Time-variable gravity field determination from GRACE Follow-On data using the Celestial Mechanics Approach extended by empirical noise modelling

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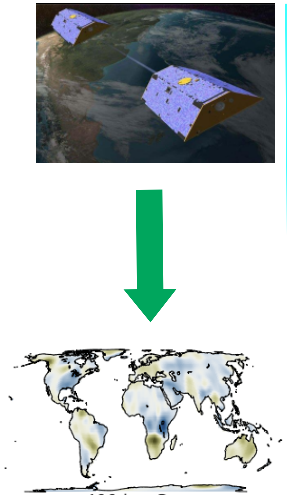
18 October 2022
Potsdam, Germany

Operational processing



Basic parametrisation

▪ initial conditions	2x[6]
▪ accelerometer bias	2x[3]
▪ accelerometer scaling	2x[3]
parameters per arc	24

Additional parameters

▪ 15 min PCA per satellite in

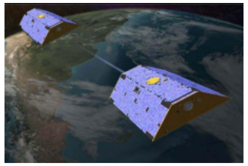
➔ radial	2x[96]
➔ along-track	2x[96]
➔ cross-track	2x[96]
parameters per arc	576

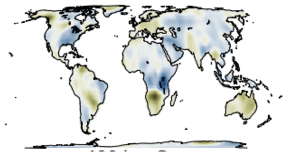
in daily arcs (30 days):

- 18000 parameters,
- 17280 for the noise model
- + gravity field

→ frequently used in the Celestial Mechanics Approach [Beutler et al., 2010]

Operational processing





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Force models	
Gravity field	Internal AIUB static GRACE field
Astromomic bodies	JPL DE421 (all planets + Pluto)
Mean pole	Linear
Solid Earth tides	IERS2010
Solid Earth pole tides	IERS2010
Ocean tides	FES2014b (+ admittances from TUG)
Ocean pole tides	Desai
Atmospheric tides	AOD RL06
Atmospheric & oeanic dealiasing	AOD RL06
Relativistic effects	IERS2010

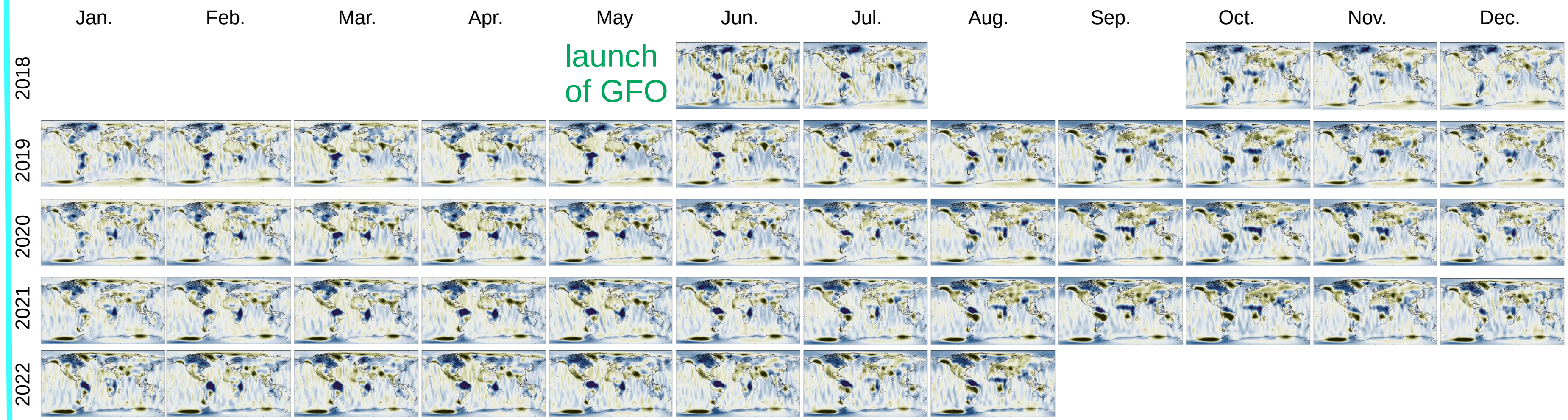
Non-conservative forces:
ACT from TUG

Processing – operational solution

$$\mathbf{N}=\left(\mathbf{A}^T\mathbf{P}\,\mathbf{A}\right)$$

$$\boldsymbol{b}=\mathbf{A}^T\mathbf{P}\,\boldsymbol{l}$$

$$\hat{x}=\mathbf{N}^{-1}\boldsymbol{b}$$



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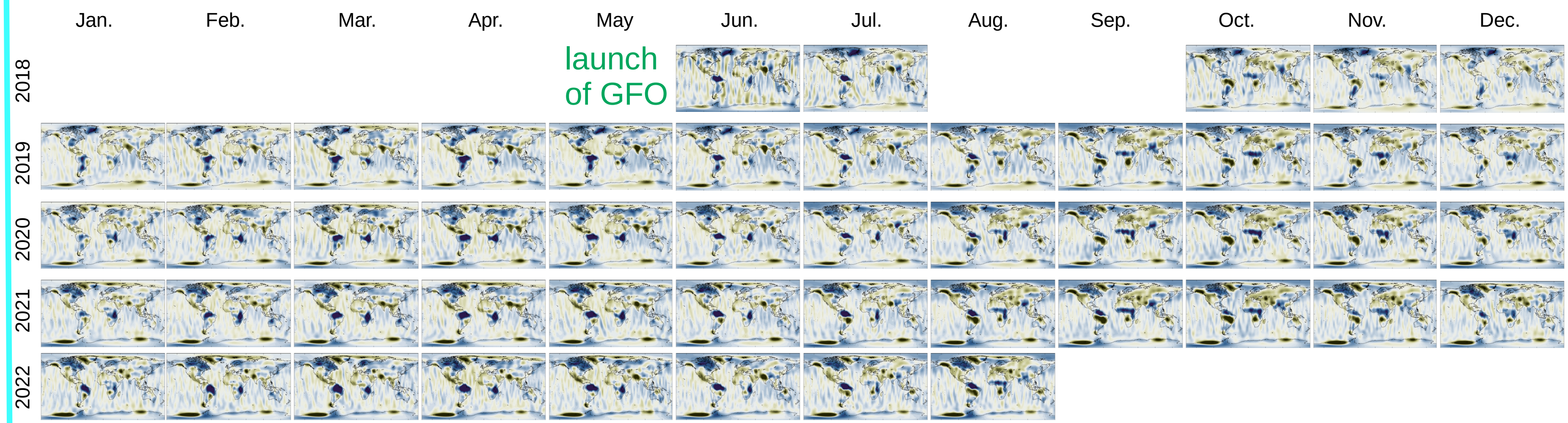


Facts

- operational since Sepember 2020
- available at ICGEM as AIUB-GRACE-FO_operational
- continuation of AIUB-RL02
- current status: 49 months from June 2018 until August 2022

Features

- updated background models
- data screening with variance component estimation
- use of alternative GF2 Level 1B ACT product from TUG

