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# EFFECTS OF BOUND ELECTRONS ON THE TRANSVERSE WAVES IN A VLASOV PLASMA

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Propagation of transverse waves is studied in a Vlasov plasma consisting of a mixture of free electrons, weakly bound electrons, free static ions and static ions associated with the bound electrons, maintaining macroscopic charge neutrality. The dependence on temperature of the phase velocity, group velocity and Thomson scattering cross-section have been investigated. Lagrangian and the Hamiltonian of a compressible plasma having bound electrons in the fluid mixture approximation have been discussed.

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## 1. Introduction

To find the linear polarization of a medium, in the classical study, electrons are regarded as charged particles harmonically bound to their nuclei [1]. The valence electrons of an atom are bound by the Coulomb field of ionic cores. For large deviation from equilibrium the anharmonicity of the electron oscillators had been used by Rayleigh [2] to explain nonlinearities in acoustic resonators. The classical study of nonlinear distortions in the path of an electron, caused by strong electromagnetic (EM) radiation, was used by Bloembergen [3,4] for analysing the electrons harmonically bound to their respective nuclei and to determine the nonlinear optical properties of atoms in the dielectric medium. The existing fluid and kinetic models for dynamics of plasmas are based on the collective effects of free electrons and bound electrons are not taken into consideration. However, the particle dynamics exists both for free electrons and bound electrons in the presence of an electromagnetic wave. The polarization concepts are important in the classical physics of bound electrons. When the presence of a population of bound electrons in a population of free electrons is taken into account, a different, more realistic response of the plasma to different kinds of perturbations is obtained, than if plasma is regarded as free from the bound electrons [5].

The manuscript is organized in the following way. In Sect. 2, the basic equations of the Vlasov-plasma kinetic theory, including the evolution equations for the bound electrons and free electrons in the phase space of position coordinates, velocity coordinates and time are briefly discussed. The dispersion relation of a plane polarized wave, having frequency $\omega$ and wave number $\vec{k}$, in such a plasma, is determined in Sect. 3. The phase velocity ($v_p$) and the group velocity $v_g$ (energy transport velocity), have been derived in Sect. 4 for the plane EM wave propagation as a function of kinetic temperature and other parameters specifying the plasma and the applied wave field. The temperature dependence of Thomson's total scattering cross-section has been considered in Sect. 5. In Sect. 6, the Lagrangian density and the Hamiltonian density, in the presence of bound electrons and acoustic fields of compressibility of all the plasma species, have been obtained. These yield the equations of momentum transfer with the help of the Euler Lagrangian equations, which follow from the application of the action principle on the Lagrangian density.

## *2. Basic equations of Vlasov plasmas including bound electrons*

In the small amplitude approximation, the field-induced average displacement of a species of particles, per unit volume, at time $t$, from the pre-field position $\vec{r}$ is denoted $\vec{\xi}$. The displacement $\vec{\xi}(\vec{r},t)$ should be finite and small. So, approximately,

$$\dot{\vec{\xi}}(\vec{r},t) = \vec{u}(\vec{r},t)\,, \tag{1}$$

where the dot denotes the partial time derivative and $\vec{u}$ is the field-induced average velocity. The displacement vector $\vec{\xi}(\vec{r},t)$ is considered at par with the other field vectors as a field variable, depending on time and the coordinates of the point of application of the wave in the field-free plasma. The relation is valid separately for each of the four components, so

$$\vec{\xi} = \vec{\xi}_{\rm eb} + \vec{\xi}_{\rm ef} + \vec{\xi}_{\rm ib} + \vec{\xi}_{\rm if}\,, \tag{2}$$

$$\vec{u} = \vec{u}_{\rm eb} + \vec{u}_{\rm ef} + \vec{u}_{\rm ib} + \vec{u}_{\rm if}\,, \tag{3}$$

where the subscript 'eb' stands for the bound electrons, 'ef' for the free electrons, 'ib' for ions of bound electrons and 'if' for ions which have released all the free electrons.

