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Dusty plasma in space

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RESUMEN

Intensos estudios de la física de plasmas realizados desde hace alrededor de 10 años han visto el surgimiento de una nueva línea de investigación: la física de plasmas polvosos, que consisten de electrones, iones y partículas de polvo cargadas. Se observan en varios medios astrofísicos como nebulosas, colas cometarias, anillos planetarios, ionosferas planetarias, etc. La presencia de partículas de polvo pesadas y cargadas puede influenciar significativamente varios parámetros del plasma al crear, por ejemplo, estructuras en su interior en un rango muy amplio de cientos a decenas de millones de kilómetros. En este trabajo presentamos una revisión y un análisis del espectro del volumen y ondas superficiales, estabilidades y solitones en un plasma polvoso.

PALABRAS CLAVE: Plasmas polvosos, ondas, inestabilidades, solitones, cristales de Coulomb.

ABSTRACT

Dusty plasmas consist of electrons, ions and charged dust particles observed in several astro-and space-physical environments such as nebulas, cometary tails, planetary rings, and planetary ionospheres. The presence of heavy, highly charged dust may significantly influence various physical processes, for example, the creation of spatial structures over a wide range of distances, from hundreds to tens of millions of kilometres. In this paper we present a review and an analysis of the spectra of volume and surface waves, instabilities and solitons in a dusty plasma.

KEY WORDS: Dusty plasma, waves, instabilities, solitons, Coulomb crystals.

DETERMINATION AND PARAMETERS OF SPACE DUSTY PLASMA

In recent years there has been a growing interest in the study of physical processes in plasma consisting of electrons, ions and charged dust grains. Dusty plasmas are rather common in space, being found in planetary rings, interstellar clouds, cometary plasma tails, and the ionosphere of the Earth and other planets (e.g. Goertz, 1989; Spitzer, 1978). In the Earth's ionosphere the origin of plasma components is from eruptions of volcanoes, meteorite showers as well as anthropogenic factors: rocket and airplane exhausts, large fires, explosions, and so on. Laboratory plasma is surrounded by dielectric walls and contains a dusty component, produced by different processes on the walls: photoionization processes are dominant. Special plasma source machines can obtain dusty plasmas with density changing over many orders of magnitude (Motley, 1975).

The size of dust grains R is usually between a fraction of a micrometer to hundreds of micrometers. The charge of the grain Q is caused by different processes on its surface: electron and ion flows, photoemission, second electron emission, chemical reactions. As in dominant flow processes, the grain has a potential $\varphi \approx 2k_BT_e/e$ and a negative charge value $Q \approx \varphi R$, where k_B is the constant of Boltzmann, T_e is the temperature of electrons, and e is the charge of the electron. The charge number $Z = Q/e \approx 2k_BT_eR/e^2$ depends on the size of the grain and the temperature of the plasma is $Z \approx 10\text{-}10^4$. When the charge is produced by processes of photoemission and second electron emission the grains will have a positive charge, as near the core of comet Halley where the cometary plasma is dense and the grains have an average potential of about -7 V. In the tail, however, the plasma is less dense, and the average grain potential is about +7 V (de Angelis *et al*., 1988). For these values of the radii of grains, their mass, m, will be in the range $m \approx (10^6\text{-}10^{12})m_p$, where m_p is the proton mass.

In accordance with first detections of charged dust particles in the Earth's ionosphere at heights from 80 to 90 km (Havnes *et al*., 1996), there are charged dusty particles with $R \approx 0.06\mu m$, $Z \approx +80$, and density $n_d \approx 10^2\text{-}10^3\ cm^{-3}$. Positive charges must be a result of photoemission. The charge number density n_dZ_d of positive dust may be larger than the electron density n_e without dust. For negative charged particles, average values of $R \approx 0.02\mu m$, $Z \approx -1$ and $n_d \approx 10^3\ cm^{-3}$ are obtained.

In cometary tails of the comets Halley and Giacobini-Zinner, there is a dusty component with parameters $R \approx 1\mu m$, $Z \approx 400\text{x}10^{10}\ cm^{-3} \leq n_d \leq 10^{-7}\ cm^{-3}$.

In the planetary rings of Jupiter and Saturn, from measurements of Voyager spacecraft, there is a charged dusty component with parameters $R \approx 1\mu m$, $Z \approx 10^2\text{-}10^4$, $10^{-2}\ cm^{-3}$

$\leq n_d \leq 10^{-1}\, cm^{-3}$. The first success of the theory of space dusty plasma was connected with the attempt to explain the observations by Voyager of the spokes of planetary rings as space charge waves in a dusty plasma in Saturn (Bliokh and Yaroshenko, 1985; Goertz, 1989).

The first paper on laboratory dusty plasma was written by Langmuir *et al*. (1924). Remarkable successes in the field of laboratory plasma were obtained with macroscopic Coulomb crystals in strongly coupled dusty plasmas (Chu and Lin, 1994). Theoretically, this effect was predicted by Ikezi (1986).

THE INFLUENCE OF CHARGED DUSTY PARTICLES ON PLASMA PROPERTIES

Consider a plasma of electrons, ions and identical dusty particles, each of mass *m* and charge *Ze*. Actually, the grains have different sizes (Aslakyan and Havnes, 1994; Northrop, 1992); however, in the first approximation one can assume the grain radius to be constant. In this approximation, the mass of a dust grain is typically 6-12 orders of magnitude larger than that of a proton. Thus the time scales associated with the dusty component, the Langmuir frequency $\omega_p = \sqrt{4\pi Z^2 e^2 n_d / m}$ and the cyclotron frequency $\omega_H = ZeB_o/mc$, will be orders of magnitude greater than those of the ionic component. Thus self-oscillations in a dusty plasma, where grain motion is important, will be characterized by very low frequencies; for example, in Saturn's rings $\omega_p \approx 10^{-3} s^{-1}$ (Bliokh and Yaroshenko,1985). Such a system will be seen as ***spatial***, with a halflife of hours or days. For many dusty plasmas such time scales are perfectly observable.

The influence of charged dust particles on plasma oscillations is manifested in two ways. For high-frequency plasma oscillations the density of electrons, n_e, and ions, n_i, will no longer be equal. Instead, $n_i = n_e + Zn_d$, which leads to a modification of the spectrum of plasma waves. For example the dispersion relation for ion-acoustic waves takes the form $\omega = kC_s$, where $C_s = \sqrt{T_e n_i / n_e m_i}$ (Shukla and Silin, 1992). Then, if $n_i >> n_e$ the condition of the existence of these waves, ($V_{Ti} << \omega/k << V_{Te}$) may be fulfilled even in an isothermal plasma, where $V_{Te,Ti}$ are the thermal velocities of electrons and ions. In the case of Alfvén waves, for $\omega << \omega_H$ the velocity is $V_A = B_o / \sqrt{4\pi(n_i m_i + n_d m)}$ (Rao, 1993). Thus when the dust density $n << n_d$, but nm $>> n_i m_i$, the Alfvén velocity will be determined by the dusty component. In addition, a whole new set of low-frequency waves, with frequencies close to the Langmuir frequency of the dusty component ω_p (dusty acoustic waves, Rao *et al.*, 1990) or to the dust's cyclotron frequency ω_H (D'Angelo,1990). Another specific feature of the dusty plasma is a new mechanism for wave damping, which is associated with the transformation of the ions into neutral atoms during ion-grain collisions (Melandso *et al*.,1993a; D'Angelo,1994).

SPECTRUM OF WAVES SUPPORTED BY A DUSTY COMPONENT

Consider waves in a dusty plasma first without an external magnetic field. For a model of a three-component electron-ion-charged grains plasma we may use common formulas for electron-ion plasma by adding a system of identical charged particles, each with mass *m* and charge *-Ze* (*Z*>0) for negative charged grains and $Z < 0$ for positive charged grains. Thus we may use the dispersion relation for electrostatic waves $\vec{E} = -\nabla\varphi$ in the hydrodynamical approximation (Akhiezer *et al.*, 1975):

$$\varepsilon_l(\omega,k) = 1 - \sum_{\alpha=e,i,d} \frac{\omega_{p\alpha}^2}{\omega^2 - k^2 V_{t\alpha}^2} = 0 \tag{1}$$

where $\omega_{pe} = \sqrt{4\pi e^2 n_e / m_e}$, $\omega_{pi} = \sqrt{4\pi e^2 n_i / m_i}$, $\omega_{pd} \equiv \omega_p = \sqrt{4\pi Z^2 e^2 n_d / m}$ are Langmuir frequencies of electrons, ions and charged particles, respectively, and $V_{T\alpha} = \sqrt{T_\alpha / m_\alpha}$ are thermal velocities (***T*** is in energetic units). The solution for waves is found in the form $\exp i(\omega t - \vec{k}\cdot\vec{r})$. Densities n_α are connected by the electroneutrality condition

$$n_i = n_e + Z_{nd} \quad . \tag{2}$$

(a) Ion-acoustic and dusty acoustic waves

For the case $\omega << kV_{Te}$, $\omega >> kV_{Ti}$, $\omega_{pd}^2 / \omega_{pi}^2 \approx Z^2 m_i / m << 1$, equation (1) becomes:

$$1 + \frac{\omega_{pe}^2}{k^2 V_{Te}^2} - \frac{\omega_{pi}^2}{\omega^2} \approx 0 \quad .$$

From here we obtain the solution for frequency in the form:

$$\omega = \frac{kC_s}{\sqrt{1 + k^2 a_e^2}} \quad , \tag{3}$$

where $C_s = \sqrt{T_e n_i / m_i n_e}$, $a_e = V_{Te}/\omega_{pe}$ is the Debye radius of electrons. Formula (3) describes ion-acoustic waves in a dusty plasma as first obtained by Shukla and Silin (1992). As seen from (3) the presence of negatively charged grains increases the velocity of ion-acoustic waves in comparison with electron-ion plasma. At $n_i >> n_e$ (as in Saturn's rings), the condition for the existence of ion-acoustic waves $V_{Ti} << C_s << V_{Te}$ may be fulfilled even in the case of an isothermal plasma $T_e \approx T_i$. The influence of charged particles on ion-acoustic waves may be also obtained from the decrement of the Landau damping of these waves (Akhiezer *et al*.,1975):

$$\gamma \approx \sqrt{\frac{\pi}{8}\frac{m_e}{m_i}\frac{n_i}{n_e}\frac{kC_s}{(1+k^2a_e^2)^2}} \quad .$$

At $ka_e >> 1$ we have $\gamma \propto n_e n_i \propto n_i\ (n_i\text{-}Zn_{nd})$; thus the presence of negative grains (or negative ions) decreases Landau damping of ion-acoustic waves. Both results were experimentally demonstrated (Barkan *et al.*,1996). A similar effect in a negative ion plasma was predicted by D'Angelo *et al.* (1966) and observed in experiment (Song *et al.*, 1991).

