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OPTIMAL PLACEMENT OF WAVE GLIDERS FOR ANTI-SUBMARINE WARFARE

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THESIS

OPTIMAL PLACEMENT OF WAVE GLIDERS FOR ANTI-SUBMARINE WARFARE

by

Rohan Kulkarni

March 2023

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13. ABSTRACT (maximum 200 words)

Many nations must contend with the need to keep pathways in the oceans secure against an increasing number of maritime challenges under the constraints of limited capital to acquire naval platforms. Unmanned platforms such as Wave Gliders may help to address this problem. A Wave Glider is an unmanned underwater vehicle that can be equipped with a passive array and can remain deployed in the area of interest (AOI) for extended durations. They are capable of providing a layered defense to prevent adversaries from transiting the area undetected, thereby providing a low cost, persistent, antisubmarine warfare (ASW) solution. The capabilities of ASW Wave Gliders to successfully track a manned submarine were demonstrated during the Unmanned Warrior exercise in 2016, led by the British Royal Navy. Yet, the question of how to deploy a given number of Wave Gliders to detect a transiting adversary submarine remains relatively unexplored. This thesis aims to develop a model to determine the detection capability of deployed Wave Gliders that accounts for variables associated with the detection of underwater contacts, using passive sonar in an acoustically challenging underwater environment and with constraints on deploying unmanned assets. The model prescribes an optimal number of Wave Gliders required to achieve a given probability of detection and provides a reference for their placement in the AOI to minimize the probability of an adversary submarine transiting the area undetected.

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OPTIMAL PLACEMENT OF WAVE GLIDERS FOR ANTI-SUBMARINE WARFARE

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ABSTRACT

Many nations must contend with the need to keep pathways in the oceans secure against an increasing number of maritime challenges under the constraints of limited capital to acquire naval platforms. Unmanned platforms such as Wave Gliders may help to address this problem. A Wave Glider is an unmanned underwater vehicle that can be equipped with a passive array and can remain deployed in the area of interest (AOI) for extended durations. They are capable of providing a layered defense to prevent adversaries from transiting the area undetected, thereby providing a low cost, persistent, antisubmarine warfare (ASW) solution. The capabilities of ASW Wave Gliders to successfully track a manned submarine were demonstrated during the Unmanned Warrior exercise in 2016, led by the British Royal Navy. Yet, the question of how to deploy a given number of Wave Gliders to detect a transiting adversary submarine remains relatively unexplored. This thesis aims to develop a model to determine the detection capability of deployed Wave Gliders that accounts for variables associated with the detection of underwater contacts, using passive sonar in an acoustically challenging underwater environment and with constraints on deploying unmanned assets. The model prescribes an optimal number of Wave Gliders required to achieve a given probability of detection and provides a reference for their placement in the AOI to minimize the probability of an adversary submarine transiting the area undetected.

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LIST OF ACRONYMS AND ABBREVIATIONS

AIP	Air Independent Propulsion
AOI	Area of Interest
AOO	Area of Operations
ASW	Antisubmarine Warfare
CBG	Carrier Battle Group
CDF	Continuous Distribution Function
CDP	Cumulative Detection Probability
CPA	Closest Point of Approach
CTF	Carrier Task Force
DI	Directivity Index
DoE	Design of Experiment
DT	Detection Threshold
ESR	Estimated Sonar Range
FOM	Figure of Merit
HVU	High Value Unit
IPD	Instantaneous Probability of Detection
ISR	Intelligence, Surveillance and Reconnaissance
LE	Line Efficiency
MOE	Measure of Effectiveness
MOP	Measure of Performance
MPA	Maritime Patrol Aircraft
NL	Noise Level
NM	Nautical Mile
NPS	Naval Postgraduate School
OR	Operations Research
PLA (N)	People's Liberation Army (Navy)
RNL	Radiated Noise Level

SLOC	Sea Lines of Communications
SoA	Speed of Advance
SONAR	Sound Navigation and Ranging
SPA	Submarine Probability Area
SSK	Conventional Attack Submarine
SSN	Ship Submersible Nuclear (Nuclear Submarine)
TL	Transmission Loss
TS	Target Strength
UUV	Unmanned Underwater Vehicle

EXECUTIVE SUMMARY

In order to leverage the myriad advantages offered by unmanned systems, their prominence in military operations has grown in recent years. Unmanned systems, in this case unmanned underwater vehicles (UUV), are employed for various tasks such as oceanography, mine countermeasures, and intelligence, surveillance and reconnaissance (ISR), to name a few. More recently, the use of UUVs in the field of antisubmarine warfare (ASW) has also evolved. This thesis explores the use of Wave Gliders, an UUV equipped with a passive array, for ASW. The scenario evolves around the optimal placement of ASW Wave Gliders in the AOO to maximize the probability of detecting an enemy submarine transiting the area. A model to calculate the probability of detection accrued by a given number of Wave Gliders with a particular estimated sonar range (ESR) is developed.

For the development of the model, the underwater detection characteristics of the Wave Glider fitted with a passive sonar are articulated using the passive sonar equation. Aspects such as the equipment, target, and environment characteristics are factored into the equation. Various factors that affect the propagation of sound underwater such as transmission loss and the presence of underwater noise, which impedes the overall sound received from the target, are also considered. The passive sonar equation and the parameters involved therein are used to calculate the performance of the sonar called the figure of merit (FOM) and the signal excess (SE), which tells us whether a signal emitted by the target will be detected by the sensor (Urick, 1967) on the Wave Glider. Thereafter, the Poisson Scan Model (Washburn, 2014), which models detections as occurring to a Poisson process, is used to formulate an expression for the cumulative probability of detection. The expression paves way for the lateral range function, which describes the ability of the Wave Glider to detect a target passing at a particular range in under the given environmental conditions.

In order to maximize the overall probability of detection, placement of Wave Gliders in different formations—viz, Barrier, Sector, Circular and Multiple Barrier in the AOO was explored. Experiments were undertaken by simulating instances of submarine transits through random points along the perimeter of the area. The ESR and the number

of Wave Gliders in the different formations were then varied to gain insight into the optimal placement for a particular scenario. Sensitivity analysis by varying critical parameters such as the target speed, the detection rate of the Poisson process, and FOM in the simulations was also undertaken to analyze their effect on the overall probability of detection. Results of simulations indicate that placement of Wave Gliders in the barrier formation in the AOO maximizes the probability of detecting an undersea contact transiting through the area. Though the barrier formation always provides a higher probability of detection than the multiple barrier formation, it could be used as a tactical option to keep the submarine on the defensive for an extended duration, as the submarine would have to traverse through an interspersed layer of Wave Gliders. The probability of detection increases with ESR detection rate, and keeping all other factors constant decreases with the increase in target speed.

References

Urick RJ (1967*) Principles of Underwater Sound for Engineers* (McGraw-Hill Companies, New York).

Washburn A (2014) *Search and Detection*, 5th ed. (CreateSpace Independent Publishing Platform, California).

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I. INTRODUCTION

This chapter provides a brief overview of the current geopolitical situation, the threat from adversary submarine forces to the economic security of nations, and the plausible deployment of Wave Gliders to counter that threat. The chapter also describes the problem statement and objective of the thesis.

A. BACKGROUND

Throughout history, maritime power has played a pivotal role in the development and wealth of great nations. Even today, the seas are a key enabler in the economic affluence of nations. The dominance and protection of Sea Lanes of Communication (SLOC's) 'at will' has become a prerogative in the 21st century. Against this backdrop, the series of tense events and developments in the South China Sea and the Indo-Pacific region in recent years, corresponding with the precipitous expansion of the People's Liberation Army Navy (PLA (N)), bear evidence to this fact. The overall maritime threat calculus is evolving at a rapid space. The challenges from traditional as well as non-traditional maritime threats are multifarious. Given that protection of SLOCs to preserve or increase the economic might of a nation is one of the paramount tasks of any naval force, these tasks must be achieved under various traditional threats from enemy air, surface, and subsurface platforms or a combination of any or all of them. A major factor in this realm is the undersea domain.

B. THE THREAT

With the advent of new technology and emergence of sophisticated and more capable subsurface platforms and weapons, Antisubmarine warfare (ASW) has garnered significant attention. Submarines have long been considered as the weapon of choice by a weaker force to intimidate a more powerful adversary. They revel in the advantage offered by submarines' stealth, which empowers a weaker force with the exclusive capability to undertake surprise attacks and evade counter targeting by their adversary. With the effective deployment of submarines, a nation can pose a significant threat to its adversary's ability to control and project power from the seas. Advancements in submarine technology such as improved Air Independent Propulsion (AIP) systems have enabled submarines to remain submerged for several months, adding to their inherent stealth and straining the resources of adversary forces attempting to detect and neutralize the threat. Additionally, advances in submarine weapon systems, which provide submarines the ability to carry out attacks with missiles as well as torpedoes from stand-off ranges, have greatly enhanced lethality. Submarines have emerged as the primary instrument for exercising sea denial and sea control as part of naval operations. Therefore, efficient ASW tactics and capability enhancement is necessary to ensure protection of our assets from an enemy submarine force inflicting attrition, as well as to ensure effective detection and classification of quiet, lethal submarines in challenging acoustic environments.

The increased lethality of modern submarines and dynamic geo-political circumstances have led to a significant increase in the roles and responsibilities of naval forces worldwide over the years. Nations are also grappling with an unstable economic environment, limiting their available capital for military expenditure. One way to address some of these challenges is the employment of unmanned platforms such as unmanned underwater vehicles (UUV). UUVs used for underwater missions have some inherent advantages. They are small in size, thereby reducing the probability of detection by the enemy. They can be manufactured at a significantly lower cost than manned submarines. They are easy to deploy and maintain and thus have lower life cycle costs. Additionally, they can be deployed in high-risk areas without the fear of losing human lives. Owing to these advantages, unmanned systems have gained prominence in recent years. One of many UUVs to enter the fray is the Wave Glider, developed by Liquid Robotics, a subsidiary of Boeing. Wave Gliders equipped with a passive array can remain deployed in an area of operation (AOO) for extended periods of time. Furthermore, they can contribute to a layered defense preventing enemy contacts from transiting the area undetected, thereby providing a low cost, persistent ASW capability.

C. PROBLEM STATEMENT

This thesis aims to determine, what degree of probability of detection is achievable with the strategic organization and placement of ASW Wave Gliders equipped with passive sonar arrays in a given AOO.

D. OBJECTIVE

The objective of the thesis is to develop a model to determine the detection capability of deployed Wave Gliders, catering for the various factors associated with detecting underwater contacts using a passive sonar. The model will provide guidance on:

(a) The minimum number of Wave Gliders required to achieve a certain probability of detection.

(b) The optimal placement of the Wave Gliders in an area to maximize the probability of detection.

E. THESIS ORGANIZATION

Chapter II deals with the concepts for formulating the problem statement and developing the model. Chapter III describes the Design of Experiments and Methodology employed for arriving at the optimal solution and tools involved. Chapter IV deals with simulations conducted for different problem instances, the results obtained, and the analysis of the simulations. The thesis concludes with the findings and recommendations for future work.

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II. FORMULATION

This chapter gives a detailed description of the concepts underpinning the formulation of the problem statement and development of the model. It provides insights into the capabilities of Wave Gliders and characteristics of submarine. Further, calculations for the Lateral Range Curve and the passive sonar equation, which provide the tools employed to determine the probability of detection, are specified. Assumptions, both due to practical constraints as well as limitations of the model, are also discussed.

A. OVERVIEW OF THE WAVE GLIDER

According to its manufacturer, Liquid Robotics, the Wave Glider SV3 is an unmanned underwater vehicle that offers a low-cost, silent, and persistent acoustic surveillance capability. Wave Glider is powered by wave and solar energy, which means that it is capable of being deployed at sea for long durations without the need for replenishment or turnaround. A visual depiction of the Wave Glider is shown in Figure 1. This UUV works by converting wave energy into thrust for its propulsion and has electric thrusters for additional speed and control. Being 2.2 meters in length, with a low noise propulsion system and minimum radar signature, the Wave Glider is immune to counter detection by submarines. While it can perform a variety of missions such as collecting oceanographic and environmental data, Intelligence Surveillance Reconnaissance (ISR), etc., the Wave Glider's ASW variant is fitted with a towed passive sonar array for acoustic detection. The array has 32 channels optimized for detection of submarine-emitted frequencies and can be deployed up to a maximum depth of 150 m, providing effective underwater detection capability (Liquid Robotics 2022). A Wave Glider with its passive array deployed is depicted in Figure 2.

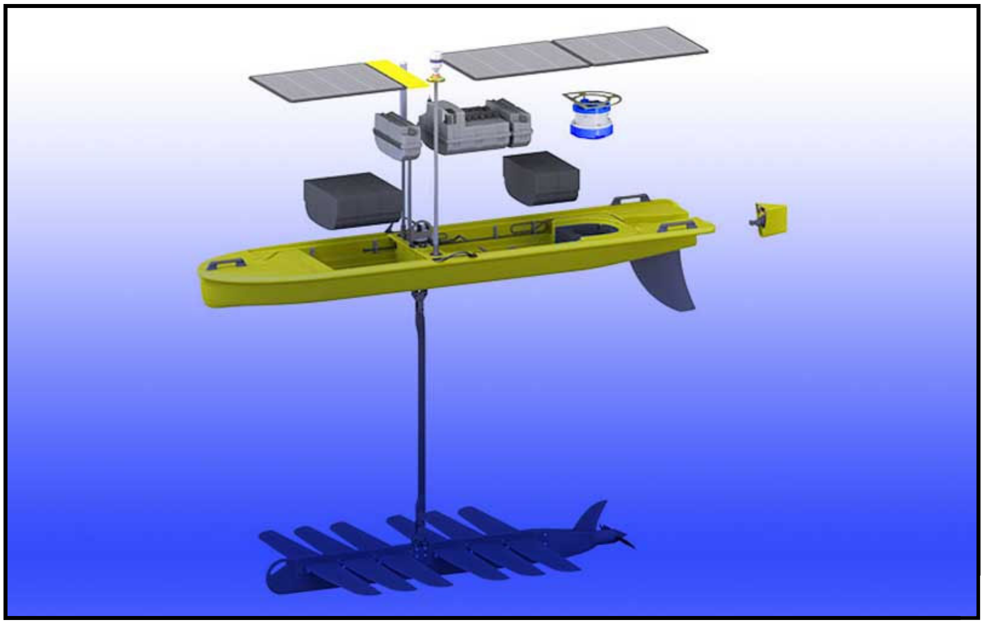

Figure 1. Visual Depiction of a Wave Glider. Source: Tobe (2014).

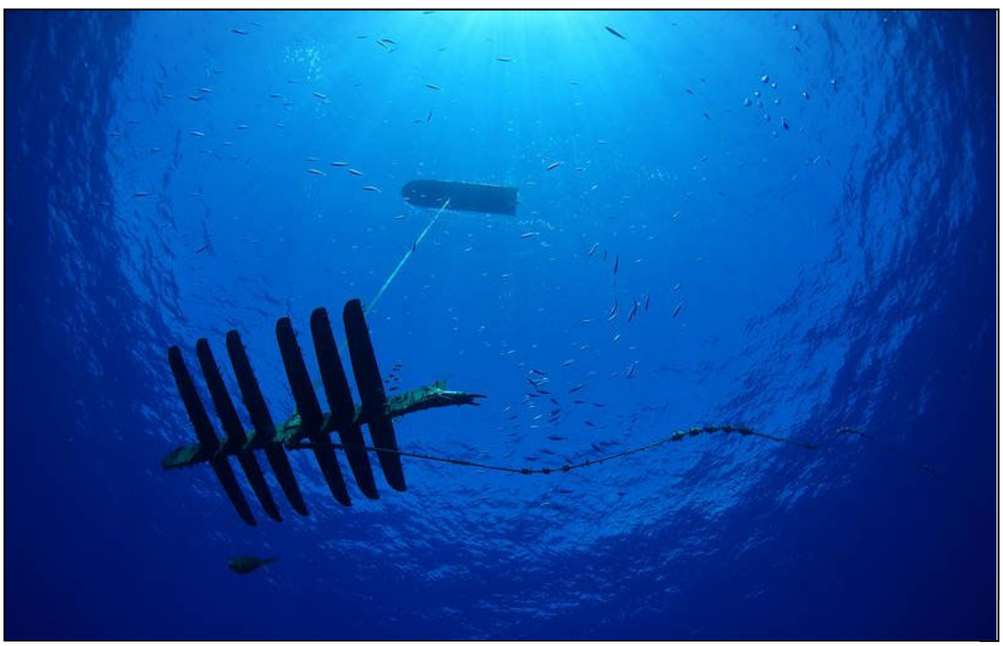

Figure 2. Wave Glider with Passive Sonar Array Deployed. Source: Eger (2017).

