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ON FOUR-MANIFOLDS FIBERING OVER SURFACES

By

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Abstract. We study closed connected topological or smooth 4-manifolds fibering over a surface in terms of classifying spaces, characteristic classes, and intersection forms.

1. Introduction.

Let F and X be closed connected oriented surfaces of genus h and g , respectively. We are going to study closed connected 4-manifolds M which admit a fibration

$$(1) \quad F \longrightarrow M \xrightarrow{\pi} X$$

with base X and fiber F .

It was shown by Meyer [11] that for a fixed $h \geq 3$ any integer $4m \in \mathbb{Z}$ may appear as signature of such a manifold M . So these manifolds provide an interesting class of 4-manifolds (see also [1] and [2] for related examples).

More recently Hillman ([5] and [6]) has proved that the necessary conditions:

$$\chi(M') = \chi(X)\chi(F)$$

and

$$\Pi_1(M') \text{ is an extension of } \Pi_1(F) \text{ by } \Pi_1(X)$$

are sufficient for the closed 4-manifold M' is homotopy equivalent to a 4-manifold M with fibration structure (1). Here χ denotes the Euler characteristic as

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usual. Moreover, if the homotopy equivalence is simple, then M' and M are topologically s -cobordant.

In this paper, we are going to study the problem of uniqueness of the fibration structure (1). If for example $M = X \times F$, then there are at least two such structures. But even if we fix the base X and the fiber F , the following question arises:

Does M admit non-isomorphic fibrations

$$F \xrightarrow{i} M \xrightarrow{\pi} X \quad \text{and} \quad F \xrightarrow{i'} M \xrightarrow{\pi'} X?$$

We are trying to construct invariants of such fibrations which are not invariants of the manifold M . A complete set of such invariants will give a classification of closed 4-manifolds fibering as in (1) in terms of homotopy classes of maps from X to certain classifying spaces described below. As a general reference on the algebraic theory of closed 4-manifolds see for example [6]. For a standard text on differential topology of fiber bundles we refer to [8].

2. The description in terms of classifying spaces.

Let X and F be given as above. We fix an orientation on $F = F_h$, where h is the genus of F .

Let

$$G = \begin{cases} \text{Aut}^+(F) & \text{with the compact-open topology} \\ \text{Diff}^+(F) & \text{with the } C^\infty\text{-topology,} \end{cases}$$

where $\text{Aut}^+(F)$ (resp. $\text{Diff}^+(F)$) is the group of orientation preserving homeomorphisms (resp. diffeomorphisms) of F .

Any fiber bundle (1), $F \rightarrow M \rightarrow X$, is classified by a map $f : X \rightarrow BG$. Isomorphism classes of bundles (1) correspond bijectively to homotopy classes of maps $X \rightarrow BG$, i.e. to elements of $[X, BG]$.

The component $G_0 \subset G$ of the identity is (weakly) contractible if $h \geq 2$ (see [3] and [4]). Classical results, due to Nielsen, Dehn, and Birman, imply the following canonical group isomorphisms:

$$G/G_0 \cong \text{Aut}^+(\Pi_1(F_h))/\text{Inn}(\Pi_1(F_h)) \cong E^+(F_h),$$

where

$$E^+(F_h) = \{[\varphi] \in [F_h, F_h] : \varphi \text{ orientation preserving homotopy equivalence of } F_h\}.$$

Note that $\Gamma_h := E^+(F_h)$ is just the *Teichmüller group* of F_h .

It follows that G/G_0 is a discrete group. Since $G_0 \simeq \{*\}$ (at least weakly), the fibration

$$G/G_0 \longrightarrow BG_0 \longrightarrow BG$$

implies that $BG \simeq K(\Gamma_h, 1) = B\Gamma_h$.

Assuming that the genus of X is ≥ 2 , i.e. $X = K(\Pi_1(X), 1) = B\Pi_1(X)$, we obtain

$$[X, BG] \xrightarrow{\cong} [B\Pi_1(X), B\Gamma_h] \cong \text{Hom}(\Pi_1(X), \Gamma_h)/\Gamma_h,$$

where Γ_h acts on $\text{Hom}(\Pi_1(X), \Gamma_h)$ by setting $(\gamma\alpha)(c) = \gamma\alpha(c)\gamma^{-1}$ for any $\gamma \in \Gamma_h$, $\alpha \in \text{Hom}(\Pi_1(X), \Gamma_h)$, and $c \in \Pi_1(X)$. The last isomorphism holds for discrete groups $\Pi = \Pi_1(X)$ and $\Gamma = \Gamma_h$. Under this isomorphism the classifying map $f : X \rightarrow BG$ of the fibration (1) goes to the induced homomorphism

$$f_* : \Pi_1(X) \rightarrow \Pi_1(BG) \cong \Pi_0(G/G_0) \cong \Gamma_h.$$

In particular, we note that the obvious map

$$(2) \quad \text{Diff}^+(F) \rightarrow E^+(F)$$

is a homotopy equivalence (see [4]).

