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Global ionospheric models in three dimensions from GPS measurements: Numerical simulation

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RESUMEN

Gracias a las observaciones GPS en doble – frecuencia de receptores distribuidos uniformemente sobre la superficie terrestre es posible hacer un análisis de la ionosfera como puede verse en los trabajos de Mannucci A. *et al.*, 1993; Beutler G., 1995 y Brunini C. *et al.*, 1997 entre otros.

Este análisis consiste básicamente en el modelado de las variaciones del contenido total de electrones en función de 2 coordenadas que bien pueden ser la latitud y la longitud en un sistema sol fijo. Estos modelos asumen que la ionosfera puede representarse por una capa esférica de espesor despreciable, situada a una determinada altura (entre los 300 y 400 km), donde se concentra la totalidad de los electrones libres. Para modelar la distribución espacial de la concentración total de electrones en la delgada capa, se han utilizado series de Taylor en dos dimensiones o armónicos esféricos entre otros.

En abril de 1995, gracias al lanzamiento de la misión GPS-MET, se pudo hacer realidad la obtención de observaciones GPS desde un receptor en el espacio. GPS-MET es un experimento dirigido por el UCAR (University Corporation of Atmospheric Research), cuyo objetivo es el sondeo de la atmósfera terrestre mediante observaciones GPS colectadas por un receptor de alto rendimiento situado a bordo del satélite MicroLab I (MLI). Este satélite describe una órbita circular a 730 km de altura (LEO=Low Elevation Orbit), con una inclinación de 60°. Las observaciones colectadas por este receptor están disponibles vía ftp en una base de datos administrada por el UCAR.

Este satélite de baja altura con receptor GPS de doble frecuencia nos brinda la posibilidad de contar con señales GPS que atraviesan la ionosfera a diferentes alturas.

Este trabajo no apunta a discutir un modelo ionosférico en sí mismo, sino más bien a analizar las posibilidades de utilizar mediciones GPS para extraer información sobre el comportamiento vertical de la densidad electrónica, basadas en un modelo medio y global.

En este trabajo emplearemos simulaciones numéricas con el objetivo de analizar si las observaciones del Microlab I son suficientemente sensibles a las variaciones en altura de la ionosfera. Afortunadamente y pese a la limitación en la geometría del problema, ya que sólo contamos con un satélite de órbita fija, se concluye que el receptor espacial nos brinda información fundamental para el modelado en altura de la densidad de electrones

PALABRAS CLAVE: Ionosfera, modelos, simulación numérica.

ABSTRACT

A network of globally distributed dual frequency Global Position System (GPS) receivers is used to compute global maps of the vertical total electron content (VTEC) distribution of the ionosphere. A mapping function is used to convert the total electron content along the path of the signal into vertical total electron content, which can be represented by two dimensional Taylor series or spherical functions.

This vertical variation cannot be recovered directly from GPS data measured with ground based receivers. The technique was tested with the launch of the GPS - MET mission on April 3, 1995. Combining the trans-ionospheric measurements from ground based receivers with the data collected in the GPS-MET experiment we attempt to model the electron content based on the global behaviour of the ionosphere. Using least square estimation we determine the unknown coefficients of the electron content model. The results suggest that, even without a global geometrical distribution, the space receiver is useful to model the mean high electron content variation.

KEY WORDS: Ionosphere, models, numerical simulation.

1. INTRODUCTION

Using space and ground sensors it is possible to study the ionosphere in detail (Hargreaves, 1995). The main aspect of any ranging system is the speed of propagation of the signal. This speed multiplied by the measured propagation time interval that provides a measure of the range. Electromagnetic signals propagate with the vacuum speed of light (c = 299792.458 km/s) – at all frequencies. However, in the case of GPS satellites, the signals must pass through the Earth's atmosphere on their way to ground. The signal interacts with charged particles and neutral atoms and molecules of the atmosphere with the result that their signals are refracted. This work is a feasibility study of GPS measurements to extract useful information from a signal which traverses the ionosphere.

