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NAVAL POSTGRADUATE SCHOOL

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THEATER TORPEDO INVENTORY OPTIMIZATION

by

Dr. Javier Salmeron, Dr. Moshe Kress, and LT Violeta Lopez

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14. ABSTRACT
When making torpedo loadout decisions, planners must consider the capacities and capabilities of different anti-submarine warfare (ASW) units, a limited budget, and diverse adversary submarine fleets. Currently, loadout decisions for the Mk-54 lightweight torpedo are made manually, and without a systematic approach to deal with threat uncertainty. The research seeks to inform these decisions by using stochastic optimization to determine the type and quantity of torpedoes to load on U.S. surface ships, fixed-wing aircraft, and helicopters, in order to face an uncertain submarine threat with a desired probability of kill. We develop two formulations of the Torpedo Allocation Stochastic Optimization Model (TASOM): TASOM-1, which minimizes the number of missed submarines; and TASOM-2, which minimizes the deviation below the probability of kill threshold. To show the value of the stochastic programming approach over the typical deterministic planning, we present a notional case designed to represent an operation where ASW units are patrolling an area for adversary submarines. We randomly generate 100 threat scenarios where the number and class of submarines deployed to the area vary. The TASOM-2 loadout notably outperforms the deterministic average loadout. Our models combined with an accessible user interface provide planners with a decision aid tool to conduct sensitivity analysis to guide torpedo allocation and budget decisions under uncertainty.

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I. INTRODUCTION

Anti-submarine warfare (ASW) is defined as "operations conducted with the intention of denying the enemy the effective use of submarines" (Joint Chiefs of Staff 2021, p. IV-10). These operations include locating, tracking, and neutralizing enemy submarines. This research focuses on the last task. As adversaries continue to modernize and grow their submarine fleets, we seek to optimally equip U.S. Navy's ASW platforms with weapons that can effectively target those submarines.

A. BACKGROUND

ASW is primarily executed by maritime patrol aircraft, surface combatants and their embarked helicopters, and submarines. Communication restrictions and water space management requirements typically preclude submarines from operating collaboratively with other platform types. We assume that friendly submarines will conduct ASW operations in areas that do not overlap with surface and air assets. Submarine operations will not be discussed further in this report.

Both cruisers and destroyers can fire lightweight torpedoes from their surface vessel torpedo tubes (SVTT) and vertical launch anti-submarine rocket (ASROC) systems.

The P-8 Poseidon is a multi-mission maritime patrol aircraft. When conducting ASW, it can be equipped with lightweight torpedoes for engaging adversary submarines. Compared to surface platforms, the P-8 can cover a greater area when searching for submarines and can engage without the threat of enemy torpedoes. A P-8 squadron consists of six or seven aircraft and a detachment consists of four or five aircraft. Squadrons and detachments can deploy and operate out of U.S., allied, and partner air bases worldwide.

MH-60R Seahawk helicopters share the same advantages over surface platforms as the P-8, but can carry fewer torpedoes and have a much shorter operational range. MH-60R detachments can be embarked on Flight IIA Arleigh-Burke guided missile destroyers, Ticonderoga guided missile cruisers, both Independence and Freedom class littoral combat ships, and aircraft carriers. Destroyers, cruisers, and littoral combat ships have space to embark a maximum of two MH-60Rs.

The U.S. Navy must be prepared to face a very diverse threat. According to Janes (Janes 2021a), there are 15 different submarine classes in the People's Liberation Army Navy (PLAN) and 27 in the Russian Navy.

The divergence of the composition of the Russian and Chinese submarine fleets poses a complicated challenge for defense planning.

The Mk-54 lightweight torpedo is fired from surface ships from the SVTT and the ASROC systems. It can also be loaded onto the MH-60R and the P-8 when conducting ASW operations. We consider the allocation of a torpedo inventory comprised of mod 0, mod 1, and mod 2 variants.

B. STATE OF THE ART AND MOTIVATION

The models we develop in this research are two-stage stochastic models with recourse. Specifically, we allocate torpedoes to ASW units in the first (weapon allocation) stage, and assign torpedoes to submarines in the second (weapon target assignment, WTA) stage. Weapon allocation decisions are often made without complete knowledge of the threat, which motivates stochastic optimization and simulation.

Since the introduction of the WTA problem by Manne (1958), significant work has been done in weapon allocation and WTA. Page (1991) develops a mixed integer programming model for obtaining optimal mix of artillery systems and munitions. Jarek (1994) uses simulation to obtain the number of shipboard air defense missiles needed for air warfare. Tutton (2003) develops a sensor allocation model using stochastic optimization to assign search packages to targets under uncertain enemy order of battle. Avital (2004) develops a two-period stochastic supply chain model to determine how many antiship cruise missiles should be procured and how to allocate them given an uncertain target demand. Uryasev and Pardalos (2004) show the lack of robustness in the deterministic weapon assignment decisions compared to the stochastic counterpart. Buss and Ahner (2006) develop a combat simulation, called DFAS, which evaluates future combat systems for the Army (Havens 2002). DFAS is a discrete-event simulation that represents entity movement, detection, and weapon effects events. It also includes a periodical optimization to revise WTAs. Hattaway (2008) adapts DFAS for a naval warfare application by accounting for radar and electronic sensors, and naval ordnance. Laird (2016) considers a mix of weapons to assign against swarming threats from air, surface, and sub-surface. Cai (2018) uses an agent-based, time-stepped based simulation to find an effective mix of precision and area artillery munitions for offensive operations in an urban environment. Brown and Kline (2021) consider mission coverage instead of target engagement in order to determine an optimal weapon loadout for VLS ships. Different types of missiles, each employed in different missions (strike, air defense, or anti-submarine warfare) can be accommodated in a VLS cell. Adamah et al. (2021) develop a nonlinear optimization model for determining the type and quantity of Mk-48 heavyweight torpedoes to allocate to submarines conducting ASW operations. Templin (2021) considers a derivative of the WTA problem solved heuristically, with the simplifying assumption that there is only one target to engage. The focus of the research is to inform firing policy, specifically the amount and type of missiles to use on a threat.

Among the weapon allocation models from the abovementioned literature, and in relation with this research, we note that Page (1991) and Avital (2004) both use a commander's specified threshold for desired success; however, they focus on minimizing weapon cost in their models and look at targets as an aggregate demand. The simulations in Jarek (1994) and Cai (2018) provide general recommendations for total missiles required or munition composition, but not a closed-form solution that can be taken as an actionable loadout plan. Tutton's (2003) model allocates sensors to units, which differs from torpedo allocation in sensors, which are not expended (after use) on targets. Brown and Kline (2021) consider mission coverage instead of targets, which is not an appropriate approach for our problem as torpedoes are employed solely for engaging adversary submarines (or countering adversary submarine torpedoes). Only Adamah et al. (2021) involve torpedoes as the weapon type; however, their model is nonlinear and also does not recommend a torpedo loadout plan that accounts for multiple targets.

Also, except for DAFS, the WTA models reviewed above only consider one shooter. While we desire to plan for an uncertain threat, where different types and quantities of targets are present in a scenario, Uryasey and Paradalos (2004) plan for one scenario but an uncertain probability of kill of the weapon. Like the other simulation work reviewed, DAFS (Havens 2002; Buss and Ahner 2006; Hattaway 2008) does not supply a closed-form solution of how weapons should be assigned to targets or allocated to units.

Both Laird (2016) and Templin (2021) plan for a given threat and do not account for any uncertainty in the threat scenario.

Despite the vast literature in weapon allocation and assignment modeling, we note that most models do not use stochastic optimization. Moreover, currently, torpedo loadout decisions are made manually. The goal of this research is to aid torpedo allocation decisions using formal mathematical optimization. Specifically, stochastic optimization will allow decision makers to plan for an uncertain threat. Planning for uncertainty in threat composition is realistic as typically loadout decision must be made in advance of detection of an adversary submarine or even deployment of an ASW unit.

II. STOCHASTIC OPTIMIZATION MODEL

This chapter introduces the two formulation variants of our Torpedo Allocation Stochastic Optimization Model (TASOM). A key parameter in both variants is the probability of kill of a submarine, and the main difference between them relates to the assessment of engagements that do not reach a desired probability threshold. The first model, TASOM-1, minimizes the expected *number* of missed submarines, where either an ASW unit fully meets the probability of kill threshold, or it will not engage at all. The second model, TASOM-2, minimizes the expected *shortfall* from (i.e., deviation below) the probability of kill threshold, where engagements may still occur even if they not fully achieve the desired threshold. Both models also consider the cost of the additional weapons acquired (not in inventory) for the recommended loadouts.

In what follows, we will use the following terminology:

- **Red target**—An individual submarine of a particular class that could be deployed.
- **Blue shooter—**An individual P-8 squadron or detachment, an MH-60R detachment, or a surface ship with VLS capability that is available for ASW operations.
- **Scenario—**A configuration of deployed red targets, surviving (not killed) blue shooters, and subsets of blue shooters that can engage a certain red target.

A. ASSUMPTIONS

A blue shooter is an individual P-8 squadron or detachment, or an MH-60R detachment, or surface ship with VLS capability that is available for ASW operations. Engagements of multiple red targets from an individual squadron or detachment can be interpreted as separate flights from the same squadron or detachment.

TASOM only considers offensive engagements. For a ship, this means that we only seek to determine the allocation of torpedoes to be used as ASROCs from the VLS and not those that may be employed from the SVTT as reactive shots.

To ensure engagements are feasible, particularly for the aircraft, it is necessary to differentiate between capacity and maximum number of torpedoes a blue shooter can expend on a target (also known as maximum "salvo" size). Capacity for ships would be VLS cells available for ASROCs, a ship magazine for embarked helicopters, and a shoreside magazine for P-8 squadron. The maximum salvo size would be what the aircraft would physically carry.

