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Differences-in-Differences with multiple Treatments under Control

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Abstract

Numerous quasi-experimental identification strategies making use of the difference-in-differences setup suffer from multiple treatments which can be separated into sequential and simultaneous treatments. While for causal inferences under sequential treatments a staggered difference-in-differences approach might be applied, for causal inferences under simultaneous treatments the standard differences-in-differences approach is normally not applicable. Accordingly, we present an adjusted differences-in-differences identification strategy that can neutralize the effects of additional treatments implemented simultaneously through the definition and the specific composition of the control group and an amended common trend assumption. Even though the adjusted difference-in-differences strategy identifies the average treatment effect on the treated, we also show that the adjusted strategy is capable of identifying the average treatment effect under stronger common trend assumptions and the absence of interaction effects between the treatments.

Key words: Econometrics, Semiparametric and Nonparametric Methods, Treatment Effect Models

JEL classification: C01, C14; C21

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1 Introduction

In order to identify and quantify the causal impact of a certain treatment based on observational data, economists make frequent use of quasi-experimental identification strategies.[1] In particular, the difference-in-differences (DiD) setup is frequently applied to determine the average treatment effect of an intervention. However, in practical contexts these research designs are particularly prone to multiple treatments which can be segmented into sequential treatments and simultaneous treatments. Whilst for the identification of sequential treatment effects a staggered DiD setup might be applied, for the delineation of separate treatment effects under simultaneous treatments the standard DiD is normally not applicable. This paper sets out the detailed necessary and sufficient conditions in order to neutralize and eliminate the effects of additional treatments implemented simultaneously as part of an adjusted DiD strategy.

As highlighted previously, the effects of *sequential treatments* can be identified based on a staggered DiD setup (e.g. Wooldridge, 2021). In essence, the staggered DiD estimates "are variance-weighted averages of many different 2x2 DiDs, each involving the comparison between a treated and an effective control group in a window before and after the treated group" (Baker et al., 2022, p. 371).[2] Under the condition that the adoption dates of the treatments are randomly assigned, the "standard DiD estimator is an unbiased estimator of a particular weighted average causal effect" (Athey and Imbens, 2022). However, according to Baker et al. (2022), Bailey et al. (2021) and Goodman-Bacon (2021), deriving static treatment effects based on a staggered DiD setup is problematic since the treatment effects might change over time, potentially even translating into reversed treatment effects. Moreover, making use of partially treated control groups in a staggered DiD setup might lead to a problem of "bad comparisons". In light of the weaknesses, de Chaisemartin and d'Haultfoeuille (2020a) propose a slightly amended two-way fixed effects estimator in order to solve the bad comparisons problem.

In contrast to a DiD setup with sequential treatments, causal identification and the delineation of causal effects under *simultaneous treatments* is even more challenging. Roller and Steinberg (2020) assess the impact of two simultaneous treatments, the introduction of centralized school examinations and preponed school tracking, on PISA achievement test scores of German students. By making use of additional common trend assumptions, the authors utilize a partially treated control group in order to neutralize the effect of the additional treatment. In addition, de Chaisemartin and d'Haultfoeuille (2020b) applies the amended version of the two way fixed effects highlighted above to a setup with several treatments. Moreover, Fricke (2017) analyzes the DiD strategy in a setup in which the treatment group is exposed to a treatment and the control group is exposed to a same or similar treatment but to a lower extent. Hence, both the

[1] Compare e.g. Card and Krueger (1993) as well as Imbens and Wooldridge (2009).

[2] Further contributions regarding two way fixed effects approaches under multiple treatments originate from Callaway and Sant'Anna (2021) and Hull (2018).

treatment and the control group are exposed to a similar treatment but the intensity of the exposition differs across both groups. According to the approach proposed by Fricke (2017), "instead of the difference between two treatments, the DiD estimand can be interpreted as the lower bound in magnitude of an average treatment effect on the treated in comparison to the non-treatment state" (p. 430). Accordingly, the author makes use of bounding treatment effects as part of the identification strategy under multiple treatments. Rather than comparing two states implementing the same treatment but with differing intensities, we provide an adjusted DiD approach according to which the treatment group is exposed to totally different treatments. Additionally, Frölich (2004) discusses program evaluations under multiple treatments as well. However, the author does not lay out a detailed identification strategy in order to determine the effect of several interventions on an outcome. Rather, the author provides a general discussion of various non-parametric approaches. We complement the papers proposed by Fricke (2017) and Frölich (2004) by laying out necessary and sufficient conditions for the application of the DiD strategy under multiple treatments.

In particular, we postulate that the treatment group is exposed to two treatments, a treatment of interest and a treatment not of interest, which are implemented simultaneously in a particular group. In practical contexts, several interventions like labor market programs are often put in place in parallel in order to strengthen the corresponding effects, e.g. mitigating unemployment. To separate the effects of both treatments on an outcome variable, similar to Roller and Steinberg (2020) and de Chaisemartin and d'Haultfoeuille (2020b), our adjusted DiD strategy relies on a specific definition and composition of the control group. If one group was treated with only one out of the two treatments, it restricts the control group to this specific group to neutralize the impact of the additional treatment which is not of interest to the researcher. Under a set of traditional assumptions and a modified common trend assumption, the adjusted DiD strategy identifies the average treatment effect on the treated (ATET). The identification of the average treatment effect (ATE) relies on several additional common trend assumptions and the requirement that both treatments do not interact with each other, i.e. their effects are additive. A practical application of the adjusted DiD framework in educational economics is presented in Roller and Steinberg (2020).