The effect of the atmosphere on signals can be considered as a systematic and random noise which must be removed, or as useful data that can be analysed. In the first case the aim is to remove the atmospheric effect by filtering, but in the second case the situation is quite the opposite.

The advantage of GPS observations is that we have permanent tracking stations with data accessible by Internet and we have a space receiver as Microlab I satellite. The goal of the work is to investigate the capabilities of the GPS observations to be useful as an independent source of information for ionospheric research.

The main parameter to be analysed with GPS observations is the total electron content (TEC). Thanks to the permanent tracking station service (IGS, International GPS Service for Geodynamics) global and regional maps of the total vertical electron content (VTEC) have been generated. These maps describe the average ionosphere conditions for a certain period of time (Brunini *et al.,* 1996). With the additional information available from observations obtained with a space bound receiver (MicroLab I), it is possible to obtain the variation of the electron density, called electron content (EC). The advantage is that we get information of the EC at different heights, as in these observations the signal does not go through the whole atmosphere, as in the case of earth bound receivers.

The development of ionosphere models has a great impact both in geodetic and geophysical aspects. Regarding geodesy, the ionosphere introduces an error in the scale factor of up to 10 ppm when single frequency receivers are used to measure a net. Ionosphere maps are useful to minimise this kind of errors. Regarding geophysics, the GPS observations allow to monitor the total electron content and its variations in the ionosphere.

2. DESCRIPTION OF THE MODEL

GPS satellites broadcast in 2 sinusoidal carrying waves, called L1 (1575.42 MHz) and L2 (1227.60 MHz). They are both modulated in phase by two pseudo-random codes: the P code and the C/A code, the latter one exclusively modulated in L1.

Bearing in mind the dispersive nature of the ionosphere we will use, among all the possible combinations of code and phase, the combination called 'free of geometry' (Kleusberg A. and Teunissen P., 1996). By using this observable, all frequency independent terms are removed, and we are left only with the ionospheric delay and the hardware differential delay.

The observable that we use is the geometry free signal, P_4, that we can obtain from the P-code (difference between the codes P2 and P1). The observation equation is (in meter):

$$k\,P_4 = I_1 + k\,10^{-9}\,c\left(\tau_R + \tau^s\right) + \varepsilon_R \qquad (1)$$

c is the light speed, τ_R is the hardware delay due to the receiver's clock, τ^s is the hardware delay due to the satellite's clock, ε_R measurement error, *k* is a dimensionless constant (1.546).

The ionospheric delay in L1 can be expressed in the following form:

$$I_1 = k_1\,TEC, \quad k_1 = -\frac{40,28}{f_1^2} \qquad f_1 \cong 1.5\ Ghz \qquad (2)$$

where the TEC quantity is the number of electrons in a unit column along the line of propagation:

$$TEC = \int_{path} EC\,ds \qquad \left[10^{16}\,electrons/m^2\right] \qquad (3)$$

where EC is the volumetric density of free electrons.

Using these expressions, we can obtain the observation equation that links the observable quantity P_4 with TEC, which we wish to model:

$$k\,P_4 = \frac{-40.28}{f_1^2}\,TEC + kc\left(\tau_R + \tau^s\right) + \varepsilon\ . \qquad (4)$$

2.1 Three dimensional model

To analyse ionospheric model in three dimensions, we use observations obtained with ground based receivers (IGS stations) and observations obtained with a space based receiver.

The ground based receivers allow us to model the ionosphere, but only in two dimensions. The signal crosses the ionosphere from the upper layer to the lower layer, so we only can describe it in terms of geographic latitude and longitude (ϕ and λ).

On April 3, 1995, the MicroLab I satellite was launched into a circular orbit of about 730 km altitude and 60° of inclination. On board the satellite is a high performance GPS receiver, observing signals travelling horizontally through the Earth's atmosphere. This experiment named GPS-MET is managed by the University Corporation Atmospheric Research (UCAR). Although GPS-MET is primarily focused on studying the lower neutral atmosphere, it is also able to contribute significantly to the study of the vertical structure of the ionosphere.