Another assumption is that time is not a component in TASOM. The decisions of which blue shooters engage which red targets and with what torpedoes are made at once: if a blue shooter can engage multiple red targets during a deployment, weapon assignment decisions are made as if engagements were simultaneous.

Similarly, shooter to target decisions are made when multiple blue shooters can engage the same red. Both TASOM-1 and TASOM-2 have the capability for this type of "cooperative engagement." However, in practice, this is a rare event and our computational experience is limited to a single shooter per target.

Lastly, the scenario information defines whether or not a blue shooter may engage a target. If so, we assume the target can be tracked regardless of the salvo size.

B. TASOM-1

This model focuses on minimizing the cost of purchasing new torpedoes and the expected "cost" of red targets for which the desired probability of kill is not achieved (a planner's input). Specifically, shooters will only engage targets for which the desired probability of kill is achieved; other targets are abandoned. The assessed cost for not meeting a target probability of kill threshold is assumed to be a planner's input. In our test case, we set the cost of completely missing a target to $100M.

1. Indices and Sets [Size]

$t \in T$ torpedo variants [~3];

$b \in B$	individual blue shooters [~10];
$p_b \in P$	platform type of *b* shooter [~3];
$r \in R$	individual red targets [~20];
$s_r \in S$	submarine type of *r* target [~3];
$\omega \in \Omega$	scenarios defining red force composition [~50].

2. Scenario-dependent Sets

$R^{D,\omega} \subset R$	subset of individual red targets that are deployed, under scenario ω;
$B^{E,\omega} \subset B$	subset of individual blue shooters that exist, i.e., not killed, under scenario ω;
$B_r^{\omega} \subset B^{E,\omega}$	subset of individual existing blue shooters that can engage target $r \in R^{D,\omega}$ under scenario ω.

3. Parameters [Units]

$cost$	nominal cost of loading a torpedo from inventory [$];
$cost'_t$	cost of purchasing and loading torpedo variant *t* [$];
$budget$	budget for purchasing torpedoes [$];
inv	inventory of torpedoes variant *t* [torpedoes];
$thres_r$	probability of kill threshold for target *r* [unitless];
ε_r	cost for not meeting probability of kill threshold for target *r* [$]; in our test cases we set it to $100M for all targets;
cap_b	number of torpedoes shooter *b* can carry [torpedoes];
$maxb_r$	maximum number of blue shooters that can shoot at target *r* [shooters];

$maxt_b$ maximum number of torpedoes shooter b can expend on a target [torpedoes];

$pkill_{t,p_b,s_r}$ probability of killing submarine type s_r by platform type p_b with torpedo variant t [unitless].

4. Scenario-dependent Parameters [Units]

$prob^{\omega}$ probability of scenario ω occurring [unitless].

5. Derived Sets and Parameters [Units]

$R^{T,\omega} \subset R^{D,\omega}$ subset of individual red targets that are targetable, i.e., can be engaged by an existing blue shooter: $R^{T,\omega} = \{r \in R^{D,\omega} \mid \exists b \in B_r^{\omega}\}$;

$R^{\varnothing,\omega} \subset R^{D,\omega}$ subset of individual red targets that are not targetable: $R^{\varnothing,\omega} = R^{D,\omega} \setminus R^{T,\omega}$;

$miss_r$ value of $-\ln(1 - thres_r)$ (where ln refers to natural logarithm) for target r [unitless];

$pkill'^{\omega}_{t,b,r}$ probability of target r of submarine type s_r being killed by shooter b of platform type p_b with torpedo variant t [unitless]:

$$pkill'^{\omega}_{t,b,r} = pkill_{t,p,c} \text{ for } b \in B_p, r \in R_c^{D,\omega} \text{ if } b \in B_r^{\omega}.$$

6. Decision Variables [Units]

$X_{t,b}$ number of torpedoes of variant t from inventory to be loaded onto shooter b [torpedoes];

$X'_{t,b}$ number of torpedoes of variant t purchased to be loaded onto shooter b [torpedoes];

$Y^{\omega}_{t,b,r}$ number of torpedoes of variant t expended by shooter b on target r under scenario ω [torpedoes];

$B_{b,r}^{\omega}$ equals 1 if shooter b engages target r under scenario ω, and 0 otherwise [unitless];

D_r^{ω} equals 1 if probability of kill threshold for target r is not met under scenario ω, and 0 otherwise [unitless].

7. Formulation

$$\min_{X,X',D} \sum_t \sum_b (cost\, X_{t,b} + cost'_t\, X'_{t,b}) + \sum_{\omega} \sum_{r \in R^{T,\omega}} prob_{\omega} \varepsilon_r thres_r D_r^{\omega} + \sum_{\omega} \sum_{r \in R^{\varnothing,\omega}} prob_{\omega} \varepsilon_r thres_r \tag{1}$$

s.t.

$$\sum_t \sum_b cost'_t\, X'_{t,b} \le budget \tag{2}$$

$$\sum_b X_{t,b} \le inv_t \quad \forall t \in T \tag{3}$$

$$\sum_t X_{t,b} + X'_{t,b} \le cap_b \quad \forall b \in B \tag{4}$$

$$\sum_{r \in R^{T,\omega} | b \in B_r^{\omega}} Y_{t,b,r}^{\omega} \le X_{t,b} + X'_{t,b} \quad \forall t \in T, b \in B, \omega \in \Omega \tag{5}$$

$$\sum_{b \in B_r^{\omega}} B_{b,r}^{\omega} \le maxb_r \quad \forall r \in R^{T,\omega}, \omega \in \Omega \tag{6}$$

$$\sum_t Y_{t,b,r}^{\omega} \le maxt_b B_{b,r}^{\omega} \quad \forall b, r \in R^{T,\omega} \text{ if } b \in B_r^{\omega}, \forall \omega \in \Omega \tag{7}$$

$$\sum_t \sum_{b \in B_r^{\omega}} \ln(1 - pkill'_{t,b,r}) Y_{t,b,r}^{\omega} \le \ln(1 - thres_r) + miss_r D_r^{\omega} \quad \forall r \in R^{T,\omega}, \omega \in \Omega \tag{8}$$

$$Y_{t,b,r}^{\omega} \ge 0 \text{ and integer} \quad \forall t \in T, b \in B, r \in R^{T,\omega}, \omega \in \Omega \tag{9}$$

$$X_{t,b}, X'_{t,b} \ge 0 \text{ and integer} \quad \forall t \in T, b \in B \tag{10}$$

$$D_r^{\omega} \in \{0,1\} \quad \forall \mathrm{r} \in \mathrm{R}^{T,\omega}, \omega \in \Omega \tag{11}$$

$$B_{b,r}^{\omega} \in \{0,1\} \quad \forall \mathrm{r} \in \mathrm{R}^{T}, b \in B, \omega \in \Omega \tag{12}$$

8. Discussion

The objective function (1) minimizes the cost of torpedoes allocated, from inventory or purchased, and the expected cost of not meeting the probability of kill threshold, including red submarines that are targetable and not targetable. The first term (cost of loading torpedoes already in inventory) is usually negligible compared to the cost of loading torpedoes that need to be purchased. Since a shooter must fully meet the probability of kill threshold in an engagement, we apply the cost of not meeting a threshold to the desired threshold itself. This is done for comparison purposes with TASOM-2. We also note that the last term in the objective function is a constant. It penalizes individual red targets guaranteed to survive (due to the lack of blue shooters capable of targeting them). We keep it in order to measure total damage (not just from the targetable red submarines).

Constraint (2) ensures the total cost of torpedoes purchased does not exceed the budget. Constraints (3) ensure that all torpedoes that are allocated, when not purchased, are in inventory. Constraints (4) limit the total number of torpedoes carried by each blue shooter to its capacity. Constraints (5) limit the total number of torpedoes expended in engagements by a blue shooter for whichever scenario occurs to the number of torpedoes allocated to that shooter. Constraints (6) limit cooperative engagements: a limited number of blue shooters is allowed to engage each red target in any given scenario. Constraints (7) ensure each shooter expends torpedoes only on targets that it is engaging and further limits the number of torpedoes it can expend on the target.

Constraints (8) ensures the mix of torpedoes expended on a target achieves a specified minimum probability of kill for any given scenario, or deems the submarine engagement under that scenario as not targetable. Note: original constraints to enforce meeting all thresholds are:

$$\prod_{t}\prod_{b\in B_r^{\omega}}(1-pkill'_{t,b,r})^{Y_{t,b,r}^{\omega}} \le 1-thres_r \quad \forall r\in R^{T,\omega}, \omega\in\Omega .$$

Constraints (8) are derived after taking natural logarithms on both sides, for an equivalent expression. We then add $miss_r D_r^{\omega}$ to the right-hand side in order to signal when an original constraint was not met.

Lastly, Constraints (9) to (12) include variable domains.

C. TASOM-2

This model focuses on minimizing the cost of purchasing new torpedoes and the cost due to expected total deviation from the desired probability of kill threshold. Thus, as opposed to TASOM-1, TASOM-2 only penalizes the unmet fraction of the threshold. Unfortunately, the linearization that led to Constraints (8) cannot be used to measure this deviation. Instead, engagements are pre-generated as candidate "packages," where each package consists of a torpedo mix. Thus, we may also pre-calculate the probability that a candidate package kills a red target, and we may penalize the shortfall from the probability threshold as desired (e.g., via functions that are linear, quadratic, logarithmic, etc.) In our computational experience we use a linear penalty.

Albeit not a model limitation, for the computational experience we do not pre-generate "cooperative engagements" on the same target from different blue shooters. Limiting the candidate engagements to torpedo mixes from a single blue shooter reduces the computational complexity which, in turn, helps us to solve this model faster.

For brevity, we only show additional sets and parameters used in TASOM-2 with respect to TASOM-1. However, we still list all decision variables.