Empirical Noise Modelling

Empirical noise modelling based on post-fit residuals

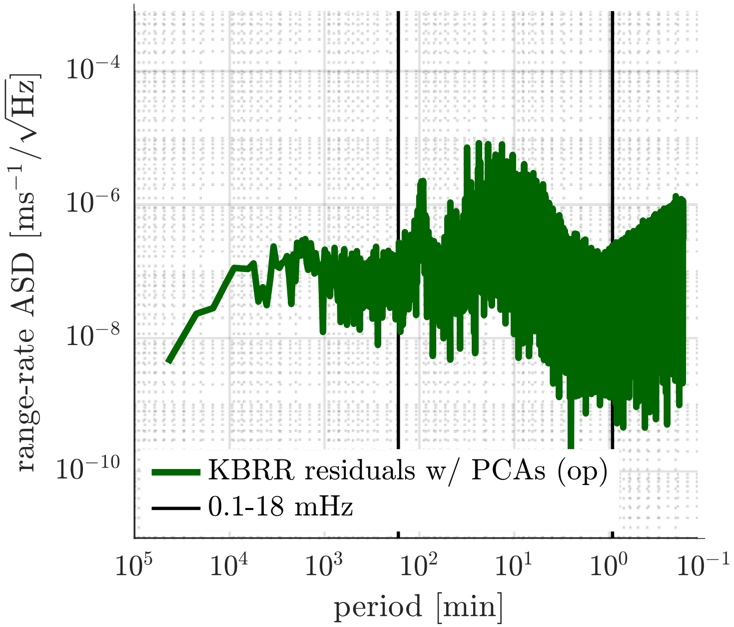
Least-squares

$$\mathbf{N}=(\mathbf{A}^T\mathbf{P}\,\mathbf{A})$$

$$\boldsymbol{b}=\mathbf{A}^T\mathbf{P}\,\boldsymbol{l}$$

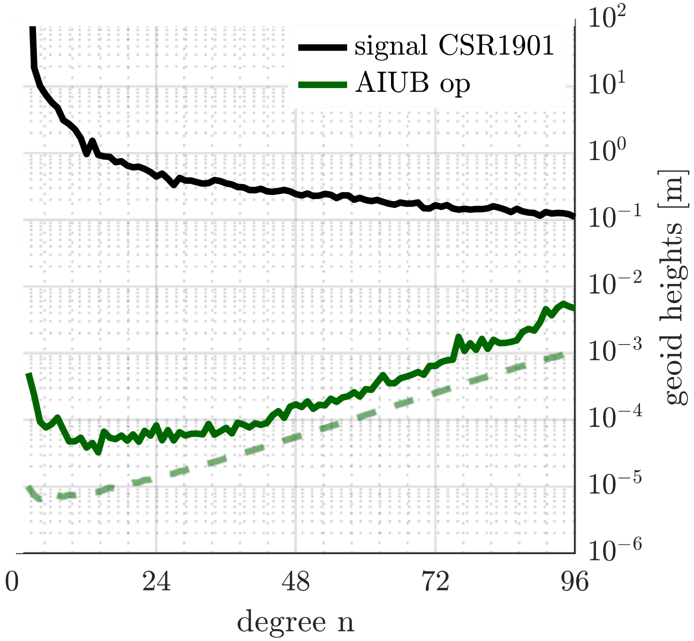
$$\hat{\boldsymbol{x}}=\mathbf{N}^{-1}\boldsymbol{b}$$

$$\hat{\boldsymbol{e}}=\boldsymbol{l}-\mathbf{A}\,\hat{\boldsymbol{x}}$$



- information in the residuals (noise?)

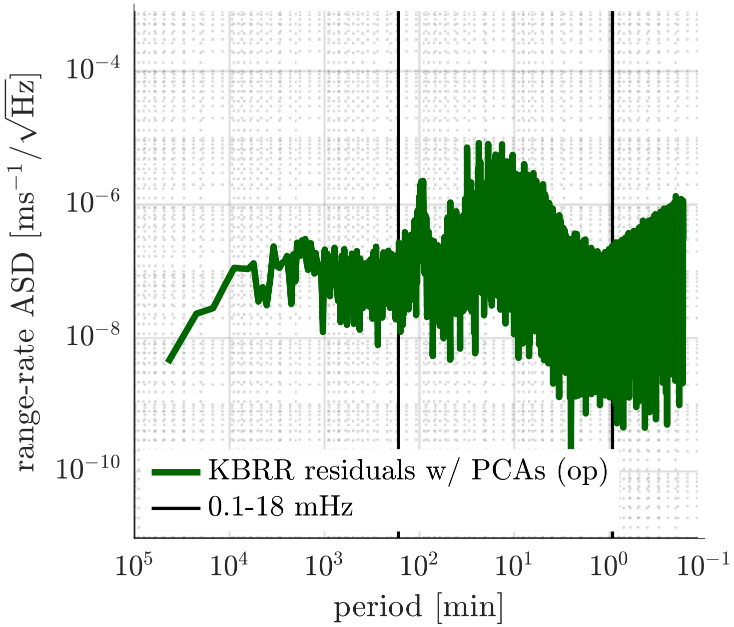
$$\hat{\boldsymbol{x}}=\mathbf{N}^{-1}\boldsymbol{b}$$



- information in the parameters (signal?)

What do we expect from residuals?

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$$



The estimator is BLUE
(best – linear – unbiased) if

- $\mathrm{E}(\hat{\boldsymbol{x}}) = \boldsymbol{x}$
- $\mathrm{E}(\boldsymbol{e}|\boldsymbol{x}) = \mathrm{E}(\hat{\boldsymbol{e}}) = \mathbf{0}$
- $\mathrm{D}(\boldsymbol{e}|\boldsymbol{x}, \sigma_0^2) = \mathrm{D}(\boldsymbol{l}|\sigma_0^2) = \sigma_0^2\,\mathbf{P}^{-1}$
  $= \sigma_0^2\,\mathbf{Q}_{\boldsymbol{ee}} = \sigma_0^2\,\mathbf{Q}_{\boldsymbol{ll}}$

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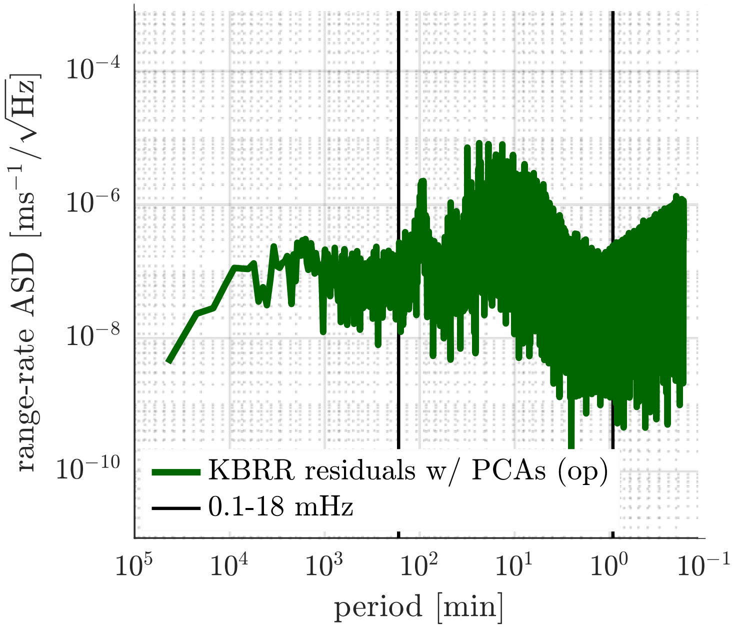

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$$\mathbf{P} = \frac{1}{\sigma_0^2}\mathbf{I}$$

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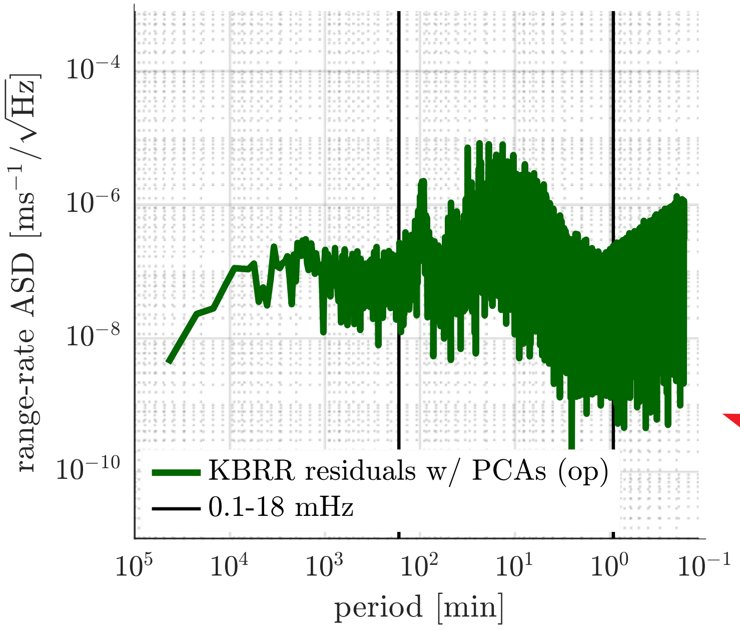

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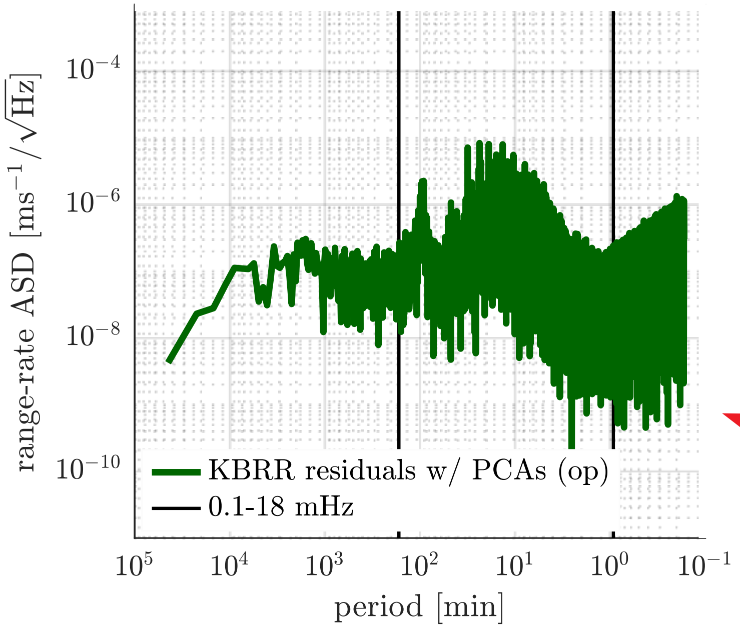

