

Histeretical behaviour of two-dimensional vortex shedding past the NACA0012 airfoil at ultra-low Reynolds number.

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We unfold the complex transition route from steady to chaotic dynamics of the two-dimensional flow past the NACA0012 airfoil upon increasing the angle of attack α at the ultra-low Reynolds number $Re = 5300$. Hopf, forward and backward period-doubling cascades, Neimark-Sacker and saddle-node bifurcations are found along the way, the latter producing hysteretical regions where two distinct two-dimensional vortex-shedding modes coexist. Although the advent of three-dimensionality obliterates the two-dimensional path here addressed and deteriorates aerodynamic performances shortly after the advent of time-dependence at the first Hopf bifurcation, active flow control might assist in postponing the disruption of spanwise invariance to some extent, thus partially recovering the two-dimensional scenario.

INTRODUCTION

The airfoil constitutes the core design element of any discipline that involves aerodynamic lift generation [9]. Atmospheric flight (from insects to aircraft), or atmospheric and under-water propulsion (fins, propellers, gas turbines) are but two notable examples of applications involving lifting-surfaces that rely on the efficient operation of airfoils.

The nature of the phenomena that might be observed crucially depends on scale effects, and the need for the development of ever smaller air vehicles in the past few decades has decisively spurred the research on low Reynolds number airfoil aerodynamics [12, 13].

The key parameter for airfoil aerodynamics at low Reynolds number and high angle of attack concerns the type of flow separation that occurs on the upper surface. Separation might be classified into one of three types, depending on whether laminar-turbulent transition occurs upstream from the separation point (turbulent separation), or downstream, in which case the re-energised separated boundary layer might reattach (transitional separation) or continue downstream as a turbulent shear layer (laminar separation). Separation bubbles, i.e. regions of recirculating flow enclosed between the separation and reattachment points in the case of transitional separation, might be observed at Reynolds numbers in the order $\sim O(10^4 - 10^6)$, such that separation is always laminar at very low Reynolds numbers and reattachment does not occur [15].

Several numerical studies have aimed at characterising the flow past airfoils at these very low Reynolds number regimes in order to trace their aerodynamic performances to the wake topology and flow phenomena involved [3, 5, 6]. For very low angles of attack, the flow remains two-dimensional and steady, while vortex shedding and three-dimensional spanwise structures characterise the wake at sufficiently large angles of attack [6]. These structures may initially retain high spanwise coherence, but the wake becomes turbulent (spatio-temporally chaotic) beyond a certain threshold.

In a purely two-dimensional setting, airfoils at very low Reynolds numbers may exhibit a wealth of distinct wake regimes as the angle of attack or Reynolds number are

increased, including steady, periodic, quasiperiodic and chaotic solutions [7, 8]. The relations and transitions among the different states has been recently analysed to some extent both by varying the Reynolds number [2, 14] or the angle of attack [4, 10], but the discontinuities observed in the aerodynamic characteristic curves and the coexistence of different vortex shedding regimes within some ranges of the parameters (hysteretical regions), remains mostly unexplained. Figure 1 depicts two distinct two-dimensional vortex-shedding modes at the same $Re = 5300$ and $\alpha = 9^\circ$.

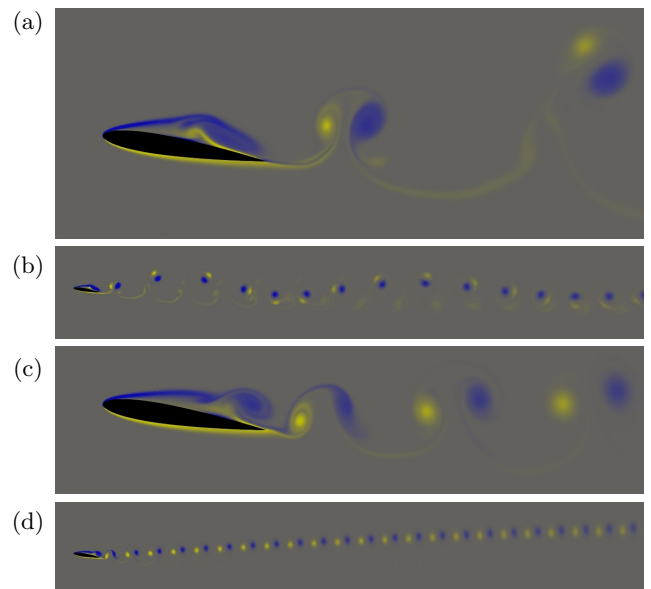


FIG. 1. Two distinct vortex-shedding modes at $Re = 5300$ and $\alpha = 9^\circ$, visualised through spanwise vorticity colourmaps (ω_z). (a) Near wake and (b) vortex street of solution obtained by decreasing α from above. (c) Near wake and (d) vortex street of state resulting from increasing α from below.

TWO-DIMENSIONAL VORTEX SHEDDING REGIMES

We conduct a thorough continuation of solutions in (Re, α) parameter space in order to explain the inter-

connections among the various distinct vortex-shedding modes and analyse the intervening transitions in the light of nonlinear dynamical systems and bifurcation theory. In particular, we address the coexistence of different vortex shedding modes and the sharp transitions back and forth from one another in terms of double saddle-node bifurcations originating at neighbouring cusps bifurcations. The analysis requires an extensive parametric exploration of two-dimensional airfoil configurations, which will be performed using the incompressible Navier-Stokes solver of the tensor-product-based spectral/finite element package Nektar++ [1, 11].

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