Three Essays on
Financial Crises and Economic Recessions

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This thesis contains 65,866 words, including all figures, tables, and footnotes.

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Thesis Abstract

This thesis investigates the dynamics of financial and macroeconomic indicators during and in the aftermath of financial crises and economic recessions. Within the scope of this thesis, I identify three major phenomena: (1) the U.S. slow recovery and the U.K. productivity puzzle and their relationship following the 2007–09 financial crisis; (2) asymmetric co-fluctuations of output and unemployment in the U.S. by integrating Friedman's plucking model and Okun's law; and finally (3) the asymmetric deviation of market prices from efficient prices, which I call inefficient plunges.

The first essay, "Slow recovery of output after the 2007−09 financial crisis: U.S. shortfall spillovers and the U.K. productivity puzzle," explores the slow recovery by explaining why output in the U.S. and the U.K. recovered slowly after the recession trough even though unemployment rates returned to pre-crisis levels. To explain this mismatch, we examine the effect of instability in the empirical relationship between output and the unemployment rate on the slow recovery. Using a difference version of Okun's law and a dynamic factor model to estimate the counterfactual, we identify a change in regime in the aftermath of the financial crisis as the main determinant of the slow recovery. Further, by applying a trend-cycle decomposition that allows for time-variation in the parameters, we distinguish between three driving forces of the slow recovery: (1) a declining trend growth, which started in the 1960s; (2) an unprecedented trend deceleration, which began during the financial crisis; and (3) the sluggish cyclical recovery known as hysteresis effects. In the second part of this paper, we answer this question: what would the output recovery in the U.K. have been if there was no slow recovery in the U.S.? Indeed, we demonstrate that spillovers of real activity shortfall from the U.S. explain the productivity puzzle in the U.K.

The second essay, "Friedman's plucking model and Okun's law," integrates Friedman's plucking model and the gap version of Okun's law by embedding U.S. output and the unemployment rate in a bivariate unobserved components model with Markov-switching to capture their asymmetric co-fluctuations. We demonstrate that the plucking property of the unemployment rate, through a stable gap version of Okun's law, transmits to U.S. output. The asymmetric bivariate model also deciphers two puzzling dilemmas in trend-cycle decomposition: First, by considering stochastic rather than deterministic trend growth, we identify an unprecedented deceleration in U.S. potential output in the aftermath of the 2007−09 financial crisis. Second, including unemployment as an auxiliary variable in the bivariate model yields a robust estimation of parameters and components with insignificant correlation, which we refer to as correlation irrelevance.

In the third essay titled, "Asymmetric Fads, inefficient plunges, and efficient market hypothesis," I define the concept of "inefficient plunges" to characterize the asymmetric deviation of market prices from efficient prices with the aim of examining the efficient market hypothesis. To measure market inefficiency, I present an asymmetric Fads model, which allows for both inefficient plunges in the transitory component and a switching variance in the permanent component by employing a Markov-switching process. Applying the model to the S&P 500 and the FTSE 250 shows that inefficient plunges are deep and transient. Market inefficiency is therefore a regime-dependent and asymmetric phenomenon, meaning that although the U.S. and U.K. stock markets are adequately efficient during normal times, they are far below efficient prices during crises.

Declaration of Originality

I declare that no portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

Sincerely, Mohammad Dehghani

Copyright Statement

Acknowledgments and Dedication

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Chapter 0: Thesis Introduction

How can the dynamics of financial and macroeconomic indicators be characterized during and in the aftermath of disastrous rare events? Addressing this question is what this thesis, **"Three Essays on Financial Crises and Economic Recessions,"** is built on. In this regard, I conducted three studies, each aimed at disentangling a phenomenon, including the slow recovery of output after the global financial crisis; asymmetric co-fluctuations of output and the unemployment rate; and inefficient plunges in stock markets. This journal-format thesis, therefore, consists of three papers (also called essays), explained briefly here and presented thoroughly in Chapters (1), (2), and (3).

The topics of these three research papers are as follows: (1) slow recovery of output after the 2007−09 financial crisis: U.S. shortfall spillovers and the U.K. productivity puzzle; (2) Friedman's plucking model and Okun's law; (3) asymmetric Fads, inefficient plunges, and efficient market hypothesis. Since my supervisors, Stuart Hyde and Sungjun Cho, are co-authors of the first and second papers, I employ a first-person plural narrative throughout Chapters (1) and (2), whereas Chapter (3) is written in first-person singular as the third paper is a single-author paper.

0.1. The first essay

Output in the U.S. and the U.K. has recovered slowly following the 2007−09 global financial crisis even though the unemployment rate returned to pre-crisis levels. To explain this mismatch, Fernald et al. (2017), by presuming the stability of Okun's law, identify a declining output trend growth as the only determinant of the slow recovery. They argue that since the decline in trend growth was in play before the financial crisis, the slow recovery seems unrelated to the 2007−09 financial crisis. Nevertheless, in the first paper, **"Slow recovery of output after the 2007−09 financial crisis: U.S. shortfall spillovers and the U.K. productivity puzzle,"** by employing Okun's law and a dynamic factor model (DFM) to estimate the counterfactual, we identify a structural break in both the Okun's law intercept and coefficient during the 2007−09 financial crisis as the main determinant of the slow recovery in the U.S. and the U.K.

To capture the slow recovery, we identify the shortfall by comparing the actual observed recovery with a counterfactual recovery derived by one of the two methods. The first method specifies the counterfactual as the post-crisis fitted output, which is estimated by two approaches: Okun's law and a DFM. The slow recovery in this method is attributable to the structural break in the Okun's intercept and coefficient as well as cyclical factor loadings. We identify a sharp break in the parameters of Okun's law and DFM that occurred during the 2007−09 financial crisis. Our findings also document a significant shortfall of at least 1.32 and 0.83 percentage points per year in the U.S. and the U.K.

We therefore highlight a change in regime in the aftermath of the 2007–09 financial crisis that is revealed in the form of a break in the parameters.

In the second method, we consider three previous recoveries as counterfactual. Since the depth of the Great Recession is different from that of the three previous recessions, it is necessary to control for the recession depth through a cyclical adjustment. We follow the same two approaches employed in the first method for trend-cycle decomposition. By comparing the trend and cyclical components of the recovery following the Great Recession with their counterparts in previous recoveries, we distinguish between three distinct driving forces of the slow recovery. The first is a declining trend growth that started in the mid-1960s. This driver accords with the finding of a gradual slowdown in potential output by Antolin-Diaz et al. (2017), Fernald et al. (2017), and Zhang (2019). The second driver is an unprecedented trend deceleration in the aftermath of the 2007−09 financial crisis, which confirms the work of Patterson et al. (2016), Van Ark and Jäger (2017), Oulton (2019), and Bauer et al. (2020), among others. And the third one is an unusually sluggish recovery of the U.S. output gap (cyclical component) that is related to constrained demand and hysteresis effects suggested by Fatás and Mihov (2013), Michau (2018), Anzoategui et al. (2019), Fontanari et al. (2020), and Cerra et al. (2022), among others.

Finally, the concurrent occurrence of the change in regimes in two countries during the 2007−09 financial crisis motivates us to explore the connection between the slow recovery in the U.S. and the productivity puzzle in the U.K. In this sense, given the tight relationship between the two countries, we are concerned about measuring spillovers to the U.K. economy originating directly from the U.S. We thus develop a two-country DFM to provide policy recommendations for the U.K. as a dominant receiver of spillovers from the U.S. economy. We define the shortfall in the U.K. as the difference between actual and counterfactual recoveries. To derive the counterfactual for the U.K., we answer this simple question: what would the normal recovery path of the U.K. output have been if there was a normal recovery in the U.S.? In this framework, therefore, we measure the magnitude of the shortfall spillovers to the U.K. conditional on the shortfall in the U.S.

Consistent with the literature on the transmission of international shocks (Dees et al., 2007; Mumtaz and Surico, 2009; Fadejeva et al., 2017; Georgiadis, 2017; among others), we find that the magnitude of the shortfall spillovers from the U.S. to the U.K., measured for output per capita, is 0.62 percentage points per year. This result underscores the economic diversification in the U.K. as a small country in response to the long-term productivity slowdown and hysteresis effects in the U.S. As a result, the U.K. must shift its economy away from a mostly bilateral economic relationship toward a more multilateral economic relationship.

