \\ = -i \frac{\delta^{(5)}(X-X')}{\sqrt{-G}} (G_{mm'}G_{nn'} - G_{mn'}G_{nm'}) \,, \end{aligned} \tag{3.8}$$

[6] Our normalization of the B–field differs from that in [41] by a factor of 2.

where M is either M_1 or M_2. The solution was obtained in [42] and takes the form

$$\begin{aligned} D_{mn,m'n'} &= T^1_{mn,m'n'}(D(u)+2H(u)) + T^2_{mn,m'n'}H'(u) + T^3_{mn,m'n'}K(u) \\ &= T^1_{mnm'n'}D(u) - \partial_m V_{n,m'n'} + \partial_n V_{m,m'n'} + T^3_{mn,m'n'}K(u)\,, \end{aligned} \tag{3.9}$$

where

$$T^1_{mn,m'n'} = R^4(\partial_m\partial_{m'}u\partial_n\partial_{n'}u - \partial_m\partial_{n'}u\partial_n\partial_{m'}u)\,, \tag{3.10}$$

$$\begin{aligned} T^2_{mn,m'n'} = R^4(&\partial_m\partial_{m'}u\partial_n u\partial_{n'}u - \partial_m\partial_{n'}u\partial_n u\partial_{m'}u \\ &-\partial_n\partial_{m'}u\partial_m u\partial_{n'}u + \partial_n\partial_{n'}u\partial_m u\partial_{m'}u)\,, \end{aligned} \tag{3.11}$$

$$T^3_{mn,m'n'} = R^5\epsilon_{mn}{}^{lpq}\partial_l\partial_{m'}u\partial_p\partial_{n'}u\partial_q u\,, \tag{3.12}$$

$$V_{m,m'n'} = R^4H(u)(\partial_m\partial_{m'}u\partial_{n'}u - \partial_m\partial_{n'}u\partial_{m'}u)\,, \tag{3.13}$$

and u is the chordal distance in AdS_5

$$u = \frac{(z-z')^2 + (x_\perp - x'_\perp)^2 - 2(x^+ - x'^+)(x^- - x'^-)}{2zz'}\,. \tag{3.14}$$

We shall focus on the $(mn, m'n') = (+z, -'z')$ component. One can check that $T^3_{+z,-'z'} = 0$, and

$$T^1_{+z,-'z'} = \frac{R^4}{z^2z'^2}\left(1 + v - \frac{z}{z'} - \frac{z'}{z}\right) = \frac{R^4}{zz'}\partial_z\partial_{z'}v\,, \tag{3.15}$$

where

$$v = \frac{(z-z')^2 + (x_\perp - x'_\perp)^2}{2zz'}\,, \tag{3.16}$$

is the chordal distance in H_3. As for terms involving V, the first term $-\partial_+V_{z,-'z'}$ can be neglected since we integrate the propagator over x^+ (see (3.5)), while the other term is given by

$$\partial_z V_{+,-'z'} = \partial_z\left(\frac{H(u)}{zz'}\partial_{z'}v\right)\,. \tag{3.17}$$

The function $D(u)$ satisfies the following equation

$$\frac{1}{R^2}(z^2\partial_z^2 - 3z\partial_z + z^2\partial^2 + 4 - M^2)D(u) = iM\frac{z^5}{R^5}\delta^{(5)}(X - X')\,, \tag{3.18}$$

and $H(u)$, $K(u)$ are given in terms of $D(u)$

$$H(u) = -\frac{1}{M^2}\left(2D(u) + (u+1)D'(u)\right)\,, \quad K(u) = -\frac{i}{M}D'(u)\,. \tag{3.19}$$