1. Derived Sets and Parameters [Units]

$c \in C$ all potential combinations of torpedo variants [combinations];

$n_{c,t}$ number of torpedoes of variant t in combination c [torpedoes];

$C_b \subset C$ subset of combinations that can be carried by shooter b:

$$C_b = \{c \in C \mid \sum_t n_{c,t} \le maxt_b\};$$

$pkill'^{\omega}_{c,b,r}$ probability of target r of submarine type s_r by shooter b of platform type p_b with torpedo combination c [unitless]:

$$pkill'^{\omega}_{c,b,r} = 1 - \prod_t (1 - pkill_{t,p_b,s_r})^{n_{c,t}}, \text{ if } b \in B^{E,\omega} \cap B^{\omega}_r \text{ and } c \in C_b.$$

2. Decision Variables [Units]

$X_{t,b}$ — number of torpedoes of variant *t* purchased to be loaded onto shooter *b* [torpedoes];

$X'_{t,b}$ — number of torpedoes of variant *t* purchased to be loaded onto shooter *b* [torpedoes];

$Y^{\omega}_{c,b,r}$ — equals 1 if shooter *b* engages target *r* with torpedo combination c under scenario ω, and 0 otherwise [unitless];

δ^{ω}_{r} — shortfall of probability of kill threshold for target *r* under scenario ω [unitless].

3. Formulation

$$\min_{X,X',\delta} \sum_{t}\sum_{b}(cost\, X_{t,b} + cost_t X'_{t,b}) + \sum_{\omega}\sum_{r\in R^{T,\omega}} prob_{\omega}\varepsilon_r\delta^{\omega}_{r} + \sum_{\omega}\sum_{r\in R^{\varnothing,\omega}} prob_{\omega}\varepsilon_r thres_r \quad (13)$$

$$\text{s.t.} \quad \sum_{t}\sum_{b} cost'_t\, X'_{t,b} \le budget \quad (14)$$

$$\sum_{b} X_{t,b} \le inv_t \quad \forall t\in T \quad (15)$$

$$\sum_{t} X_{t,b} + X'_{t,b} \le cap_b \quad \forall b\in B \quad (16)$$

$$\sum_{r\in R^{T,\omega}}\sum_{c\in C_b} n_{c,t}Y^{\omega}_{c,b,r} \le X_{t,b} + X'_{t,b} \quad \forall t\in T, b\in B, \omega\in\Omega \quad (17)$$

$$\sum_{b\in B^{\omega}_{r}}\sum_{c\in C_b} pkill'^{\omega}_{c,b,r}\, Y^{\omega}_{c,b,r} \ge thres_r - \delta^{\omega}_{r} \quad \forall r\in R^{T} \in R^{T,\omega}, \omega\in\Omega \quad (18)$$

$$\sum_{b\in B^{\omega}_{r}}\sum_{c\in C_b} Y^{\omega}_{c,b,r} \le 1 \quad \forall r\in R^{T,\omega}, \omega\in\Omega \quad (19)$$

$$Y^{\omega}_{c,b,r} \in \{0,1\} \quad \forall c\in C, b\in B, r\in R^{T,\omega}, \omega\in\Omega \quad (20)$$

$$X_{t,b}, X'_{t,b} \ge 0 \text{ and integer} \quad \forall t\in T, b\in B \quad (21)$$

$$\delta^{\omega}_{r} \ge 0 \quad \forall \mathrm{r}\in \mathrm{R}^{T,\omega}, \omega\in\Omega \quad (22)$$

4. Discussion

The objective function (13) minimizes the cost of allocated torpedoes, from inventory or purchased, and the expected cost of not meeting the probability of kill threshold, including red submarines that are targetable and not targetable. The latter expected cost is based on the deviation between the achieved probability of kill and the desired threshold.

Constraint (14) ensures the total cost of torpedoes purchased does not exceed the budget. Constraints (15) ensure that all torpedoes that are allocated, when not purchased, are in inventory. Constraints (16) limit the total number of torpedoes carried by a *b* shooter to its capacity. Constraints (17) limit the number of torpedoes within the torpedo combinations used in engagements by a blue shooter for whichever scenario occurs to the number of torpedoes allocated to that shooter. Constraints (18) calculates the deviation when the combination of torpedoes expended on a red target fails to achieve a specified probability of kill for any given scenario. Of note, this equation could be eliminated and the deviation δ_r^ω easily precalculated for every candidate combination *c*: deviation is 0 if *c* meets or exceeds the threshold, or the shortfall otherwise. That would allow for any deviation to be penalized nonlinearly (which we do not in our computational experience, but is a straightforward extension). Constraints (19) ensure that at most one combination of torpedoes is used to engage a red target for any scenario.

Lastly, Constraints (20) to (22) include variable domains.

D. DETERMINISTIC MODELS

Later in this report, we compare the solutions from the stochastic models to deterministic solutions found by solving versions of TASOM-1 and TASOM-2 for only one representative scenario. In the deterministic version of both TASOM-1 and TASOM-2, the scenario set is a singleton, $\Omega = \{\hat{\omega}\}$, and $prob^{\hat{\omega}} = 1$. That is, we assume "perfect information" about a certain scenario $\hat{\omega}$. Sometimes $\hat{\omega}$ may be one of the original scenarios (in order to analyze the value of perfect information); others, $\hat{\omega}$ may refer to a hybrid scenario (such as to analyze the value of the stochastic solution). The objective function for the deterministic version of TASOM-1 minimizes the cost of torpedoes allocated, from inventory or purchased, and the cost (instead of expected cost) of not meeting the probability of kill threshold in the assumed scenario. Similarly, for the deterministic version of TASOM-2, the objective function minimizes the cost of allocated torpedoes and the cost of not meeting the probability of kill threshold.

III. RESULTS AND ANALYSIS

All input data for the models are contained in a multi-sheet Microsoft Excel workbook (Microsoft Corporation 2022). Inputs can easily be changed by the user to consider more torpedo variants, shooter platforms, target types, individual blues, and individual reds.

We present an ASW operation where several ASW units are patrolling an area for adversary submarines. We randomly generate 100 scenarios where different types of adversary submarines appear to the ASW units. We use unclassified, notional data of inventories, costs, capabilities, and threats. Using this test case, we first determine the value of using either TASOM over the deterministic approach of planning. We then explore how our models can be used to inform torpedo allocation and budgeting decisions.

Our notional case considers three different torpedo variants, ten individual blue shooters available for the campaign, and twenty individual red targets (of three different types) that can deploy in a scenario. Mod 0 and mod 1 torpedoes have a pre-existing inventory; any mod 2 torpedoes must be purchased.

All the specifics about baseline data inputs, scenario generation and output display can be found in the Appendix. The remainder of this chapter is devoted to analyzing the value of TASOM compared to deterministic planning, and performing sensitivity analysis on the baseline data.

A. IMPLEMENTATION

Both TASOM models have been implemented using Pyomo, a Python optimization modeling package (Hart et al. 2011, 2017), using CPLEX (IBM 2022) as the core solver. Hardware includes a personal laptop with 16 gigabytes of random-access memory available and eight processors.

In our notional, baseline test case with three torpedo variants, ten blue shooters, and twenty red targets, containing 100 scenarios, the number of variables and constraints in both models are listed in Table 1.

TASOM-1 solves to optimality in 112 seconds; TASOM-2 solves in 15 seconds. The computational runtimes in seconds are listed in Table 2. Additional runtimes for other test cases with the same assets but 500 and 1,000 scenarios, respectively, are included for comparison.

Table 1. Computational data for baseline case with 100 scenarios

	continuous variables	binary variables	integer variables	constraints
TASOM-1	0	4,548	11,433	8,320
TASOM-2	758	15,9263	60	4,529

Table 2. Comparison of computation times for baseline case with various number of scenarios

number of scenarios	optimality gap	TASOM-1	TASOM-2
		runtime (seconds)	
100	5%	17	6
	0%	112	15
500	5%	1,657	90
1,000	5%	3,010	158

B. COMPARISON OF DETERMINISITC VS. STOCHASTIC MODELING

We first determine the value of (hypothetical) complete knowledge of the adversary before making loadout decisions. To do that, we optimize for one scenario at a time (therefore obtaining different loadouts, each one suited to a particular scenario). This solution by scenario is not implementable in real life, given the loadouts must be established before the actual scenario unveils. Nevertheless, it gives us an ideal objective value (by individual scenario and in expected value). We then compare it to the loadout plan from the stochastic solution, which is a compromise among all 100 scenarios. In addition, we consider two implementable loadout solutions, derived from typical deterministic manual planning: the "average loadout" plan and the loadout plan from the "all-targets" scenario.

5. Expected Value of Perfect Information

The expected value of perfect information (EVPI) assesses how much better (in terms of expected cost of purchasing torpedoes and missing targets) a torpedo loadout plan would be if a planner had perfect scenario information beforehand (Avriel and Williams 1970). The EVPI is found by subtracting the expected optimal deterministic cost from the optimal stochastic cost. By denoting z^{Ω} as the optimal objective function value to a given stochastic model (such as TASOM-1 or TASOM-2) with scenarios in set Ω (and, accordingly, $z^{\{\omega\}}$ being such objective for a deterministic problem for single scenario ω), EVPI can be defined as:

$$EVPI = z^{\Omega} - \sum_{\omega \in \Omega} prob^{\omega} z^{\{\omega\}}.$$

The EVPI for our notional case, and budgets of \$25M and \$10M, respectively, is presented in Table 3. If, for example, a planner is using TASOM-1 and has a purchasing budget of \$25M, and perfect information could be attained for less than \$14.12M, that would be a worthwhile investment. Otherwise, using the stochastic loadout is more cost efficient. Note that, for this problem, having perfect information for planning purposes would involve knowing with certainty all of the following: how many and which red submarines are deployed to an area; which blue shooters will detect and engage them; and, *then*, appropriately allocating torpedoes to ships and squadrons.