0.2. The second essay

In contrast to the mainstream view in the business cycle literature, the plucking model proposed by Milton Friedman (1964, 1993) suggests an asymmetric cyclical component, meaning that output does not fluctuate around a trend but instead is steeply plucked down below a ceiling, known as potential output, during recessions and gradually returns toward the ceiling during recoveries. Likewise, the U.S. unemployment rate does not fluctuate around the trend but is characterized by steep jumps above the natural rate of unemployment during recessions and gradual decrements to its natural level during recoveries. Moreover, Okun's law, first proposed by Arthur Okun (1962), is an empirical correlation between the U.S. output and the unemployment rate gaps. Considering Friedman's plucking model and Okun's law together, asymmetric fluctuations appear to be a common feature of both U.S. output and the unemployment rate. We call this phenomenon "asymmetric co-fluctuations" and characterize it in the second paper, **"Friedman's plucking model and Okun's law."**

To capture the asymmetric co-fluctuation, we present a novel model for trend-cycle decomposition. We integrate Friedman's plucking model and a gap version of Okun's law by embedding both U.S. output and the unemployment rate into a bivariate unobserved components (UC) model, in which the asymmetry is accommodated by employing a single Markov-switching process of Hamilton (1989). Concerning four specification aspects of trend-cycle decomposition, we disentangle four puzzling dilemmas as follows: (1) whether the shocks to the cyclical component are asymmetric or symmetric; (2) whether or not the unemployment rate should be included as an auxiliary within a bivariate model; (3) whether the trend growth of U.S. output is stochastic or deterministic; and finally (4) whether the correlation between shocks to the trend and cyclical component is zero or not.

I cast the model in state-space form to estimate the parameters and components, similar to the method applied by Kim and Nelson (1999). The estimation results establish the presence of asymmetric co-fluctuations; indeed, grounded on the stable gap version of Okun's law, we find that output and the unemployment rate in the U.S. are synchronously and proportionally characterized by the plucking property. The model captures the plucking property in both indicators as the estimated coefficients of the plucking property and Okun's law are substantial ($\pi_u = 0.70$ and $\beta = -1.45$). Also, estimated gaps that are large in magnitude and often negative for output and positive for unemployment verify the ceiling effect. The U.S. output rarely ascends above the ceiling (potential output), and likewise, the U.S. unemployment rate rarely descends below the floor (natural rate). The expected duration is about 3 quarters for recessions and 28 quarters for recoveries, implying short recessions and long recoveries. Overall, the co-fluctuations tend to be asymmetric in amplitude, speed, and duration, which supports the idea that deep, steep, and transitory recessions will be followed by commensurate, gradual, and permanent recoveries. Our other findings related to each of the specification aspects are presented below.

First, to test for asymmetry, we compare the log likelihood value of -11.9 for the asymmetric bivariate model with the value of -57.5 for its symmetric counterpart. Since the corresponding likelihood ratio of 91.2 is substantially greater than the critical value of 10.8 for a conservative 0.1% significance level, in line with the results of Kim and Nelson (1999), Sinclair (2010), and Eo and Morley (2022), our findings favour Friedman's plucking model over symmetric alternatives.

Second, consistent with the findings of Ball et al. (2017) and Michail (2019), among others, the gap version of Okun's law is stable. Accordingly, in view of the fact that Ferraro (2018), Dupraz et al. (2019), and Ferraro and Fiori (2022) identify the U.S. labour market as the source of the plucking property, we conclude that the plucking property transmits from the unemployment rate to output. Moreover, the bivariate model yields components insensitive to the choice of the other specification aspects. In particular, independent from the assumption about the correlation between shocks, the estimated cyclical component has substantial amplitude, which is in contrast to Beveridge and Nelson (1981), Nelson and Plosser (1982), Morley et al. (2003), Grant and Chan (2017b), Kim and Chon (2020) and Kim and Kim (2020), who say that the cyclical component is small and noisy. The reason behind this counter-intuitive finding is that their models impose two restrictions: fluctuations are assumed to be symmetric and the unemployment rate is excluded from the model.

Third, we document a time-variation in output trend growth in the U.S. in the form of both a gradual decline, which began in the 1960s, and a structural break following the 2007–09 financial crisis. The former is in agreement with Antolin-Diaz et al. (2017), Fernald et al. (2017), Grant and Chan (2017a), and Kim and Chon (2020), and the latter is consistent with the finding of a structural break in trend growth around or after 2007 by Eo and Morley (2022) and Dehghani et al. (2022).

Fourth, we conclude that the correlation irrelevance can be achieved through accounting for both the asymmetry and co-fluctuations in the asymmetric bivariate model, although each of the asymmetric univariate model and the symmetric bivariate model helps to mitigate the sensitivity of the UC model to the assumption about correlation. Conclusively, the asymmetric bivariate model yields robust results with an insignificant correlation between shocks to the trend and cyclical components.

Finally, by jointly estimating the trends of output and the unemployment rate and accounting for the plucking property in both indicators, our model provides a substitute for the Non-Accelerating Inflation Rate of Unemployment (NAIRU) to measure the natural rate of unemployment. We call this new measure the Zero Output Gap Rate of Unemployment (ZOGRU), the unemployment rate at which the output gap is zero.

0.3. The third essay

In the third paper, **"Asymmetric Fads, inefficient plunges, and the efficient market hypothesis,"** I develop an asymmetric Fads model by using an unobserved components (UC) model with Markov-switching to address unresolved questions about the efficient market hypothesis. I seek to show that the deviation of market prices from efficient prices is asymmetric: downside deviations during crisis periods tend to be deep and steep, while upside deviations during non-crisis periods are negligible.[1] I call the gap between actual and efficient prices **"inefficient plunges,"** a term I use to convey the concept of regime-dependent and asymmetric deviation from rationality during crises.[2]

This study lies at the crossroads of four branches in the literature on asset pricing, which are discussed as follows. First, the Efficient Market Hypothesis (EMH), proposed by Samuelson (1965) and Fama (1970), maintains that market prices reflect all available information, and thus future prices are purely unpredictable. Accordingly, market prices must follow a random walk process with a drift, which is referred to as the Random Walk Hypothesis (RWH). This claim is based on the Rational Expectations Hypothesis (REH) introduced by Muth (1961) and Lucas (1978), which states that all investors have rational expectations. In contrast, behavioural finance and economics argue that a sizeable portion of investors are not always rational (Simon, 1955; Arrow, 1982; among others); thus, the market cannot always be efficient (Russell and Thaler, 1985; Lo, 2004).

Second, to reconcile the EMH with behavioural economics, Lo (2004, 2019) introduced the Adaptive Market Hypothesis (AMH), a framework in which rationality and irrationality coexist and investors are not unboundedly rational. This framework implies that market inefficiency is not constant but instead evolves over time. In this regard, a growing literature supports AMH and casts doubt on EMH and RWH, particularly during crises (see, e.g., Lim and Brook, 2008; Anagnostidis et al., 2016; Ito et al., 2016; Hill and Motegi, 2019).

Third, regarding speculative bubbles, Blanchard and Watson (1983) and Diba and Grossman (1988) present a model of "rational bubbles" to rationalize the formation of speculative bubbles. According to rational bubbles, the occurrence of speculative bubbles is entirely consistent with the REH and EMH. As a result, the market price that might contain positive bubbles is still efficient and hence must follow a random walk with a drift. However, as stated by Emery (2021), this model ignores the possibility of negative bubbles during crises, which is one of the main culprits of market inefficiency.

[1] See Ito and Sugiyama (2009), Anagnostidis et al. (2016), and Hill and Motegi (2019), who suggest that the market price deviation from the efficient price is asymmetric.

[2] The concept of inefficient plunges means an excessive price drop during crashes, where the actual market price negatively deviates from the efficient price. Indeed, during crashes, the actual price drops so sharply that the efficient price, which is modelled as a random walk, cannot explain the whole drop completely. This concept is similar to that of negative bubbles, which are the opposite of positive bubbles.

Fourth, the Fads model, adopted by Shiller et al. (1984), Summers (1986), Fama and French (1988), and Poterba and Summers (1988), is a trend-cycle decomposition that aims to capture the possibility of deviation of the price from its fundamental caused by noisy traders, who trade based on fashions, fads, and sentiments. This model, however, does not distinguish between the positive and negative bubbles since it rules out two possibilities. First, the Fads model ignores the presence of positive bubbles inside the permanent component. However, if the transversality condition does not hold, the permanent component (efficient price) exceeds the fundamental by the size of speculative bubbles. Second, the conventional Fads model dismisses the asymmetry in the deviation of market prices from efficient prices, while there is sufficient evidence for asymmetry.

I answer four questions related to four branches of literature. The first question is whether the market price is efficient and reflects all available information. Does the market price follow a random walk with drift? Based on the existing literature, there is some agreement about the answer to this question: the market is not always efficient and market inefficiency evolves over time (Noda, 2016; Hill and Motegi, 2019; among others). However, there are still some unresolved questions about the level and dynamics of market inefficiency. The second question thus concerns the extent to which the market is inefficient and whether the level of market inefficiency is regime-dependent or not. Regarding the concept of negative bubbles, I explore whether the deviation of market prices from efficient prices that measures market inefficiency is asymmetric or not. Finally, to estimate the inefficient plunges, I relax the symmetric assumption in the conventional Fads model by using a UC model with Markov-switching.