1. Concept of Operations

The Wave Glider's concept of operations calls for strategically positioning a Wave Glider network, with each glider having a passive towed array, in an area to detect underwater contacts of interest.

Whenever a Wave Glider detects a contact, it relays the acoustic information and positional data such as line of bearing, closest point of approach, and doppler information in real time to a remote operator at a shore station via a satellite link. This information can then be used to cue ASW assets to further classify and neutralize the threat.

2. Submarine Capabilities

This research uses the Type 093 Shang Class submarine of the PLA (N), which is the Red submarine in the scenario described at Figure 3. Whilst the other parameters of the submarine have less significance, it is necessary to consider the submarine's radiated noise level of 110 dB (Erickson and Goldstein 2007). This value plays a role in determining the range at which it could be detected by the Wave Gliders.

3. Scenario

This research considers a scenario in which a Blue Carrier Task Force (CTF) is operating at sea and has to traverse a choke point in order to proceed to its next deployment area. Intelligence indicates that a Red submarine equipped with advanced weapons and sensors is likely to be deployed to search for and prosecute the Blue CTF. Authorities have identified a 30 x 30 Nm area in which the submarine is most likely to transit in order to impede the safe transit of the Blue Force. The commander has at his disposal a given number of ASW Wave Gliders which need to be optimally placed in the area to maximize the probability of detecting the transiting enemy submarine. The scenario is represented in Figure 3.

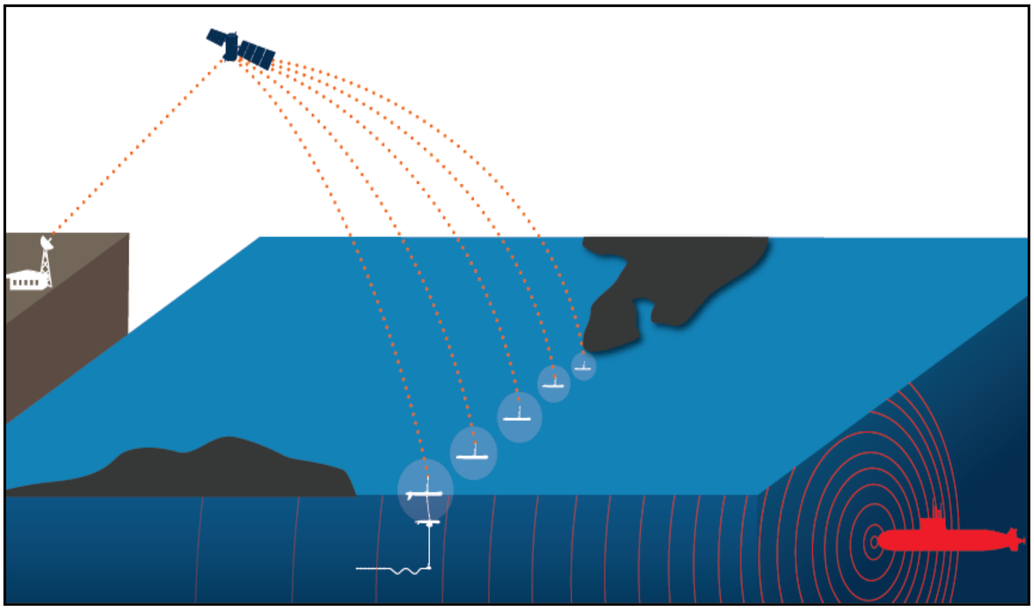

Figure 3. Scenario for Deployment of Wave Gliders

4. Assumptions

The assumptions made are as follows:

(a) The ASW Wave Gliders are pre-deployed in the area or can reach the area in sufficient time to commence operations prior to the submarine's arrival.

(b) An area of 30 x 30 Nm has been designated as the submarine probability area (SPA).

(c) Communication with the Wave Gliders from the shore station from which they are controlled is working satisfactorily.

(d) Sea states are conducive for Wave Glider deployment and operations.

(e) The threat axis is assumed as True North (000).

(f) The location of the Wave Glider deployment is not known to the submarine.

B. FORMULATION OF PARAMETERS

With the preceding description of the scenario and capabilities of the entities involved, the parameters which form the basis of the model are elucidated in the succeeding paragraphs.

1. Estimated Sonar Range (ESR)

One of the major factors that determines the detection capability of underwater sensors is the Estimated Sonar Range (ESR). ESR in the simplest form is the sweep width, or the estimated swath of ocean covered by the sensor of a searching unit (Wagner et al. 1999). Submarine tactics are also an important factor in determining the sweep width. The submarine will most often seek its best depth to avoid detection. Consequently, the sweep width that is used to evaluate the detection capability of the sensor is a function of the below layer ESR, it being the depth at which the submarine will most likely operate to avoid detection. (Layer is the depth in the sea column at which the temperature starts decreasing or the gradient between the temperature and depth becomes negative from positive. The terms above layer and below layer refer to the water above and below this depth, respectively). The ESR is established at the range at which (on average) the returned echo from a suspected target submarine will be detected 50% of the time; i.e., there is a 50% probability of detection. This factor varies with several things, such as target aspect, target depth (above or below layer), sonar characteristics, environmental conditions, operator efficiency, and speed of the target. The effect of taking the below layer ESR, which is generally the lesser range compared to the above layer ESR, is to provide a more conservative solution.

2. Lateral Range Curve

During the search for a target, either the searcher or the target, or both, would be in motion. The searcher is able to detect the target because the relative motion of both objects gets them sufficiently close to each other for detection to occur. If the relative path of the objects is known, the probability of detection of the target could be easily determined. Consider a target which is moving along a straight-line path which causes it to pass at a particular distance from the searcher as described in the scenario in the third paragraph of

Section A. The cumulative chance of detecting the target increases as it enters the detection circle of the sensor until its time of departure. The distance at which the target is closest to the sensor, or its closest point of approach (CPA), is called the Lateral Range (illustrated in Figure 4), and the graphical representation of the cumulative probability of detection is called the Lateral Range Curve (Washburn 2014). The curve describes the ability of a sensor to detect a target passing at a given range, under a particular set of environmental conditions. As these parameters vary, the Lateral Range Curve would also be different.

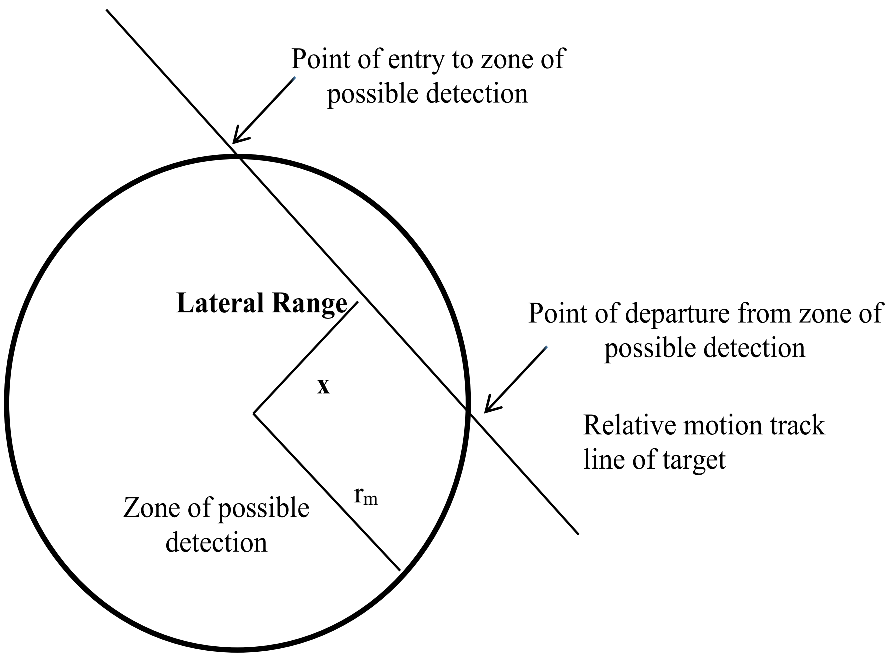

Figure 4. Lateral Range of the Target

In Figure 4, the zone of possible detection is assumed to be the region enclosed by a circle centered on the sensor, having a radius equal to the maximum possible detection range r_m. If the target is moving along a line that will cause it to pass at some lateral range *x* within the detection zone of the sensor, the cumulative chance of detecting the target will increase as it enters the detection zone until it reaches the point of departure from the zone. The searcher's path is assumed to be x=0, with the path to the searcher's right being $x > 0$ and to the searcher's left being $x < 0$. A typical Lateral Range Curve is depicted in

Figure 5. This Range vs. Probability graph is the Lateral Range Curve for a sonar constructed for a 50 % probability of detection as is considered while calculating the ESR. The sweep width for a particular sensor is equal to the area W under the Lateral Range Curve.

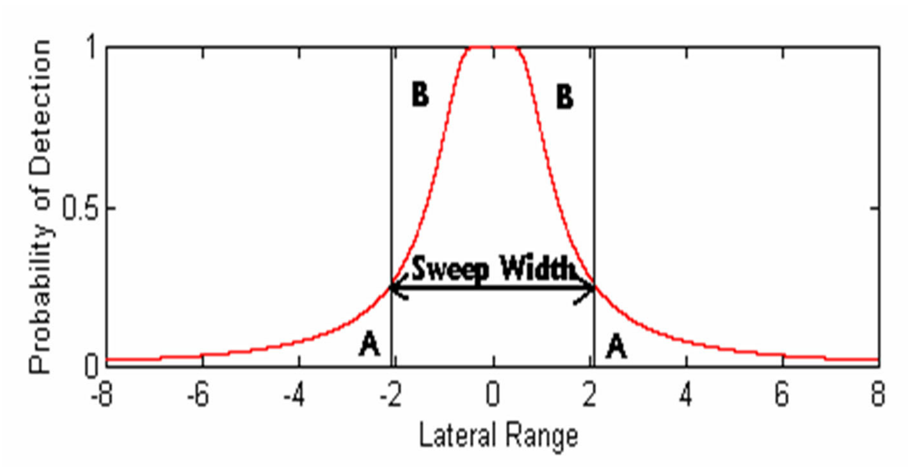

Figure 5. Typical Lateral Range Curve. Source: Koester (2004).

C. SEARCH MODELS

The search models explored in order to determine the probability of detection are as described in the succeeding paragraphs.

1. Random Search

Since the position of the submarine is unknown, it is assumed to be uniformly distributed over the entire region; i.e., the submarine is as likely to be found in one part of the region as in any other. In this case, it is assumed that the target is searched in a random manner. Therefore, the probability that the target will be detected by a particular sensor is given by (Wagner et al. 1999, chapter 7):

$$P\ (D) = 1 - e^{-\,wvt/A}$$

where

P (D)	=	Probability of Detection
W	=	Sweep Width of the sensor (Estimated Sonar Range)
V	=	Search Speed
T	=	Time of Search
A	=	Area of Search

This method describes the computation of P(D), when the least amount of information is available about the target. It is often the *conservative* approach and provides a lower bound on the probability of detection. In case some further information about the target is available, an equivalent search effort would yield a better result. It is pertinent to mention that the quantity wvt/A can also be described as the *Coverage Factor*, which is the ratio of the sweep width to the track spacing employed for the search.

2. Inverse Cube Law

In a situation, where it is estimated that the target is likely to pass or be forced to pass between two sensors or a line of sensors, the inverse cube law of detection can be used. It is also useful when a systematic rather than random search for the target is being undertaken. The method of estimating the probability of detection of a target passing between two sensors in a line of identical sensors, when only their sweep width is known, is called the inverse cube law of detection and given by (Wagner et al. 1999, chapter 7):

$$P(D) = 2\int_0^z \Phi(t)dt$$

where

Φ = Standardized, normal probability function with Mean (μ) = 0 and Var (σ^2) = 1,

Z = $(\sqrt{\Pi/2})$ x W / S,

in which

W = Sweep width,

S = Track Spacing, and

W/S = Coverage Factor.

3. Comparison of Detection Laws

The graph shown in Figure 6 depicts the comparison of Probability of Detection (Y–Axis) wrt the Coverage Factor (X–Axis) for both search methods. For a well conducted search, Inverse Cube Law gives a higher probability of detection than Random Search for the given coverage factor. As the position of the submarine is not known and in order to provide a conservative estimate, this thesis uses the random search model as the basis for computations.

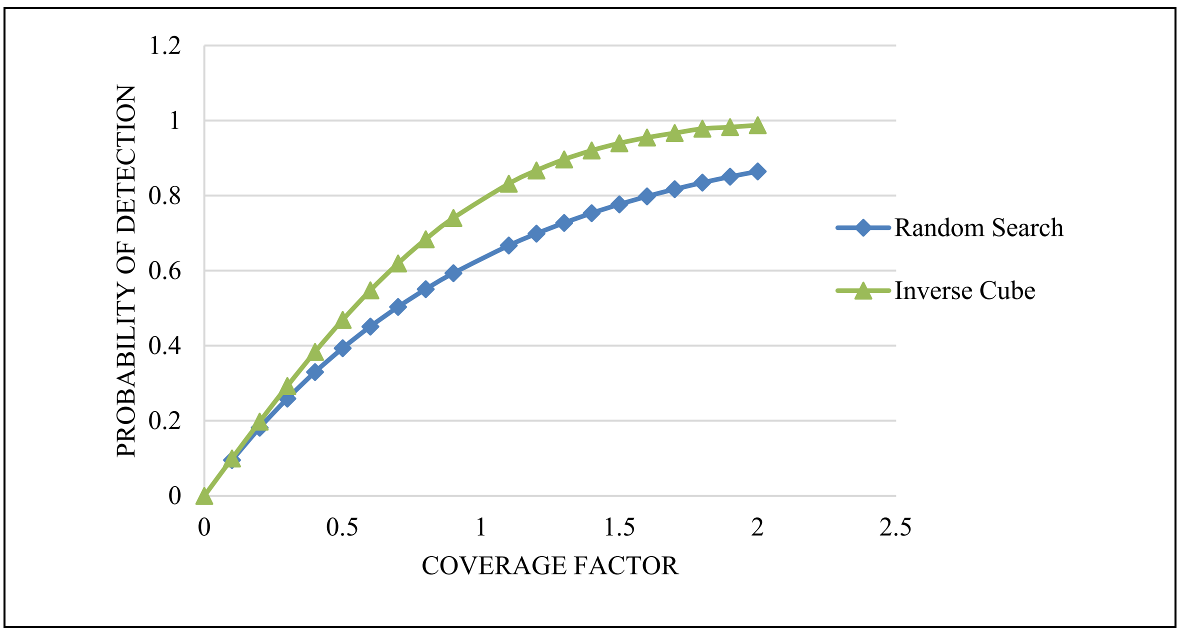

Figure 6. Comparison of Detection Laws

D. UNDERWATER DETECTION CHARACTERISTICS

The phenomena and peculiarities associated with the propagation of underwater sound have myriad effects on the design and operation of sonar equipment. These diverse effects can be logically grouped into equations called the sonar equations, which depend on the target, equipment, and environmental characteristics. The major parameters that play a role in detection of an underwater object by a passive sensor as is fitted on a Wave Glider are discussed in this section.

1. Passive Sonar Equation

A target can be detected by sonar when the signal strength received from the target at the sensor is greater than the strength of the background noise present in the area around the sensor, i.e., when the signal/*noise* ratio is greater than 1 (Urick 1967). When the sensor is passive, detection is dependent on the signal produced by the target itself. An illustrative image of the passive sonar equation is provided in Figure 7. The passive sonar equation to achieve a probability of detection of 0.5 is given by (Urick 1967):

$$\mathrm{SL} - \mathrm{TL} - (\mathrm{NL} - \mathrm{DI}) = \mathrm{DT}$$

where

SL = Source Level – Amount of sound radiated by the target,

TL = Transmission Loss – Weakening of sound as it travels from the source in the medium,

NL = Noise Level – Noise level at the receiver hydrophone,

DI = Directivity Index – Ability of the equipment to detect incoming signals, and

DT = Detection Threshold – Value above which the signal would be detected.