Table 3. Expected cost from stochastic and deterministic models, and value of perfect information, for budget levels of \$25M and \$10M

	TASOM-1			TASOM-2		
	Stochastic	Deterministic	EVPI	Stochastic	Deterministic	EVPI
Budget \$25M	\$114.70M	\$100.58M	\$14.12M	\$33.04M	\$22.35M	\$10.68M
Budget \$10M	\$199.90M	\$194.56M	\$5.34M	\$38.61M	\$27.90M	\$10.70M

6. Average Loadout vs. Stochastic Loadout

Typically, the value of the stochastic solution (VSS) is assessed by comparing the objective value of the stochastic solution to the value of the deterministic solution that replaces the uncertain variable in the problem with its expected value (Birge and Louveaux 1997). The typical approach consists of: (a) defining an "average-demand scenario," which we denote $\bar{\omega}$; (b) calculating its first-stage solution, $\bar{X}$ (loadouts) using a deterministic version of TASOM; (c) fixing that loadout solution in TASOM in order to calculate the second-stage (WTA) solution by scenario; and (d) comparing each of those individually, and on the aggregate, to the full stochastic (TASOM) solution.

However, in our problem, the uncertainty relates to which red targets appear and which blue shooters can engage them; thus, finding the "average" scenario is somewhat arbitrary: we could consider $\bar{\omega}$ where blue shooters need to kill a fraction of each red target; but, that does not seem to represent a sensible scenario in itself. Or, $\bar{\omega}$ where blue shooters need to kill an "average subset" of red targets (such as those that appear most frequently); but, that makes the deterministic solution myopic to the existence of other red targets.

Thus, we replace steps (a) and (b) above by a single step that calculates the average loadout $\bar{X}$ across all scenarios directly, when each scenario is (deterministically) solved for optimality. Specifically, we use the average number of torpedoes allocated to each blue shooter in the 100 scenarios, and refer to this as the "average loadout plan," $\bar{X}$.

Loadout plans (total torpedoes by type) for TASOM-1 and TASOM-2 are summarized in Table 4, including: (i) the stochastic loadout with the baseline budget of \$25M; (ii) the stochastic loadout where the budget is limited based on the cost of torpedoes purchased in the abovementioned (deterministic) average loadout plan, $\bar{X}$; and (iii) the average loadout plan itself.

Here, the VSS is found by subtracting the optimal stochastic cost from the expected cost of using the average loadout:

$$VSS = \sum_{\omega \in \Omega} prob^{\omega} z^{\{\omega\}}(\bar{X}) - z^{\Omega},$$

where $z^{\{\omega\}}(\bar{X})$ still refers to the optimal objective function value of a (deterministic) TASOM for single scenario ω, but with its loadout variables fixed to $\bar{X}$.

The larger the VSS, the greater the justification to use a stochastic programming approach in planning instead of using the simpler approach of planning for the average scenario. The VSS for our notional case, and budgets of $25M and $10M, respectively, is presented in Table 5. For example, when planning with TASOM-1 and a $25M budget, using the average loadout will cost an additional $92.60M than if the stochastic loadout was used.

Table 4. Summary (total torpedoes) for: deterministic, fixed average loadout plan with unlimited budget; stochastic loadout plan when budget is limited by the deterministic solution; and stochastic, for a $25M budget

	TASOM-1			TASOM-2		
	Deterministic (fixed average loadout)	Stochastic (budget as in deterministic)	Stochastic ($25M budget)	Deterministic (fixed average loadout)	Stochastic (budget as in deterministic)	Stochastic ($25M budget)
mod 0	2	2	15	5	14	30
mod 1	23	26	30	24	64	36
mod 2	6	6	11	5	4	9

Table 5. Value of stochastic solution over manual planning with deterministic, fixed average loadout plan, for budget limited by the deterministic solution, and for budget of $25M

	TASOM-1			TASOM-2		
	Stochastic	Deterministic	VSS	Stochastic	Deterministic	VSS
Limited budget	$179.40M	$207.30M	$27.90M	$40.50M	$103.24M	$62.74M
$25M	$114.70M	$207.30M	$92.60M	$33.04M	$103.24M	$70.21M

A more detailed comparison of each loadout performance for TASOM-1 is shown in Figure 1. Performance is evaluated in TASOM-1 by the number of targets where the probability of kill threshold is not met at all, or by the number of targets missed, in a scenario. Plotted are exceedance functions that show the probability (percentage of scenarios) where the number of targets missed exceeds a given value. The stochastic loadout with a budget of $25M performs, as expected, significantly better than the average loadout which only expends $12M: two or more submarines are missed in 65% of the scenarios under the deterministic loadout; only 25% under the stochastic with the baseline budget. However, when TASOM-1 is forced to use the budget dictated by the deterministic model, its improvement over the average loadout plan is diminished. Nevertheless, it is important to remark that the deterministic solution is unable to take advantage of additional budget, whereas TASOM-1 does.

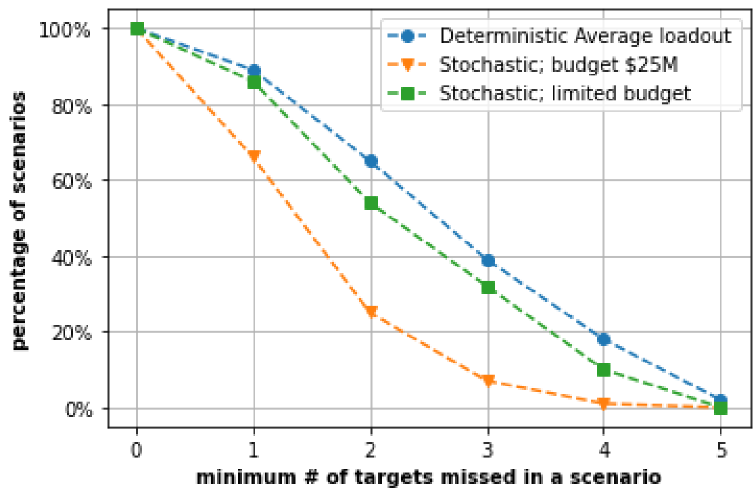

Figure 1. TASOM-1: Exceedance functions for misses for: deterministic, fixed average loadout plan with unlimited budget; stochastic loadout plan when budget is limited by the deterministic solution; and stochastic loadout plan for a $25M budget

The comparison for each loadout performance for TASOM-2 is shown in Figure 2. Note for TASOM-2, the performance is evaluated by the average shortfall in meeting the probability of kill thresholds for all targets in a scenario. Plotted are exceedance functions that show the probability the average shortfall exceeds a value, under a given loadout. The difference between the stochastic, even with a similar budget of $10M, and the average loadout performance is significant: the average probability of kill shortfall is 0.1 or greater for 50% of the scenarios under the average loadout; only 2% of the scenarios under the stochastic with a limited budget. The expected shortfall is 0.1144 under the average loadout and 0.0365 under the stochastic with a limited budget.

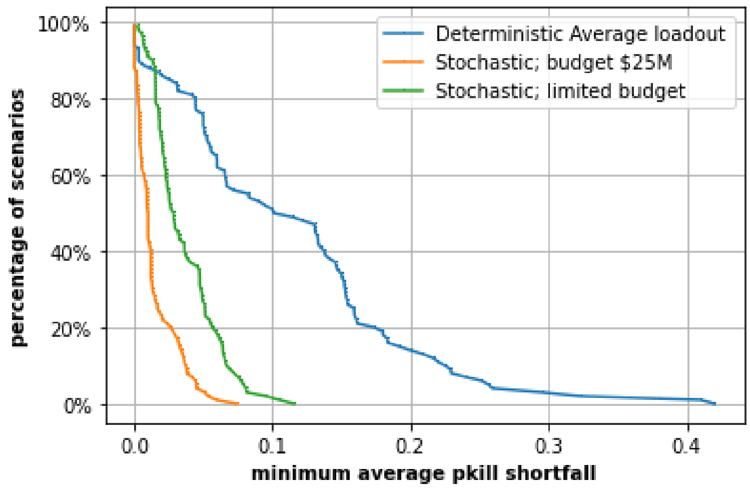

Figure 2. TASOM-2: Exceedance functions for average probability of kill shortfall: deterministic, fixed average loadout plan with unlimited budget; stochastic loadout plan when budget is limited by the deterministic solution; and stochastic loadout plan for a $25M budget

7. Loadout from All-Targets Scenario vs. Stochastic Loadout

Another important study, from a planner's perspective, consists of calculating a solution that plans against a scenario consisting of all targets. The "all-targets" scenario is defined as a deterministic scenario where all blue shooters are alive and have detected and able to engage all twenty red targets.

We compare the deterministic solution that plans for the all-targets scenario with the TASOM-1 and TASOM-2 solutions for the same 100 simulated scenarios as above. We use budgets of $25M and $10M. The loadouts recommended by this analysis are summarized in Table 6. (Of course, TASOM solutions for a budget level of $25M are unchanged with respect to the previous analysis.)

Table 6. Summary (total torpedoes) of the stochastic loadout plan and the loadout plan from the all-targets scenario

		TASOM-1		TASOM-2	
		Stochastic	Deterministic (all-targets loadout)	Stochastic	Deterministic (all-targets loadout)
Budget $25M	mod 0	15	26	30	35
	mod 1	30	30	36	55
	mod 2	11	12	9	10
Budget $10M	mod 0	10	15	30	22
	mod 1	24	15	30	72
	mod 2	5	5	5	3

As shown in Figure 3, for TASOM-1, with the budget of $25M, the stochastic loadout only performs slightly better than the deterministic loadout planned for an all-targets scenario. However, the performance increase is more significant when the solutions with the $10M budget are considered: two or more submarines are missed in 70% of the scenarios under the deterministic loadout; only 60% under the stochastic.