Let us define

$$R^2\int\frac{dx^+dx'^-}{zz'}D(u) = D^{(3)}(v)\,. \tag{3.20}$$

Then

$$R^2 \int \frac{dx^+ dx'^-}{zz'} H(u) = -\frac{1}{M^2} D^{(3)}(v)\,, \tag{3.21}$$

where we integrated by parts. From (3.18), one has

$$\frac{1}{R^2}(z^2\partial_z^2 - z\partial_z + z^2\partial_\perp^2 + 1 - M^2) D^{(3)} = iM\frac{z^3}{R^3}\delta(z-z')\delta^{(2)}(x_\perp - x'_\perp)\,, \tag{3.22}$$

and the solution can be conveniently written as

$$D^{(3)}(v) = \frac{-iM}{4\pi R}\frac{e^{-M\xi}}{\sinh\xi}\,, \tag{3.23}$$

where $\xi \equiv \cosh^{-1}(1+v)$. Note that $D^{(3)}$ vanishes when $M=0$, which explicitly shows the decoupling of the mode $M_1 = k = 0$. [The V terms become gauge artifacts.] In the following we consider only the case $M \geq 1$ and obtain

$$\begin{aligned}\int dx^+ dx'^- \langle B^{(k)}_{+z}(x,z) B^{(k)}_{-z}(x',z')\rangle &= \sum_I \frac{1}{k+2}\frac{\kappa^2}{R^5\pi^3}R^2 \\ &\times \left[\left(1-\frac{1}{M_I^2}\right) D^{(3)}(v)\partial_z\partial_{z'}v - \frac{1}{M_I^2}\partial_v D^{(3)}(v)\partial_z v\partial_{z'}v\right]. \end{aligned}\tag{3.24}$$

3.3 The total cross section difference

Equation (3.24) is the contribution from the bare B–field, and as such, the corresponding amplitude $\mathcal{A}$ is purely real. In order to get an imaginary part, one has to Reggeize the B–field by doing analytic continuation in j. This boils down to replacing [14][7]

$$D^{(3)}(v) \to D_j^{(3)}(v) \equiv \frac{-iM}{4\pi R}\frac{e^{-M_j\xi}}{\sinh\xi}\,, \tag{3.25}$$

where

$$M_j^2 = M^2 + 2\sqrt{\lambda}(j-1) = 2\sqrt{\lambda}(j-j_O)\,, \tag{3.26}$$

and $j_O \equiv 1 - \frac{M^2}{2\sqrt{\lambda}}$ is the Odderon intercept (2.7). Let us consider the first term on the right hand side of (3.24). Summing over odd values of j in the form of the contour integral, as dictated by the presence of the odd signature factor in (2.3), one reaches the following expression

$$\int \frac{dj}{4i}\frac{1-e^{-i\pi j}}{\sin\pi j}\left(\frac{\alpha'\tilde{s}}{4}\right)^{j-1}\left(1+v-\frac{z}{z'}-\frac{z'}{z}\right)\frac{D_j^{(3)}(v)}{zz'}\,. \tag{3.27}$$

[7]Note that we do not replace $M \to M_j$ in the prefactor. This factor of M comes from the right hand side of (3.22) and is clearly not associated with the pole of the t–channel propagator. Incidentally, Ref. [14] leaves open the possibility that the $M=0$ mode becomes physical after the analytic continuation. In our approach this might correspond to replacing $M \to M_j$ also in the prefactor so that the coupling apparently becomes nonvanishing. However, such a procedure is somewhat arbitrary, and induces uncertainties in the subsequent calculations. Thus, although we think it is an interesting possibility, we do not pursue it in the present paper.

The j–integral goes around the positive real axis encircling poles at odd integers $j = 1, 3, 5, \cdots$ clockwise. In order to obtain the forward scattering amplitude $\mathcal{A}(s, t = 0)$ one needs to take the Fourier transform of (3.27) with respect to $b = x_\perp - x'_\perp$ and take the limit $t \to 0$.[8] Using $d^2b = 2\pi zz' \sinh \xi d\xi$, one obtains

$$2\pi zz' \int_{\xi_0}^{\infty} d\xi \int \frac{dj}{4i} \frac{1 - e^{-i\pi j}}{\sin \pi j} \left(\frac{zz's}{4\sqrt{\lambda}}\right)^{j-1} \left(\cosh \xi - \frac{z}{z'} - \frac{z'}{z}\right) \frac{-iMe^{-M_j\xi}}{4\pi Rzz'}$$
$$\approx \frac{-iM}{4R} \sqrt{\frac{\pi\sqrt{\lambda}}{2\tau^3}} \frac{1 - e^{-i\pi j_O}}{\sin \pi j_O} e^{(j_O-1)\tau} \int_{\xi_0}^{\infty} d\xi \left(\cosh \xi - \frac{z}{z'} - \frac{z'}{z}\right) \xi e^{-\frac{\sqrt{\lambda}\xi^2}{2\tau}}, \quad (3.28)$$