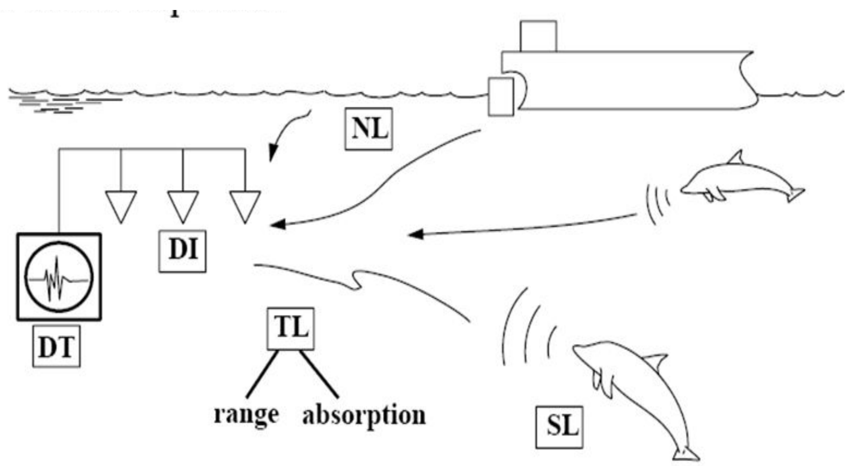

Figure 7. Illustrative Image of Passive Sonar Equation. Source: Inner Space Center – The University of Rhode Island (2002).

In the preceding equation, the left side indicates the ability of the sonar to detect a particular signal within the noise present in the environment. The right side indicates a threshold value which needs to be exceeded to enable detection with a probability of 50%. If the threshold is kept too low, it increases the probability of detecting more signals; however, it also increases the amount of noise received by the hydrophone, thereby increasing unwanted signals or false alarms. If the threshold is high, it reduces the number of false alarms but, in turn, will also reduce the probability of detecting the desired signals as well. Therefore, a fine balance for the threshold needs to be achieved depending on the underwater environmental conditions and the sonar design. The receiver operating characteristics curve for a sonar are depicted in Figure 8.

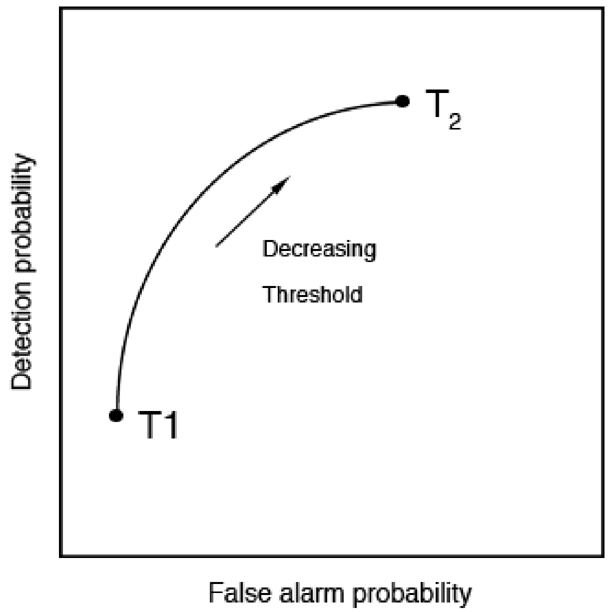

Figure 8. ROC Curve for Sonar. Source: Inner Space Center – The University of Rhode Island (2002).

2. Transmission Loss

The way sound propagates through water is a complex mixture of various parameters such as temperature, depth, salinity, and ambient noise, to name a few. As it travels through its medium, sound gets attenuated, distorted, and weakened. The weakening of sound between a point of 1 yd. from the source and a point at a particular distance in the sea as it travels through water is described as the transmission loss (Urick 1967). It, as a single equation, summarizes the overall effect of the propagation losses in the sea as is given by (Urick 1967):

$$TL = 10 \log (I_0/I_1) \text{ dB}$$

where

I_0 = Intensity of sound at source, and

I_1 = Intensity of source at a distant point.

When correlated in terms of range in water, transmission loss may be said to occur due to spreading and as loss due to absorption. Spreading loss can be expressed in terms of logarithm of range with a certain number of decibels per twice the distance. Attenuation combines the effects of absorption and scattering and varies linearly with range. Spreading loss can further be quantified as spherical spreading, where it is assumed that sound emitted by the source is spread equally in all directions, embodying a sphere and cylindrical spreading in which sound is assumed to spread between a lower and upper boundary embodying a cylinder. The equations for spreading loss are as follows (Urick 1967):

Spherical Spreading: TL = 20 log r

Cylindrical Spreading: TL = 10 log r

Loss due to absorption is due to the conversion of sound energy into heat as it passes through water. The absorption of sound in water depends on various factors, such as frequency, viscosity, depth, chemical reactions with ions in water, salinity, and temperature. The various factors are quantified in terms of an absorption coefficient which gives the loss in the transmitted signal with respect to range.

The physical conditions in sea water impede the incorporation of all factors into an all-encompassing formula. However, the combined loss due to spherical spreading and absorption has been deemed to provide a best estimate of transmission loss at sea. It can be represented by (Urick 1967):

$$TL = 20 \log r + \alpha r \times 10^{-3}$$

where

r = Range of target, and

α = Absorption Coefficient.

E. NOISE CONSIDERATIONS

The important factors that contribute to the background noise present in the underwater environment and impede detection of the signal generated by a source are discussed below.

1. Self-Noise

Underwater noise generated by the platform adds to the noise level present at the receiver hydrophone, which impedes the overall signal received from the target. As mentioned in the description of the Wave Glider, it converts wave motion into thrust for its propulsion. Therefore, sound produced during its movement in water would only be mechanical in nature and depend on wave motion at that particular moment. While the Wave Glider has electrical thrusters powered by solar energy to provide enhanced speed and movement, their use is limited and therefore would not significantly impact the self-noise of the platform. In addition, the sub-glider and the towed array (parts of the glider) are placed at a distance from each other, thereby leading to a further reduced effect of self-noise of the platform.

2. Flow Noise

Flow noise also contributes to the self-noise and is produced due to the relative motion of the object and the water around it. The flow noise is highly dependent on the speed of the object in water (Urick 1967). As the speed through water increases, friction between the object and the water leads to a turbulent flow of water around it that increases the intensity of noise around the platform. As Wave Gliders proceed at a very low speed and often in consonance with wave motion, their flow noise is assumed to be insignificant compared to the other background noises around the hydrophone.

3. Figure of Merit

The Figure of Merit (FOM) defines the ability of a particular sonar to detect a target. It combines the equipment and target parameters to provide an estimate of sonar performance (Urick 1967). The FOM equals the maximum allowable one-way transmission loss in passive sonars for a target to be detected. If the value for the FOM is greater than the transmission loss, the target would be detected by the sonar. The equation for FOM is (Urick 1967):

$$\mathbf{FOM = SL - (NL - DI + DT)}$$

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III. MODEL DEVELOPMENT AND DESIGN OF EXPERIMENTS

Based on the concepts that were described in the previous chapter, development of the model used for optimizing the probability of detection and the design of experiments is discussed in this chapter.

A. MODEL DEVELOPMENT

This sections describes the methodology, analytical tools and the fundamentals used for calculating the overall probability of detecting an underwater contact by the Wave Glider.

1. Signal Excess

The signal excess indicates whether a signal emitted by the target will be detected by the receiver hydrophone (Forest 1987). A signal is detected by the sonar only if the signal excess is greater than or equal to zero for a probability of detection of 50%. It can be quantitatively described as the difference between the present signal-to-noise ratio and the ratio required for detection. It is expressed as (Wagner et al. 1999):

$$\boxed{SE = FOM - TL}$$

2. Mean Signal Excess

When the passive sonar equation is being used as a basis to predict the probability of detection in future, the terms of the passive sonar equation can be construed to be independent random variables distributed normally. Each term of the equation would have a mean and standard deviation (Wagner et al. 1999, chapter 5). In the model, the passive sonar equation is assumed to pave way for calculating the mean signal excess as:

$$\text{Mean SE} = \overline{\text{SE}} = \text{FOM} - \text{TL}$$

The standard deviation of the SE calculated by taking the respective standard deviations of the terms in the equation is assumed to be between 3 and 9 dB (Kim 2009). Therefore, the SE is modeled as a normally distributed random variable, as SE ~ N (Mean SE, σ^2).

3. Instantaneous Probability of Detection

The Instantaneous Probability of Detection (IPD) is the probability of detection at a particular instant. The instant is a short duration and independent of any other instant in time. For a continuous looking sensor such as a passive sonar, the cumulative detection probability (cdp) over a period of time is an amalgamation of independent IPDs at various intervals within the period. As detection only occurs when SE > 0, the IPD could be described as (Washburn 2014):

$$\text{IPD} = \Phi\left(\frac{\overline{\text{SE}}}{\sigma}\right)$$

where Φ is the normal cumulative distribution function and mean SE and sigma that have been calculated as just described.

Examples of normal curves to represent a distribution of different SE values is depicted in Figures 9–12, where the probability of detection; i.e., the area under the curve to the right of the zero mean is displayed by the shaded portion in the plots in the figures:

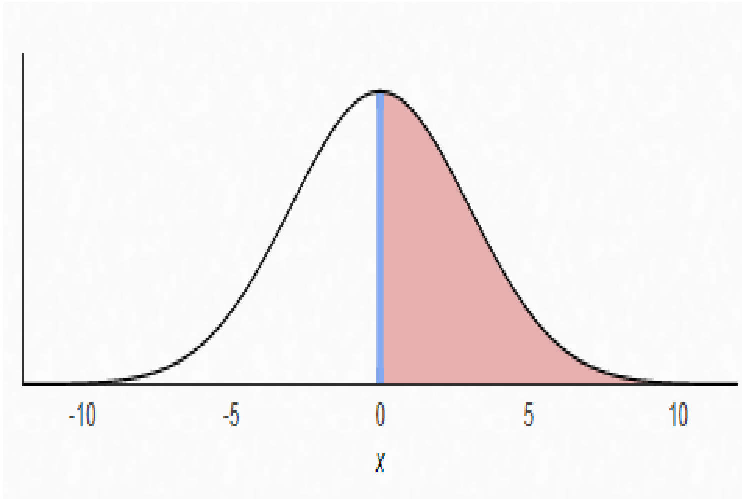

Figure 9. IPD – SE ~ N (0, 3^2); Pd = 0.5

Figure 9 depicts the IPD when the SE is normally distributed with a mean of 0 and standard deviation of 3. It results in a probability of detection of 0.5 at that instant.

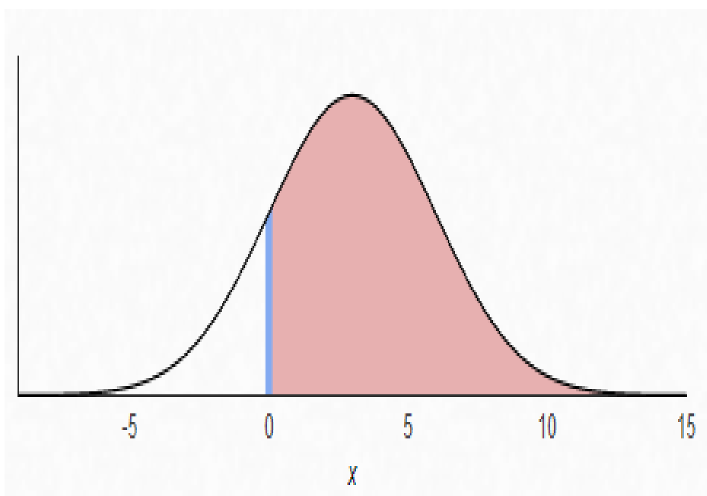

Figure 10. IPD – SE ~ N (3, 3^2); Pd = 0.84

Figure 10 depicts the IPD when the SE is normally distributed with a mean of 3 and standard deviation of 3. It is evident that with a higher value of mean for the SE, the probability of detection increases to 0.84 at that instant.

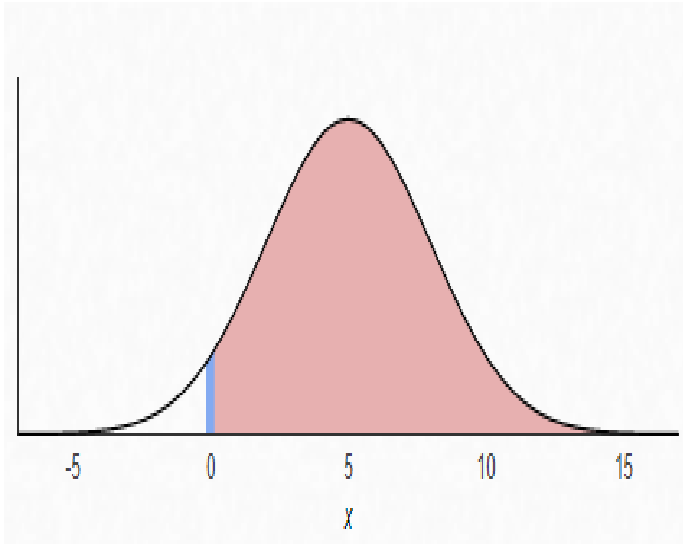

Figure 11. IPD – SE ~ N (5, 3^2); Pd = 0.95

Figure 11 depicts the IPD when SE is normally distributed with a mean of 5 and standard deviation of 3. The probability of detection increases to 0.95.

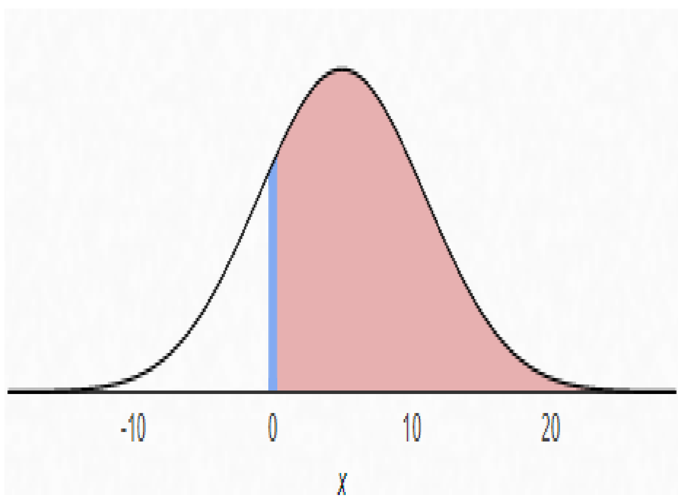

Figure 12. IPD – SE ~ N (5, 6^2); Pd = 0.80

Finally, Figure 12 depicts the IPD when SE is normally distributed with a mean of 5 and standard deviation of 6. The probability of detection in this case decreases to 0.80 as the standard deviation of SE increases.

4. Poisson Scan Model

The instantaneous probability of detection expresses the probability of detection at a given instant. However, by using detection rates and modeling detections as occurring according to a Poisson process with a particular rate, it is possible to extend the concept for a continuous looking sensor and calculate the cumulative detection probability over a specified period. For this, the following is assumed (Kim 2009):

(a) Detections occur with a Poisson rate λ (1/unit time).

(b) Detections in non-overlapping time intervals are independent.

Therefore, the IPD can now be expressed as:

$$\mathrm{IPD}(t) = \Phi\left(\frac{\overline{\mathrm{SE}}(t)}{\sigma}\right)$$

From the preceding assumptions the detection rate can be expressed as:

$$\gamma(t) = \lambda\Phi\left(\frac{\overline{\mathrm{SE}}(t)}{\sigma}\right)$$

Therefore, from the properties of a non-homogenous Poisson process, the cumulative probability of detection is expressed as (Wagner et al. 1999):

$$F_d(t) = 1 - \exp(-\int_0^t \gamma(v)dv)$$

5. Lateral Range Curve

As was described in the previous chapter, the distance to the target at its CPA is called the Lateral Range and the graphical representation of the cumulative probability of detection is called the Lateral Range Curve. Let us consider a scenario where the Wave Glider is deployed at a particular location in the AOO, and the target submarine is approaching the area with a *speed* = *v* in a direction that will lead it into the possible zone of detection of the Wave Glider as described in Figure 13. Suppose we set the clock to zero at the time at which the submarine enters the detection zone and let *time* = *t* be the time at which it leaves the detection zone. Also let *x* be the Lateral Range of the submarine from the Wave Glider.