For the case of very low-frequency slow waves $\omega << kV_{Te,i}$, Eq. (1) provides the solution for ω in the form

$$\omega^2 = \frac{k^2}{m}\left\{T_d + \frac{Z^2T_eT_in_d}{n_iT_e + n_eT_i(1+k^2a_e^2)}\right\} \quad . \tag{4}$$

This expression was first obtained by Rao *et al.* (1990); it describes the so called *dusty acoustic waves.* Eq. (4) may also be obtained from the system of equations

$$\begin{aligned}
&\Delta\varphi = 4\pi e(n_e + Zn_d - n_i)\\
&n_e = n_{e0}e^{e\varphi/T_e} \approx n_{eo}(1+e\varphi/T_e),\\
&n_i = n_{io}e^{-e\varphi/T_i} \approx n_{io}(1+e\varphi/T_i)\\
&\frac{\partial n_d}{\partial t} + div(n_d\vec{V}) = 0,\ \frac{\partial\vec{V}}{\partial t} = \frac{Ze}{m}\nabla\varphi - \frac{T_d}{n_d m}\nabla n_d
\end{aligned} \tag{5}$$

where $n_d = n_o + \tilde{n}_d$, $|\tilde{n}_d| << n_o$, n_o and $\tilde{n}_d$ are the equilibrium and variable densities of charged grains, respectively. In other words, oscillations (4) are so slow that electrons as well as ions distribute according to Boltzmann's formula.

(b) Cyclotron oscillations

Consider electrostatic oscillations of cold electron-ion-charged grain plasma in the presence of a magnetic field $\vec{B}_o = \{0,0,B_o\}$.We will use Poisson's equation

$$\Delta\varphi = 4\pi e\ (n_e + Zn_d - n_i) \tag{6}$$

and the equations of continuity and motion for each type of particles:

$$\frac{\partial n_\alpha}{\partial t} + div(n_\alpha\vec{V}_\alpha) = 0,\quad \frac{\partial\vec{V}_\alpha}{\partial t} = \frac{e_\alpha}{m_\alpha}\left(\vec{E} + \frac{\vec{V}\times\vec{B}_o}{c}\right) \tag{7}$$

where $e_d = -Ze$, $m_d = m$. Writing the density of particles n_α in the form $n_\alpha = n_{\alpha o} + \tilde{n}_\alpha$ and assuming $|\tilde{n}_\alpha| << n_{\alpha o}$ in the linear approximation for waves with wave vector $\vec{k} = k\{\sin\theta, 0, \cos\theta\}$, we obtain from (6) and (7) the dispersion relation in the form

$$1 - \sum_\alpha \omega_{p\alpha}^2\left(\frac{\sin^2\theta}{\omega^2 - \omega_{H\alpha}^2} + \frac{\cos^2\theta}{\omega^2}\right) = 0 \quad , \tag{8}$$

where $\omega_{He} = eB_o / m_ec$, $\omega_{Hi} = eB_o / m_ic$, $\omega_{Hd} = ZeB_o / mc \equiv \omega_H$ are cyclotron frequencies of electrons, ions and grains, respectively. For low frequencies $\omega << \omega_{Hi}$ we obtain from (8)

$$\omega \approx \omega_H\left(1 - \frac{\omega_p^2}{2\omega_{pe}^2}tg^2\theta\right) \quad . \tag{9}$$

Equation (9) describes the oscillations of the cyclotron frequency of charged grains (D'Angelo, 1990). Experimentally these oscillations were investigated by Barkan *et al.*(1995).

In the range of higher frequencies $\omega_{\mathrm{Hi}} << \omega << \omega_{He}$ we have from (8) the solution for ω in the form

$$\omega^2 \approx \frac{\omega_{pe}^2\cos^2\theta + \omega_{pi}^2}{1 + \frac{\omega_{pe}^2}{\omega_{He}^2}\sin^2\theta} \quad . \tag{10}$$

For the case $\omega_{pe} >> \omega_{He}$ and $\theta \approx \pi/2$ (if $\omega_{pe}^2\cos^2\theta << \omega_{pi}^2$), from (10) we obtain the solution for the lower hybrid mode:

$$\omega \approx \sqrt{\frac{n_{oi}}{n_{oe}}\sqrt{\omega_{He}\omega_{Hi}}} \quad . \tag{11}$$

From (11) it is seen that the frequency of the lower hybrid wave in dusty plasma with negative grains is larger by a factor of $\sqrt{n_{oi}/n_{oe}} > 1$ in comparison with that in an electron-ion plasma without dust grains.

(c) Low-frequency electromagnetic waves

We use the common dispersion equation for waves in a cold collisionless magnetized plasma, for a frequency $\omega << \omega_{pe}$ (Akhiezer *et al.*, 1975):

$$n_4\cos^2\theta - n^2\varepsilon_1(1+\cos^2\theta) + \varepsilon_1^2 - \varepsilon_2^2 = 0 \ , \tag{12}$$

where $n = ck/\omega$, θ is the angle between the wave vector $\vec{k}$ and the direction of external magnetic field $\vec{\mathrm{B}}_0$, and ε_1 and ε_2 are components of the permittivity tensor:

$$\varepsilon_1 = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{He}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{Hi}^2} - \frac{\omega_p^2}{\omega^2 - \omega_H^2}$$

$$\varepsilon_2 = -\frac{\omega_{He}\omega_{pe}^2}{\omega(\omega^2 - \omega_{He}^2)} + \frac{\omega_{Hi}\omega_{pi}^2}{\omega(\omega^2 - \omega_{Hi}^2)} - \frac{\omega_H\omega_p^2}{\omega(\omega^2 - \omega_H^2)} \quad .$$

For $\omega << \omega_{H,Hi,He}$, the value of ε_1 is $\varepsilon_1 \approx 1 + c^2/V_A^2 >> |\varepsilon_2|$, where $V_A = B_0/\sqrt{4\pi(n_im_i + n_em_e + nm)}$. From (12) we find the solutions

$$\omega = \frac{kV_A\cos\theta}{\sqrt{1+V_A^2/c^2}} \quad , \quad \omega = \frac{kV_A}{\sqrt{1+V_A^2/c^2}} \tag{13}$$

for Alfvén and fast magnetoacoustic waves, respectively. The dusty plasma contributes only at the Alfvén speed. The case of frequencies $\omega << \omega_{Hi}$ is of more interesting, when we have

$$\varepsilon_1 \approx \varepsilon_0 - \frac{\omega_p^2}{\omega^2 - \omega_H^2}, \quad \varepsilon_1 \pm \varepsilon_2 \approx \varepsilon_0 \mp \frac{\omega_p^2}{\omega_H(\omega \mp \omega_H)} , \quad (14)$$

and the value of the permittivity where is $\varepsilon_0 = 1 + c^2 / C_A^2$, where $C_A = B_0 / \sqrt{4\pi(n_i m_i + n_e m_e)}$ is Alfvén's speed in absence of the dusty component. To obtain $\varepsilon_1 \pm \varepsilon_2$, it is necessary to use the condition $n_i - n_e = Zn$ and therefore the difference of electron and ion terms $4\pi e^2 c(n_i - n_e) / e\omega B_0 \equiv \omega_p^2 / \omega\omega_H$ yields the same contribution as the grains. Then $\varepsilon_1 \pm \varepsilon_2$ depends on frequency only through $\omega \mp \omega_H$. First consider the separate cases of waves propagating along and across the magnetic field direction (Kotsarenko, 1997).

At $\theta = 0$ from (12) and (13) we have $n^2 = \varepsilon_1 \pm \varepsilon_2$ or

$$k = \frac{\omega}{c}\sqrt{\varepsilon_0 \mp \frac{\omega_p^2}{\omega_H(\omega \mp \omega_H)}} \quad . \quad (15)$$

If the upper signs are taken, the wave has left rotation and the solution for spreading waves presents two frequency ranges:

(a) $0 \le \omega < \omega_H$. If $\omega << \omega_H$, the dispersion equation (15) transforms into (13). When $\omega \to \omega_H$ the wave vector k increases (Figure 1a). This is a slow mode with phase velocity $V_f = \omega / k < c / \sqrt{\varepsilon_0}$.

(b) $\omega \ge \omega_s^2 / \omega_H$, where $\omega_s^2 = \omega_H^2 + \omega_p^2 / \varepsilon_0$. In this range of frequency, the wave is fast with a phase velocity $V_f > c / \sqrt{\varepsilon_0}$ (Figure 1b).

In the case of the lower signs the fast wave has right rotation and exists in all ranges of frequency $0 \le \omega < \infty$ (Figure 1b). At an angle $\theta = \pi / 2$ from (12) and (14) we have $n^2 = (\varepsilon_1^2 - \varepsilon_2^2) / \varepsilon_1$ or the solution for $k(\omega)$:

$$k = \frac{\omega}{c}\sqrt{\varepsilon_0}\sqrt{\frac{\omega^2 - \omega_s^4 / \omega_H^2}{\omega^2 - \omega_s^2}} \quad . \quad (16)$$

In this case, for the range $0 \le \omega < \omega_s$, we have the slow mode, and for the range $\omega \ge \omega_s^2 / \omega_H$ we have the fast mode (Figure 2).

In order to study waves of arbitrary θ we rewrite equation (12) in the form

$$k^4 - k^2 \frac{\omega^2}{c^2}\varepsilon_0 \frac{1+\cos^2\theta}{\cos^2\theta}\frac{\omega^2 - \omega_s^2}{\omega^2 - \omega_H^2} +$$

$$\frac{\omega^4}{c^4}\frac{\varepsilon_0^2}{\cos^2\theta}\frac{\omega^2 - \omega_s^4 / \omega_H^2}{\omega^2 - \omega_H^2} = 0 \quad . \quad (17)$$

Equation (17) defines two solutions $k_{1,2}(\omega)$ for a fixed frequency ω and angle θ.