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$\hat{\boldsymbol{x}} = \mathbf{N}^{-1}\boldsymbol{b}$	unbiased
$\mathbf{C}_{\hat{x}\hat{x}} = \sigma_0^2(\mathbf{A}^T\mathbf{P}\,\mathbf{A})^{-1}$	biased

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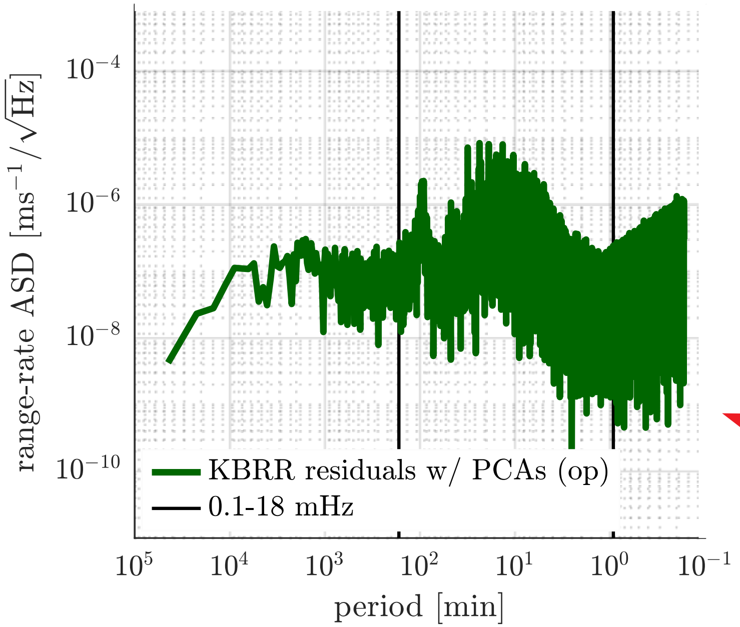

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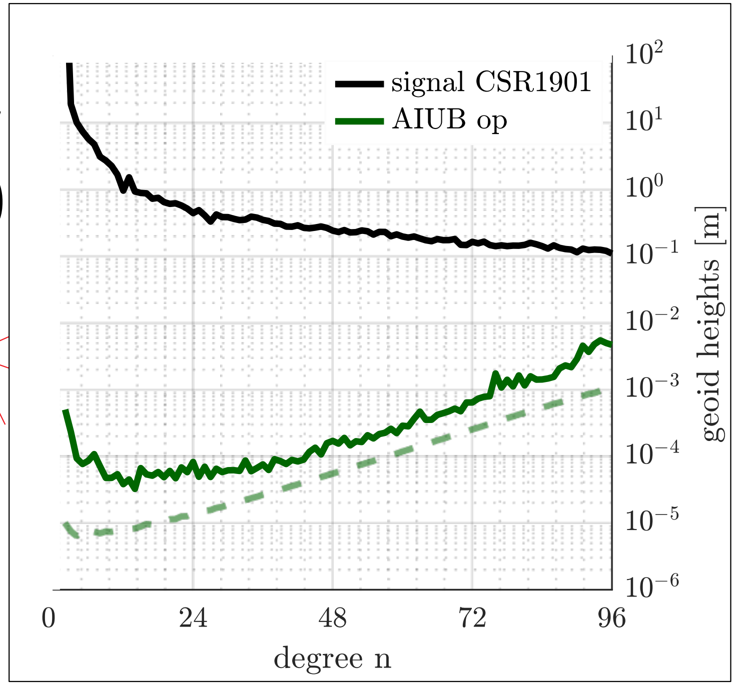

$$\hat{\boldsymbol{x}} = \mathbf{N}^{-1}\boldsymbol{b}$$ unbiased

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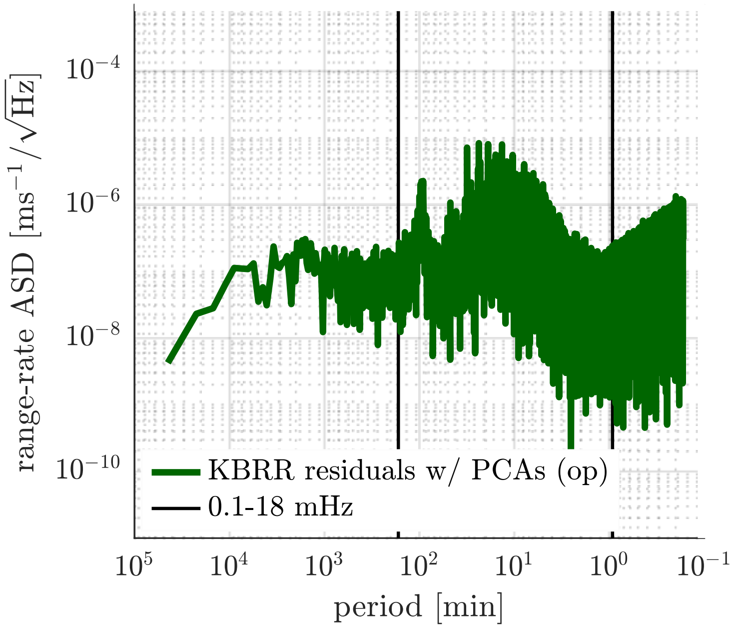
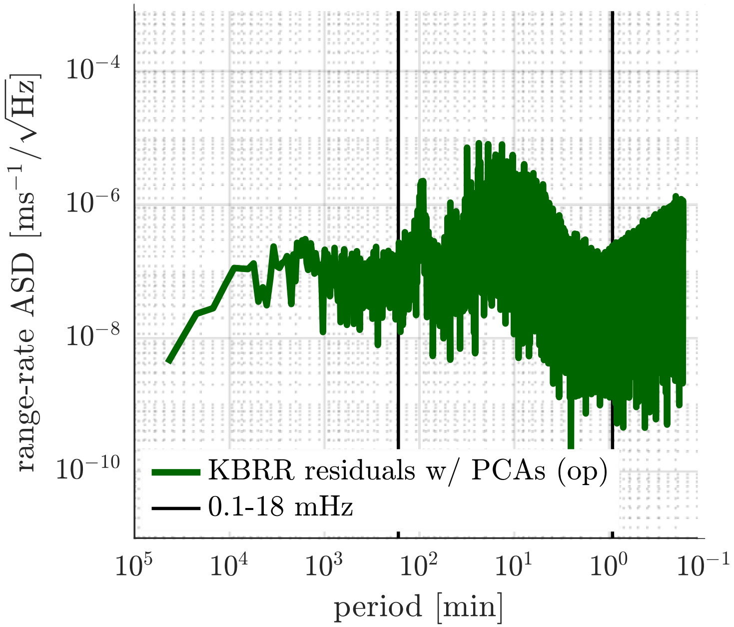
Auto-covariance function



$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\hat{\boldsymbol{x}}$$ (post-fit residuals)

$$\text{cov}(\Delta t_k) = \frac{1}{N} \sum_{i=0}^{N} \hat{e}(t_i)\hat{e}(t_i + \Delta t_k)$$

Auto-covariance function



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- stationarity assumed
- biased estimation of auto-covariance
  → covariance matrix nondegenerate

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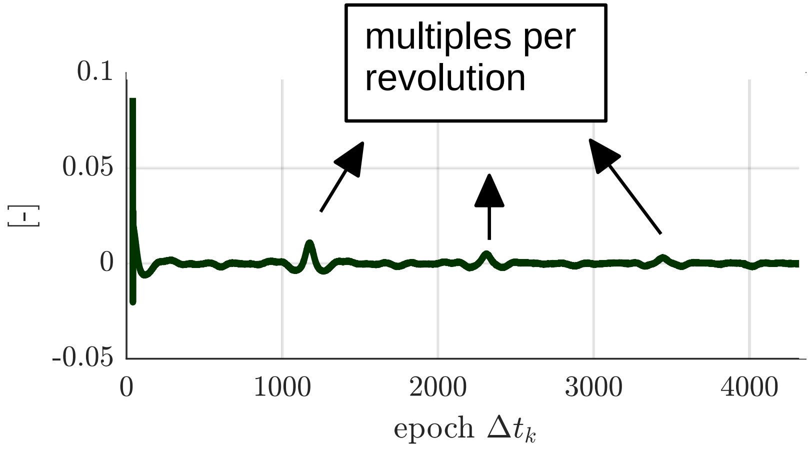

$$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\hat{\boldsymbol{x}} \quad \text{(post-fit residuals)}$$