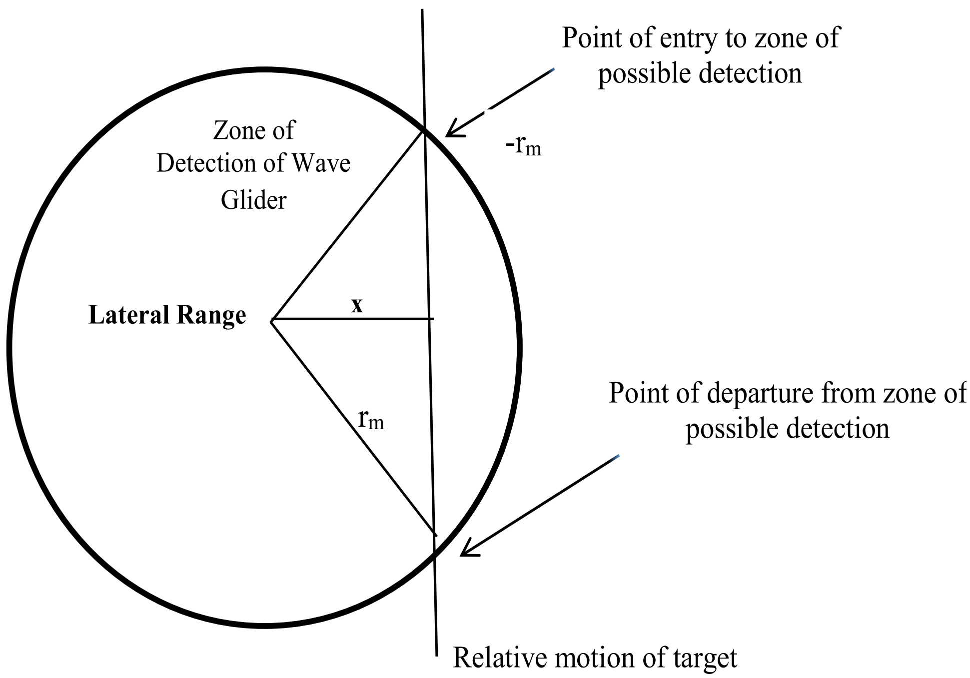

Figure 13. Movement of Target wrt Sensor

Therefore, the probability of detecting the submarine before it leaves the detection zone can be expressed as:

$$\mathrm{P\{detection\ by\ time\ t\}} = F_d(t) = 1 - \exp(-\int_0^t \gamma(s)ds)$$

where

$$\gamma(s) = \lambda\phi(\overline{SE}(s)/\sigma).$$

Considering the scenario depicted in the Figure 13, the point at which the target enters and leaves the zone of detection could be expressed as $\sqrt{(-r_m)^2 - x^2}$ and, $\sqrt{(r_m)^2 - x^2}$ respectively. Therefore, the total time that the target remains in the detection zone can be encapsulated as $2\sqrt{{r_m}^2 - x^2}/v$, thereby eliminating the dependency on time. Thus, the equation for lateral range function can now be expressed as (Wagner et al. 1999),

$$p_l(x) = 1 - \exp(-\int_0^{\frac{2\sqrt{r_m^2 - x^2}}{v}} \gamma(s)ds)$$

when

$|x| < r_m$ and 0 otherwise.

The Lateral Range Curve constructed using the preceding equation and an arbitrary ESR of 1 Nm is as shown in Figure 14:

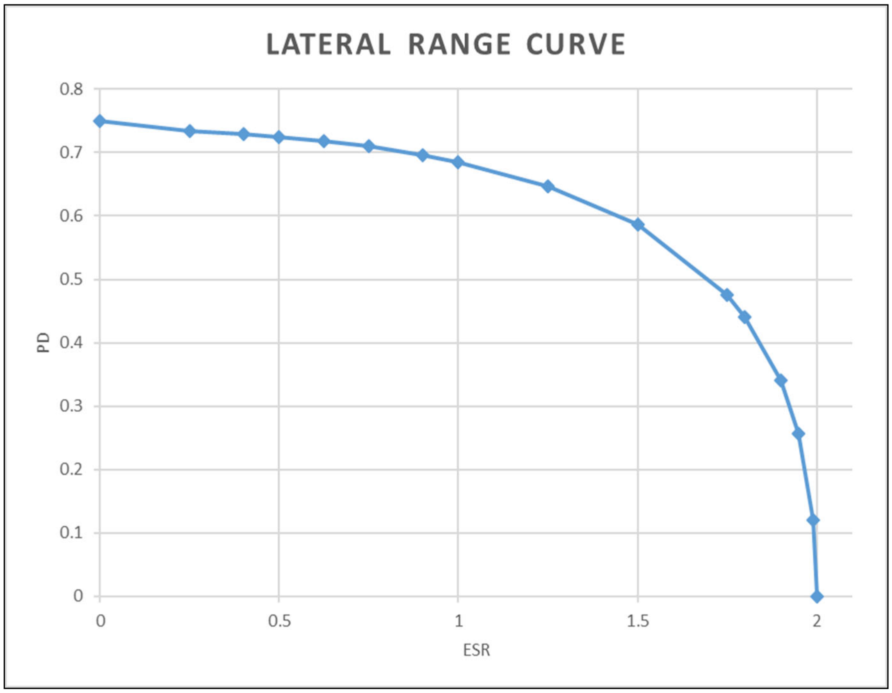

Figure 14. Lateral Range Curve – 1 ESR in Nautical Miles

6. Probability of Detection

In the scenario that was described in Chapter II, it was assumed that the target is likely to enter through any point along the upper boundary of the given area and follow a straight-line course until it exits the area. Since exact information about the target movement is not likely to be available, we assume that it would enter through any point along the perimeter of the area from its direction of approach. Therefore, the target is equally likely to pass through all lateral ranges within $-r_m$ to r_m. Thus, the random variable X, defined in subsection 4 (Poisson Scan Model), as the lateral range to the target will have a uniform probability distribution over these range of values. If $f(x)$ is the probability density function of X, then the probability of detecting the target passing at a lateral range x is expressed as (Wagner et al. 1999, chapter 7):

$$P\{detection\} = \int_{-\infty}^{\infty} p_l(x)f(x)dx)$$

where

$p_l\,(x)$ = Lateral range function of x, and

$f\,(x)$ = Probability density function of x.

Assuming that the length of the frontage that the Wave Gliders are deployed to protect is *l*, and the target is equally likely to enter through any point along the frontage, the probability density function of the lateral range of x, distributed uniformly, as described in the preceding paragraph can be given by,

$$f(x) = \begin{cases} 1/l, & \text{for } |x| \leq l/2 \\ 0, & \text{otherwise.} \end{cases}$$

If $l/2 > r_m$ and considering the probability of detection expressed previously, the probability can now be given as:

$$P\{detection\} = \frac{1}{l}\int_{-r_m}^{r_m} p_l(x)dx$$

B. INPUT PARAMETERS

The input parameters that were used for the experiments are described in the succeeding paragraphs.

1. Control Variables

The following parameters have been designated as user inputs as the user will have the best appreciation of the scenario and envisaged threats in the AOO:

(a) **Submarine Speed**. The transit speed of the submarine will vary by the type of threat. For our experiments, we use a submarine speed of 6 knots.

(b) **Number of Wave Gliders**. The number of wave gliders presently available for deployment is also a user input as the number actually available will only be known to the commander of the forces. For the purpose of the model, experiments with the number of Wave Gliders ranging between five and 20 were conducted.

(c) **Estimated Sonar Range**. The estimated sonar range, which is calculated based on the prevailing environmental conditions in the area, is also a user input as the forces presently deployed in the AOO will have the best estimate of the prevailing conditions to feed the relevant parameters in the model. For the purpose of the model, experiments with the ESR ranging between 0.5 and 2.5 Nm have been conducted. Being a small passive array with 32 hydrophones, the range between 0.5 and 2.5 Nm is likely to cover the maximum ESR that the Wave Gliders could be expected to have on any particular day.

2. Figure of Merit

As mentioned in Chapter II, the FOM equals the maximum allowable one-way transmission loss in passive sonars for a target to be detected. The FOM is required to calculate the signal excess that is present at the receiver hydrophone, which can be used to compute the probability of detection. The following provides an illustration of the methodology for calculating a particular FOM:

(a) **Target Source Level (SL)**. The SL for a Shang Class submarine as part of the *Red* force has been assumed to be *110 dB,* as mentioned in subsection 4 of Chapter II.A.

(b) **Noise Level (NL)**. The noise level at the receiver hydrophone is a combination of various sources, such as ambient noise, self-noise, flow-noise, etc. This value is assumed to be *50 dB*.

(c) **<u>Directivity Index (DI)</u>**. The value of the directivity index for the passive sonar of the Wave Glider is a design parameter of a particular sonar, and in this case, it is assumed to be *15 dB*.

(d) **<u>Detection Threshold (DT)</u>**. The detection threshold determines the amount of signal required to enable a 50% probability of detecting the target over the background noise in the environment. The DT is also a feature of the sonar design itself. For the Wave Glider, its value is assumed to be *-15 dB*.

Using the values just presented, the FOM is calculated to be ***90 dB***. Value of the FOM was also varied in the experiments to analyze its effect on the overall probability of detection.

3. Transmission Loss

Transmission loss is the reduction in the intensity of the signal as it travels through water. As mentioned in Chapter II, it can be calculated as $TL = 20 \log r + \alpha r \times 10^{-3}$ (Urick 1967), where *r* is the range of the target which would be determined through simulation in the model. The value of α represents the absorption coefficient when the loss is due to absorption of the sound waves. The value of the absorption coefficients for various water bodies as determined by Ainslie and McColm (1998) is shown in Figure 15. The value of α is assumed to be ***10 dB*** in the model.

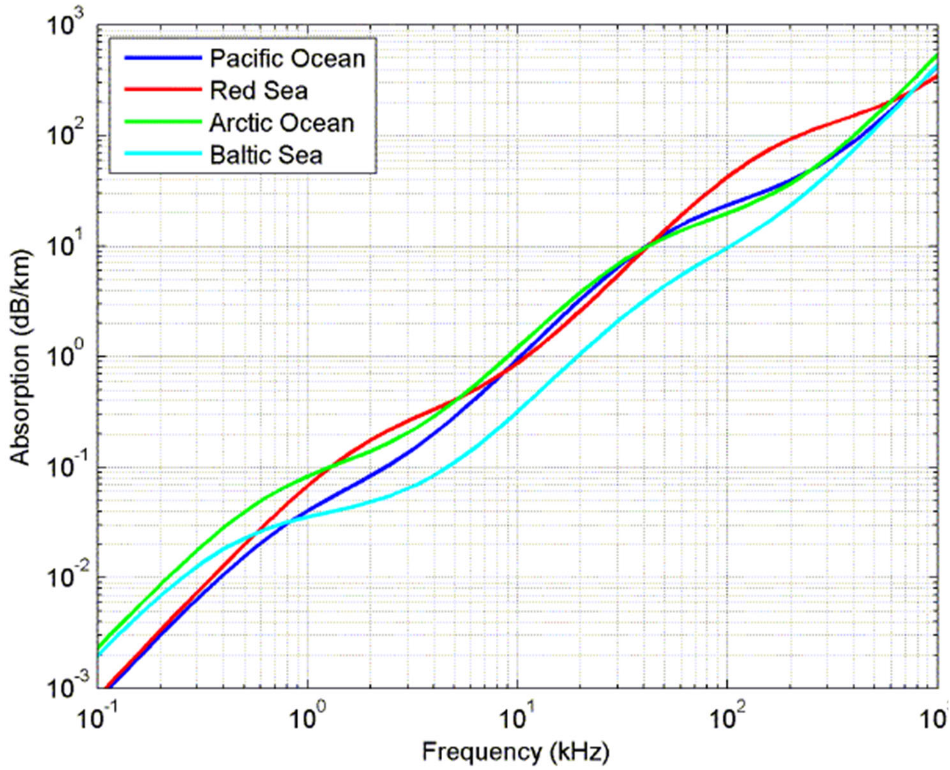

Figure 15. Illustration of Variation in Absorption Coefficient. Source: Ainslie and McColm (1998).

Based upon the preceding calculations, the mean signal excess is determined as Mean $SE = FOM - TL$ $(= 80$ $dB)$. This is used to calculate the detection rate as well as probability of detection.

4. Placement of Wave Gliders

As the Wave Gliders are small unmanned systems with a length of 2.2m, there is a restriction on the amount of payload they can carry. They have a small passive array of 32 channels with minimal post-processing capabilities on the platform itself. Though Wave Gliders offer numerous advantages, as described earlier in the thesis, a single wave glider has much fewer detection capabilities compared to sonars onboard ships and submarines. Therefore, a network of Wave Gliders is recommended for deployment to increase the probability of detecting an undersea contact attempting to transit through the AOO. The aspects explored for placement of Wave Gliders are described in the succeeding paragraphs.

Among the many mathematical methods available, perturbation theory provides a basis for finding an approximate solution to a complicated problem by starting from the exact solution of a similar but simpler problem. It is closely related to the methodologies used in solving problems through numerical analysis. A core of this technique is to break the problem into two parts, one which can be solved easily and for which an exact solution can be obtained, and the other which is demanding to solve. Solving the problem by this technique thus leads us to an expression in the form of deviations from the quantifiable terms of the solvable part of the problem, which can then be applied to find an approximate solution to the problem. It is applicable if the problem at hand cannot be solved exactly, but an accurate solution could be developed from its mathematical description. A similar approach was used in the model for determining the placement of Wave Gliders in the given area.

5. Circle Packing

Circle packing uses an area search model to represent the acoustic coverage of cookie-cutter sensors, each of which covers an area equivalent to its detection radius, *R*. Given *R* and an area *A*, it is possible to determine probability of detection accrued when a specified number of circles are packed in an area, or the number of circles required to achieve the desired probability of detection (Washburn 2014).

Figure 16 represents an attempt to pack an area with circles of radius *R*. The triangle acts as a recurring element; thus, the fraction of search area covered is the same as the fraction of the triangle covered. The area of the equilateral triangle is expressed as (Washburn 2014):

$$(1/2)(\text{Base})(\text{Height}) = (1/2)\,(2R\cos\theta)\,(\sqrt{3}R\cos\theta) = \sqrt{3}R^2\cos^2(\theta).$$

Also, the area within the triangle is given by 3A1 + 3A2, where

A1 = (2R cos θ) (R sin θ)/ 2 = R^2cos θ sin θ, and

A2 = (πR^2) (π/3 − 2θ) / (2π) = R^2 π/6 − R^2 θ.

Therefore,

$$
\begin{aligned}
P_d(\theta) &= 3(A1 + A2) / \sqrt{3}R2 \cos2\theta \\
&= 3(R^2 \cos\theta \sin\theta + R^2\pi/6 - R^2\theta)/ \sqrt{3}R2 \cos2\theta \\
&= \sqrt{3}(\cos\theta \sin\theta + \pi/6 - \theta) /\cos^2\theta.
\end{aligned}
$$

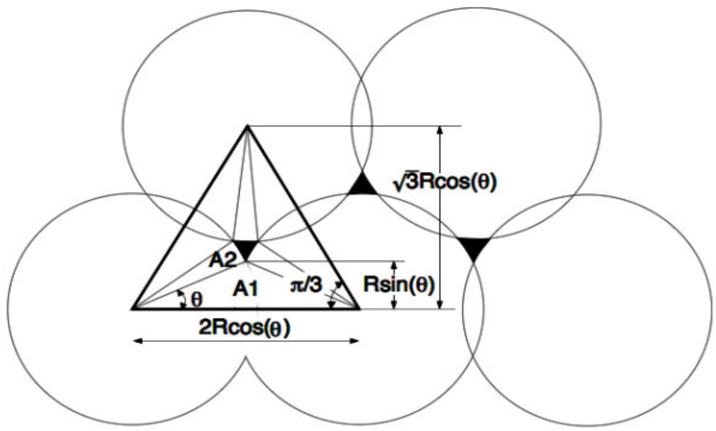

Figure 16. Circle Packing Geometry. Source: (Washburn 2014).

The quantity $n\pi R^2/A$ is defined as the coverage factor (CF), with n being the number of circles, in the following cases:

(a) θ = 0, circles do not overlap, $P_d = n\pi R^2/A$.

(b) θ = π/6, then there are no coverage gaps, P_d = 1.

(c) θ ∈ (0, π/6), partial overlap,

$$P_d = \sqrt{3}(\cos\theta \sin\theta + \pi/6 - \theta) /\cos^2\theta \leq CF$$

Using the same conventions as those just presented, θ can also be used to determine the number of circles required to cover the search area A. Thus,

$$n(\theta) \begin{cases} = A P_d/(\pi R^2),\ \theta = 0 \text{ (no overlap)} \\ = A/(2\sqrt{3}R^2 \cos^2\theta),\ \theta \in (0, \pi/6) \text{ (partial overlap)} \\ = 0.3849A/R^2,\ \theta = \pi/6 \text{ (no gaps).} \end{cases}$$

The graph in Figure 17 illustrates the circle packing efficiency in terms of probability of detection and coverage factor.

