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Control theory tools for best understanding brain Learning Disorders

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***Abstract*—Recent developments in cognitive neuroscience have made it possible to take a step forward in understanding the processes involved in the development of learning. Sometimes there are alterations in learning processes that cause structural and functional dysfunctions at the brain level called learning disorders. These disorders are brain-specific disorders that for detection, are needed specific screeners are needed, rather than more general tests of learning abilities.**

One of the brain disorders that is little studied and that causes great damage to those who suffer from it is dyscalculia. The lack of study on this alteration means that many cases remain undiagnosed or misdiagnosed, and therefore remain untreated. In order to make a correct diagnosis for the correct treatment we have been able to prove that by performing a BAEP and analyzing wave VI, the presence or absence of a learning disorder can be objectified. Another problem that arises is that of the evolution of the disorder, for which a good tool could be the modeling of brain activity through dynamic systems. In this work, in addition to delving into how to make a correct diagnosis, the evolution and possible control of linear dynamic systems that model the process in order to infer it are analyzed, taking advantage of brain plasticity that can facilitate the control that allows improvement of the dysfunction

***Index Terms*—Learning disabilities, cognition, neuroscience, neuromodulation, neural networks.**

I. Introduction

Learning considered as knowledge acquisition, is closely related to the correct activity of neural circuits.

Sometimes there are alterations in synaptic transmission that cause structural and functional dysfunctions at the brain level. These alterations are what we know as learning difficulties. Learning disorders are manifested in various ways in the activities of daily life, hindering to a varying degree for each person, their ability to understand, reason, speak, read, write, calculate..., actions all of which are necessary for the development of a normal life. Xiang in [8] has been shown that cognitive control and the ability to control brain dynamics holds great suggestive of improvement of cognitive functions and reversing the possible disorder in learning processes. The main goal is to try to establish as accurately as possible each type of cognitive impairment to start an adequate rehabilitation as quickly as possible to achieve the best integration of the person in society.

Learning disorders are brain-specific disorders, it follows that for detection, specific screeners are needed, rather than more general tests of learning abilities. To explain how the brain works in learning processes, we use the recording of brain bioelectric activity under normal conditions and through the application of an auditory stimulus. This brain activity is translated into responses that represent the variations in voltage of the neurons, which allows us to objectively assess their activity and helps us to detect alterations in neuronal function in the areas involved in the numerical process. Concretely, our proposal is to use the neuroelectric response of the auditory system to a sound stimulus, this response is called the acoustic evoked potential of the brainstem. The study of these evoked potentials is of great clinical interest since it allows diagnosing, by comparison with the responses considered normal, various pathologies or dysfunctions of the nerve pathways. This technique has highlighted how brain processes interact with each other, allowing us the detection of possible alterations in learning. The auditory brainstem response is recorded from the scalp, and it consists of a series of vertex-positive waves within the first 10 ms after an auditory stimulus. We have proven that the wave VI show clear differences between results obtained in students with difficulties and students who do not, [1].

The brain structure is a complex recurrent neuronal network that can be readily described by a graph where the nodes represent brain areas and the edges the strength of connections between these areas that emerge when certain tasks are performed. An open problem is the detection of the neural networks that are activated in the individual when he is faced with a situation of learning. The Knowledge of these networks and their mathematical modelling will allow them to be controlled and thus modulate the individual's response to a possible disorder of learning.

Several authors use artificial neural networks to study the brain problems as tumors, concretely Isselmou et al. [2], tried the high performance of a Convolutional Neural Network model, and they strongly recommend using it as a computer brain aide technique for early brain tumor detection and reducing the number of deaths.

Other authors, such as Ashwani Kumar Aggarwal [3] study learning disabilities caused by brain tumors using magnetic resonance imaging (MRI). Of note, unlike MRIs, evoked potentials do not radiate to the child.

In an informal way, the control problem consists of select-

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ing, from a specific or arbitrary set of elements (or parameters, configurations, functions, etc), those that, applied to a fixed system, make it behave in a predetermined way. It is a challenge to know which elements are valid for the system to be controllable and even more so if these elements have the minimum number of inputs. So, it is of interest to recognize the minimum set of independent driver nodes needed to achieve full control of networks having arbitrary structures and link-weight distributions.

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II. Brain Auditory Evoked Potentials

The graphic expression of the response of the central nervous system to an acoustic stimulus are a quantitative and qualitative method of recording the activity of the peripheral and central auditory pathway generated in certain brain areas that are involved in the cognitive process, through the application of an auditory stimulus. The application of the auditory stimulus upon reaching the organ of Corti is transformed into an electrical stimulus that reaches the auditory nerve (VIII cranial nerve) and spreads along the auditory pathway until it reaches the temporal cortex, [4]. Along the way, the arrival at the different brain nuclei manifests itself in the form of waves with a positive vertex, which are what we call waves I, II, III, IV, V, VI and VII, which are collected in the first 12 msg. Most of the auditory information reaches the left primary auditory cortex where it is decoded and completed with the intervention of other neuronal groups (Reticular System, cerebrum, diencephalon) located in both hemispheres.