where $\xi_0 = |\ln z/z'|$ and $\frac{zz's}{4\sqrt{\lambda}} \equiv e^\tau$. Here we have deformed the contour so as to surround the branch cut beginning at $j = j_O$, and evaluated the j–integral using the saddle point approximation which is valid when $\tau/\sqrt{\lambda}$ is large. The ξ integral then gives

$$\frac{-iM}{8R} \sqrt{\frac{\pi}{2\tau\sqrt{\lambda}}} \frac{1 - e^{-i\pi j_O}}{\sin \pi j_O} e^{(j_O-1)\tau} \left(e^{\frac{\tau}{2\sqrt{\lambda}}} \int_{\xi_0 - \frac{\tau}{\sqrt{\lambda}}}^{\xi_0 + \frac{\tau}{\sqrt{\lambda}}} d\xi \, e^{-\frac{\sqrt{\lambda}\xi^2}{2\tau}} - \left(\frac{z}{z'} + \frac{z'}{z}\right) e^{-\frac{\sqrt{\lambda}\xi_0^2}{2\tau}} \right). \quad (3.29)$$

When $\tau/\sqrt{\lambda}$ is large, the first term in the brackets dominates. Noting also that typically ξ_0 should be small since we are scattering objects of the same size, the limits of the Gaussian integral can be extended to $\pm\infty$ and we obtain

$$\frac{-iM\pi}{8R\sqrt{\lambda}} \frac{1 - e^{-i\pi j_O}}{\sin \pi j_O} \left(\frac{zz's}{4\sqrt{\lambda}}\right)^{\alpha_O(0)-1}, \quad (3.30)$$

where

$$\alpha_O(0) = j_O + \frac{1}{2\sqrt{\lambda}} = 1 - \frac{M^2 - 1}{2\sqrt{\lambda}}. \quad (3.31)$$

We now see explicitly that the intercept has shifted by a small amount after the b–integration—a possibility we pointed out at the end of section 2.3. This is due to the $v \sim b^2$ factor in $T^{(1)}$ which modifies the large–b behavior of the amplitude. Such a shift would not occur if one had exchanged the transverse component of the B–field $B_{\pm\perp}$. A curious consequence of (3.31) is that the $M_1 = k = 1$ mode has $\alpha_O(0) = 1$, the value commonly associated with the phenomenological Odderon. At the moment we do not have a deep understanding of this coincidence, especially in terms of the corresponding operator (2.14).

Essentially the same shift occurs in the second term of (3.24). After some algebra, one finds that the net effect of the second term is simply to replace the prefactor $1 - \frac{1}{M^2}$ with

[8] It is important to notice that the limit $t \to 0$ has to be taken *after* the j–integration, which means that the order of the j and ξ integrations in (3.28) cannot be interchanged in general. This is because in Regge theory the amplitude is obtained by an analytic continuation from the region $t \gg |s|$.

$1+\frac{1}{M^2}$. We thus find,

$$
\begin{aligned}
i\int d^2b\,\mathcal{A}^{\pm}(s,b) &\approx \pm i\left(\frac{N_c}{2\pi\alpha'\mathrm{Vol}_{S^4}}\right)^2\sum_{I,k}\frac{M_I}{k+2}\left(1+\frac{1}{M_I^2}\right)\int dzd\Omega_4Y^{(k)}(\Omega_5)\\
&\quad\times\int dz'd\Omega'_4Y^{(k)}(\Omega'_5)\frac{\kappa^2}{8R^4\pi^2\sqrt{\lambda}}\frac{1-e^{-i\pi j_O}}{\sin\pi j_O}\left(\frac{zz's}{4\sqrt{\lambda}}\right)^{\alpha_O(0)-1}\\
&=\pm\frac{i\sqrt{\lambda}\pi}{8(\mathrm{Vol}_{S^4})^2}\sum_{I,k}\frac{M_I+\frac{1}{M_I}}{k+2}\int dzd\Omega_4Y^{(k)}(\Omega_5)\\
&\quad\times\int dz'd\Omega'_4Y^{(k)}(\Omega'_5)\frac{1-e^{-i\pi j_O}}{\sin\pi j_O}\left(\frac{zz's}{4\sqrt{\lambda}}\right)^{\alpha_O(0)-1}.
\end{aligned}
\tag{3.32}
$$