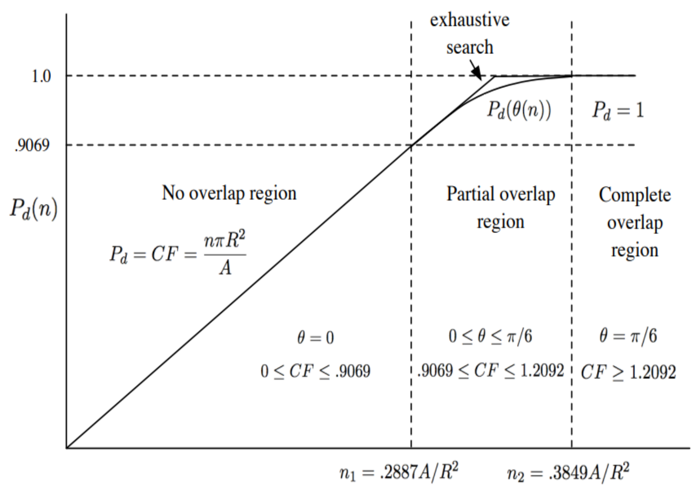

Figure 17. Circle Packing Efficiency. Source: (Washburn 2014).

If we solve for the number of Wave Gliders required in our area of 30 x 30 Nm to achieve a probability of detection of 50%, with each Wave Glider having a detection radius of 2 Nm, we get

$A = 900\ Nm^2$, $R = 2$ Nm, P_d (required) = 0.5

$0.5 \le .9069$ = no overlap required ($\theta = 0$ and $P_d = CF$).

With no overlap,

$n = P_d A/(\pi R^2)$

$= (900)(.5)/(\pi 2^2) = 35.80 \sim$ *36 Wave Gliders.*

Circle packing would be an efficient solution when the detection radius of the sensor is effectively high. However, as the detection capability of the Wave Gliders is restricted, a large number of Wave Gliders would be required to cover the entire area using the circle packing model. Therefore, we consider the probability of detection when a fixed number of Wave Gliders is arranged in specific formations.

6. Formations of Wave Gliders

For our experiments, we consider the following Wave Glider formations:

(a) **<u>Circular Formation</u>**. The Wave Gliders are placed in a circle with a given radius from the center of the area. The placement of Wave Gliders is governed by the ESR or their detection radius.

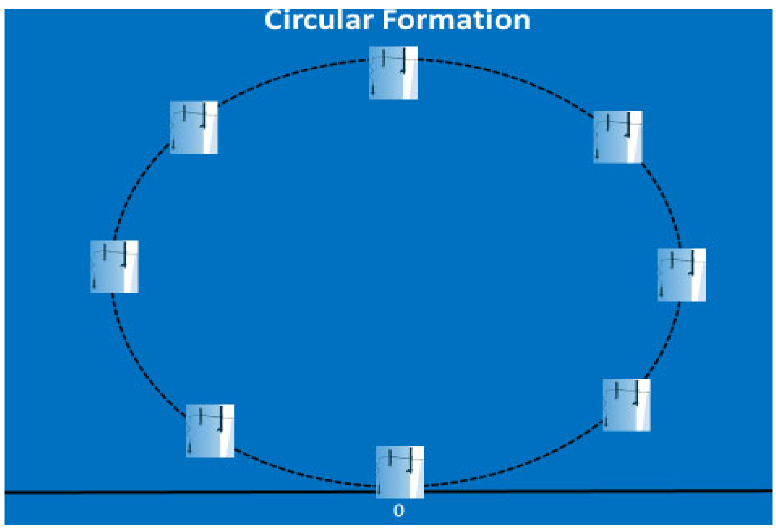

Figure 18. Illustration of Circular Formation

(b) **Barrier Formation**. In the barrier formation, the Wave Gliders are placed on a straight line aligned perpendicularly to the direction of approach of the enemy submarine.

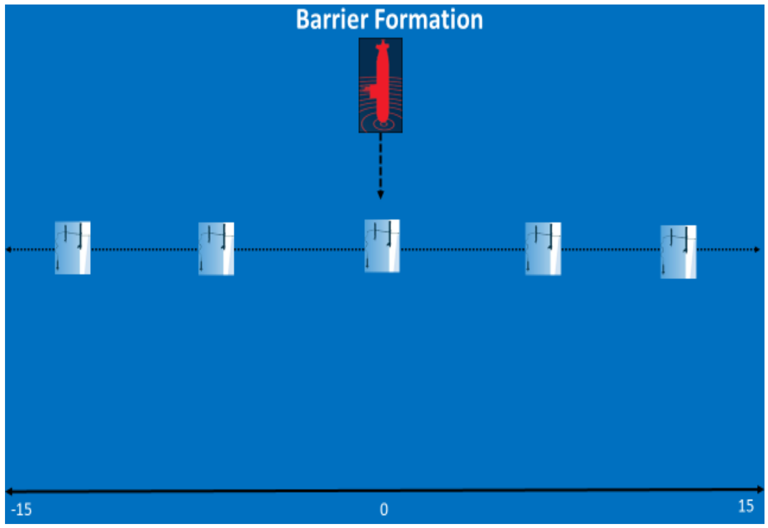

Figure 19. Illustration of Barrier Formation

(c) **Sector Formation**. In the sector formation, the Wave Gliders are arranged on a sector line oriented towards the direction of threat.

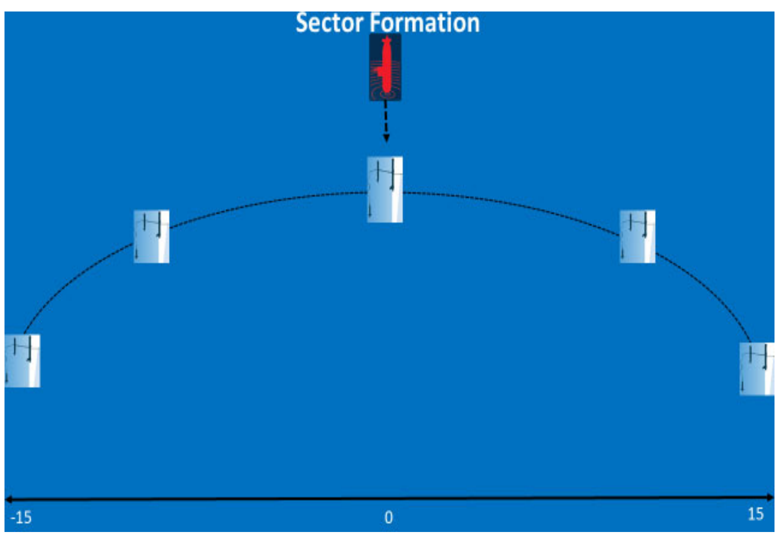

Figure 20. Illustration of Sector Formation

(d) **Multiple Barrier Formation**. The multiple barrier formation consists of Wave Gliders placed in two barriers perpendicular to the direction of threat.

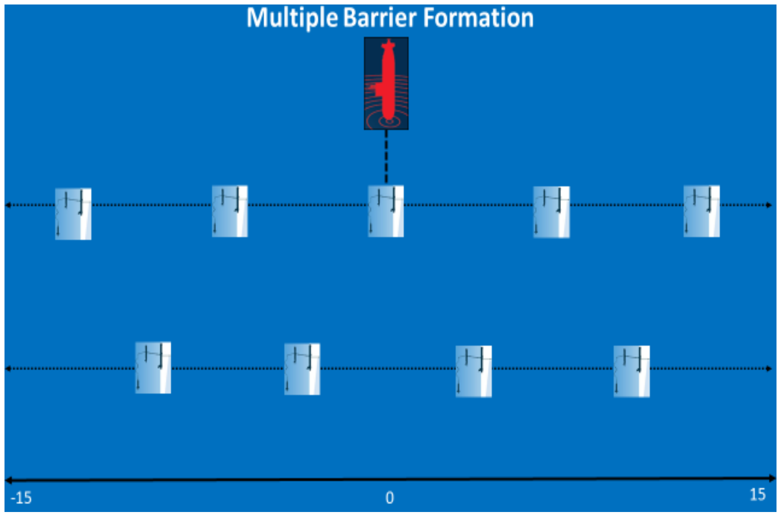

Figure 21. Illustration of Multiple Barrier Formation

The probability of detection achieved when the Wave Gliders are arranged in the preceding formations is discussed in the next chapter.

IV. RESULTS AND ANALYSIS

This chapter provides a description of the simulation experiments undertaken to estimate the probability of detection when a certain number of Wave Gliders is deployed in a given formation. A comparison of the different values for the probability of detection obtained by varying different inputs, such as number of Wave Gliders, estimated sonar ranges, and formations, is provided. The different results have been compared to arrive at an optimal placement of Wave Gliders in different scenarios depending upon the target parameters, environment parameters, and the number of available wave gliders. *Python 3* was used to develop the model and undertake simulations.

The simulations were undertaken with a FOM of *90 dB*, with detections occurring as a Poisson process with $\lambda = 1$ and signal excess assumed to be a normally distributed random variable with SE ~ N (Mean SE, $\sigma^2 = 6$). The number of replications for all experiments was set to 10,000.

A. THEORETICAL CALCULATIONS

Prior to the results of simulations, to verify the semblance of simulations theoretical calculations were undertaken to determine the probability of detection. For this purpose, four Wave Gliders, each with an ESR of 2 Nm, were assumed to be deployed in the barrier formation in the AOO, with the length of the barrier being 30 Nm. The arrangement of Wave Glides for the scenario is described in Figure 22.

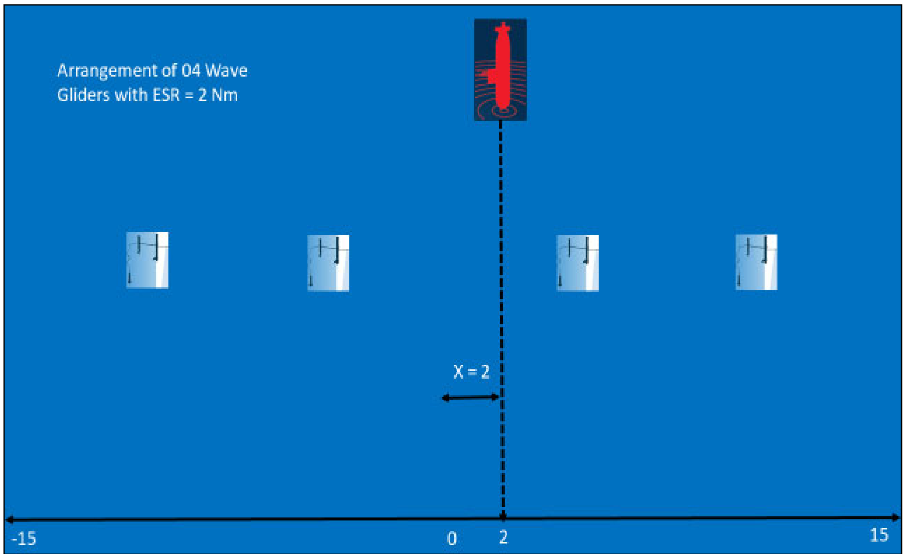

Figure 22. Arrangement of Wave Gliders

We assume that on its current path the submarine will pass at a distance of x = 2 Nm from the center of the area. The Wave Gliders are arranged equidistant from one another as depicted in the figure. The lateral range function as developed in the previous chapter is used for calculations:

$$p_l(x) = 1 - \exp(-\int_0^{\frac{2\sqrt{r_m^2 - x^2}}{v}} \gamma(s)ds)$$

Under the assumptions just stated, the overall probability of detection will be a combination of the probabilities of detection of each sensor calculated independently. Therefore, for with *d* being the range at which the Wave Glider passes from each sensor, the independent probabilities can be defined as:

- Sensor 1 -> $P_d = p_l(x - d1)$.
- Sensor 2 -> $P_d = p_l(x - d2)$.
- Sensor 3 -> $P_d = p_l(x - d3)$.
- Sensor 4 -> $P_d = p_l(x - d4)$.

Thus, the probability of detection of all four sensors is

$$P\{detection|x\} = 1 - \prod_{i=1}^{4}\left(1 - p_l\ (x - di)\right)$$

Calculating the probability of detection using the values previously mentioned, we get $P_d(x) = 0.69$. The model with four sensors and an ESR of 2 Nm, with the assumptions as stated above, gives $P_d(x) = 0.66$ as the output. The values appear to be comparable to the theoretical calculations and within the 95% confidence interval for a low number of replications. However, as the number of replications is increased, thereby reducing the width of the confidence interval, the value tends to fall outside the interval.

B. BASE CASE SIMULATIONS

The various simulation experiments conducted and the results obtained are described in the succeeding paragraphs with tabulated probabilities and relevant plots.

1. Probability of Detection Results with Varying ESRs

The probability of detection achieved in the formations by varying the ESR for a given number of Wave Gliders is elucidated below.

a. Probability of Detection Results with ESR of 1000 Yds

Table 1. Values of P(D) – ESR = 1000 Yds, $\lambda = 1$, SE ~ N (Mean SE, $\sigma^2 = 6$).

No. of Wave Gliders	**Barrier**	**Circular**	**Sector**	**Multiple Barrier**
5	0.02(±.06)	0.0045(±.03)	0.01(±.04)	0.01(±.001)
6	0.03(±.04)	0.0057(±.01)	0.012(±.004)	0.02(±.04)
7	0.054(±.09)	0.016(±.05)	0.02(±.09)	0.03(±.003)
8	0.059(±.11)	0.019(±.03)	0.03(±.02)	0.05(±.001)
9	0.06 (±.06)	0.02(±.04)	0.04(±.01)	0.055 (±.006)
10	0.072(±.12)	0.03(±.04)	0.05(±.12)	0.06(±.002)
15	0.14(±.18)	0.02(±.14)	0.07(±.14)	0.10(±.001)
20	0.24(±.18)	0.05(±.10)	0.10(±.19)	0.19 (±.01)

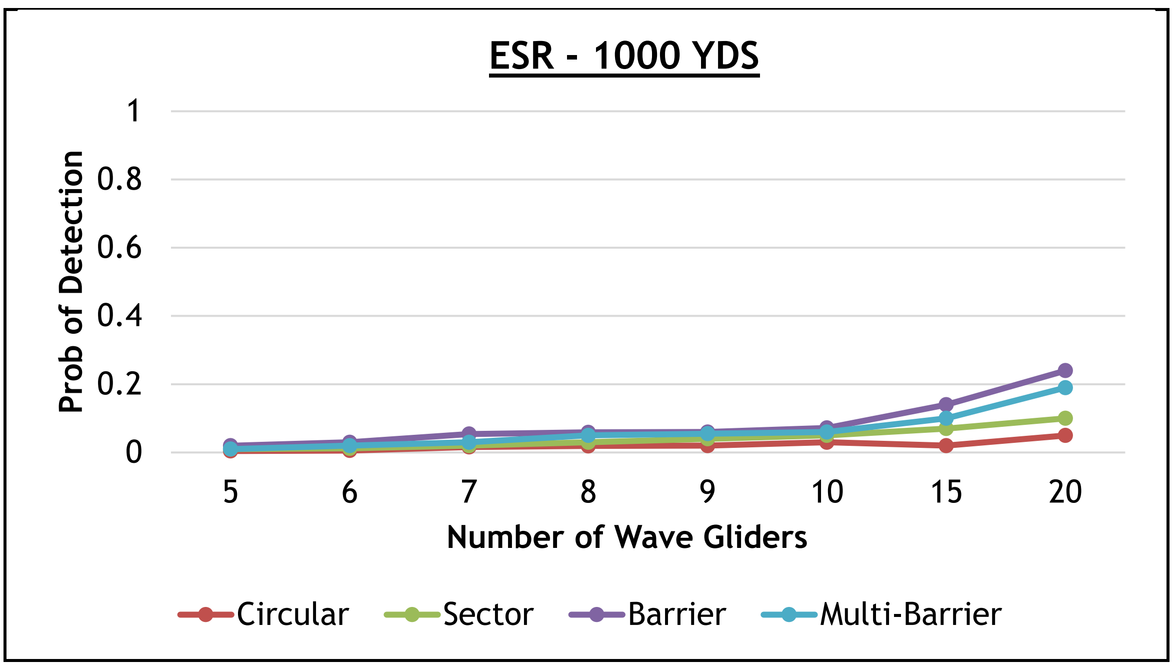

Figure 23. Plot of Probability of Detection – ESR 1000 Yds

b. Probability of Detection Results with ESR of 2000 Yds

Table 2. Values of P(D) – ESR = 2000 Yds, $\lambda = 1$, SE ~ N (Mean SE, $\sigma^2 = 6$).

