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Do internal capital markets in business groups mitigate firms' financial constraints?

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ABSTRACT

We develop a new rationale for capital allocation in business groups' internal capital markets. We show that productivity and pledgeable income jointly drive capital allocation within an internal capital market. In financially constrained business groups, an efficient internal capital market can allocate marginal funds to firms that have high pledgeability of income because of a multiplier effect: a dollar of internal funds generates a bigger increase in investment. This result has important implications for the business group affiliation strategy. Whether or not a financially constrained but highly productive firm will benefit from group affiliation depends on its borrowing capacity vis-à-vis other affiliates.

1. Introduction

Business groups—groups of legally independent private and publicly listed firms with limited liability and autonomous access to external capital markets—are present around the world, including emerging economies in Latin America (Chong and López-de Silanes, 2007) and Asia (Claessens et al., 2002; Carney and Child, 2012) and developed economies in Europe (Faccio and Lang, 2002). In a business group, a controlling shareholder—an individual, a founding family, or the state—controls firms through a pyramidal organizational structure, that is, a chain of ownership relations in which the controlling shareholder directly controls a firm that, in turn, controls another firm, and so on (Bianco and Casavola, 1999; Almeida and Wolfenzon, 2006b). This structure allows the controlling shareholder at the top of the pyramid to achieve legal control of the decisions in the firms down the ownership chain despite owning only a small amount of cash flow rights (Morck et al., 2005), fundamentally differing from the conglomerates or multidivisional organizations of fully owned subsidiaries or divisions.

In spite of the fundamental differences between business groups and conglomerates, there is empirical evidence of the development of internal capital markets in both types of organizational forms. Perhaps because of this commonality, most theoretical models of internal capital markets have focused on conglomerates (with a few exceptions, e.g., Cestone and Fumagalli, 2005), which are commonplace in the United States (Kandel et al., 2018). Thus, this article attempts to model the allocation of internal resources in business groups, aiming to shed new light on the effects of the idiosyncrasies of the business groups' organizational structure on corporate finance. More specifically, we aim to answer the following questions: (a) How do business groups allocate resources in their internal capital markets? (b) And do the internal capital markets alleviate the financial constraints of affiliate firms that have limited access to external finance?

The independence of business groups' affiliated firms allows them to directly access external capital markets and to secure financing on their own merits. We contend that the resource allocation within a business group could be related to the same factors that drive resource allocation in external capital markets. We therefore develop a simple model of investment in business groups subject to moral hazard, proposing that a firm's productivity and pledgeable income (external financing capacity) jointly explain the (efficient) allocation of internal resources in business groups. The central result is that, if two companies have different amounts of pledgeable income, it could be better to allocate resources to the

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firm with the greatest ability to multiply its wealth than to the most productive firm.

Thus, financially constrained business groups could decide to finance investment opportunities that allow them to increase their external financing—that is, where internal financing has a higher multiplier effect over external finance—instead of financing the most profitable opportunities. In other words, both winner picking and cross-subsidization strategies for internal capital markets discussed in the corporate finance literature can arise, but for a different reason.

For example, suppose that firms 1 and 2 belong to a business group. Firm 1 has an investment with a net present value (NPV) of \$0.15 per unit of a dollar and can raise \$0.80 from outside investors per unit of internal wealth. Firm 2 has an investment with an NPV of \$0.20 per unit of a dollar and can raise \$0.30 from outside investors per unit of internal wealth. If an entrepreneur has equal cash flow rights in both firms, the entrepreneur maximizes wealth by allocating the maximum possible amount of internal resources to firm 1. For each \$1 of internal wealth, firm 1 generates an economic surplus of $\$0.27 = (1 + 0.8) \times 0.15$, whereas firm 2 generates a surplus of $\$0.26 = (1 + 0.3) \times 0.20$.

This example illustrates our central result in two ways. First, productivity alone should not explain the resource allocation of internal capital markets within a business group. Second, pledgeable income is an important factor (if not the most important) in financing investments across firms within business groups.

In other words, our model's predictions question the argument that the efficiency of an internal capital market is related to the allocation of resources to high-productivity firms alone. Our key point is that, in a financially constrained business group, an efficient internal capital market can allocate marginal funds to firms that have high pledgeability because of a credit multiplier effect, where a dollar of internal funds generates a bigger increase in investment. In other words, we propose that an efficient internal capital market in constrained business groups can allocate resources from financially weak to financially strong firms, or, more precisely, from firms with low multipliers to firms with high multipliers.

According to our model, if productivity varies little relative to pledgeable income across firms in the same business group, pledgeable income tends to be the most critical driver of resource allocation within business groups. Thus, if one analyzes the determinants of the resource allocation, taking pledgeable income for granted, one could conclude that the internal capital markets are inefficient. This conclusion is especially troubling if there is a negative correlation between productivity and pledgeable income (as in the example above).

Indeed, Shin and Park (1999) and Lee et al. (2009) have concluded that internal capital markets do not improve the efficiency of resource allocation in Korean business groups (*chaebols*), showing that chaebols invest more than their non-chaebol counterparts in firms with poor growth opportunities (i.e., low-productivity firms). However, their analysis does not take into account variables associated with pledgeable income such as private benefits of control, tangible assets, and risk shifting. Therefore, their results do not exclude the alternative explanation of the multiplier effect of internal financing over external funds. In other words, the authors conclude that the internal capital markets of chaebols are inefficient, whereas, in reality, the reduction of financial constraints for the group as a whole could be the desired (efficient) outcome.

Our contribution to the corporate finance literature is twofold. First, we build a new model showing that internal capital markets in business groups resemble external financial markets. In other words, our model implies that the same factors that limit a firm's access to external finance also reduce its access to financial resources in internal capital markets. According to the literature, a company that has considerable private benefits, few tangible assets, and/or high risk-shifting problems can face difficulties in raising external finance (e.g., Stiglitz and Weis, 1981; Bernanke and Gertler, 1989; Gertler and Gilchrist, 1994; Holmstrom and Tirole, 1997; Kiyotaki and Moore, 1997; Almeida and Campello, 2007). We propose that, if a firm with the same above-mentioned characteristics is affiliated with a business group, it will also face financial constraints in the internal capital market. More specifically, this focal firm is likely to be a provider rather than a receiver of finance within the business group. This theoretical prediction contradicts the view in which the internal capital market of business groups can mitigate the negative effect of the failures of external financial markets (Khanna and Palepu, 2000; Khanna and Yafeh, 2007).

Second, our model generates new testable predictions. For example, we distinguish between receivers and providers of intra-group loans. Only the investment of receivers is sensitive to other affiliates' cash flows, because receivers benefit from the internal capital market whereas providers support it. As pledgeable income enables firms to multiply internal wealth and increase investment spending, the investment sensitivity to other affiliates' cash flows tends to be positive and to increase with pledgeable income.

Moreover, the likelihood that a firm will receive intra-group loans increases with productivity, pledgeable income, and controlling shareholder cash flow rights. In other words, the same factors that make a firm a good candidate for external finance also increase its odds of accessing the internal capital market. Finally, if financially strong firms are those that receive resources from other affiliates in business groups, these firms will be able to invest more than their standalone counterparts. On the flip side, if the financially weak firms in a business group tend to support the internal capital market by sharing their positive cash flow with other affiliates, these firms will have fewer resources available, and, consequently, they will invest less than similar non–business group firms.

The article proceeds as follows. In the next section, we discuss the related literature on resource allocation in business groups' internal capital markets. Next, in Section 3, we develop our model for financial resource allocation in business groups. In Section 4, we discuss the key results and relate them to the literature. Next, in Section 5, we state the main empirical implications derived from our model and provide some guidance to test them in future research. Finally, Section 6 concludes.

2. Related literature

Our article rests on both theoretical and empirical research on resource allocation in internal capital markets. From a theoretical standpoint, Stein (1997) develops an investment model in conglomerates (multidivisional firms) in which headquarters with proper incentives and power to freely transfer resources between divisions engage in a winner-picking strategy, taking scarce funds from low-productivity divisions to give to high-productivity divisions, improving overall performance. Although a winner-picking strategy could emerge in our model, this will not always be the case. In our model, productivity and pledgeable income interact to determine internal resource allocation, and, in some cases, it will be optimal for business groups to allocate more resources to an affiliated firm with higher pledgeable income, even if this unit is not the most productive. These differences in predictions arise because business group affiliates are independent legal entities and must approach outside investors based on their own merits, whereas, in Stein's (1997) model, the conglomerate's headquarters approaches outside investors for funding and then allocates resources across divisions, with the entire conglomerate being liable for the debt repayment.

In addition, Cestone and Fumagalli (2005) provide one of the first attempts to model the resource allocation decisions in business groups' internal capital markets.[1] Our model shares several of their assumptions[2] and, in a broader sense, some of the results. For example, in both our model and theirs, winner picking and cross-subsidization can arise in business groups' internal capital markets. However, there are also notable differences. In Cestone and Fumagalli's model, the outcome depends mainly on the amount of internal wealth available (the intensity of financial constraints at the group level), winner picking (cross-subsidization) being more likely if the business group suffers strict (loose) financial constraints. In our model the direction of resources in the internal capital market depends mainly on each group's affiliated firms' characteristics, such as private benefits of control, asset tangibility, risk-shifting problems, controlling shareholder cash flow rights, and productivity.

The differences in allocations in business groups between Cestone and Fumagalli's (2005) and our model arise because of different assumptions regarding the investment decisions of the business group affiliates. First, Cestone and Fumagalli (2005) assume a fixed investment size, whereas we opt for a model with continuous investment. This explains why their results are based on the amount of internal wealth and ours is not (as long as there are financial constraints). Second, in Cestone and Fumagalli's model, as moral hazard is in the form of costly and unobservable managerial effort, productivity and pledgeable income go hand in hand; that is, the most productive firm is also the one with the highest income to pledge to outside investors. In our model, we disentangle these constructs, allowing pledgeable income to interact with other variables besides productivity. This approach explains why, in our case, the direction of resources in the internal capital market depends on all these group-affiliated firm variables and why resources can be shifted to firms of low productivity but high pledgeable income. In this sense, our model innovates as it shows how productivity and variables related to financial capacity interact to determine resource allocation in business groups' internal capital markets, extending prior literature on conglomerates (Stein, 1997) and business groups (Cestone and Fumagalli, 2005).

Lastly, Samphantharak (2006) develops a dynamic investment model for business groups with costly external finance. In that model, assuming that a controlling shareholder can freely transfer resources within the group, including funds raised in the external financial markets, all firms in the group will borrow until their marginal costs of external finance are equal, giving rise to an "insurance effect" across affiliated firms. That is, through internal transfers, the entire business group absorbs an idiosyncratic shock affecting the cost of external finance in one particular firm. These transfers also give rise to a "tunneling effect" in which firms with lower costs of external finance provide resources to firms with higher costs of capital. These predictions confirm prior literature on business groups that claims that internal capital markets can mitigate firms' financial constraints (Khanna and Palepu, 2000; Khanna and Yafeh, 2007).