We use two types of GPS measurements to model the ionosphere:

- The measurements that are taken from the Earth where the signals go through the ionosphere, and
- The measurements which are taken from the GPS satellites by the LEO satellite.

Finally, it is more convenient to write down equation 4 in terms of the electron content (EC). Introducing the TEC definition (equation 3), we can express the observable P_4 in terms of EC as follows:

$$k\,P_4 = \frac{-40.28}{f_1^2}\int_{path} EC\,ds + kc\,\left(\tau_R + \tau^S\right) + \varepsilon . \qquad (5)$$

EC is the volumetric density of free electrons in the ionosphere. The Sun is the main agent that produces ionospheric changes, so we use a sun-fixed reference frame to model this function. This system is geocentric and the Z-axis points towards the North Pole, and the X - Z plane contains the Sun direction.

If we use an interval smaller than 12 hours we don't consider the temporal variations of the EC in this system, so the temporal variation of the coefficients can be ignored.

To describe the spatial variation, we propose:

- A spherical harmonic expansion to model the longitude and latitude variations (Brunini and Kleusberg, 1996).
- A linear combination of functions to model the height variations (h), which we will describe. We chose a Chapman function with fixed parameters (Meza *et al.*, 1997).

EC can be written as:

$$EC(h,\lambda,\varphi) = \sum_{l=1}^{L}\sum_{m=l}^{M}\sum_{k=0}^{K} \mathrm{f_k}(h)\left[a_{lmk}\cos(2\pi m\lambda) + \right.$$
$$\left. + b_{lmk}\sin(2\pi m\lambda)\right] P_l^m(\cos\varphi) . \qquad (6)$$

After replacing equation 6 in 5 we have:

$$k\,P_4 = \frac{-40.28}{f_1^2}\int_{path}\sum_{l=1}^{L}\sum_{m=l}^{M}\sum_{k=0}^{K} \mathrm{f_k}(h)\,\left[a_{lmk}\cos(2\pi m\lambda) + \right.$$

$$\left. b_{lmk}\sin(2\pi m\lambda)\right] P_l^m(\cos\varphi)\,ds + kc\,\left(\tau_R + \tau^S\right) + \varepsilon . \qquad (7)$$

Where $f_k(h)$, is expressed as:

$$f_k(h') = A\,\exp\left[1 - h' - \exp(-h')\right] \qquad (8)$$

where $h' = \frac{h - h_m}{H}$, H is the height scale; h_m is the height of maximum production rate.

Taking into account the typical vertical profile of ionospheric electron density and the Chapman layer model we propose to estimate the amplitude A (by least square method) and we fix the parameters h_m = 300 km and H = 90 km.

Finally, the equation 7 can be written as:

$$k\,P_4 = \frac{-40.28}{f_1^2}\sum_{l=1}^{L}\sum_{m=l}^{M}\sum_{k=0}^{0} a_{lmk}\int_{path} F_{lmk}^{a}(h,\lambda,\varphi)\,ds +$$
$$b_{lmk}\int_{path} F_{lmk}^{b}(h,\lambda,\varphi)\,ds + kc\,\left(\tau_R + \tau^S\right) + \varepsilon \qquad (9)$$

where a_{lmk} and b_{lmk} are the development coefficients and P_{lm} are the Legendre polynomials. They can be determined by a least square method, together with the hardware delay. The integrals depend on the path of the signal, in other words we need only the GPS and receiver positions to solve them. In this work we consider L=6.

3. NUMERICAL SIMULATION

As we wrote before, the equation of observation is:

$k\,P_4 = I_1 + k\,10^{-9}\,c\left(\tau_R + \tau^s\right) + \varepsilon_R$ where τ_R and τ^s are expressed in nanosecond.