First we note that the cut-off frequency of the waves, $\omega_c \equiv \omega(k=0)$ at any angle θ is $\omega_c = \omega_s^2 / \omega_H$. For ω near cut-off frequency ω_c, from (17) we find two solutions: a first solution with $k \to 0$ at $\omega \to \omega_c$:

$$k^2 \approx \frac{\omega^2 - \omega_c^2}{c^2}\frac{\omega_s^2}{\omega_p^2(1+\cos^2\theta)} \quad (18)$$

describing the fast wave, and a second solution with $k \ne 0$ at $\omega \to \omega_c$:

$$k^2 \approx \frac{\omega^2}{c^2}\varepsilon_0\frac{1+\cos^2\theta}{\cos^2\theta}\frac{\omega_s^2\omega_p^2}{\omega_s^4\omega_H^4} \quad . \quad (19)$$

Analogously, for $\omega \le \omega_H$, it is possible to find a first solution from (17) for $k \to \infty$ at $\omega \to \omega_H$-0:

$$k^2 \approx \frac{\omega^2}{c^2}\frac{1+\cos^2\theta}{\cos^2\theta}\frac{\omega_p^2}{\omega_H^2 - \omega^2} \quad (20)$$

and a second solution limited at $\omega \to \omega_H$:

$$k^2 \approx \frac{\omega^2}{c^2}\frac{\varepsilon_0^2}{1+\cos^2\theta}\frac{\omega_s^4 - \omega_H^4}{\omega_H^2\omega_p^2} \quad . \quad (21)$$

If $\omega \to \infty$, from (17) we obtain the solutions $k \approx \omega\sqrt{\varepsilon_0} / c\cos\theta$ and $k \approx \omega\sqrt{\varepsilon_0} / c$, describing Alfvén and fast magnetoacoustic waves in electron-ion plasma. At low frequencies $\omega << \omega_H$, as mentioned above, we obtain again solution (13) defining Alfvén and magnetoacoustic waves in a dusty plasma taking into account the contribution of a charged dusty component.

These particular solutions yield the qualitative possibility to reconstruct the dependence $k(\omega)$ for any angle θ for all range of frequency range (Figure 3).

From Figure 3 it is seen that in a dusty plasma in the low-frequency range $\omega << \omega_{Hi}$, there are two characteristic Alfvén speeds: a low velocity in the dusty plasma V_A and a high one in the electron-ion plasma C_A. Two modes have velocities less than $c\cos\theta / \sqrt{\varepsilon_0}$, one mode is fast with phase velocity $V_f > c / \sqrt{\varepsilon_0}$.

(d) Kinetic Alfvén waves

Kinetic Alfvén waves (KAW) are interesting for the following reason. For ordinary Alfvén waves we have $\omega = k_z V_A$,

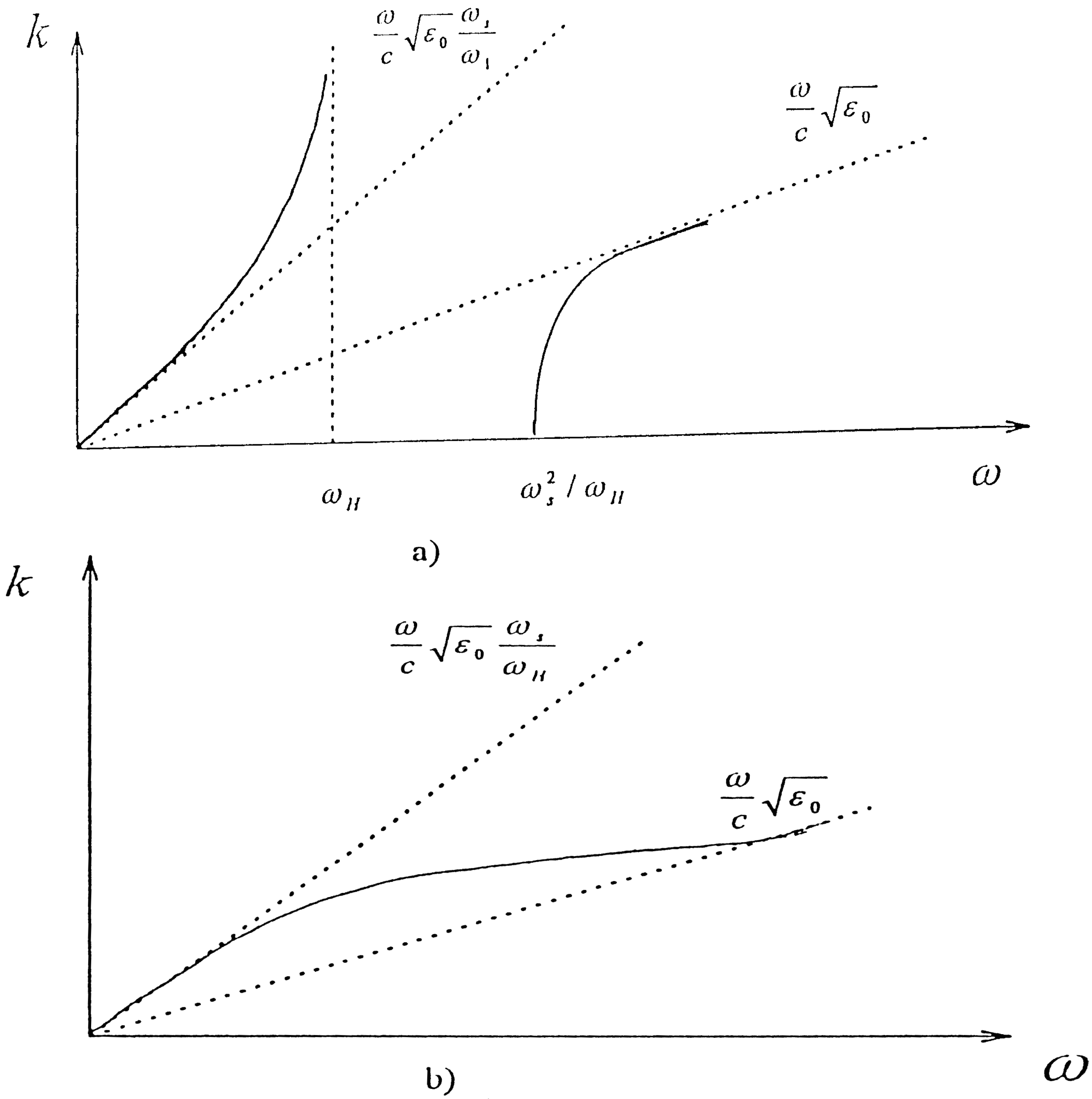

Fig. 1. Low frequency electromagnetic waves for the angle $\theta = 0$. (a) Left-rotating: in frequency ranges $0 \le \omega < \omega_H$ the wave is slow; in the frequency range $\omega \ge \omega_s^2 / \omega_H$ the wave is fast; (b) right rotating: the wave exists in frequency range $0 < \omega < \infty$.

the group velocity is $\vec{V}_g = \partial\omega / \partial\vec{k}$ and it is strongly oriented along the magnetic field $\vec{V}_g = V_A \vec{B}_0 / B_0$. For kinetic Alfvén waves, taking into account the temperature of electrons (Hasegawa *et al.*, 1989), the law of dispersion has the form

$$\omega^2 = k_z^2 V_A^2 (1 + k_\perp^2 C_{si}^2 / \omega_{Hi}^2) \quad , \qquad (22)$$

where $C_{si} = \sqrt{T_e / m_i}$. Hence, a component of the group velocity perpendicular to the magnetic field and a longitudinal component of the electric field of the wave appear. This is very important for processes of interaction of charged flows with KAW and for the contribution of the KAW in processes of transformation of waves in space plasma, first of all, for the process of heating of the solar corona by means of Alfvén waves (Kotsarenko *et al.*, 1993).

In order to study KAW in a dusty plasma, we start from Maxwell's equations

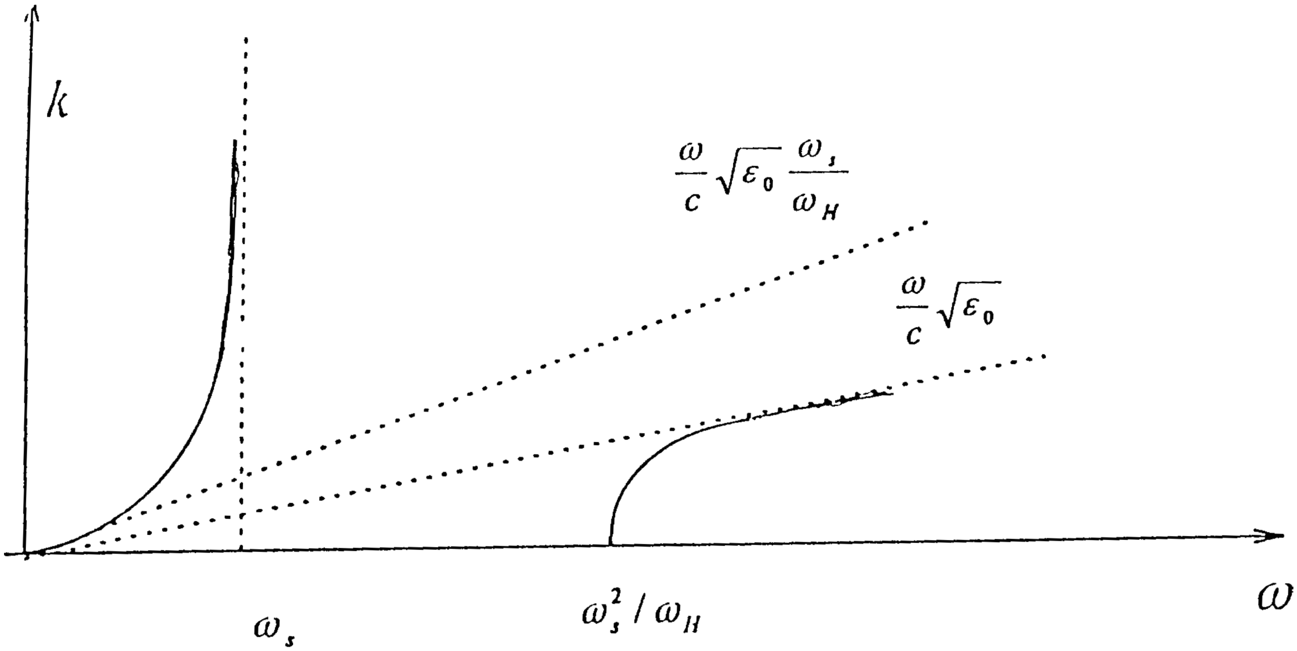

Fig. 2. Low-frequency electromagnetic waves for angle $\theta = \pi/2$. In the frequency range $0 \le \omega < \omega_s$ the wave is slow; in the frequency range $\omega \ge \omega_s^2/\omega_H$ the wave is fast.