On the other hand, any fibration $F \rightarrow M \rightarrow X$ defines an element in

$$\text{Ext}(\Pi_1(F), \Pi_1(X))$$

by the sequence:

$$1 \longrightarrow \Pi_1(F) \longrightarrow \Pi_1(M) \longrightarrow \Pi_1(X) \longrightarrow 1.$$

Conversely, given

$$[1 \rightarrow \Pi_1(F) \rightarrow \Pi \rightarrow \Pi_1(X) \rightarrow 1] \in \text{Ext}(\Pi_1(F), \Pi_1(X)),$$

where $h = \text{genus } F \geq 2$ and $g = \text{genus } X \geq 2$, we obtain a homotopy fibration $F \rightarrow B\Pi \rightarrow X$ and a classifying map $f_1 : X \rightarrow BE^+(F)$. Using (2), we find a unique DIFF fiber bundle (up to bundle isomorphism) $F \rightarrow E \rightarrow X$ inducing the sequence:

$$1 \longrightarrow \Pi_1(F) \longrightarrow \Pi_1(E) = \Pi \longrightarrow \Pi_1(X) \longrightarrow 1.$$

Let $f : X \rightarrow BG$ be its classifying map. Now it is well-known that the set of extensions

$$[1 \rightarrow \Pi_1(F) \rightarrow \Pi \rightarrow \Pi_1(X) \rightarrow 1] \in \text{Ext}(\Pi_1(F), \Pi_1(X)),$$

inducing the same homomorphism $f_* : \Pi_1(X) \rightarrow \Pi_1(BG) \cong \Gamma_h$, is isomorphic to $H^2(X; \zeta\Pi_1(F))$, where $\zeta\Pi_1(F)$ denotes the center of $\Pi_1(F)$ (see [10], p. 128). But $\zeta\Pi_1(F) = \{1\}$ if $h \geq 3$. Hence we conclude that the fibration structure $F \rightarrow M \rightarrow X$ is uniquely defined by the element

$$(3) \quad [1 \rightarrow \Pi_1(F) \rightarrow \Pi \rightarrow \Pi_1(X) \rightarrow 1] \in \text{Ext}(\Pi_1(F), \Pi_1(X))$$

if $h = \text{genus } F \geq 3$.

In this case our question is equivalent to:

In how many elements of $\text{Ext}(\Pi_1(F), \Pi_1(X))$ can a given group Π occur (as in (3))?

As pointed out by Hillman in [7], a closely related question was considered by Johnson in [9] from a purely algebraic point of view. He proved that if the homomorphism f_* is not injective but has infinite image, then the extension is unique; if f_* has finite image, there are at most two distinct extension structures, and that there are such groups Π with two extension structures. If Π has one extension structure with f_* injective, then all have this property, but he does not settle the question completely in this case. Note also that $\text{Wh}(\Pi) = 0$ in all cases (see for example [6], V.1, p. 68).

3. Characteristic classes.

Let $F \rightarrow M \xrightarrow{\pi} X$ be given as in Section 1. Let $\xi \subset TM$ be the subbundle of vertical vectors, i.e. vectors tangent to the fibers. We assume that $\xi \rightarrow M$ is an orientable bundle. Such fibrations $F \rightarrow M \rightarrow X$ are called *orientable* in [12] and [13]. Let $e(\xi) \in H^2(M; \mathbb{Z})$ be the Euler class of ξ . Note that $e(M) = e(\xi)\pi^*(e(X))$, where $e(M) \in H^4(M; \mathbb{Z})$ and $e(X) \in H^2(X; \mathbb{Z})$ denote the Euler classes of M and X , respectively.