Our model diverges from that of Samphantharak (2006) in several ways. First, our assumptions make external finance (and investment) proportional to the internal resources available to each group firm, whereas Samphantharak (2006) uses a costly external borrowing approach in which external finance becomes more expensive as it increases, but its availability is not directly related to the amount of internal resources available to each affiliated firm. In short, our financial constraints are in terms of *quantity* and those of Samphantharak (2006) are in terms of *cost*.

Second, neither the insurance effect nor the tunneling effect show up in our model. In contrast, in our model, an external financial supply shock affects a focal firm's pledgeable income, reducing the firm's likelihood of obtaining resources in the internal capital market. In other words, the external shock is amplified within the business group. Third, in our model we do not allow joint responsibility for loans. Each group firm has its own budget constraint and must raise external finance based on its own merits. We also require that affiliated firms that received resources in the internal capital markets are liable for repayment, that is, if a firm receives resources today, it must repay an (interest-adjusted) amount in the future. Our assumption differs from Samphantharak's (2006) model in which an affiliated firm has no liability to repay, and the controlling shareholder can freely shift resources across group firms, as long as they add up to zero. Samphantharak's (2006) assumption makes the business group behave as if it had only one joint business constraint.[3] Altogether, these differences lead to different outcomes and implications for the resource allocation and efficiency in business groups' internal capital markets, as discussed in the next section.

From an empirical perspective, this study relates to those of Almeida and Wolfenzon (2006a,b). Assuming that internal capital markets mitigate the limited pledgeability problem that characterizes external financial markets, Almeida and Wolfenzon (2006a) show that conglomerates' internal capital markets can reduce the efficiency of economy-wide capital allocation. This result is especially salient in countries with intermediate levels of investor protection. Even though we do not look for such an economy-wide equilibrium effect, our model suggests that internal capital markets in business groups could bear the same characteristics as external markets. That is, internal capital markets in business groups might not mitigate the limited pledgeability problem. If this is the case (as we predict), there could be an even greater loss of efficiency in economy-wide capital allocation than that noted by Almeida and Wolfenzon (2006a).

Finally, Almeida and Wolfenzon (2006b) provide a theoretical rationale for the formation of pyramidal ownership in family business groups. They show, for example, that family business groups should be more common in countries with low levels of investor protection, because families can use resources from firms they already control to finance new ones. The authors argue that this financing advantage over other entrepreneurs is more important in countries with weak investor protection, where pledgeable income tends to be lower. Almeida and Wolfenzon (2006b) suggest that financial factors can foster the formation of family business groups in weak–investor protection environments. Our model suggests that these same factors could be the key drivers of resource allocation in the internal capital markets in business groups.

3. The model

We develop a simple model in the spirit of Tirole (2006) to derive empirical implications about the investment behavior and external/internal financing in business group–affiliated firms. We propose a one-period model in which a risk-neutral entrepreneur entirely (and directly) owns a firm U (up). Firm U, along with outside investors (also risk neutral), owns a second firm, called D (down). The entrepreneur controls these two firms and owns a fraction β of the capital (economic rights) of firm D (directly and indirectly through firm U).

[1] The main focus of this study is the interaction between internal capital markets and product market competition.

[2] For example, after receiving their internal capital allocations, business groups members raise additional resources from outside investors for investment; most importantly, the rest of the group is not liable for this external debt.

[3] It is worth noting that the power to freely shift resources in the internal capital markets can harm the interests of outside investors of the donor firm.

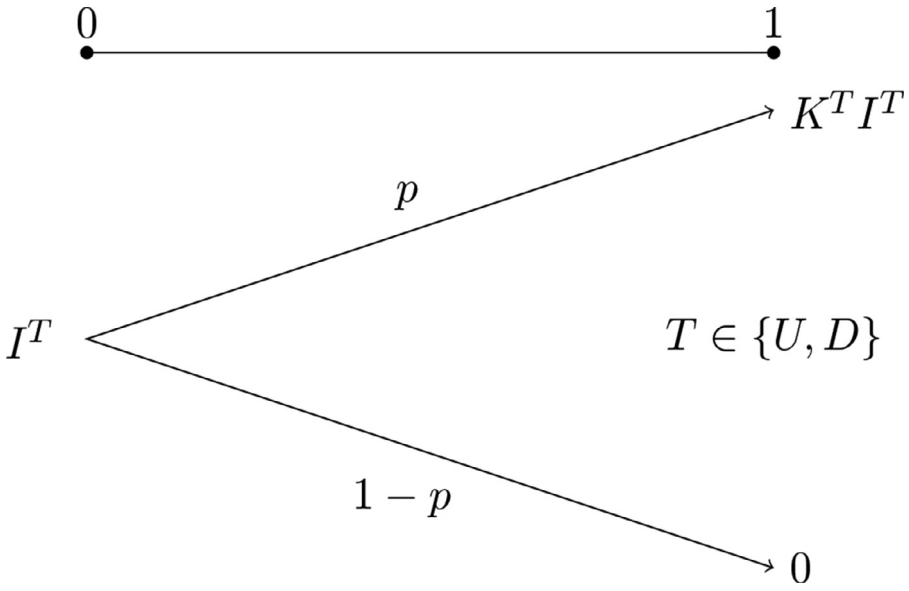

Fig. 1. Timing of the model.

The entrepreneur is assumed to retain control over firm D, whatever the size of β.[4] The introduction of a wedge between control and cash flow rights has at least two relevant roles in our model. First, as we will see below, the entrepreneur's cash flow rights in firm D influences the direction of internal transfers. The higher the entrepreneur's cash flow rights in firm D, the higher are firm D's pledgeable income and its odds of getting support in the internal capital market. Second, it helps us to draw attention to another important difference between a business group and a conglomerate. In the latter there is no difference in cash flow rights between its divisions. In the former cash flow rights tend to vary a lot between its affiliates (see, for example, the evidence in Buchuk et al., 2014), since those affiliate firms do not usually share exactly the same minority shareholders and their shares in the group firms may differ. Therefore, as the entrepreneur's cash flow rights matter for the direction of internal transfers, the internal allocation of capital in a business group may be very different from that of a conglomerate with similar divisions.

On date 0, both firms have opportunities to invest. If firm U invests I^U on date 0, it will receive a cash flow of $K^U I^U$ with probability p (success), or zero with probability $1 - p$ (failure) on date 1. Similarly, if firm D invests I^D on date 0, it will receive a cash flow of $K^D I^D$ with probability p, or zero with probability $(1 - p)$ on date 1 (the two projects are independent). The timing of the model is shown in Fig. 1. The production function of both firms is linear, with K^U and K^D being the proportionality constants. We also allow that the firms may have a technology with decreasing returns to scale. We show in the Appendix that our model's implications about the resource allocation in business groups essentially hold in this extension of a more general production function.

To introduce moral hazard, we assume that the probability of success (of each project) depends on the entrepreneur's efforts. Therefore, if the entrepreneur behaves (exerts effort), the probability of success is p_H and there are no private benefits. If the entrepreneur misbehaves, the probability of success is $p_L < p_H = p_L + \Delta_p$ and the private benefits are B^U (B^D) per unit of investment in firm U (D).[5] That being said, as long as the projects are funded, the entrepreneur can work on either one or both, or cheat on both. Only projects with a probability p_H of success are considered socially desirable. In other words, p_L is assumed such that, if the entrepreneur misbehaves, the expected NPV (social surplus) per unit of investment is negative, even if private benefits are considered.

$$\begin{array}{ll} p_H K^U & > 1, \\ p_L K^U + B^U & < 1, \\ p_H K^D & > 1, \\ p_L K^D + B^D \div \beta & < 1. \end{array} \tag{1}$$

To achieve a finite level of optimum investment, we need to make an additional assumption about the productivity of investment and the extent of moral hazard (regarding pledgeable income). Following Tirole (2006), the expected NPV per unit of investment is lower than the per-unit agency cost related to the entrepreneur's misbehavior (i.e., the expected minimal income per unit of investment that is incentive compatible):

$$\begin{array}{ll} p_H\left(K^U - \frac{B^U}{\Delta_p}\right) & < 1, \\ p_H\left(K^D - \frac{B^D}{\beta \Delta_p}\right) & < 1. \end{array} \tag{2}$$

Therefore, there is a limit to the value that firms can raise from external investors, imposing a specific investment level, even though infinite levels of investment are optimal under no moral hazard. Assumption 2 is key to our model, as it implies financial constraints at the firm level: for each unit of investment, the income that can be pledged to outside investors, that is, the expected cash flow less the expected minimal income that ensures the entrepreneur will behave, is less than one, and firms must therefore supplement this amount with internal resources to finance this unit of investment. In other words, the amount that firms can raise in external capital markets and the level of investment depend partly on the internal resources available. This dependence on internal wealth is at the core of the investment models with moral hazard developed by Tirole (2006).

[4] Mechanisms like dual class shares and pyramidal structures allow the entrepreneur to exercise control over a group firm even with a small fraction of its total capital (Morck et al., 2005). It is worth noting that we opted for a pyramidal structure of control, but the results will be the same if we use a horizontal structure.

[5] Note that we are assuming that private benefits are asset specific, not human specific. Although we recognize that business groups can transfer human resources across affiliates, in our model changing the entrepreneur does not change the private benefits associated with each firm in the group.

Continuing, on date 0, firm U (D) has liquid assets (i.e., cash holdings) of A^U (A^D), and there is an internal capital market in which firms U and D can transfer resources between themselves on date 0 in exchange for an income on date 1. We denote by $(1-\alpha^T)$, with $T \in \{U, D\}$, the (observable) fraction of cash on date 0 that is transferred from one firm to another. An upper bound on the internal transfers (perhaps as a result of legal and statutory limits) is imposed,[6] requiring that $\alpha^T \in [\alpha, 1]$, with $0 < \alpha < 1$.

We also assume that business groups use direct loans to make internal transfers across affiliated firms. A direct loan is a common mechanism for allocating resources within business groups (Gopalan et al., 2007; Buchuk et al., 2014). As Buchuk et al. (2014) point out, the widespread existence of preemptive rights is the main reason why direct loans (internal debt) are often more convenient than internal equity (cross-ownership) as a way of transferring resources within a business group. In part, this is because preemptive rights give current shareholders the right to buy new shares issued by the firm, protecting them against the dilution of control of their shares.

We now assume that date 1 income from internal transfers cannot be contracted out of the business group. In other words, the lending firm cannot pledge this income to outside investors. For simplicity, interest rates are set to zero (no time discount). Under these conditions, the borrower, say, U, needs to promise an amount $(1-\alpha^D)A^D \div p_H$ on date 1, in the case of success, in exchange for a loan of $(1-\alpha^D)A^D$ on date 0 (we opt for a conditional debt contract between firms).

