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Unsteady Boundary Layer Flow over a Sphere in a Porous Medium

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Abstract. This study focuses on the problem of unsteady boundary layer flow over a sphere in a porous medium. The governing equations which consists of a system of dimensional partial differential equations is applied with dimensionless parameter in order to attain non-dimensional partial differential equations. Later, the similarity transformation is performed in order to attain nonsimilar governing equations. Afterwards, the nonsimilar governing equations are solved numerically by using the Keller-Box method in Octave programme. The effect of porosity parameter is examined on separation time, velocity profile and skin friction of the unsteady flow. The results attained are presented in the form of table and graph.

INTRODUCTION

Porous medium can be defined as a substance of solid matrix with an interconnected void which allow the fluid flow through. There are two types of porous medium in general, which are natural and manmade. Natural porous medium consists of beach sand, wood, and human lung. Meanwhile, manmade porous medium are ceramics and composite materials [1]. Natural porous medium has the irregular shape and size of the distribution of pores. Porous medium received widely attention among the researchers due to the many useful applications, for instance to conserve the temperature of a heated body [2], ore leaching, and fluidized bed combustion [3]. Early work of the theory porous medium was founded by Reinhard Woltman in 1794 [4] and later expanded by numerous researchers [5]-[8].

The fluid flows via porous medium commonly apply a general form of integral relationship which is known as Kozeny or Darcy's law [5]. The pressure gradient in this law acts as the function of flow rate and as the geometrical properties matrix. The fluid flow model over the various geometrical surfaces had been investigated by several researchers. Ahmad and Pop [6] studied the boundary layer flow of a porous medium that filled with nanofluid from

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a vertical flat plate. Later, Kasim et al. [7] solved the problem of flow in porous medium filled with viscoelastic fluid over a circular cylinder.

There are large volumes of published studies describing steady flow past sphere in porous medium. Yamamoto [2] discussed the effect of natural convection in a porous medium. Next, Haber and Mauri [9] studied the boundary condition for Darcy's flow through porous media using a sample problem of flow fields exterior to a porous spherical particle. Barman [10] analyzed the flow of a porous medium filled with Newtonian fluid. Another study about mixed convection flow with Newtonian heating was done by Salleh et al. [11]. In addition, Kimura and Pop [12] published a paper on conjugate convection in a porous medium. The MHD flow of porous medium embedded with viscoelastic fluid with Newtonian heating was studied by Kasim et al. [13].

The research about porous medium filled with a nanofluid past a sphere was carried out by Chamkha et al. [14]. Tham and Nazar [15] studied the flow of a porous medium filled with a nanofluid over a sphere using the Brinkman model which is the extension of the Darcy's law. The flow over a sphere buried in a porous media was analyzed by Delgado and Vázquez da Silva [3]. Recently, a study by Saryazli et al. [8] shows numerical solutions of forced convective flow in nanofluid flowing inside a straight circular pipe filled with a saturated porous medium. They found that increasing the porosity increase the permeability which lead to the inertial and viscous effect. Those studies mentioned had been done for flat plate, cylinder, and sphere but for steady flow condition.

While for unsteady flow, changes that occur in time leads to the velocity and skin friction that differ over time. Sano [16] mentioned the significant relationship of the unsteady convection around a sphere embedded in a porous medium. Aurangzaib et al. [17] examined the unsteady MHD mixed convection flow in a porous medium saturated with micropolar fluid over a vertical surface. Another study by Aurangzaib et al. [18] was focused on the relationship of the MHD flow with heat and mass transfer of a micropolar fluid embedded in porous medium over a vertical plate at the unsteady state. Furthermore, the steady and unsteady boundary layers in a porous medium was solved by Ishak and Nazar [19] using the Darcy-Brinkman equation model. However, the works mentioned above applied several similarity variables or transformation that somehow reduces the number of independent variables. This produces unclearness of how the solutions obtained changes over time. Therefore, this study aims to analyze how the separation times, velocity and skin friction changes over time in the problem of unsteady boundary layer flow over a sphere in fluid-saturated porous medium.

