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NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

CLANDESTINE MINE COUNTERMEASURES OPTIMIZATION FOR AUTONOMY AND RISK ASSESSMENT

by

Sean P. Kragelund
Isaac I. Kaminer

December 2022

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Mines are inexpensive, easily deployed, and put distributed maritime operations (DMO) at high-risk, particularly as Great Power Competition (GPC) requires naval forces to operate in contested environments. Autonomous underwater vehicles (AUVs) will play an increasingly important role in mine countermeasures (MCM), but research is required to optimize their performance when support from surface or airborne assets is denied or severely limited by the constraints of GPC. This project investigated methods for AUVs to conduct entirely clandestine MCM. It examined whether a conventional MCM search problem could be inverted: instead of conducting sequential operations to find and neutralize mines in a predefined transit lane, an AUV can find a navigable mine-free route that maximizes its probability of survival, potentially decreasing MCM mission timelines. Preliminary results suggest that this framework can also be used to prioritize mines for neutralization to achieve acceptable risk levels. Additional student thesis research examined methods for object detection and size determination with forward-looking sonar (FLS) to enable more efficient AUV path planning.

15. SUBJECT TERMS

optimal search, mine countermeasures, MCM, mine hunting, sonar, mission planning, motion planning, optimal control, autonomous vehicles, unmanned vehicles, unmanned surface vessel, USV, unmanned underwater vehicle, UUV, autonomous underwater vehicle, AUV

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ABSTRACT

Mines are inexpensive, easily deployed, and put distributed maritime operations (DMO) at high-risk, particularly as Great Power Competition (GPC) requires naval forces to operate in contested environments. Autonomous underwater vehicles (AUVs) will play an increasingly important role in mine countermeasures (MCM), but research is required to optimize their performance when support from surface or airborne assets is denied or severely limited by the constraints of GPC. This project investigated methods for AUVs to conduct entirely clandestine MCM. It examined whether a conventional MCM search problem could be inverted: instead of conducting sequential operations to find and neutralize mines in a predefined transit lane, an AUV can find a navigable mine-free route that maximizes its probability of survival, potentially decreasing MCM mission timelines. Preliminary results suggest that this framework can also be used to prioritize mines for neutralization to achieve acceptable risk levels. Additional student thesis research examined methods for object detection and size determination with forward-looking sonar (FLS) to enable more efficient AUV path planning.

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TABLE OF CONTENTS

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LIST OF FIGURES

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I. EXECUTIVE SUMMARY

A. PROJECT SUMMARY

Mines are inexpensive, easily deployed, and put distributed maritime operations (DMO) at high-risk, particularly as Great Power Competition (GPC) requires naval forces to operate in contested environments. Autonomous underwater vehicles (AUVs) will play an increasingly important role in mine countermeasures (MCM), but research is required to optimize their performance when support from surface or airborne assets is denied or severely limited by the constraints of GPC. This project investigated methods for AUVs to conduct entirely clandestine MCM. It examined whether a conventional MCM search problem could be inverted: instead of conducting sequential operations to find and neutralize mines in a predefined transit lane, an AUV can find a navigable mine-free route that maximizes its probability of survival, potentially decreasing MCM mission timelines. Preliminary results suggest that this framework can also be used to prioritize mines for neutralization to achieve acceptable risk levels. Additional student thesis research examined methods for object detection and size determination with forward-looking sonar (FLS) to enable more efficient AUV path planning.

B. BACKGROUND

Today's MCM systems still rely on surface and airborne assets for vehicle support, data analysis, and mission planning. In the contested battlespace of the future, it may not be possible to establish and maintain the permissive environment that current systems require. While AUVs are capable of clandestine operations, research is required to identify and assess new methods for conducting entirely clandestine MCM—without support from vulnerable surface assets.

Many MCM missions are search problems, and recent advances in computational optimal control have made it possible to optimize search functions performed by AUVs. Examples include area search to determine optimal track line geometry and collaborative search to detect, localize, identify, and (when necessary) neutralize mines. Past research [1] and [2] has shown that these capabilities can improve upon conventional MCM methods that rely on sequential "lawnmower" search missions to clear a designated transit lane. By using targeted rather than exhaustive search, clandestine MCM has potential to reduce MCM timelines even further. Based on prior Naval Research Program research conducted for the topic sponsor, this study topic was developed in consultation with Navy stakeholders at the sponsor's organization, the Naval Surface and Mine Warfighting Development Center Mine Warfare Division (SMWDC-MIW).

For this study, Naval Postgraduate School (NPS) researchers contacted subject matter experts to develop realistic assumptions about MCM vehicles, sensors, and operations. Contacts included topic sponsors at SMWDC-MIW, AUV operators in the expeditionary MCM and explosive ordnance disposal communities, and engineers at Naval Information Warfare Center-Pacific responsible for fielding automated target recognition algorithms on MCM AUVs. A brief literature review of search theory, optimal trajectory generation, and coordinated path following algorithms was also conducted. Finally, NPS reviewed papers by MCM planning experts at the Naval Surface Warfare Center-Panama City covering new methods for calculating and assessing MCM risk.