Or we can write it as follows:

$$k\,P_4 = \frac{-40.28}{f_1^2}\int_{path} EC\,ds + k\,10^{-9}\,c\left(\tau_R + \tau^S\right) + \varepsilon$$

Using the equation 9 it can be able to generate the simulated observations, "P_4", as follow:

$$k''P_4'' = k_4\sum\sum\sum\left(a_{lmk}'\,I_{lmk}^{c} + b_{lmk}'\,I_{lmk}^{s}\right) + \tau_r' + \tau^{s'} + \varepsilon_R';k = 0 \qquad (10)$$

where, a_{lmk}' and b_{lmk}' are values that are introduced *a priori*, τ_r' and τ^s' are values that are introduced *a priori*, $I^c{}_{lmk}$ and $I^s{}_{lmk}$ are the integrals that can be solved using the satellites and receivers positions, ε_R' are values that are calculated using normal distribution, with σ=0.2 m.

In few words we generate the observations using a deterministic model. So if we have the receivers and satellites positions we can calculate the ionosphere delay. To be more 'real' these simulated observations, we add noise which has a normal distribution.

In the analysis the receivers on the Earth are chosen with a good geometrical distribution and the space-receivers are generated at equidistant nodes.

We suppose that the ionosphere behaviour is known perfectly, so we can produce the 'real' observable P4 (for each receiver-satellite).

Step by step, we have:

a) The coefficients a_{lmk} and b_{lmk} are selected. This is the 'real' model for the Ionosphere.

b) The Model for the Ionosphere is generated.

c) The 'measurements' for IGS+LEOs (receivers) are computed.

d) The noise (with a normal distribution) is added to the 'measurements'.
After producing the 'measurements', we have to check different models to different receivers' configuration.

e) We propose a model for the Ionosphere.

f) Least Squares Method is used to calculate the unknowns of the model.

g) We calculate the Total Electron Content (TEC), and the vertical profiles of the Electron Content (EC).

h) Finally, the inner agreement is checked.

3.1 General considerations

First we will write about the temporal distribution of the observations:

We worked with the day 033, 1996, at temporal interval: 9 hours, the temporal step is 15 minutes for GPS and 2 minutes for LEO

Model used to the ionosphere is:

-A spherical harmonic expansion to model the longitude and latitude (in the Sun Fixed Reference Frame (SFRF)) (Brunini and Kleusberg, 1996).

-Chapman model, the hm (height of maximum production rate) is fixed, and the amplitude is fitted using least square method (Ratcliffe, 1972).

$$EC(h,\lambda,\varphi) = \sum_{l=1}^{L}\sum_{m=l}^{M}\sum_{k=0}^{K} \mathrm{f}_{\mathrm{k}}(h)\left[a_{lmk}\cos(2\pi m\lambda) + b_{lmk}\sin(2\pi m\lambda)\right]P_l^m(\cos\varphi)$$

this model is used by k=0.

The integral along the signal path is TEC and can be written as:

$$\int_{path} ECds = \sum_{l=1}^{L}\sum_{m=l}^{M}\sum_{k=0}^{K}\left[a_{lmk}\int_{P1}^{P2} F^a_{lmk}(h,\lambda,\varphi)\,ds + b_{lmk}\int_{P1}^{P2} F^b_{lmk}(h,\lambda,\varphi)\,ds\right] = \sum_{l}^{L}\sum_{m}^{M}\sum_{k}^{K}\left[a_{lmk}I^a_{lmk} + b_{lmk}I^b_{lmk}\right]. \tag{11}$$

In the equation (5) we have the coefficients, which are the unknowns of the problem, and the integrals, which depend on the receivers and GPS positions.

We will work with 35 stations, which belong to IGS (International Geodetic Service) and 6 LEOs: one of them has the position of the real LEO (Microlab I) and the other five have 'virtual positions' at equidistant nodes.

We will use the precise ephemeris of the GPS satellites and Microlab I satellite. The positions of the other LEO satellites are obtaining from the position of the real LEO (GPS-MET).