The difference between the total cross sections is then given by

$$
\begin{aligned}
\Delta\sigma &= \sigma^{B\bar{B}}-\sigma^{BB} = 2\int d^2b\,Im\,\mathcal{A}^{-}(s,b)-2\int d^2b\,Im\,\mathcal{A}^{+}(s,b)\\
&= -\frac{\pi\sqrt{\lambda}}{4(\mathrm{Vol}_{S^4})^2}\sum_{I,k}\frac{M_I+\frac{1}{M_I}}{k+2}\int dzd\Omega_4Y^{(k)}(\Omega_5)\int dz'd\Omega'_4Y^{(k)}(\Omega'_5)\left(\frac{zz's}{4\sqrt{\lambda}}\right)^{\alpha_O(0)-1}.
\end{aligned}
\tag{3.33}
$$

This is the main result of this paper. We remind the reader that the z–integrals are bounded $z\le r_h$, see (3.2).

4. Discussion

Surprisingly, the right hand side of (3.33) is *negative*, which means that the baryon–baryon cross section is *larger* than the baryon–anti-baryon cross section. This immediately raises both theoretical and practical questions.

Theoretically, one would expect the exchange of an antisymmetric field to generate a repulsive force between like charges, as in the well–known cancellation of the attractive NS-NS and repulsive R-R forces between parallel D–branes. However, in the above calculation the B–field exchange effectively generates an attraction between like charges and repulsion between opposite charges. The sign change can be traced back to the first term in equation (3.10) where the z and z' derivatives both act on the *denominator* of the chordal distance u, resulting in the first two terms of (3.15). These are positive and dominate over the (expected) negative contributions because of the b^2 factor which is amplified by the b–integration. Thus the attraction originates from a combined effect of the warp factor of AdS_5 and the particular component $B_{\pm z}$ we have used. Had we exchanged the transverse component $B_{\pm\perp}$, we would have obtained an opposite sign in the final result.

Practically, the available experimental data show that the total cross section is larger in $p\bar{p}$ collisions than in pp collisions. However, before comparing this fact with our result, the following three remarks are in order:
(i) The dominance of the first term over the second term in (3.29) is safely claimed only at very high energies, $\tau\gg\sqrt{\lambda}$, where the small shift $1/2\sqrt{\lambda}$ in the intercept becomes

noticeable. On the other hand the highest energy at which $\Delta\sigma$ has been measured is the ISR energy $\sqrt{s} = 53$ GeV which is not that high. We would probably need to at least go to the Tevatron $\sqrt{s} = 2$ TeV, or the LHC energies $\sqrt{s} = 14$ TeV.

(ii) Our result pertains to a particular choice of the D5–brane embedding which has no dependence on S^4. This means that the corresponding baryon is a singlet under the $SO(5)$ subgroup of the 'flavor' $SO(6) \cong SU(4)$ group. As we have seen, this has resulted in the decoupling of the vector meson, or the Reggeon contribution which plays an important role in QCD. By considering baryons which belong to a larger representation of the flavor group, one should be able to find nonvanishing coupling to the Reggeon. In practice, this amounts to adding electric fluxes in the S^4 direction. The point is that, by a simple generalization of the argument in a previous paragraph, one is guaranteed that the exchange of the vector mode $b_\pm$ gives a normal sign. Therefore, in more realistic situations where both the Odderon and the Reggeon couple, the sign of $\Delta\sigma$ is determined by a competition between the two exchanges. The Reggeon has the intercepts[9]