Because firms U and D are *legally independent*, we assume no cross-pledging, where one firm could potentially pledge another affiliate's income to external investors (lenders). This means that each group firm has its own budget constraint. Had we allowed the extreme case of "full" cross-pledging, as in Tirole (2006, section 4.2), the entire business group would behave as if it had a single joint budget constraint and one incentive compatibility constraint. Even in this limit case our main results still hold, that is, firms with high productivity and low private benefits need to invest more to scale up pledgeable internal resources.[7] Finally, we assume that the lender sector is competitive. Therefore, by having control over both firms, the entrepreneur will offer a contract to outside investors as follows:

- Firms' income in each state of the world (success, S, or failure, F): $R_S^T \geq 0$ and $R_F^T \geq 0$, with $T \in \{U, D\}$; that is, the borrower's limited liability imply that firms will receive zero in the case of failure.
- Each firm's level of investment: $I^T \geq 0$, with $T \in \{U, D\}$.
- Internal transfers from one firm to another: $(1-\alpha^T)A^T$, with $T \in \{U, D\}$.

The contract will solve the following problem (for details, see the Appendix):

$$\max_{\{R_S^T, R_F^T, I^T, \alpha^T\}} \; p_H\left(R_S^U - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H}\right) + (1-p_H)R_F^U + p_H\left(\beta R_S^D + (1-\beta)\frac{(1-\alpha^U)A^U}{p_H}\right) + (1-p_H)\beta R_F^D,$$

subject to four constraints that are binding at the optimal solution. The "investor rationality" constraints, IR^U and IR^D, require that, on average, outside investors get back their investment:

$$p_H\left(K^U I^U - R_S^U\right) - (1-p_H)R_F^U \geq I^U - \alpha^U A^U - (1-\alpha^D)A^D, \tag{3}$$

$$p_H\left(K^D I^D - R_S^D\right) - (1-p_H)R_F^D \geq I^D - \alpha^D A^D - (1-\alpha^U)A^U, \tag{4}$$

and the incentive compatibility constraints, IC^U and IC^D, ensure that the entrepreneur will choose to behave well in both projects:

$$\Delta_p\left((R_S^U - R_F^U) - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H}\right) \geq B^U I^U, \tag{5}$$

$$\Delta_p\left(\beta(R_S^D - R_F^D) + (1-\beta)\frac{(1-\alpha^U)A^U}{p_H}\right) \geq B^D I^D. \tag{6}$$

The non-negativity and internal transfer limitation constraints are expressed as follows:

$$R_S^U \geq 0,\ R_F^U \geq 0,\ R_S^D \geq 0,\ R_F^D \geq 0,\ I^U \geq 0,\ I^D \geq 0,$$

$$\alpha^U \in [\alpha, 1],\ \alpha^D \in [\alpha, 1].$$

Because the lender sector is competitive, the firms will earn the entire surplus. Under our assumptions about the productivity of investments and moral hazard in (1) and (2) and the limits of internal transfers, it is optimal that both firms invest: $I^U > 0$ and $I^D > 0$.

The investor's rationality constraints are binding; otherwise, firms could increase their payoffs without violating the incentive compatibility constraints. To show that the incentive compatibility constraints are also binding at the optimum, suppose that (5) is not binding (the same applies to (6)). Then, R_S^U, R_F^U, and I^U could be increased as long as the difference $R_S^U - R_F^U$ is constant and the increase in the payoffs is limited to $(p_H K^U - 1)$ times the increase in I^U. These changes will increase the value of the objective function without violating the investors' rationality constraint, so this solution cannot be optimal.

With strictly positive investment, the incentive compatibility constraint (5) implies that $R_S^U > R_F^U \geq 0$ (again, the same applies to firm D). Because the entrepreneur is risk neutral and will earn the entire social surplus of the investment, it is best for the entrepreneur to set the firm payoffs at a level that maximizes the pledgeable income. From the investors' rationality constraint (3), the pledgeable income is given by

$$p_H K^U I^U - p_H(R_S^U - R_F^U) - R_F^U.$$

Maintaining the difference $R_S^U - R_F^U$ to satisfy the incentive compatibility constraint and setting $R_F^U = 0$ maximizes the pledgeable income.[8] Under these conditions, the incentive compatibility constraints can be used to determine the payoffs in the case of success:

$$R_S^U = \frac{B^U I^U}{\Delta_p} + \frac{(1-\beta)(1-\alpha^D)A^D}{p_H}, \tag{1}$$

$$R_S^D = \frac{B^D I^D}{\beta \Delta_p} - \frac{(1-\beta)(1-\alpha^U)A^U}{\beta p_H}. \tag{2}$$

The investors' rationality constraints determine the level of investment of each firm (after replacing R_S^U and R_S^D by (1) and (2)), as follows:

$$I^U = \frac{\alpha^U A^U + \beta\left(1-\alpha^D\right)A^D}{\left[1 - p_H\left(K^U - \frac{B^U}{\Delta_p}\right)\right]} = M^U \times \left(\alpha^U A^U + \beta\left(1-\alpha^D\right)A^D\right), \tag{3}$$

[6] Actually, assumption 2 will constrain α^T to be greater or equal to zero. We require α^T to be strictly positive so both firms invest. In the Appendix, we comment on this assumption and show that our predictions do not change.

[7] One can also argue that business groups could raise more resources than a comparable portfolio of standalone firms, due to coinsurance effects, for example. As pointed out by Berger and Ofek (1995) and Scharfstein and Stein (2000) for diversified conglomerates, these effects are of trivial importance.

[8] In the Appendix, we provide proof that, at the optimum, $R_F^U = 0$ and $R_F^D = 0$, using the Karush–Kuhn–Tucker multipliers.

$$I^D = \frac{\beta\alpha^D A^D + \left(1-\alpha^U\right)A^U}{\beta\left[1 - p_H\left(K^D - \frac{B^D}{\beta\Delta_p}\right)\right]} = M^D \times \left(\beta\alpha^D A^D + \left(1-\alpha^U\right)A^U\right). \quad (4)$$

In (3) and (4), respectively, M^U and M^D are the equity multipliers, where equity refers to the entrepreneur's wealth ($A^U + \beta A^D$), split between firms by internal transfers. One can see that, under assumptions (1) and (2), both multipliers are greater than one but finite. They are finite because, under assumption (2), the minimal income that is incentive compatible increases faster than the NPV when investment is increased. Therefore, investors' rationality constraints bind with finite levels of investments. In short, moral hazard implies limits to the investment level, reducing the entrepreneur's utility.

It is worth noting that, if firms U and D were standalone entities, their equity multipliers would be the same as in (3) and (4), respectively. However, in this case, each firm can only rely on the wealth of its own entrepreneur. Internal capital markets in business groups can transfer the entrepreneur's wealth across group firms. Hence, with the appropriate incentives, the entrepreneur can increase the total output (over what it would be if the group firms were standalone entities).

The entrepreneur will earn the surplus from investment according to the number of shares owned in each firm and will thus benefit from higher multipliers. Taking the partial derivatives of the multipliers with respect to the exogenous parameters, we have:

- Multipliers increase with p_H and Δ_p. All else being equal, the higher p_H (Δ_p), the greater the income that can be pledged to outside investors and the lower the minimal income that requires the entrepreneur to behave.
- The term M^U (M^D) increases with K^U (K^D). All else being equal, more productive investment attracts more external finance.
- The term M^U (M^D) decreases with B^U (B^D). The minimal income that the entrepreneur needs to behave increases with private benefits, reducing the pledgeable income.
- The term M^D (but not M^U) increases with β. All else being equal, the higher the entrepreneur's cash flow rights in firm D, the lower the minimal income the entrepreneur needs to behave in this firm and, consequently, the higher firm D's pledgeable income. Remember that the entrepreneur receives a fraction β of the cash flows of firm D in the case of success, but gets 100% of the private benefits when she misbehaves. If cash flow rights are low, the entrepreneur cannot promise much of the firm income to external investors because she needs too much of that income in order to have incentives to exert sufficient effort and give up private benefits. In other words, the smaller β is the larger the moral hazard problem in firm D is.[9]

What remains to be determined are the internal transfers between firms, α^U and α^D. To show how the internal capital market works, the entrepreneur's problem is rewritten using the optimal values of the endogenous variables, except α^U and α^D. The entrepreneur's expected total income equals the expected NPV of the investment in firm U plus a fraction β of the expected NPV of the investment in firm D plus the entrepreneur's initial wealth, $A^U + \beta A^D$, as follows:

$$(p_H K^U - 1)I^U + \beta(p_H K^D - 1)I^D + A^U + \beta A^D. \quad (5)$$

Substituting (3) and (4) into I^U and I^D, respectively, the entrepreneur's objective function becomes:

$$F(\alpha^U, \alpha^D) = (p_H K^U - 1)\frac{\alpha^U A^U + \beta\left(1-\alpha^D\right)A^D}{\left[1 - p_H\left(K^U - \frac{B^U}{\Delta_p}\right)\right]} + (p_H K^D - 1)\frac{\beta\alpha^D A^D + \left(1-\alpha^U\right)A^U}{\left[1 - p_H\left(K^D - \frac{B^D}{\beta\Delta_p}\right)\right]} + A^U + \beta A^D. \quad (6)$$

Next, it is possible to determine how the entrepreneur's expected total income changes when α^U or α^D increases:

$$\frac{\partial F(\alpha^U, \alpha^D)}{\partial \alpha^U} = \frac{(p_H K^U - 1)A^U}{\left[1 - p_H\left(K^U - \frac{B^U}{\Delta_p}\right)\right]} - \frac{(p_H K^D - 1)A^U}{\left[1 - p_H\left(K^D - \frac{B^D}{\beta\Delta_p}\right)\right]}, \quad (7)$$

$$\frac{\partial F(\alpha^U, \alpha^D)}{\partial \alpha^D} = \frac{(p_H K^D - 1)\beta A^D}{\left[1 - p_H\left(K^D - \frac{B^D}{\beta\Delta_p}\right)\right]} - \frac{(p_H K^U - 1)\beta A^D}{\left[1 - p_H\left(K^U - \frac{B^U}{\Delta_p}\right)\right]}. \quad (8)$$

Note that the partial derivatives depend only on the exogenous parameters and, if (7) is positive [negative] (zero), then (8) is negative [positive] (zero), and vice versa. Thus, there are three possible alternatives of internal transfers in business groups that we shall discuss in turn.

3.1. Internal transfers from D to U

Internal transfers from D to U occur if and only if

$$B^D(p_H K^U - 1) > B^U\beta(p_H K^D - 1).$$

If this condition holds, (7) is positive, (8) is negative, and the entrepreneur's expected total income increases with α^U and decreases with α^D. Three factors can contribute to this result: (a) the investment productivity of firm U, K^U, is higher than that of firm D, K^D; (b) there are fewer private benefits associated with firm U's investment, B^U, than with firm D's investment, B^D — that is, all else being equal, the minimal income that motivates the entrepreneur to behave is lower and, therefore, pledgeable income is higher in firm U vis-à-vis firm D — and (c) entrepreneur cash flow rights in firm D, β, are low enough to distort the socially efficient capital allocation.[10]

As the entrepreneur's expected total income increases (decreases) with α^U (α^D), the internal transfer goes from firm D to firm U, up to the upper bond of internal transfers in which $\alpha^U = 1$ and $\alpha^D = \alpha$. In this case, the sensitivities of the firm's investment to its cash flow and to the other firm's cash flow are[11]:

$$\frac{\partial I^U}{\partial A^U} = M^U > 0,$$
$$\frac{\partial I^U}{\partial A^D} = M^U \times \beta(1-\alpha) > 0,$$
$$\frac{\partial I^D}{\partial A^U} = 0,$$
$$\frac{\partial I^D}{\partial A^D} = M^D \times \beta\alpha > 0.$$

[9] Our assumption is that the entrepreneur has all the cash flow rights in firm U. If we assume that the cash flow rights are of size β^U, the equity multiplier of this firm, M^U, will also increase with β^U.