The paper is structured as follows: Section 2.1 lays out the notation for the subsequent sections while section 2.2 introduces the main definitions and section 2.3 the main assumptions. Section 2.4 specifies the adjusted DiD identification strategy under multiple treatments and section 2.5 is devoted to the identification of population treatment effects. Section 2.6 discusses indicative tests of the underlying assumptions from a practical perspective and section 3 concludes.

2 Differences-in-Differences with multiple treatments

2.1 Notation

We follow the notation as laid out in Lechner (2011) for a standard DiD setup and assume that a researcher is interested in the effect of a binary treatment $D = d$, $d \in \{0; 1\}$, which is implemented between the pre-treatment period $t = 0$ and the post-treatment period $t = 1$, $t \in T = \{0; 1\}$. Simultaneously, an additional binary treatment $E = e$, $e \in \{0, 1\}$, might be implemented in which the researcher is not interested. Accordingly, the researcher observes an outcome variable in each period t, Y_t, while the potential outcome variable in period t is denoted as $Y_t^{d,e}$. In addition, the researcher might observe covariates, X, which could serve as time invariant fixed effects.[3]
In the pre-treatment period, neither treatment D nor treatment E is implemented while in the post-treatment period either treatment E or treatment D or both treatments D and E are implemented simultaneously. It is also possible that no treatment is implemented at all. Thus, in the post-treatment period, the set of possible observation groups which are potentially composed of one or several states might be exposed to the following treatments:

- Group 1: $D = 0$, $E = 0$ (no treatments)
- Group 2: $D = 1$, $E = 0$ (singular treatment D)
- Group 3: $D = 0$, $E = 1$ (singular treatment E)
- Group 4: $D = 1$, $E = 1$ (multiple treatments D and E)

In the following subsections, we lay out detailed definitions, i.e. we specify the treatment effect of D (section 2.2.1), the treatment effect of E (section 2.2.2) as well as the treatment effect of D and E (section 2.2.3), respectively.

2.2 Definitions

2.2.1 Average treatment effect of D

The ATE of treatment D in period t is defined as the sum of the ATET of the treatment of interest, D, conditional on the treatment not of interest, E, weighted with the respective

[3] The setup can easily be generalized to a scenario with more than two treatments and time varying covariates.

probabilities, P:

$$\begin{aligned} ATE_t^D =& P(D=1,E=1)ATET_{t|D=1,E=1}^{D|E=1} \\ &+ P(D=1,E=0)ATET_{t|D=1,E=0}^{D|E=0} \\ &+ P(D=0,E=1)ATET_{t|D=0,E=1}^{D|E=1} \\ &+ P(D=0,E=0)ATET_{t|D=0,E=0}^{D|E=0} \end{aligned} \tag{1}$$

The ATET of D for those who are actually exposed to treatments D and E in period 1, conditional on receiving the additional treatment E[4], is defined as the expected difference in the potential outcomes, $Y_1^{1,1} - Y_1^{0,1}$:

$$\begin{aligned} ATET_{t|D=1,E=1}^{D|E=1} =& E[Y_1^{1,1} - Y_1^{0,1}|D=1,E=1] \\ =& E\left\{ \underbrace{E[Y_1^{1,1} - Y_1^{0,1}|X=x,D=1,E=1]}_{\delta_t^1(x)} |D=1,E=1 \right\} \\ =& E_{X|D=1,E=1}\delta_t^{1,1}(x) \end{aligned} \tag{2}$$

The additional ATET which are part of the definition of ATE can be specified in a consistent manner. In particular, $ATET_{t|D=1,E=0}^{D|E=0}$ is defined as the expected difference in the potential outcomes, $Y_1^{1,0} - Y_1^{0,0}$, conditional on receiving the treatment D but not treatment E:

$$\begin{aligned} ATET_{t|D=1,E=0}^{D|E=0} =& E[Y_1^{1,0} - Y_1^{0,0}|D=1,E=0] \\ =& E\left\{ \underbrace{E[Y_1^{1,0} - Y_1^{0,0}|X=x,D=1,E=0]}_{\delta_t^{1,0}(x)} |D=1,E=0 \right\} \\ =& E_{X|D=1,E=0}\delta_t^{1,0}(x) \end{aligned} \tag{3}$$

$ATET_{t|D=0,E=1}^{D|E=1}$ is defined as the expected difference in the potential outcomes, $Y_1^{1,1} - Y_1^{0,1}$,

[4]Theoretically, there exists also an ATET of treatment D for this group given that they do not receive treatment E, $ATET_{t|D=1,E=1}^{D|E=0} = E[Y_1^{1,0} - Y_1^{0,0}|D=1,E=1]$. But there is no corresponding population, so we can neglect it.

conditional on receiving the treatment E but not treatment D:

$$\begin{aligned} ATET^{D|E=1}_{t|D=0,E=1} =& E[Y_1^{1,1} - Y_1^{0,1}|D=0,E=1] \\ =& E\left\{ \underbrace{E[Y_1^{1,1} - Y_1^{0,1}|X=x,D=0,E=1]}_{\delta_t^{0,1}(x)} |D=0,E=1 \right\} \\ =& E_{X|D=0,E=1}\delta_t^{0,1}(x) \end{aligned} \tag{4}$$

$ATET^{D|E=0}_{t|D=0,E=0}$ is defined as the expected difference in the potential outcomes, $Y_1^{1,1} - Y_1^{0,0}$, conditional on receiving neither the treatment E nor the treatment D:

$$\begin{aligned} ATET^{D|E=0}_{t|D=0,E=0} =& E[Y_1^{1,0} - Y_1^{0,0}|D=0,E=0] \\ =& E\left\{ \underbrace{E[Y_1^{1,0} - Y_1^{0,0}|X=x,D=0,E=0]}_{\delta_t^{0,0}(x)} |D=0,E=0 \right\} \\ =& E_{X|D=0,E=0}\delta_t^{0,0}(x) \end{aligned} \tag{5}$$