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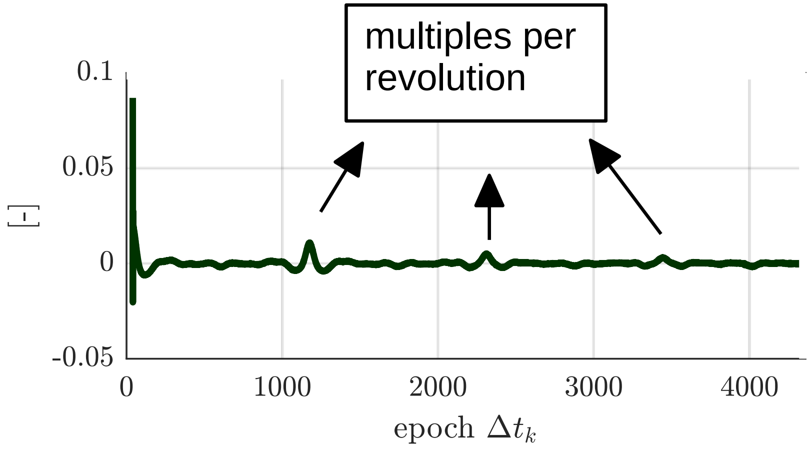
Auto-covariance function and weight matrix



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- stationarity assumed
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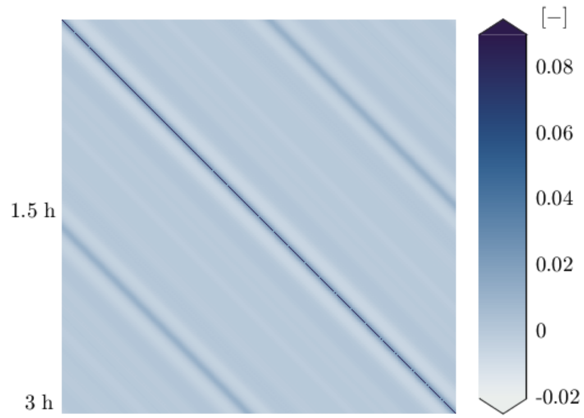
  → covariance matrix nondegenerate

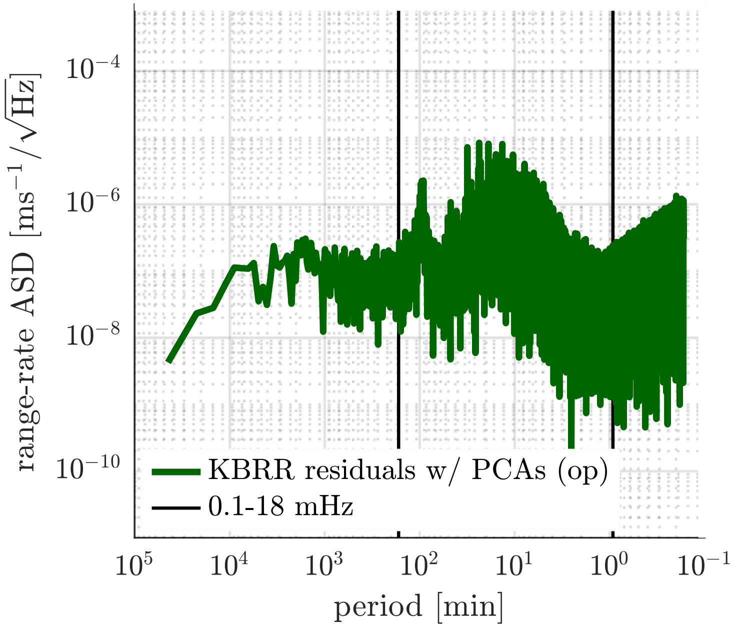
block Toeplitz matrix



→ **P**

Auto-covariance function – examples

$\hat{\boldsymbol{e}} = \boldsymbol{l} - \mathbf{A}\,\hat{\boldsymbol{x}}$



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Auto-covariance function – examples

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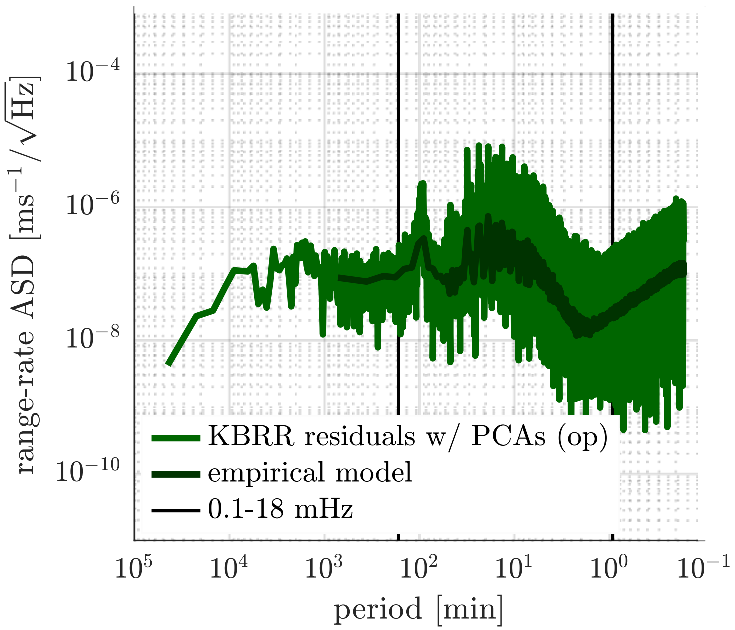

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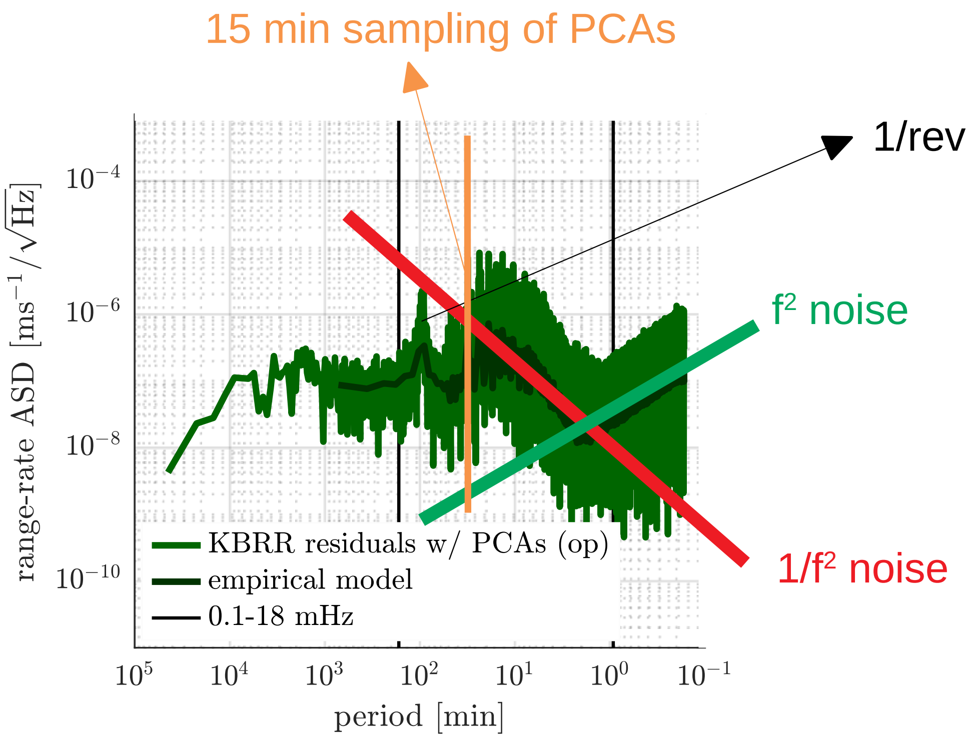