I design a model to decompose the market price into its permanent and transitory components, which represent efficient prices and inefficient plunges, respectively. I specify the permanent component as a random walk process with stochastic drift to characterize efficient prices that might contain positive bubbles. To account for the asymmetric price deviation, I include inefficient plunges, which measure the level of market inefficiency, in the transitory component and a concomitant switching variance in the permanent component by embedding a Markov-switching process as in Hamilton (1989) in the UC model.

By applying the model to the monthly inflation-adjusted S&P500 and the FTSE100, the estimation results substantiate the presence of inefficient plunges during crises, which accords with Cao et al. (2016), Goetzmann and Kim (2018), Acharya and Naqvi (2019), and Emery (2021), who corroborate the presence of negative bubbles. The transitory component is indeed large in amplitude and short in duration, implying that inefficient plunges are deep and steep. Market inefficiency is not constant but instead is a regime-dependent and asymmetric phenomenon, meaning that although the U.S. and U.K. stock markets are adequately efficient during normal times, they are below the efficient price during crises. These results are consistent with the findings of time-variation in market inefficiency by Noda (2016), Hill and Motegi (2019), Le Tran and Leirvik (2019), and Mattera and Di Sciorio (2022), among others.

Overall, unlike the conventional Fads model, I state that the deviation from the efficient price is asymmetric. Inefficient plunges, estimated as an asymmetric Fads component, are indeed the foremost determinant of market inefficiency. In this sense, this study supports the AMH of Lo (2004) against the EMH of Fama (1970), meaning that arbitrage opportunities and predictability exist during crisis periods, although inefficient plunges are transient and market price reverts to the efficient level after a couple of months.

0.4. Methodologies

In this thesis, I have employed a diverse range of econometric models, including principal component analysis, dynamic factor models, several structural break tests, trend-cycle decomposition, state-space models with Markov-switching, as well as counterfactual analysis.

The approach I adopt in my papers is unique in two directions. In the first paper, to satisfy the ceteris paribus condition in the counterfactual analysis, I identify the intervention factor as a structural break in the empirical relationship between two variables. I construct the counterfactual based on the post-event fitted values, where the stability of parameters is the intervention factor. This differs from the traditional approach in which counterfactuals are constructed based on retrospective or prospective data, where the intervention factor is built on the assumption that the future is a repetition of the past. In this traditional approach, however, unwanted confounding factors, namely, forecasting errors, cyclicality, or episode-dependence of variables, interfere in the measurement of the cause and effect relationship.

In the second and third papers, I specify the model in levels following the approach in UC models, instead of specifying it in differences, which is conventional in regression, vector auto regressive, and some UC models. This matters because modelling economic indicators in differences is subject to two substantial caveats. First, there is no consensus on the correct choice of order of integration for output and market price considering that the output growth (the differences in logs) of advanced economies and the return of stock market indices are not necessarily stationary. Second, it is likely that the potential over-differencing eliminates important information in the permanent components of output and market price.

Lastly, to characterize asymmetric fluctuations of variables, I apply a univariate or multivariate state-space model with Markov-switching, where both models are specified in levels. To the best of my knowledge, the asymmetric bivariate model has never been applied in any other study.

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Chapter 1

Slow Recovery of Output after the 2007−09 Financial Crisis: U.S. Shortfall Spillovers and the U.K. Productivity Puzzle

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Abstract

Advanced economies experienced a slow recovery in output after the 2007−09 global financial crisis. In particular, output in the U.S. and the U.K. recovered slowly after the recession trough even though unemployment rates returned to pre-crisis levels. To explain this mismatch, we examine the effect of instability in the empirical relationship between output and unemployment on the slow recovery. Using a difference version of Okun's law and a dynamic factor model to estimate the counterfactual, we identify a change in regime in the aftermath of the financial crisis as the main determinant of the slow recovery. Additionally, by applying a trend-cycle decomposition that allows for a time-variation in the Okun's law intercept and a break in the Okun's law coefficient, we distinguish between three driving forces of the slow recovery: declining trend growth, unprecedented trend deceleration, and sluggish cyclical recovery. The first is the downward growth of potential output (trend component), which started in the 1960s. The second one refers to the unprecedented slowdown in potential output, which began during the 2007−09 financial crisis, and the third is an unusually persistent output gap (cyclical component) known as hysteresis effects. Further, the growth-accounting decomposition implies the contribution of slow growth in total factor productivity and capital input to the output shortfall. We also develop an open-economy hierarchical dynamic factor model to demonstrate that spillovers of real activity shortfall from the U.S. explain the productivity puzzle in the U.K.

Keywords: Slow Recovery, Productivity Puzzle, Business Cycles, Trend-Cycle Decomposition, Okun's Law, Dynamic Factor Model (DFM), Structural Break**,** Open-economy Macroeconomics.
JEL Classification: C32, C38, E32, E37, F41, F44, G01, O47.

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1.1. Introduction

Over a decade after the 2007−09 global financial crisis, the slow recovery of output in the U.S. and the U.K. is now well documented. However, at least until 2015, both the survey of professional forecasters (SPF) and the summary of economic projections (SEP) repeatedly over-predicted the output recovery in the U.S. (Lansing and Pyle, 2015). Forecasts for other advanced economies conducted by the OECD and Office for Budget Responsibility were also over-optimistic during the same period (Lewis and Pain, 2015).[1] In fact, there was a negative systematic forecast error after the 2007−09 financial crisis. Likewise, after a long delay, the slow recovery was recognized by only a very few studies (see, e.g., Gordon, 2014; Fernald, 2015; Hall, 2015).

Since the 2007−09 economic recession was deep, predicting a strong recovery is consistent with Friedman's plucking model (1964, 1993), suggesting a strong recovery after deep recessions.[2] In addition, there were no such persistent negative forecast errors for previous recoveries. As a result, this contradiction—the persistent over-prediction of output despite slow recovery—points to a need to better understand the drivers behind the slow recovery after the 2007−09 financial crisis and economic recession. What is the reason behind this contradiction? Is it caused by poor forecasting or an unprecedented event during the 2007−09 financial crisis? To answer these questions, a growing literature attempts to identify and discuss the potential drivers of the fall in the growth of output and productivity, known as the slow recovery or productivity puzzle, in the U.S., the U.K., and other advanced economies.

The three culprits of the productivity puzzle discussed in the existing literature are as follows: (1) a gradual slowdown in total factor productivity (TFP) growth that began before the 2007−09 financial crisis (see, e.g., Fernald, 2015; Van Ark, 2016; Antolin-Diaz et al., 2017; Fernald et al., 2017; Zhang, 2019); (2) a sharp fall in TFP growth and technological progress observed during the financial crisis (see, e.g., Van Ark, 2016; Van Ark and Jäger, 2017; Crafts, 2018; Oulton, 2019; Bauer et al., 2020); and (3) an ineffective monetary and fiscal policy resulted from the zero lower bound, and consequent hysteresis effects and constrained demand in the aftermath of the crisis (see, e.g., Reifschneider et al., 2015; Michau, 2018; Anzoategui et al., 2019; Fontanari et al., 2020; Cerra et al., 2022).

[1] See Figures 1.1 and 1.2 for evidence of the slow recovery in the U.S., the U.K. and other advanced economies.
[2] Friedman's plucking model (1964, 1993) is an asymmetric real business cycle model in which, by considering a ceiling of maximum output, explains that business cycles are the result of a negative shock to the economy during the recession and a self-corrective response known as the "bounce-back" effect during the recovery. The main stylized fact of this model states that deeper recessions breed stronger recoveries and has been empirically confirmed by different methodologies in the U.S. (see, among others, Bordo and Haubrich, 2016; Wynne and Balke, 1992; Beaudry and Koop, 1993).

Concerning the slow recovery in the U.S., Fernald et al. (2017) show that U.S. output expanded more slowly after 2009 compared to the unemployment recovery. While, by 2017, the unemployment rate had fully recovered and returned to its pre-crisis rate, output has not yet returned to its long-run trend path. Investigating this mismatch between recovery paths of output and unemployment, they suggest that slow growth of TFP and a decline in labour force participation (LFP) have mostly contributed to the slow recovery. The former is consistent with Gordon (2014), Hall (2015), Antolin-Diaz et al. (2017), and Zhang (2019), who state that innovation reduction and routine biased technological changes account for the slow recovery in the U.S. The latter result, suggesting the contribution of the decline in LFP to the slow recovery, also accords with that of Gordon (2014) and Hall (2015).