The plasma is taken to be unmagnetised, homogeneous and collision-free, in which electrons are mobile and ions provide a static neutralizing charge background. The self-consistent system of Vlasov equations for the plasma, including the distribution function for the bound electrons, is

$$\partial f_l/\partial t + (\vec{v}\cdot\vec{\nabla}_r)f_l + \left\{(\vec{F}_l/m_l)\cdot\vec{\nabla}_v\right\}f_l = 0\,, \tag{4}$$

$$\vec{\nabla}\times\vec{H} = \frac{1}{c}(\partial\vec{D}/\partial t) + \frac{4\pi e}{c}\iiint\limits_v \vec{v}[f_{\mathrm{if}} + f_{\mathrm{ib}} - f_{\mathrm{ef}} - f_{\mathrm{eb}}]\mathrm{d}\vec{v}\,, \tag{5}$$

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}(\partial\vec{H}/\partial t)\,, \tag{6}$$

$$\vec{\nabla}\cdot\vec{E} = 4\pi e\iiint\limits_v [f_{\mathrm{if}} + f_{\mathrm{ib}} - f_{\mathrm{ef}} - f_{\mathrm{eb}}]\mathrm{d}v\,, \tag{7}$$

$$\vec{\nabla}\cdot\vec{B} = 0\,, \tag{8}$$

where $\vec{F}_l$ is the force per unit mass acting on the different species, $f_l$ is the distribution function of the l-th charged species and l = eb, ef, if or ib. $\vec{E}$, $\vec{B}(\vec{H})$, $\vec{D}$ and $e$ are the electric field, the magnetic field, the electric displacement vector and the charge of the electron, respectively.

This closed system of nonlinear integro-differential equations permits a variety of solutions, which are linear, nonlinear, non-singular as well as singular. Longitudinal-wave solutions would be Landau damped due to singularity at resonances. Here, for simplicity, we consider the propagation in unmagnetised plasmas of a high frequency and transverse wave of very small amplitude.

The distribution function of the charged species, $f_l$, is the sum of the normalized isotropic Maxwell velocity distribution function, $f_l^0(v^2)$, and the space-time dependent perturbed infinitesimally small distribution $f_l^{'}$ [6],

$$f_l(r,v,t) = N_l^0(f_l^0(v^2) + f_l^{'}(z,v,t))\,, \tag{9}$$

where

$$f_l^0(v^2) = (m_l/2\pi K_{\mathrm{B}}T_l)\,\exp(-m_l v^2/2K_{\mathrm{B}}T_l)\,, \tag{10}$$

$|f_l(r,v,t)| >> |f_l^{'}(z,v,t)|$, and $T_l$, $m_l$, $N_l^0$ and $K_{\mathrm{B}}$ are the kinetic temperature, mass, equilibrium number density of the l-th species and the Boltzmann constant, respectively.

The Lorentz force for the bound electrons and free electrons and the binding force of the bound electrons [7,8] are given by

$$\vec{F}_{\rm eb} = -e\vec{E} - e(\dot{\vec{\xi}}_{\rm eb} \times \vec{H})/c - m\,\omega_0^2\vec{\xi}_{\rm eb}\,, \tag{11}$$

$$\vec{F}_{\rm ef} = -e\vec{E} - e(\dot{\vec{\xi}}_{\rm ef} \times \vec{H})/c\,, \tag{12}$$

where $\omega_0$ is the classical or orbital frequency. In the absence of the magnetic field $\vec{H}$, we have