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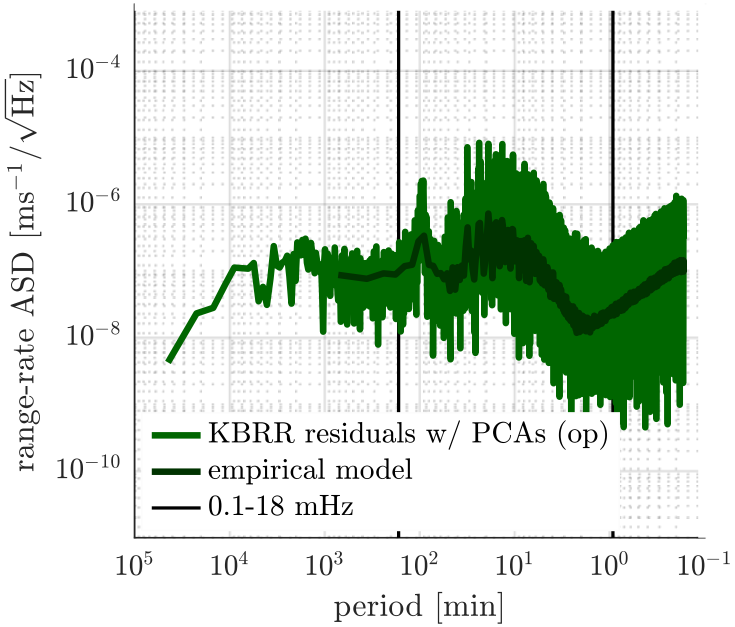
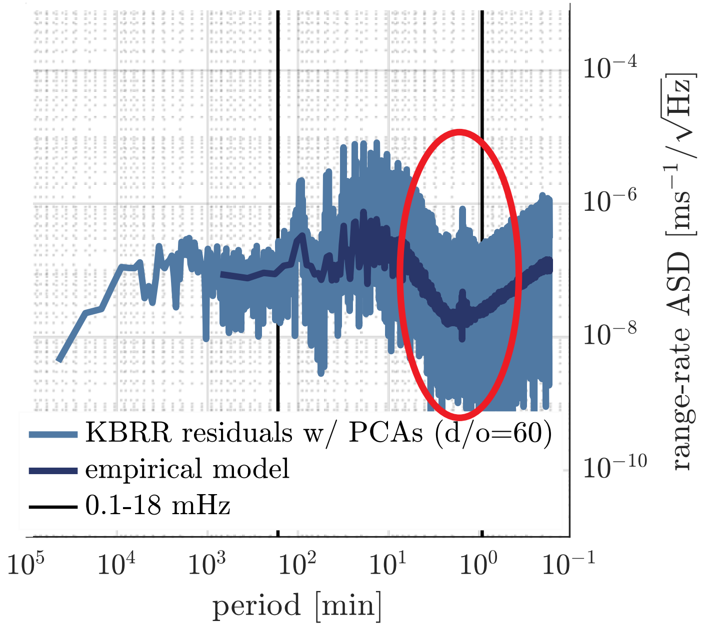
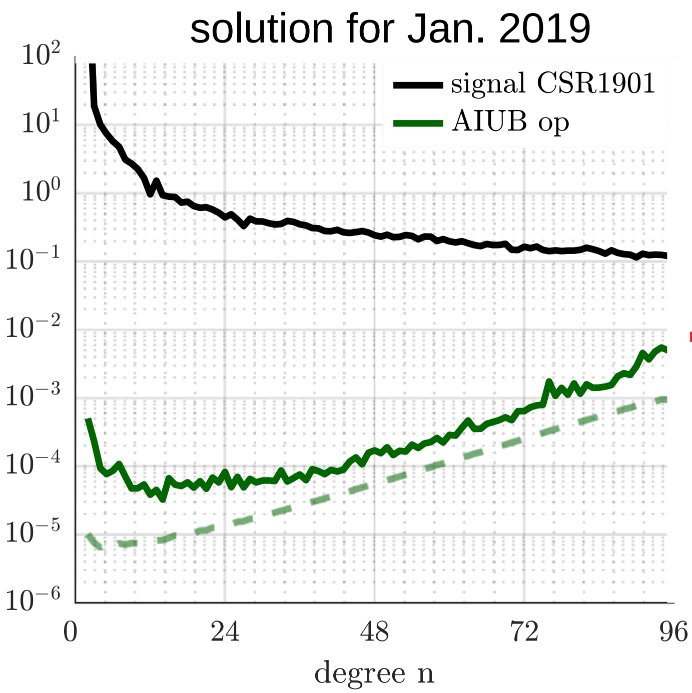
truncation at d/o=60 (artefact → bias) is reflected





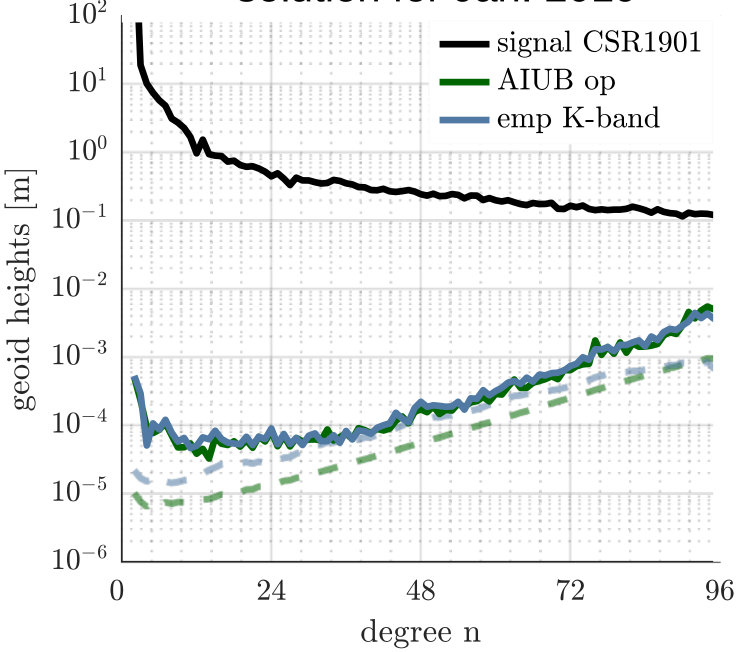
Results of empirical modelling – gravity field



- formal errors too optimistic compared to the assessed noise

Results of empirical modelling (K-band) – gravity field

solution for Jan. 2019



- formal errors reflect assessed noise very well
- including features of resonance orders

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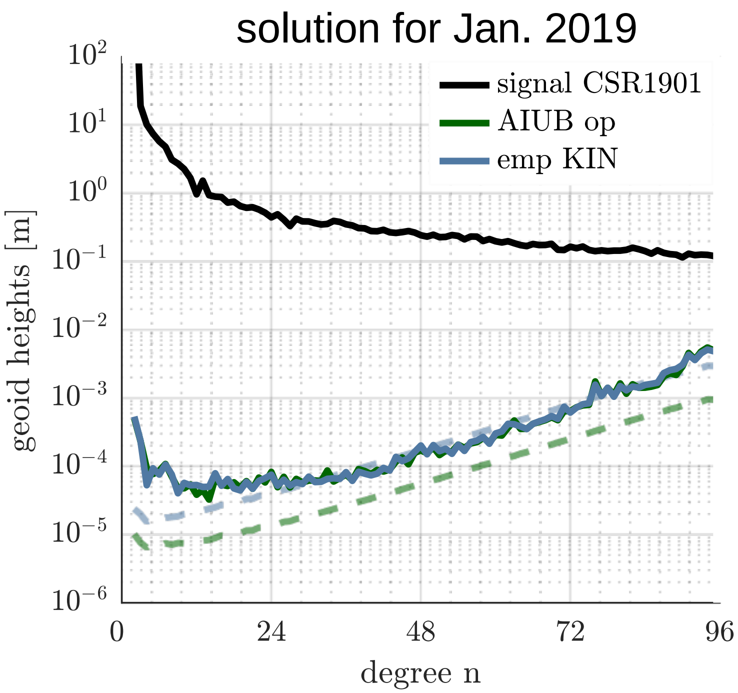
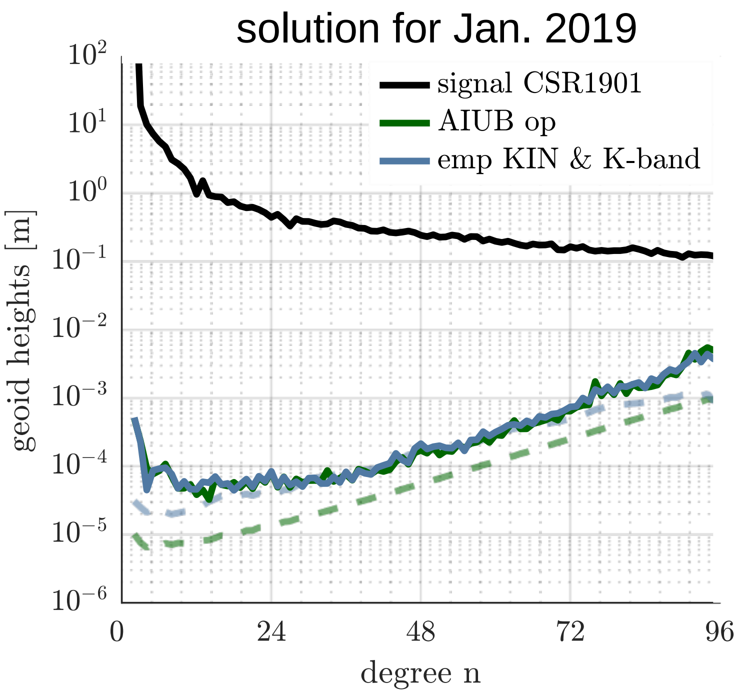
Results of empirical modelling (KIN) – gravity field



- formal errors reflect assessed noise very well
- basically the formal error curve is shifted