To date, BAEPs have been used mainly for the assessment of auditory alterations through the study of the first 5 waves and they had not been taken into account for the detection of learning problems since latency, which is the time of appearance of a certain wave after applying the auditory stimulus, does not vary significantly between normal people and those with some type of learning dysfunction, taking into account that they have normal hearing.

Of all the waves obtained in the graph, we have focused on the study of the functionality of wave VI of which we have observed its possible clinical usefulness in this type of cognitive alterations.

The analyzes of the results obtained by studying wave VI show differences between normal people and those with some type of learning disorder as we can see in the following figure 1.

A relevant piece of information that we have reached is that we have observed that normal patients have a latency between 7.5-7.8. miliseconds and people with some learning difficulty, the value obtained is, in all cases, less than 7.5. Therefore, the use of BAEPs is a useful, objective test because of the child does not participate in the explanation, non-invasive because of the child was not irradiated, easily reproducible and economical tool to determine the functionality of part of the cerebral neuronal groups that intervenes in the learning process through the assessment of wave VI.

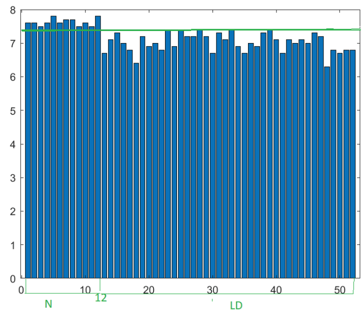

Fig. 1. BAEP's results

The evoked potentials present a limitation that the child is still since any movement of the child can cause a cable to come loose and the test to be interrupted. In the case of very nervous children, one solution is sedation.

III. Controllability of Multiagent Neural Networks

It has been shown that cognitive control and the ability to control brain dynamics are highly suggestive of improving cognitive functions and reversing the possible disorder in learning processes. The human brain seems to be able to travel between diverse cognitive states. Its most imposing role is in connecting multiple sources of information in large-scale networks that are required to solve complex cognitive problems and strengthen memory.

Due to the existence of multiple factors that involved in learning processus, the neuronal system that is activated is a network of interconnected networks whose controllability we need to study.

If the system is controllable, it is possible, through drivers, to obtain predetermined responses. In our case, it is of special interest when there is some dysfunction in the neural network.

Learning disorders can be characterized by consistently altered intrinsic connectivity between and/or within networks.

In this work, we study the controllability of multiagent neural networks by simulating possible brain networks.

Due to the complexity of the network we have thought about the possibility of seeing this network as different interconnected networks, modeling it as a multi-agent system with an intercommunication topology.

García-Planas in [5] showed that a noise-free multisystem of linear discrete-time and time-invariant model

$$\left.\begin{aligned} \dot{x}^1(t) &= A_1x^1(t) + B_1u^1(t) \\ &\vdots \\ \dot{x}^k(t) &= A_kx^k(t) + B_ku^k(t) \end{aligned}\right\} \qquad (1)$$

where $A_i \in M_n(\mathbb{R})$, $B_i \in M_{n\times m}(\mathbb{R})$, $x^i(t) \in \mathbb{R}^n$, $u^i(t) \in \mathbb{R}^m$, $1 \leq i \leq k$, could be employed to describe the neuronal dynamics.

Fig. 2. Abstract brain neural network

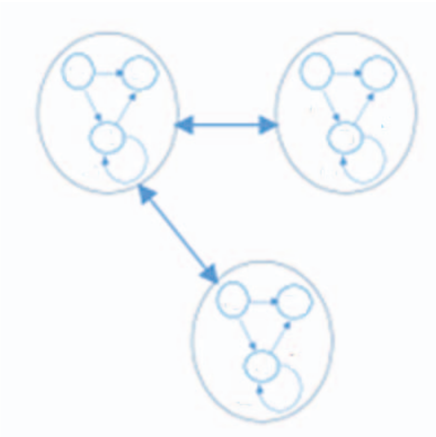

Fig. 3. Multiagent graph

Controllability consists in analyzing whether the solution of the multiagent system can be driven to a given final target by means of a control applied on the boundary or on a subdomain of the domain in which the equation evolves. In other words, controllability consists of the ability of a dynamic system to reach a specific state previously fixed from an initial state in a finite time. It is a significant condition of control systems since it defines the behavior of said control systems.