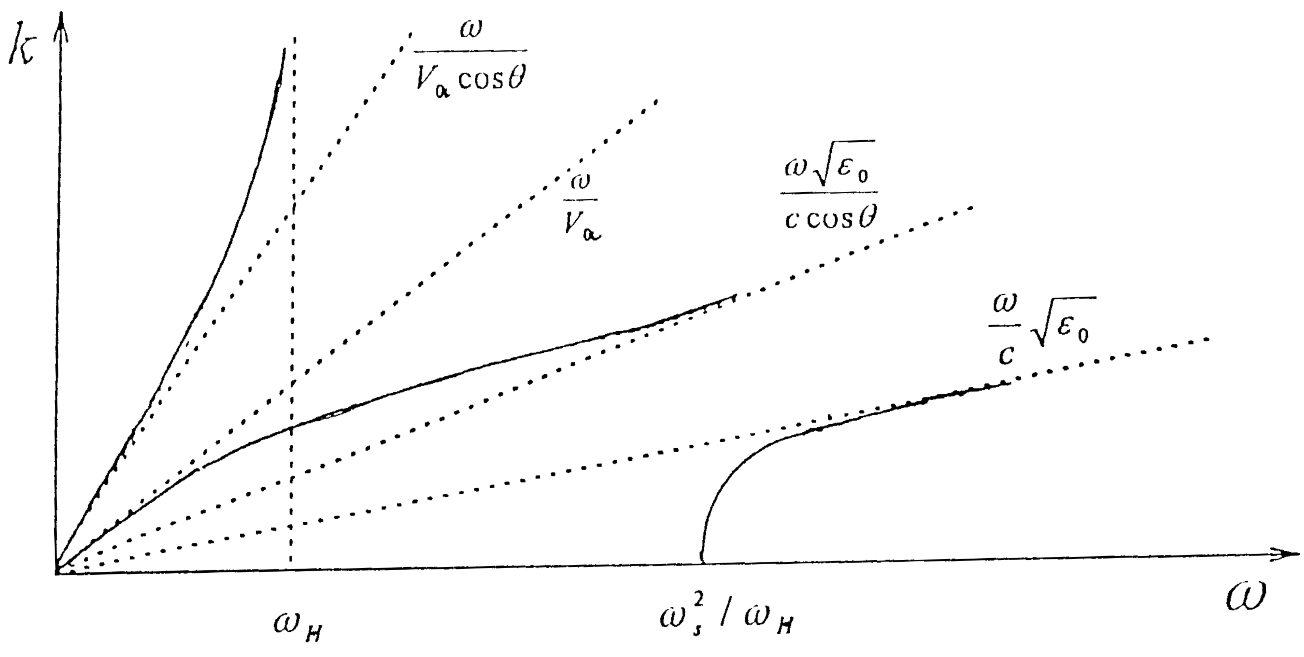

Fig. 3. Alfvén and magnetoacoustic waves.

$$rot\vec{B} = \frac{4\pi}{c}\vec{j}, \qquad rot\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$
$$\vec{j} = -e(n_e\vec{V}_e + Zn\vec{V} - n_i\vec{V}_i), \tag{23}$$

the equations of motion for charged particles

$$\frac{\partial \vec{V}_i}{\partial t} = \frac{e}{m_i}\vec{E} + \vec{V}_i \times \vec{\omega}_{Hi} - \frac{T_i}{n_i m_i}\nabla n_i$$

$$\frac{\partial \vec{V}}{\partial t} = -\frac{Ze}{m}\vec{E} - \vec{V} \times \vec{\omega}_H \quad , \tag{24}$$

and the equation of continuity for a charged dusty component

$$\frac{\partial n}{\partial t} + div(n\vec{V}) = 0 \quad . \tag{25}$$

It is convenient to use scalar φ and vector $\vec{A}$ potentials

$$\vec{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}, \qquad \vec{H} = rot\vec{A}. \qquad (26)$$

As these potentials are not uniquely determined, we may choose $A_{x,y}$=0. The solutions for φ and A_z are found in the form

$$\varphi, A_z \sim \exp i(\omega t - k_z z - \vec{k}_\perp \cdot \vec{r}_\perp) \ .$$

Writing the densities of electrons and ions as

$$n_{e,i} = n_{e,io} + \delta n_{e,i}\left(\left|\delta n_{e,i}\right| << n_{e,io}\right)$$

and assuming $\omega/k_z << V_{Te,i}$, from the z-component of first two equations (24), we obtain:

$$\delta n_e = \frac{en_{oe}}{k_z T_e}\left(k_z\varphi - \frac{\omega}{c}A_z\right),\ \delta n_i = -\frac{en_{oi}}{k_z T_i}\left(k_z\varphi - \frac{\omega}{c}A_z\right) \ . \quad (27)$$

The variable density of a dusty component δn ($n = n_o + \delta n$) is obtained from (25) and the last equation (24):

$$\delta n = i\frac{n_o}{\omega}div\vec{V} = -\frac{Zen_o}{m\omega^2}\left(k_z^2\varphi + \frac{\omega^2 k_\perp^2}{\omega^2 - \omega_H^2}\varphi - \frac{\omega k_z}{c}A_z\right) \ . \qquad (28)$$

Assuming condition $ka_e << 1$ from the neutrality condition $\delta n_i = \delta n_e + Z\delta n$, and using (27) and (28), we obtain the first equation for φ and A_z:

$$\varphi\left(1 - \frac{k_z^2 C_d^2}{\omega^2} - \frac{k_\perp^2 C_d^2}{\omega^2 - \omega_H^2}\right) = \frac{\omega}{k_z c}\left(1 - \frac{k_z^2 C_d^2}{\omega^2}\right)A_z \ , \qquad (29)$$

where $C_d = \sqrt{Z^2 T_e T_i n_o / (n_{oe}T_i + n_{oi}T_e)m}$ is the speed of volume dusty acoustic waves (Rao *et al*., 1990). To obtain a second equation for potentials φ and A_z, we use the equation for $\vec{E}$, obtained from (23):

$$\nabla div\vec{E} - \Delta\vec{E} = -\frac{4\pi\omega i}{c^2}\vec{j} \ . \qquad (30)$$

Operating on the formula (19) with $\nabla_\perp\cdot$, we have :

$$\nabla_\perp div\vec{E} - \Delta\nabla_\perp \cdot \vec{E}_\perp = -\frac{4\pi\omega i}{c^2}\nabla_\perp \cdot \vec{j}_\perp \ . \qquad (31)$$

From (24) the solutions for the velocities of ions and charged grains for $\omega << \omega_{Hi}$ are

$$\nabla_\perp \cdot \vec{V}_{\perp i} \approx \frac{ie\omega}{m_i\omega_{Hi}^2}\nabla_\perp \cdot \vec{E}_\perp,$$

$$\nabla_\perp \cdot \vec{V}_\perp \approx \frac{iZe\omega}{m(\omega^2 - \omega_H^2)}\nabla_\perp \cdot \vec{E}_\perp \ .$$

Next we obtain from (31) a second equation for φ and A_z:

$$A_z = \frac{\omega}{ck_z}\left(\frac{\omega_{pi}^2}{\omega_{Hi}^2} - \frac{\omega_p^2}{\omega^2 - \omega_H^2}\right)\varphi \ . \qquad (32)$$

From (29) and (32) the dispersion equation follows in the form

$$1 - \frac{C_d^2 k_z^2}{\omega^2} - \frac{C_d^2 k_\perp^2}{\omega^2 - \omega_H^2} = \frac{\omega^2}{ck_z^2}\left(1 - \frac{k_z^2 C_d^2}{\omega^2}\right)\left(\frac{\omega_{pi}^2}{\omega_{Hi}^2} - \frac{\omega_p^2}{\omega^2 - \omega_H^2}\right). \qquad (33)$$

From here, for the case $\omega << \omega_{Hi}$, $C_d << V_A$, we obtain the solution

$$\omega^2 \approx k_z^2 V_A^2\left(1 + k_z^2 C_d^2 / \omega_H^2\right) \ , \qquad (34)$$

which is a natural generalization of (22) for the case of a dusty plasma. This formula, for the case $n_e << n_i$ when $C_d \cong \sqrt{ZT_i / m}$, was obtained by Shukla and Rahman (1996). In the intermediate range of frequencies $\omega_H << \omega << \omega_{Hi}$ investigated above, the dispersion equation (33) has the form

$$\left(\omega^2 - \Omega_p^2\right)\left(\omega^2 - k_z^2 C_d^2\right) = \left(\omega^2 - k^2 C_d^2\right)k_z^2 C_A^2, \qquad (35)$$

where $\Omega_p = \omega_p\omega_{Hi}/\omega_{pi}$ is a new characteristic frequency.

If $\omega >> \Omega_p$ from (35) at $C_d << C_A$, the solution for the frequency is

$$\omega^2 \approx k_z^2 C_A^2 - k_\perp^2 C_d^2 \qquad (36)$$

which resembles KAW in an electron-ion plasma with the negative term $-k_\perp^2 C_d^2$ for frequency. For the case $\omega \approx \Omega_p$ and, as before, $C_d << C_A$ the solution (35) for frequency has the form

$$\omega^2 \approx \Omega_p^2 + k_z^2 C_A^2\left(1 - \frac{k_\perp^2 C_d^2}{\Omega_p^2 + k_z^2 C_A^2}\right) \ . \qquad (37)$$

Notice that the condition $\omega \approx \Omega_p >> \omega_H$ used to obtain (35) and (37) requires the fulfilment of inequality $n_o m >> n_i m_i$, which obtains only for some types of dusty plasma.

Thus, in a cold dusty plasma for frequencies lower than the cyclotron frequency of grains $\omega << \omega_H$, only slow Alfvén and magnetoacoustic waves (13) may propagate, where Alfvén's speed V_A is defined by the masses of all three plasma components: ions, electrons and grains. In the frequency range $\omega_H << \omega << \omega_{Hi}$ for fixed ω and angle θ, the propagation of two waves takes place (or one in the frequency range $\omega_H < \omega < \omega_s^2 / \omega_H$). The characteristic velocities of waves C_A in the case $\omega > \omega_H$ are defined only by the densities of electrons and ions. Not far from the cutoff frequency $\omega_c = \omega_s^2 / \omega_H$ which does not depend on angle θ, the waves may have velocities larger than V_A or C_A.