No. of Wave Gliders	Barrier	Circular	Sector	Multiple Barrier
5	0.27(±.003)	0.02(±.07)	0.18(±.17)	0.18(±.001)
6	0.32(±.003)	0.04(±.003)	0.22(±.003)	0.28(±.001)
7	0.37(±.001)	0.06(±.005)	0.25(±.005)	0.33(±.001)
8	0.40(±.02)	0.07(±.12)	0.28(±.007)	0.38(±.001)
9	0.45(±.01)	0.08(±.001)	0.32(±.01)	0.42(±.002)
10	0.49(±.005)	0.09(±.17)	0.35(±.008)	0.46(±.001)
15	0.63(±.001)	0.13(±.26)	0.49(±.01)	0.62(±.002)
20	0.74(±.008)	0.35(±.01)	0.59(±.006)	0.73(±.001)

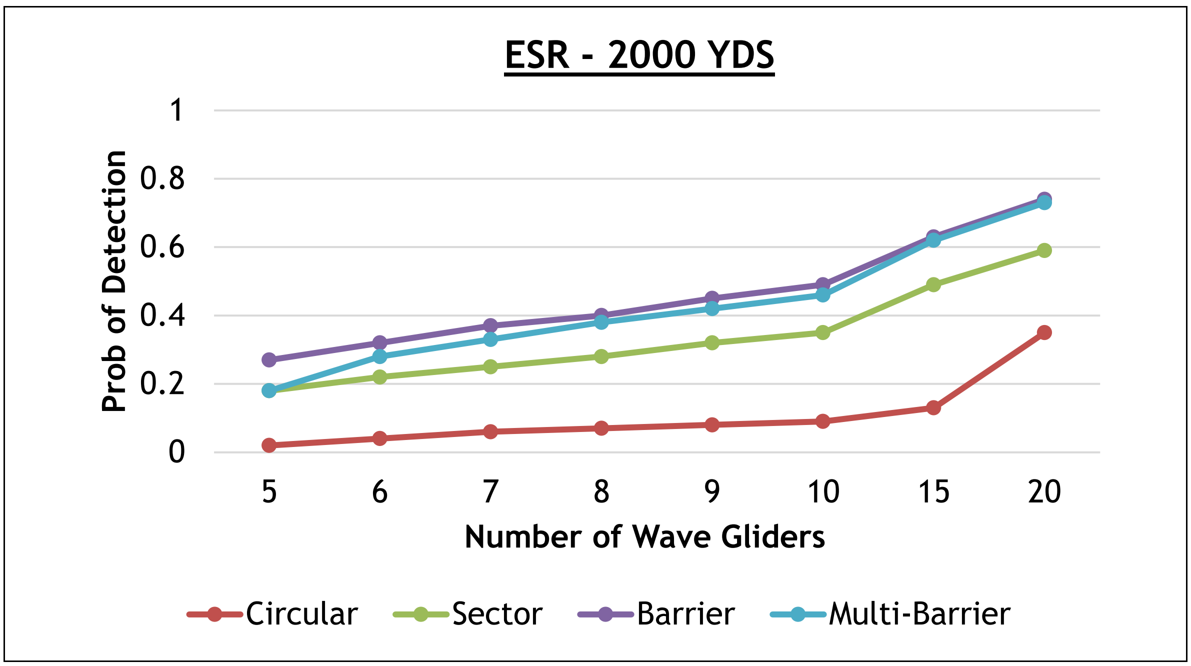

Figure 24. Plot of Probability of Detection – ESR 2000 Yds

The multiple barrier formation provides the commander with a tactically prudent option enabling another opportunity of tracking the undersea contact and confirming its presence. It can also be used to keep the submarine on the defensive as a layer of Wave Gliders would now be deployed in the area, thereby enabling detection by forcing the submarine commander to pursue actions detrimental for transiting undetected through the area. From the preceding results, it is evident that the barrier formation always provides a higher probability of detection than the multiple barrier formation, which employs the same methodology of deploying the given Wave Gliders, albeit by dividing them into two barriers. Therefore, the multiple barrier formation was not been explored in further experiments, though the option exists.

c. *Probability of Detection Results with ESR of 3000 Yds*

Table 3. Values of P(D) – ESR = 3000 Yds, $\lambda = 1$, SE ~ N (Mean SE, $\sigma^2 = 6$).

No. of Wave Gliders	Barrier	Circular	Sector
5	0.53(±.006)	0.18(±.001)	0.38(±.004)
6	0.60(±.002)	0.23(±.01)	0.46(±.002)
7	0.65(±.002)	0.26(±.01)	0.52(±.003)
8	0.69(±.009)	0.29(±.004)	0.57(±.02)
9	0.72(±.005)	0.34(±.001)	0.62(±.01)
10	0.77(±.001)	0.37(±.003)	0.65(±.002)
15	0.89(±.001)	0.55(±.01)	0.79(±.001)
20	0.94(±.001)	0.66(±.009)	0.87(±.001)

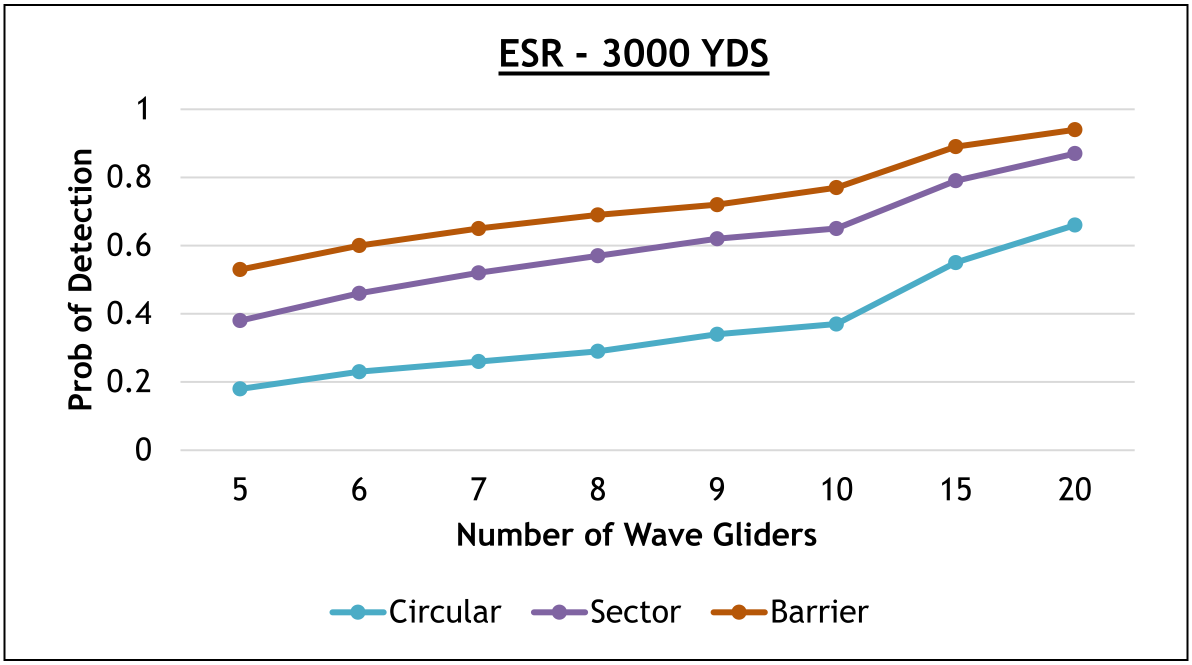

Figure 25. Plot of Probability of Detection – ESR 3000 Yds

d. Probability of Detection Results with ESR of 4,000 Yds

Table 4. Values of P(D) – ESR = 4000 Yds, $\lambda = 1$, SE ~ N (Mean SE, $\sigma^2 = 6$).

No. of Wave Gliders	**Barrier**	**Circular**	**Sector**
5	0.73(±.001)	0.29(±.008)	0.62(±.003)
6	0.78(±.002)	0.38(±.001)	0.70(±.001)
7	0.83(±.001)	0.42(±.002)	0.74(±.001)
8	0.86(±.001)	0.49(±.004)	0.78(±.006)
9	0.89(±.001)	0.56(±.004)	0.81(±.003)
10	0.92(±.001)	0.61(±.01)	0.85(±.001)
15	0.97	0.77(±.006)	0.94(±.001)
20	0.99(±.002)	0.86(±.002)	0.97(±.001)

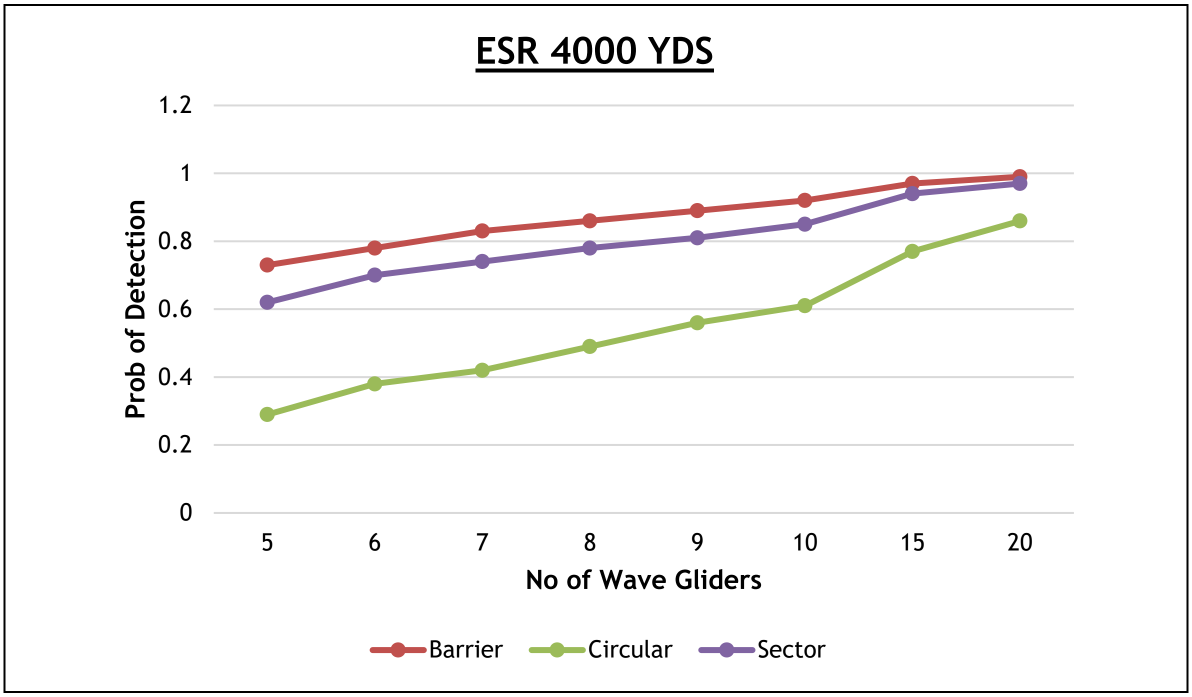

Figure 26. Plot of Probability of Detection – ESR 4,000 Yds

e. Probability of Detection Results with ESR of 5,000 Yds

Table 5. Values of P(D) – ESR = 5000 Yds, $\lambda = 1$, SE ~ N (Mean SE, $\sigma^2 = 6$).

No. of Wave Gliders	Barrier	Circular	Sector
5	0.85(±.001)	0.46(±.004)	0.80(±.001)
6	0.90(±.001)	0.55(±.001)	0.84(±.006)
7	0.93(±.001)	0.62(±.005)	0.88
8	0.95(±.002)	0.71(±.01)	0.91(±.002)
9	0.96(±.001)	0.77(±.002)	0.93(±.001)
10	0.97(±.001)	0.80(±.003)	0.95(±.001)
15	0.995	0.91(±.001)	0.98(±.001)
20	0.999(±.001)	0.96(±.001)	0.997(±.001)

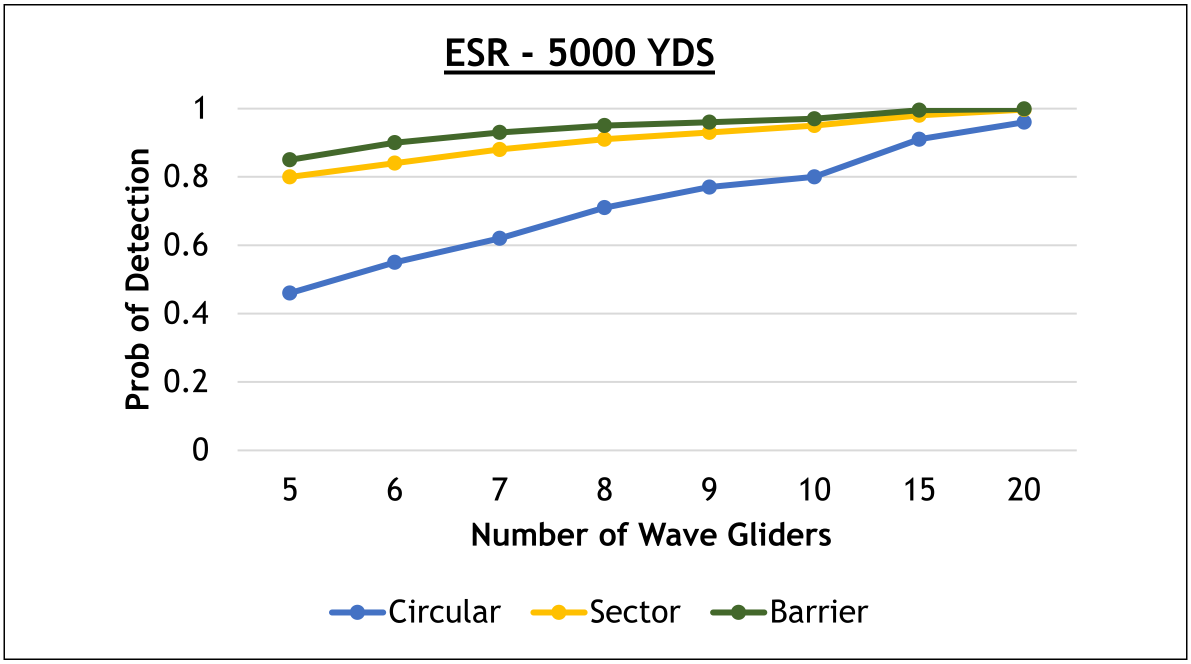

Figure 27. Plot of Probability of Detection – ESR 5,000 Yds

2. Output of Simulations – Placement of Wave Gliders

The output of the simulations is illustrated in the diagrams in Figures 28–31. The figures depict the arrangement of Wave Gliders for the different formations in the given area and the associated probability of detection vis-à-vis the number of Wave Gliders.