Moreover, Fernald (2015), Antolin-Diaz et al. (2017), and Fernald et al. (2017) suggest that since the slow growth of TFP commenced prior to the onset of the 2007−09 financial crisis, this driver is mostly unrelated to the financial crisis. However, the presence of a pre-crisis negative shock to the TFP does not necessarily rule out the possibility of a larger post-crisis negative shock to the TFP. In fact, while the slowdown in U.S. TFP growth was modest and predictable before 2007, it was substantial and unpredictable after the Great Recession in 2009 (Van Ark and Jäger, 2017; Crafts and Mills, 2017). Furthermore, the claim that the 2007−09 financial crisis was irrelevant to the slow recovery contradicts the findings of Fatás and Mihov (2013), Reinhart and Rogoff (2014), Patterson et al. (2016), Bordo and Haubrich (2016), Grant (2018), Crafts (2018), Oulton (2019), Anzoategui (2019), Fontanari et al. (2020), and Cerra et al. (2022) who point to the contribution of the 2007−09 financial crisis to the U.S. slow recovery and the U.K. productivity puzzle.

Concerning the productivity puzzle in the U.K. and other advanced economies, there is a consensus in recent studies that attributes the productivity slowdown to the 2007−09 financial crisis and the economic recession. Van Ark (2016) and Van Ark and Jäger (2017) identify a drop in the growth of information and communication technology (ICT) investment that occurred after the Great Recession as the driver of the U.K. productivity puzzle. Patterson et al. (2016) propose that the sectoral labour misallocation (a type of job polarization) in the aftermath of the Great Recession explains up to two thirds of the U.K. productivity puzzle. Oulton (2019) states that the constrained foreign demand for U.K. exports in the aftermath of the Great Recession due to a bad regime during the recovery coupled with a rapid rate of immigration explains the collapse in U.K. productivity growth.

Additionally, based on the difference version of Okun's law (1962), there should be a stable empirical relationship between the log difference of output and the difference of unemployment (Daly et al., 2014; Economou and Psarianos, 2016; Ball et al., 2017; Michail, 2019). As a result, the slow recovery of output following the financial crisis despite full recovery of the unemployment rate, prompts questions on reasons for the mismatch between the output and unemployment recoveries in the U.S.

and the U.K. In this regard, some studies report a structural break in Okun's law during the Great Recession (Owyang and Sekhposyan, 2012; Basu and Foley, 2013; Grant, 2018). Since these studies do not account for time-variation in trend growth of output, we pose the following questions: Is there a structural break in Okun's law or a structural break in trend growth? Are the instability of Okun's law, the slow recovery, and the productivity puzzle associated phenomena? We believe that they are two different interpretations of the same phenomenon from different perspectives. Nevertheless, regardless of the specification, there is a change in the regime during the 2007–09 financial crisis that needs to be specified as a break in either Okun's law or trend growth.[3]

In this study, we investigate the U.S. slow recovery and the U.K. productivity puzzle following the years after the 2007−09 financial crisis and economic recession. The two mentioned conflicts in the literature on the contribution of the financial crisis to the slow recovery and the instability of Okun's law are our motivation to explore a change in regime in the aftermath of the financial crisis. The main question, hence, is whether a change in the regime after the 2007–09 financial crisis caused the actual output to fall short of the normal recovery path. In fact, we examine the possibility of a change in regime after the financial crisis by exploring a structural break in the parameters of the difference version of Okun's law (1962) and a dynamic factor model (DFM).[4] In addition to measuring the magnitude of the shortfall and establishing the extent of the slow recovery, we seek to identify the drivers of any such slow recovery and productivity puzzle. Finally, we analyze whether the spillovers of real activity shortfall (RAS) from the U.S. affect the recovery in the U.K.

To capture the slow recovery, we identify the shortfall by comparing the actual recovery with a counterfactual recovery derived by one of the two methods. The first method asks: what would the counterfactual recovery of output per capita have been given the 2009–16 recovery of unemployment or cyclical factors? We specify the counterfactual recovery as the post-crisis fitted output per capita, which is estimated by two complementary approaches: the Okun's law regression and the DFM. In generating the counterfactual recovery, in order to capture the impact of the structural break on the output shortfall, we assume that the parameters of Okun's law as well as the DFM are stable. In this

[3] The original specification of Okun's law is the gap version, which characterizes the relationship between the deviation of output from the potential output (output gap) and the deviation of unemployment from its natural rate (unemployment gap). The difference version of Okun's law can be derived from the gap version of Okun's law by taking the difference and imposing three assumptions, including a constant trend growth, a constant natural rate of unemployment, and a stable Okun's coefficient (see Appendix 1.F).

[4] In this study, structural breaks are estimated by different methods, including the chow test (1960) designed to test the presence of a known break, the Quandt likelihood ratio test applied by Andrews (1993) and Andrews and Ploberger (1994), and Hansen (2000) to estimate an unknown break. Also, the Bai and Perron test (2003) is used to identify the number of breaks in the case of the possibility of multiple breaks.

method, therefore, the slow recovery is attributable to the structural break in the Okun's law intercept and Okun's law coefficient as well as the intercept and cyclical factor loadings.[5]

The second method considers the three previous recoveries as counterfactual. Since the depth of the Great Recession is different from that of the three previous recessions, it is necessary to control for the recession depth through a cyclical adjustment. We follow the same two approaches used in the first method for the trend-cycle decomposition. In both approaches, a structural break in the Okun's coefficient and cyclical factor loadings is allowed for to distinguish between the contribution of the trend and cyclical components to the slow recovery. By comparing the trend and cyclical components of the recovery after the Great Recession with their counterparts after the previous recessions, in the case of a significant shortfall, the model can identify the declining growth of the trend component (potential output), an unprecedented deceleration in the trend component, and an unusually sluggish recovery of the cyclical component (output gap) as three distinct driving forces of the slow recovery.

Finally, to measure the magnitude of the spillovers of U.S. real activity shortfall (U.S. RAS) by using a counterfactual analysis based on an open-economy hierarchical DFM, we answer this question: what would the output recovery in the U.K. have been, if there was no slow recovery in the U.S.?

This paper makes four contributions to the literature. First, without relying on any trend-cycle decomposition, it examines the slow recovery by taking account of potential structural breaks in the intercept and coefficient as the intervention factor within the framework of counterfactual analysis.[6] Second, in the trend-cycle decomposition, allowing for a potential structural break in the Okun's intercept and coefficient helps better identify the declining trend growth, the fall in trend growth, and an unusual slow cyclical recovery as the three distinct drivers of the slow recovery.[7] Third, very few studies have applied a DFM to the U.S. series to explain the U.S. slow recovery (see, e.g., Fernald et al., 2017; Antolin-Diaz et al., 2017), and to our knowledge, no study has analyzed the U.K. productivity puzzle by applying a DFM.[8] Fourth, considering the relationship between the two phenomena, the slow recovery in the U.S. and the U.K. productivity puzzle, the magnitude of the shortfall spillovers from the U.S. to the U.K. has not yet been measured. In this sense, we estimate a novel counterfactual over the whole period of recovery based on a reduced DFM under the stability

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[5] Because the Principal Component Analysis (PCA) is performed on stationary and locally demeaned series, we refer to factors and factor loadings as the cyclical factors and cyclical factor loadings. For a detailed explanation, see data and methodology section and Appendix 1.C.

[6] Since our first method does not take previous recoveries as the benchmark for the counterfactual recovery, in contrast to Fernald et al. (2017) and Grant (2018), it does not rely on trend-cycle decomposition. Thus, this method is not susceptible to the uncertainty in the estimation of the unobserved components (trend and cycle), which is the result of the unaccounted for instability in the coefficient of the difference version of Okun's law.

[7] While Fernald et al. (2017) and Antolin-Diaz et al. (2017) identify the downward trend growth, they fail to identify two additional drivers: unprecedented trend deceleration, and sluggish cycle recovery.

[8] It is worth noting that Marcellino et al. (2003) and Artis et al. (2005) are two of the few studies that used a DFM on the U.K. series for forecasting purposes.

assumption of parameters, rather than, estimating an impulse response to a shock of interest that hits the economy at a single point based on a structural DFM.