Kinetic Alfvén waves in the frequency range $\omega << \omega_H$ have the same dispersion law as in the case of an electron-ion plasma, but their Alfvén speed is defined by the masses of all three components of the plasma, including grains. The role of the ion cyclotron radius has the parameter C_d/ω_H, where C_d is the speed of the volume dusty acoustic wave. But in the frequency range $\omega_H \ll \omega \ll \omega_{Hi}$, the character of waves changes and a new characteristic frequency appears: $\Omega_p = \omega_p \omega_{Hi}/\omega_{pi}$. In the frequency range $\omega \gg \Omega_p$ the wave is like a kinetic Alfvén wave in an electron-ion plasma with negative term $-k_\perp^2$.

(e) Surface waves

A space dusty plasma is often a semiinfinite system with a sharp boundary between vacuum and expanding plasma. Therefore, the investigation of properties of low-frequency surface waves of a semiinfinite multicomponent dusty plasma is a subject of great interest.

Let us consider a uniform dusty plasma filling the half space $x \leq 0$ (region I), and assume empty space in region II, $x \geq 0$. We assume the wave frequencies are so low ($\omega \ll kV_{Te,i}$), that Boltzmann's distribution (5) can be used for both electron and ion component. We use Poisson's equation

$$div\vec{E} = -\Delta\varphi = 4\pi e(n_e + Zn = n_i) \tag{38}$$

and the equations of continuity and motion for charged grains (density of grains $n = n_o + \tilde{n}$, $|\tilde{n}| \ll n_o$):

$$\frac{\partial\tilde{n}}{\partial t} + n_o div\vec{V} = 0 \ , \ \frac{\partial\vec{V}}{\partial t} = -\frac{Ze}{m}\vec{E}\,\frac{Ze}{m}\nabla\varphi \quad . \tag{39}$$

We find a solution for $\tilde{n}$, φ in the form $f(x)\exp i(\omega t - kz)$ at $\partial/\partial y = 0$. Then from (39) we have:

$$\tilde{n} = i\frac{n_o}{\omega}div\vec{V}, \qquad \vec{V} = i\frac{Ze}{m\omega}\vec{E} \quad . \tag{40}$$

Using (5) and (40), equation (38) may be rewritten in the form:

$$div\vec{D} = \varphi / a_D^2 \tag{41}$$

where the vector induction is $\vec{D} = \varepsilon_d\vec{E}$; $\varepsilon_d = 1 - \omega_p^2/\omega^2$; ω_p is the Langmuir frequency of grains, and $a_D = \sqrt{T_eT_i / 4\pi e^2(n_{eo}T_i + n_{io}T_e)}$. From (41) we obtain a first boundary condition

$$D_x^I = D_x^{II}\Big|_{x=0} \qquad \text{or} \qquad \varepsilon_d\varphi^I = \varphi^{Ii}\Big|_{x=0} \quad . \tag{42}$$

A second boundary condition is the usual condition of continuity of potential φ:

$$\varphi^I = \varphi^{II}\Big|_{x=0} \quad . \tag{43}$$

We may write equation (41) as

$$\frac{\partial^2\varphi^I}{\partial x^2} - \gamma^2\varphi^I = 0 \quad , \tag{44}$$

where $\gamma^2 = k^2 + 1/\varepsilon_d a_D^2$. Thus, the solution for φ^I is:

$$\varphi^I = Ae^{\gamma x + i(\omega t - kz)} \qquad x \leq 0 \quad . \tag{45}$$

Similarly, for a vacuum in (44) we will have instead a γ^2 value of k^2 and the solution for φ^{II} will be

$$\varphi^{II} = Be^{-kx + i(\omega t - kz)} \qquad x \geq 0 \quad , \tag{46}$$

where A, B are arbitrary constants. Substituting (45) and (46) into (43), (44), we obtain the dispersion equation for surface waves:

$$\gamma\varepsilon_d = -k \quad . \tag{47}$$

Thus, for γ, $k > 0$ must be $\varepsilon_d < 0$ or $\omega < \omega_p$. From (47) we may obtain the relation $k(\omega)$:

$$k = \frac{\omega}{C_d}\sqrt{\frac{\varepsilon_d}{1+\varepsilon_d}} = \frac{\omega}{C_d}\sqrt{\frac{\omega^2 - \omega_p^2}{2\omega^2 - \omega_p^2}}, \tag{48}$$

where C_d is the speed of the volume dusty acoustic waves. As $\varepsilon_d < 0$ from (48) it follows that $1+\varepsilon_d < 0$ or $\omega < \omega_p/\sqrt{2}$. Thus, surface dusty acoustic waves exist in the frequency range $0 \leq \omega < \omega_p/\sqrt{2}$. The law of dispersion (48) was first obtained by Bharuthram and Shukla (1993). Surface dusty acoustic waves (48) are like low-frequency surface waves in electron-ion plasma first analysed by Kondratenko (1965) and Romanov (1965).

In the presence of an external magnetic field $\vec{B}_o$, instead of the second equation (39) we will use the equation

$$\frac{\partial\vec{V}}{\partial t} = -\frac{Ze}{m}\vec{E} - \vec{V} \times \vec{\omega}_H$$

where ω_H is the cyclotron frequency of grains. In this case the meaning of γ in (44) and the components of vector induction $\vec{D}$ will be determined by the orientation of the magnetic field $\vec{B}_o$. Let us consider three cases of interest (A. Kotsarenko *et al.*, 1997).

(1) The magnetic field $\vec{B}_o$ is parallel to Z-axis, i.e. $\vec{B}_o II \vec{k}$. Then

$$D_x = \varepsilon_\perp E_x, D_z = \varepsilon_\| E_z, \gamma^2 = \frac{k^2\varepsilon_\|}{\varepsilon_\perp} + \frac{1}{a_D^2\varepsilon_\perp}, \tag{49}$$

where $\varepsilon_\perp = 1 - \frac{\omega_p^2}{\omega^2 - \omega_H^2}$, $\varepsilon_\| = 1 - \frac{\omega_p^2}{\omega^2}$.

The solution for potential φ has the form (45), (46). Inserting this solution into boundary condition (42), (43), we obtain the dispersion equation in the form

$$\varepsilon_\perp\gamma = -k \quad . \tag{50}$$

Since the values of γ, k are positive, it follows from equations (50) and (49) that $\varepsilon_\perp < 0$ and $\varepsilon_\| < 0$. The surface waves exist only if $\omega_H<\omega_p$. So, we find that surface waves may exist only in the plasma with grain Langmuir frequency greater than a cyclotron frequency. The frequency region for the waves is defined by

$$\omega_H < \omega < \omega_s / \sqrt{2} \quad ,$$

where $\omega_s = \sqrt{\omega_H^2 + \omega_p^2}$. The evident dependence $k(\omega)$ follows from the equation (50):

$$k = \frac{\omega}{C_d}\sqrt{\frac{\omega^2 - \omega_s^2}{2\omega^2 - \omega_s^2}} \quad . \qquad (51)$$

The dependence $k(\omega)$ from (51) is presented in Figure 4. The value of the minimum wave vector k_0 is equal to

$$k_0 = \frac{\omega_H \omega_p}{C_d\sqrt{\omega_p^2 - \omega_H^2}} \quad . \qquad (52)$$

It follows from equation (52) that the maximum length of the surface wave is $2\pi/k_0$. The upper limit for the value of k follows from the condition $\omega >> kV_T$. This restriction follows from our approximation which neglects the dust grain thermal motion. Here $V_T = \sqrt{T/m}$, T is the dust component temperature. In the case $\omega_H = 0$, formula (51) transforms into (48).

(2) The magnetic field $\vec{B}_o$ is parallel to the Y-axis, i.e. $\vec{B}_o \perp \vec{k} \perp \vec{N}$, where $\vec{N}$ is the vector normal to the plasma boundary. In this case we should introduce in formulas (41) and (44) $\vec{D} = \varepsilon_\perp \vec{E}$, $\gamma^2 = k^2 + 1/\varepsilon_\perp a_D^2$. The dispersion relation takes the form:

$$k = \frac{1}{a_D}\sqrt{\frac{\varepsilon_\perp}{1-\varepsilon_\perp^2}} = \frac{1}{C_d}\sqrt{\frac{\left(\omega^2 - \omega_H^2\right)\left(\omega^2 - \omega_s^2\right)}{\left(2\omega^2 - 2\omega_H^2 - \omega_p^2\right)}} \quad . \qquad (53)$$

The frequencies of the surface waves follow from the conditions $\varepsilon_\perp < 0,\ 1-\varepsilon_\perp^2 < 0$:

$$\omega_H \le \omega \le \sqrt{\omega_H^2 + \omega_p^2/2} \quad . \qquad (54)$$

These waves exist for any relation between ω_H and ω_p. The dependence $k(\omega)$ according to equation (53) is shown in Figure 5. For the limit $k \to 0$ ($\omega \approx \omega_H$), the solution (53) may be rewritten as $\omega \approx \omega_H + k^2 C_d^2 / 2\omega_H$. Thus, the phase velocity of the surface wave in the case $\omega \to \omega_H$ may be much greater than the value of C_d.