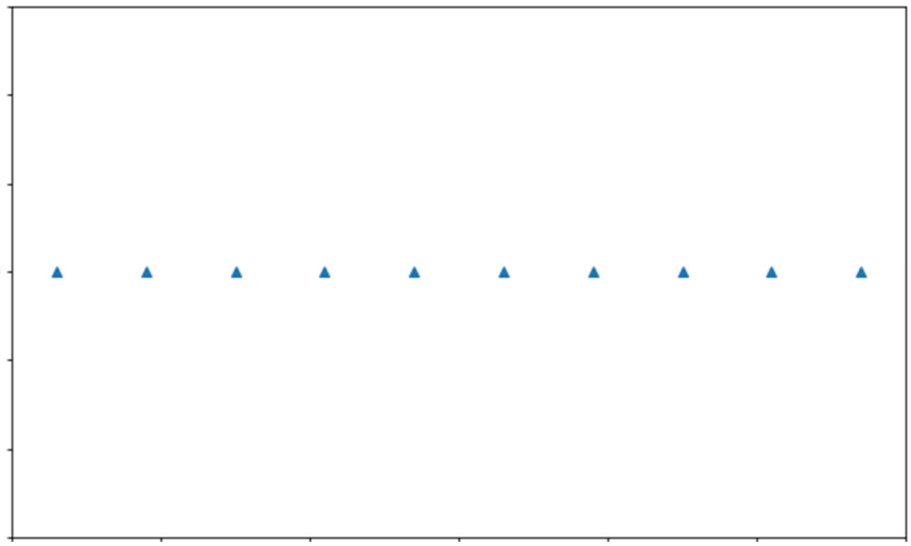

Figure 28. Output of Placement of 10 Wave Gliders in Barrier Formation. ESR = 2000 Yds, $P_d = 0.49$

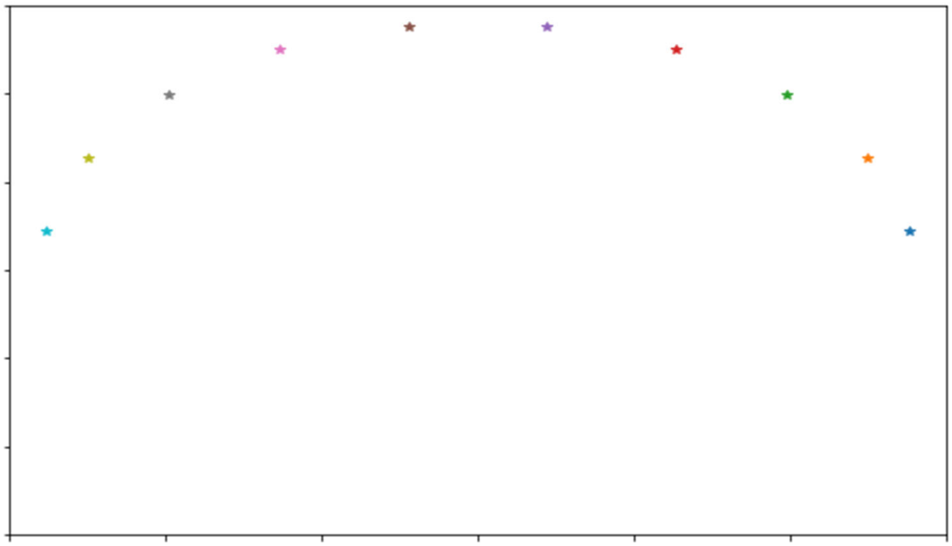

Figure 29. Output of Placement of 10 Wave Gliders for Sector Formation. ESR = 2000 Yds, $P_d = 0.35$

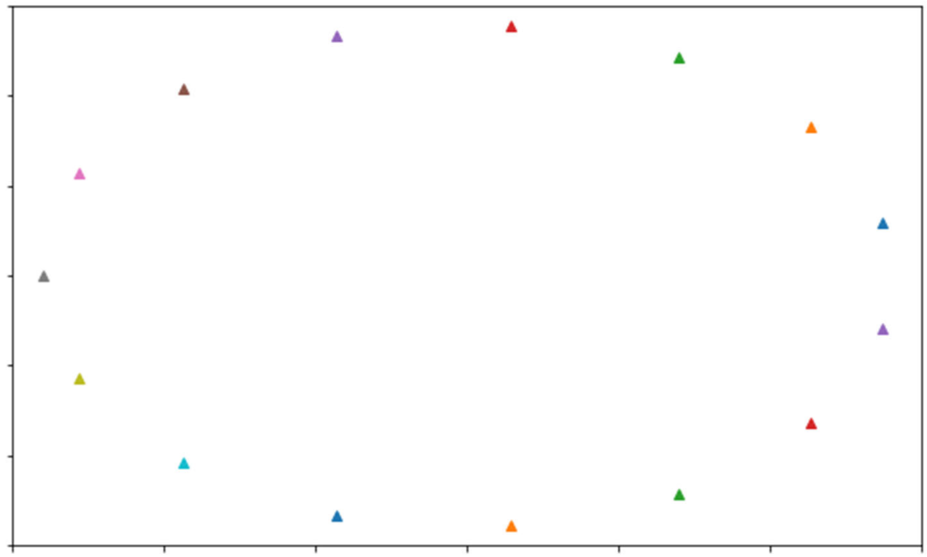

Figure 30. Output of Placement of 20 Wave Gliders in Circular Formation. ESR = 3000 Yds, $P_d = 0.66$

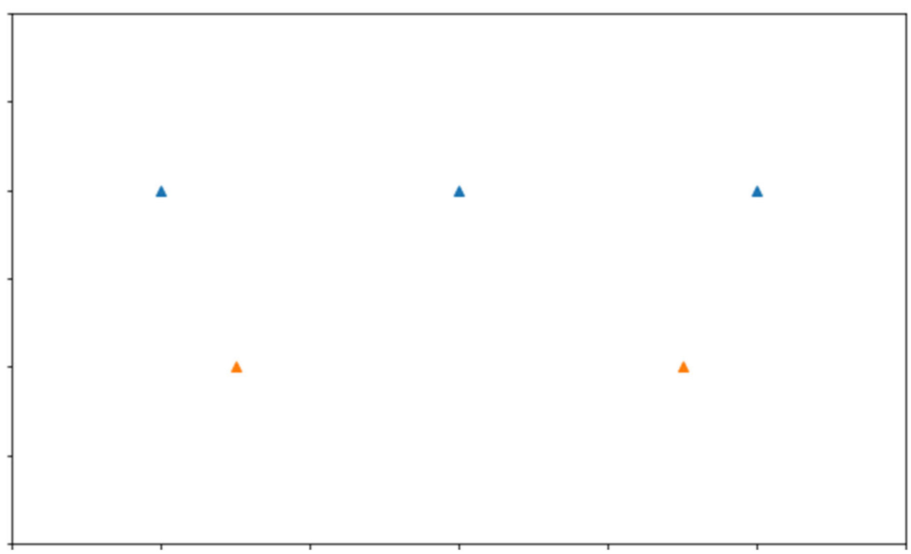

Figure 31. Output of Placement of 5 Wave Gliders in Multiple Barrier Formation. ESR = 2000 Yds, $P_d = 0.18$

C. SENSITIVITY ANALYSIS

To assess the impact of various parameters in determining the probability of detection, a few critical parameters were modified, and the results analyzed. The results are described in the following sections.

1. Figure of Merit

As described earlier, the FOM is a key factor in determining the detection rate in the Poisson Scan Model, thereby affecting the overall probability of detection. The FOM was varied in the simulations as presented in Table 6. It was observed that the detection rate remains same if we further increase the FOM above 90 dB; however, if we decrease the FOM below 60 dB, changes in detection rate begin to show.

Table 6. Values of Detection Rate and P(D) wrt FOM.

ESR = 2000 Yds, $\lambda = 1$, SE ~ N (Mean SE, $\sigma^2 = 6$)		
FOM	**Detection Rate**	**Probability of Detection**
120	1	0.49(±.005)
100	1	0.49(±.005)
90	0.99	0.49(±.005)
80	0.99	0.49(±.005)
70	0.99	0.49(±.005)
60	0.97	0.48(±.004)
55	0.88	0.44(±.003)
50	0.63	0.35(±.001)
45	0.31	0.19(±.001)
40	0.09	0.06(±.001)

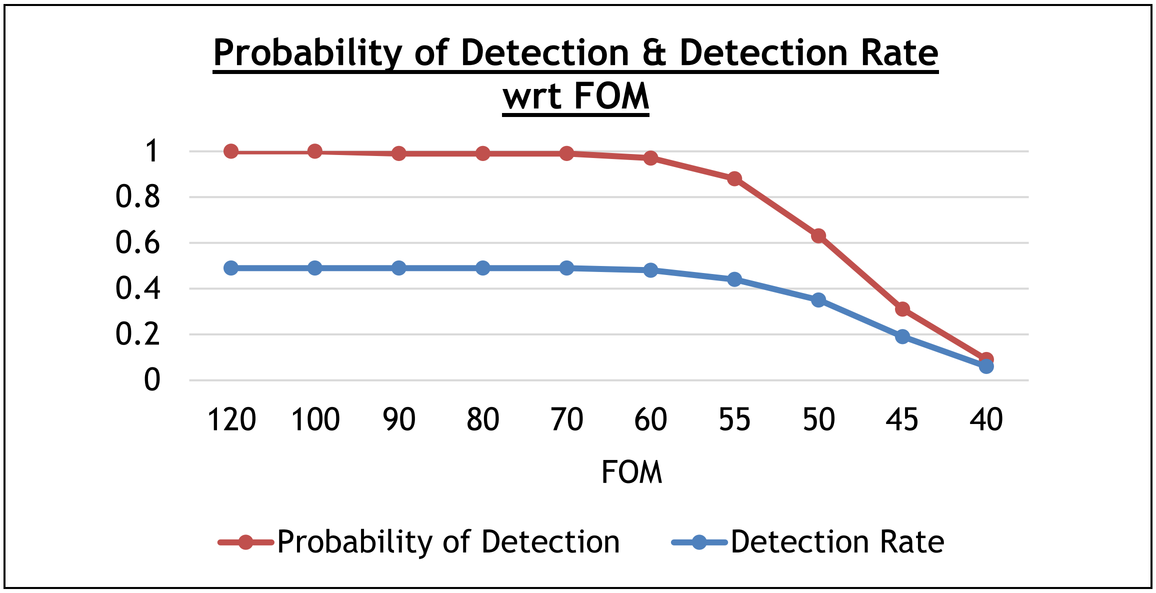

Figure 32. Change of Detection Rate and Probability of Detection with FOM of 10 Wave Gliders – Barrier Formation

As the FOM decreases, the probability of detection should also decrease. From the preceding graphs, it is evident that the probablity of detection decreases as the FOM decreases, as expected. The effect is more pronounced in the circular formation, whereas the barrier formation remains least affected.

Table 7. Probability of Detection Results with FOM of 50.

ESR = 2000 Yds, λ = 1, SE ~ N (Mean SE, σ^2 = 6)			
No. of Wave Gliders	**Barrier**	**Circular**	**Sector**
5	0.19(±.002)	0.0001(±.001)	0.018(±.17)
6	0.22(±.002)	0.0002(±.001)	0.021(±.003)
7	0.26(±.001)	0.0004(±.001)	0.0235(±.005)
8	0.29(±.02)	0.0005(±.001)	0.0238(±.007)
9	0.32(±.01)	0.0007(±.001)	0.025(±.01)
10	0.35(±.001)	0.0009(±.001)	0.026(±.008)
15	0.43(±.002)	0.0014(±.001)	0.03(±.01)
20	0.52(±.007)	0.0016(±.007)	0.04(±.006)

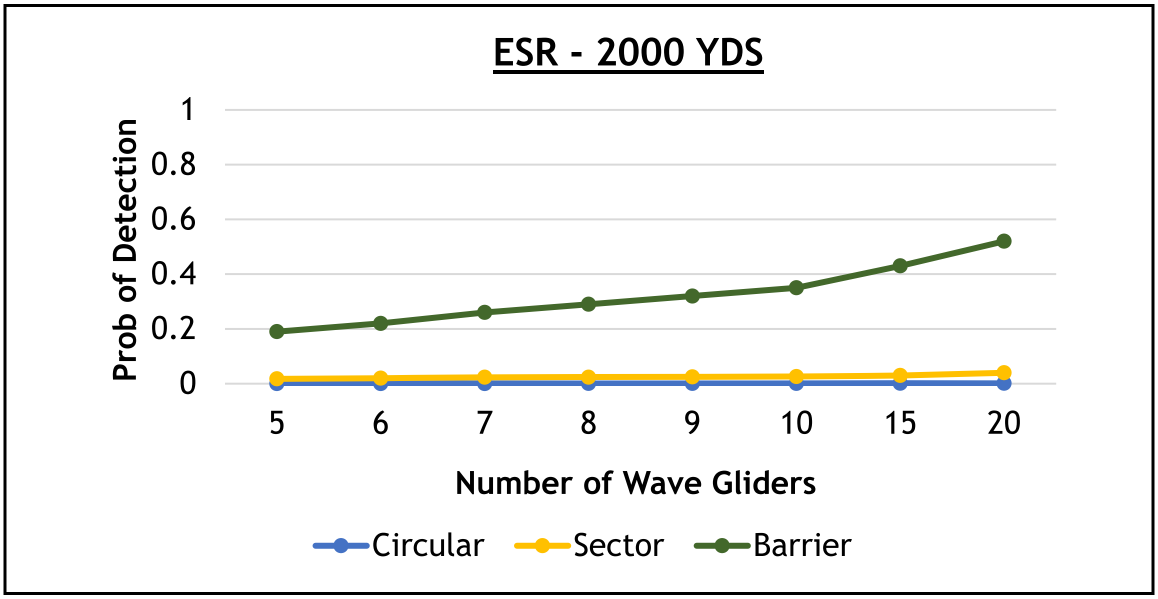

Figure 33. Change of Detection Rate and Probability of Detection with FOM

2. Poisson Process

The rate at which detection occurs was increased from λ = 1 in the base model to verify its effect on the probability of detection.

a. Probability of Detection results with λ = 2

Table 8. Values of P(D) – ESR = 2000 Yds, λ = 2, SE ~ N (Mean SE, $\sigma^2 = 6$).

No. of Wave Gliders	Barrier	Circular	Sector
5	0.42(±.004)	0.14(±.001)	0.31(±.001)
6	0.51(±.006)	0.17(±.004)	0.35(±.005)
7	0.58(±.001)	0.20(±.01)	0.39(±.008)
8	0.63(±.04)	0.23(±.02)	0.45(±.01)
9	0.69(±.02)	0.26(±.001)	0.51(±.01)
10	0.73(±.01)	0.30(±.002)	0.56(±.02)
15	0.86(±.004)	0.43(±.01)	0.73(±.02)
20	0.92(±.008)	0.55(±.03)	0.82(±.01)

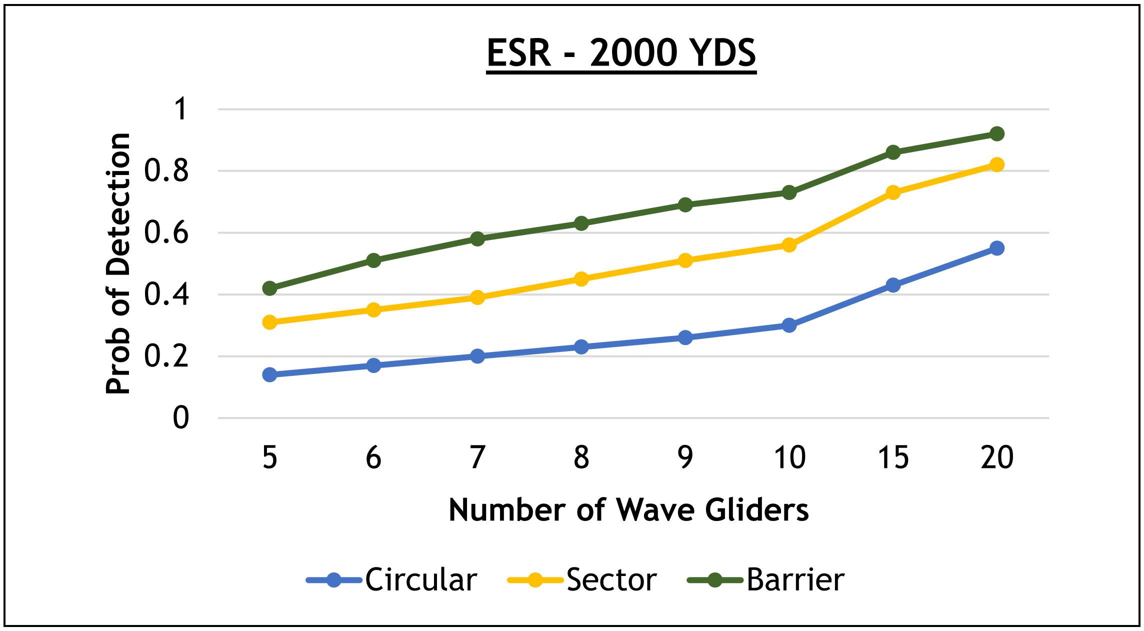

Figure 34. Probability of Detection with λ = 2

b. *Probability of Detection Results with λ = 3*

Table 9. Values of P(D) – ESR = 2000 Yds, λ = 3, SE ~ N (Mean SE, $\sigma^2 = 6$)

ESR = 2000 Yds, λ = 3, SE ~ N (Mean SE, σ^2 = 6)			
No. of Wave Gliders	**Barrier**	**Circular**	**Sector**
5	0.51(±.006)	0.18(±.001)	0.36(±.006)
6	0.62(±.008)	0.21(±.004)	0.43(±.005)
7	0.70(±.005)	0.24(±.008)	0.48(±.01)
8	0.76(±.06)	0.28(±.02)	0.54(±.01)
9	0.82(±.03)	0.31(±.009)	0.61(±.02)
10	0.85 (±.01)	0.34(±.02)	0.67(±.03)
15	0.94(±.005)	0.48(±.02)	0.85(±.03)
20	0.97(±.004)	0.67(±.05)	0.92(±.01)