MATHEMATICAL FORMULATION

Consider a laminar flow starts impulsively at rest in a porous medium saturated with incompressible Newtonian fluid over a solid sphere of radius a with ambient velocity of the fluid $(1/2)U_\infty$ far from the sphere. It is assumed that the fluid flows vertically upward. Under the Boussinesq and boundary layer approximation, the governing equations are

$$\frac{\partial(\bar{r}\bar{u})}{\partial\bar{x}}+\frac{\partial(\bar{r}\bar{v})}{\partial\bar{y}}=0 \tag{1}$$

$$\frac{\partial\bar{u}}{\partial\bar{t}}+\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}=\bar{u}_e\frac{\partial\bar{u}_e}{\partial\bar{x}}+\upsilon\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+\frac{\upsilon}{k}\left(\bar{u}_e-\bar{u}\right) \tag{2}$$

subject to

$$\begin{aligned}
\bar{t}<0: &\quad \bar{u}=\bar{v}=0 \text{ for any } \bar{x},\bar{y}\\
\bar{t}\geq 0: &\quad \bar{u}=\bar{v}=0 \text{ at } \bar{y}=0\\
&\quad \bar{u}=\bar{u}_e(\bar{x}) \text{ as } \bar{y}\to\infty
\end{aligned} \tag{3}$$

where $\bar{u}$ is velocity at $\bar{x}$-direction, $\bar{v}$ is velocity at $\bar{y}$-direction, $\bar{t}$ is time, υ is kinematic viscosity and k is permeability. Meanwhile, $\bar{r}(\bar{x}) = a\sin(\bar{x}/a)$ is radial distance from symmetrical axis to surface of the sphere and the free stream velocity $\overline{u_e}(\bar{x}) = (3/2)U_\infty \sin(\bar{x}/a)$.

Afterwards, dimensionless variables are introduced as follow:

$$x = \frac{\bar{x}}{a},\; y = \mathrm{Re}^{1/2}\frac{\bar{y}}{a},\; t = \frac{U_\infty \bar{t}}{a},\; u = \frac{\bar{u}}{U_\infty},\; v = \mathrm{Re}^{1/2}\frac{\bar{v}}{U_\infty},\; r = \frac{\bar{r}(\bar{x})}{a} \tag{4}$$

where Re is the Reynolds number. Thus, by embedding Equation (4) into Equations (1)-(2), the non-dimensional governing equations are

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \tag{5}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \upsilon\frac{\partial^2 u}{\partial y^2} + \frac{\upsilon}{k}(u_e - u) \tag{6}$$

subject to

$$\begin{aligned} t<0: &\quad u = v = 0 \text{ for any } x, y \\ t \geq 0: &\quad u = v = 0 \text{ at } y = 0 \\ &\quad u = u_e(x) \text{ as } y \to \infty \end{aligned} \tag{7}$$

Furthermore, the relations of stream function ψ as below

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y}, v = -\frac{1}{r}\frac{\partial \psi}{\partial x}, \tag{8}$$

are substituted into Equations (5)-(6) and results in the following governing equation

$$\frac{1}{r}\frac{\partial^2\psi}{\partial t\partial y} + \frac{1}{r^2}\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{1}{r^3}\frac{dr}{dx}\left(\frac{\partial\psi}{\partial y}\right)^2 - \frac{1}{r^2}\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = u_e\frac{du_e}{dx} + \frac{1}{r}\frac{\partial^3\psi}{\partial y^3} + P\left(u_e - \frac{1}{r}\frac{\partial\psi}{\partial y}\right) \tag{9}$$