This study's initial focus on AUV sensing capabilities led to a thesis by Fedorovich [3] that explored the potential to classify underwater objects using only FLS. Experiments with two different target shapes were conducted in a controlled environment to determine relationships between a target's actual size/shape and its apparent size/shape in head-on FLS imagery. Fedorovich et al. present a potential obstacle avoidance strategy for AUVs conducting MCM in these environments [4].

Finally, we developed an optimal control formulation for an AUV to find a safe route through a minefield by computing a feasible trajectory which maximizes the AUV's probability of survival. This optimal control framework can be generalized to accommodate probabilistic models for mine damage, vehicle navigation, etc.; performance; and mission objectives for numeric optimization.

C. FINDINGS AND CONCLUSIONS

While preliminary in nature, this study found qualitatively that an optimal control framework can find a safe, navigable route through a minefield. This result relied on assumptions about an AUV's ability to detect and localize mines in the environment.

These assumptions were informed by our literature review and by our thesis student's experiments with a sensor used on some actual MCM AUVs. However, additional modeling and simulation is required to assess the capabilities of actual MCM vehicle/sensor systems for clandestine MCM.

One benefit of optimal control is that it is a model-based framework. This method can generate a wealth of data for parametric studies (e.g., Kragelund et al., 2020b) by incorporating different models of the mine threat, vehicle dynamics, sensor capabilities, and mission objectives. Monte Carlo simulation can be employed to generate results for analysis. Another benefit of optimal trajectory generation is that vehicle trajectories found in this manner are feasible by definition (i.e., they can be followed by vehicle autopilots). NPS has demonstrated this on several of its autonomous vehicle systems, but rigorous experimentation with fleet MCM vehicles is needed to test this capability in the field.

In this study, we defined risk in terms of the AUV's probability of survival along its trajectory. Additional analysis is required to assess the risk to other vehicles following the first AUV's path. This risk is a function of each vehicle's navigation accuracy, acoustic/magnetic signature, etc., and was considered outside the scope of our study. However, the proposed trajectory generation framework can be modified to account for the risk to other vehicles. Alternatively, the objective function could also be modified to identify/prioritize individual mines that must be neutralized to guarantee a specified risk threshold.

In conclusion, our initial findings represent a promising approach to MCM that can be explored for future operational concepts.

II. AUV SENSOR PERFORMANCE ANALYSIS

This chapter provides a brief overview of research conducted by LT Alexander Fedorovich, USN, for his master's thesis in Mechanical Engineering. His work explored the potential to classify underwater objects using only forward-looking sonar (FLS).

A. INTRODUCTION

Mine countermeasures (MCM) research with autonomous underwater vehicles (AUVs) has been explored for decades. In 1999, Smith et al. showed that AUV mine detection in water depths of 50-70 feet with calm conditions is possible using area search tactics and side-scan sonar [5]. That research also successfully used camera video to manually correlate data from the sonar. Topple and Fawcett attempted to apply MiNet, a deep learning model, for automatic target recognition with side scan sonars on AUVs [6]. Similarly, recent research at the Naval Information Warfare Center (NIWC) has implemented deep learning algorithms trained on a vast amount of historical data with successful object detection and classification trials. However, that effort uses side-looking sonar on larger vehicles with precise navigation and maneuvering profiles. These vehicles and procedures cannot be utilized in the challenging environment of the very shallow water (VSW) zone, which is the focused of LT Fedorovich's thesis.

Automated detection with FLS does have successful implementations. For example, Horner et al. implemented reactive obstacle avoidance for an AUV with a blazed-array FLS [7]. ATR algorithms can also "flag" potential targets for human operators to classify upon mission completion. This saves vast amounts of time that would be spent analyzing sonar data from the entire mission. However, automated classification with a standalone FLS in the VSW remains an area of active research.

B. NEURAL NETWORKS FOR OBSTACLE DETECTION

Convolutional Neural Networks (CNNs) are artificial, multi-layer neural networks that are designed for use on two-dimensional data, such as camera images and videos [8]. An example application which uses a generic CNN for automated surface inspection techniques is described in [9]. In another application, Girshick et al. adapted CNNs to create Region-Based Convolutional Neural Networks (RCNNs) for improved object detection with less training data [10]. NPS researchers therefore studied the ability of a standard, pre-built RCNN algorithm to detect underwater targets with a forward-looking camera exposed to oscillatory motion. A 3D-printed mount was designed and built to restrict translation in the x, y, and z directions, but permit roll, pitch, and yaw rotations. Analysis of multiple camera trials using the RCNN algorithm determined that the standard algorithm worked exceptionally well. Even applying the maximum possible motion achievable with the camera mount did not cause the detector to fail. Although the camera images were not in perfect focus and image blurring was observed, the algorithm still successfully detected the object. This indicates that the applied RCNN algorithm was robust to image distortion, even though it had only been trained with fewer than 1000 images.