[10] Social efficiency refers to the allocation that provides the higher expected NPV. In our setting, the NPV of an investment depends not only on its productivity, but also on its capacity to attract financing.

[11] In our static one-period model, A^U and A^D can be regarded as both a flow (cash flows from existing assets) and a stock (cash holdings). We use comparative statics on A^U and A^D to derive our investment–cash flow sensitivities. To justify this, we resort to DeMarzo and Fishman (2007).

Because of the unidirectionality of internal transfers, firm U's investment increases with its cash flow and with the other firm's cash flow, and firm D's investment increases with its cash flow and is insensitive to the other firm's cash flow. In this case, the business group's resources flow toward firm U, and the investment in firm D is proportional to its cash flow and occurs only because there are limits to internal transfers.

Finally, following Tirole (2006), the sensitivity of investment to the firm's cash flow (and to the other firm's cash flow, when applicable) is reduced with the private benefits. This happens because of the negative effect of private benefits on pledgeable income and, consequently, on the equity multiplier. Therefore, in our model, firms with low agency costs will exhibit greater investment-cash flow sensitivity.

3.2. Internal transfers from U to D

Internal transfers from U to D occur if and only if

$$B^D(p_H K^U - 1) < B^U \beta (p_H K^D - 1).$$

If this condition is met, (7) is negative and (8) is positive, so the entrepreneur's expected total income decreases with α^U and increases with α^D. Again, three factors can contribute to this result: (a) The investment productivity of firm D, K^D, is higher than that of firm U, K^U; (b) there are fewer private benefits associated with firm D's investment, B^D, than with firm U's investment, B^U—that is, all else being equal, the minimal income that encourages the entrepreneur to behave is lower and, therefore, the pledgeable income is higher in firm D vis-à-vis firm U—and (c) the entrepreneur's cash flow rights in firm D, β, are high enough not to distort the socially efficient capital allocation.

As the entrepreneur's expected total income decreases (increases) with α^U (α^D), the internal transfer goes from firm U to firm D, up to the upper bond on internal transfers in which $\alpha^U = \alpha$ and $\alpha^D = 1$. In this case, the sensitivities of a firm's investment to its cash flow and to the other firm's cash flow are

$$\frac{\partial I^U}{\partial A^U} = M^U \times \alpha > 0,$$
$$\frac{\partial I^U}{\partial A^D} = 0,$$
$$\frac{\partial I^D}{\partial A^U} = M^D \times (1 - \alpha) > 0,$$
$$\frac{\partial I^D}{\partial A^D} = M^D \times \beta > 0.$$

Firm D's investment increases both with its cash flow and with the other firm's cash flow. Firm U's investment increases with its cash flow and is insensitive to the other firm's cash flow. Now, the business groups' resources flow toward firm D, and the investment in firm U is proportional to its cash flow and occurs only because there are limits to internal transfers.

As before, the sensitivity of investment to the firm's cash flow (and to the other firm's cash flow, when applicable) of both firms decreases with private benefits. This happens because of the negative effect of private benefits on pledgeable income and, thus, on the equity multiplier. Again, firms with low agency costs will exhibit greater investment–cash flow sensitivity.

3.3. No internal capital market

There is no internal capital market if and only if

$$B^D(p_H K^U - 1) = B^U \beta (p_H K^D - 1).$$

If this condition is met, (7) and (8) are equal to zero, and the entrepreneur's expected total income does not depend on α^U or α^D. This independence of the entrepreneur's income from α^U and α^D can occur if, for example, the pledgeable income and the NPV per unit of investment (from the entrepreneur's perspective) are very similar across firms.

As the entrepreneur's expected total income does not depend on α^U or α^D, the internal transfers are undetermined; that is, any admissible values of α^U and α^D are optimal. We assume that, under these circumstances, the entrepreneur will opt for the simplest contract in which there is no transfer across firms ($\alpha^U = \alpha^D = 1$). Consequently, the investment–cash flow sensitivities are

$$\frac{\partial I^U}{\partial A^U} = M^U > 0,$$
$$\frac{\partial I^U}{\partial A^D} = 0,$$
$$\frac{\partial I^D}{\partial A^U} = 0,$$
$$\frac{\partial I^D}{\partial A^D} = M^D \times \beta > 0.$$

The investments in firms U and D increase with firm cash flows and are insensitive to the other firm's cash flow. Thus, without internal transfers, the investment in each firm is proportional to the entrepreneur's cash flow in that firm, with the constant of proportionality equal to the equity multiplier. Finally, the sensitivity of investment to cash flow for both firms decreases with private benefits. Again, the investments of companies with low agency costs will be more sensitive to their cash flow.

To better illustrate how differences in investment productivity (K^U and K^D), private benefits (B^U and B^D), and entrepreneur's cash flow rights (β) affect the allocation of resources in our model of business groups' internal capital markets, we plot the outcomes of this process, that is, which firm (U or D) will receive internal resources (alternatives A to C above) as we change the values of these variables. We start from a base case in which we set the variables as follows: $p_H = 0.7$, $K^U = 1.5$, $K^D = 1.6$, $B^U = 0.048$, $B^D = 0.06$, and $\beta = 0.75$. In this base case, firm D is more productive than firm U, but due to higher private benefits it has lower pledgeable income than firm U. Moreover, the entrepreneur's cash flow rights are higher in firm U (100%) than in firm D (75%). Despite these limitations, in this base case, the resources of this internal capital market should flow toward firm D (alternative B), as its higher productivity more than compensates for its lower pledgeable income.

In Fig. 2, we let B^U and B^D be fixed at their base case values and we vary K^U (horizontal axis), K^D (vertical axis), and β (panels). All else being equal, an increase in the productivity of a firm tends to favor it as a receiver of resources in the internal capital market. For example, an increase in firm U's productivity (horizontal movement) makes this firm a better candidate to receive internal resources. The same applies to firm D (vertical movement). Regarding the entrepreneur's cash flow rights in firm D (β), the higher the β, the smaller the difference between the productivity of firm D and that of firm U must be so that the former is the receiver of resources in the internal capital market. In the hypothetical case in which both firms' productivity follows a uniform distribution in the intervals in the graphs, the probability that firm D will be the receiver of internal resources corresponds to the fraction of the total dark gray area in the figure. As shown, the more the entrepreneur's cash flow rights in firm D increase (from the left to the right panel), the higher (lower) the likelihood that firm D (U) will receive resources in the internal capital market.

In Fig. 3, we let K^U and K^D be fixed at their base case values and we vary B^U (horizontal axis), B^D (vertical axis), and β (panels). All else being equal, an increase in private benefits in one firm tends to favor other firms in the internal capital market as potential receivers of resources. This occurs because an increase in

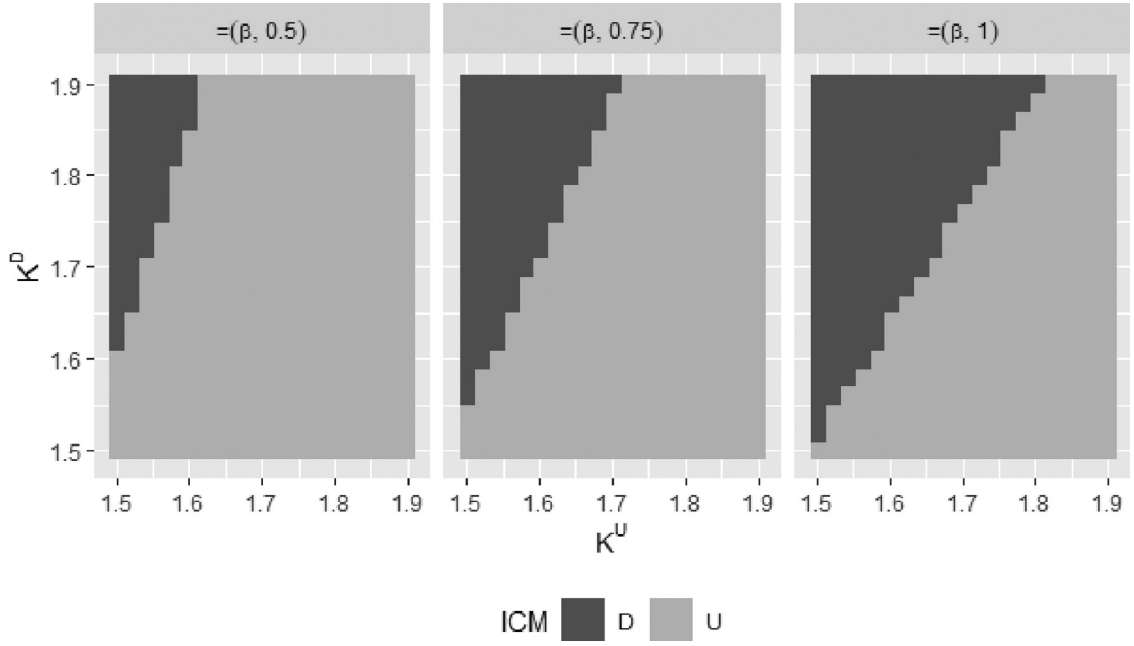

Fig. 2. Internal capital market (ICM) outcomes and productivity differences.

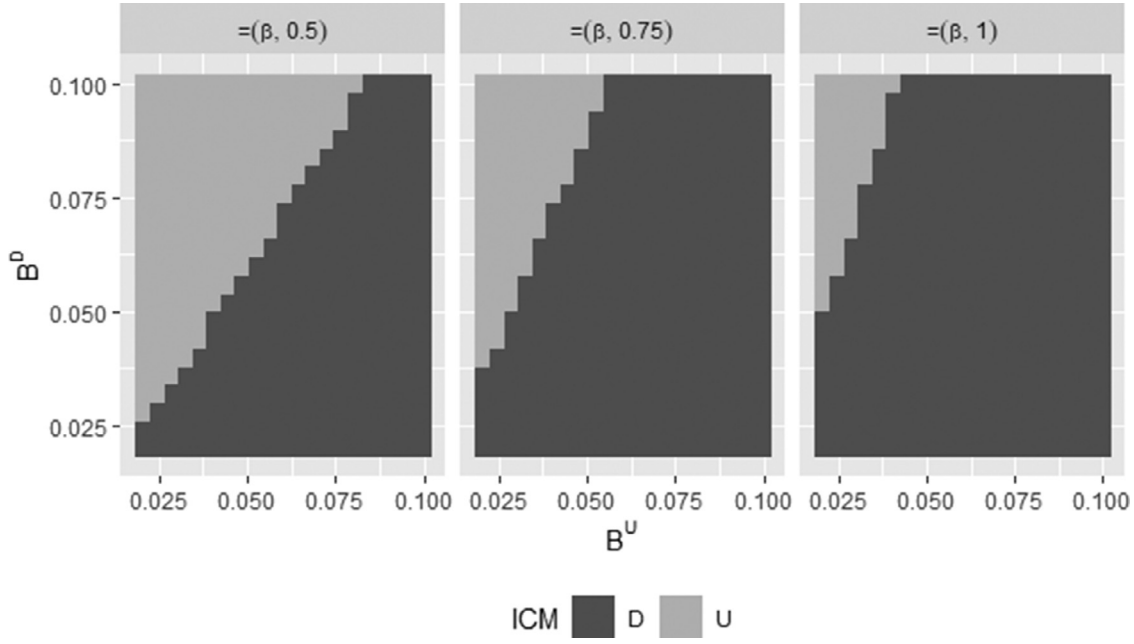

Fig. 3. Internal capital market (ICM) outcomes and private benefit differences.

private benefits means lower pledgeable income and so a lower chance of being supported by the internal capital market. For example, an increase in firm *D*'s private benefits (vertical movement) makes the other firm (*U*) a better candidate to receive internal resources. The opposite occurs if firm *U*'s private benefits are increased (horizontal movement). The higher the entrepreneur's cash flow rights in firm *D*, the higher the difference between the private benefits of firms *D* and *U* must be so that the latter is the target of resources in the internal capital market. In the hypothetical case in which both firms' private benefits follow a uniform distribution in the intervals in the graphs, the probability that firm *D* will be the receiver of internal resources corresponds to the fraction of the total dark gray area. As shown, as the entrepreneur's cash flow rights in firm D increase (from the left to the right panel), the higher (lower) the likelihood that firm *D* (*U*) will be the receiver of resources in the internal capital market.