Our findings indicate that there is a significant structural break in the parameters of Okun's law and the DFM that occurred during the 2007−09 financial crisis. By specifying the counterfactual recovery as the post-crisis fitted output per capita, estimated by a difference version of Okun's law, we document a significant annual shortfall of 1.32 and 0.83 percentage points per year in the U.S. and the U.K., respectively. Putting these together, our results highlight that a change in regime in the aftermath of the financial crisis (revealed in the form of a break in the Okun's intercept, Okun's coefficient, and cyclical factor loadings) is the main determinant of the slow recovery in the U.S. and the U.K. Therefore, in contrast to Fernald et al. (2017) and Antolin-Diaz et al. (2017), our results underline the influence of the 2007–09 financial crisis and Great Recession on the subsequent slow recovery. In addition, the growth-accounting suggests that the slow growth of TFP and a sharp fall in the growth of capital input are the main contributors to the slow recovery.

The trend-cycle decomposition supports the results of the first method, implying that the identified shortfall is robust to the choice of methods to generate the counterfactual recovery. The trend-cycle decomposition for output per capita, in which the instability of the Okun's intercept is accounted for, identifies two distinct supply-side drivers of the slow recovery: a decline in trend growth from about 3% in the 1960s to 1.6% in the mid-2000s; and a remarkable trend deceleration that led to a further drop in trend growth from 1.6% to less than 0.8%, which started following the 2007−09 financial crisis. In addition, allowing for a structural break in the Okun's coefficient helps to identify an additional demand-side driver: a sluggish cyclical recovery. Although the 2007−09 recession was deep, the average growth of the cyclical component during the recovery was less than its counterparts during the previous recoveries in the U.S. This implies that the cycle had not recovered, consistent with the stylized fact of Friedman's plucking model (1964, 1993). This finding contrasts with the result obtained by the trend-cycle decomposition proposed by Fernald et al. (2017), in which the stability of Okun's coefficient is presumed, and identifies the decline in trend growth as the only determinant of the slow recovery.

Overall, we identify three distinct driving forces for the U.S. slow recovery and the U.K. productivity puzzle. (1) A decline in the growth of potential output (trend component) that began in the 1960s. This driver is the same as the findings of a gradual slowdown in potential output by Antolin-Diaz et al. (2017), Fernald et al. (2017), and Zhang (2019). (2) An unprecedented deceleration of potential output (trend component) in the aftermath of the 2007−09 financial crisis. Identifying this driver confirms the work of Patterson (2016), Van Ark (2016), Van Ark and Jäger (2017), Oulton (2019), and Bauer et al. (2020), who all report a sharp fall in the growth of output and TFP after the Great

Recession. It also accords with the results of Reinhart and Rogoff (2014), Bordo and Haubrich (2016), Crafts (2018), and Grant (2018), who conclude that the Great Recession, which is associated with the 2007–09 financial crisis, is followed by a slower recovery compared to its counterparts. (3) An unusually sluggish recovery of the U.S. output gap (cyclical component). This driver suggests that constrained demand and hysteresis effects following the Great Recession contributed to the slow recovery, which is presented by Fatás and Mihov (2013), Reifschneider et al. (2015), Michau (2018), Anzoategui et al. (2019), Fontanari et al. (2020), and Cerra et al. (2022).

Finally, the concomitant occurrence of the U.S. slow recovery and the U.K. productivity puzzle after the global financial crisis motivates us to examine their relationship. Consistent with the literature on the transmission of international shocks (see, e.g., Dees et al., 2007; Mumtaz and Surico, 2009; Georgiadis, 2017; Fadejeva et al., 2017), our findings indicate that the slow recovery in the U.S. has a significant influence on the productivity puzzle in the U.K. By measuring the magnitude of the shortfall spillovers, on an annual basis, we demonstrate that spillovers of U.S. RAS account for 0.62 percentage points of the shortfall in U.K. output per capita. This result underscores the economic diversification in the U.K. as a small country in response to the long-term productivity slowdown and hysteresis effects in the U.S. Hence, the U.K. must shift its economy away from a mostly bilateral economic relationship toward a more multilateral economic relationship.

The rest of the paper reviews the literature on the slow recovery, the productivity puzzle, and Okun's law in Section 1.2. Section 1.3 describes the data, methodologies, and approaches used; Section 1.4 presents the results and discussion; and Section 1.5 provides conclusions.

1.2. Literature review

Following the 2007–09 global financial crisis and economic recession, researchers have focused on analysing factors causing this financial recession as well as the subsequent slow recovery. While the first strand of the literature attempts to examine the recession itself and identifies the causes of the financial crisis and their role in shaping or exacerbating the severity of the recession (see, among others, Reinhart and Rogoff, 2008, 2009; Stock and Watson, 2012; Taylor, 2014; Jorda et al., 2017; Greenwood et al., 2022), the second strand explores the aftermath of such a recession and the main causes of the post-crisis slow recovery (see, among others, Reinhart and Rogoff, 2014; Gordon, 2014; Hall, 2015; Fernald et al., 2017; Antolin-Diaz et al., 2017; Van Ark and Jäger, 2017).

Regarding the first strand, Stock and Watson (2012), by estimating a DFM with 200 U.S. variables, argue that the 2007–09 economic recession is attributable to abrupt and large changes in constructed factors based on data before 2007 rather than a change in regime (e.g., structural break in factor loadings). Indeed, a factor model applied to data covering all previous post-war recessions is capable of explaining the Great Recession by allowing larger shocks to be taken into account. Conversely, Jorda et al. (2017) document the importance of financial factors in shaping business cycles.

Regarding the second strand, it took at least five years after the 2007−09 financial crisis for the slow recovery of output to be appreciated by pioneer researchers (Gordon, 2014; Fernald, 2015; Hall, 2015). Professional forecasters have also persistently over-predicted output recovery compared to the actual output growth, while under-predicted the unemployment recovery compared to the actual unemployment rate (Lansing and Pyle, 2015; Lewis and Pain, 2015). This means that forecasters almost always over-predicted the output, and there was an unusual negative forecast error during the recovery after the financial crisis.

Is forecasting a strong recovery after the Great Recession consistent with the theory? According to Friedman's plucking model (1993), the deeper the recession, the stronger the subsequent recovery. In this regard, Ferraro (2018), Dupraz et al. (2019), and Ferraro and Fiori (2022), by developing a micro-founded model of the business cycle, establish the plucking property in the U.S. unemployment rate. Bordo and Haubrich (2016) also, by analysing 26 business cycles starting from 1882 and ending with the 2007–09 recession, substantiate this general pattern in U.S. output: a deep recession preceded by or associated with a financial crisis is prone to be followed by a stronger recovery. Since the Great Recession was deep and associated with the financial crisis, predicting a strong recovery for output and unemployment by forecasters was supported by the theory and was not because of the low quality of the forecasts. Hence, there should be an unpredicted event such as a structural break and change in regime to explain the negative systematic forecast error for output.

As preliminary evidence, Fatás and Mihov (2013) and Reinhart and Rogoff (2014) state that although the Great Recession, which was caused by the 2007−09 financial crisis, was one of the deepest post-World War II recessions, its subsequent recovery was longer than others. Bordo and Haubrich (2016) also find three exceptions to the abovementioned general pattern. They discovered that recoveries from the 1930s Great Depression, the early 1990s recession, and the 2007–09 Great Recession took longer than others. Concerning the 2007−09 recession, while the recovery of the U.S. unemployment rate is consistent with the stylized facts of the plucking model (Dupraz et al., 2019), U.S. output falls short of the recovery supposed to be based on the plucking model.

Fernald et al. (2017) detect that U.S. output recovered slowly, although the unemployment rate recovery was strong. To investigate the U.S. slow recovery, they compared the recovery following the Great Recession with three previous recoveries, which were deemed as counterfactual. Since the depth of each recession is different from the others, they control for the depth by using a cyclical adjustment, where unemployment is taken as a reliable proxy for the cycle. They document an annual shortfall of at least 1.21 percentage points in U.S. output per capita and identify the decline in trend growth as the main driving force of the slow recovery. Furthermore, by using the growth-accounting decomposition, they find that the slow growth of TFP and the decline in LFP have mostly contributed to the slow recovery. Antolin-Diaz et al. (2017) apply a small DFM with time-varying intercepts and report a significant slowdown in the long-run output growth. Zhang (2019), analogously, identifies the routine biased technological changes and structural changes that shifted investment in the late 1990s as the two drivers of the U.S. slow recovery.

Van Ark and Jäger (2017) find that the slow recovery in the U.S. and the U.K. has broadened to the manufacturing sector after the Great Recession. To explain the U.K. productivity puzzle, Oulton (2019) proposes that a bad regime during the recovery from the Great Recession has constrained the demand for U.K. exports and indirectly led to the slowdown of TFP. Patterson et al. (2016) also, by applying a labour-matching model, suggest that the labour misallocation during the recovery explains up to two thirds of the U.K. productivity puzzle.