Due to initial success with camera images, this research effort initially focused on applying the same training and detector approach to images from a FLS. This initial focus failed despite numerous trials. The failure to apply an RCNN designed for camera images to FLS images was most likely due to sensor modality. Cameras produce a two-dimensional signal that can be mapped to an array of pixels in an image. A sonar beam, however, produces a one-dimensional signal that can be mapped to its range from an object. An array of sonar beams can be used to construct a two-dimensional image based on the array's geometry and the range measurement from each beam. The resulting image, however, differs greatly from the two-dimensional signals produced by a camera. As a result, CNNs that were trained to detect color, contrast, texture, and/or spatial relationships in camera images fail for sonar images that lack this information. Deep learning with FLS, however, is an ongoing field of study.

Despite the inability to translate deep learning success from camera images to sonar images, it is important to explore the extended capabilities of a FLS (i.e., beyond detection). This will support AUV MCM missions in the VSW zone, where side-looking sonars are ineffective and forward-looking cameras cannot be used because water clarity is often poor. Therefore, the objective of this research is to determine if a single AUV, equipped with a standalone FLS and following a non-standard path around an object, can obtain FLS images with features that support the classification of that object. Utilizing lessons learned from the prior camera research, a controlled testing environment was constructed to record repeatable sonar data with targets in multiple positions and orientations. The targets varied by size, shape, and material. The recorded imagery was processed to analyze features of the returns, including temporal variation, peak intensity values, and intensity distribution relative to each target.

C. FORWARD-LOOKING SONAR EXPERIMENTATION RESULTS

AUVs must be able to detect and avoid underwater objects, a task that becomes more difficult in very shallow water. First, breaking waves generate water that is well-mixed with suspended sediment and entrained air bubbles, reducing visibility and making cameras unreliable for vision-based navigation. Second, hydrodynamic loads from waves and currents can produce unsteady vehicle motions which degrade the imagery produced by side-looking sonar systems. For these reasons, forward-looking sonar (FLS) is an effective sensor for detecting obstacles in these environments. Avoiding them, however, remains difficult because the seafloor and free surface severely limit a vehicle's ability to maneuver in the vertical plane. As a result, AUVs must be able to plan and execute obstacle avoidance maneuvers in the horizontal plane.

In dense obstacle fields, successful path planning requires the ability to determine the size of objects to be avoided. Typically, when operating in deep water, AUVs are programmed to operate at a specified altitude above the seabed. Sufficient vertical plane separation allows forward- or side-looking sonar to ensonify the sea floor with a downward grazing angle, generating sonar imagery containing spatial information about an object's size, shape, and height above the seafloor. An AUV operating in shallow water and constrained to operate in the horizontal plane, however, is likely to encounter obstacles head-on. In general, FLS images generated from head-on aspect angles provide less spatial information than images generated with a downward grazing angle. As a result, head-on FLS imagery contains significant ambiguity about an obstacle's actual physical extent. Vertical plane maneuvers to overcome this ambiguity were proposed in [11], and shallow water constraints motivate a similar approach for horizontal plane maneuvers [12]. Moreover, LT Fedorovich's experiments found that the uncertainty when predicting an object's true width, based on the number of FLS beams which composed its corresponding return in a head-on image, depended greatly on its shape [3]. Specifically, after controlling for object size and distance from the FLS, the most significant factor was whether the object had curved or planar surfaces. In the FLS imagery examined, the width of sonar returns from a flat-faced object corresponded reasonably well with its physical width. Sonar returns from a spherical object, however, suggested the object was much smaller than it actually was. This discrepancy has important ramifications for an AUV's ability to avoid head-on obstacles detected by its FLS.

One contribution of this research has been presented in [4]. FLS testing with a horizontal offset demonstrated how one can quickly determine if an object in FLS imagery has plane or curved surfaces. Maneuvering to offset the FLS from a target with planar geometry allows for successful visual identification of important geometric aspects, such as sides or corners that can be used to estimate the target's extent. Conversely, the shape of FLS returns from objects with rotational symmetry do not change with increased offset distance. The ability to identify these differences in FLS imagery, based on lateral offset distance, can aid motion planning for obstacle avoidance maneuvers constrained by shallow water environments.

III. ROUTES FOR OPTIMIZING PROBABILITY OF SURVIVAL

A. INTRODUCTION

Current mine countermeasures (MCM) systems rely on surface and airborne assets, with associated force protection burdens required to establish and maintain a permissive environment. In the future, MCM forces must be able to operate in contested environments where overt operations are denied and supporting technologies (GPS, communications, etc.) are severely limited. Autonomous underwater vehicles (AUVs) are capable of clandestine operations. At present, however, underwater MCM still requires surface forces for vehicle support, data analysis, and mission planning. Research is required to identify and assess new methods for conducting entirely clandestine MCM.