$$\alpha_R(0) = 1 - \frac{9}{2\sqrt{\lambda}}, \quad 1 - \frac{16}{2\sqrt{\lambda}}, \cdots , \tag{4.1}$$

and gives positive contributions to $\Delta\sigma$, whereas the Odderon has the intercepts

$$\alpha_O(0) = 1, \quad 1 - \frac{3}{2\sqrt{\lambda}}, \quad 1 - \frac{8}{2\sqrt{\lambda}}, \quad 1 - \frac{15}{2\sqrt{\lambda}}, \cdots . \tag{4.2}$$

and gives negative contributions at very high energies. Which effect wins is likely to become a quantitative question, rather than parametric.

(iii) We have not included unitarity corrections in our calculation. In $\mathcal{N} = 4$ SYM proper, strong unitarity corrections come from the eikonal exponentiation of the graviton (Pomeron) amplitude which is dominantly real [36, 37], and this could make any 'predictions' of the single Odderon approximation uncertain [14]. However, as far as unitarity corrections are concerned, $\mathcal{N} = 4$ SYM fares poorly as a model of QCD where the Pomeron amplitude is dominantly imaginary. In contrast, the Odderon amplitude is dominantly real both in QCD and $\mathcal{N} = 4$ SYM, and in this sense the Odderon sector of the AdS/CFT correspondence is closer to QCD than the Pomeron sector. Turning to phenomenology, in practice there is no urgent need for unitarity corrections to the Odderon, since its intercept $\alpha_O \leq 1$ does not violate any constraints from unitarity. Indeed, most of the recent phenomenological applications [3–11] are more or less based on models inspired by the single Odderon exchange.[10] In view of these circumstances, we retain the hope that the qualitative features of the single Odderon exchange can survive and give useful information to QCD.

Backed by these observations, we now come to the implications of our results to experiments. The proton belongs to the **8** of the flavor $SU(3)$ group which is not a singlet

[9] It is easy to see that the b–integral will not modify the intercept in the Reggeon case or in the transverse case $B_{\pm\perp}$ due to the absence of the $v \sim b^2$ factor in the tensorial part of the propagator.

[10] In perturbative QCD, unitarity corrections to the BLV Odderon have been included only in asymmetric collisions where one of the hadrons is small and perturbative [22, 23, 28].

under any subgroup of $SU(3)$. Therefore, it should couple to both the Reggeon and the Odderon. Then we find the following scenario rather compelling: The ISR data show that $\Delta\sigma > 0$ in the hitherto explored energy regime. This is quite naturally attributed to the $\alpha_R(0) = 1 - 9/2\sqrt{\lambda}$ Reggeon, whereas in the Odderon sector there must be some cancellation of the sort mentioned above. However, the Reggeon contribution $s^{\alpha_R(0)-1}$ dies away quickly as the energy increases. On the other hand, we see that certain components of the Odderon have much milder energy dependences. Then the Odderon contribution inevitably takes over and $\Delta\sigma$ must turn negative. As mentioned in the introduction, the possiblity of having $\Delta\sigma < 0$ was already raised in [1], and more recently also in [9] on the basis of an extrapolation of a purely phenomenological fit. It is quite remarkable that the AdS/CFT correspondence allows for an *analytical* evaluation of the cross section difference which leads to the same conclusion. Incidentally, it is amusing to notice that, if one uses $\sqrt{\lambda} = 7 \sim 8$ for the 't Hooft coupling which in QCD ($N_c = 3$) would correspond to a typical strong coupling regime $\alpha_s = 1 \sim 2$, one obtains $\alpha_R^{QCD}(0) = 0.4 \sim 0.5$ in rough accordance with the known phenomenological value. Note also that at the ISR energy the same estimate gives $\tau/\sqrt{\lambda} \approx 1$, suggesting that the Odderon contribution is not yet dominant in this regime.

Admittedly, the above scenario sidesteps many perils in naively translating the results for $\mathcal{N} = 4$ SYM to those for QCD. However, to the extent that we believe in the potential of the AdS/CFT correspondence to shed light on the otherwise inaccessible regime of gauge theories, we propose it as a very interesting and testable possibility that carries an imprint of string theory in a curved background.