Results of empirical modelling (KIN & K-band) – gravity field



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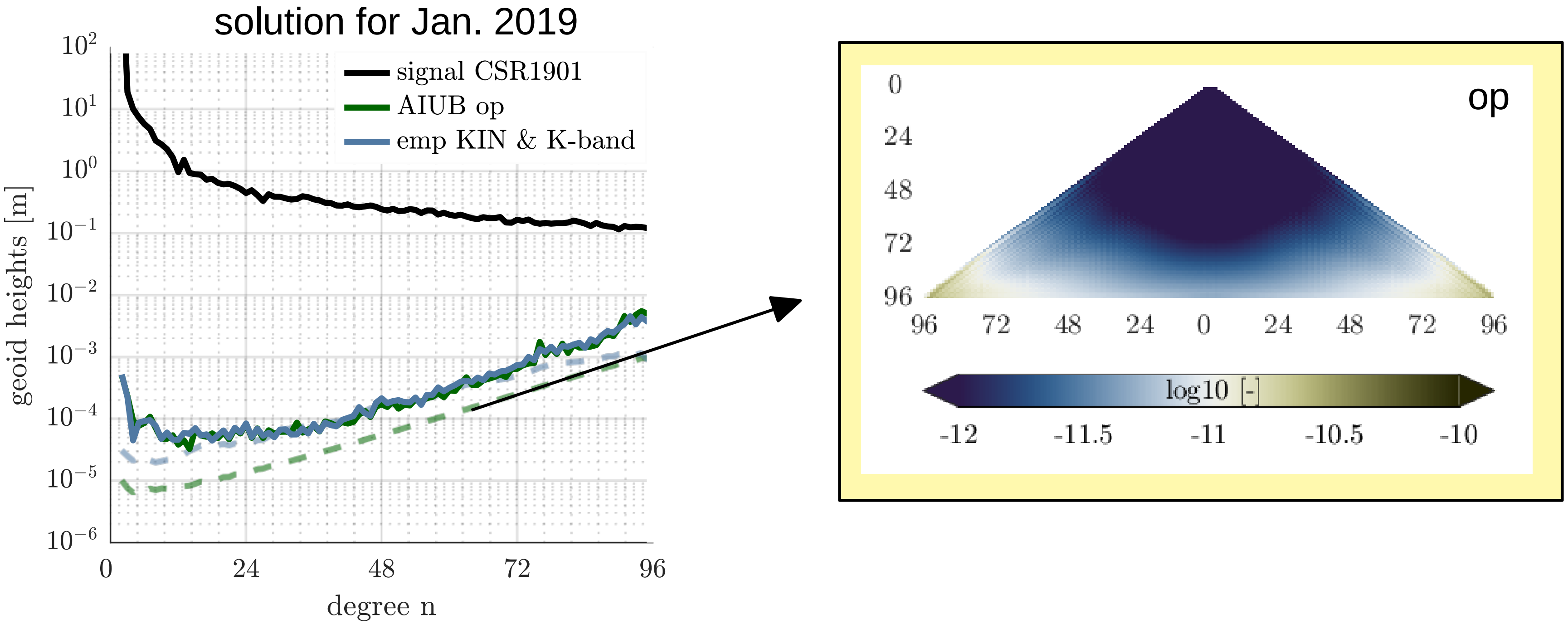
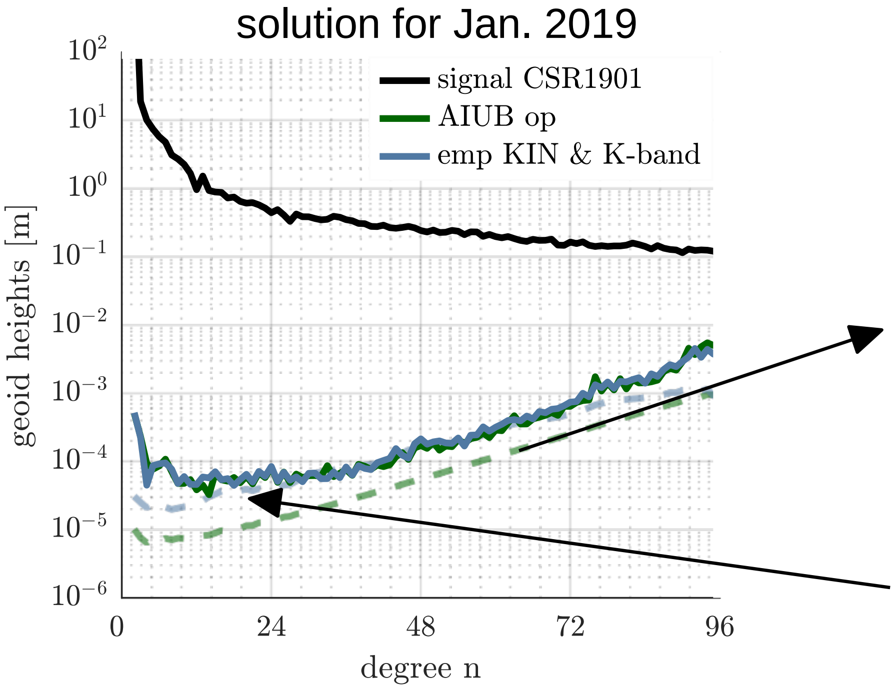
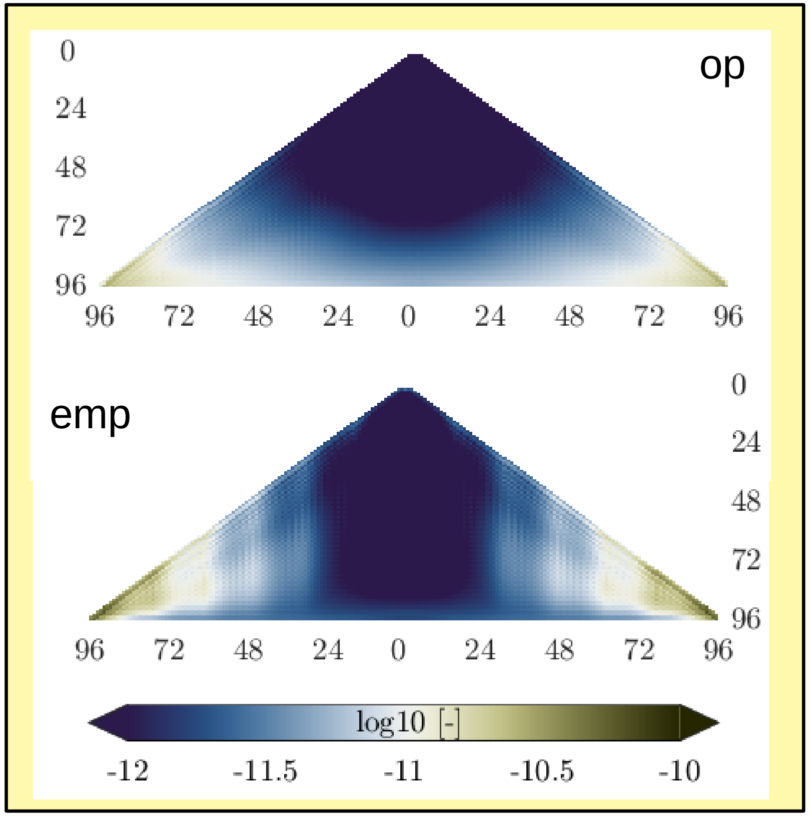
- including features of resonance orders