subject to

$$\begin{aligned} t<0: &\quad \psi = \frac{\partial\psi}{\partial y} = 0 \text{ for any } x, y \\ t \geq 0: &\quad \psi = \frac{\partial\psi}{\partial y} = 0 \text{ at } y = 0 \\ &\quad \frac{\partial\psi}{\partial y} = u_e(x) \text{ as } y \to \infty \end{aligned} \tag{10}$$

where P is porosity parameter. A boundary layer which thickness $O(\upsilon t)^{1/2}$ is developed right after the flow begin. This suggests to attain a solution for small time case $(t \leq 1)$. Thence, the governing equations are transformed by using the following similarity variables for small time case

$$\psi = t^{1/2} u_e(x) r(x) f(x, \eta, t), \text{ and } \eta = y / t^{1/2}. \tag{11}$$

Therefore, the momentum equation for small time case is

$$\begin{aligned} & t \frac{\partial^2 f}{\partial t \partial \eta} + t u_e \left(\frac{\partial f}{\partial \eta} \frac{\partial^2 f}{\partial x \partial \eta} - \frac{\partial^2 f}{\partial \eta^2} \frac{\partial f}{\partial x} - \frac{1}{r} \frac{dr}{dx} f \frac{\partial^2 f}{\partial \eta^2} \right) \\ & = \frac{\partial^3 f}{\partial \eta^3} + \frac{\eta}{2} \frac{\partial^2 f}{\partial \eta^2} + t \frac{du_e}{dx} \left(1 + f \frac{\partial^2 f}{\partial \eta^2} - \left(\frac{\partial f}{\partial \eta} \right)^2 \right) + Pt \left(1 - \frac{\partial f}{\partial \eta} \right) \end{aligned} \tag{12}$$

subject to

$$\begin{aligned} t < 0: \quad & f = \frac{\partial f}{\partial \eta} = 0 \text{ for any } x, \eta \\ t \geq 0: \quad & f = \frac{\partial f}{\partial \eta} = 0 \text{ at } \eta = 0 \\ & \frac{\partial f}{\partial \eta} = 1 \text{ as } \eta \to \infty \end{aligned} \tag{13}$$

On the other hand, the similarity variable for large time $(t > 1)$ case is

$$\psi = u_e(x) r(x) F(x, y, t). \tag{14}$$

By applying Equation (14) on Equation (9), the momentum equation obtained for large time case is

$$\begin{aligned} & \frac{\partial^2 F}{\partial t \partial y} + u_e \left(\frac{\partial F}{\partial y} \frac{\partial^2 F}{\partial x \partial y} - \frac{\partial^2 F}{\partial y^2} \frac{\partial F}{\partial x} - \frac{1}{r} \frac{dr}{dx} F \frac{\partial^2 F}{\partial y^2} \right) \\ & = \frac{\partial^3 F}{\partial y^3} + \frac{du_e}{dx} \left(1 + F \frac{\partial^2 F}{\partial y^2} - \left(\frac{\partial F}{\partial y} \right)^2 \right) + P \left(1 - \frac{\partial F}{\partial y} \right) \end{aligned} \tag{15}$$

subject to boundary conditions

$$\begin{aligned} & F = \frac{\partial F}{\partial y} = 0 \text{ at } y = 0 \\ & \frac{\partial F}{\partial y} = 1 \text{ as } y \to \infty \end{aligned} \tag{16}$$

For this problem, the skin friction is

$$C_f \operatorname{Re}^{1/2} = \frac{u_e}{t^{1/2}} \left(\frac{\partial^2 f}{\partial \eta^2} \right)_{\eta=0} \tag{17}$$

for small time case and

$$C_f \operatorname{Re}^{1/2} = u_e \left(\frac{\partial^2 F}{\partial y^2} \right)_{y=0} \tag{18}$$

for large time case.