When we consider various budget levels, the performance of TASOM-1 loadout remains only marginally better over the deterministic, as shown in Figure 4. At a budget level of $5M, the expected number of misses is 4.59 under the deterministic loadout and 5.15 under the stochastic loadout.

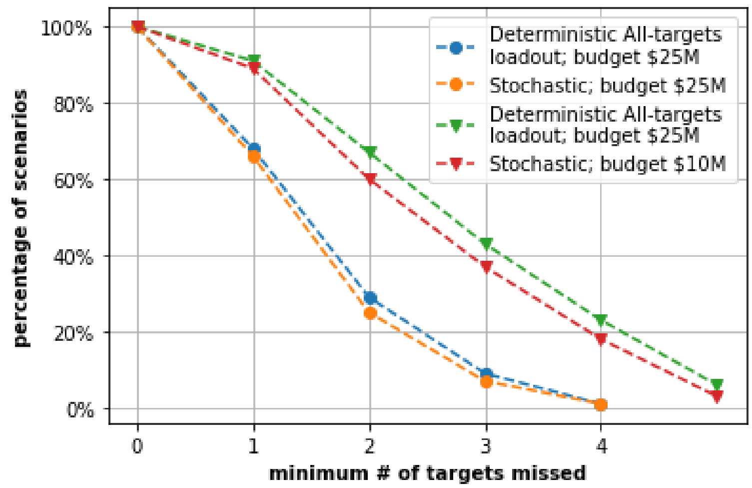

Figure 3. TASOM-1: Exceedance functions for misses for deterministic, fixed loadouts from all-targets scenario and stochastic loadouts, for $25M and $10M budgets

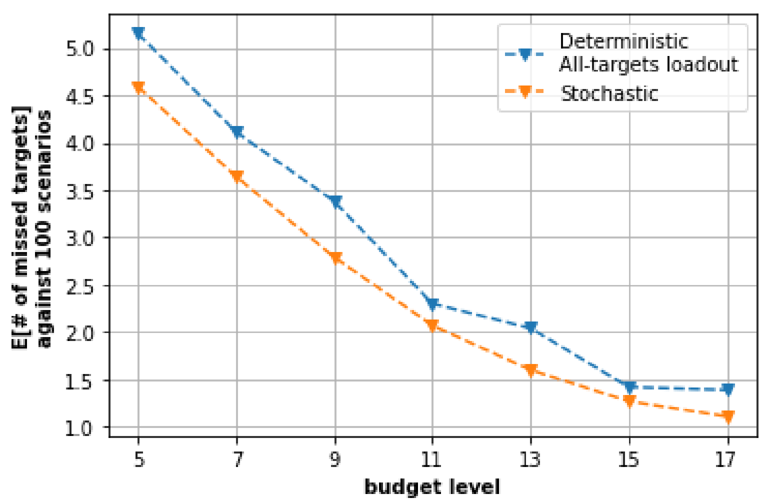

Figure 4. TASOM-1: Expected number of missed targets for various budget levels

The difference between the stochastic and the deterministic loadout performance is much more dramatic for TASOM-2, as shown in Figure 5. With a budget of $10M the deterministic loadout produces an average probability of kill shortfall of 0.1 or greater for 65% of the scenarios; in contrast, in the stochastic models it is only 2% of the scenarios for the same budget. With a budget of $10M, the expected average shortfall is 0.1421 under the deterministic loadout and 0.0317 under the stochastic loadout.

Of note, the deterministic and stochastic solutions for TASOM-2 do not have the same symmetric grouping as the solutions in TASOM-1. In all solutions using model TASOM-1, torpedoes are almost exclusively loaded out to P-8 squadrons. In our notional case, torpedoes from aircraft will result in higher probabilities of kill and P-8s have a greater max salvo size than a helicopter, an important consideration when only one ASW unit can engage a submarine. In TASOM-2, solutions utilize helicopters and ships in addition to P-8s; the deterministic solutions only use P-8s. For this reason, the stochastic loadout with a budget of $10M is still able to perform better than the deterministic loadout with the greater budget of $25M.

At all considered budget levels, the performance of TASOM-2 loadout remains significantly better over the deterministic version, as shown in Figure 6. For a budget of $5M, the expected average probability of kill shortfall is 0.1463 under the deterministic loadout and 0.0526 under the stochastic loadout.

As discussed above, the deterministic solution for TASOM-2 only loads out torpedoes to P-8s. We consequently see very similar performances of the deterministic loadouts at different budget levels and the expected average shortfall for the deterministic loadout does not consistently decrease as the budget level increases: in fact, the expected average shortfall increases when the budget is raised from $11M to $15M. This is not necessarily a contradiction: the deterministic plan is created against an all-targets scenario and shortfall for that assumed scenario (not shown) actually decreases as budget increases. However, when that plan is tested against 100 specific scenarios, the shortfall need not be monotonic.

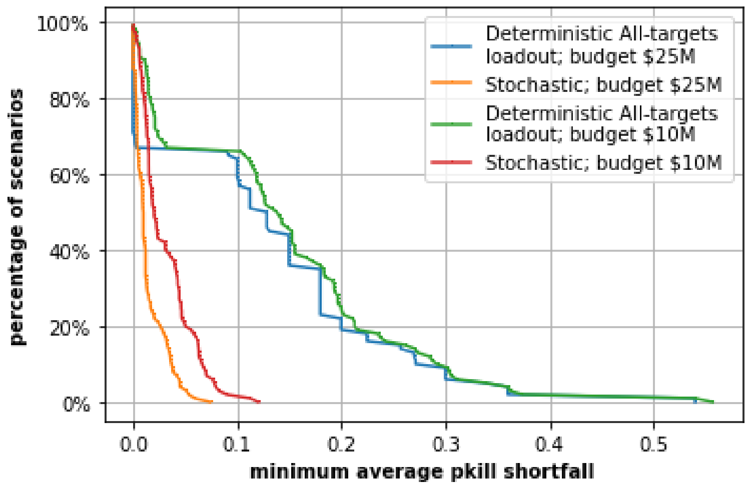

Figure 5. TASOM-2: Exceedance functions for average probability of kill shortfall for deterministic, fixed loadouts from all-targets scenario and stochastic loadouts, for $25M and $10M budgets

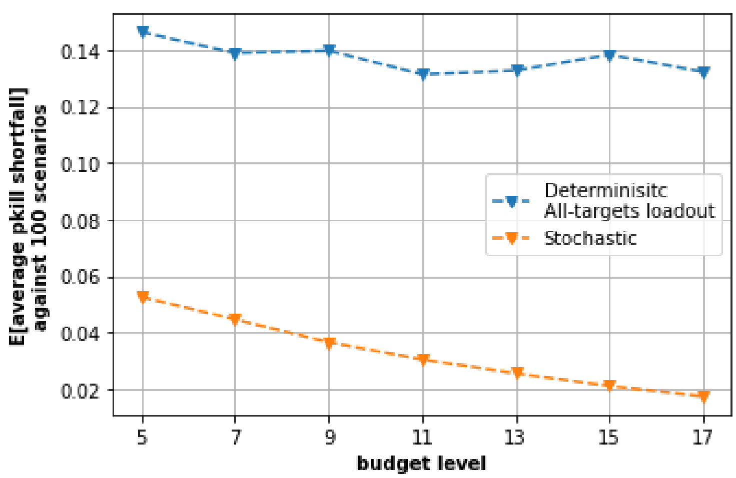

Figure 6. TASOM-2: Expected average probability of kill shortfall for various budget levels

C. SENSITIVITY ANALYSIS

8. Varying Budget Level

Performance metrics are compared at different budget levels to help inform torpedo budget decisions. In Figure 7, we see how increasing the budget yields improved performance for TASOM-2. For example, with a budget of $8M, the average shortfall for the desired probability of kill is 0.02 (or greater) in 70% of the scenarios. With a budget of $10M, that improves to 50% of the scenarios. Note that the loadout from using a budget of $25M (our baseline budget), is not plotted as its performance is almost identical to that of a loadout from using a budget of $20M.

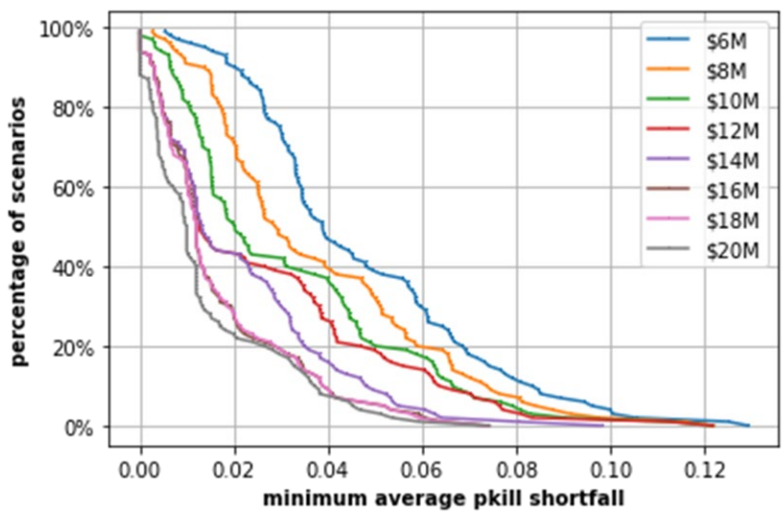

Figure 7. TASOM-2: Exceedance functions for average probability of kill shortfall for stochastic loadouts for various budgets levels

9. Varying Desired Probability of Kill

Sensitivity analysis can also be conducted on the probability of kill threshold desired for red target engagements. Here, probability of kill thresholds for all targets have been uniformly adjusted. In Figure 8, when the desired probability of kill threshold is reduced to 0.85 (from the original 0.90 in all previous test cases), the number of mod 2 torpedoes purchased in TASOM-2 loadout plan is reduced. Specifically, for all budget levels (starting at $5M), and for any probability of kill threshold (starting at 70%), a minimum of two mod 2 torpedoes are purchased by TASOM-2. This number increases to twelve mod 2 for 95% and $25M, respectively.