Results of empirical modelling – formal errors

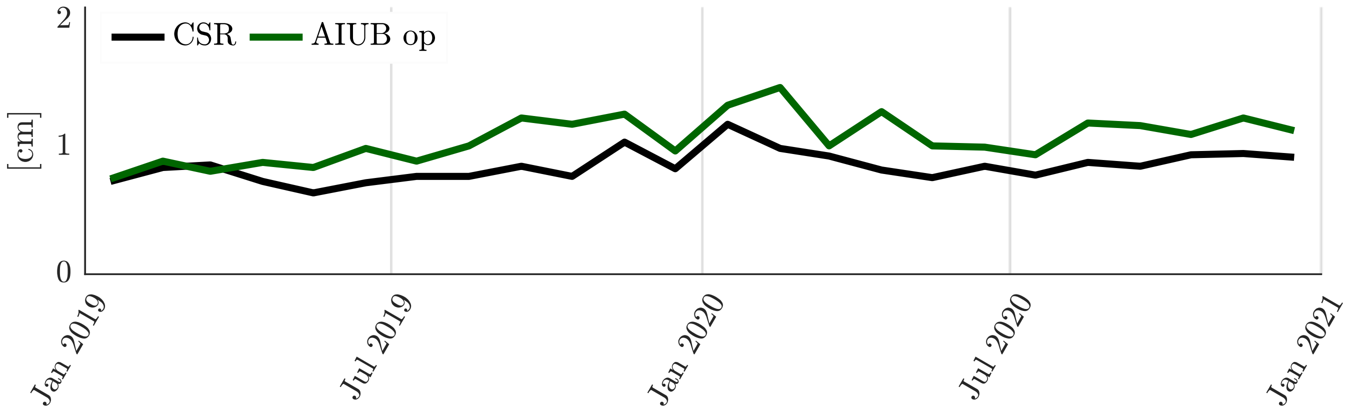


Results of empirical modelling – formal errors





Results of empirical modelling – RMS over the ocean



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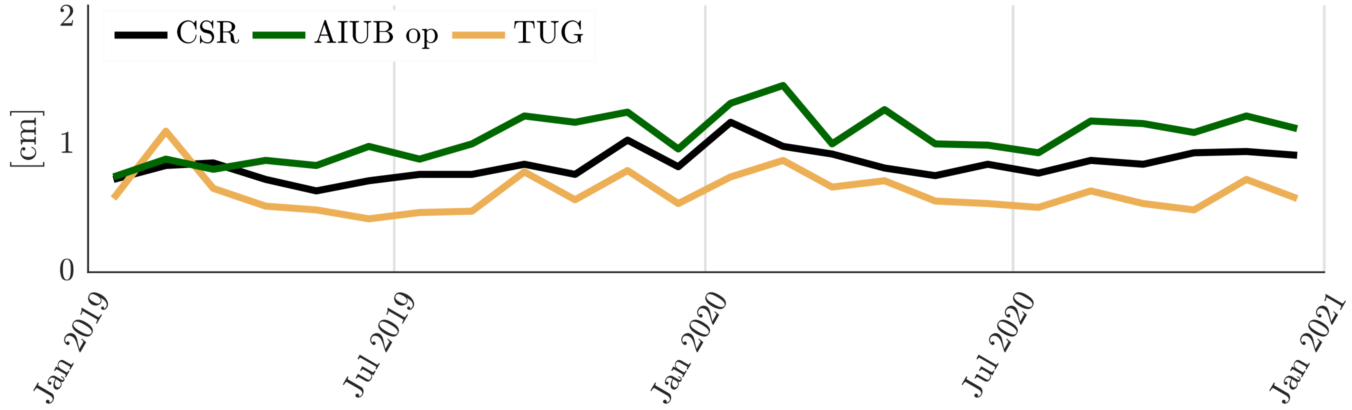

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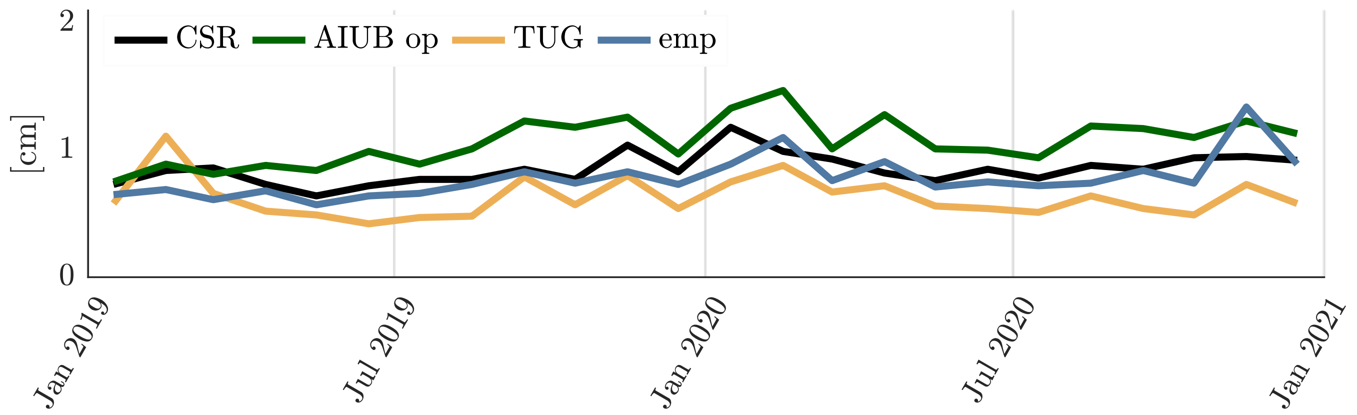
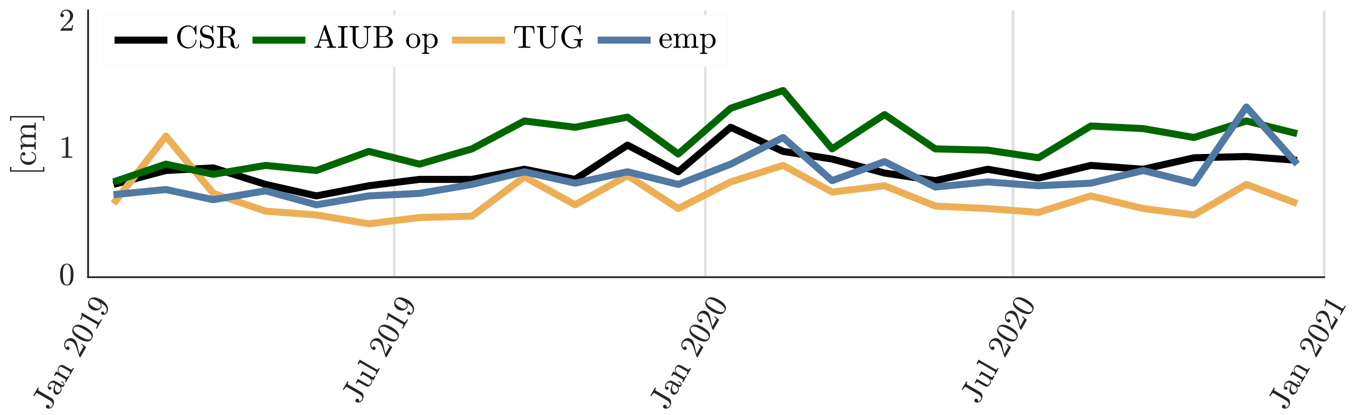


Results of empirical modelling – summary



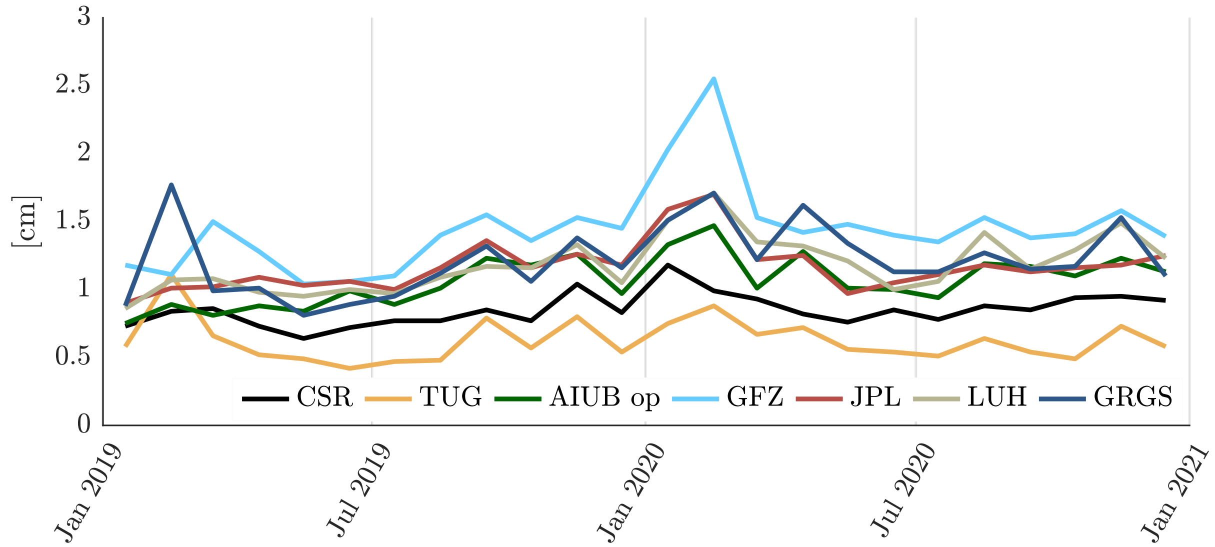
- possible on any (stationary) residuals time series
- additional parameters can be reduced as stationary behaviour can be absorbed
- formal errors much more realistic and show resonance orders (if correlation length > 3 h)
- no constraints needed
- no/few a priori knowledge needed

- iterations required (might be time consuming)
- memory consumption and inversion time dependent on length of auto-covariance function

Performance in COST-G – RMS over the ocean

COST-G
Combination Service for Time-variable Gravity Models

Combining time-variable gravity field solutions to provide for a product of improved quality, robustness and reliability