Unfortunately, experiments of *both* pp and $p\bar{p}$ collisions at similar, and high energies (beyond ISR) are lacking. $p\bar{p}$ collisions are currently not planned in the ongoing program at the LHC, while there are no pp data at the Tevatron. We hope this situation will change in the future, as we expect that the question on the fate of the Odderon will most likely be possible to answer only when data from new experiments are available. One possibility is the planned pp collisions at RHIC at around $\sqrt{s} = 500$ GeV which, though not at very high energy, would still be welcome.

There are several directions for further study. As suggested already, one can employ more realistic baryon configurations (see, e.g., [43]) which have fluxes in various directions and/or a nontrivial extension in the transverse direction. Then the modes $b_\pm$ and $B_{\pm\perp}$ will come into play. Moreover, the calculation should be extended to observables other than the total cross section difference. There are a number of processes where the Odderon, or more generally, the C–odd exchanges are involved [4–11], many of which concern the real part of the amplitude. It would be interesting to revisit these studies with inputs from the AdS/CFT correspondence. Finally, one would like to understand the difference in the total cross sections in terms of the final states, especially in the Odderon exchange channel where the relevant particle production mechanism must be such that it gives a negative contribution to $\Delta\sigma$. [See, e.g., [44, 45] for studies of final states with emphasis on the difference between pp and $p\bar{p}$ collisions.] This might be difficult to test in the total cross section measurement since the Odderon contribution is a small fraction, but perhaps there are ways to see it in less inclusive reactions.

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A. Spherical harmonics on S^5

In this appendix we review the basics of the scalar and vector spherical harmonics which appear in the decomposition (2.6). (See also [46].)

A.1 Scalar spherical harmonics

We parameterize the coordinates of $S^5 \in \mathbb{R}^6$ as

$$\begin{aligned} y^1 &= \cos\theta\,, \\ y^2 &= \sin\theta\cos\theta_2\,, \\ y^3 &= \sin\theta\sin\theta_2\cos\theta_3\,, \\ y^4 &= \sin\theta\sin\theta_2\sin\theta_3\cos\theta_4\,, \\ y^5 &= \sin\theta\sin\theta_2\sin\theta_3\sin\theta_4\cos\theta_5\,, \\ y^6 &= \sin\theta\sin\theta_2\sin\theta_3\sin\theta_4\sin\theta_5\,. \end{aligned} \tag{A.1}$$

The volume element is

$$d\Omega_5 = \sin^4\theta\sin^3\theta_2\sin^2\theta_3\sin\theta_4 d\theta d\theta_2 d\theta_3 d\theta_4 d\theta_5 = \sin^4\theta d\theta d\Omega_4\,. \tag{A.2}$$

The rank–k scalar spherical harmonics are defined as

$$Y^{(k)} = \sum C_{i_1,i_2,..i_k} y^{i_1} y^{i_2} \cdots y^{i_k} \tag{A.3}$$

where indices i runs from 1 to 6 and the tensor C is totally symmetric and traceless with respect to any pair of indices. They form a representation of $SO(6)$. Using the Dynkin label of $SU(4) \cong SO(6)$, it is

$$(0, k, 0)\,, \tag{A.4}$$

whose dimension is

$$d = \frac{(k+3)(k+2)^2(k+1)}{12}\,. \tag{A.5}$$

Because of the traceless condition, one has that

$$\int_{S^5} Y^{(k)} = 0\,. \tag{A.6}$$

For $k = 1$, there are six harmonics which are just the six coordinates $y^1, y^2, .., y^6$. Note that $Y^{(1)} \propto y^1 = \cos\theta$ is independent of the S^4 coordinates, so it gives a nonzero value after integration over S^4 as in (3.3).