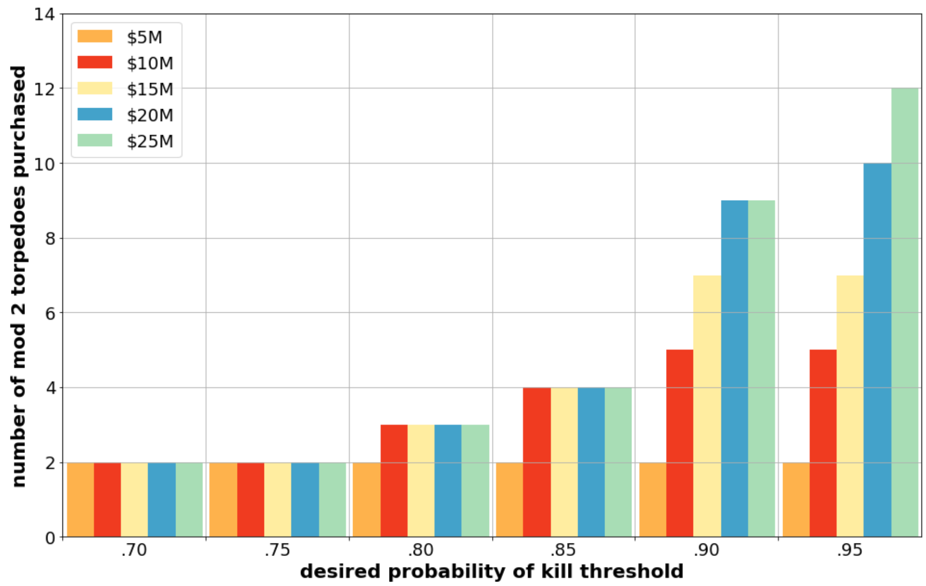

Figure 8. TASOM-2: Number of mod 2 torpedoes purchased for different budget levels and desired probability of kill thresholds

10. Varying Effectiveness of Mod 2

We specify different, hypothetical probability of kill values of the mod 2 torpedo in Table 7. Note, the percentage increase of the mod 2 compared to the mod 1 applies uniformly for ship or air platforms, but differently against each target type. The percentage value shown in the table is the percent increases in probability of kill of a mod 2 torpedo from a mod 1 torpedo against a type 1 target. Of note, that percentage is reduced against type 2 and type 3 targets. For example, if the probability of kill increases by 100% (from a mod 1 probability of kill) with the mod 2 torpedo against a type 1, the probability of kill increases by only 90% against a type 2 target and 85% against a type 3 target. This roughly follows the performance of the mod 2 that we assumed in the baseline test case; the mod 2 exact probability of kill values used in the test case are not represented but are most similar to those at the 85% level.

In Figure 9, we vary the probability of kill of a mod 2 torpedo to find the number of mod 2 torpedoes purchased under different budget levels. Here, we keep our original desired probability of kill threshold of 0.90. When the mod 2 torpedo probability of kill improves by 25%, no mod 2 torpedoes are purchased, at any budget level. If the mod 2 improves by 40%, at most five torpedoes are purchased. When the mod 2 improves by 100%, we see that is no longer beneficial to buy more than eight mod 2 torpedoes at any budget level.

Table 7. Probability of kill values of the mod 2 torpedo at various levels of percentage increase over mod 1 probability of kill value

platform	class	mod 0	mod 1	mod 2					
				25%[b]	40%	55%	70%	85%	100%
ship	type1	.13[a]	.16	.2	.22	.25	.27	.30	.32
ship	type2	.13	.16	.18	.21	.23	.26	.28	.30
ship	type3	.19	.2	.22	.25	.28	.31	.34	.37
aircraft	type1	.29	.3	.38	.42	.47	.51	.56	.6
aircraft	type2	.34	.35	.40	.46	.51	.56	.61	.67
aircraft	type3	.39	.4	.44	.5	.56	.62	.68	.74

[a] The probability of kill values for the mod 0 and mod 1 torpedoes are the same values used in previously presented test case.

[b] The percentage value represented here is the increase from the mod 1 torpedo probability of kill against a type 1 target. Against a type 2 target, the percentage increase is 10% less; against a type 3 target, the percentage increase is 15% less.

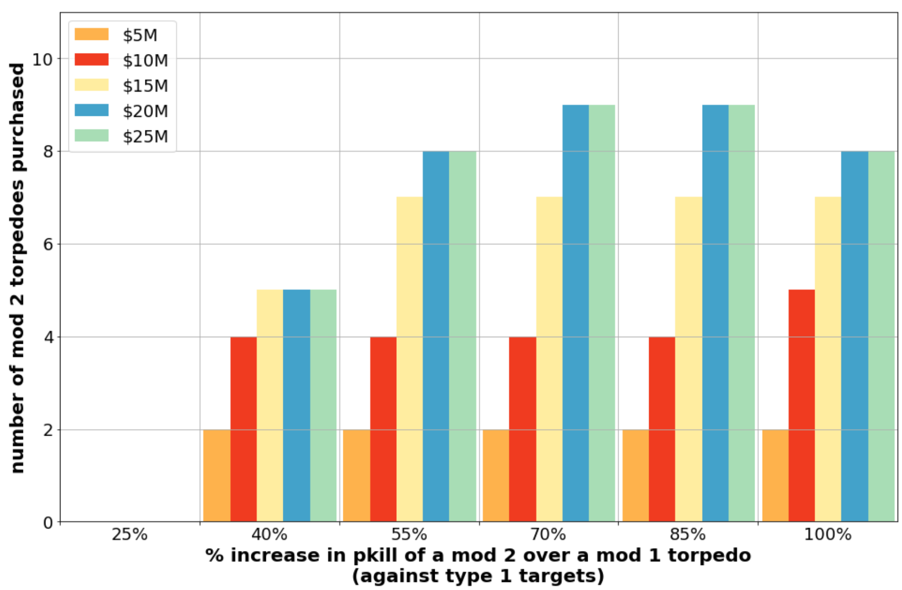

Figure 9. TASOM-2: Number of mod 2 torpedoes purchased for different budget levels and probability of kill values of a mod 2 torpedo

APPENDIX

This appendix describes all input data for the models, which are contained in a multi-sheet Microsoft Excel workbook (Microsoft Corporation 2022).

A. BASELINE DATA

Our notional case considers three different torpedo variants, ten individual blue shooters available for the campaign, and twenty individual red targets that can deploy in a scenario. Mod 0 and mod 1 torpedoes have a pre-existing inventory; any mod 2 torpedoes must be purchased (at a nominal price of $2 million per unit). There is a notional cost to load torpedoes from inventory (in order to prevent extra torpedoes from being included in a solution loadout plan); this cost is not applied to the budget limit. Torpedo inventory and purchasing costs are listed in Table 8. Torpedo loading cost is assumed the same for all torpedoes and listed in Table 9, along with budget for purchase of new torpedoes.

Table 10 lists the capacity and salvo restrictions for each of the three different blue platform types. The probability of kill of a torpedo depends on the variant, the platform type of the shooter, and the submarine type of the target; these data are listed in Table 11.

Table 12 lists the ten blue shooters considered in our test case. All ship units have an embarked helicopter detachment. Table 13 lists the individual red target data. Of the twenty reds, ten are classified as type 1 submarines, seven as type 2, and three as type 3. The commander's desired probability of kill threshold is set to 0.9 for all threats, and the dollar penalty is applied for not meeting that threshold: the entire threshold is penalized in TASOM-1, but only the shortfall (unmet fraction) is penalized in TASOM-2. In TASOM-1, the maximum number of blues that can engage the same red is explicitly set by the user. We set this value to one in all runs for our test case. This input is implicit in TASOM-2: it is dictated by the generated candidate torpedo mixes for each engagement. In our computational runs we have generated mixes limited to one blue shooter.

Table 8. Torpedo inventory and cost data

Torpedo Data		
torpedo_type	cost ($ millions)	inventory
mod_0	0.005	30
mod_1	0.008	30
mod_2	2	0

Table 9. Budget and torpedo loading cost data.

Misc Data	
purchasing budget ($ millions)	loading cost for owned torpedoes ($ millions)
25	0.0001

Table 10. Platform data

Platform Data		
platform_type	torpedo capacity	max salvo size
ship	6	6
helo	15	2
p8	50	5

Table 11. Probability of kill data

Pkill Data			
platform_type	sub_type	torpedo_type	pkill
ship	type1	mod_0	0.13
ship	type1	mod_1	0.16
ship	type1	mod_2	0.2
ship	type2	mod_0	0.13
ship	type2	mod_1	0.16
ship	type2	mod_2	0.2
ship	type3	mod_0	0.19
ship	type3	mod_1	0.2
ship	type3	mod_2	0.4
p8	type1	mod_0	0.29
p8	type1	mod_1	0.3
p8	type1	mod_2	0.6
p8	type2	mod_0	0.34
p8	type2	mod_1	0.35
p8	type2	mod_2	0.65
p8	type3	mod_0	0.39
p8	type3	mod_1	0.4
p8	type3	mod_2	0.7
helo	type1	mod_0	0.29
helo	type1	mod_1	0.3
helo	type1	mod_2	0.6
helo	type2	mod_0	0.34
helo	type2	mod_1	0.35
helo	type2	mod_2	0.65
helo	type3	mod_0	0.39
helo	type3	mod_1	0.4
helo	type3	mod_2	0.7