RESULTS AND DISCUSSION

Equations (12) and (13) together with Equations (15) and (16) are solved by using the Keller-Box method in Octave programme. The computations performed are based on step size $\Delta\eta = 0.1$ and up until $\eta_\infty = 150$. Moreover, the step size for position x is $\Delta x = \pi / 20$ and the time step is $\Delta t = 0.05$. Table 1 shows separation times at different porosity parameter P and position x. It can be seen that when the effect of porosity parameter P is considered, the separation of flow is detected starting from $x = 117°$ for small time case. Further, separation times are presented as P increases from 0 to 1.3. At each points presented in this table, the separation times are higher as the values of the porosity parameter increase. Also, increasing the values of porosity parameter lead to less points involved with separation of flow. The separation of flow can only be detected in large time when P =1.3. Based on the results discussed above, it is apparent that when the porosity is higher, the separation of flow is delayed in terms of time and distance. Moreover, at appropriate values of porosity parameter, separation of flow only exists at large time case.

TABLE 1: Separation times at various porosity parameter P and positions x

x	$P=0$	$P=0.1$	$P=0.2$	$P=0.3$	$P=0.5$	$P=1.0$	$P=1.3$
180°	0.4166	0.4375	0.4608	0.4868	0.5495	0.8298	1.2804
171°	0.4219	0.4433	0.4673	0.4941	0.5589	0.8529	1.3451
162°	0.4388	0.4621	0.4882	0.5176	0.5895	0.9314	-
153°	0.4697	0.4966	0.5269	0.5615	0.6478	1.0996	-
144°	0.5198	0.5532	0.5914	0.6357	0.7503	1.4824	-
135°	0.6001	0.6455	0.6988	0.7627	0.9393	-	-
126°	0.7349	0.8057	0.8934	1.0056	1.3636	-	-
117°	0.9952	1.1374	1.3428	-	-	-	-

Figure 1 portrays velocity profile f' at front stagnation point ($x = 0°$) with different values of P. There is no f' in negative value detected. This clarify there is no reversal of flow occurred at least up until $t = 1.5$. As the values of P increase, f' is also increases significantly. Moreover, the momentum boundary layer thickness for this case is $\eta = 2$. Furthermore, Figure 2 conveys the velocity profiles f' of rear stagnation point ($x = 180°$) at $t = 1.5$ for various $P = 0, 0.1, 0.5, 1.0, 1.5$. The momentum boundary layer thickness is much thicker than the one at front stagnation point ($x = 0°$) which is $\eta = 150$. It can also be seen that for $P = 0, 0.1, 0.5, 1.0$, there are f' in negative values near the surface of a sphere ($\eta = 0$) which indicate there exists reversal of flow around this region. While at $P = 1.5$, there is no negative values detected on f' and this means no reversal of flow detected for this value of P. Similar to Figure 1, Figure 2 also conveys that increasing the values of P increases f'. According to the velocity profiles revealed at front ($x = 0°$) and rear ($x = 180°$) stagnation points, it is clear that the higher porosity may boost the velocity of fluid flow over a sphere. At rear stagnation point ($x = 180°$), the effect of porosity can also control the separation of flow and when given appropriate permeability of porous medium, the separation of flow is no longer occured.