Following [12], we define $e_1(\xi) = \mathcal{G}_*(e(\xi)^2) \in H^2(X; \mathbb{Z})$, where \mathcal{G}_* is the Gysin homomorphism

$$\begin{array}{ccc} H^4(M; \mathbb{Z}) & \xrightarrow{\mathcal{G}_*} & H^2(X; \mathbb{Z}) \\ \text{PD} \downarrow \cong & & \cong \downarrow \text{PD} \\ H_0(M; \mathbb{Z}) & \xrightarrow{\pi_*} & H_0(X; \mathbb{Z}). \end{array}$$

It is clear that the higher classes $e_j = \mathcal{G}_*(e^{j+1}) = 0$ in our case. However there is another characteristic class. The classifying map $f_* : \Pi_1(X) \rightarrow \Gamma_h$ composes with

$\sigma_* : \Gamma_h \rightarrow \text{Sp}(2h; \mathbf{Z})$, which is induced by the action of

$$\Gamma_h = \text{Aut}(\Pi_1(F))/\text{Inn}(\Pi_1(F))$$

on $H^1(F; \mathbf{Z})$. The composition of the maps

$$(4) \quad \Pi_1(X) \rightarrow \Gamma_h \rightarrow \text{Sp}(2h; \mathbf{Z}) \subset \text{Sp}(2h; \mathbf{R})$$

induces $X \rightarrow B\text{Sp}(2h; \mathbf{R})$. Here we continue to assume that the genus of X is ≥ 2 .

Let us consider the bundle η associated to the fiber bundle $F \rightarrow M \rightarrow X$. For every $x \in X$, the fiber of η over x is the real cohomology of the fiber F_x . Since the unitary group $\mathcal{U}(h)$ is a maximal torus in $\text{Sp}(2h; \mathbf{R})$, the structure group of this bundle can be reduced to the unitary group, i.e. η can be considered as a complex vector bundle over X .

Then we have the first Chern class $c_1(\eta) \in H^2(X; \mathbf{Z})$.

Now from [12], p. 555, it follows that $e_1(\xi) = -12c_1(\eta)$.

One of the results proved by Meyer in [11], p. 246, is

$$\text{Sign}(M) = -\langle 4c_1(\eta), [X] \rangle,$$

where $\text{Sign}(M)$ denotes the signature of M .

Furthermore, assuming $h \geq 3$, any class $c_1(\eta)$ can be realized by a fibration $F \rightarrow M \rightarrow X$. Here $h \geq 3$ is fixed, but X may vary.

Summarizing we have

PROPOSITION 3.1. *The characteristic classes $c_1(\eta)$ and $e(\xi)$ of the fibration*

$$F \rightarrow M \rightarrow X$$

are uniquely defined by the signature of M . Moreover, since $\xi \rightarrow M$ is a bundle of dimension two, it is also defined by the signature (because it is defined by $e(\xi)$).

Now we can state the following problem: *define characteristic classes of the fibration $F \rightarrow M \rightarrow X$ using other representation of $\Pi_1(X)$ instead of (4).*

4. The spectral sequence and the intersection form.

The spectral sequence of the fibration $F \xrightarrow{i} M \xrightarrow{\pi} X$ is of the following type:

$$E_2^{p,q} = H^p(X; \overline{H^q(F)}) \Rightarrow H^{p+q}(M; \mathbf{Z}),$$

where $\overline{H^q(F)} = \tilde{X} \times_{\Pi_1(X)} H^q(F; \mathbf{Z})$ is the coefficient system over X induced by the homomorphism $f_* : \Pi_1(X) \rightarrow \text{Sp}(2h; \mathbf{Z})$.

Since we continue to assume that $F \rightarrow M \rightarrow X$ is oriented, we have

$$E_2^{0,2} = H^0(X; \overline{H^2(F)}) \cong H^2(F)^{\Pi_1(X)} \cong H^2(F) \cong \mathbf{Z}$$

and

$$E_2^{p,0} = H^p(X; \overline{H^0(F)}) \cong H^p(X; H^0(F)) \cong H^p(X; \mathbf{Z}).$$

The only possible non-trivial differentials are

$$d_2^{0,2} : E_2^{0,2} \cong \mathbf{Z} \rightarrow E_2^{2,1} \quad \text{and} \quad d_2^{0,1} : E_2^{0,1} \rightarrow E_2^{2,0} \cong H^2(X; \mathbf{Z}).$$

There is the following commutative diagram

$$\begin{array}{ccc} H^2(M; \mathbf{Z}) & \xrightarrow{i^*} & H^2(F; \mathbf{Z}) \cong \mathbf{Z} \\ \text{epi} \downarrow & & \parallel \\ E_\infty^{0,2} & \longrightarrow & E_2^{0,2} \cong \mathbf{Z}. \end{array}$$

Since $i^*(e(\xi)) = e(F_h) \neq 0$, $h \geq 2$, it follows that $E_\infty^{0,2} \cong \mathbf{Z}$, which implies $d_2^{0,2} = 0$.