(3) The magnetic field $\vec{B}_o$ is parallel to vector $\vec{N}\left(\vec{B}_o \| \vec{N}\right)$, i.e, the magnetic field is directed along the X-axis. In this case $\vec{D} = \left\{\varepsilon_\| E_x, 0, \varepsilon_\perp E_z\right\}$, $\gamma^2 = k^2 \frac{\varepsilon_\perp}{\varepsilon_\|} + \frac{1}{\varepsilon_\| a_D^2}$. The dispersion relation has the form $\varepsilon_\| \gamma = -k$, so as in case 1, $\varepsilon_{\|,\perp}<0$. This leads to the condition $\omega_H<\omega_p$ and the dispersion relation

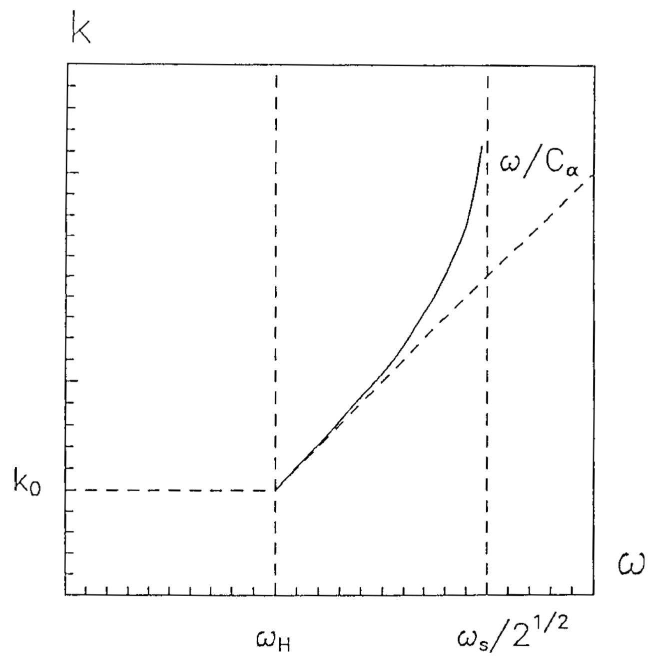

Fig. 4. Dependence $k(\omega)$ for the surface wave in the case $\vec{B}_o \| \vec{k}$.

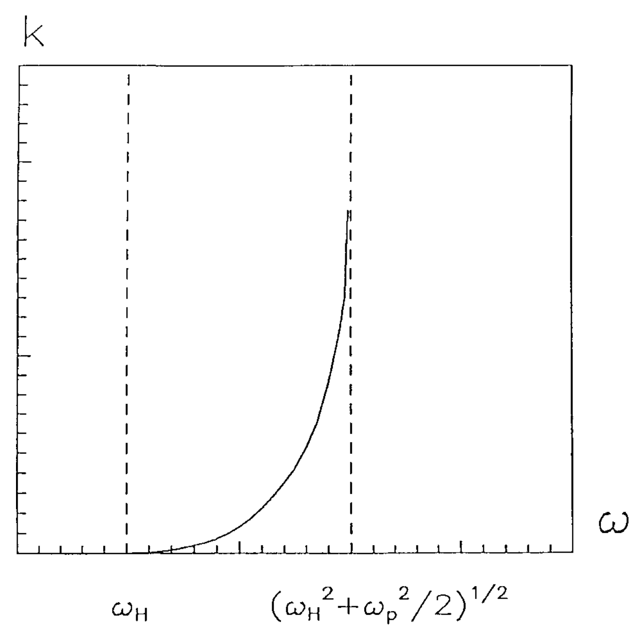

Fig. 5. Dependence $k(\omega)$ for the surface wave in the case $\vec{B}_o \perp \vec{k} \perp \vec{N}$.

$$k = \frac{1}{a_D}\sqrt{\frac{\varepsilon_\|}{1-\varepsilon_\| \varepsilon_\perp}} = \frac{1}{C_d}\sqrt{\frac{(\omega^2 - \omega_H^2)(\omega^2 - \omega_p^2)}{(2\omega^2 - 2\omega_H^2 - \omega_p^2)}} \quad . \qquad (55)$$

The possible frequencies of the surface waves in this case follow from the conditions $\varepsilon_\|<0$, $\varepsilon_\| \varepsilon_\perp>1$, that leads to condition (54). The dependence $k(\omega)$ is similar to that shown in Figure 5, if one replaces the longitudinal frequency $\sqrt{\omega_H^2 + \omega_p^2/2}$ with $\omega_s/2$.

Thus, low-frequency electrostatic surface waves in a magnetized dusty plasma caused by the charged grains motion have been examined. We have found that the action of the electron and ion components, where the Boltzmann distribution for electrons and ions is assumed, is reduced only to the shield of the potential. It has been shown that the surface-wave range of frequencies is defined by the Langmuir ω_p and cyclotron ω_H frequencies of charged grains. The frequencies of surface waves propagating along the magnetic field, and in the case of magnetic field which is normal to the plasma boundary, are limited by the relation $\omega_H<\omega_p$, which is common for a dusty space plasma. In the first case a minimum wave number exists, and the surface waves are slow, i.e. their phase velocity is less than the velocity of volume dusty acoustic waves C_d. If a magnetic field is normal to the plasma boundary, the phase velocity of surface waves near minimum frequency ω_H may be arbitrarily greater than C_d.

Instabilities in dusty plasma

Different mechanisms of instabilities in a dusty plasma have been investigated. First there are two-stream or drift instabilities (Bharuthram *et al*., 1992a; 1996; Chow *et al*., 1995; Havnes, 1988; Lakhina *et al*., 1987; 1988; Melandso *et al*., 1993b; Reddy *et al*., 1996; Rosenberg, 1993, 1994, 1995; Verheest, 1987), the Rayleigh-Taylor instability (D'Angelo, 1993), the Kelvin-Helmholtz instability (D'Angelo *et al*., 1990) and so on. According to these papers we will consider typical cases of kinetic and hydrodynamical instabilities. By investigation of kinetic instabilities we may use the common kinetic dispersion equation for waves $\propto \exp i(\omega t\text{-}kz)$ in a multicomponent plasma (Akhiezer *et al*., 1975):

$$\varepsilon_l(\omega,k) = 1 + \sum_{\alpha=e,i,ib,d} \frac{1}{K^2 a_\alpha^2}\left\{1-\varphi(Z_\alpha) - i\sqrt{\pi} z_\alpha e^{-z_\alpha^2}\right\} = 0 \quad , \tag{56}$$

where $a_{e,i,ib} = \sqrt{T_{e,i,ib}/4\pi n_{oe,i,ib}e^2}$ are Debye radii of electrons, ions and ion beam respectively, $a_d = \sqrt{T_d/4\pi Z^2 e^2 n_d}$ is the Debye radius of charged grains, $z_\alpha = (\omega - kU_\alpha)/ks_\alpha$, $s_\alpha = \sqrt{2T_\alpha/m_\alpha}$ is the thermal velocity of the α-th kind of particles, U_α are velocities of drift, $\varphi(z)$ is plasma dispersion function ($\varphi(z) \approx 2z^2$ at $|z|<<1$, $\varphi(z) \approx 1 + \frac{1}{2z^2} + \frac{3}{4z^4} +$ ...at $|z|>>1$). The dispersion relation (64) may be used for different plasma models.

Let us consider a plasma consisting of stationary electrons of number density n_{oe}, ions of number density n_{oi}, negatively charged dust particles of number density n_d and charge Ze, and an ion beam of density n_{ib}, thermal velocity s_{ib} and drift velocity U_o. The condition of electroneutrality has the form

$$n_{oi} + n_{ib} = n_{oe} + Zn_d \ .$$

For perturbations with phase velocity satisfying conditions

$|\omega/k| << s_{e,i}$, $|\omega/k| \le s_{ib}$, $|\omega/k| \le s_d$ we obtain from (56):

$$1 + \frac{1}{k^2 a_e^2} + \frac{1}{k^2 a_i^2} - \frac{\omega_p^2}{\omega^2} - i\frac{\sqrt{\pi}\omega}{k^3 s_e a_e^2} - i\frac{\sqrt{\pi}\omega}{k^3 s_i a_i^2} - i\sqrt{\pi}\frac{\omega - kU_o}{k^3 a_{ib}^2 s_{ib}} e^{-(\omega - KU_o)^2/k^2 s_{ib}^e} \approx 0 \tag{57}$$

where ω_p is the Langmuir frequency of grains. As in (57) the imaginary part is small, we may find a solution for frequency ω in the form $\omega = \omega' + i\omega''$, where $\omega'' << \omega''$. Then from equation (57) we obtain

$$\omega' = \frac{\omega_p}{\sqrt{1 + \frac{1}{k^2 a_e^2} + \frac{1}{k^2 a_i^2}}} \tag{58}$$

$$\frac{\omega''}{\omega'} = \frac{\sqrt{\pi}\omega'^2}{2k^3\omega_p^2}\left\{\frac{\omega'}{s_i a_i^2} + \frac{\omega' - kU_o}{s_{ib} a_{ib}^2} e^{-(\omega' - kU_o)/k^2 s_{ib}^2}\right\} . \tag{59}$$

Equations (58), (59) determine real and imaginary parts of frequency of dusty acoustic wave. In obtaining formula (58) we have neglected the imaginary electron term which is of order $\sqrt{m_e/m_i}$ as compared with unity. As seen from formula (59), at large velocity of drift $U_o>U_{oc}$ the imaginary part of frequency $\omega''<0$ and there will be an instability of the dusty acoustic wave caused by ion flow. Such a situation takes place in the rings of giant planets, plasma cometary tails and so on. Supposing an exponent in formula (59) approximately unity we obtain a value for U_{oc} as $U_{oc} \approx \frac{\omega'}{k}\left(1 + \frac{s_{ib} a_{ib}^2}{s_i a_i^2}\right) \propto C_d$, where C_d is the velocity of the dusty acoustic wave. Formulas (58), (59) are not valid for "cold" ion beam, when $(\omega' - kU_o)/k^2 s_{ib}^2 \le 1$.

Consider now the case of hydrodynamical instability, when in formula (56) $z_{e,i} << 1$, $z_d >> 1$ and $z_{ib} >> 1$, i.e. the ion beam is "cold". We neglect also Landau damping of waves caused by electrons and ions. Then the dispersion relation (56) becomes

$$1 + \frac{1}{k^2 a_e^2} + \frac{1}{k^2 a_i^2} - \frac{\omega_p^2}{\omega^2} - \frac{\omega_{pib}^2}{(\omega - kU_o)^2} = 0 \tag{60}$$

where ω_{pib} is the Langmuir frequency of the ion beam. The dispersion equation (57) is familiar from the theory of plasma (Krall *et al*., 1973). The criterion of instability may be written in the form

$$kU_o < \sqrt{\frac{\omega_p^2}{1+\frac{1}{k^2a_D^2}}\left[1+\left(\frac{\omega_{pib}}{\omega_p}\right)^{2/3}\right]^{3/2}} \tag{61}$$

where $a_D^2 = a_e^2 a_i^2 / (a_e^2 + a_i^2)$. If, as usual, $\omega_{pib} >> \omega_p$, criterion (61) may be simplified as:

$$kU_o < \omega_{pib} / \sqrt{1+\frac{1}{k^2a_D^2}} \quad . \tag{62}$$

In addition, when $ka_D << 1$ we may further simplify to

$$U_o < \omega_{pib} a_D \quad . \tag{63}$$

Note that equations (60) and (63) are valid for $U_o >> s_{ib}$, so the inequality (63) should be understood as $s_{ib} << U_o < \omega_{pib} a_D$. For $\omega << kU_o$ solution (60) for ω may be written as:

$$\omega \approx \pm\omega_p / \sqrt{1+\frac{1}{k^2a_D^2}-\frac{\omega_{pib}^2}{k^2U_o^2}} \quad . \tag{64}$$

For long-wavelength perturbations ($ka_D<<1$) Eq. (64) becomes

$$\omega \approx k\omega_p a_D / \sqrt{1-\omega_{pib}^2 a_D^2 / U_o^2} \quad , \tag{65}$$

and for condition (63) we have an aperiodical instability with increment

$$\omega = -ik\omega_p a_D / \sqrt{\omega_{pib}^2 a_D^2 / U_o^2 - 1} \quad . \tag{66}$$

In conclusion, short-wavelength perturbations are more unstable than long-wavelength ones.