Since there is evidence supporting the occurrence of a break in TFP in 2006 before the onset of the financial crisis, Luo and Startz (2014) and Fernald et al. (2017) conclude that the adverse driving forces of the slow recovery are largely unrelated to the financial crisis. Antolin-Diaz et al. (2017) likewise state that the decline in the growth of labour productivity mostly occurred prior to the Great Recession. This, however, contrasts with the results of Fatás and Mihov (2013), Reinhart and Rogoff (2014), Bordo and Haubrich (2016), Patterson et al. (2016), Van Ark and Jäger (2017), Crafts (2018), Oulton (2019), Anzoategui (2019), Fontanari et al. (2020), and Cerra et al. (2022) who all underscore the influence of the 2007–09 financial crisis on the slow recovery. To give an example, Anzoategui (2019) states that although the productivity slowdown started in 2005, the subsequent drop in TFP after the financial crisis was an endogenous response to the zero lower bound and constrained demand. Fontanari et al. (2020) conclude that a prolonged low-demand period following the Great Recession explains the slowdown in potential output.

Let us turn to the difference version of Okun's law, the empirical relationship between the growth of output and the change in unemployment, which is well-established for different countries (Ball et al., 2017). There is still a dispute about the stability of the Okun's law coefficient in the U.S. and other advanced economies. Those studies, in which the time-variation in trend growth is unaccounted for, report the instability of the Okun's coefficient in the form of a break during the Great Recession (Owyang and Sekhposyan, 2012; Basu and Foley, 2013; Grant, 2018). By contrast, others who consider a time-varying trend growth, conclude in favour of the stability of Okun's law (Daly et al., 2014; Economou and Psarianos, 2016; Ball et al., 2017; Michail, 2019). These conflicting results appear to be the consequence of different specifications for trend growth.

1.3. Data and methodology

This study uses Okun's law and a DFM to generate the counterfactual. To estimate the Okun's law regressions, we use quarterly data on real gross domestic product (GDP) per capita and the quarterly unemployment rate for people aged 16 and over. For the DFM, we use data consisting of 123 U.S. series from 1960Q1 to 2016Q4 and 48 U.K. series from 1971Q1 to 2016Q4. To estimate four cyclical factor loadings, following Stock and Watson (2016) and Fernald et al. (2017), we perform Principal Component Analysis (PCA) on the stationary series that are locally demeaned by using Tukey's bi-weight filter with a bandwidth of 100 quarters. Appendices 1.A-D describe series, transformations required for each series to achieve stationarity, and seasonal adjustment where appropriate. To generate the counterfactual, we estimate the Okun's law parameters and cyclical factor loadings for both the U.S. and the U.K. over the sample period of interest, which is 1981Q1 to 2016Q2. The start date is chosen to align with the early 1980s recession.

To examine whether the economy has experienced a slow recovery or not, like the ex-post static evaluation presented by Pesaran and Smith (2016), we compare the actual recovery with a counterfactual recovery. A counterfactual is what would be expected to happen in the absence of a specific determinant factor (Pearl, 2009). We then define the shortfall as the difference between the counterfactual and actual recoveries. Thus, by definition, if the ceteris paribus condition holds, a significant shortfall suggests the causal effect of the determinant factor on the observed event (Heckman, 2000; Hoover, 2001).[9] In this study, the determinant factors which we are interested in analysing their causal effects on the output shortfall are: (1) structural breaks in the Okun's law intercept, Okun's law coefficient, and cyclical factor loadings during the 2007−09 financial crisis in the first method; and (2) the downward trend growth, unprecedented trend deceleration, and sluggish cyclical recovery in the second method.

The existing literature applies three methods to capture the slow recovery. Below, we will briefly discuss these methods and their shortcomings. First, Van Ark and Jäger (2017) and Oulton (2019) compare the recovery after the Great Recession with previous recoveries, deemed as a counterfactual. They impose that the current recovery must follow previous recoveries, although the depth of the Great Recession is different from the others. To deal with this issue, Fernald et al. (2017) use a trend-cycle decomposition to control for depth. They measure the cyclically adjusted trend as the residuals of the Okun's law regression, assuming a constant intercept and coefficient. Accordingly, any time-variation in the intercept that represents trend growth will be captured by the residuals. However,

[9] Based on the ceteris paribus condition, a credible counterfactual recovery should be the same as the actual recovery in all factors except a specific determinant factor to avoid any unwanted interference from the impact of other confounding factors (King and Zeng, 2006). See Appendix 1.E for more information.

since they do not allow for a potential break in the coefficient, the effect of any potential structural break in the Okun's coefficient will inevitably permeate into the residuals, contaminating the measurement of the trend and invalidating the conclusion about the influence of the trend and cycle on the slow recovery.[10]

Second, Antolin-Diaz et al. (2017) apply a small DFM with 24 series to the U.S. and other advanced economies to explain the slowdown in long-run growth. They employ a single factor model with time-varying intercepts in order to accommodate the decline in trend growth. Fernald et al. (2017) comparably utilize a DFM with 123 U.S. series to explain the slow recovery. However, both studies do not accommodate a structural break in cyclical factor loadings, while there is evidence suggesting a break in them during the 2007–09 financial crisis.[11] Also, as explained in Appendix 1.J, since Fernald et al. (2017) generate the counterfactual recovery by forecasting cyclical components for a long horizon of 28 quarters, their method does not satisfy the ceteris paribus condition.

Third, the gap and difference versions of Okun's law employed by Basu and Foley (2013), Grant (2018), and Fontanari et al. (2020) identify a structural break in the Okun's law coefficient. They consider a constant Okun's law intercept that imposes a constant trend growth on the output. But this presumption has been called into question because there is strong evidence supporting a gradual decline in trend growth that began in the 1960s and a sharp fall in trend growth following the Great Recession. The unaccounted for time-variation in trend growth, unavoidably, appears in the form of an instability in the Okun's coefficient.[12]

To address the shortcomings discussed above, we propose two methods to estimate the counterfactual recovery. In the first method, the counterfactual is measured by the post-crisis fitted output per capita, which is estimated by the following two approaches. In the first approach, the fitted output per capita is estimated by Okun's law, while the Okun's intercept and coefficient are assumed to be stable. This method answers the question of what the counterfactual recovery of output per capita would have been given the 2009–16 recovery of unemployment rate under the assumption of the stability of Okun's law. Although unemployment is a reliable proxy for the cyclical component, there are other competing variables such as price and the cyclical principal component (Morley and Wong, 2020; Gonzalez and Roberts, 2022). Hence, in the second approach, the fitted output per capita is estimated

[10] See Tables 1.1.3 and 1.2.3, which document the occurrence of a break in the Okun's law coefficient during the 2007−09 financial crisis for the U.S. and the U.K. The model which does not allow a break, tends to overestimate the contribution of the trend and underestimate the contribution of the cycle to the slow recovery by attributing the break in the Okun's coefficient to the trend while it is attributable to the cyclical component.

[11] See Tables 1.1.4 and 1.2.4 that indicate the occurrence of a structural break in the cyclical factor loadings for the GDP per capita and its components around the 2007−09 financial crisis for the U.S. and the U.K.

[12] When the time-variation in trend growth is not accounted for, our findings presented in Figures 1.3.3 and 1.4.3 support the presence of a structural break in the difference version of Okun's law in 2009 and 2008 in the U.S. and the U.K.

by a DFM, where the stability of the cyclical factor loadings is the assumption on which the counterfactual is built. In the case of the significant shortfall, the slow recovery is attributable to the instability of the parameters of Okun's law and the DFM in the first and second approaches, respectively, implying a change in regime in the aftermath of the 2007–09 financial crisis. This method appears to be more reliable than the methods applied by previous studies since it does not rely on the trend-cycle decomposition.

The second method, like Fernald et al. (2017), considers previous recoveries as counterfactual. Since the depth of the Great Recession is different from that of the previous recessions, we control for the depth by using a trend-cycle decomposition performed based on Okun's law. In contrast to Fernald et al. (2017), the structural break in the Okun's coefficient, which is identified by break tests, is accommodated with the aim of distinguishing between trend-related and cycle-related driving forces of the slow recovery. Alternatively, for trend-cycle decomposition, we use a DFM that, in contrast to Antolin-Diaz et al. (2017), allows for a structural break in the cyclical factor loadings.

In all of the above methods, as explained in Appendix 1.I, a growth-accounting decomposition of the output growth into the change in TFP, capital input, and labour input is conducted to find the main contributors to the slow recovery.