$$\vec{F}_{\rm eb} = -e\vec{E} - m\omega_0^2\vec{\xi}_{\rm eb}\,, \tag{11$a$}$$

$$\vec{F}_{\rm ef} = -e\vec{E}\,, \tag{12$a$}$$

and

$$\vec{D} = 4\pi\vec{\varepsilon}\vec{E}\,. \tag{13}$$

In the linear approximation, the polarization vector is

$$\vec{\varepsilon} = -eN_{\rm eb}^0\vec{\xi}_{\rm eb}\,. \tag{14}$$

Equation (5) is replaced by the Ampère-Maxwell equation

$$\vec{\nabla} \times \vec{H} = \frac{1}{c}(\partial\vec{E}/\partial t) - \frac{4\pi e}{c}\left[\iiint \vec{v} f_{\rm eb}{\rm d}\vec{v} + \iiint \vec{v} f_{\rm ef}{\rm d}\vec{v}\right], \tag{15}$$

where $\vec{j}_b(= -e\iiint \vec{v} f_{\rm eb}{\rm d}\vec{v})$ is the induced current of the bound electrons due to polarized atoms present in the plasma and $\vec{j}_f(= -e\iiint \vec{v} f_{\rm ef}{\rm d}\vec{v})$ is the induced current of free electrons. The vectors $\vec{\varepsilon}$ (polarization) and $\vec{D}$ (electric displacement) are the fields of the displacement of the bound charges in the classical concept of the quantum mechanics of nonlinear polarization. The incident plane-polarized wave, propagating along the $z$-direction (purely transverse), is

$$\vec{E} = (E_0\,\exp({\rm i}\theta),\,0,\,0)\,, \tag{16}$$

where

$$\theta = (k\,z - \omega t)\,, \tag{17}$$

and $\omega$ and $k$ are the wave frequency and the wave number, respectively. The perturbed distribution function is

$$f_1^{'} = g_1(v)\,\exp({\rm i}\theta)\,, \tag{18}$$

where $g(v)$ is its velocity-dependent amplitude factor. The displacement of bound electrons, induced by the external incident field, is

$$\vec{\xi}_1 = (\xi_0\,\exp({\rm i}\theta),\,0,\,0)\,. \tag{19}$$

## 3. Dispersion relation for a transverse wave

Taking curl of Eq. (6) and then using Eqs. (5) and (15), we obtained the relation

$$(\frac{1}{c^2}\ddot{E} - E^{''}) = \frac{4\pi e}{c^2}\frac{\partial}{\partial t}\left[\iiint v f_{\rm eb}{\rm d}v + \iiint v f_{\rm ef}{\rm d}v\right], \tag{20}$$

where the dot and the prime denote the partial time and the space derivative, respectively. From the Vlasov Eq. (9), with the help of Eqs. (11a) to (12b) and Eqs. (16) to (18), we get

$$g_{\rm eb}(v) = \frac{1}{{\rm i}(kv_z - \omega)}\left[\omega_0^2\xi_0 + \frac{eE_0}{m}\right]\frac{\partial f_{\rm eb}^0}{\partial v_x}, \tag{21}$$

$$g_{\rm ef}(v) = \frac{1}{{\rm i}(kv_z - \omega)}\left[\frac{eE_0}{m}\right]\frac{\partial f_{\rm ef}^0}{\partial v_x}, \tag{22}$$

where $\omega_0^2\xi_0 = \chi E_0$ and $\chi$ is the electrical susceptibility of this medium. On using Eq. (18) and Eqs. (21) and (22) in Eq. (20), one obtains

$$(k^2c^2 - \omega^2) = \left[\left\{\omega\omega_{\rm pb}^2(1 + \frac{m\chi}{e})\iiint \frac{\partial F_{\rm eb}^0}{\partial v_x} v_x \frac{1}{(kv_z - \omega)}{\rm d}v\right\}\right.$$
$$\left. + \left\{\omega\omega_{\rm pf}^2 \iiint \frac{\partial F_{\rm ef}^0}{\partial v_x} v_x \frac{1}{(kv_z - \omega)}{\rm d}v\right\}\right], \tag{23}$$