From this we obtain

$$(5) \quad H^2(M; \mathbf{Z}) \cong H^2(F; \mathbf{Z}) \oplus H^1(X; \overline{H^1(F)}) \oplus E_2^{2,0} / \text{Im } d_2^{0,1}.$$

Since $E_2^{2,0} = H^2(X; \mathbf{Z}) \cong \mathbf{Z}$, there are three possibilities:

$$E_2^{2,0} / \text{Im } d_2^{0,1} \cong \begin{cases} \mathbf{Z} \\ \mathbf{Z}/k\mathbf{Z} \\ 0. \end{cases}$$

Meyer has proved in [11] that

$$\text{Sign}(M) = \text{Sign}(H^1(X; \overline{H^1(F)}), \text{ with respect to the obvious pairing}) = 4m$$

for some integer m . This implies that $\text{rank } H^2(M; \mathbf{Z})$ and $\text{rank } H^1(X; \overline{H^1(F)})$ are even. Then it follows from (5) that $E_2^{2,0} / \text{Im } d_2^{0,1} \cong \mathbf{Z}$, hence $d_2^{0,1} = 0$.

So we have proved the following result.

THEOREM 4.1. *If $F \rightarrow M \rightarrow X$ is an oriented fibration (i.e. the vertical subbundle $\xi \subset TM$ is oriented), then the spectral sequence*

$$E_2^{p,q} = H^p(X; \overline{H^q(F)}) \Rightarrow H^{p+q}(M; \mathbf{Z})$$

collapses. In particular, we have

$$H^1(M; \mathbf{Z}) \cong H^1(F; \mathbf{Z})^{\Pi_1(X)} \oplus H^1(X; \mathbf{Z})$$

and

$$H^2(M; \mathbf{Z}) \cong H^2(F; \mathbf{Z}) \oplus H^1(X; \overline{H^1(F)}) \oplus H^2(X; \mathbf{Z}).$$

We remark that in [12] Morita proved that the spectral sequence of the rational cohomology of any surface bundle collapses.

As a consequence, the homomorphism $\pi^* : H^2(X; \mathbf{Z}) \rightarrow H^2(M; \mathbf{Z})$ is not trivial, and hence the quadratic form on $H^2(M; \mathbf{Z})$ is *indefinite*. Then we must distinguish two cases according to the intersection form is even or odd (in the second case there exists an element $x \in H^2(M; \mathbf{Z})$ such that $x^2 \not\equiv 0 \pmod{2}$).

Let us consider now the second Stiefel-Whitney class

$$w_2(M) = w_2(TM) \in H^2(M; \mathbf{Z}_2).$$

Let $\xi \subset TM$ be as above the subbundle of vectors tangent to the fibers of $M \xrightarrow{\pi} X$. Then we have $TM/\xi \cong \pi^*(TX)$, i.e. $w_2(M) = w_2(\xi) + \pi^*(w_2(X)) = w_2(\xi)$. Since $w_2(\xi) \equiv e(\xi) \pmod{2}$, we conclude with the following implications:

$$\begin{aligned} w_2(M) = 0 &\Rightarrow e(\xi) = 2x \in H^2(M; \mathbf{Z}) \\ &\Rightarrow e_1(\xi) = \mathcal{G}_*(e(\xi)^2) \equiv 0 \pmod{4} \text{ in } H^2(X; \mathbf{Z}) \cong \mathbf{Z} \\ &\Rightarrow c_1(\eta) \equiv 0 \pmod{4} \\ &\Rightarrow \text{Sign}(M) = -4\langle c_1(\eta), [X] \rangle \equiv 0 \pmod{16}. \end{aligned}$$

In particular, one reobtains Rohlin's theorem for the special class of 4-manifolds fibering over surfaces.

THEOREM 4.2. *The closed TOP or DIFF 4-manifolds M considered above satisfy the Rohlin theorem, i.e. $w_2(M) = 0$ implies that $\text{Sign}(M) \equiv 0 \pmod{16}$. In this case the integral intersection form μ_M is always even.*

On the other hand, if $w_2(M) \neq 0$, then $e(\xi)^2 \not\equiv 0 \pmod{2}$, hence the intersection form μ_M is odd, i.e. it is of type:

$$\mu_M \cong (1) \oplus \cdots \oplus (1) \oplus (-1) \oplus \cdots \oplus (-1).$$

Finally, let us calculate the Euler characteristics for oriented fibrations

$$F \rightarrow M \rightarrow X,$$

i.e. ξ is oriented.