Many MCM missions are search problems, and recent advances in computational optimal control make it possible to optimize search functions performed by AUVs. Examples include area search to determine optimal MCM geometries and collaborative search to detect, localize, identify, and (when necessary) neutralize mines. These capabilities can improve upon conventional MCM methods that rely on sequential "lawnmower" search missions to clear a designated transit lane. By using targeted rather than exhaustive search, clandestine MCM has potential to reduce MCM timelines even further, while preserving the element of surprise.

This study will investigate novel methods for conducting clandestine mine countermeasures. In short, it will examine whether the MCM search problem can be inverted: instead of searching to find and remove mines within a predefined transit lane, can search assets find a navigable mine-free route that ships can follow? This capability requires AUVs to acquire, process, and evaluate information obtained from their environment. The primary contribution of this study is assessing the utility of an optimal control framework for maximizing the probability of survival of an AUV trajectory toward enabling clandestine MCM operations.

B. PROBLEM FORMULATION

In this study we address the problem of determining whether a vehicle, either an AUV or a follow-on vessel, can traverse a given mine field safely. We assume that 1) a search-classify-map mission has been conducted to precisely determine mine locations; and 2) the lethality radius of each mine is known. A typical scenario is shown in Figure 1, where mine locations and their lethality radii are shown in red, while the AUV's trajectory is shown in blue. Note that the scale of the x- and y- axes are scaled such that mine lethality circles appear as ovals instead.

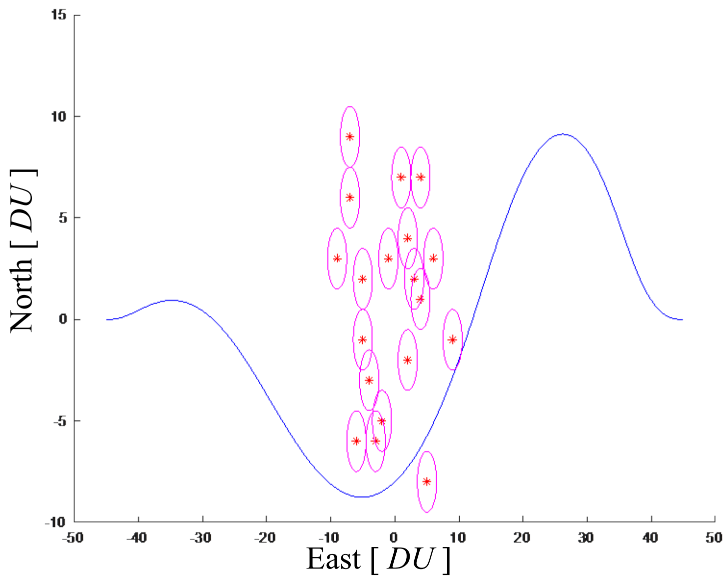

Figure 1 A map of the minefield with mine locations and their lethality radii shown in red, and a safe candidate vehicle trajectory shown in blue.

C. MINE LETHALITY MODEL

We assume that the lethality of each mine can be represented using a so-called damage function, see for example [13] in which the authors introduce mathematical models for a weapon's lethality. The idea is to capture how the weapon's lethality depends on the relative positions of the weapon and a target. For example, a mine's lethality can be described by a function shown in the left plot of Figure 2. For a mine placed at the origin, the target's probability of destruction is equal to 1.0 and 0.0, respectively, inside and outside of the shaded region. The right plot of Figure 2 shows a smooth approximation of this effectiveness model generated using a Poisson scan model. Next, we will use this smoothed approximation of the mine damage function to obtain an expression for a vehicle's probability of survival as it traverses a minefield.

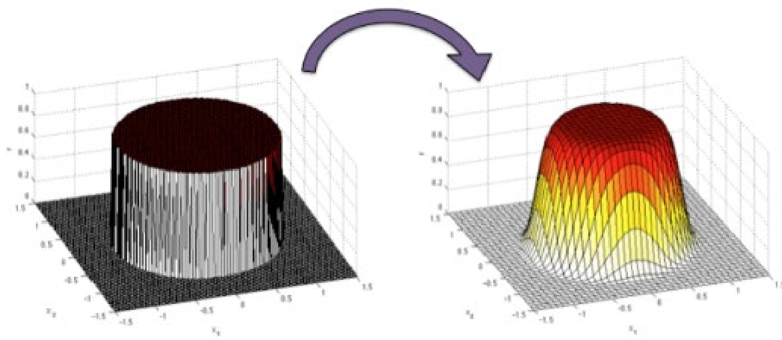

Figure 2 Discrete mine lethality vs. range (left) and smoothed using the Poisson scan model (right). Source: [13]

Let Φ_a represent the Poisson scan model function, and let the parameters λ_a, F_a, σ_a and α_a represent the lethality characteristics of an individual mine such as ship counter, effective range, and inflicted damage. These parameters can be adjusted to match the steepness of the mine damage function to represent mine capabilities as a function of distance from a target vehicle, e.g., a high value unit (HVU). In the following equations, we denote the damage rate of the i^{th} mine against the HVU as d_i^{hvu}:

$$d_i^{hvu} = \lambda_a \Phi_\alpha \left(\frac{F_a - a_a [r_i - r_{hvu}]^2}{\sigma_a} \right). \tag{1}$$