1.3.1. The first method: post-crisis estimation of output

1.3.1.1. The first approach: post-crisis estimation of output based on Okun's law

In the first approach, we estimate fitted output per capita as a measure of the counterfactual recovery after the 2007–09 financial crisis by using a difference version of Okun's law, in which the unemployment rate is placed on the right-hand:

$$y_t = \alpha + \beta(L)(\Delta U_t) + e_t = \alpha + \sum_{l=-p}^{p} \beta_l(\Delta U_{t+l}) + e_t \qquad (1.1)$$

where y_t is the percentage growth rate (differences of log) of output per capita on an annual basis, ΔU_t is the change in unemployment, α is the Okun's intercept, $\beta(L)$ represents the Okun's coefficient, which is a lag polynomial with p leads and lags, and e_t is a white noise error term.[13] We estimate Eq. (1.1) over the period from 1981Q1 to 2016Q2, assuming that the Okun's law is stable. Alternatively, we estimate parameters over the period 1981–2009 and assume that the Okun's law parameters during the post-crisis recovery are equal to those before the 2007–09 financial crisis. The post-crisis fitted output per capita ($\hat{y}_t$) from 2009Q4 to 2016Q2 is obtained by using the post-crisis

[13] The sum of the lag coefficients is called the generalized Okun's law coefficient. To include the lead-lag effect between output and unemployment in the benchmark model, we estimate Eq. (1.1) with $p = 1$ lead and lag. To estimate the Okun's coefficient without a lead-lag effect, we also estimate the model with $p = 0$.

unemployment rate. The difference between the counterfactual growth of output per capita ($\hat{y}_t$) and the actual growth of output per capita (y_t) after the Great Recession is defined as the shortfall:

$$shortfall_t = \hat{y}_t - y_t \tag{1.2}$$

for $t = 2009Q4, \dots, 2016Q2$. Shortfall in Eq. (1.2) is a mean-zero error term if the assumption of the stability of Okun's law holds, i.e. $\hat{\alpha}^{81-16} = \alpha^{09-16}$ and $\hat{\beta}(L)^{81-16} = \beta(L)^{09-16}$, and is equal to Eq. (1.3) if the assumption does not hold:

$$shortfall_t = (\hat{\alpha}^{81-16} - \alpha^{09-16}) + \left(\hat{\beta}(L)^{81-16} - \beta(L)^{09-16}\right)\Delta U_t - e_t \tag{1.3}$$

In fact, if the shortfall is a mean-zero error term, the assumption of the stability of Okun's law would be correct. Conversely, if the shortfall is statistically different from zero, the stability of Okun's law after the recession trough (2009Q3) would not hold and the observed shortfall is attributable to the instability of Okun's law. Since a weaker relationship between output and unemployment after 2009 has been suggested, we use the Wilcoxon signed rank test to determine whether the shortfall is zero for the null or positive for the alternative hypothesis.[14] To distinguish between the contributions of instability of the Okun's intercept and Okun's coefficient, we relax the assumption of stability of the Okun's intercept by estimating Eq. (1.4) and measuring the shortfall for the new setup:

$$y_t = \alpha_t + \beta(L)(\Delta U_t) + e_t \tag{1.4}$$

where α_t is a time-varying intercept that represents the trend growth. Since the growth of potential output (trend component) is downward for the U.S. and the U.K., we model the Okun's intercept as a random walk process. We estimate Eq. (1.4) by casting it into the state-space form and applying Kalman's (1960) filter explained in Appendices 1.F and 1.G. If incorporating time-varying intercept reduces the shortfall, we infer the causal effect of the decrease in trend growth on the slow recovery; whereas, the remaining shortfall is attributable to the instability of the Okun's coefficient. We alternatively measure the shortfall by incorporating a structural break in the Okun's coefficient:

$$y_t = \alpha + [\beta(L) + \delta(L)D_t]\Delta U_t + e_t \tag{1.5}$$

where $\delta(L)$ is a lag polynomial and D_t is a dummy variable that is equal to one after the identified structural break (2009Q1) and zero otherwise. If including the break in the Okun's coefficient sharply decreases the shortfall, we infer the causal effect of the structural break in the Okun's coefficient on the slow recovery. Indeed, the magnitude of the decrease in the shortfall by including the break represents the contribution of the structural break in the Okun's coefficient to the slow recovery. We also measure the shortfall when the structural break in both intercept and coefficient is allowed for.

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[14] We test H_0: $shortfall_t = 0$ against H_1: $shortfall_t > 0$ for $t = 2009Q4, \dots, 2016Q2$ and report the Wilcoxon one-sided p-value to examine whether the shortfall is statistically different from zero or not. In addition, similar to Fernald et al. (2017), we simply calculate the average of the shortfall and compare it with zero.

1.3.1.2. The second approach: post-crisis estimation of output based on DFM

In the second approach, under the assumption of stability of the intercept and cyclical factor loadings, we take the fitted series estimated by a DFM as a measure of the counterfactual recovery.[15] Consider the factor model in Eq. (1.6), which specifies each time series as a linear combination of an intercept, unobserved cyclical factors, and an idiosyncratic error term:

$$X_t = A + \Lambda F_t + e_t \tag{1.6}$$

where X_t denotes a vector of time series, A is a vector of intercepts, Λ is an $n \times r$ matrix of cyclical factor loadings, F_t is a vector of cyclical factors capturing the cyclical movements of the series and e_t is a vector of idiosyncratic white noise error terms. For a better illustration, Eq. (1.7) shows the extended form of each matrix:

$$\begin{bmatrix} X_{1,t} \\ X_{2,t} \\ \cdots \\ X_{n,t} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \cdots \\ \alpha_n \end{bmatrix} + \begin{bmatrix} \Lambda_{11} & \cdots & \Lambda_{1r} \\ \Lambda_{21} & \cdots & \Lambda_{2r} \\ \cdots & \cdots & \cdots \\ \Lambda_{n1} & \cdots & \Lambda_{nr} \end{bmatrix} \begin{bmatrix} F_{1,t} \\ F_{2,t} \\ \cdots \\ F_{r,t} \end{bmatrix} + \begin{bmatrix} e_{1,t} \\ e_{2,t} \\ \cdots \\ e_{n,t} \end{bmatrix} \tag{1.7}$$

where $X_{i,t}$ denotes the i^{th} time series, α_i is an intercept, Λ_{ij} is a cyclical factor loading, capturing the relationship between j^{th} cyclical factor and i^{th} variable, and $F_{j,t}$ is the j^{th} cyclical factor. To capture the slow recovery, we took the following steps: First, we estimate the cyclical factors by using PCA on all of the locally demeaned series over the whole sample period.[16] Second, we treat the estimated cyclical factors as data to estimate the cyclical factor loadings (Λ_{ij}) by regressing each series ($X_{i,t}$) on an intercept and the cyclical factors:

$$\hat{X}_{i,t} = \hat{\alpha}_i + \hat{\Lambda}_i^{81-16} \hat{F}_t^{81-16} \tag{1.8}$$

We estimate Eq. (1.8) in the period 1981Q1–2016Q2, under the assumption of parameter stability.[17] In this equation, $\hat{\Lambda}_i^{81-16} = (\hat{\Lambda}_{i1}^{81-16}, \hat{\Lambda}_{i2}^{81-16}, \dots, \hat{\Lambda}_{ir}^{81-16})$ denotes the i^{th} row of the matrix of cyclical

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[15] The DFM model utilises 123 quarterly time series for the U.S. and 48 quarterly time series for the U.K. All series included in the factor model are seasonally adjusted and stationary. More importantly, they are locally demeaned by using Tukey's bi-weight filter with a bandwidth of 100 quarters. The motivation for applying PCA to the locally demeaned series is to derive a proxy for the business cycle, which we call cyclical factors, in order to enable the model to distinguish between the contribution of the instability of the intercept related to a downward trend growth and the instability of the cyclical factor loadings related to the unusual sluggish recovery of the cyclical component. Stock and Watson (2016) also provide other reasons for using a low-pass filter to remove any remaining low-frequency movements. See Appendices 1.A, 1.B, and 1.C for more information on the names of series, the transformation, and seasonal adjustment necessary for each series, etc.

[16] We apply PCA to 123 U.S. series from 1960Q1 to 2016Q4, and to 48 U.K. series from 1971Q1 to 2016Q4. We derive four cyclical factors for each of the U.S. and U.K. data. The number of factors (r) is determined by Bai and Ng (2002) information criteria and marginal R^2 presented in Tables 1.L.3 and 1.L.4 in Appendix 1.L. Since the number of series (n) is large, the estimated factors are consistent and can be treated as true factors when used in the second-step regressions (Artis et al., 2005; Bai and Ng, 2006; 2008; Stock and Watson, 2002).