For $k = 2$, the allowed tensor is, for example,

$$C_{ij} = \delta_{i1}\delta_{j2} + \delta_{i2}\delta_{j1}\,, \tag{A.7}$$

which gives

$$Y^{(2)} \propto y^1 y^2\,. \tag{A.8}$$

This vanishes upon integration over S^4. Another possibility is

$$C_{ij} = \delta_{i1}\delta_{j1} - \frac{1}{6}\delta_{ij}\,, \tag{A.9}$$

so that

$$Y^{(2)} \propto \cos^2\theta - \frac{1}{6}\,. \tag{A.10}$$

This is nonzero after integrating over S^4. For any value of k there is at least one spherical harmonics which does not vanish after integrated over S^4.

A.2 Vector spherical harmonics

The rank-k vector spherical harmonics are defined by

$$Y_i^{(k)} = \sum C^i_{i_1,i_2,..i_k} y^{i_1} y^{i_2} \cdots y^{i_k} \tag{A.11}$$

with the conditions that

$$\partial_i Y_i = y_i Y_i = 0\,. \tag{A.12}$$

Projecting on S^5, one finds

$$Y_\alpha^{(k)} = \frac{\partial y^i}{\partial \theta^\alpha} Y_i^{(k)}\,. \tag{A.13}$$

Useful identities are

$$\nabla^\alpha Y_\alpha^{(k)} = 0\,, \qquad \nabla^2 Y_\alpha^{(k)} = -\big(k(k+4) - 1\big) Y_\alpha^{(k)}\,. \tag{A.14}$$

One sees that a vector of the form (A.11) is a direct product of the fundamental representation of $SO(6)$ and the totally symmetric rank–k tensor. Using the Dynkin label, one has the decomposition

$$(0,1,0) \otimes (0,k,0) = (0,k+1,0) \oplus (1,k-1,1) \oplus (0,k-1,0)\,. \tag{A.15}$$

$(1, k-1, 1)$ is the rank–k vector spherical harmonics whose dimension is

$$d = \frac{k(k+2)^2(k+4)}{3}\,. \tag{A.16}$$

For $k = 1$, $d = 15$ which is the number of the Killing vectors. The conditions (A.12) become

$$y^i C^i_j y^j = C^i_i = 0\,. \tag{A.17}$$

The solution is, for example,

$$C^i_j = \delta_{i1}\delta_{j2} - \delta_{i2}\delta_{j1}\,, \tag{A.18}$$

so that

$$Y^{(1)}_\alpha \propto y_2 \partial_\alpha y_1 - y_1 \partial_\alpha y_2\,. \tag{A.19}$$

This is indeed a Killing vector. It is easy to show that

$$\int_{S^4} Y^{(1)}_\theta = 0\,, \tag{A.20}$$

for all the 15 Killing vectors.

For $k = 2$, we have 64 harmonics which appears in the decomposition

$$6 \otimes 20 = 50 \oplus 64 \oplus 6\,. \tag{A.21}$$

They can be constructed as follows. Starting from an arbitrary tensor A^i_{jk} which is symmetric and traceless in j and k, one has the decomposition

$$\begin{aligned} A^i_{jk} = &\frac{1}{3}\left(A^i_{jk} + A^j_{ik} + A^k_{ij} - \frac{\delta_{ij}}{4}A^l_{lk} - \frac{\delta_{ik}}{4}A^l_{lj} - \frac{\delta_{jk}}{4}A^l_{li}\right) \\ &+ \frac{1}{3}\left(2A^i_{jk} - A^j_{ik} - A^k_{ij}\right) - \frac{1}{15}\left(\delta_{ij}A^l_{lk} + \delta_{ik}A^l_{lj} - 2\delta_{jk}A^l_{li}\right) \\ &+ \frac{1}{20}\left(3\delta_{ij}A^l_{lk} + 3\delta_{ik}A^l_{lj} - \delta_{jk}A^l_{li}\right)\,. \end{aligned} \tag{A.22}$$

The first line is totally symmetric and traceless in ijk, so this belongs to the $k = 3$ scalar spherical harmonics $(0, 3, 0)$. Projecting on S^5, it becomes $\nabla_\alpha Y^{(3)}$. The last line is $(0, 1, 0)$ which becomes $\nabla_\alpha Y^{(1)}$ on S^5.