In (1) r_i represents the position of the i^{th} mine and r_{HVU} represents the position of the HVU. Let $P(t)$ represent the probability of survival of the HVU up to time t. Then the HVU's probability of survival at time $t + \Delta t$, due to damage caused by the i^{th} mine can be expressed as follows:

$$P(t + \Delta t) = P(t)(1 - d_i^{hvu} \Delta t). \tag{2}$$

In (2) we assume that the HVU's probability of survival on the interval $[t, t + \Delta t]$ is independent of the HVU's probability of survival on the interval $[0, t]$. Using (2) and an assumption that the damage to the HVU from a given mine is independent of the damage caused by all other mines yields:

$$P(t + \Delta t) = P(t) \prod_{i=1}^{M} (1 - d_i^{HVU} \Delta t) \tag{3}$$

where $\prod$ denotes the product operator and M denotes the number of mines. This result represents the probability of survival for the HVU at the time instant $t + \Delta t$, after considering the potential damage from all of the mines in the minefield.

Next, let t_f denote the time it takes the vehicle to traverse a mine field. (Note that t_f is not fixed but will be used as an optimization parameter). Then the vehicle's probability of destruction at t_f is given by the relationship:

$$J = 1 - P(t_f) \tag{4}$$

The scalar quantity J can be used to define a cost function to be minimized over all possible dynamically feasible trajectories that a vehicle can take through the mine field. In this report, "dynamically feasible" means that a vehicle can follow the trajectory without exceeding it dynamic constraints on minimum/maximum speed and maximum turn rate. These quantities will be defined precisely in the next section.

D. VEHICLE DYNAMIC MODEL

For a transiting vehicle of interest, let v denote its speed and ψ denote its heading, and let u_1 and u_2 denote its turn rate and acceleration commands, respectively. Furthermore, we let the vehicle's two-dimensional position be expressed by the components of vector $r = [x_1\ x_2]^T$. Using this notation, we obtain a system of ordinary differential equations (ODEs) describing the relationship between the vehicle's position, speed, heading, turn rate and acceleration:

$$\begin{aligned} \dot{x}_1 &= v cos(\psi) \\ \dot{x}_2 &= v sin(\psi) \\ \dot{\psi} &= u_1 \\ \dot{v} &= u_2 \end{aligned} \quad . \tag{5}$$

This system of ODEs can be expressed in a more compact form using the state vector $x = [x_1\ x_2\ \ \psi\ \ v]^T$ and the control input vector $u = [u_1\ u_2]^T$:

$$\dot{x} = f(x,u) = \begin{cases} v\ \cos\psi \\ v\ \sin\psi \\ \quad u_1 \\ \quad u_2 \end{cases} . \tag{6}$$

Combining (4) and (6) and placing additional constraints on the vehicle's speed, turn rate, and acceleration results in a well-known optimal control problem [14]. A detailed formulation of the Optimal Control Problem and its numerical solution utilized for this study are discussed in the next section.

E. OPTIMAL CONTROL PROBLEM AND ITS NUMERICAL SOLUTION

The analytical form of the optimal control problem (OCP) is used to **determine** the state vector $x(t)$ and the control vector $u(t)$ which **minimize** the cost function, expressed as:

$$J = E(x(0), x(t_f)) + \int_0^{t_f} F(x(t), u(t))dt, \tag{7}$$

where $E(x(0), x(t_f))$ is called the terminal cost and $\int_0^{t_f} F(x(t), u(t))dt$ is called the running cost. In our problem formulation, the running cost corresponds to the vehicle's probability of destruction. In other words, a gradient-based constrained optimization solver (e.g., MATLAB's *fmincon* function) can be used to compute a trajectory with locally minimum cost, as long as the problem and its constraints have been formulated appropriately [15]. The problem is **subject** to the system dynamics, equality, and inequality constraints expressed in the equations:

$$\dot{x} = f(x(t), u(t)), \forall t \in [0, t_f] \tag{8}$$

$$e(x(0), x(t_f)) = 0 \tag{9}$$

$$h(x(t), u(t)) \leq 0, \forall t \in [0, t_f] \tag{10}$$

In most cases, analytical solutions to such non-linear problems are not easy to find. Hence, in our case, we implement a numerical framework in which we discretize the time interval, the state vector $x(t)$, and the control vector $u(t)$, and exploit the numerical stability of Bernstein polynomials to express these vectors in terms of Bernstein coefficients. Conceptually, this process is illustrated in for the problem of finding the shortest route between an initial point (A_1 or A_2) and a goal point (B_1 or B_2) which avoids navigational obstacles or keep-out zones. Consequently, we discretize the problem, find the optimal solution, and finally interpolate between time nodes.