Next, we discuss the key results of our model.

4. Discussion

There are two leading explanations in the literature for the dynamics of business groups' internal capital markets. The institutional voids hypothesis suggests that business groups may work as a second-best solution to underdeveloped external capital markets (Almeida and Wolfenzon, 2006a; 2006b; Khanna and Yafeh, 2007; Kali, 2003). The tunneling hypothesis argues that majority shareholders use pyramidal business groups to magnify their control position and potentially extract private benefits at the ex-

pense of minority shareholders (Baek et al., 2006; Bebchuk et al., 2000; Bertrand et al., 2002). Accordingly, the direction of resources' transfers inside business groups depends upon two factors that vary across business groups' affiliated firms: investment productivity and the entrepreneur's cash flow rights.

These two factors are useful but limited to explain how internal capital markets work. For example, Larrain and Urzña (2016) show that business groups remained mostly unchanged despite a deep economic transformation in Chile, casting doubt on the institutional voids hypothesis. In addition, Siegel and Choudhury (2012) show that internal transfers are driven by business strategies that differ remarkably across business group firms and standalone firms, finding no evidence of tunneling in Indian business groups. More recently, Belenzon et al. (2019) show that many groups in Western European economies are largely wholly-owned, suggesting that the tunneling hypothesis may not be the primary reason for internal capital markets.

Our model extends this literature to provide a third and novel factor – the pledgeable income's differences across affiliated firms. To the best of our knowledge, we still do not have a theory that explains how pledgeable income influences the direction of resource allocation in the internal capital market. In a business group, each firm is a legally independent entity with direct access to the external capital market. To access this market, each group-affiliated firm can rely only on its merits and its pledgeable income. As Eq. (6) shows, the surplus of an investment depends on the interaction between its marginal expected net present value (productivity) and its equity multiplier (pledgeable income). Therefore, the entrepreneur will direct resources to the firm with the higher product of investment productivity and pledgeable income. That is, pledgeable income matters in financing decisions in both internal and external capital markets.

It is worth emphasizing that, in our model, pledgeable income is a function of the following three parameters: cash flow rights, productivity, and private benefits. Pledgeable income is positively related to entrepreneur's cash flow rights (β) and a firm's productivity (K^U and K^D), and negatively related to private benefits (B^U and B^D). An increase in cash flow rights lowers the minimal income needed for the entrepreneur to behave well in firm D, boosting the pledgeable income and the likelihood of this firm receiving funds in the internal capital market.[12]

Also, an increase in private benefits raises the minimal income necessary for the entrepreneur to behave and thus reduces the pledgeable income. Consequently, lower private benefits increase the likelihood of financing new investment in both internal and external capital markets. Similarly, higher investment productivity increases the pledgeable income and raises the odds of obtaining both resources from the other business group affiliates and external finance.

In sum, we argue that any factor affecting the firm's pledgeable income, that is the ability to raise external finance, also affects the firm's likelihood of obtaining financing in the internal capital market. To prove this main result, assume that one unit of investment in firm U (D) requires raising $\tau^U \geq 1$ ($\tau^D \geq 1$) units of internal or external money. We can think of τ^T as a proxy for factors that reduce the firm's ability to finance its projects, including low pledgeable assets (collateral), a high probability of risk shifting, and high levels of asymmetric information. Assuming that the investment in both firms is still profitable, we can show that:

$$I^U = \frac{\alpha^U A^U + \beta\left(1-\alpha^D\right)A^D}{\left[\tau^U - p_H\left(K^U - \frac{B^U}{\Delta_p}\right)\right]} = M'^U \times \left(\alpha^U A^U + \beta\left(1-\alpha^D\right)A^D\right), \quad (9)$$

$$I^D = \frac{\beta\alpha^D A^D + \left(1-\alpha^U\right)A^U}{\beta\left[\tau^D - p_H\left(K^D - \frac{B^D}{\beta\Delta_p}\right)\right]} = M'^D \times \left(\beta\alpha^D A^D + \left(1-\alpha^U\right)A^U\right). \quad (10)$$

Hence, under financial constraints, the equity multiplier of both firms is reduced; that is, if $\tau^U > 1$, then $M'^U < M^U$, and if $\tau^D > 1$, then $M'^D < M^D$. This is the result of a reduction in the pledgeable income of the firms and implies a lower level of investment. The direction of resources inside the business group will now depend on the following inequality:

$$B^D\left(p_H K^U - \tau^U\right) \gtreqless B^U\beta\left(p_H K^D - \tau^D\right).$$

All else being equal, the higher the τ^U (τ^D), the lower the chance that internal resources will flow from firm D (U) to firm U (D). In other words, the same factors that limit a firm's access to external finance also reduce the likelihood that the same firm will receive resources in the internal capital market. As far as we know, this is a novel prediction, shedding new light on our understanding of the formation and functioning of business groups. If internal capital markets reproduce the same logic of external capital markets, as our model suggests, we should question the idea that they can alleviate the financial constraints of affiliated firms (Khanna and Palepu, 2000; Khanna and Yafeh, 2007), especially for financially weak ones.

Finally, the prediction that productivity and pledgeable income jointly determine the direction of resources partially explains the evidence of socialist cross-subsidization in business groups. If productivity and pledgeable income are weakly associated and the latter varies more than the former, our model predicts that pledgeable income will be the most important factor in explaining resource allocation within a business group. This outcome implies that the omission of pledgeable income can produce a conclusion biased toward the socialist cross-subsidization hypothesis.

5. Empirical implications

In this section, we state the main empirical implications derived from our model. Testing these implications requires detailed data from business groups and standalone firms, including internal transfers of financial resources across firms in business groups. In order to enhance the empirical relevance of our results, we also present a road-map to test each implication. Lastly, we relate our main empirical implications with the findings of the existing literature.

The main implication of our model is that the same factors that limit companies' access to external finance also reduce their chance to obtain internal resources in business groups. Firms with high levels of pledgeable income (and thus easy access to external finance) will be more likely to benefit from resource allocation within a business group. In other words, internal capital markets tend to support the financially strong firms in a group, just as outside lenders would, reproducing the same financial constraints that plague external financial markets, leading to the following implication:

Empirical Implication 1. The likelihood of obtaining resources in the internal capital market increases with productivity, pledgeable income, and the entrepreneur's cash flow rights.

According to our model, receivers of intra-group loans tend to be those group firms that (relative to other firms in the same group) have high productivity, high cash flow rights of controlling shareholders, low private benefits, low risk-shifting problems, and high asset tangibility. Except for the productivity and cash flow rights factors, this is a novel hypothesis, and it was discussed at length in the final part of the previous section.

[12] Again, if we assume that the entrepreneur's cash flow rights in firm U is β^U, this same effect will be present in firm U.

To test this empirical implication one needs market and financial data from firms affiliated to business groups, including information regarding the internal transfers between these firms and the composition of business groups over time. In each business group, affiliated firms can be classified as either providers or receivers of internal resources. This classification refers to the dependent variable and could be computed following Gopalan et al.'s (2007) and Buchuk et al.'s (2014) approaches. The main explanatory variables should include proxies for productivity, pledgeable income, and controller shareholder's cash flow rights.

Since our empirical implication relies on the relative position of firms within the business group, in contrast to Gopalan et al. (2007) and Buchuk et al. (2014), one should measure the explanatory variables relative to the other affiliates of the same business group. For example, in the case of a productivity measure one can employ a categorical variable that classifies firms above and below the median of Tobin's Q in the group in a given year (as in Shin and Park, 1999; Lee et al., 2009). Alternatively, one could follow Almeida et al. (2015) and use the original explanatory variables and group-year fixed effects in an OLS estimation. As one needs to focus on firm-level variation within a group in the same year, we propose to use group-year fixed effects, which differs from Gopalan et al. (2007)'s approach that uses group and year fixed effects independently.

Finally, when it comes to the measure of pledgeable income, one could follow Almeida and Campello (2007) and Campello and Hackbarth (2012). These studies proxy pledgeable income using asset tangibility. In general, fixed assets can serve as collateral in debt agreements and so increase a firm's ability to raise external capital. The use of tangibility as a proxy for pledgeable income is also supported in countries with low levels of financial development where collateral requirements tend to be higher and target non-specific assets (Liberti and Mian, 2010).

Empirical Implication 2. The size of transfers in the internal capital market increases with productivity, pledgeable income, and the entrepreneur's cash flow rights.

Under decreasing marginal productivity (see our extended model in the Appendix), the magnitude of resources transferred to receivers increases with the differences in productivity, pledgeable income, and controlling shareholder cash flow rights between group affiliates. The intuition behind this result is that the controlling shareholder will transfer resources to eliminate the marginal productivity and pledgeable income gap between affiliated firms. Therefore, the wider the gap, the greater the amount is that must be transferred to eliminate it.

This implication is an extension of the *Empirical Implication 1*. Differences in productivity, pledgeable income, and controlling shareholder cash flow rights determine both the direction (extensive margin) and the size of resources (intensive margin) being transferred in an active internal capital market. One could test this implication by following Gopalan et al. (2007) and investigating the intensive margin using as a dependent variable the ratio of net intra-group loans received to total assets.

If *Empirical Implication 1* is true, financially strong firms in business groups will be able to raise more resources and, consequently, will invest more than their standalone counterparts. The contrary occurs with financially weak firms that are more likely to be lenders, supporting the internal capital market and relying on only a fraction of their wealth to finance their investments. Thus, financially weak affiliated firms will have fewer available resources and will tend to invest less than their standalone counterparts that do not have related firms to finance. This reasoning leads to the following implication:

Empirical Implication 3. Financially strong (weak) firms in the business group tend to invest more (less) than their standalone counterparts, because these firms tend to benefit from (support) internal capital markets. In addition, the wider the gap in productivity, pledgeable income, and controlling shareholder cash flow rights within business groups, the higher the absolute difference in investment intensity between group affiliated and standalone firms.