2.2.2 Average treatment effect of E

The ATE of treatment E in which the researcher is not particularly interested can be analogously derived to the previous section and is defined as:

$$\epsilon^{D,E} \tag{6}$$

2.2.3 Average treatment effect of D and E

The ATE of treatments D and E in period t is defined as the sum of the ATET of treatment D and E, weighted with the respective probabilities:

$$\begin{aligned} ATE_t^{D,E} =& P(D=1,E=1)ATET^{D,E}_{t|D=1,E=1} \\ &+ P(D=1,E=0)ATET^{D,E}_{t|D=1,E=0} \\ &+ P(D=0,E=1)ATET^{D,E}_{t|D=0,E=1} \\ &+ P(D=0,E=0)ATET^{D,E}_{t|D=0,E=0} \end{aligned} \tag{7}$$

$ATET^{D,E}_{t|D=1,E=1}$ can be specified as the expected difference in the potential outcomes, $Y_1^{1,1} - Y_1^{0,0}$, conditional on receiving both treatment E and treatment D:

$$\begin{aligned} ATET^{D,E}_{t|D=1,E=1} =& E[Y_1^{1,1} - Y_1^{0,0} | D=1, E=1] \\ =& E\left\{ \underbrace{E[Y_1^{1,1} - Y_1^{0,0} | X=x, D=1, E=1]}_{\gamma_t^{1,1}(x)} | D=1, E=1 \right\} \\ =& E_{X|D=1,E=1} \gamma_t^{1,1}(x) \end{aligned} \tag{8}$$

$ATET^{D,E}_{t|D=1,E=0}$ can be specified as the expected difference in the potential outcomes, $Y_1^{1,1} - Y_1^{0,0}$, conditional on receiving treatment D but not treatment E:

$$\begin{aligned} ATET^{D,E}_{t|D=1,E=0} =& E[Y_1^{1,1} - Y_1^{0,0} | D=1, E=0] \\ =& E\left\{ \underbrace{E[Y_1^{1,1} - Y_1^{0,0} | X=x, D=1, E=0]}_{\gamma_t^{1,0}(x)} | D=1, E=0 \right\} \\ =& E_{X|D=1,E=0} \gamma_t^{1,0}(x) \end{aligned} \tag{9}$$

$ATET^{D,E}_{t|D=0,E=1}$ can be specified as the expected difference in the potential outcomes, $Y_1^{1,1} - Y_1^{0,0}$, conditional on receiving treatment E but not treatment D:

$$\begin{aligned} ATET^{D,E}_{t|D=0,E=1} =& E[Y_1^{1,1} - Y_1^{0,0} | D=0, E=1] \\ =& E\left\{ \underbrace{E[Y_1^{1,1} - Y_1^{0,0} | X=x, D=0, E=1]}_{\gamma_t^{0,1}(x)} | D=0, E=1 \right\} \\ =& E_{X|D=0,E=1} \gamma_t^{0,1}(x) \end{aligned} \tag{10}$$

Finally, $ATET^{D,E}_{t|D=0,E=0}$ can be specified as the expected difference in the potential outcomes, $Y_1^{1,1} - Y_1^{0,0}$, conditional on receiving neither treatment E nor treatment D:

$$\begin{aligned} ATET^{D,E}_{t|D=0,E=0} =& E[Y_1^{1,1} - Y_1^{0,0} | D=0, E=0] \\ =& E\left\{ \underbrace{E[Y_1^{1,1} - Y_1^{0,0} | X=x, D=0, E=0]}_{\gamma_t^{0,0}(x)} | D=0, E=0 \right\} \\ =& E_{X|D=0,E=0} \gamma_t^{0,0}(x) \end{aligned} \tag{11}$$

The following section introduces the assumptions which we utilize for the identification of the

treatment effects in a multiple treatment context.

2.3 Assumptions

2.3.1 Stable unit treatment value assumption (SUTVA)

In general, the stable unit treatment value assumption (SUTVA) states that only one of the potential outcomes is observed, i.e. either the potential outcome when treated or the potential outcome when untreated (c.f. Rubin, 1986). Applied to a setup with multiple treatment, the SUTVA states that a potential outcome is observed in case of multiple treatments D and E, a singular treatment D or E or no treatment at all. Formally, four combinations are observed:

$$Y_t = deY_t^{1,1} + (1-d)eY_t^{0,1} + d(1-e)Y_t^{1,0} + (1-d)(1-e)Y_t^{0,0} \tag{12}$$

We are going to relax this assumption in subsequent sections in order to determine the treatment effects under additional assumptions.

2.3.2 Exogeneity (EXO)

The exogeneity assumption states that both treatments D and E are unrelated to the covariates X. Hence, the covariates are equal for all potential combinations of treatments D and E. Formally,

$$X^{1,1} = X^{0,1} = X^{1,0} = X^{0,0} \tag{13}$$

2.3.3 No effect on the pre-treatment population (NOPT)

We further postulate that the treatments D and E do not have an impact on the outcome in the pre-treatment population. Formally,

$$E[Y_0^{1,0} - Y_0^{0,0}|X = x, D = d, E = e] = 0 \qquad \forall d \in [0,1] \& \forall e \in [0,1] \tag{14}$$

$$E[Y_0^{0,1} - Y_0^{0,0}|X = x, D = d, E = e] = 0 \qquad \forall d \in [0,1] \& \forall e \in [0,1] \tag{15}$$

$$E[Y_0^{1,1} - Y_0^{0,0}|X = x, D = d, E = e] = 0 \qquad \forall d \in [0,1] \& \forall e \in [0,1] \tag{16}$$