Solitons in dusty plasmas

As we have shown above, in dusty plasmas wave excitation mechanisms will often give rise to large-amplitude waves, where nonlinear effects will be important. One of the more interesting nonlinear effects is the possibility of formation of solitons. Ion-acoustic and dusty acoustic solitons in dusty plasmas have been investigated (Rao *et al*., 1990; Bharuthram *et al*., 1992b, 1992c; Verheest, 1992; Shukla, 1992; Lakshmi *et al*., 1994; Mamun *et al*., 1996). It turns out that ion-acoustic solitons may be compressional or dilatational in the presence of negatively charged grains (Rao *et al*., 1990), if the density of grains is large enough. In a dusty plasma compressional and dilatational dusty acoustic solitons will occur depending on the sign of the charge number Z and the density of grains.

Ion-acoustic solitons

For studying ion-acoustic solitons we use Poisson's equation

$$\frac{\partial^2\varphi}{\partial z^2} = 4\pi e(n_e + Zn - n_i) \quad , \tag{67}$$

and the equations of motion and continuity for ions:

$$\frac{\partial V_i}{\partial t} + V_i\frac{\partial V_i}{\partial z} = -\frac{e}{m_i}\frac{\partial\varphi}{\partial z} \tag{68}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial z}(n_i V_i) = 0 \quad . \tag{69}$$

The electron number density n_e is given by the Boltzmann distribution

$$n_e = n_{eo} e^{e\varphi/T_e} \quad . \tag{70}$$

The condition of electroneutrality in equilibrium leads to

$$n_{io} = n_{eo} + Zn_o \tag{71}$$

where n_{eo}, n_{io} and n_o are the equilibrium number densities of electrons, ions and dust particles respectively. Grains are assumed motionless, so in equation (67) $n_o = const$. Solutions of the equations (67)-(70) take the form $f(z,t) = f(z-Ut) \equiv f(z')$, where U is an arbitrary velocity to be defined later. Then from (68) and (69) we obtain expressions for V_i and n_i:

$$V_i = U - \sqrt{U^2 - \frac{2e\varphi}{m_i}}, \quad n_i = n_{io}\frac{U}{U-V_i} = \frac{n_{io}}{\sqrt{1-\frac{2e\varphi}{m_iU^2}}} \quad . \tag{72}$$

Here we used the conditions: $V_i \to O$, $n_i \to n_{io}$ at $|z| \to \infty$. Substituting n_i from (72) and n_e from (71) into equation (67) we obtain the equation for the potential φ:

$$\frac{d^2\varphi}{dz'^2} = 4\pi e\left\{n_{eo}e^{e\varphi/Te} - \frac{n_{io}}{\sqrt{1-\frac{2e\varphi}{m_iU^2}}} + Zn_o\right\} \quad . \tag{73}$$

It is convenient to use the dimensionless variables $\phi = e\phi/T_e$, $\xi = z'/a_e$, $\delta = n_{io}/n_{eo}$, $M = U/C_s$, where $a_e = \sqrt{4\pi e^2 n_{eo} / m_e}$ is the Debye radius, $C_s = \sqrt{T_e / m_i}$ is the speed of ion sound in plasma, and M is the Mach number. Then equation (73) becomes

$$\frac{d^2\varphi}{d\xi^2} = e^{\phi} - \frac{\delta}{\sqrt{1-\frac{2\phi}{M^2}}} + \delta - 1 \quad . \tag{74}$$

After multiplying by $d\phi/d\xi$ and integrating over the appropriate boundary conditions for localized solutions, i.e. ϕ, $d\phi/d\xi \to 0$ at $|\xi| \to \infty$, we find from Eq (74)

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 = e^{\phi} - 1 + \delta M^2\left(\sqrt{1-\frac{2\phi}{M^2}} - 1\right) + (\delta - 1)\phi \equiv \Psi(\phi). \tag{75}$$

We consider only the case of a low potential, when $\phi << 1$, $\phi/M^2 << 1$. Then function $\Psi(\phi) \approx \frac{1}{2}(A\phi^2 - B\phi^3)$ is exact, where $A = 1 - \delta/M^2$, and $B = \frac{1}{3}(\frac{3\delta}{M^4} - 1)$, and from equation (75) we obtain the equation for ϕ in the form

$$\frac{d\phi}{d\xi} = \pm\sqrt{A\phi^2 - B\phi^3} \quad . \tag{76}$$

We may determine the amplitude ϕ_m of the soliton from the condition $\phi = \phi_m$ at $d\phi / d\xi = 0$, and from equation (76)

$$\phi_m = A / B \quad . \tag{77}$$

This equation yields a relation between the amplitude ϕ_m and the velocity U of the soliton. From equation (76) we find the solution for $\phi(\xi)$:

$$\phi(\xi) = \frac{\phi_m}{ch^2\left(\frac{\sqrt{A}}{2}\xi\right)} = \frac{\phi_m}{ch^2\left(\frac{\sqrt{A}}{2}\frac{(z-Ut)}{a_e}\right)} \quad . \tag{78}$$

For $\delta = 1 (n_o = 0)$ equation (78) describes the well-known ion-acoustic soliton in an electron-ion plasma, where according to equation (77) $M^2 = U^2 / C_s^2 \approx 1 + \frac{2}{3}\phi_m$ and $A = \frac{2}{3}\phi_m$. If $n_o \neq 0$ (dusty plasma) we obtain from equation (77)

$$M^2 \approx \delta + \frac{3-\delta}{\delta}\phi_m \ , \ A = \frac{3-\delta}{3\delta}\phi_m \quad . \tag{79}$$

Since in (78), A must be positive (A>0), we have two cases from Eq (79):

(a) If $\delta = \frac{n_{io}}{n_{eo}} < 3$, then A>0 at $\phi_m > 0$. In this case, Eq. (78), (70) and (72) determine a compressional soliton with $n_e(\xi) > n_{eo}$, $n_i(\xi) > n_{io}$, which is similar to the ion-acoustic soliton in an electron-ion plasma.

(b) If $\delta > 3$, then $A > 0$ at $\phi_m < 0$. According to Eq. (70) and (72) we obtain a dilatational soliton, which is not found in an electron-ion plasma. This result was obtained by Bharuthram and Shukla (1992).

Dusty acoustic solitons

For treating dusty acoustic solitons we use Poisson's equation (67), where electrons and ions are assumed to be distributed by Boltzmann's law:

$$n_e = n_{eo}e^{e\varphi/T_e}, \quad n_i = n_{io}e^{-e\varphi/T_i} \quad . \tag{80}$$

For the grains we use the equations of motion and continuity:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial z} = \frac{Ze}{m}\frac{\partial\varphi}{\partial z}, \quad \frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(nV) = 0 \tag{81}$$

where V and n are velocity and number density of the charged grains. For a solution of the kind $f(z-Ut) \equiv f(z')$ we find the density of grains from (81):

$$n = \frac{n_o}{\sqrt{1 + \frac{2Ze}{mU^2}\varphi}} \quad . \tag{82}$$

Substituting densities $n_{e,i}$ and n from (80) and (82) into Eq. (67) we obtain an equation for the potential φ in the form

$$\frac{d^2\varphi}{dz^2} = 4\pi e\left\{n_{eo}e^{e\varphi/T_e} - n_{io}e^{-e\varphi/T_i} + \frac{Zn_o}{\sqrt{1+\frac{2Ze}{mU^2}\varphi}}\right\}, \tag{83}$$

and after integrating with conditions φ, $d\varphi/d\xi \to 0$ at $|z| \to \infty$:

$$\frac{1}{2}\left(\frac{d\varphi}{dz'}\right)^2 = 4\pi\Big\{ n_{eo}T_e(e^{e\varphi/T_e} - 1) +$$

$$n_{io}T_i(e^{-e\varphi/T_i} - 1) + n_o mU^2\sqrt{1+\frac{2Ze}{mU^2}\varphi - 1}\Big\} \quad . \tag{84}$$

Using the dimensionless variables $\phi = e\varphi/T_e$, $\xi = z'/a_e$, $\delta = n_{io}/n_{eo}$, $\eta = T_e/T_i$, $M^2 = mU^2/ZT_e$, equation (84) becomes

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 = e^{\phi} - 1 + \frac{\delta}{\eta}(e^{\eta\phi} - 1) + M^2(\delta - 1)\left(\sqrt{1+\frac{2\phi}{M^2}} - 1\right). \tag{85}$$

Assuming $\phi \ll 1$, $\phi/M^2 \ll 1$, $\eta\phi \ll 1$ we can write the equation (85) in the form (76), where the coefficients A and B are

$$A = 1 + \delta\eta - \frac{\delta - 1}{M^2}, \ B = \frac{1}{3}\left(\delta\eta - 1 - 3\frac{\delta-1}{M^4}\right) \quad . \tag{86}$$

Thus solution for potential φ has the form (78). From Eq. (77) and (86) we find

$$M^2 = \frac{\delta - 1}{1 + \delta\eta}(1 - \Delta\varphi_m) \ , \tag{87}$$

where $$\Delta = \frac{2(1+\delta^2\eta^2) + \delta(1+\eta^2) + 6\delta\eta}{3(\delta - 1)(1+\delta\eta)} \quad . \tag{88}$$

In dimensional form, from Eq. (87) we may write an expression for the speed of a dusty acoustic soliton:

$$U^2 = C_d^2(1 - \Delta\phi_m) \tag{89}$$

where $C_d = \sqrt{Z^2 n_o T_e T_i / m(n_{oe}T_i + n_{oi}T_e)}$ is the speed of the dusty acoustic wave. From Eq. (86) and (87) we find the meaning $\sqrt{A}$, which determines the width of the soliton $l = 2a_e / \sqrt{A}$:

$$\sqrt{A} = \sqrt{-(1+\delta\eta)\Delta\phi_m} \quad . \tag{90}$$

Thus for A>0 we have two cases:

(a) Z>0 (the grains have negative charge). Then from (79) $\delta = \frac{n_{io}}{n_{eo}} > 1$, according to (88) and (90) Δ>0 and we must have ϕ_m<0. Here we have a soliton of negative potential φ (78), in the field of the soliton there is an increasing number density of grains (82) and ions, and a decreasing number density of electrons (80).