where $\omega_{\rm ps}$ $[= (4\pi N_{\rm ps}^0 e^2/m)^{1/2},\ {\rm s} = {\rm b, f}]$ is the plasma frequency and $F_{\rm l}^0 = N_{\rm l}^0 f_{\rm l}^0$, l = eb, ef. Here, for simplicity, we consider the propagation of a high frequency, transverse wave of infinitesimally small amplitude in unmagnetised plasma. Then the wave phase velocity $\omega/k$ is large enough to make physically ineffective the singularity at $\omega/k = v_z$ for wave propagation parallel to the $z$-axis. The definite integrals are determined by ignoring the resonance $\omega/k = v_z$ for relativistic reasons. In this case, expansion in positive integral powers of $(kv_z/\omega)$ is permissible. By expanding the factor $(1/(kv_z - \omega)$ in the integral in powers of $(kv_z/\omega)$, we obtain

$$\frac{1}{kv_z - \omega} = \left(-\frac{1}{\omega}\right)\left(1 + \frac{kv_z}{\omega} + \frac{k^2v_z^2}{\omega^2}\right). \tag{24}$$

Since the terms containing kinetic temperature $T$ are less than 1 for the high-frequency transverse wave, we keep only the terms up to the first power of $T$. Therefore, we find from Eq. (23) by using Eqs.(10) and (24) that

$$\iiint \frac{\partial F_{\rm eb}^0}{\partial v_x} v_x \frac{1}{(kv_z - \omega)}{\rm d}v = \left(-\frac{1}{\omega}\right)\left(1 + \frac{k^2 K_{\rm B} T_{\rm eb}}{\omega^2 m}\right), \tag{25}$$

$$\iiint \frac{\partial F_{\rm ef}^0}{\partial v_x} v_x \frac{1}{(kv_z - \omega)} {\rm d}v = \left(-\frac{1}{\omega}\right)\left(1 + \frac{k^2 K_{\rm B} T_{\rm ef}}{\omega^2 m}\right) . \qquad (26)$$

From Eq. (23), by using Eqs. (24) to (26), we get the dispersion relation

$$\omega^2 - k^2c^2 - (\omega_{pb}^2 + \omega_{pf}^2) = [(k^2/\omega^2)(T_{\rm eb}\omega_{pb}^2 + T_{\rm ef}\omega_{pf}^2)(K_{\rm B}/m)$$

$$+(m\chi\omega_{pb}^2/e)\left\{1 + (k^2/\omega^2)(K_{\rm B}T_{\rm eb}/m)\right\}] . \qquad (27)$$

The cut-off frequency is determined by putting $k = 0$ in Eq. (27). The expression in Eq. (27) give the results obtained in the fluid approximation for each of these species if we keep the terms of the first power and ignore completely those of higher powers of the kinetic temperatures.

## 4. *Phase velocity and group velocity*

Rayleigh [2] defined the group velocity as the velocity of the envelope of a beat constructed by two wave patterns; for two waves having a small difference of frequencies it reduces to $v_g = \partial\omega/\partial k$ [9]. From the dispersion relation (27) for transverse waves, we determine the phase velocity, $v_p = \omega/k$, and the group velocity, $v_g$. In an unmagnetised plasma in presence of bound electrons, including the kinetic temperature effects and the radiation damping effect of charged species, we find that

$$v_p = \frac{[\omega^2c^2 + (K_{\rm b}/m)(T_{\rm eb}\omega_{\rm pb}^2 + T_{\rm ef}\omega_{\rm pf}^2) + \omega_{\rm pb}^2 K_{\rm b}T_{\rm eb}(\chi/e)]^{1/2}}{[\omega^2 - (\omega_{\rm pb}^2 + \omega_{\rm pf}^2) - m\omega_{\rm pb}^2\chi/e]^{1/2}} , \qquad (28)$$

$$v_g = \frac{[\omega^2c^2 + (K_{\rm b}/m)(T_{\rm eb}\omega_{\rm pb}^2 + T_{\rm ef}\omega_{\rm pf}^2) + \omega_{\rm pb}^2 K_{\rm b}T_{\rm eb}(\chi/e)]}{[\omega^2 v_{\rm p} + (K_{\rm b}/mv_{\rm p})(T_{\rm eb}\omega_{\rm pb}^2 + T_{\rm ef}\omega_{\rm pf}^2 + T_{\rm eb}\omega_{\rm pb}^2 m\chi/e)]} . \qquad (29)$$