2.3.4 Common trends (CT)

Consistently with the established DiD approach under consideration of a singular treatment, we utilize several common trend assumptions in the follow sections. In general, the common trend assumptions state that the development of the differences in the expected potential untreated outcomes conditional on X is unrelated to belonging to the treated or control group in the post-treatment period (compare Lechner (2011)). In light of the multiple treatment setup, the

identification of the different effects requests different common trend assumption. Therefore, we specify the following five common trend assumptions:
The first common assumption, CT1, states that the development of the differences in the expected potential untreated outcomes conditional on X is unrelated to belonging to the group getting both treatments or the group getting no treatment in the post-treatment period:

a) CT1

$$\begin{aligned} & E[Y_1^{0,0}|X=x,D=1,E=1]-E[Y_0^{0,0}|X=x,D=1,E=1] \\ =&E[Y_1^{0,0}|X=x,D=0,E=0]-E[Y_0^{0,0}|X=x,D=0,E=0] \qquad \forall x\in\mathcal{X} \end{aligned} \tag{17}$$

The second common assumption, CT2, states that the development of the differences in the expected potential untreated outcomes conditional on X is unrelated to belonging to the group getting only treatment D or the group getting no treatment in the post-treatment period:

b) CT2

$$\begin{aligned} & E[Y_1^{0,0}|X=x,D=1,E=0]-E[Y_0^{0,0}|X=x,D=1,E=0] \\ =&E[Y_1^{0,0}|X=x,D=0,E=0]-E[Y_0^{0,0}|X=x,D=0,E=0] \\ =&E[Y_1^{0,0}|X=x,E=0]-E[Y_0^{0,0}|X=x,E=0] \qquad \forall x\in\mathcal{X} \end{aligned} \tag{18}$$

The third common assumption, CT3, states that the development of the differences in the expected potential treated (with E) outcomes conditional on X is unrelated to belonging to the group getting both treatments or the group getting only treatment E in the post-treatment period:

c) CT3

$$\begin{aligned} & E[Y_1^{0,1}|X=x,D=1,E=1]-E[Y_0^{0,1}|X=x,D=1,E=1] \\ =&E[Y_1^{0,1}|X=x,D=0,E=1]-E[Y_0^{0,1}|X=x,D=0,E=1] \\ =&E[Y_1^{0,1}|X=x,E=1]-E[Y_0^{0,1}|X=x,E=1] \qquad \forall x\in\mathcal{X} \end{aligned} \tag{19}$$

The fourth common assumption, CT4, states that the development of the differences in the expected potential untreated outcomes conditional on X is unrelated to belonging to the group getting only treatment D or the group getting no treatment at all in the post-treatment period:

d) CT4

$$
\begin{aligned}
&E[Y_1^{0,0}|X=x, D=0, E=1] - E[Y_0^{0,0}|X=x, D=0, E=1] \\
=&E[Y_1^{0,0}|X=x, D=0, E=0] - E[Y_0^{0,0}|X=x, D=0, E=0] \\
=&E[Y_1^{0,0}|X=x, D=0] - E[Y_0^{0,0}|X=x, D=0] \qquad \forall x \in \mathcal{X} \qquad (20)
\end{aligned}
$$

The fifth common assumption, CT5, states that the development of the differences in the expected potential treated (with D) outcomes conditional on X is unrelated to belonging to the group getting both treatments or the group getting only treatment E in the post-treatment period:

e) CT5

$$
\begin{aligned}
&E[Y_1^{1,0}|X=x, D=1, E=1] - E[Y_0^{1,0}|X=x, D=1, E=1] \\
=&E[Y_1^{1,0}|X=x, D=1, E=0] - E[Y_0^{1,0}|X=x, D=1, E=0] \\
=&E[Y_1^{1,0}|X=x, D=1] - E[Y_0^{1,0}|X=x, D=1] \qquad \forall x \in \mathcal{X} \qquad (21)
\end{aligned}
$$

In the follow section, we utilize the standard assumptions introduced above in order to identify the ATET under multiple treatments.

2.4 Identification under standard assumptions

Based on the standard assumptions laid out in the previous section, i.e. CT, NOPT, EXO and SUTVA, we can identify several ATET. The identification of the ATET are laid our below with respect to singular treatments D or E and regarding multiple treatments D and E.

2.4.1 Average treatment effect of treatment D

The ATET of treatment D can be identified both in case of a singular treatment and in case of a multiple treatment setup. For the former setup the standard DiD setup can be utilized while making use of the standard common trend assumption (CT2). As highlighted above, this assumption states that the potential outcome of the treatment group exposed to treatment D (i.e. treatment group: D=1, E=0) would have moved in parallel to the outcome of a control group not exposed to any treatments (i.e. control group: D=0, E=0), in the absence of treatment D in the treatment group. This baseline scenario is already well established in the literature (see e.g. Athey and Imbens, 2006; Lechner, 2011). Proposition 1 below summarizes the identifying assumptions accordingly.