The second line is the $k = 2$ vector spherical harmonics

$$C^i_{jk} = \frac{1}{3}\left(2A^i_{jk} - A^j_{ik} - A^k_{ij}\right) - \frac{1}{15}\left(\delta_{ij}A^l_{lk} + \delta_{ik}A^l_{lj} - 2\delta_{jk}A^l_{li}\right)\,, \tag{A.23}$$

constructed such that it satisfies (A.12) which reads

$$C^i_{jk} y^i y^j y^k = 0, \qquad C^i_{ij} y^j = 0\,. \tag{A.24}$$

We are now ready to prove that

$$\int_{S^4} Y^{(2)}_\theta = 0\,. \tag{A.25}$$

Using the notation $y^a = y^{2,3,4,5,6}$, we have

$$\begin{aligned} Y_\theta^{(2)} &= \frac{\partial y^i}{\partial \theta} C^i_{jk} y^j y^k = \frac{\partial y^1}{\partial \theta} C^1_{jk} y^j y^k + \frac{\partial y^a}{\partial \theta} C^a_{jk} y^j y^k \\ &= -\sin\theta C^1_{jk} y^j y^k + \frac{\cos\theta}{\sin\theta} y^a C^a_{jk} y^j y^k = \left(-\sin\theta - \frac{\cos^2\theta}{\sin\theta}\right) C^1_{jk} y^j y^k \,, \end{aligned} \tag{A.26}$$

where in the last equality we used the first condition of (A.24). The last term can be written as

$$C^1_{jk} y^j y^k = \cos^2\theta C^1_{11} + 2\cos\theta\sin\theta C^1_{1a}\hat{y}^a + \sin^2\theta C^1_{ab}\hat{y}^a\hat{y}^b \,, \tag{A.27}$$

where we have denoted $y^a = \sin\theta\, \hat{y}^a$. The first term is zero because $C^1_{11} = 0$ as is evident from (A.23). The second term is $k = 1$ scalar spherical harmonics on S^4, so it vanishes after integrating over S^4. The last term gives, after S^4 integration,

$$\int C^1_{ab}\hat{y}^a\hat{y}^b \propto C^1_{ab}\delta_{ab} = C^1_{aa} \,. \tag{A.28}$$

However, this is zero because of the traceless condition $C^1_{jj} = C^1_{11} + C^1_{aa} = 0$. Thus we have proven that the $k = 2$ vector spherical harmonics do not couple to the D–brane.

For the $k = 3$ vector spherical harmonics, we have similarly,

$$Y_\theta^{(3)} \sim C^1_{111}\cos^3\theta + 3\cos^2\theta\sin\theta C^1_{11a}\hat{y}^a + 3\cos\theta\sin^2\theta C^1_{1ab}\hat{y}^a\hat{y}^b + \sin^3\theta C^1_{abc}\hat{y}^a\hat{y}^b\hat{y}^c \,.$$

The second and the fourth terms give zero after integrating over S^4. The third term gives C^1_{1aa}. This is zero if $C^1_{111} = 0$ (because of the traceless condition $C^1_{1jj} = 0$), in which case the first term also vanishes. To see this is indeed the case, note that (A.12) reads

$$0 = C^i_{jkl} y^i y^j y^k y^l = \cos^4\theta C^1_{111} + \cos^3\theta\sin\theta(3C^1_{11a} + C^a_{111})\hat{y}^a + \cdots \,. \tag{A.29}$$

In order for this to hold identically, one must have that $C^1_{111} = 0$, $3C^1_{11a} + C^a_{111} = 0$, and so on.

The situation is similar for higher k values. First one has $C^1_{1111\cdots} = 0$. After integrating over S^4, one gets terms like

$$C^1_{1aabbcc\ldots} \,, \quad C^1_{111aabb\ldots} \,, \tag{A.30}$$

for k odd, and

$$C^1_{aabbcc\ldots} \,, \quad C^1_{11aabb\ldots} \,, \tag{A.31}$$

for k even. These are all zero because, for instance,

$$C^1_{1aabbcc} = -C^1_{111bbcc} = C^1_{11111cc} = -C^1_{1111111} = 0 \,. \tag{A.32}$$

Thus we have proven that the integral (3.4) vanishes for all k.

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