3.2 Analysis of the geometry

We have only the information of the TEC along the path and from this observation we would like to find the height behaviour of the EC. So to compare the different results we use the TEC map obtained after fitting the coefficients and clock delays (eq. 5) by L S M (Least Squares Method). As we wrote before, we chose a co-ordinates system where the Sun position changes as slowly as possible. This system is called the Sun Fix Reference Frame (S F RF).

We work under the following conditions:

- 35 stations which belong to IGS.
- 6 LEO satellites.

Ionosphere model:

-The spherical harmonic expansion up to 6th degree.
-We choose hm equal to 300 km and H equal to 90km.

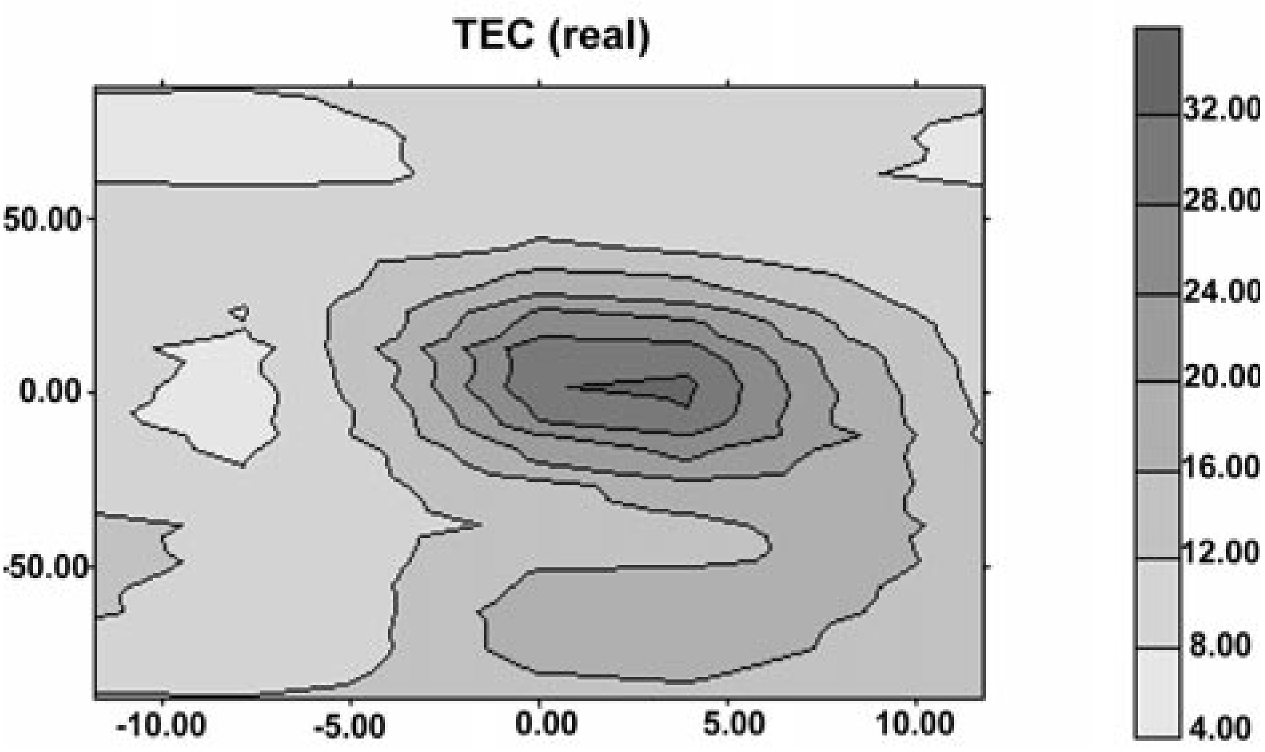

Fig. 1. The difference between the real values of the TEC (TEC(real)) and its values obtained after fitting the ionosphere model, give us the spatial behaviour of the errors. These variations are written down in maximum TEC value units, in other words if maximum TEC value is equal to 34 TECU (TEC Unity) and the errors is about ± 3.4 TECU, the range of variations is approximately ± 10% of the maximum TEC value. With 6 LEO and 35 earth stations, the errors is about ± 0.2 %, and with only 1 LEO the error is about ±1%. With these values we conclude that any variation larger than the pervious one, is due to the error of ionosphere model.