Proposition 1: *The ATET of treatment D, $ATET^{D|E=0}_{t|D=1,E=0}$, can be identified by utilizing the following assumptions:*

- *CT2;*
- *NOPT;*
- *EXO;*
- *SUTVA.*

Proof: Based on the linearity of expectations, $\delta_t^{1,0}(x)$ can be expressed as follows:

$$\delta_t^{1,0}(x) = E[Y_1^{1,0}|X = x, D = 1, E = 0] - E[Y_1^{0,0}|X = x, D = 1, E = 0] \tag{22}$$

Utilizing the SUTVA assumption in the minuend yields:

$$\delta_t^{1,0}(x) = E[Y_1|X = x, D = 1, E = 0] - E[Y_1^{0,0}|X = x, D = 1, E = 0] \tag{23}$$

Making use of the common trend assumption CT2 in the subtrahend gives us:

$$\begin{aligned} E[Y_1^{0,0}|X = x, D = 1, E = 0] =& E[Y_0^{0,0}|X = x, D = 1, E = 0] \\ &+ E[Y_1^{0,0}|X = x, D = 0, E = 0] \\ &- E[Y_0^{0,0}|X = x, D = 0, E = 0] \end{aligned} \tag{24}$$

Again, applying the SUTVA assumption leads to:

$$\begin{aligned} E[Y_1^{0,1}|X = x, D = 1, E = 1] =& E[Y_0^{0,0}|X = x, D = 1, E = 0] \\ &+ E[Y_1|X = x, D = 0, E = 0] \\ &- E[Y_0|X = x, D = 0, E = 0] \end{aligned} \tag{25}$$

Applying the NOPT assumption to the minuend yields:

$$\begin{aligned} E[Y_0^{0,0}|X = x, D = 1, E = 0] =& E[Y_0^{1,0}|X = x, D = 1, E = 0] \\ &- \underbrace{\left\{E[Y_0^{1,0}|X = x, D = 1, E = 0] - E[Y_0^{0,0}|X = x, D = 1, E = 0]\right\}}_{=\delta_0^0(x)=0} \\ =& E[Y_0^{1,0}|X = x, D = 1, E = 0] \end{aligned} \tag{26}$$

Based on the SUTVA assumption, it holds:

$$E[Y_0^{1,0}|X=x,D=1,E=0] = E[Y_0|X=x,D=1,E=0] \tag{27}$$

Consolidating the previous elements, leads to the well established DiD identification strategy:

$$\begin{aligned}\delta_t^{1,0}(x) = {} & E[Y_1|X=x,D=1,E=0] - E[Y_0|X=x,D=1,E=0] \\ & - \{E[Y_1|X=x,D=0,E=0] - E[Y_0|X=x,D=0,E=0]\}\end{aligned} \tag{28}$$

■

Proposition 1 was devoted to the baseline scenario which is characterized by a treatment group which was exclusively exposed to treatment D and a control group which was not exposed to any treatment at all. In this case, the standard DiD strategy has to be applied in order to identify the ATET of treatment D.
In addition, proposition 2 is devoted to a multiple treatment scenario in which the treatment group is exposed to both treatments D and E (i.e. treatment group: D=1, E=1). In this case an adjusted DiD strategy can be utilized under certain assumptions as laid out in proposition 2. Based on this adjusted DiD strategy, the effect of the additional treatment E can be neutralized and eliminated by relying on an adjusted control group which is exposed to treatment E but not to treatment D (i.e. control group: D=0, E=1).

Proposition 2: *The ATET of treatment D,* $ATET_{t|D=1,E=1}^{D|E=1}$*, can be identified by utilizing the following assumptions:*

- *CT3;*
- *SUTVA;*
- *NOPT;*
- *EXO.*

Proof: Based on the linearity of expectations, $\delta_t^{1,1}(x)$ can be expressed as follows:

$$\delta_t^{1,1}(x) = E[Y_1^{1,1}|X=x,D=1,E=1] - E[Y_1^{0,1}|X=x,D=1,E=1] \tag{29}$$

Utilizing the SUTVA assumption in the minuend yields:

$$\delta_t^{1,1}(x) = E[Y_1|X=x,D=1,E=1] - E[Y_1^{0,1}|X=x,D=1,E=1] \tag{30}$$

Making use of the common trend assumption CT3 in the subtrahend gives us:

$$\begin{aligned} E[Y_1^{0,1}|X=x,D=1,E=1] =& E[Y_0^{0,1}|X=x,D=1,E=1] \\ &+ E[Y_1^{0,1}|X=x,D=0,E=1] \\ &- E[Y_0^{0,1}|X=x,D=0,E=1] \end{aligned} \tag{31}$$

Again, applying the SUTVA assumption leads to:

$$\begin{aligned} E[Y_1^{0,1}|X=x,D=1,E=1] =& E[Y_0^{0,1}|X=x,D=1,E=1] \\ &+ E[Y_1|X=x,D=0,E=1] \\ &- E[Y_0|X=x,D=0,E=1] \end{aligned} \tag{32}$$

Applying the NOPT assumption to the minuend yields:

$$\begin{aligned} E[Y_0^{0,1}|X=x,D=1,E=1] =& E[Y_0^{1,1}|X=x,D=1,E=1] \\ &- \underbrace{\left\{E[Y_0^{1,1}|X=x,D=1,E=1] - E[Y_0^{0,1}|X=x,D=1,E=1]\right\}}_{=\delta_0(x)=0} \\ =& E[Y_0^{1,1}|X=x,D=1,E=1] \end{aligned} \tag{33}$$

Based on the SUTVA assumption, it holds:

$$E[Y_0^{1,1}|X=x,D=1,E=1] = E[Y_0|X=x,D=1,E=1] \tag{34}$$

Consolidating the previous elements, leads to the adjusted DiD strategy:

$$\begin{aligned} \delta_t^{1,1}(x) =& E[Y_1|X=x,D=1,E=1] - E[Y_0|X=x,D=1,E=1] \\ &- \{E[Y_1|X=x,D=0,E=1] - E[Y_0|X=x,D=0,E=1]\} \end{aligned} \tag{35}$$

■

According to proposition 2, the adjusted DiD strategy eliminates the effect of the additional treatment E in the treatment group by relying on a control group which is exposed to treatment E but not to treatment D. As a consequence, the effect of treatment D on the outcome variable Y can be extracted.