If true, this implication raises questions about the effectiveness of internal capital markets in overcoming external capital markets' failures, as hypothesized by Khanna and Palepu (2000) and Khanna and Yafeh (2007). Given that productivity and pledgeable income have a positive effect on the likelihood of getting resources from internal capital markets, the factors that improve a firm's ability to get external finance, such as asset tangibility, also increase the likelihood of internal financing in business groups.

To test this empirical implication one could compare how the differences in investment intensity between group affiliated and standalone firms depends on the financial status (financially strong/weak) of the affiliated firms. According to the view that internal capital markets overcome the failures of external financial markets (Khanna and Palepu, 2000; Khanna and Yafeh, 2007), one should expect that business group affiliates invest more than standalone firms, especially in the case of financially weak firms. Our model challenges this hypothesis and posits that: i) within financially weak firms, business group affiliates invest less than standalone firms; and ii) within financially strong firms, business group affiliates invest more than standalone firms.

In our model, the magnitude of internal transfers increases with the gap in productivity, pledgeable income, and controlling shareholder cash flow rights within business groups. Therefore, one should observe higher differences in investment intensity between group affiliated and standalone firms in groups with wider gaps. Namely, the investment absolute difference relative to standalone firms should be higher the wider the gap is in productivity, pledgeable income, and controlling shareholder cash flow rights within the business group.

To contrast group affiliates and standalone firms, in addition to the data described above, one needs to collect financial and market data from standalone firms with relevant characteristics similar to those of the firms affiliated to business groups.

As already discussed, tangibility of assets can be used as a proxy for pledgeable income and thus to split business groups affiliates in the two groups: financially strong and financially weak firms (Almeida and Campello, 2007; Campello and Hackbarth, 2012; Liberti and Mian, 2010). For example, one could classify as financially strong (weak) the affiliated firms with tangibility above (below) the median of tangibility in the focal business group in a given year.

In the institutional voids perspective, internal capital markets work as the second best financing solutions to external capital markets failures. Our model makes the opposite prediction, that is, internal capital markets work similarly to external capital markets and so will magnify the difference in investment rates of business groups affiliates with similar growth-opportunities and cash flows, but different pledgeable income. New empirical results will shed light on our understanding about the role of internal capital markets in business groups.

Next, we discuss the empirical implications related to the investment sensitivity to the focal firm's cash flow and other group-affiliated firms' cash flow.

Empirical Implication 4. Investment–cash flow sensitivity is positive and increases with pledgeable income (e.g., firms with high productivity, high controlling shareholder cash flow rights, low private benefits, low risk-shifting problems, and high asset tangibility will exhibit greater investment–cash flow sensitivity).

This implication is not particularly new. Theoretically, it has been derived from Tirole's (2006) models. Empirically, several authors, including Fazzari et al. (1988), Hoshi et al. (1991), and Kaplan and Zingales (1997), have documented the positive ef-

fect of cash flow on firm investment. The cross-sectional variation in the sensitivity of investment to cash flow, however, has been the subject of debate in the corporate finance literature. Although Fazzari et al. (1988) present evidence that investment–cash flow sensitivity increases with the degree of financial constraint, Kaplan and Zingales (1997) challenge this view, both theoretically and empirically.

We suggest that investment becomes less sensitive to cash flow with the degree of financial constraints, which is consistent with Kaplan and Zingales (1997). For example, if firms with low private benefits and high asset tangibility are less financially constrained, then our model implies that their investment–cash flow sensitivity will be high. This hypothesis is also consistent with Almeida and Campello's (2007) findings. These authors propose that tangible (pledgeable) assets support more borrowing, allowing for further investment in tangible assets, giving rise to a credit multiplier. They show that the sensitivity of investment to cash flow increases with asset tangibility for financially constrained firms, as suggested by the credit multiplier rationale.

Empirical Implication 5. The sensitivity of investment to other group-affiliated firms' cash flow is positive (null) for receivers (providers) of intra-group loans and increases with their level of pledgeable income.

According to our model, the sensitivity of investment to other group firms' cash flow is the outcome of an active internal capital market within business groups. Lamont (1997), Shin and Stulz (1998), Shin and Park (1999), and Lee et al. (2009) use this logic to motivate their empirical analyses and to interpret their results.[13] They report that the cash flow of other segments (firms) in the same conglomerate (business group) positively affects firm investment. This evidence supports the internal capital markets hypothesis.

As far as we know, no study differentiates between the investment–cash flow sensitivity of receivers and providers of capital within business groups. Therefore, our model explicitly makes this distinction and predicts that only the receivers' investment is positively affected by the cash flow of other firms in the business group, a novel empirical implication.

The extent of the effect of other group firms' cash flow on investment depends on the receivers' equity multiplier, that is, their productivity and pledgeable income. Receivers with high levels of pledgeable income can leverage internal wealth to a greater degree (higher multiplier), and their investment therefore responds more strongly to other firms' cash flow than the investment of receivers with low pledgeable income. Thus, our model suggests that the investment of receivers who have high productivity, high cash flow rights of controlling shareholders, low private benefits, low risk-shifting problems, and high asset tangibility will be more sensitive to the cash flow of other group firms.

Further, the empirical literature presents mixed evidence on the effect of productivity on the sensitivity of investment to other group firms' cash flow. On the one hand, the results of Shin and Stulz (1998) suggest that the sensitivity of a segment's investment to the cash flow of other segments does not depend on whether its investment opportunities are better than those of the other segments. For business groups, Shin and Park (1999) suggest a lower investment sensitivity to other group firms' cash flow for firms with strong growth opportunities. On the other hand, Lee et al. (2009) find the opposite results in the period preceding the 1997 Asian crisis.

As stated in this empirical implication, in addition to productivity, our model accounts for the sensitivity of investment to other firms' cash flow variations with controlling shareholder's cash flow rights, private benefits, risk-shifting problems or asset tangibility, opening a new avenue for future empirical research.

To test *Empirical Implications 4 and 5* one can use financial and market data from business groups affiliates to estimate the sensitivities of investment to their own cash flow and to the other group-affiliated firms' cash flow, allowing both to vary with productivity and pledgeable income (equity multiplier). This approach is similar to those of Shin and Stulz (1998), Shin and Park (1999), and Lee et al. (2009), with some important differences. First, one should allow the effect of the cash flow on investment to vary with factors related to pledgeable income – asset tangibility – and controlling shareholder's cash flow rights. Second, one should be able to test whether the effect of other group-affiliates' cash flow on investment is positive for receivers and equal to zero for providers.

Before we conclude this section, we discuss the findings of the existing empirical literature. First, consistent with *Empirical Implications 1 and 2*, Santioni et al. (2020) find that firms' productivity and cash flows are the main drivers of the directions of funds in Italian business groups, specially in crisis periods. Along the same line, Buchuk et al. (2014) show that net receivers of intra-group loans tend to be the firms with the most growth opportunities (Tobin's Q) and high asset tangibility (property, plant, and equipment). However, Buchuk et al. (2014) also find that firm size is negatively associated with the odds of being a net receiver of internal loans. One alternative explanation for this conflicting result could be the existence of unobservable factors that are positively related to firm productivity and negatively associated with firm size. Indeed, small firms are more likely to be financially constrained (e.g., Gertler and Gilchrist, 1994; Almeida and Campello, 2007; Hadlock and Pierce, 2010); therefore, they exhibit higher marginal productivity as they cannot attain their first-best level of investment (Almeida and Campello, 2007).

Second, to the best of our knowledge, *Empirical Implication 3* has not been tested yet. New evidence can extend our understanding about the role of internal capital markets in business groups, as it challenges the hypothesis of internal capital markets being effective in overcoming external capital markets' failures (Khanna and Palepu, 2000; Khanna and Yafeh, 2007). According to this literature, we should expect a negative relationship between financial market development and the existence of business groups. However, Larrain and Urzña (2016) show that, despite substantial financial market development, Chilean business groups remain structurally unchanged and prevalent.

Third, regarding *Empirical Implications 4 and 5* several papers have investigated the main factors determining the sensitivity of investment to firm cash flow in business groups (e.g., Shin and Park, 1999; Lee et al., 2009). However, there is no evidence on how financial and agency factors (e.g., asset tangibility, risk-shifting, and entrepreneurs' private benefits) affect the investment–cash flow sensitivity of group-affiliated firms. Furthermore, previous studies have not examined how a firm's investment sensitivity to the other group-affiliated firms' cash flow depends on its status (receiver or provider of internal resources).

Finally, an important aspect for testing all the empirical implications concerns the existence of formal control rights, since they are necessary for the allocation of resources in internal capital markets (Stein, 1997). More recently, Buchuk et al. (2020) and Allen et al. (2021) show that ownership network centrality is key for capital intermediation. In this sense, our empirical implications should be more pronounced in business groups with more central firms in the ownership structure.

Moreover, as our model shows, pledgeable income increases with the controlling shareholder's cash flow rights. At the same time, the cash flow rights influence on the direction of resources is also consistent with tunneling: resources may be diverted to firms

[13] Lamont (1997) and Shin and Stulz (1998) address conglomerates, whereas Shin and Park (1999) and Lee et al. (2009) address business groups.

in which the controlling shareholder holds higher stakes. Therefore, the main driver of the direction and intensity of resources is an empirical question.

If over time a receiver borrows, invests, and profits more than a similar standalone firm, it suggests that pledgeable income is the main driver. While the existing empirical evidence is broadly consistent with our model predictions, it does not rule out tunneling as a potential explanation (Buchuk et al., 2014; 2020). This is because findings may be influenced by the institutional and legal environment of empirical analysis. One potential alternative to overcome this limitation is to examine natural experiments derived from changes in law and public policies that affect the economic incentives for tunneling. For example, there has been a recent change in the European Model Companies Act's rules that substantially reduced minority shareholder protection against tunneling (Enriques and Gilotta, 2022). Following an increase in tunneling incentives, we should expect a reduction in the performance differences between receivers and similar standalone firms.

6. Concluding remarks

We provide a new rationale for investment in business groups subject to moral hazard in order to answer two related questions: (1) How do business groups allocate resources in internal capital markets? (2) And do the internal capital markets alleviate the financial constraints of affiliate firms that have limited access to external finance?

To answer the first question, our model suggests that productivity and pledgeable income jointly determine the allocation of resources in business groups' internal capital markets. That is, funds within groups tend to flow in the direction of firms with high productivity and high pledgeable income. This means that, if productivity varies little relative to pledgeable income across firms within a group, pledgeable income will be the key driver of resources.

To answer the second question, our model predicts that internal capital markets in business groups tend to favor financially strong over financially weak firms. This result casts doubt on the ability of internal capital markets to alleviate the financial constraints of group firms that have limited access to external finance, as hypothesized by Khanna and Palepu (2000) and Khanna and Yafeh (2007). We propose that capital allocation is driven by productivity *and* pledgeable income. Therefore, our model could rationalize empirical evidence suggesting socialist cross-subsidization in business groups.