The phase velocity for only free electrons is

$$v_{\rm p}^{\rm f} = \frac{[\omega^2c^2 + (K_{\rm b}T_{\rm ef}/m)\omega_{\rm pf}^2]^{1/2}}{[\omega^2 - \omega_{\rm pf}^2]^{1/2}} . \qquad (30)$$

The ratio of the two phase velocities is

$$(v_{\rm p}/v_{\rm p}^{\rm f}) = \frac{[\omega^2c^2 + (K_{\rm b}/m)(T_{\rm eb}\omega_{\rm pb}^2 + T_{\rm ef}\omega_{\rm pf}^2) + \omega_{\rm pb}^2 K_{\rm b}T_{\rm eb}(\chi/e)]^{1/2}}{[\omega^2 - (\omega_{\rm pb}^2 + \omega_{\rm pf}^2) - m\omega_{\rm pb}^2\chi/e]^{1/2}}$$

$$\times \frac{[\omega^2 - \omega_{\rm pf}^2]^{1/2}}{[\omega^2c^2 + (K_{\rm b}T_{\rm ef}/m)\omega_{\rm pf}^2]^{1/2}} . \qquad (31)$$

When $\omega_{\rm pb} = \omega_{\rm pf}$ and $T_{\rm eb} = T_{\rm ef}$, and the number density of bound electrons is comparable to that of the free electrons, Eq. (31) gives

$$(v_{\rm p}/v_{\rm p}^{\rm f}) = \frac{\left[\left\{\omega^2 c^2 + (K_{\rm b}/m)2T_{\rm ef}\omega_{\rm pf}^2 + \omega_{\rm pf}^2 K_{\rm b} T_{\rm eb}(\chi/e)\right\}\left\{\omega^2 - \omega_{\rm pf}^2\right\}\right]^{1/2}}{\left[\left\{\omega^2 - 2\omega_{\rm pf}^2 - m\omega_{\rm pf}^2\chi/e\right\}\left\{\omega^2 c^2 + (K_{\rm b}T_{\rm eb}/m)\omega_{\rm pf}^2\right\}\right]^{1/2}} . \qquad (32)$$

If also $m\chi/e$ is small, Eq. (32) can be written as

$$(v_{\rm p}/v_{\rm p}^{\rm f}) = \frac{1+2g_1}{1-2g_2} \times \frac{1-g_2}{1+g_1} , \qquad (33)$$

where

$$g_1 = K_{\rm b}T_{\rm eb}\omega_{\rm pf}^2/m\omega^2 c^2 \qquad \text{and} \qquad g_2 = \omega_{\rm pf}^2/\omega^2 . \qquad (34)$$

This shows that

$$v_{\rm p} >> v_{\rm p}^{\rm f} , \qquad (35)$$

i.e., the presence of bound electron increases the phase velocity of the transverse wave.

But if $\omega_{\rm pf} \gg \omega_{\rm pb}$, then from Eq. (32) we obtain

$$v_{\rm p} = v_{\rm p}^{\rm f} , \qquad (36)$$

meaning that when the number density of free electrons is greater than the number density of bound electrons, the phase velocity of the transverse wave due to bound electrons is equal to the phase velocity of free electrons.