2.4.2 Average treatment effect of treatment E

The ATET regarding treatment E under multiple treatments (i.e. treatment group: D=1, E=1) can be identified consistently with the adjusted DiD strategy introduced in proposition

2. Instead of a control group which is exclusively exposed to treatment E as highlighted in the previous section, a control group has to be defined which is exclusively exposed to treatment D and not exposed to treatment E. The adjusted control group neutralizes the additional treatment D (i.e. control group: D=1, E=0) as part of the adjusted approach.

2.4.3 Average treatment effect of treatments D and E

In some cases, researchers might be interested in the additive treatment D and E on an outcome variable Y (i.e. treatment group: D=1, E=1). In this case the researcher has to rely on a control group which is exposed to neither treatment D nor treatment E (i.e. control group: D=0, E=0). The detailed identifying assumptions are laid out in proposition 3.

Proposition 3: *The ATET of treatments D and E,* $ATET^{D,E}_{t|D=1,E=1}$*, can be identified by utilizing the following assumptions:*

- *CT1;*
- *SUTVA;*
- *NOPT;*
- *EXO.*

Proof: Based on the linearity of expectations, $\gamma_t^{1,1}(x)$ can be expressed as follows:

$$\gamma_t^{1,1}(x) = E[Y_1^{1,1}|X = x, D = 1, E = 1] - E[Y_1^{0,0}|X = x, D = 1, E = 1] \tag{36}$$

Utilizing the SUTVA assumption in the minuend yields:

$$\gamma_t^{1,1}(x) = E[Y_1|X = x, D = 1, E = 1] - E[Y_1^{0,0}|X = x, D = 1, E = 1] \tag{37}$$

Making use of the common trend assumption CT1 in the subtrahend gives us:

$$\begin{aligned} E[Y_1^{0,0}|X = x, D = 1, E = 1] =& E[Y_0^{0,0}|X = x, D = 1, E = 1] \\ &+ E[Y_1^{0,0}|X = x, D = 0, E = 0] \\ &- E[Y_0^{0,0}|X = x, D = 0, E = 0] \end{aligned} \tag{38}$$

Again, applying the SUTVA assumption leads to:

$$\begin{aligned} E[Y_1^{0,0}|X=x,D=1,E=1] =& E[Y_0^{0,0}|X=x,D=1,E=1] \\ &+ E[Y_1|X=x,D=0,E=0] \\ &- E[Y_0|X=x,D=0,E=0] \end{aligned} \tag{39}$$

Applying the NOPT assumption to the minuend yields:

$$\begin{aligned} E[Y_0^{0,0}|X=x,D=1,E=1] =& E[Y_0^{1,1}|X=x,D=1,E=1] \\ &- \underbrace{\left\{E[Y_0^{1,1}|X=x,D=1,E=1] - E[Y_0^{0,0}|X=x,D=1,E=1]\right\}}_{=\gamma_0(x)=0} \\ =& E[Y_0^{1,1}|X=x,D=1,E=1] \end{aligned} \tag{40}$$

Based on the SUTVA assumption, it holds:

$$E[Y_0^{1,1}|X=x,D=1,E=1] = E[Y_0|X=x,D=1,E=1] \tag{41}$$

Combining the previous elements leads to the DiD strategy:

$$\begin{aligned} \gamma_t^{1,1}(x) = & E[Y_1|X=x,D=1,E=1] - E[Y_0|X=x,D=1,E=1] \\ & - \{E[Y_1|X=x,D=0,E=0] - E[Y_0|X=x,D=0,E=0]\} \end{aligned} \tag{42}$$

■

While the previous section was devoted to the ATET under multiple treatments, the following section refers to population treatment effects, i.e. average treatment effects of the untreated (ATEU).

2.5 Population treatment effects

With respect to the treatment of interest D, we laid out the respective strategies for identifying $\delta_t^{1,0}(x)$ and $\delta_t^{1,1}(x)$ in the previous section. However, we were not able to identify the effects $\delta_t^{0,1}(x)$ and $\delta_t^{0,0}(x)$ under the set of standard assumptions. As a remedy, this section shows under which assumptions we identify the additional population treatment effects.

2.5.1 Additional assumptions

The following section introduces additional assumptions which we utilize for the identification of the population treatment effects below. Apart from additional common trend assumptions, CT6

- CT8, the additional assumptions comprise a no interaction assumption between treatment D and treatment E.
The no interaction assumption, postulates that the treatments D and E do not unfold interaction effects in their impact on the outcome variable Y. Formally, this assumption can be formulated as the equality in the expected differences between the potential outcomes in case of no additional treatment E and the expected difference in the potential outcomes in case of an additional treatment E:

No interaction assumption (NOINT):

$$E[Y_1^{1,0} - Y_1^{0,0}|X = x, D = 0, E = 0] = E[Y_1^{1,1} - Y_1^{0,1}|X = x, D = 0, E = 0] \tag{43}$$

CT6: The sixth common assumption, CT6, states that the development of the differences in the expected potential treated (with D and E) outcomes conditional on X is unrelated to belonging to the group getting only treatment E or the group getting both treatments in the post-treatment period:

$$E[Y_1^{1,1} - Y_0^{1,1}|X = x, D = 0, E = 1] = E[Y_1^{1,1} - Y_0^{1,1}|X = x, D = 1, E = 1] \tag{44}$$

The seventh common assumption, CT7, states that the development of the differences in the expected potential treated (with D and E) outcomes conditional on X is unrelated to belonging to the group getting both treatments or the group getting no treatment in the post-treatment period:

CT7:

$$E[Y_1^{1,1} - Y_0^{1,1}|X = x, D = 0, E = 0] = E[Y_1^{1,1} - Y_0^{1,1}|X = x, D = 1, E = 1] \tag{45}$$

The eighth common assumption, CT8, states that the development of the differences in the expected potential treated (with E) outcomes conditional on X is unrelated to belonging to the group getting both treatments or the group getting no treatment in the post-treatment period.