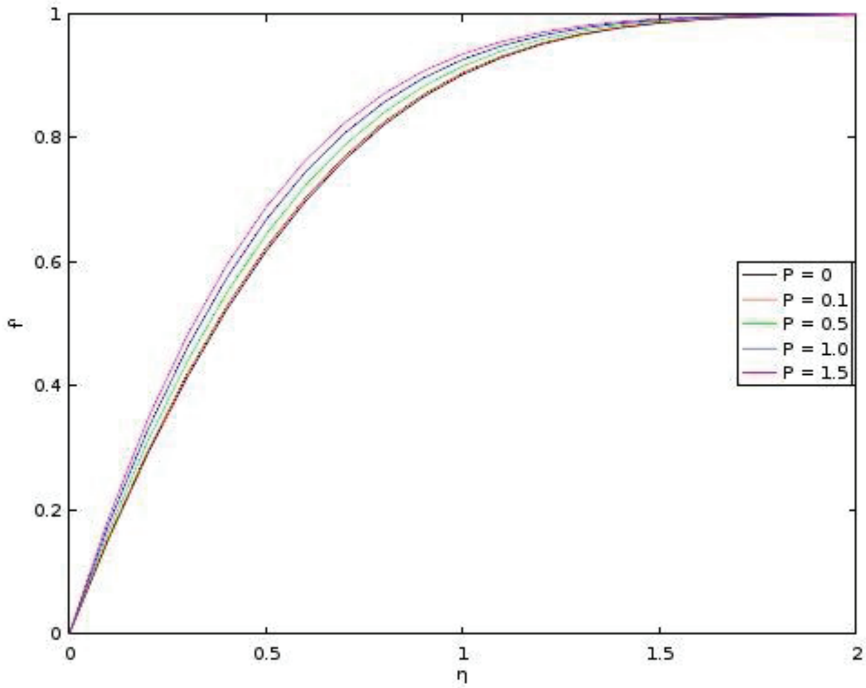

FIGURE 1. Velocity profiles at front stagnation point ($x = 0°$) with various values of porosity parameter P

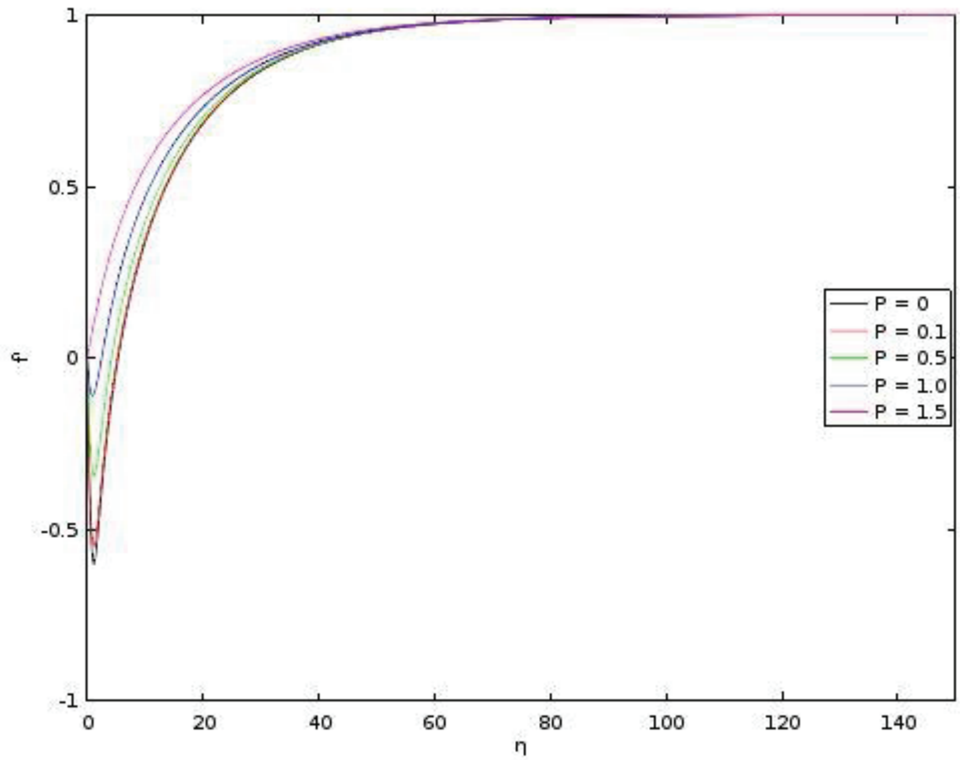

FIGURE 2. Velocity profiles at rear stagnation point $(x = 180°)$ with various values of porosity parameter P