In order to analyze the controllability of the mutilinear systems we write the system (1) in the form

$$\dot{\mathcal{X}}(t) = \mathcal{A}\mathcal{X}(t) + \mathcal{B}\mathcal{U}(t).$$

where

$$\mathcal{X}(t) = \begin{pmatrix} x^1(t) & & \\ & \ddots & \\ & & x^k(t) \end{pmatrix}, \; \dot{\mathcal{X}}(t) = \begin{pmatrix} \dot{x}^1(t) & & \\ & \ddots & \\ & & \dot{x}^k(t) \end{pmatrix},$$

$$\mathcal{U}(t) = \begin{pmatrix} u^1(t) & & \\ & \ddots & \\ & & u^k(t) \end{pmatrix},$$

$$\mathcal{A} = \begin{pmatrix} A_1 & & \\ & \ddots & \\ & & A_k \end{pmatrix}, \; \mathcal{B} = \begin{pmatrix} B_1 & & \\ & \ddots & \\ & & B_k \end{pmatrix},$$

and the topology, defined as a graph $\mathcal{G}$ with

i) Set of Vertices: $V = \{1, \ldots, k\}$

ii) Set of Edges: $E = \{(i, j) \mid i, j \in V\} \subset V \times V$,

is introduced through the control $\mathcal{U}$ with

$$u^i(t) = F_i \sum_{j \in \mathcal{N}_i} (x^i(t) - x^j(t)), \; 1 \leq i \leq k \tag{2}$$

That in a matrix description is

$$\mathcal{F}\mathcal{U}(t) = \mathcal{F}(\mathcal{L} \otimes I_n)\mathcal{X}(t)$$

where $\mathcal{F} = \begin{pmatrix} F_1 & & \\ & \ddots & \\ & & F_k \end{pmatrix}$ and $\mathcal{L}$ is the Laplacian matrix associated to the graph

$$\mathcal{L} = (l_{ij}) = \begin{cases} |\mathcal{N}_i| & \text{if } i = j \\ -1 & \text{if } j \in \mathcal{N}_i \\ 0 & \text{otherwise} \end{cases}$$

(See [7] for Kronecker product properties) Then, the multisystem with interrelation control is described as:

$$\dot{\mathcal{X}}(t) = \mathcal{A}\mathcal{X}(t) + \mathcal{B}\mathcal{F}\mathcal{U}(t) = (\mathcal{A} + \mathcal{B}\mathcal{F}(\mathcal{L} \otimes I_n))\mathcal{X}(t). \tag{3}$$

the controllability of the system (1) translates into ensuring the existence of a matrix $\mathcal{F}$ such that the system (3) has predetermined eigenvalues.

It is in our interest to be able to externally modify the agents so that we can control the goal to be achieved by the system.

The agent dynamics perturbed by external disturbances are described by

$$\begin{aligned} \dot{\mathcal{X}}(t) &= \mathcal{A}\mathcal{X}(t) + \mathcal{B}\mathcal{F}\mathcal{U}(t) + \mathcal{E}\mathcal{U}_2(t) \\ &= (\mathcal{A} + \mathcal{B}\mathcal{F}(\mathcal{L} \otimes I_n))\mathcal{X}(t) + \mathcal{E}\mathcal{U}_2(t). \end{aligned} \tag{4}$$

with

$$\mathcal{D} = \begin{pmatrix} E_1 & & \\ & \ddots & \\ & & E_k \end{pmatrix}, \text{ and } \mathcal{U}_2(t) = \begin{pmatrix} u_2^1 & & \\ & \ddots & \\ & & u_2^k \end{pmatrix}.$$

In this particular setup the controllability of the multisystem can be described as follows:

Proposition 1: The multilinear system 4 is controllable if and only if, there exists a particular matrix $\mathcal{F}$ in such away that

$$\text{rank } \left(sI_{nk} - \mathcal{A} + \mathcal{B}\mathcal{F}(\mathcal{L} \otimes I_n \quad \mathcal{E} \right) = kn$$

for all $s \in \mathbb{R}$.

Suppose now, the multisystem formed by the identical agents and the graph being undirected, so there exists an orthogonal matrix $P \in Gl(k;\mathbb{R})$ such that $P\mathcal{L}P^t = D = \operatorname{diag}(\lambda_1,\dots,\lambda_k)$, $(\lambda_1 \geq \ldots \geq \lambda_k)$, [6].