Table 12. Individual shooter data

Individual Blue Data			
blue name	embarked unit	platform_type	area
p1		p8	theater1
p2		p8	theater1
ddg1	helo1	ship	theater1
helo1		helo	theater1
ddg2	helo2	ship	theater1
helo2		helo	theater1
ddg3	helo3	ship	theater1
helo3		helo	theater1
ddg4	helo4	ship	theater1
helo4		helo	theater1

Table 13. Individual target data

Individual Red Data					
red name	sub_type	area	desired pkill	max # of blues that can engage the same red sub	penalty for not reaching desired pkill ($ million)
r1	type1	theater1	0.9	1	100
r2	type1	theater1	0.9	1	100
r3	type1	theater1	0.9	1	100
r4	type1	theater1	0.9	1	100
r5	type1	theater1	0.9	1	100
r6	type1	theater1	0.9	1	100
r7	type1	theater1	0.9	1	100
r8	type1	theater1	0.9	1	100
r9	type1	theater1	0.9	1	100
r10	type1	theater1	0.9	1	100
r11	type2	theater1	0.9	1	100
r12	type2	theater1	0.9	1	100
r13	type2	theater1	0.9	1	100
r14	type2	theater1	0.9	1	100
r15	type2	theater1	0.9	1	100
r16	type2	theater1	0.9	1	100
r17	type2	theater1	0.9	1	100
r18	type3	theater1	0.9	1	100
r19	type3	theater1	0.9	1	100
r20	type3	theater1	0.9	1	100

B. SCENARIO GENERATION

From the abovementioned baseline data, the user can generate scenarios either manually or randomly. Manually creating a scenario may be preferable if only a few defined scenarios are to be considered and probabilities of occurrence are known. For other cases, we have developed a random scenario generator function. This function allows many scenarios to be created according to some additional parameters provided by the user via the input file. The scenario generator function writes the scenarios to its own data sheet in the same excel workbook as the rest of the input data for the test case. The user can then review and modify the generated scenario data, if desired. For each scenario, the scenario generator function determines (and records):

How many and which specific red targets deploy.

Which blue shooters, if classified as a ship, are preemptively killed. Note: if the killed shooter is a blue ship, its embarked helicopter is also considered killed.

For every surviving blue shooter and deployed red target, whether the shooter detects and can engage the target. Note: if a ship can engage a target, its embarked helicopter can also engage it.

All the targets each shooter could kill, the targets that are "undetected" (i.e., untargeted), and which shooters are killed.

For our baseline test case we generate 100 equally probable scenarios according to the parameters defined for the case, summarized in Table 14. In this example, the given inputs specify that, from the fleet of twenty red targets, a random selection of five to ten targets can deploy in every scenario. Any blue ship is killed prior to engagement with 10% probability. Among those surviving, every blue ship detects a red target in the same area with 40% probability, and the same occurs for every MH-60R. For the P-8s, the probability is 60%. We assume only platforms that have detected the target are allowed to engage it.

A sample of resulting scenario data output is presented in Figure 10. Each scenario is defined in a row and each blue shooter has a column where the reds that it can engage (for that scenario) are listed in the corresponding cell, separated by commas. Note, in this figure, the columns for "ddg2," "helo2," "ddg3," "helo3," and "ddg4" are hidden and only the first thirty scenarios are visible. For example, in scenario "1," platform "ddg1" can shoot at red targets "r3," "r6," and "r2;" all red targets are detected (thus can be attacked) by some blue shooter; and, two blue shooters ("ddg2" and "helo2" are killed, noting that the latter was embarked on the former). On the other hand, in scenario "15," red target "r13" is in the area, but undetected; and, blue platform "ddg3" and its "helo3" are killed. All scenarios have probability 1%. After the generation process, the planner may edit these scenario inputs in Excel (along with any of the other input data) before running TASOM.

Table 14. Scenario generator inputs

Scenario Generator Inputs							
seed #	number of scenarios	min reds deployed	max reds deployed	probability blue ship killed	probability blue ship detects	probability blue helo detects	probability p8 detects
1234	100	5	10	0.1	0.4	0.4	0.6

	prob	p1	p2	ddg1	helo1	helo4	undetected	blues killed
1	0.01	r3,r2,r18,r13,r17	r3,r18,r1,r4,r6,r13,r17	r3,r6,r2	r3,r2,r18,r1,r6,r17	r3,r18,r1,r4,r13,r17		helo2,ddg2
2	0.01	r13,r17,r16	r9,r13,r16	r9,r17,r4,r16	r9,r17,r4,r16	r13,r4,r16		
3	0.01	r11,r5,r18,r12,r6	r3,r14,r5,r11,r18,r7,r6	r3,r11,r5,r7,r4	r3,r11,r5,r7,r4	r14,r11,r5,r18,r4		
4	0.01	r6,r17	r19,r11,r17	r3,r13,r17	r3,r19,r6,r13,r17	r6,r19,r13		
5	0.01	r3,r14,r20,r1,r6,r15	r3,r14,r20,r1,r4,r6,r8,r15	r6,r15	r14,r5,r4,r6,r8,r15	r3,r14,r19,r20,r4,r1,r8		
6	0.01	r18,r17,r1,r15	r13,r10,r1,r15	r3,r14,r10,r13,r15	r3,r9,r14,r10,r1,r13,r15	r14,r17,r10,r1		
7	0.01	r11,r10,r18,r5,r1,r7	r5,r20,r12,r1,r7,r6	r6,r11,r12	r11,r10,r20,r12,r6	r5,r10,r18,r12,r1,r7,r6		
8	0.01	r6,r8,r12,r18	r6,r19,r8	r11	r3,r8,r11,r7	r3,r11,r18,r12,r7,r6		
9	0.01	r3,r20,r14,r6	r14,r6,r17,r8,r15	r6,r20,r8	r20,r6,r17,r8,r15	r3,r14,r20,r17,r8,r15		helo3,ddg3
10	0.01	r3,r14,r10,r20,r4,r6	r3,r14,r4	r6,r14,r20,r17	r14,r10,r20,r12,r6,r17	r12,r20,r10,r4		
11	0.01	r14,r8,r18,r15	r8,r20,r18	r14,r15	r14,r18,r15	r14,r20,r18		
12	0.01	r1	r1,r15	r17	r14,r17,r15	r2,r17,r1,r15		
13	0.01	r5,r10,r2,r12,r13	r10,r13,r5,r2	r11,r13,r5,r12	r11,r10,r5,r12,r13	r11,r5		
14	0.01	r19,r5,r4	r3,r14,r12,r1,r17	r19,r17,r12,r4	r3,r14,r19,r12,r4,r17	r14,r19,r5,r1,r17,r8		
15	0.01	r9,r14,r10,r18,r6,r15	r6,r9,r4	r6,r10	r6,r10	r10,r12,r4,r6,r8,r15	r13	helo3,ddg3
16	0.01	r9,r5,r15	r9,r15	r9,r5	r9,r5,r8,r15	r3,r5,r8		
17	0.01	r3,r14,r2,r12,r1	r9,r2,r12,r4,r6	r9,r1,r4	r3,r9,r11,r2,r12,r1,r4,r6	r3,r11,r2,r12,r6		
18	0.01	r9,r1,r7,r13,r17	r9,r11,r1,r6,r17	r6,r11,r10,r7	r6,r11,r10,r7	r9,r10,r7,r6,r13,r17,r15		
19	0.01	r3,r19,r11	r3,r11,r10,r12	r3,r11,r5,r12	r3,r11,r5,r12	r19,r5,r12		
20	0.01	r11,r5,r18	r9,r5,r18,r16,r17,r15	r18,r15	r9,r11,r18,r16,r6,r17,r15	r9,r5,r18,r17,r15		
21	0.01	r1,r17,r7	r1,r7,r8,r15	r13,r17	r13,r17	r1,r7,r13,r17,r8		helo3,ddg3
22	0.01	r6,r14,r17,r15	r14,r11,r2,r12,r1,r17,r15	r14,r12,r6,r17,r15	r14,r12,r1,r6,r17,r15	r14,r10,r2,r12,r1,r6,r17,r15		
23	0.01	r9,r14,r20,r12,r6,r8	r9,r20,r12,r8	r5	r9,r5,r20,r4,r8	r6,r9,r14,r5		
24	0.01	r3,r5,r16,r20,r12,r8	r3,r5,r2,r18,r20,r12,r6,r8	r8	r3,r5,r2,r7,r6,r8	r6,r2,r16		
25	0.01	r8,r17,r2,r12	r3,r10,r20,r4,r7,r6,r17,r8	r10,r6,r2,r4	r10,r2,r20,r4,r6,r17	r17,r2,r4,r7		
26	0.01	r1,r4,r6,r17,r8	r16,r1,r4,r6,r17	r6	r6,r4,r16	r18,r16,r1,r17,r8		
27	0.01	r3,r19,r16,r1,r7	r6,r5,r17,r1	r3,r5,r16	r3,r5,r16	r3,r16,r1,r7,r6		
28	0.01	r3,r5,r18,r12,r13	r13,r5,r18	r5,r12	r3,r5,r18,r12,r13	r3,r18		
29	0.01	r5,r10,r18,r20,r12,r1,r4,r6	r20,r10,r1	r6	r5,r10,r18,r1,r6	r5,r10,r18,r12,r6,r8		helo3,ddg3
30	0.01	r14,r10,r18,r7,r6,r17	r14,r5,r17,r18	r6,r14,r5,r17	r14,r5,r6,r13,r17	r14,r5,r10,r6,r13,r17		

Figure 10. Sample of scenario data

C. OUTPUT DISPLAY

Results are written to a separate multi-sheet Excel workbook. The optimal "torpedo loadout plan," or the type and quantity of torpedoes assigned to each individual blue shooter for the baseline budget of $25M, is listed in Table 15. For example, TASOM-1 allocates five mod 0, fifteen mod 1, and four mod 2 torpedoes to the P-8 squadron "p1." TASOM-2 allocates twelve mod 0, ten mod 1, and three mod 2 torpedoes to "p1." Of note, TASOM-1 only allocates two torpedoes to a unit that is not a P-8 ("helo3") while TASOM-2 allocates torpedoes to eight of the ten blue shooters.