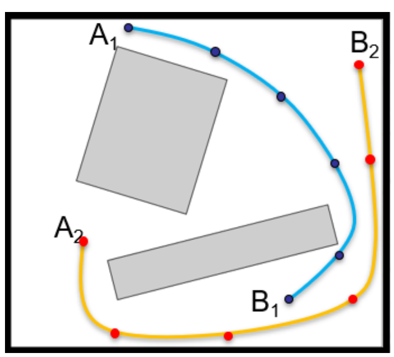

Figure 3 Discretization and interpolation to solve a nonlinear optimal control problem to obtain obstacle avoidance trajectories. Source: [16]

We now present details of the Bernstein discretization scheme described in [16] and [17]. A Bernstein polynomial of degree N is given by:

$$x_N(t) = \sum_{k=0}^{N} c_k b_{k,N}(t) \quad , \tag{11}$$

where $b_{k,N}(t)$ are the basis of the Bernstein polynomial:

$$b_{k,N} = \binom{N}{k} t^N (t_f - t)^{N-k}, t \in [0, t_f], \tag{12}$$

and $c_k \in \Re^n$ are the Bernstein coefficients for an n-dimensional trajectory. The numerical algorithm for solving a discretized Optimal Control Problem using the Bernstein polynomial approximation of the state and control input trajectories proceeds as follows:

- Discretize the time interval into time nodes:

$$t_j = j\frac{t_f}{N}, j = 0.....N, \tag{13}$$

- Apply Bernstein approximation of the state and control vectors [16]

$$x(t) \approx x_N(t) = \sum_{k=0}^{N} c_k b_{k,N}(t) \tag{14}$$

$$u(t) \approx u_N(t) = \sum_{k=0}^{N} c_{u,k} b_{k,N}(t) \tag{15}$$

- Differentiate the state vector via the differentiation matrix D:

$$\dot{x} \approx \dot{x}_N(t) = \sum_{j=0}^{N-1} (\sum_{i=0}^{N} c_i D_{ij}) b_{j,N(t)}, \tag{16}$$

$$\text{where } D = \left|\begin{matrix} -\frac{N}{t_f} & 0 & \cdots & 0 \\ \frac{N}{t_f} & \ddots & \cdots & \vdots \\ 0 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & -\frac{N}{t_f} \\ 0 & \cdots & \cdots & \frac{N}{t_f} \end{matrix}\right|. \tag{17}$$

- Using degree elevation, which is implemented through the matrix D, obtain equivalent higher-order Bernstein polynomials.
- Approximate the running cost:

$$\int_0^{t_f} F(x(t),u(t))dt \approx \sum_{i=0}^{N} w_i F(x_N(t_i),u_N(t_i)), w_i = \frac{t_f}{N+1}. \tag{18}$$

Equations (11) through (18) can be used to formulate a nonlinear programming (NLP) problem that can be efficiently solved with off-the-shelf numerical optimization solvers. Our NLP problem can be expressed as follows.

NLP Problem. Given a positive integer N and $\delta_p > 0$, determine the coefficients c and $c_{u,k}$ that minimize the approximate cost J^N of the cost J defined in (4) and (7):

$$J^N = E(\boldsymbol{x}_N(0), \boldsymbol{x}_N(t_N)) + w \sum_{j=0}^{N} F(\boldsymbol{x}_N(t_j), \boldsymbol{u}_N(t_j)),$$

subject to

$$\begin{aligned}
&\|\dot{\boldsymbol{x}}_N(t_j) - \boldsymbol{f}(\boldsymbol{x}_N(t_j), \boldsymbol{u}_N(t_j))\| \leq N^{-\delta_P}, \\
&\qquad \forall j = 0, \ldots, N, \\
&\boldsymbol{e}(\boldsymbol{x}_N(0), \boldsymbol{x}_N(t_N)) = \boldsymbol{0}, \\
&\boldsymbol{h}(\boldsymbol{x}_N(t_j), \boldsymbol{u}_N(t_j)) \leq N^{-\delta_P}\boldsymbol{1}, \quad \forall j = 0, \ldots, N.
\end{aligned}$$

with $w = \frac{t_f}{N+1}$.

For this study, MATLAB's *fmincon* solver [15] was used to solve the NLP Problem formulated above. Numerical results are presented in the next section.

F. RESULTS

We obtained several numerical solutions to the NLP Problem introduced in the previous section. For this problem, we modeled the minefield using a set of randomly generated mines in a 10 x 10 area defined by non-dimensional distance units (DU).10mX10m. Each mine was assumed to have a lethality radius of 5 DU. For this study, the total number of mines ranged from 20 to 35, which translated into mine densities from 0.2 mines per DU^2 to 0.35 mines per per DU^2. Figure 4 through Figure 6 depict locally optimal trajectories for minefields with 20, 25, and 30 mines respectively.