We have

$$\chi(M) = \chi(F)\chi(X) = (2 - 2h)(2 - 2g)$$

and

$$\chi(M) = 2 - 2(\text{rank } H^1(F)^{\Pi_1(X)} + 2g) + \text{rank } H^1(X; \overline{H^1(F)}) + 2,$$

hence

$$4gh - 4h = \text{rank } H^1(X; \overline{H^1(F)}) - 2 \text{rank } H^1(F)^{\Pi_1(X)} \geq \text{rank } H^1(X; \overline{H^1(F)}) - 4h.$$

Thus we have the following

PROPOSITION 4.3. $\text{rank } H^1(X; \overline{H^1(F)}) \leq 4gh$ and equality holds if and only if $H^1(F)^{\Pi_1(X)} \cong H^1(F)$, i.e. if and only if the classifying map

$$\Pi_1(X) \xrightarrow{f_*} \Gamma_h \xrightarrow{\sigma_*} \text{Sp}(2h; \mathbf{Z})$$

vanishes.

5. The Pontryagin and Euler classes.

In dimension four the Hirzebruch formula for the signature writes as follows:

$$\text{Sign}(M) = (1/3)\langle p_1(M), [M] \rangle.$$

Now the formula of Meyer [11]

$$\text{Sign}(M) = -4\langle c_1(\eta), [X] \rangle$$

and the above calculated relations

$$\mathcal{G}_*(e(\xi)^2) = e_1(\xi) = -12c_1(\eta)$$

give

$$\text{Sign}(M) = (1/3)\langle e_1(\xi), [X] \rangle.$$

The commutative diagram in Section 3 implies that

$$\pi_*\langle e(\xi)^2, [M] \rangle = \langle e_1(\xi), [X] \rangle,$$

where $\pi_* : H_0(M; \mathbf{Z}) \xrightarrow{\cong} H_0(X; \mathbf{Z})$.

But π_* = identity via the identification $H_0(M; \mathbf{Z}) \cong \mathbf{Z} \cong H_0(X; \mathbf{Z})$, hence

$$(1/3)\langle e(\xi)^2, [M] \rangle = \text{Sign}(M) = (1/3)\langle p_1(M), [M] \rangle,$$

i.e.

$$p_1(M) = e(\xi)^2.$$

This is a well known relation between p_1 and e^2 .

For the Euler class we have

$$e(M) = e(\xi)\pi^*(e(X)).$$

Now recall the product $BSO(4) = BSO(3) \times BSU(2)$ induced by the fibration (which has a section):

$$SO(3) \longrightarrow SO(4) \longrightarrow S^3 = SU(2).$$

Hence we have $p_1 \in H^4(BSO(3); \mathbb{Z}) \cong \mathbb{Z}$ and $H^4(BSO(4); \mathbb{Z}) \cong \mathbb{Z}[p_1] \oplus \mathbb{Z}[e]$, so $p_1(M)$ and $e(M)$ determine the tangent bundle TM .

REMARK 1: If $e(\xi) = 0$, then $p_1(M) = e(M) = 0$. Since $i^*(e(\xi)) = e(\xi|_F) = 0$, it follows that $h = 1$, i.e. $F = S^1 \times S^1$. In this case we have a map

$$\Pi_1(X) \rightarrow \Gamma_1 = SL(2; \mathbb{Z}) = Sp(2; \mathbb{Z}).$$

Assume that the composition $\Pi_1(X) \rightarrow \Gamma_1 = SL(2; \mathbb{Z}) \rightarrow SL(2; \mathbb{R})$ induces the constant map. Does it follow that $\Pi_1(X) \rightarrow SL(2; \mathbb{Z})$ is trivial? (in other words: Is then $M = X \times F$?). As remarked by Hillman in [7], M need not be a product. Let N be an orientable S^1 -bundle over the torus T with nonzero Euler class. Assume that $\Pi_1(T)$ acts trivially on the fibre. Then $N \times S^1$ is a T -bundle over T of the requested type.

REMARK 2: It is well-known that an aspherical 4-manifold M which fibres over a surface admits the geometry $H^2 \times H^2$ if and only if the map $f_* : \Pi_1(M) \rightarrow \Pi_1(BG)$ (our notation in Section 2) has finite image (see for example [6]).

OPEN PROBLEM (J. A. Hillman) Find examples of aspherical surface bundles over surfaces which admit one of the geometries H^4 or $H^2(\mathbb{C})$ (note that f_* must be injective in these cases).

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