A sample from the blue-red engagement results, with the type and quantity of torpedoes that each shooter expends on each engaged target, by scenario, is presented in Table 16. Both the TASOM-1 and TASOM-2 engagement results are structured similarly: each row contains an engagement, and engagements are ordered and grouped by scenario. For example, for the TASOM-1 results, in scenario "1," "p1" engages "r3," "r13," "r17," and "r2." Specifically, "p1" engages "r3" with two mod 0, two mod 1, and one mod 2 torpedoes.

A sample from the "miss" results is presented in Table 17. For TASOM-1, the miss results indicate whether a detected red target was missed or not in each scenario. For example, in scenario "1," every detected red target is engaged and meets the desired probability of kill threshold. In scenario "2," however, "r4" is detected, but not engaged. For TASOM-2, the miss results report the shortfall from the desired probability of kill threshold for every detected red target in each scenario. For example, in scenario "2," "r13," "r17," "r16," and "r9" are engaged with a torpedo combination that meets the desired probability of kill threshold, but "r4" is engaged with a combination that only achieves a probability of 0.84, or a shortfall of 0.06. For both models, the miss results do not list the undetected red targets (that are automatically missed); that information is reported in the input scenario data.

A sample from the summary results is presented in Table 18. For TASOM-1, the objective value (in millions of dollars) and the total number of red targets missed are reported for each scenario. Recall, the objective value for each scenario is the sum of (i) the total cost of torpedoes purchased; (ii) the total cost of loading torpedoes from inventory;

and (iii) the penalty incurred from missing any red targets (detected or undetected). For example, in scenario "1," the objective value is $22.0045M. $22M is the cost for purchasing eleven mod 2 torpedoes and the remainder is the cost of loading out 45 torpedoes from inventory as there were no missed targets in this scenario. The last row reports the expected objective value, the number of each torpedo type used from inventory, the number of each torpedo type purchased, and the average number of red targets missed in a scenario.

For TASOM-2, the objective value (in millions of dollars) and the average shortfall from the desired probability of kill are reported for each scenario. Recall, the objective value for each scenario is the sum of (i) the total cost of torpedoes purchased; (ii) the total cost of loading torpedoes from inventory; and (iii) the penalty incurred from any probability of kill shortfalls for any red targets (for an undetected target, the value of the entire threshold is penalized). For example, in scenario "2," the objective value is $24.054M. $18.048M is the cost for purchasing nine mod 2 torpedoes and six mod 1 torpedoes, $6M is the penalty for the 0.06 shortfall on the one under-targeted engagement, and the remainder is the cost of loading out 60 torpedoes from inventory. The last row reports the expected objective value, the number of each torpedo type used from inventory, the number of each torpedo type purchased, and the expected average probability of kill shortfall.

Table 15. Stochastic loadout plan for budget level of $25M

TASOM-1				TASOM-2			
blue	**mod_0**	**mod_1**	**mod_2**	**blue**	**mod_0**	**mod_1**	**mod_2**
p1	5	15	4	p1	12	10	3
p2	10	15	5	p2	10	12	2
ddg1	0	0	0	ddg1	0	0	0
helo1	0	0	0	helo1	2	0	2
ddg2	0	0	0	ddg2	6	0	0
helo2	0	0	0	helo2	0	0	2
ddg3	0	0	0	ddg3	0	6	0
helo3	0	0	2	helo3	0	0	0
ddg4	0	0	0	ddg4	0	6	0
helo4	0	0	0	helo4	0	2	0

Table 16. Sample of engagement results

TASOM-1

	scenario	blue	red	mod_0	mod_1	mod_2
2	1	p1	r3	2	2	1
3	1	p1	r13	0	3	1
4	1	p1	r17	0	4	1
5	1	p1	r2	0	4	1
6	1	p2	r6	0	4	1
7	1	p2	r4	0	4	1
8	1	p2	r18	5	0	0
9	1	p2	r1	0	4	1
10	2	p1	r16	0	0	3
11	2	p1	r17	0	4	1
12	2	p2	r13	4	0	1
13	2	p2	r9	2	0	2
14	3	p1	r6	0	4	1
15	3	p1	r5	3	0	2
16	3	p1	r18	0	5	0
17	3	p1	r12	0	3	1
18	3	p2	r3	0	4	1
19	3	p2	r14	0	4	1
20	3	p2	r7	0	4	1
21	3	p2	r11	3	0	2
22	4	p1	r6	0	4	1
23	4	p1	r17	0	0	3
24	4	p2	r19	0	5	0
25	4	p2	r11	4	0	1
26	5	p1	r3	0	4	1
27	5	p1	r14	0	3	1
28	5	p1	r15	0	3	1
29	5	p1	r1	0	4	1
30	5	p2	r8	0	4	1
31	5	p2	r6	0	4	1
32	5	p2	r4	0	4	1
33	5	p2	r20	5	0	0
34	5	helo3	r19	0	0	2
35	6	p1	r18	0	5	0
36	6	p1	r17	2	1	1

Torpedo Allocation | Engagements

TASOM-2

	scenario	blue	red	mod_0	mod_1	mod_2
2	1	p1	r18	2	3	0
3	1	p1	r2	2	2	1
4	1	p1	r3	2	2	1
5	1	p1	r17	4	0	1
6	1	p2	r4	2	2	1
7	1	p2	r13	0	5	0
8	1	p2	r6	2	2	1
9	1	helo1	r1	0	0	2
10	2	p1	r13	4	0	1
11	2	p1	r17	4	0	1
12	2	p1	r16	2	1	1
13	2	p2	r9	0	4	1
14	2	helo1	r4	0	0	2
15	3	p1	r11	0	5	0
16	3	p1	r12	4	0	1
17	3	p1	r6	2	2	1
18	3	p1	r5	2	2	1
19	3	p2	r18	5	0	0
20	3	p2	r3	2	2	1
21	3	p2	r14	0	5	0
22	3	p2	r7	0	4	1
23	3	helo1	r4	0	0	2
24	3	helo2	r2	0	0	2
25	4	p1	r17	4	0	1
26	4	p1	r6	0	4	1
27	4	p2	r11	0	3	1
28	4	p2	r19	0	5	0
29	4	helo1	r3	0	0	2
30	4	helo2	r13	0	0	2
31	5	p1	r3	2	2	1
32	5	p1	r1	3	1	1
33	5	p1	r6	2	2	1
34	5	p1	r15	0	5	0
35	5	p2	r4	2	2	1
36	5	p2	r20	5	0	0

Torpedo Allocation | Engagements

Table 17. Sample of miss results

TASOM-1

	scenario	red	Not Engaged
2	1	r3	0
3	1	r6	0
4	1	r4	0
5	1	r13	0
6	1	r18	0
7	1	r17	0
8	1	r2	0
9	1	r1	0
10	2	r4	1
11	2	r13	0
12	2	r16	0
13	2	r9	0
14	2	r17	0
15	3	r3	0
16	3	r14	0
17	3	r6	0
18	3	r5	0
19	3	r7	0
20	3	r11	0
21	3	r4	1
22	3	r18	0
23	3	r2	1
24	3	r12	0
25	4	r3	1
26	4	r6	0
27	4	r19	0
28	4	r11	0
29	4	r13	1
30	4	r17	0
31	5	r8	0
32	5	r3	0
33	5	r14	0
34	5	r6	0

Torpedo Allocation | Misses

TASOM-2

	scenario	red	Deviation from Desired Pkill
2	1	r18	0
3	1	r2	0
4	1	r4	0
5	1	r3	0
6	1	r13	0.016029063
7	1	r17	0
8	1	r1	0.06
9	1	r6	0
10	2	r4	0.06
11	2	r13	0
12	2	r17	0
13	2	r16	0
14	2	r9	0
15	3	r18	0
16	3	r11	0.016029063
17	3	r2	0.06
18	3	r4	0.06
19	3	r12	0
20	3	r3	0
21	3	r14	0.016029063
22	3	r7	0
23	3	r6	0
24	3	r5	0
25	4	r11	0
26	4	r3	0.06
27	4	r13	0.0225
28	4	r17	0
29	4	r6	0
30	4	r19	0
31	5	r4	0
32	5	r19	0
33	5	r3	0

Torpedo Allocation | Misses

Table 18. Sample of summary results

TASOM-1

	A	B	C	D	E	F	G	H	I
1		obj value ($ millions)	mod_0 from inv	mod_1 from inv	mod_2 from inv	mod_0 purchased	mod_1 purchased	mod_2 purchased	Total Reds Not Engaged
2	1	22.0045							0
3	2	112.0045							1
4	3	202.0045							2
5	4	202.0045							2
6	5	112.0045							1
96	95	112.0045							1
97	96	202.0045							2
98	97	112.0045							1
99	98	22.0045							0
100	99	202.0045							2
101	100	292.0045							3
102		Expected: 114.7045	15	30	0	0	0	11	0.99

TASOM-2

	A	B	C	D	E	F	G	H	I
1		obj value ($ millions)	mod_0 from inv	mod_1 from inv	mod_2 from inv	mod_0 purchased	mod_1 purchased	mod_2 purchased	Avg Pkill Deviation
2	1	25.65691							0.0095
3	2	24.054							0.012
4	3	33.25981							0.01521
5	4	26.304							0.01375
6	5	27.28132							0.00923
96	95	24.054							0.012
97	96	34.90573							0.01685
98	97	43.1838							0.0359
99	98	19.65691							0.00321
100	99	49.1838							0.04447
101	100	53.03671							0.03498
102		Expected: 33.03639	30	30	0	0	6	9	0.014334

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