CT8:

$$\begin{aligned} &E[Y_0^{0,1}|X = x, D = 0, E = 0] - E[Y_1^{0,1}|X = x, D = 0, E = 0] \\ &= E[Y_0^{0,1}|X = x, D = 1, E = 1] - E[Y_1^{0,1}|X = x, D = 1, E = 1] \end{aligned} \tag{46}$$

2.5.2 Identification

The following sections are utilizing the initial and additional assumptions in order to identify the ATE in the population, $\delta_t^{1,0}(x)$ and $\delta_t^{1,1}(x)$, which were not identified in the previous section. With respect to the treatment D we have to separate two scenarios. In the first scenario the ATE of treatment D is identified while treatment E is observed consistently in both periods.

Proposition 4: *The ATET of treatment D,* $ATET_{t|D=0,E=1}^{D|E=1}$*, can be identified by utilizing the following assumptions:*

- *CT6;*
- *SUTVA;*
- *NOPT;*
- *EXO;*
- *NOINT.*

Proof: Making use of the common trend assumption CT6 and the SUTVA assumption yields:

$$\begin{aligned} E[Y_1^{1,1} - Y_1^{0,1}|X=x, D=0, E=1] =& E[Y_1^{1,1}|X=x, D=1, E=1] - E[Y_0^{1,1}|X=x, D=1, E=1] \\ &+ E[Y_0^{1,1}|X=x, D=0, E=1] - E[Y_1^{0,1}|X=x, D=0, E=1] \\ =& E[Y_1|X=x, D=1, E=1] - E[Y_0|X=x, D=1, E=1] \\ &+ E[Y_0^{1,1}|X=x, D=0, E=1] - E[Y_1|X=x, D=0, E=1] \end{aligned} \tag{47}$$

From the NOPT assumption introduced in the previous section follows:

$$\begin{aligned} E[Y_0^{1,1}|X=x, D=0, E=1] =& E[Y_0^{0,1}|X=x, D=0, E=1] \\ &- \underbrace{\left\{E[Y_0^{1,1}|X=x, D=0, E=1] - E[Y_0^{0,1}|X=x, D=0, E=1]\right\}}_{=\gamma_0(x)=0} \\ =& E[Y_0^{0,1}|X=x, D=0, E=1] \end{aligned} \tag{48}$$

Again utilizing the SUTVA assumption gives us:

$$E[Y_0^{0,1}|X=x, D=0, E=1] = E[Y_0|X=x, D=0, E=1] \tag{49}$$

Thus, it follows:

$$\begin{aligned}\delta_t^{0,1}(x) =& E[Y_1|X=x, D=1, E=1] - E[Y_0|X=x, D=1, E=1] \\ &+ E[Y_0|X=x, D=0, E=1] - E[Y_1|X=x, D=0, E=1] \end{aligned} \tag{50}$$

■

Based on proposition 4, the identification of $\delta_t^{1,0}(x)$, is based primarily on additional common trend assumptions. In order to identify $\delta_t^{1,1}(x)$, we assume that a singular treatment D is implemented.

Proposition 5: *The ATET of treatment D, $ATET_{t|D=0,E=0}^{D|E=0}$, can be identified by utilizing the following assumptions:*

- *CT7;*
- *CT8;*
- *SUTVA;*
- *NOPT;*
- *EXO;*
- *NOINT.*

Proof: Utilizing the assumptions NOINT1 and CT7 yields:

$$\begin{aligned}E[Y_1^{1,0} - Y_1^{0,0}|X=x, D=0, E=0] &= E[Y_1^{1,1}|X=x, D=0, E=0] - E[Y_1^{0,1}|X=x, D=0, E=0] \\ &= E[Y_0^{1,1}|X=x, D=0, E=0] + E[Y_1^{1,1}|X=x, D=1, E=1] - E[Y_0^{1,1}|X=x, D=1, E=1] \\ &\quad - E[Y_1^{0,1}|X=x, D=0, E=0] \end{aligned} \tag{51}$$

Utilizing the NOPT assumption gives us:

$$\begin{aligned}E[Y_0^{1,1}|X=x, D=0, E=0] =& E[Y_0^{0,1}|X=x, D=0, E=0] \\ &+ \underbrace{E[Y_0^{0,1}|X=x, D=0, E=0] - E[Y_0^{1,1}|X=x, D=0, E=0]}_{=0} \\ =& E[Y_0^{0,1}|X=x, D=0, E=0] \end{aligned} \tag{52}$$

Based on the common trend assumption CT8 it follows:

$$\begin{aligned} & E[Y_0^{0,1}|X=x, D=1, E=1] - E[Y_1^{0,1}|X=x, D=1, E=1] \\ = & E[Y_0^{0,1}|X=x, D=0, E=1] - E[Y_1^{0,1}|X=x, D=0, E=1] \end{aligned} \tag{53}$$

Consolidating leads to:

$$\begin{aligned} \delta_t^0(x) = & \{E[Y_1^{1,1}|X=x, D=1, E=1] - E[Y_0^{1,1}|X=x, D=1, E=1]\} \\ & - \{E[Y_1^{0,1}|X=x, D=0, E=1] - E[Y_0^{0,1}|X=x, D=0, E=1]\} \end{aligned} \tag{54}$$

Applying the SUTVA assumption gives us:

$$\begin{aligned} \delta_t^0(x) = & \{E[Y_1|X=x, D=1, E=1] - E[Y_0|X=x, D=1, E=1]\} \\ & - \{E[Y_1|X=x, D=0, E=1] - E[Y_0|X=x, D=0, E=1]\} \end{aligned} \tag{55}$$

■

The following section provides an overview of the underlying assumptions for each case and describes the corresponding identification strategies based on empirical data.