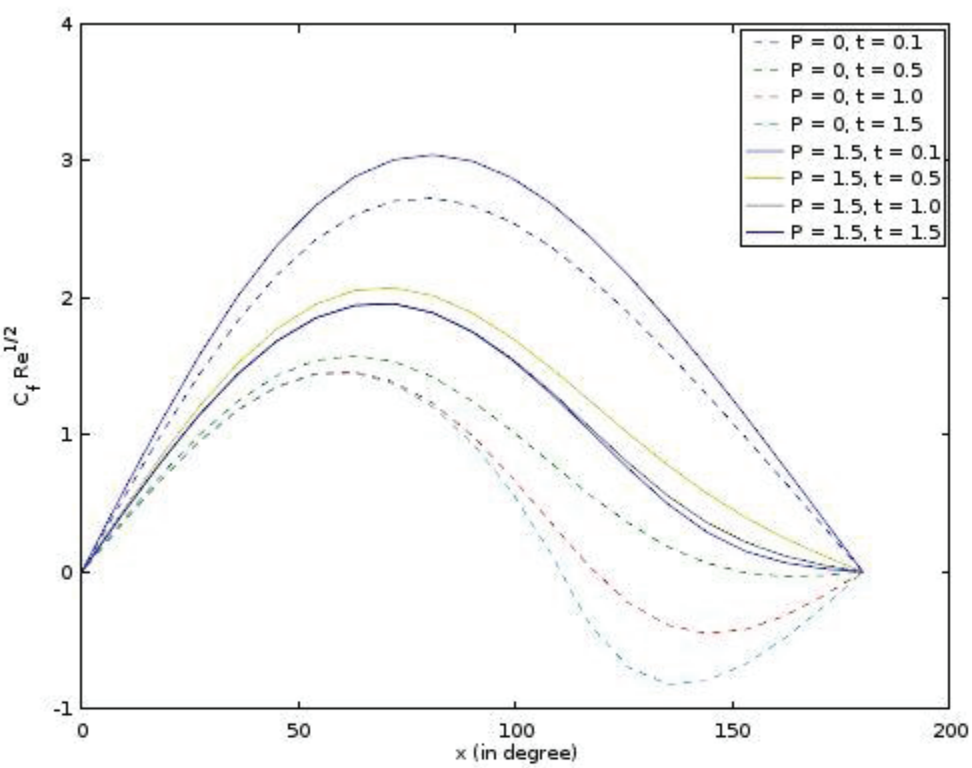

FIGURE 3. Skin friction coefficient $C_f Re^{1/2}$ when porosity parameter $P = 0$ and 1.5 at various positions x

Figure 3 depicts skin friction coefficients $C_f Re^{1/2}$ variate with position x around a sphere when there is no effect of porosity ($P = 0$) and the porosity parameter $P = 1.5$. It is found that as t increases, the values of $C_f Re^{1/2}$ are decreased. In addition, the position of maximum value of $C_f Re^{1/2}$ for $t \geq 0.5$ under the effect of porosity is about $x = 70°$ which is moved $10°$ from the position of without porosity effect ($P = 0$). It can be seen that at $P = 0$, there is $C_f Re^{1/2}$ in negative values. This means the reversal of flow occurs in this case. Further, when P is increased to 1.5, the values of $C_f Re^{1/2}$ are in the range of 0 to 3.5. This means there is no shear stress applied on opposite direction detected and thus no reversal of flow occurred up to time $t = 1.5$.

CONCLUSION

In this paper, the mathematical model as well as initial and boundary conditions are derived for unsteady boundary layer flow past a sphere in a porous medium. The model for small time case (12)-(13) and as well as model for large time case (15)-(16) are solved by the Keller-Box method in Octave programme. Afterwards, the numerical results consist of separation times, velocity profile and skin friction coefficients are presented and discussed. The results show that the effect of porosity is able to delay separation of flow, control reversal of flow due to shear stress, and increase velocity of a fluid-saturated porous medium at both front and rear stagnation points.

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