## 5. *Temperature dependence of Thomson scattering*

The Thomson scattering by free electrons in plasmas is obviously an energy loss for waves. We derive the temperature dependence of the scattering formula in the presence of the bound species. At a high frequency, the atoms emit radiation as free particles [8], but in a low frequency limit, the incident frequency is not large compared to the frequency of the binding. So, including the binding effect, we find the total scattering cross-section. In a laser-irradiated plasma, from an over-dense layer, this type of radiation cannot be neglected. For pulses of a finite duration, they behave like quasi-free particles. The total radiated power from the Larmor formula for non-relativistic charges, is

$$P = \frac{2e^2}{3c^3} < \dot{v}_{\rm eb} >^2 , \qquad (37)$$

where $< v_{\rm eb} >$ is the average velocity of bound electrons in distorted atoms in a very small, but finite time, induced by the applied field. From Eq. (21), the average induced velocity, neglecting the small wave damping term, is

$$< v_{\rm eb} >= \frac{1}{N_{\rm eb}^0} \iiint v g_{\rm eb} {\rm d}v$$

$$= [{\rm i}E_0 \, \exp({\rm i}\theta) \, \{\chi + e/m\} \, \{1 + (k^2/\omega^2)(K_{\rm B}T_{\rm eb}/m)\}] \, . \tag{38}$$

By taking the time derivative of Eq. (38) and using Eq. (37), we get the real part of the total power radiated by bound electrons in distorted atoms in the plasma,

$$< P >= (2e^2\omega E_0^2/3c^3)[\{\chi + e/m\} \, \{1 + (k^2/\omega^2)(K_{\rm b}T_{\rm eb}/m)\}]^2 \, . \tag{39}$$

The total scattering cross-section, $\sigma_{\rm t}$, is defined as the ratio of the total radiated power and the total incident flux, which is obtained from the Poynting vector averaged over the time period $2\pi/\omega$. Since $< S >= c/8\pi n E_0^2$,

$$\sigma_{\rm t} = (< P > / < S >)$$

$$= (16\pi e^2\omega/3nc^4)[\{(\chi + e/m)\} \, \{1 + (k^2/\omega^2)(K_{\rm b}T_{\rm eb}/m)\}]^2 \, , \tag{40}$$

where $n(= kc/\omega)$ is the refractive index of the medium.

## 6. *Lagrangian and Hamiltonian of the system*

In a plasma, the Lagrangian density provides information on the parameters of the medium, the fields and their behaviour in terms of the mechanical energy, and on the conserved properties of the system. The Hamiltonian density singles out the time variables, in contrast to the Lagrangian function for which the independent variables (time and space coordinates) are involved symmetrically. The Lagrangian density for a mixture of compressible fluids of bound electrons and free electrons in the presence of applied wave fields, both electro-magnetic and acoustic [10], is

$$L = \Bigg[ \sum_{\rm l} \frac{1}{2} m_{\rm l} \, N_{\rm l}^0 \ddot{\xi}_{\rm l}^2 + \frac{1}{c} \sum_{\rm l} \vec{A} \cdot \vec{j}_{\rm l} - e\phi - \frac{1}{2} N_{\rm eb}^0 \omega_0^2 \xi_{\rm eb}^2 + \frac{1}{8\pi}(E^2 - H^2)$$

$$- (C_s^2/\tau) \sum_{\rm l} N_{\rm l} \Bigg] \, , \quad {\rm l} \; = \; {\rm eb, \, ef} \, . \tag{41}$$

The usual field equations may be recovered by using this Lagrangian in the Euler-Lagrangian equation of motion

$$\frac{\partial}{\partial t}(\frac{\partial L}{\partial \dot{\xi}}) - \frac{\partial L}{\partial \xi} = 0 \tag{42}$$

The Hamiltonian density is then

$$H = \left[ \sum_{l} \frac{1}{2} m_i N_l^0 \dot{\xi}_l^2 + e\phi + \frac{1}{2} N_{\rm eb}^0 \omega_0^2 \xi_{\rm eb}^2 - \frac{1}{8\pi}(E^2 - H^2) - (C_s^2/\tau) \sum_{l} N_l \right] , \quad (43)$$

where $A$ and $\phi$ are, respectively, the vector and scalar potentials of the EM field, $N$ is the perturbed number density, $\tau = c_{\rm p}/c_{\rm v}$ is the ratio of the specific heat capacities and the other symbols have already been defined.