2.6 Discussion

In the previous sections, we laid out five propositions which facilitate the identification of treatment effects in a multiple treatment setup. Depending on the particular treatment effect, we made use of adjusted treatment and control groups which are summarized in table 1. The identification of the effect of treatment D, the main treatment of interest, requires either a treatment group which is exclusively exposed to treatment D and a control group which is not exposed to any treatment, consistently with the standard setup (column 1 in the table 1), or a treatment group which is simultaneously exposed to treatment D and E and a control group which is exclusively exposed to treatment E (column 2). The latter was referred to as adjusted DiD strategy as part of this paper. The exposition to a partial treatment in the control group as part of the adjusted strategy is necessary to eliminate and neutralize the effect of the additional treatment E on the outcome variable.

In addition, identifying the joint effect of the treatments D and E requires a treatment group which is exposed to the joint treatment D and E and a control group which receives not a treatment at all (column 3). For the identification of the remaining group specific treatment effects (ATEU) that are necessary to identify the average treatment effect in the population (ATE), the treatment group is consistently composed of observations receiving both treatments

D and E and the control group is consistently composed of observations receiving exclusively treatment E (columns (4) and (5)).

Table 1: Treatment and control groups underlying the DiD approach with multiple treatments.

	ATET			ATEU	
	(1)	(2)	(3)	(4)	(5)
	$\delta_t^{1,0}(x)$	$\delta_t^{1,1}(x)$	$\gamma_t^{1,1}(x)$	$\delta_t^{0,1}(x)$	$\delta_t^{0,0}(x)$
$D=0,\ E=0$	Control		Control		
$D=1,\ E=0$	Treatment				
$D=0,\ E=1$		Control		Control	Control
$D=1,\ E=1$		Treatment	Treatment	Treatment	Treatment

Notes: This table reports the respective treatment and control groups for the adjusted DiD identification strategy under multiple treatments. Column (1) highlights the treatment and control group for the standard strategy under a singular treatment. Column (2) reports the treatment and control group for the adjusted strategy under multiple treatments. Column (3) highlights the treatment and control group for a strategy for the identification of the joint effect of treatments D and E. Columns (4) and (5) depict the treatment and control groups for the identification of population treatment effects.

Complementary to the overview of the treatment and control group underlying the propositions 1 to 5, table 2 provides a detailed list of assumptions for the identification of the respective average treatment effects (ATET/ATEU) of treatment D. In contrast to the standard identification strategy which mainly rests on a common trend assumption (column 1 in table 2), the adjusted identification strategy requires a different common trend assumption depending on the effect of interest (columns (2) to (5) in table 2). For the identification of the average treatment effect in the population the adjusted identification strategy additionally requires that the treatments D and E unfold separate effects on the outcome variable Y and do not unfold interaction effects among each other. The absence of interaction effects implies that the treatments D and E are additive in their impact on the outcome variable Y. This assumption can be indicatively tested in a setup with staggered policy interventions. In particular, if the interventions D and E have been implemented sequentially in the treatment group, it can in fact be verified whether the effects on the outcome variable are additive and whether interaction effects on the outcome variable are significant. As indicated above, especially in federal contexts, policy interventions are often put in place sequentially rather than simultaneously. The adjusted common trend assumptions can be indicatively tested based on parallel pre-treatment trends, however, part of the common trend assumptions require a sequential implementation of the treatments as well

in order to perform pre-treatment tests.

Table 2: Identifying assumptions for the DiD approach with multiple treatments.

	ATET			ATEU	
	(1)	(2)	(3)	(4)	(5)
	$\delta_t^{1,0}(x)$	$\delta_t^{1,1}(x)$	$\gamma_t^{1,1}(x)$	$\delta_t^{0,1}(x)$	$\delta_t^{0,0}(x)$
SUTVA	✓	✓	✓	✓	✓
NOPT	✓	✓	✓	✓	✓
EXO	✓	✓	✓	✓	✓
NOINT1				✓	✓
CT1			✓		
CT2	✓				
CT3		✓			
CT6				✓	
CT7					✓
CT8					✓

Notes: This table reports the identifying assumptions of the adjusted DiD identification strategy under multiple treatments. Columns (1) to (3) refer to the identifying assumptions of the average treatment effect on the treated while columns (4) and (5) highlight the identifying assumptions of the population treatment effects.

The adjusted DiD strategy can be implemented in consistency with the standard DiD approach by making use of an OLS estimator with the respective dummy variables, i.e. a regional dummy variable which equals 1 for the treatment group and 0 for the control group, a time dummy variable which equals 1 after the implementation of the treatment and 0 otherwise as well as an interaction effect of the time and regional dummy variable on the outcome variable.

3 Conclusion

A common problem in the field of policy evaluation is that several treatments or policy interventions are put in place simultaneously. These multiple treatments undermine the partial identification of treatment effects while utilizing the standard DiD strategy. As a remedy, this paper introduces an adjusted DiD identification strategy which is particularly tailored to the

causal identification of treatment effects under multiple treatments. We show that under standard DiD assumptions and an adjusted common trend assumption the ATET of a singular treatment can be identified if there is a control group available that is exclusively exposed to the additional treatment which is not of interest for the researcher. The partial treatment of the control group allows to neutralize and eliminate the effect of the additional treatment. The identification of the average treatment effect in the population, however, rests on stronger assumptions, i.e. additional common trend assumptions and the requirement that both treatments do not interact with each other. The adjusted DiD identification strategy is particularly relevant for the investigation of political reforms since these are rarely put in place without accompanying reforms (see e.g. Roller and Steinberg, 2020).

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