Noise:

-We suppose observational noise, with normal distribution and $\sigma = 0.20$ m (1,2 TECU) “Reference”

In the analysis we compare the TEC calculated using the *a priori* coefficients, which we call TEC(real), and TEC calculated using the fitted coefficients. The figure below shows the behaviour of TEC(real):

3.3 Analysis of the ionosphere’s model

As we know, the simulated observations can be written down as:

$$k\text{“}P_4\text{”} = k_4\Sigma\Sigma\Sigma\,(a_{lmk}'I^c_{lmk}+b_{lmk}'I^s_{lmk})+\tau_r'+\tau^s{}'+\varepsilon_R'\ ;\quad k=0 \qquad (12)$$

where : $I_{lmk} = \int f_1\,(\lambda, \varphi)\, f_2\,(h)\, ds$.

There are two aspects of ionosphere model:

- Latitude and longitude model: spherical harmonic expansion.
- Height model: Chapman model.

a- Modification in the spherical harmonic expansion (up to n degree).

$$f_1\,(\lambda,\varphi) = \Sigma^l\,\Sigma^m\,c_{lm}f_1'\,(\lambda,\varphi)$$

b- Modification in the Chapman model parameters.

$$f_2\,(h^*) = \mathrm{A}\,f_2'(h^*),\quad h^*=(h-h_m)/H\ .$$

The parameters are: h_m (height of maximum production rate)
H (height scale).

First we analyse the latitude and longitude variations, assuming that we have the real height model. When we use the spherical harmonic model up to 4 degree, the range of error is about ±17 % when we use 6 LEO and 26 % when we use only 1 LEO (in both case we use the 35 earth stations). The error behaviour shows the 5-degree missing component (in spherical harmonic).

Then we analyse the height variations, assuming the latitude and longitude model perfectly known. We have two choices: we can change the h_m parameter, keeping the real value of H parameter or do it the other way around.

Table

Variations of the ionosphere model

Change in the original model and number of stellite receivers	Variations (TECU)	Comments
• up 4 degree and 6 LEO	-6 to 6	5th degree component
• up 4 degree and 1 LEO	-9 to 6	Idem
• up 4 degree without LEO	-6 to 6	Idem
• h_m = 400km and 1 LEO	-5 to 5	—-
• h_m = 400km and without LEO	-2 to 0	Strong latitudinal component
• h_m = 200km and 1 LEO	-8 to 3	—-
• h_m = 200km and without LEO	0.5 to 2.5	Strong latitudinal component
• H = 120km and 1 LEO	-2.5 to 0	—-
• H = 120km and without LEO	0.5 to 0	—-
• H = 60km and 1 LEO	0 to 4.5	—-
• H = 60km and without LEO	-0.5 to 0.5	—-

The height model that we propose is very simple and it is not too close to the true vertical profile of the ionosphere, the reason for choosing this kind of model is only for numerical purposes.

If we propose h_m= 400 km (Δh_m=+100 km) the error is between 0 TECU to 8 TECU when we use 6 LEO and 35 Earth stations. With only 1 LEO (and the Earth stations) the error is between –5 TECU and +5 TECU. If we only use the observation of the 35 Earth receivers, the error is five times smaller than if we work with the Earth stations and 1 LEO. That means that the results obtained in the first case are five times less sensitive of the h_m variations than the second one.

As we can see in the table below, when we change the H parameter the range error is 4 times smaller than in the case before.

We can conclude that in both cases the LEO observations give us additional information about the height variation of the ionosphere, we can say that the LEO satellite is sensitive to the height variation of the ionosphere.

General conclusion:

- Earth stations with a “good geometry” give us enough information to model the ionosphere in latitude and longitude (SFRF)
- GPS - MET satellite give us enough information to model the ionosphere in height as well as in latitude and longitude.

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