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Dirac Monopole and Spin Hall Conductance for Anisotropic Superconductivities

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Abstract

Concept of the topological order is useful to characterize anisotropic superconductivities. The spin Hall conductance distinguishes superconductivities with the same symmetry. The Chern number for the spin Hall conductance is given by the covering degree of a closed surface around the Dirac monopole. It gives a clear illustration for the on-critical condition for the $d_{x^2-y^2}$ supreconductivitiy with non zero chemical potential. Non trivial topological orders on the triangular lattice are also presented and demonstrated by the topological objects.

Key words: Spin Hall Conductance; Topological Invariant; Dirac Monopole; The Chern number

Topological quantum phase transition is conceptually interesting since it is a zero temperature phase transition without symmetry breaking[1]. It is associated with a change of topological characterization for the quantum mechanical ground state. One of the widely established examples is a quantum Hall plateau transition between different Hall states. Symmetries are the same but only the Hall conductances are different among them. The Hall conductance has an intrinsic topological character and the plateau transition is specified by the change of such topological objects. Typical realizations of them are the Chern numbers, vortices and edges states[2,3]. Also special form of a selection rule is expected and the possible stability of the phase which puts special importance on the topological transitions discriminates from other quantum phase transitions.

Recently, there have been trials to extend the concept of this topological phase transition to unconventional singlet superconductivity.[6,7]The superconducting system is mapped into the Hall system and analogy between them is used to characterize the superconducting ground state. Here the "spin" Hall conductance plays a main role which is given by the Chern numbers. Recently proposed superconductivity with time-reversal symmetry breaking is a non trivial example with the topological order. [5,4,6,7].

The topological phase transition in superconductivity raises important theoretical questions how the topological character restricts transition types. Starting from the generalized lattice Bogoliuvov-de Gennes (B-dG) hamiltonian for superconductivity, we discuss the topological quantum phase transition and demonstrate the topological character[8]. In this paper how the gap opening and the topological objects are related is focused. Extension to the triangular lattice which is the simplest system with frustration is also discussed.

We start from the following lattice B-dG hamiltonian for superconducting quasi-particles $H = \sum_{ij}(t_{ij}c^\dagger_{i\sigma}c_{j\sigma} + \Delta_{ij}c^\dagger_{i\uparrow}c^\dagger_{j\downarrow} + \Delta^*_{ij}c_{j\downarrow}c_{i\uparrow}) - \mu\sum_{i\sigma}c^\dagger_{i\sigma}c_{i\sigma}$. We assume the system is translational invariant as $t_{ij} = t_{i-j}$, $\Delta_{ij} = \Delta_{i-j}$ and further require $t_{ij} = t_{ji}$ ($\epsilon(\boldsymbol{k}) = \epsilon(-\boldsymbol{k})$). Then $H = \sum_k \boldsymbol{c}^\dagger(\boldsymbol{k})\boldsymbol{h}(\boldsymbol{k})\boldsymbol{c}$ with $\boldsymbol{h}(\boldsymbol{k}) = \boldsymbol{R}(\boldsymbol{k})\cdot\boldsymbol{\sigma}$ where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices and $\boldsymbol{R} = \boldsymbol{R}(\boldsymbol{k}) =$

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$(R_x, R_y, R_z) = (\mathrm{Re}\,\Delta(\boldsymbol{k}), -\mathrm{Im}\,\Delta(\boldsymbol{k}), \epsilon(\boldsymbol{k}))$, where $\boldsymbol{c}^\dagger(\boldsymbol{k}) = (c^\dagger_\uparrow(\boldsymbol{k}), c_\downarrow(\boldsymbol{k}))$, $\epsilon(\boldsymbol{k}) = \sum_j e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_j} t_j - \mu$ and $\Delta(\boldsymbol{k}) = \sum_j e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_j}\Delta_j$. The "spin" Hall conductance of the superconducting state on a lattice is given by the generalized Thouless-Kohmoto-Nightingale-den Nijs (TKNN) formula as $\sigma = -\frac{e^2}{h}C = \frac{1}{2\pi i}\int_{T^2} d\boldsymbol{S}_k \cdot \mathrm{rot}_k\,\boldsymbol{A}_k = \int_{T^2} dk_x \wedge dk_y (\langle\partial_x\boldsymbol{k}|\partial_y\boldsymbol{k}\rangle - \langle\partial_y\boldsymbol{k}|\partial_x\boldsymbol{k}\rangle)$ where $\boldsymbol{A}_k = \langle\boldsymbol{k}|\boldsymbol{\nabla}_k\boldsymbol{k}\rangle$, $\boldsymbol{h}(\boldsymbol{k})|\boldsymbol{k}\rangle = -E(\boldsymbol{k})|\boldsymbol{k}\rangle$, $E(\boldsymbol{k}) = \sqrt{\epsilon(\boldsymbol{k})^2 + |\Delta(\boldsymbol{k})|^2}$ [2,3,7].This expression for the Chern number is rewritten in $\boldsymbol{R}$ space as $C = \frac{1}{2\pi i}\int_{R(T^2)} d\boldsymbol{S}_R \cdot \mathrm{rot}_R\,\boldsymbol{A}_R$ where $\boldsymbol{A}_R = \langle\boldsymbol{R}|\boldsymbol{\nabla}_R\boldsymbol{R}\rangle$. [8] In a particular gauge, $|\boldsymbol{R}\rangle = {}^t(-\sin\frac{\theta}{2}e^{i\phi}\cos\frac{\theta}{2})$ and $\boldsymbol{A}_R = i\frac{\sin\theta}{2R(1-\cos\theta)}\hat{e}_\phi$ where (R,θ,ϕ) is a polar coordinate of $\boldsymbol{R}$. The integral region $R(T^2)$ is a closed surface in three dimensional parameter space mapped from the Brillouin zone T^2. The corresponding vector potential defines a magnetic monopole at the origin as $\mathrm{div}\,\mathrm{rot}\,\boldsymbol{A}_R = -2\pi i\,\delta_R(\boldsymbol{R})$ [9]. Therefore, by the Gauss' theorem, we have another expression for the Chern number as