## 7. Discussion

The plasma models, including the bound electrons considered in our paper, are more realistic and useful for laboratory and space plasmas than those of plasmas consisting of only free electrons. The cut-off frequency of the wave from the dispersion relation Eq. (27) is found to increase due to the presence of bound electrons, thermal effects and radiation damping. The existence of this cut-off frequency should be experimentally verified for laboratory plasmas and also for ionospheric plasmas. Equation (27) becomes the dispersion equation for transverse waves in a cold, free-electron plasma if we put $T_{\rm eb}, T_{\rm ef}, \chi$ and $\omega_{\rm pb}$ equal to zero. The phase velocity, $v_{\rm p}$, of both species depends on their number density. If these two species have the same number density, the phase velocity of free electrons, $v_{\rm p}^{\rm f}$, is less than the phase velocity of the transverse wave, $v_{\rm p}$. But for a small number density of bound electrons, compared to that of free electrons, the phase velocity of free electrons is equal to the phase velocity of the transverse wave. The presence of bound electrons enhances the phase velocity of the wave.

The total scattering cross-section, $\sigma_{\rm t}$, for Thomson scattering has been obtained from the kinetic theory of the plasma. Equation (40) shows that $\sigma_{\rm t}$ depends on the characteristics of the binding force for Rayleigh-scattering susceptibility (dielectric constant) and the temperature of the species. It varies parabolically with the temperature. The rotational-energy term of the bound electrons and the acoustic-effect terms of compressibility have been included in our Lagrangian and Hamiltonian densities. The relativistic generalization of mathematical models of plasma containing a population of bound electrons has not been considered, because of the lack of knowledge on how to proceed after such an ad hoc inclusion of effects of bound electrons.

## 8. Summary and concluding remarks

In the present paper, we have investigated the effects of bound electrons on the propagations of transverse waves in a Vlasov plasma consisting of free electrons, free ions, weakly bound electrons and ions. The dispersion relation of a plane polarized wave has been derived and then the phase velocity and group velocity have been obtained in terms of kinetic temperature. It is found that the cut-off frequency of the wave is higher in presence of bound electrons. Moreover, the phase velocity in the presence of only free electrons is less than the phase velocity of the wave in the presence of bound electrons. From the expression of Thomson scattering, it is found that the scattering cross section is dependent on the Rayleigh scattering susceptibility. Lagrangian and Hamiltonian density have also been obtained in the plasma considering the effect of bound electrons.

However, in our present analysis, we have not considered the collisional effects between free electrons and bound electrons, etc. Studies on the propagation of transverse waves in a magnetized plasma taking the effect of bound electrons would yield some interesting results. The magnetic moment field in such a plasma can be found which will generalize the results of previous authors [4]. We like to study in detail the propagation of transverse waves in the Vlasov plasma considering the effects of the above plasma parameters from which interesting as well as important results would be obtained.

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## UTJECAJ VEZANIH ELEKTRONA NA POPREČNE VALOVE U VLASOVLJEVOJ PLAZMI

Proučavamo poprečne valove u Vlasovljevoj plazmi koja se sastoji od smjese slobodnih elektrona, slabo vezanih elektrona, slobodnih mirnih iona i mirnih iona pridruženih vezanim elektronima, uz održavanje makroskopske neutralnosti. Istražuju se temperaturna ovisnost fazne i grupne brzine, te Thomsonovog udarnog presjeka. Raspravlja se Lagrangian i Hamiltonian stlačive plazme koja sadrži i vezane elektrone.