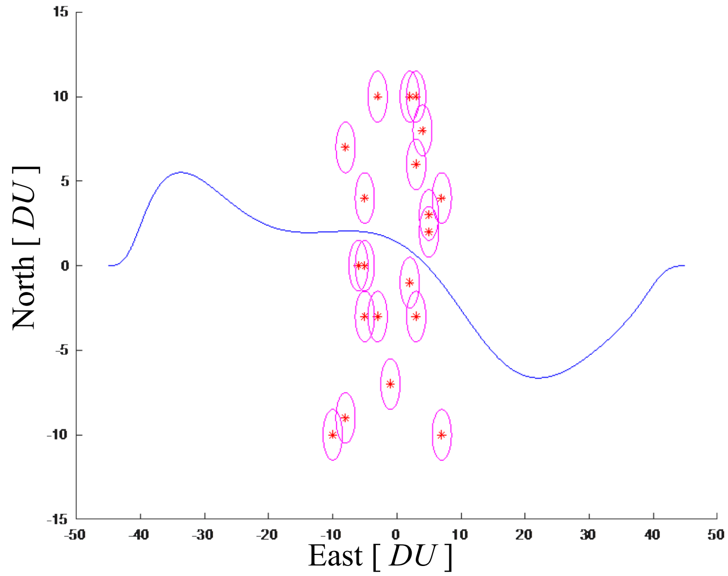

Figure 4 Trajectory through a randomly generated minefield of 20 mines. Probability of survival is 1.

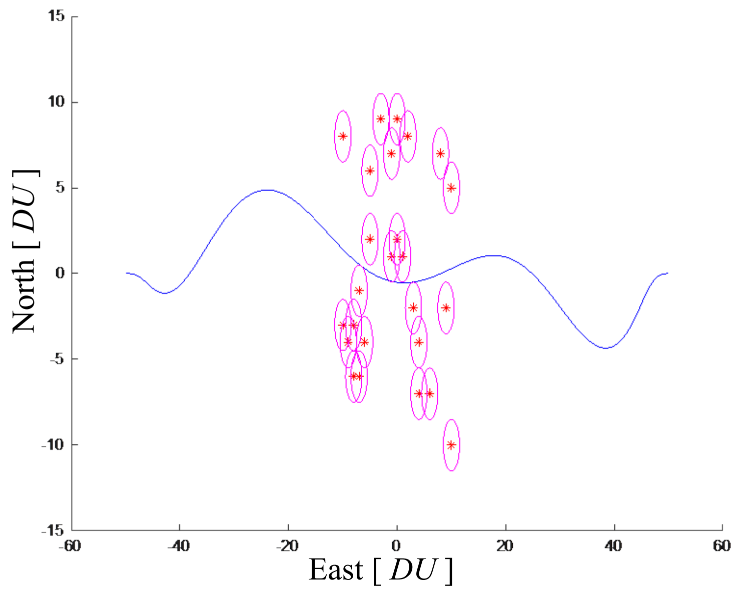

Figure 5 Trajectory through a randomly generated minefield of 25 mines. Probability of survival is 1.

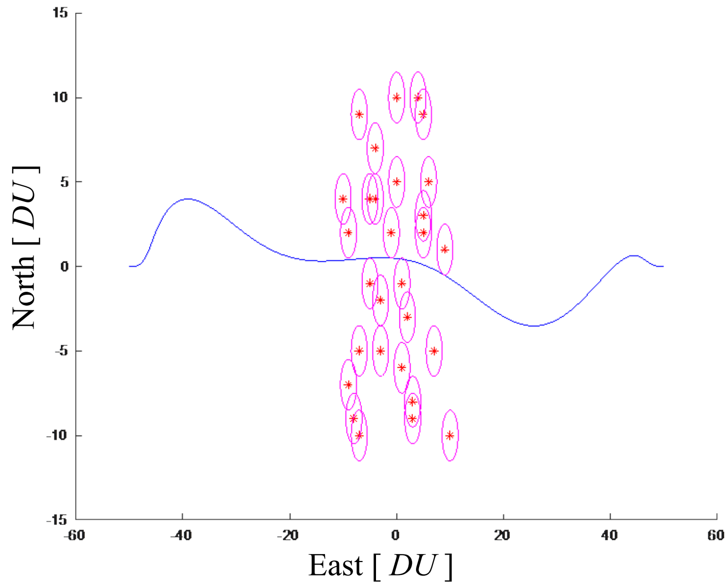

Figure 6 Trajectory through a randomly generated minefield of 30 mines. Probability of survival is 1.

In each of these cases, the solver found a trajectory that allows the vehicle to traverse the minefield with a probability of survival equal to 1. For the case of 35 mines, however, the solver fails to find a satisfactory solution. Indeed, the trajectory shown in Figure 7, has a probability of survival equal to 0.08, i.e., there is only an 8% chance that the vehicle can successfully traverse this minefield.