(b) $Z<0$ (the grains have positive charge). Then from (71) $\delta<1$, and according to (88) and (90) $\Delta<0$ and for $A>0$ we must have $\phi_m>0$. Here we have a soliton of positive potential; in the field of the soliton there is a decreasing number density of grains and ions and an increasing density of electrons.

Dusty acoustic solitons were first investigated by Rao, Shukla and Yu (1990).

Astrophysical applications

One of the first space structures associated with the presence of dusty plasma were the radial spokes in Saturn's B ring (Figure 5), which were observed by Voyager (Smith *et al*., 1981, 1982) (Figure 6). The theory of these spokes was developed by Bliokh and Yaroshenko (1985), Goertz (1989) and other. The explanation concerns the excitation of space charge waves in the dusty plasma of Saturn's rings. Such waves have a dispersion relation $k=\omega/V_o$, where $V_o \propto 10^5$ cms^{-1} is the grain velocity. Bliokh and Yaroshenko estimated the frequency as the inverse of the halflife of the spokes, yielding $\omega = \tau^{-1} \approx 10^{-3}s^{-1}$ and obtaining a spatial dimension of $l = 2\pi/k \approx 1000km$ which is close to observation. But another cause of formation of such structures is possible. In Saturn's rings different two-stream instabilities may develop. In the rings lighter plasma particles (electrons and ions) actually move together azimuthally at a speed U_c near the corotating speed, and heavy dust grains move azimuthally near the Kepler speed $U_k \neq U_c$. The relative drift speed U_c- U_k may be comparable to or greater than the ion thermal speed.

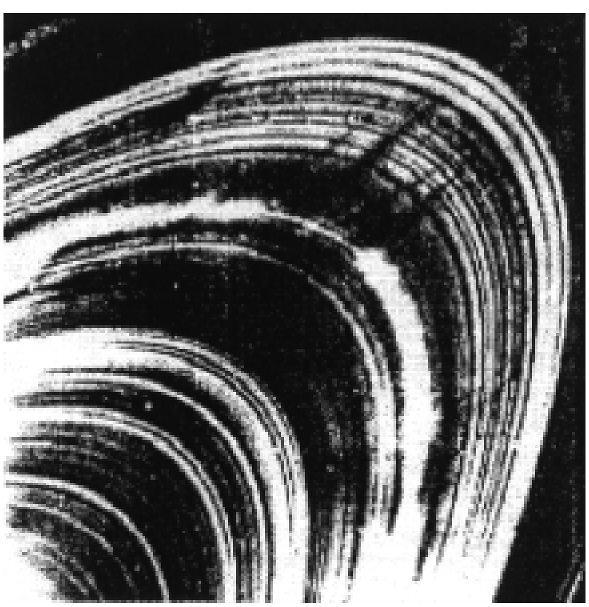

Fig. 6. The picture of the radial spokes in Saturn's ring. The azimuthal width of a spoke is typically a few thousand kilometers.

In this case instabilities like that of Buneman (1958) may occur (Rosenberg *et al*., 1995; Melandso *et al*., 1993). These instabilities may form spatial structures from hundreds to tens of millions of kilometers. Similary large-scale spatial structures are formed in the dusty plasma of cometary tails by interaction with the solar wind (Havnes, 1988; Reddy *et al*., 1996). In particular, they may be a cause of formation of the ray structures of plasma cometary tails and condensations.

Coulomb crystals in laboratory dusty plasma

In recent years interesting and surprising results have been obtained in laboratory dusty plasma. For ordinary classical plasma the ratio between the average Coulomb energy W_p and the kinetic energy W_k of a charged particle system is

$$\Gamma = W_p / W_k << 1 \tag{91}$$

where $W_p = q^2/\bar{r}$, $W_k \approx T$. Here q is the electrical charge of particle, $\bar{r}$ is the interparticle distance such that $\bar{r} = (3/4\pi n)^{1/3}$, which is determined by the particle density n, and T is the temperature.

It is known (Slattery *et al*., 1980; Ichimaru, 1982) that when Γ exceeds a critical value $\Gamma_c \approx 170$, a Coulomb lattice or Coulomb crystal is formed (Figure 7). In the electron-ion plasma, where q equals the charge of an electron e, the solidification condition of

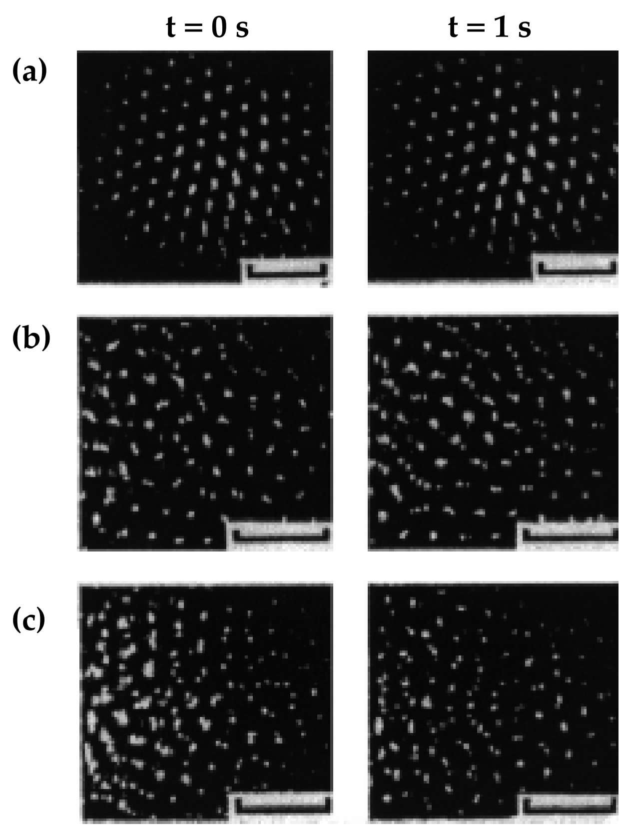

Fig. 7. Micrographs of hexagonal structures and more disordered structures at different power. The bars correspond to 500 μm.

$$\Gamma > \Gamma_c \tag{92}$$

requires a high-density and low-temperature system, and it is difficult to satisfy such conditions. However, for dusty plasma $q=Ze$ and an average potential energy of charged dust particles $W_d \approx \frac{Z^2e^2}{\bar{r}_d} \approx Z^2e^2\sqrt[3]{n_d}$ for condition $Z^2\sqrt[3]{n_d} >> \sqrt[3]{n_e}$ may greatly exceed the average potential energy of electron and ions. Then condition (92) may be satisfied and conditions for the formation of Coulomb crystals exist. The condition (92) for dusty plasma, or $Z^2e^2\sqrt[3]{n_d}\,/\,T > 170$ are easily met in laboratory conditions. Direct observation of Coulomb crystals in strongly coupled dusty plasma was first obtained by Chu and Lin (1994) using a discharge system. The strongly coupled dusty plasma was formed by suspending fine negatively charged SiO_2 particles with $10\mu m$ diameter in weakly ionized Ar discharges. The Coulomb crystals were directly observed for the first time using an optical microscope. By properly controlling the system parameters, hexagonal structures and solids with coexisting different crystal structures could be formed. In Figure 8 some ordered structures are shown.

During the last two years Coulomb crystals in dusty plasma have been obtained in a few laboratories of the world (Japan, Germany, Russia, India).

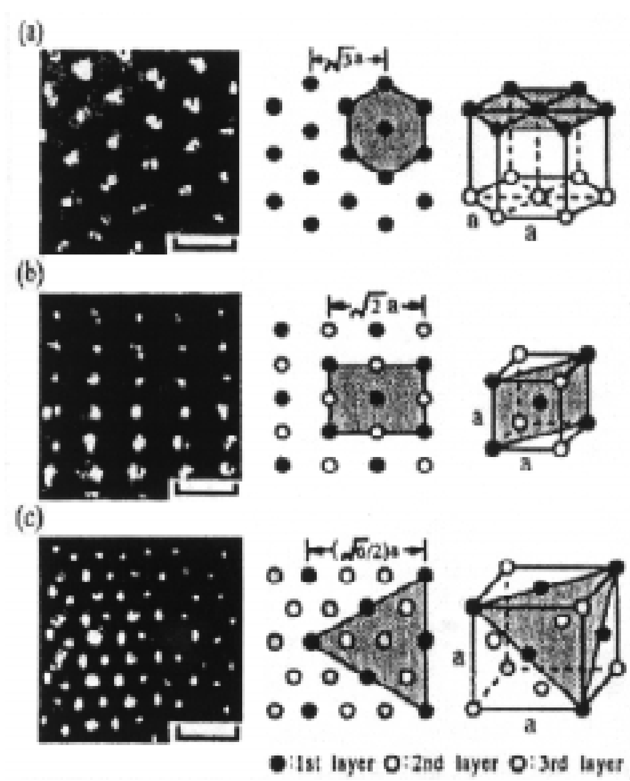

Fig. 8. Micrographs and sketches of the different crystal structures: (a) hexagonal; (b) bcc; (c) fcc. The center column corresponds to the structures in the micrographs. The graded areas in the sketches are normal to the optical axis. The bars correspond to 200 μm.

CONCLUSIONS

Dusty plasma is a very interesting object in space and in the laboratory. Even at low number densities of charged dust particles as compared with the number density of electrons and ions, the dusty component changes the spectrum of linear and nonlinear plasma waves, causing the appearance of new very slow waves which form spatial structures. In conclusion we may explain some observed space structures by the influence of the dusty component of plasma. In the laboratory it is easy to obtain strongly coupled dusty plasma with Coulomb potential energy much larger than the kinetic energy of plasma. Such plasma has a tendency to self-organization, i.e. formation of Coulomb crystals of different symmetries.

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