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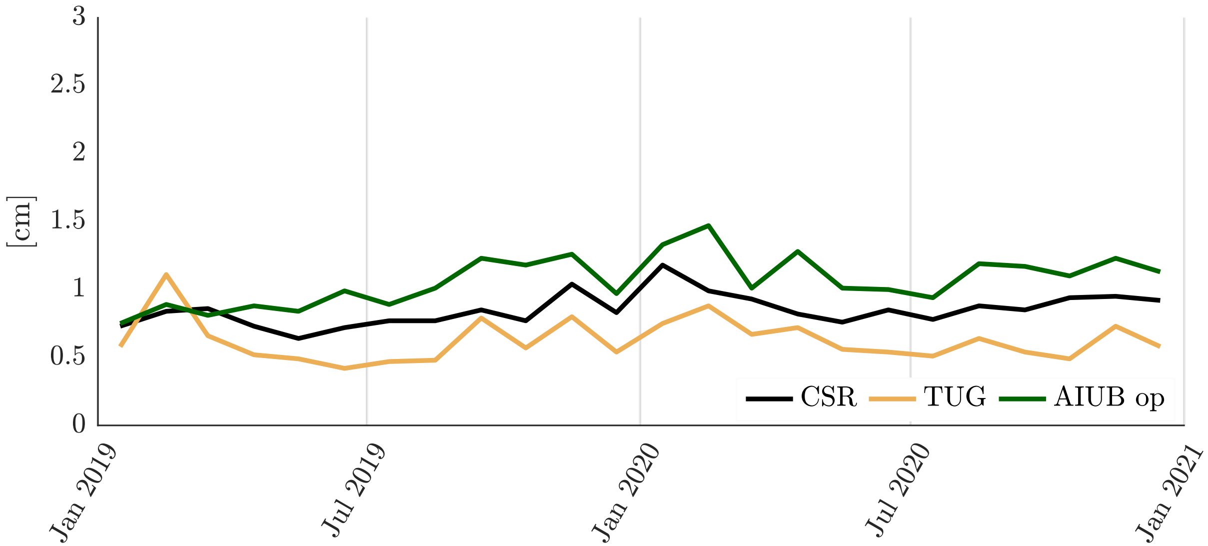
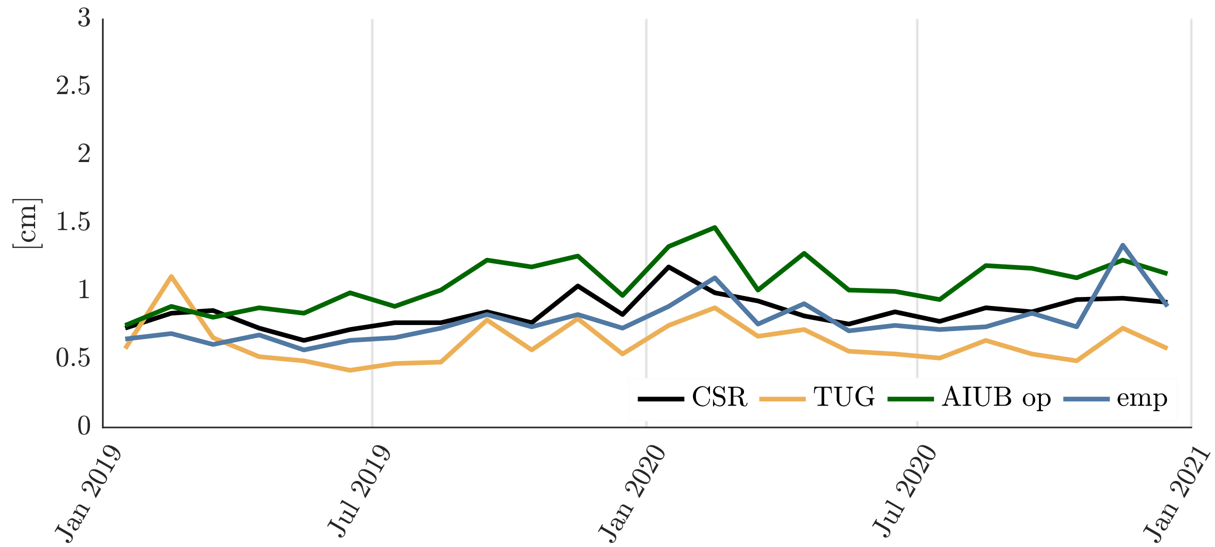


Performance in COST-G – RMS over the ocean

Combining time-variable gravity field solutions to provide for a product of improved quality, robustness and reliability



Performance in COST-G – RMS over the ocean

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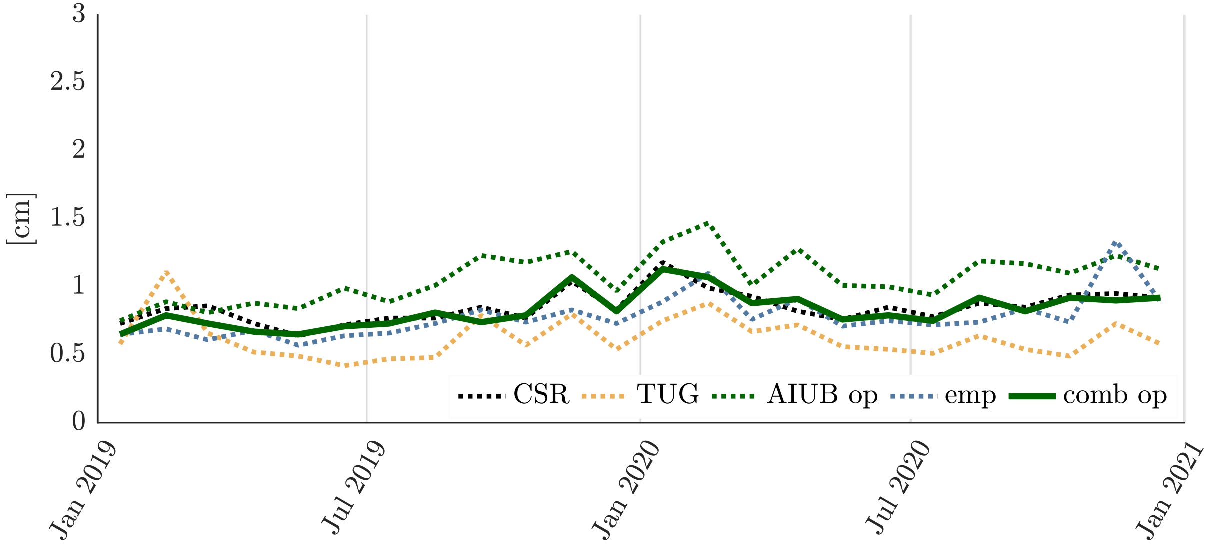
Combining time-variable gravity field solutions to provide for a product of improved quality, robustness and reliability

- Combination not at the level of the best individual solutions



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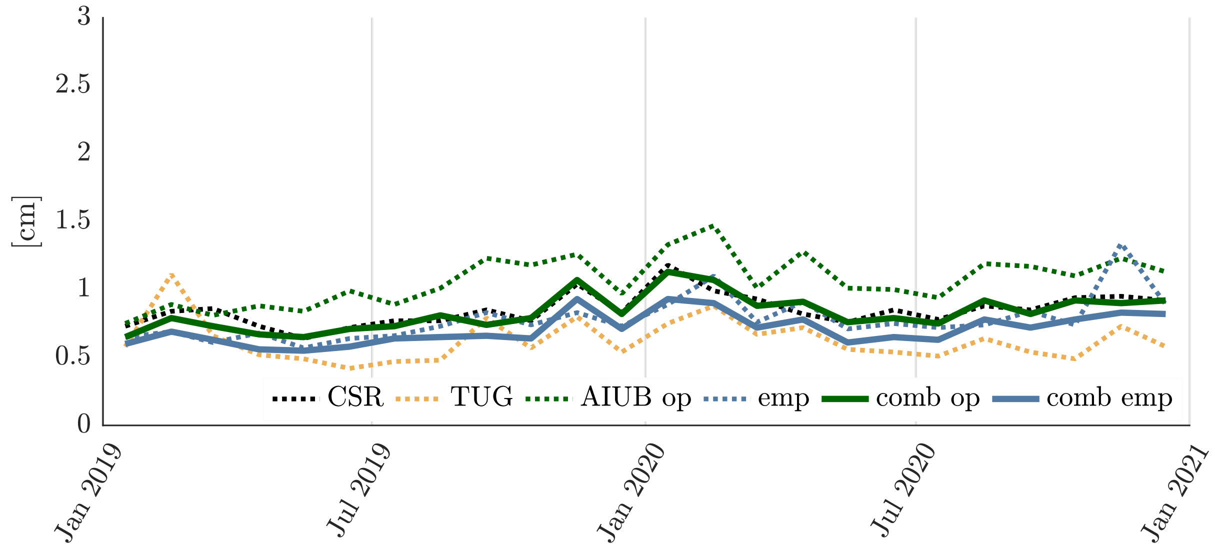
Performance in COST-G

COST-G
Combination Service for Time-variable Gravity Models

Combining time-variable gravity field solutions to provide for a product of improved quality, robustness and reliability

- (Combination not at the level of the best individual solutions)
- Combination significantly improved by the new processing scheme of one analysis centre



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Thank you for your attention

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