Then, the closed-loop system can be described in terms of the matrices A, B, the feedbacks F and the eigenvalues of $\mathcal{L}$ in the following manner

$$\dot{\widehat{\mathcal{X}}} = ((I_k \otimes A) + (I_k \otimes BK)(D \otimes I_n))\widehat{\mathcal{X}}$$

and the system with external modification

$$\dot{\widehat{\mathcal{X}}} = ((I_k \otimes A) + (I_k \otimes BK)(D \otimes I_n))\widehat{\mathcal{X}} + \mathcal{E}\widehat{\mathcal{U}}_2.$$

where

$$\widehat{\mathcal{X}} = (P \otimes I_n)\mathcal{X} = \begin{pmatrix} Px^1 \\ \vdots \\ Px^k \end{pmatrix}, \ \widehat{\mathcal{U}}_2 = (P \otimes I_n)\mathcal{U}_2$$

and the controllability character is writing in a simpler way

Proposition 2: The multilinear system 4 with the topology defined by an undirected graph is controllable if and only if, there exists a particular matrix $\mathcal{F}$ in such away that

$$\operatorname{rank}\begin{pmatrix} A+\lambda_1 BF & & E & \\ & \ddots & & \ddots \\ & A+\lambda_k BF & & E \end{pmatrix} = \quad (5)$$

$kn.$

Equivalently,

The multiagent linear system 1 with oidentical agents and the topology defined by an undirected graph is controllable if and only if the linear systems $(A+\lambda_i BF, E)$ are controllable for some matrix F where λ_i are the eigenvalues of the Laplacian associated to the graph.

Taking into account that in our particular setup, zero is an eigenvalue of $\mathcal{L}$ we have the following result.

Corollary 1: A necessary condition for the multilinear system 4 with identical agents and the topology defined by an undirected graph be controllable is that the system $\dot{x}(t) = Ax(t) + Eu_2(t)$ be controllable.

Example 1:

We consider 3 identical agents with the following dynamics of each agent

$$\begin{array}{ll} \dot{x}^1 & = Ax^1 + Bu^1 \\ \dot{x}^2 & = Ax^2 + Bu^2 \\ \dot{x}^3 & = Ax^3 + Bu^3 \end{array} \quad (6)$$

with $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

The communication topology is defined by the graph (V,E):

$V = \{1,2,3\}$

$E = \{(i,j) \mid i,j \in V\} = \{(1,2),(1,3)\} \subset V \times V$

and the adjacency matrix:

$$G = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

The neighbors of the parent nodes are $\mathcal{N}_1 = \{2,3\}$, $\mathcal{N}_2 = \{1\}$, $\mathcal{N}_3 = \{1\}$.

The Laplacian matrix of the graph is

$$\mathcal{L} = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

with eigenvalues $\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_3 = 3$.

The matrix F is $F = \begin{pmatrix} f_1 & f_2 \end{pmatrix}$.

It is easy to observe that in the system (3) with the matrix $\mathcal{F}$ we can only select two eigenvalues. Then for complete control of the system we need to introduce an external modification.

Now, we try to obtain an external modification of the system with a minimal entries

The first condition is to ensure the controllability of the sysetm (A,E) with a minimal entries.

For that, it is necessary to consider $E = \begin{pmatrix} m \\ n \end{pmatrix}$ with $n \neq 0$, now we need to ensure the controllability of $(A+BF,E)$ and $(A+3BF,E)$, and this is reached when

$$\begin{array}{c} f_1 m + n(1+f_2) \neq 0 \\ 3f_1 m + n(1+3f_2) \neq 0 \end{array}$$

IV. Discussion

In this work, the use of brainstem auditory evoked potentials has been proposed for the detection of learning difficulties by analyzing wave VI. This work is pioneering since, to date, the few authors who had tried to use these potentials only analyzed up to wave V, so they did not obtain results. On the other hand, applying the Network Theory, simplified models of cognitive neuronal functionality have been presented to help, using controls, correct the possible learning difficulties detected. Authors like Horas et al. [9] have used P300 Cognitive Evoked Potentials to improve medical diagnosis and analyze learning evolution through artificial neural networks. The attempt to model neural networks to deal with neural disorders such as learning disorders or other neurodegenerative problems [10] is gaining interest because they will allow personalized treatments and improve diagnoses of patients with mental and neurological diseases.

V. Conclusion

In neuroscience, the word control implies action and reflects the human effort to intervene in the environment that surrounds it to, on the one hand, guarantee its survival and, on the other, achieve a permanent improvement in the quality of life. Many of the control problems can be analyzed through a mathematical model that describes the physical system under consideration through equations that show the state of the system. One of the problems in which we are interested is the analysis of learning difficulties.

Due to the existence of multiple factors that cause learning disorders, the neuronal system that is activated is a network of interconnected networks whose controllability we need to study. In this work, we have studied the controllability of multiagent neural networks by simulating possible brain networks.

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