Our main results can also help explain why business groups are still prevalent in some countries, despite significant improvement in capital markets. The institutional voids perspective (Khanna and Yafeh, 2007) holds that business groups facilitate resource sharing and alleviate financial constraints. Therefore, we should observe a reduction in the prevalence of business groups following improvements in external financial markets. On the contrary, historical evidence suggests that business groups remain mostly unchanged in the wake of the development of capital markets (Larrain and Urzña, 2016). This is consistent with our model, which predicts that business groups are not well suited to substitute missing external capital markets.

The fact that these groups are still prevalent both in emerging and developed markets may be due to other factors. For example, Belenzon et al. (2021) provide an organizational law perspective of an asset partitioning process of constant redrawing firm boundaries to maximize total group value. Other studies argue that regulatory changes (Kandel et al., 2018) and the ability of business groups to adjust to the institutional environment (Carney et al., 2018) are key to understanding their existence.

Industrial organization approaches have also been provided to explain business groups' dominant organizational form. For example, in Japan, Nishiguchi (1994) shows that close ties between buyers and suppliers within a business group reduce transaction costs and maximized inter-organizational learning. Cestone and Fumagalli (2005) provide a model to show that a group decision to enter or exit a market depends on internal resources, the group's response to competition, and to internal resources allocations.

Finally, we believe that examination of the welfare effects of internal capital markets in business groups provides a unique contribution to the corporate finance literature in emerging economies, where capital markets are less developed and business groups are ubiquitous. Along with the insights of Almeida and Wolfenzon (2006a,b), our study's testable implications can inspire further theoretical and empirical work aimed at gaining a better understanding of the equilibrium effects of business groups and the policies needed to improve the efficiency of economy-wide capital allocation.

CRediT authorship contribution statement

Luiz Ricardo Kabbach-de-Castro: Conceptualization, Writing – original draft. **Guilherme Kirch:** Conceptualization, Methodology, Writing – original draft. **Rafael Matta:** Conceptualization, Writing – review & editing.

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Appendix A. Appendix

In this Appendix, we develop a more general model than the one presented in the main text. It is worth noting that our key predictions about the resource allocation in business groups do not depend on the assumption of the production technology. Instead, our model is based on the assumption that pledgeable income is not enough to fund new investments and firms must complemented with internal resources. In other words, firms are financially constrained in the sense that external finance and investment depend on the amount of internal resources available to the firms. For business groups, this assumption implies a limit on the amount of the resources that can be transferred across group firms. We also assume that when a group-affiliated firm approaches outside investors it depends solely on its own merits. Thus, it may be optimal for business groups to allocate internal wealth to firms with the greatest capacity to multiply this wealth; in other words, firms with high productivity (high profitability) **and** pledgeable income (external finance capacity).

Specifically, we assume now that the investment cash flow at date 1 in case of success is $f^U(I^U)$ for firm U and $f^D(I^D)$ for firm D. Except for some adjustments in the assumptions (1) and (2), everything else in the model setup remain the same. Regarding the production technology, we will assume the following:

$$\begin{aligned} &f^U(0)=0,\ \ f^U_I(\cdot)>0,\ \ f^U_{II}(\cdot)\le 0,\ \text{ and }\ p_H f^U_I(0)>1,\\ &f^D(0)=0,\ \ f^D_I(\cdot)>0,\ \ f^D_{II}(\cdot)\le 0,\ \text{ and }\ p_H f^D_I(0)>1. \end{aligned} \tag{A.1}$$

In the case of decreasing returns to scale, namely, $f^T_{II}(\cdot)<0$ for $T\in\{U,D\}$, we define the first-best investments as the ones that satisfy $p_H f^U_I(I^{U,FB})=1$ for firm U and $p_H f^D_I(I^{D,FB})=1$ for firm D. For $I^U<I^{U,FB}$ and $I^D<I^{D,FB}$, $p_H f^U_I(I^U)>1$ and $p_H f^D_I(I^D)>$

1 under (A.1). With these definitions and changes, assumption (1) needs to be replaced by:

$$\begin{aligned} p_L f_I^U(I^U) + B^U &< 1, \\ p_L f_I^D(I^D) + B^D \div \beta &< 1. \end{aligned} \tag{A.2}$$

In order to have solutions in which firms are financially constrained, assumption (2) also requires modifications being replaced by:

$$\begin{aligned} p_H\left(f_I^U(0) - \tfrac{B^U}{\Delta_p}\right) &< 1, \\ p_H\left(f_I^D(0) - \tfrac{B^D}{\beta\Delta_p}\right) &< 1. \end{aligned} \tag{A.3}$$

The investment of firm U (D) will be financed by a fraction α^U (α^D) of its cash flow A^U (A^D), by a fraction $(1-\alpha^D)$ ($(1-\alpha^U)$) of the cash flow of firm D (U), and the remaining by external finance. Under direct loan, the borrowing firm, say U, needs to promise an amount of $(1-\alpha^D)A^D \div p_H$ in the case of success at date 1 in exchange of a loan of $(1-\alpha^D)A^D$ at date 0. The incentive compatibility constraint of the entrepreneur in the case of firm U is:

$$p_H\left(R_S^U - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H}\right) + (1-p_H)R_F^U \geq$$
$$p_L\left(R_S^U - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H}\right) + (1-p_L)R_F^U + B^U I^U.$$

Simplifying:

$$\Delta_p\left((R_S^U - R_F^U) - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H}\right) \geq B^U I^U. \tag{A.4}$$

The entrepreneur's incentive compatibility constraint, in the case of firm D, is:

$$p_H\left(\beta R_S^D + (1-\beta)\frac{(1-\alpha^U)A^U}{p_H}\right) + (1-p_H)\beta R_F^D \geq$$
$$p_L\left(\beta R_S^D + (1-\beta)\frac{(1-\alpha^U)A^U}{p_H}\right) + (1-p_L)\beta R_F^D + B^D I^D.$$

Simplifying:

$$\Delta_p\left(\beta(R_S^D - R_F^D) + (1-\beta)\frac{(1-\alpha^U)A^U}{p_H}\right) \geq B^D I^D. \tag{A.5}$$

The investor's rationale constraint, in the case of firm U, is:

$$p_H\left(f^U(I^U) - R_S^U\right) - (1-p_H)R_F^U \geq I^U - \alpha^U A^U - (1-\alpha^D)A^D. \tag{A.6}$$

The investor;s rationale constraint, in the case of firm D, is:

$$p_H\left(f^D(I^D) - R_S^D\right) - (1-p_H)R_F^D \geq I^D - \alpha^D A^D - (1-\alpha^U)A^U. \tag{A.7}$$

Finally, the entrepreneur wants to maximize its expected total income:

$$p_H\left(R_S^U - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H}\right) + (1-p_H)R_F^U +$$
$$p_H\left(\beta R_S^D + (1-\beta)\frac{(1-\alpha^U)A^U}{p_H}\right) + (1-p_H)\beta R_F^D.$$

The Lagrangian of the problem:

$$\begin{aligned} L = {} & p_H\left(R_S^U - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H}\right) + (1-p_H)R_F^U \\ & + p_H\left(\beta R_S^D + (1-\beta)\frac{(1-\alpha^U)A^U}{p_H}\right) + (1-p_H)\beta R_F^D \\ & - \lambda^U\left[B^U I^U - \Delta_p\left((R_S^U - R_F^U) - (1-\beta)\frac{(1-\alpha^D)A^D}{p_H}\right)\right] \\ & - \lambda^D\left[B^D I^D - \Delta_p\left(\beta(R_S^D - R_F^D) + (1-\beta)\frac{(1-\alpha^U)A^U}{p_H}\right)\right] \\ & - \theta^U\left[I^U - \alpha^U A^U - (1-\alpha^D)A^D - p_H\left(f^U(I^U) - R_S^U\right) + (1-p_H)R_F^U\right] \\ & - \theta^D\left[I^D - \alpha^D A^D - (1-\alpha^U)A^U - p_H\left(f^D(I^D) - R_S^D\right) + (1-p_H)R_F^D\right] \\ & + \pi_S^U R_S^U + \pi_F^U R_F^U + \pi_S^D R_S^D + \pi_F^D R_F^D + \psi^U I^U + \psi^D I^D \\ & - \delta^U(\alpha - \alpha^U) - \delta^D(\alpha - \alpha^D) - \phi^U(\alpha^U - 1) - \phi^D(\alpha^D - 1). \end{aligned}$$

The First-Order Conditions (FOCs):

$$\frac{\partial L}{\partial R_S^U} = p_H + \lambda^U\Delta_p - \theta^U p_H + \pi_S^U = 0, \tag{A.1}$$

$$\frac{\partial L}{\partial R_F^U} = (1-p_H) - \lambda^U\Delta_p - \theta^U(1-p_H) + \pi_F^U = 0, \tag{A.2}$$

$$\frac{\partial L}{\partial R_S^D} = p_H\beta + \lambda^D\Delta_p\beta - \theta^D p_H + \pi_S^D = 0, \tag{A.3}$$

$$\frac{\partial L}{\partial R_F^D} = (1-p_H)\beta - \lambda^D\Delta_p\beta - \theta^D(1-p_H) + \pi_F^D = 0, \tag{A.4}$$

$$\frac{\partial L}{\partial I^U} = -\lambda^U B^U - \theta^U\left(1 - p_H f_I^U(I^U)\right) + \psi^U = 0, \tag{A.5}$$

$$\frac{\partial L}{\partial I^D} = -\lambda^D B^D - \theta^D\left(1 - p_H f_I^D(I^D)\right) + \psi^D = 0, \tag{A.6}$$

$$\frac{\partial L}{\partial \alpha^U} = -(1-\beta)A^U - \frac{\lambda^D\Delta_p(1-\beta)A^U}{p_H} + \theta^U A^U - \theta^D A^U + \delta^U - \phi^U = 0, \tag{A.7}$$

$$\frac{\partial L}{\partial \alpha^D} = (1-\beta)A^D + \frac{\lambda^U\Delta_p(1-\beta)A^D}{p_H} - \theta^U A^D + \theta^D A^D + \delta^D - \phi^D = 0. \tag{A.8}$$

Since the lender sector is competitive, firms will earn the entire surplus. Under our assumptions, this means that both firms invest up to their first-best levels, $0 < I^U \leq I^{U,FB}$ and $0 < I^D \leq I^{D,FB}$, and that the investor's rationality constraints are binding at the optimum. Regarding the firms payoffs, in each state (success of failure), there is four possible cases: 1) $R_S^T = R_F^T = 0$, 2) $R_S^T > 0$ and $R_F^T > 0$, 3) $R_S^T = 0$ and $R_F^T > 0$, and 4) $R_S^T > 0$ and $R_F^T = 0$, $T \in \{U, D\}$. The first case is clearly not optimal. If we make the additional assumption that both firms are financial constrained, internal wealth $(A^U + A^D)$ and external finance are low enough to not allow firms to invest at the first-best levels[14], we can show that the second and third cases are also ruled out. If $R_S^U > (=)\ 0$ and $R_F^U > 0$, then $\pi_S^U = (\geq)\ 0$ and $\pi_F^U = 0$. Hence, from (A.1) and (A.2):

$$\begin{aligned} p_H\left(\theta^U - 1\right) &= (\geq)\ \lambda^U\Delta_p \geq 0, \\ (1-p_H)\left(\theta^U - 1\right) &= -\lambda^U\Delta_p \leq 0. \end{aligned}$$

These conditions can only be satisfied if $\lambda^U = 0$, that is, if the incentive compatibility constraint (A.4) is not binding. As we will show below, this only happens when firm U is financial unconstrained. In the more interesting scenario of financial constraints (and even more realistic!), $\lambda^U > 0$ implying that $R_S^U > 0$ and $R_F^U = 0$ (the forth case above). The same applies to firm D. If $R_S^D > (=)\ 0$ and $R_F^D > 0$, then $\pi_S^D = (\geq)\ 0$ and $\pi_F^D = 0$. Hence, from (A.3) and (A.4):

$$\begin{aligned} p_H\left(\theta^D - \beta\right) &= (\geq)\ \lambda^D\Delta_p\beta \geq 0, \\ (1-p_H)\left(\theta^D - \beta\right) &= -\lambda^D\Delta_p\beta \leq 0. \end{aligned}$$

[14] Under constant returns to scale (main text), firms are always financial constrained.