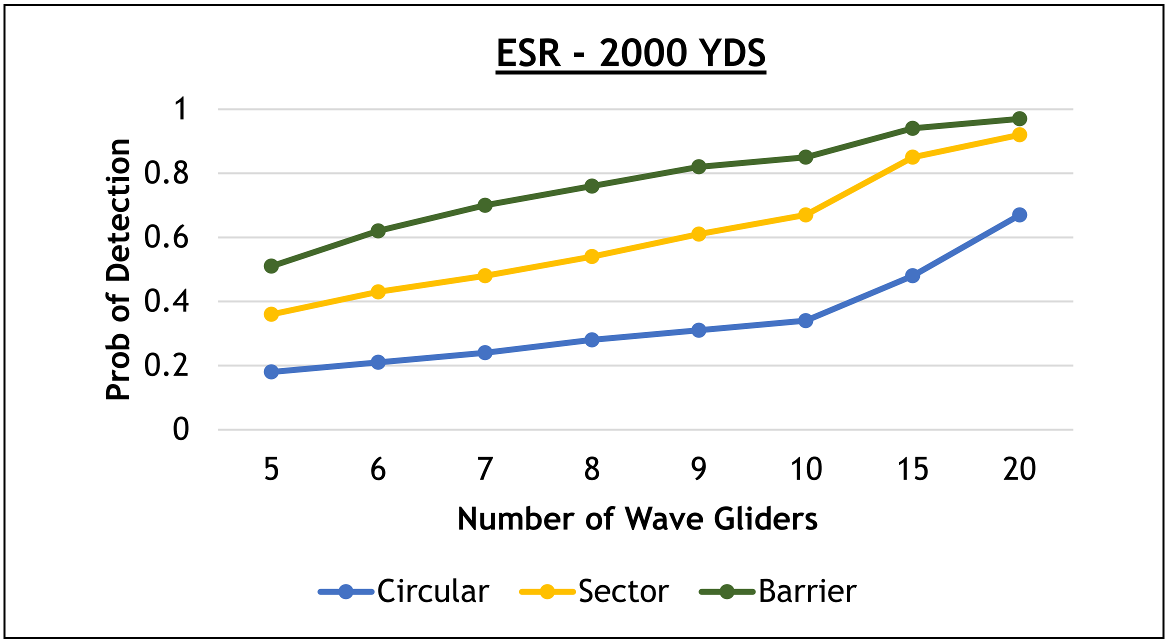

Figure 35. Probability of Detection with λ = 3

It is evident that as the rate at which the detections occur increases, the probability of detection also increases.

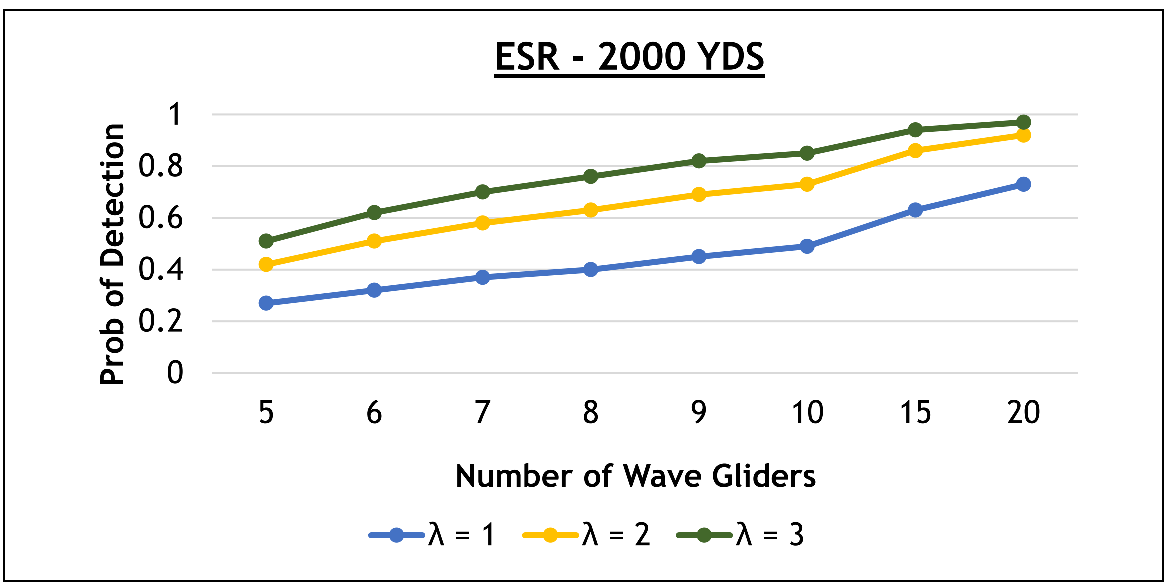

Figure 36. Change of Probability of Detection with λ – Barrier Formation

3. Target Speed

The speed of the target was doubled and halved to verify its effect on the detection capability of the Wave Gliders.

Table 10. Values of P(D) – ESR = 2000 Yds, $\lambda = 1$, SE ~ N (Mean SE, $\sigma^2 = 6$), Submarine Speed = 12 Kn.

No. of Wave Gliders	Barrier	Circular	Sector
5	0.15(±.001)	0.01(±.01)	0.10(±.01)
6	0.18 (±.001)	0.02(±.05)	0.12(±.003)
7	0.21(±.001)	0.03(±.07)	0.14(±.003)
8	0.23(±.01)	0.08(±.007)	0.16(±.003)
9	0.26(±.008)	0.09(±.002)	0.18(±.007)
10	0.28	0.10(±.002)	0.20(±.002)
15	0.39(±.002)	0.15(±.005)	0.29(±.003)
20	0.48(±.006)	0.20(±.006)	0.36(±.001)

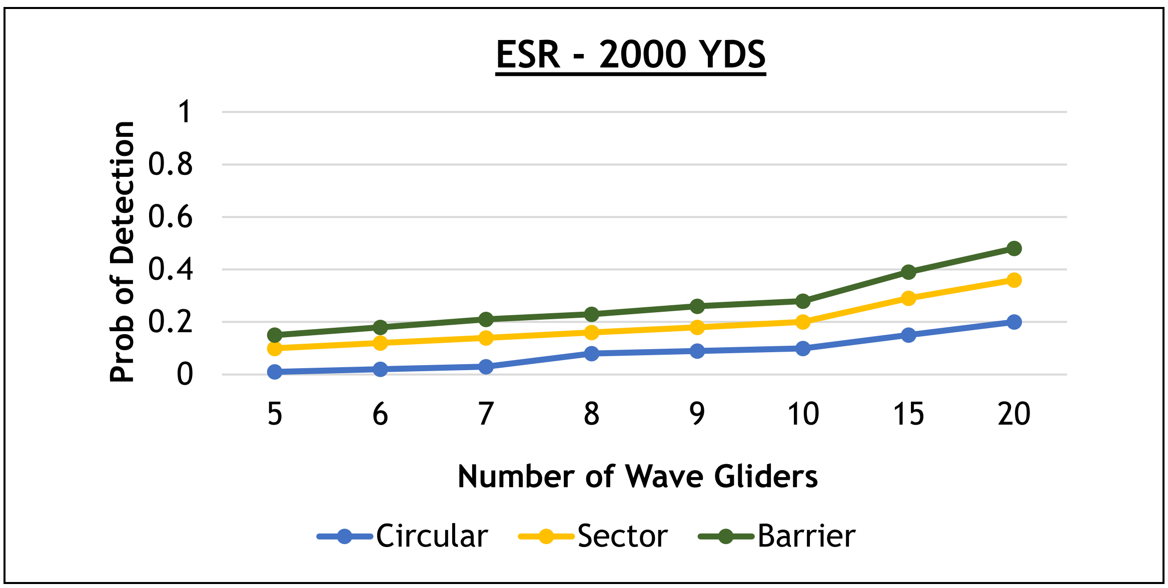

Figure 37. Probability of Detection – Increase in Target Speed

Table 11. Values of P(D) – ESR = 2000 Yds, $\lambda = 1$, SE ~ N (Mean SE, $\sigma^2 = 6$), Submarine Speed = 3 Kn.

No. of Wave Gliders	**Barrier**	**Circular**	**Sector**
5	0.42(±.002)	0.12(±.01)	0.31(±.001)
6	0.51(±.006)	0.16(±.10)	0.35(±.005)
7	0.58(±.002)	0.20(±.19)	0.39(±.008)
8	0.63(±.04)	0.23(±.02)	0.45(±.01)
9	0.66(±.02)	0.25(±.007)	0.51(±.01)
10	0.73(±.01)	0.28(±.002)	0.56(±.02)
15	0.86(±.004)	0.43(±.01)	0.73(±.02)
20	0.92(±.008)	0.55(±.03)	0.82(±.01)

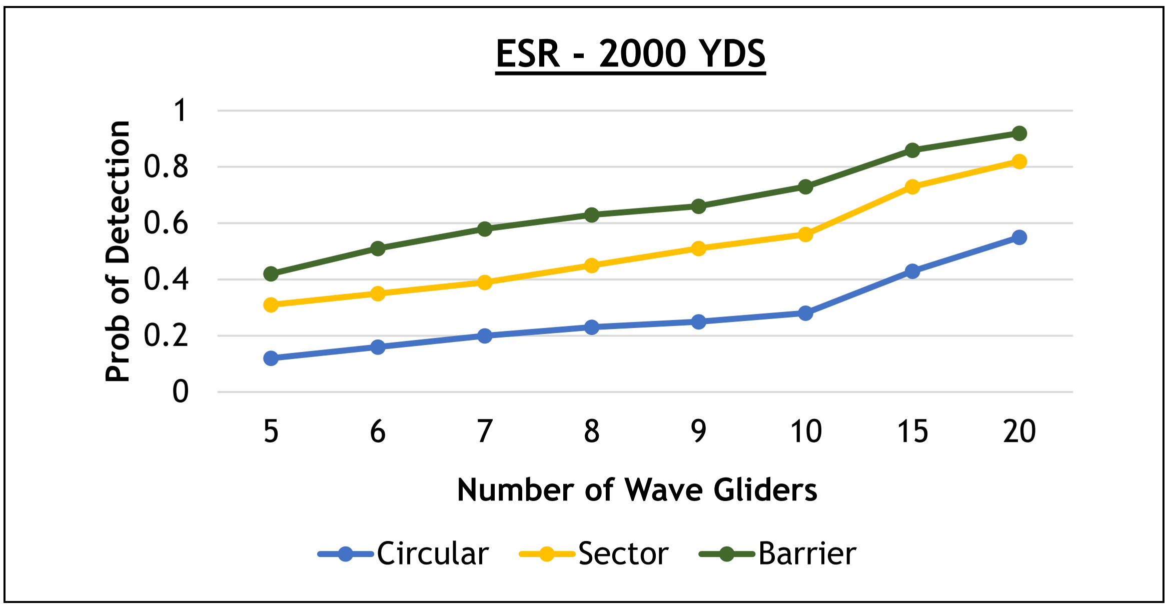

Figure 38. Probability of Detection – Decrease in Target Speed

It is evident that the higher the target speed, the lower the probability of detection will be, based purely on the detection time it offers. However, a greater target speed will also entail a higher radiated noise level from the submarine, thereby increasing the signal-to-noise ratio and thereby the FOM leading to better performance of the sonar and, consequently, an increased probability of detection. However, as these parameters remain dynamic and not easily determinable, the FOM for the experiment was assumed to be constant.

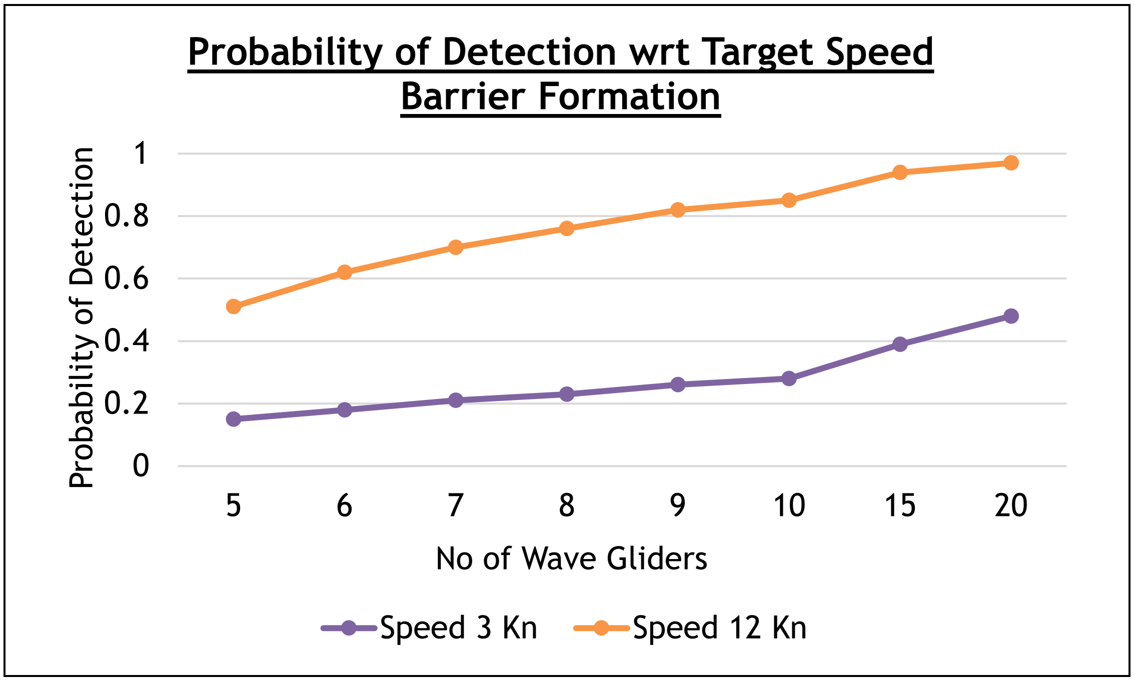

Figure 39. Change of Probability of Detection wrt Target Speed

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V. CONCLUSION

The advent of advanced technology for underwater platforms, weapons, and sensors has complicated the art of sub-surface warfare, particularly with the effective employment of unmanned systems. The problem becomes more difficult as acoustic conditions become less favorable. The availability of fewer resources further limits the detection capability against submarines. Unmanned platforms offer a potential way forward, but the question of how to effectively employ them remains open.

A. ANALYSIS

The critical analysis of the results obtained from simulations in this research leads to the following conclusions:

(a) As expected, the detection probability increases as the number of Wave Gliders increases. The detection probability also increases as the Estimated Sonar Range or Sweep Width increases. Nevertheless, as Wave Gliders need to be controlled from the shore station, the maximum number viable for deployment in the AOO must be accounted for during planning in order to avoid cluttering the underwater domain awareness.

(b) A network consisting of a given number of Wave Gliders placed in the barrier formation provides the highest probability of detection when the direction of approach of the target is known.

(c) When the AOO has restricted sea room, the sector formation would be a better fit as a higher number of Wave Gliders can be placed in the area.

(d) If the direction of approach of the target is not known, the circular formation may provide the highest probability of detection, depending upon the ESR and the number of Wave Gliders available.

(e) The barrier formation always provides a higher probability of detection than the multiple barrier formation. However, the multiple barrier formation could be used as a tactical option to keep the submarine on the defensive for an extended duration.

(f) The probablity of detection decreases as the FOM of the sonar decreases. The effect is more pronounced in the circular formation, whereas the barrier formation remains least affected.

(g) As the detection rate increases, thereby enabling more opportunities for detection of the submarine by the Wave Gliders, the probability of detection also increases.

(h) All other factors being constant, the probability of detection decreases as the target speed increases.

The model thus presents several scenarios that can act as a reference for the commander at sea to optimize the placement of Wave Gliders for Anti-Submarine Warfare.

B. FUTURE WORK

The thesis explores a few scenarios based on the assumptions stated at the outset. In future work, other factors could be incorporated, such as the non-availability of a particular Wave Glider that would create gaps in the network, the lack of available satellite communication on the Wave Gliders to relay target information, randomized target movement in the area, and the presence of more than one target, etc. In addition, the model can be extended to apply to other unmanned systems capable of undertaking similar missions by varying the critical parameters associated with their deployment.

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