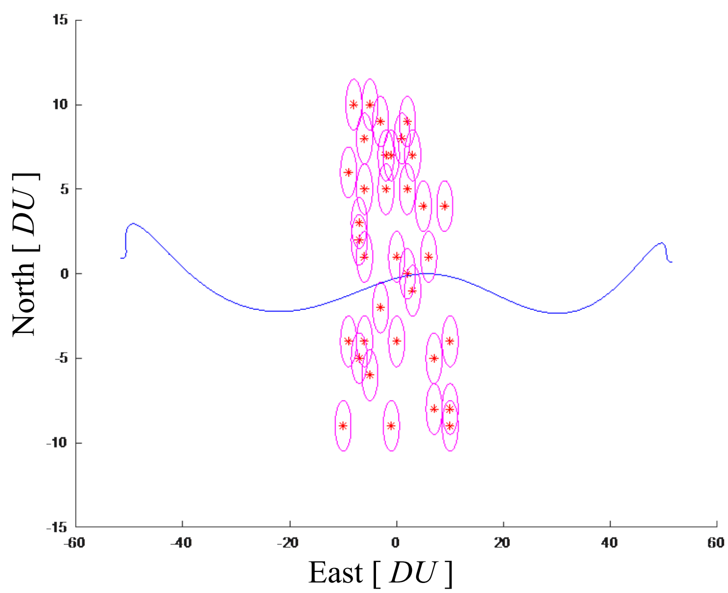

Figure 7 Trajectory through a randomly generated minefield of 35 mines. Probability of survival is 0.08.

This unfortunate result naturally leads to the following question: what is the minimum number of mines that must be neutralized to guarantee survival? One simple answer is to remove five mines which lie within 5 DU of the vehicle's trajectory. These mines that are located at (-3, -2), (0,1), (2,0), (3,-1), and (6,1) in Figure 7. This clears the path shown as shown in Figure 8.

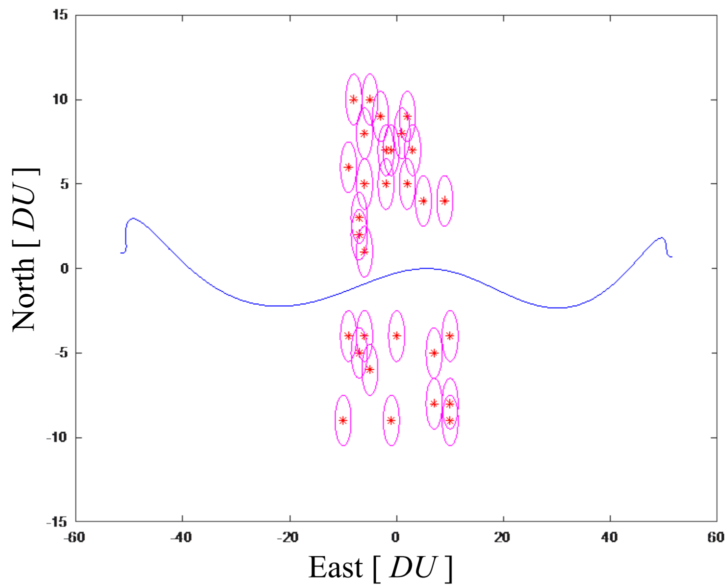

Figure 8   Trajectory through a randomly generated minefield of 35 mines after five neutralizing five mines. Probability of survival is 1.

This idea can be formalized as follows. Define indicator weights w_i, $i = 1, \ldots, M$ that can adopt the binary values 0 or 1. The number of indicator weights is equal to the number of mines. These weights will allow the optimization to indicate whether a mine must be neutralized or not. In this formulation, $w_i = 1$ implies the mine must be neutralized, whereas $w_i = 0$ implies that the mine does not directly threaten the vehicle. Using indicator weights, we can rewrite (3) as:

$$P(t + \Delta t) = P(t) \prod_{i=1}^{M} (1 - w_i d_i^{HVU} \Delta t) \quad , \qquad (19)$$

This allows the NLP problem to be reformulated as a hybrid NLP problem with cost function:

$$\begin{aligned} &J^N = \textstyle\sum_{i=1}^{M} w_i \\ &\text{where} \\ &w_i \in (0,1), i = 1, M. \end{aligned} \qquad (20)$$

For this problem, the vehicle's probability of survival computed using (19), but all other NLP constraints defined previously remain the same.

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IV. RECOMMENDATIONS FOR FUTURE RESEARCH

The preliminary results from this study can be improved and expanded in several ways. First and foremost, a complete definition and rigorous assessment of risk is needed for clandestine mine countermeasures (MCM) operations. Methods have been developed to calculate the risk to vessels transiting through a mined area, both before and after MCM operations have been performed. Whereas these methods compute the risk to follow-on vessels transiting through a relatively large area, clandestine MCM concentrates search effort in a much smaller area to find a specific route with a much lower level of risk, provided other vessels can accurately follow it. Additional research should focus on adapting existing risk models to fit this paradigm, including the risk due to navigation errors—especially in contested environments where GPS may not be available. One way to create a common risk assessment baseline for future research is to utilize the same minefield simulation software developed by Naval Surface Warfare Center-Panama City for both concepts of operation.

Another area to explore in future research are the tradeoffs associated with multiple, cooperating vehicles in contested environments. Simulation and analysis could help determine optimal AUV team compositions, assess tradeoffs between information sharing and the additional motion constraints it requires, as well as evaluating their mission performance metrics, among other factors.

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