Again, if firm D is financial constrained then $\lambda^D > 0$ implying that $R_S^D > 0$ and $R_F^D = 0$. That is, if firms are financial constrained they are reward only in the case of success, providing the right incentives for the entrepreneur to behave. From now on we will assume that this is the case. Knowing that $\psi^U = \psi^D = \pi_S^U = \pi_S^D = 0$, we can use the first order-conditions (A.1)-(A.6) to solve for θ^U, θ^D, λ^U, λ^D, π_F^U, and π_F^D:

$$\theta^U = \frac{p_H B^U/\Delta_p}{1 - p_H\left[f_I^U(I^U) - \frac{B^U}{\Delta_p}\right]}, \tag{A.9}$$

$$\theta^D = \frac{p_H B^D/\Delta_p}{1 - p_H\left[f_I^D(I^D) - \frac{B^D}{\beta\Delta_p}\right]}, \tag{A.10}$$

$$\lambda^U = \frac{p_H}{\Delta_p} \times \frac{p_H f_I^U(I^U) - 1}{1 - p_H\left[f_I^U(I^U) - \frac{B^U}{\Delta_p}\right]}, \tag{A.11}$$

$$\lambda^D = \frac{p_H}{\Delta_p} \times \frac{p_H f_I^D(I^D) - 1}{1 - p_H\left[f_I^D(I^D) - \frac{B^D}{\beta\Delta_p}\right]}, \tag{A.12}$$

$$\pi_F^U = \frac{p_H f_I^U(I^U) - 1}{1 - p_H\left[f_I^U(I^U) - \frac{B^U}{\Delta_p}\right]}, \tag{A.13}$$

$$\pi_F^D = \frac{\beta\left[p_H f_I^D(I^D) - 1\right]}{1 - p_H\left[f_I^D(I^D) - \frac{B^D}{\beta\Delta_p}\right]}. \tag{A.14}$$

The incentive compatibility constraints, (A.4) and (A.5), are binding at the optimal solution and determine the firms payoffs in case of success:

$$R_S^U = \frac{B^U I^U}{\Delta_p} + \frac{(1-\beta)(1-\alpha^D)A^D}{p_H}, \tag{A.15}$$

$$R_S^D = \frac{B^D I^D}{\beta\Delta_p} - \frac{(1-\beta)(1-\alpha^U)A^U}{\beta p_H}. \tag{A.16}$$

After replacing R_S^U and R_S^D by the values given in (A.15) and (A.16), respectively, the investor rationality constraints, (A.6) and (A.7), determine (implicitly) the firms investment levels:

$$I^U - p_H\left[f^U(I^U) - \frac{B^U I^U}{\Delta_p}\right] = \alpha^U A^U + \beta(1-\alpha^D)A^D, \tag{A.17}$$

$$\beta\left[I^D - p_H\left[f^D(I^D) - \frac{B^D I^D}{\beta\Delta_p}\right]\right] = \beta\alpha^D A^D + (1-\alpha^U)A^U. \tag{A.18}$$

At this point, it is worth to make some comments about the Lagrange multipliers of our problem (see Eqs. (A.9)-(A.14)). As we know, they measure the increase in the entrepreneur's expected total income if we could relax their respective constraints by a unit (in other words, the shadow values of the constraints). Thus, for example, π_F^U measure (approx.) the value to the entrepreneur if we could set the firm U payoff in case of failure, R_F^U, to the value minus one (instead of zero). More interesting to our analysis are the values of the Lagrange multipliers associated with the investor rationality constraints: θ^U and θ^D. It is easy to show that if the entrepreneur wealth $(A^U + \beta A^D)$ is increased by a unit and this amount is allocated in firm U (D) then the entrepreneur expected total income will increase by θ^U ($\theta^D \div \beta$). The value added by this marginal wealth allocated in firm U or in firm D (henceforth, value added) is, respectively:

$$\theta^U - 1 = \left(p_H f_I^U(I^U) - 1\right) \times \frac{1}{1 - p_H\left[f_I^U(I^U) - \frac{B^U}{\Delta_p}\right]}, \tag{A.19}$$

$$\frac{\theta^D}{\beta} - 1 = \beta\left(p_H f_I^D(I^D) - 1\right) \times \frac{1}{\beta\left[1 - p_H\left[f_I^D(I^D) - \frac{B^D}{\beta\Delta_p}\right]\right]}. \tag{A.20}$$

Thus, the value added is the product of two factors: the first is the expected NPV per unit of investment (at the entrepreneur's perspective) and the second is the firm incremental investment per unit of additional entrepreneur wealth allocated to it (the equity multiplier in the language of Tirole (2006)). As the firms are financially constrained, their investments are restricted to the amounts of internal and external wealth they can attract. So, this last factor depends positively on the pledgeable income of the firm. The amount the firm can raise in the external capital market per unit of entrepreneur wealth allocated to it. With this in mind, the value added is the result of the interaction between productivity (expected NPV) and pledgeable income. As we will see below, the resources in the internal capital market flow to the firm with higher value added.

Now, we need to pin down the optimal values of α^U and α^D. The first-order conditions (A.7) and (A.8) show us how internal transfers impact the entrepreneur expected total income. To see this more clearly, we rewrite these first-order conditions as follows:

$$A^U \times \left[\theta^U - \frac{\theta^D}{\beta}\right] = \phi^U - \delta^U, \tag{A.21}$$

$$\beta A^D \times \left[\frac{\theta^D}{\beta} - \theta^U\right] = \phi^D - \delta^D. \tag{A.22}$$

The left sides of these equations measure the increase in the entrepreneur expected total income if we augment α^U and α^D, respectively, by one unit. Suppose that with no internal transfers ($\alpha^U = \alpha^D = 1$) the value added is higher in firm U than in firm D, that is, $\theta^U > \theta^D \div \beta$. Then Eqs. (A.21) and (A.22) tell us that is worth (in the entrepreneur eyes) transferring some wealth from firm D to firm U, that is, to decrease α^D. As we can see from (A.17) and (A.18), as α^D decreases firm U investment increases, firm D investment decreases, and so the gap between θ^U and $\theta^D \div \beta$ shrinks (see Eqs. (A.9) and (A.10)). The entrepreneur will continue to transfer internal resources from firm D to firm U until this gap vanishes or until the limit to internal transfers is reached (whichever happens first). In this last case, $\delta^U = \phi^D = 0$ and Eqs. (A.21) and (A.22) determine the values of ϕ^U and δ^D, respectively. Now, suppose that with no internal transfers $\theta^U < \theta^D \div \beta$. Then the direction of resources will be reversed, namely, from firm U to firm D. If the limit to internal transfers is reached before the gap between θ^U and $\theta^D \div \beta$ vanishes, then $\phi^U = \delta^D = 0$ and Eqs. (A.21) and (A.22) determine the values of δ^U and ϕ^D, respectively. Finally, suppose that with no internal transfers $\theta^U = \theta^D \div \beta$. Then the entrepreneur expected total income cannot be increased by internal transfers and the simplest contract is the one with $\alpha^U = \alpha^D = 1$. It is easy to see that in this case $\phi^U = \delta^U = \phi^D = \delta^D = 0$. Therefore, as in the main text, productivity and pledgeable income jointly determine the allocation of resources in the internal capital markets. All of our empirical implications remain valid in this more general context.

Note that we set the upper bond of internal transfers, α, to be greater than zero. This assumption prevents all wealth from one firm from being transferred to the other in the internal capital market and so assures that both firms invest. If we do not impose such a limit, resources will be transferred from one group firm to the other until the gap between their values added vanishes or until there is no more internal wealth to transfer ($\alpha^T = 0$), whichever comes first. In this second case, the "donor" firm does not invest, since it has no wealth to raise resources in the external capital market. However, even in this case, all our predictions

remain valid, except those related to the investment-cash flow sensitivities of the "donor" firm.

To better compare the expression used here and the one used in the main text to determine the direction of resources in the internal capital market, it is interesting to note that the expression in brackets on the left side of (A.21) has the same sign as the following expression:

$$B^D\left[p_H f_I^U(I^U)-1\right]-B^U\beta\left[p_H f_I^D(I^D)-1\right].$$

In the case of linear technology (constant returns to scale), the expression above is exactly the same as the one used in the main text. Lastly, we check the second-order conditions for a local maximum. If the limit of internal transfers is reached at the optimal solution (no matter the direction of resources), only one condition must be satisfied: the determinant of the (respective) bordered Hessian matrix is positive, that is:

$$\left(\Delta_p\left(1-p_H\left[f_I^U(I^U)-\frac{B^U}{\Delta}\right]\right)\right)^2\times\left(\beta\Delta_p\left(1-p_H\left[f_I^D(I^D)-\frac{B^D}{\beta\Delta_p}\right]\right)\right)^2>0.$$

One can see that this condition is satisfied. If the limit of internal transfers is not reached at the optimal solution, then two conditions must be satisfied: the determinant of the (respective) bordered Hessian matrix is positive and the second last leading principal minor is negative. The first condition is not satisfied since this determinant is null. As we can see in the following expression, the second condition is met.

$$p_H(\Delta_p^2A^U)^2\left[\theta^U f_{II}^U(I^U)\left(\beta\left(1-p_H\left[f_I^D(I^D)-\frac{B^D}{\beta\Delta_p}\right]\right)\right)^2\right.$$
$$\left.+\theta^D f_{II}^D(I^D)\left(1-p_H\left[f_I^U(I^U)-\frac{B^U}{\Delta_p}\right]\right)^2\right]<0.$$

Hence, there is not a single optimal solution in the sense that once the value added of both firms are equated, changes to α^U and α^D that do not alter firms investment, and neither therefore, the entrepreneur expected total income, are also optimal solutions. We opt for the most parsimonious solution, the one in which the flow of resources is unidirectional.

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