$$C = -\int_{R(T^2)} dV\delta_R(\boldsymbol{R}) = -N(R(T^2), O) = -N_{covering}$$

where $N_{covering} = N(R(T^2), O)$ is a degree of covering by the closed surface $R(T^2)$ around the origin O [8].

We use the above topological expression to discuss the gap closing (on-critical) condition for the order parameters. We assume a form of the order parameter used in ref[8]. It tells $\epsilon(\boldsymbol{k}) = -2t(\cos k_x + \cos k_y) - \mu$, $\Delta(\boldsymbol{k}) = \Delta_0 + 2\Delta_{x^2-y^2}(\cos k_x - \cos k_y) + 2i\Delta_{xy}(\cos(k_x + k_y) - \cos(k_x - k_y))$. In the singlet case, the monopole, is doubly covered since $\boldsymbol{R}(\boldsymbol{k}) = \boldsymbol{R}(-\boldsymbol{k})$ which implies the selection rule $\Delta C = \pm 2$[7]. One of the interesting cases is given by the $\Delta_{xy} = 0$. In this situation, the energy gap collapses. Then due to the gapless Dirac dispersion, one can not determine the Chern number without ambiguity (when $|\mu|$ is small). This situation is clear by the present demonstration since the surface $R(T^2)$ is collapsed into a diamond shaped two dimensional region (See Fig.1). When the Dirac monopole O is on this region, the Chern number is ill-defined since the dispersion is gapless. This condition is clearly given by $|\mu| \leq 2t$. Otherwise, there is an energy gap and the Chern number is zero.

Another interesting situation can be on the triangular lattice. Starting from the t-J model on the triangular lattice which is a typical correlated system with frustration, we get the B-dG equation on the triangular lattice. We take mean field ansatz, as the hopping are non zero for $t_{i,i+\hat{x}} = t_{i+\hat{x},i+\hat{y}} = t_{i,i+\hat{y}} = t$ and a possible order parameters for the superconductivity on the triangular lattice as $\Delta_{i,i+\hat{x}} = e^{i2\pi/3}\Delta_{i+\hat{x},i+\hat{y}} = e^{i4\pi/3}\Delta_{i,i+\hat{y}} = \Delta$. Then $\epsilon(\boldsymbol{k}) = -2t(\cos k_x + \cos k_y + \cos(k_x - k_y))$ and $\Delta(\boldsymbol{k}) = 2\Delta(\cos k_x + e^{i2\pi/3}\cos k_y + e^{i4\pi/3}\cos(k_x - k_y))$. It clearly shows that the covering degree of mapping and the intersection number is 2 (The Chern number is 2). It suggests there may exist a non trivial topological order on the possible superconductivity on the triangular lattice. (Detailed discussions will be given elsewhere.)

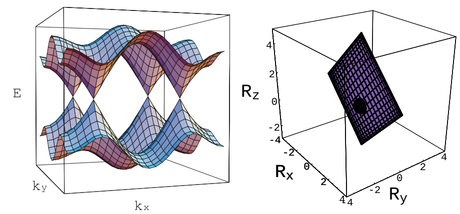

Fig. 1. The energy dispersion and the mapped surface $R(T^2)$ with Dirac monopole. ($t = \Delta_{x^2-y^2} = 1, \mu = -t$)

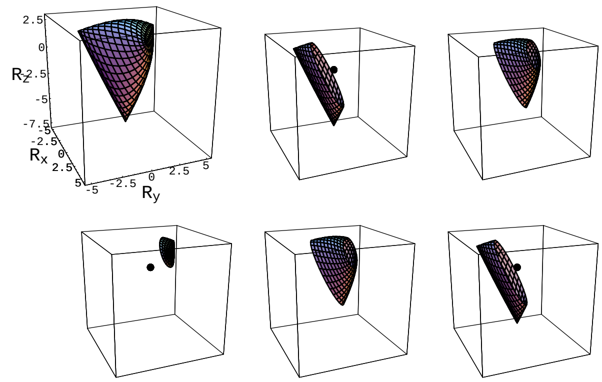

Fig. 2. Mapped Brillouin zone $R(T^2)$ for the triangular lattice. The monopole at the origin O is also shown. (a) is the total surface $R(T^2)$, $k_x \in (0, 2\pi]$, $k_y \in (0, 2\pi]$ and (b)-(f) are parts of the surface to show how it covers the origin. $\Delta_0 = 0$, $\Delta_x = \Delta_y = \Delta_{xy} = t$, $\mu = 0$.

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