[17] Alternatively, we estimate parameters over the period 1981–2009 and assume that the cyclical factor loadings during the post-crisis recovery are equal to those before the 2007–09 financial crisis. The regression of each series is estimated by the Newey-West estimator to handle the potential autocorrelation and heteroscedasticity in the error terms.

factor loadings and $\hat{F}_t^{81-16}$ is the vector of the estimated cyclical factors. Third, by subtracting the values of a single series of Eq. (1.7) from its common component estimated in Eq. (1.8), we measure the shortfall after the Great Recession that must be equal to a mean-zero error term if the stability of the intercept and cyclical factor loadings holds. By contrast, if the stability assumption does not hold, the shortfall is equal to:

$$shortfall_t = \left(\hat{\alpha}_i{}^{81-16} - \alpha_i{}^{09-16}\right) + \left(\hat{\Lambda}_i^{81-16}\,\hat{F}_t^{81-16} - \Lambda_i{}^{09-16}F_t^{09-16}\right) - e_{i,t} \tag{1.9}$$

for $t = 2009Q4, \ldots, 2016Q2$. Thus, if the shortfall is significant, the slow recovery is attributable to the instability of the intercept and cyclical factor loadings. Similar to Section 1.3.1.1, we relax the assumption of stability of the intercept and measure the new shortfall by estimating Eq. (1.10) in which the time-variation in the intercept is accommodated as follows:

$$X_{i,t} = \alpha_{i,t} + \Lambda_i F_t + e_{i,t} \tag{1.10}$$

where $\alpha_{i,t}$ is a time-varying intercept of the i^{th} time series that corresponds to the trend growth.[18] We expect this amendment reduces the shortfall, which corroborates the contribution of the slowdown in the trend component to the slow recovery. Yet, the remaining shortfall, which Eq. (1.10) is not able to eliminate, is associated with the instability of the cyclical factor loadings.

Alternatively, to substantiate the causal effect of the structural break in the cyclical factor loadings on the slow recovery, we estimate the cyclical factors by two separate PCAs over two regimes before and after 2009Q1, which is close to the break date identified by the break tests, to examine whether including the structural break in the cyclical factor loadings sharply decreases the shortfall or not. The post-crisis fitted series is then estimated as follows:

$$X_t = A + (\Lambda + \Delta D_t)F_t + e_t \tag{1.11}$$

In Eq. (1.11), F_t stands for the cyclical factors estimated separately in two regimes, Λ and Δ are $n \times r$ matrices of cyclical factor loadings, and D_t is a $r \times r$ dummy matrix that equals the identity matrix after the break and a zero matrix otherwise. As both approaches explained in Eq. (1.1) to Eq. (1.11) are applicable to the output per capita as well as all of its growth-accounting components, three versions of growth-accounting decompositions are being conducted to find the primary contributors to the slow recovery.

[18] Similar to Section 1.3.1.1, we specify the intercept (trend growth) in the form of a random walk process. After casting the model into the state-space model, we estimate Eq. (1.10) by using Kalman's (1960) filter. Since PCA is performed on locally demeaned series, the cyclical factors do not contain the dynamics of trend growth. Thus, by construction, we expect that the estimated time-varying intercept for each series will be analogous to the idiosyncratic local mean of each series. In Appendix 1.H, we explain the details of different estimation strategies to accommodate the time-varying trend growth.

1.3.2. The second method: trend-cycle decomposition

1.3.2.1. The first approach: trend-cycle decomposition based on Okun's law

The second method defines the shortfall as the difference between the counterfactual growth of output per capita and the actual growth of output per capita after the Great Recession. However, this method considers the three previous recoveries as counterfactual recovery. Since the depth of each recession is different from the others, we control for the depth of recessions by using a trend-cycle decomposition. We decompose the growth of each series into the growth of its trend and cyclical components in the form of Eq. (1.12):

$$y_t = \mu_t + c_t \tag{1.12}$$

where y_t is the percentage growth rate (differences of log) for the observable series on an annual basis, μ_t represents the growth rate of the trend, and c_t is the growth rate of the cyclical component. If we apply the above model to the output per capita, then the trend and cyclical components correspond to the potential output and output gap for GDP per capita, respectively.

We employ two approaches for the trend-cycle decomposition. In the first approach, based on Okun's law, we specify the growth of the cyclical component of each series of interest as a linear regression of the change in the unemployment rate.[19] Since there is evidence implying instability of the Okun's law coefficient in models in which the asymmetry in the cyclical component and time-variation in trend growth are unaccounted for (see, e.g., Owyang and Sekhposyan, 2012; Basu and Foley, 2013; Grant, 2018), the accuracy of the trend-cycle decomposition offered by Fernald et al. (2017) is under question. Hence, to distinguish between the three candidate drivers of the slow recovery (downward trend growth, unprecedented trend deceleration, and sluggish cyclical recovery), we include a dummy variable to allow for the structural break in the Okun's coefficient:

$$c_t = [\beta(L) + \delta(L)D_t]\Delta U_t + e_t \tag{1.13}$$

where ΔU_t is the change in unemployment, $\beta(L)$ and $\delta(L)$ are two lag polynomials, D_t is a dummy variable that equals one after the break and zero otherwise, and e_t is the irregular part or white noise error term.[20] By substituting Eq. (1.13) into Eq. (1.12), we derive Eq. (1.14) that is presented below:

$$y_t = \mu_t + [\beta(L) + \delta(L)D_t]\Delta U_t + e_t \tag{1.14}$$

where μ_t is a time-varying intercept and is assumed to evolve according to a random walk process to characterize the trend growth dynamics. To derive the growth of potential output (μ_t), we estimate

[19] Gonzalez and Roberts (2022) document the superiority of the unemployment rate over other auxiliary variables such as the inflation rate and interest rate in the trend−cycle decomposition for output.

[20] The irregular part contains the higher frequency movement in the variable that is uncorrelated with the cyclical component.

Eq. (1.14) for output per capita over the period of 1981–2016 by using the Kalman's (1960) filter (See Appendix 1.F and 1.G). Since this method is computationally burdensome and requires setting initial values for each series, we do not apply it to other series. Instead, to derive the growth of trend and cyclical components for other series, we adopt a method similar to Fernald et al. (2017) by regressing the growth of each series (y_t) on a constant intercept and the change in the unemployment rate, while the break in the Okun's coefficient is allowed for:

$$y_t = \alpha + [\beta(L) + \delta(L)D_t]\Delta U_t + \varepsilon_t \tag{1.15}$$

where ε_t is called Okun's law residual, and the growth of the cyclical component is the product of the Okun's coefficient and the change in unemployment. By comparing Eq. (1.14) and Eq. (1.15), we find that Okun's law residuals absorb the time-variation in trend growth (μ_t) around a baseline intercept (α) accompanied by the irregular part (e_t), as follows:

$$y_t - [\beta(L) + \delta(L)D_t]\Delta U_t = \alpha + \varepsilon_t = \mu_t + e_t \tag{1.16}$$

Finally, since the estimated residual is the sum of the trend growth and irregular part, we pass the estimated Okun's law residuals on the left hand side of Eq. (1.16) through a low-pass filter, represented by $\kappa(L)$, to eliminate the high frequency movements within the irregular part:

$$\hat{\mu}_t = \kappa(L)\big(y_t - [\hat{\beta}(L) + \hat{\delta}(L)D_t]\Delta U_t\big) \tag{1.17}$$

We use Tukey's bi-weight filter with a bandwidth of 60 quarters to derive a growth of smooth trend ($\hat{\mu}_t$), which by construction, it is supposed to be similar to its counterpart (μ_t) in Eq. (1.14). By conducting the above exercise for each series, we obtain the growth of the smooth trend for each series and compare its average during the current recovery with that of the previous recoveries.

1.3.2.2. The second approach: trend-cycle decomposition based on DFM

In the second approach, we use PCA for estimation of the cyclical factors as a proxy for the cyclical component. Instead of Eq. (1.13), by considering each row of the extended form of matrices in Eq. (1.11), we specify the growth of the cyclical component as a linear regression of the cyclical factors:

$$c_t = \big(\Lambda_i + \nabla_i D_{i,t}\big)F_t + e_{i,t} \tag{1.18}$$

where Λ_j and ∇_j denote the i^{th} row of the matrix of cyclical factor loadings, and $D_{i,t}$ is a dummy that equals the $r \times 1$ vector with one in its elements for the dates after the break, and is a vector of zeros otherwise. This setting accommodates a potential structural break in the cyclical factor loadings.

1.3.3. The U.S. slow recovery and the U.K. productivity puzzle

1.3.3.1. An open-economy hierarchical DFM

To capture the influence of the U.S. slow recovery on the U.K. productivity puzzle, we develop an open-economy DFM that aims to quantify the magnitude of the shortfall spillovers from the U.S. to the U.K. economy. Since the U.K. is small relative to the U.S. economy, similar to the global vector autoregressive (VAR) model introduced by Dees et al. (2007), we assume U.S. factors are block-exogenous to the U.K. factors. Indeed, shocks to U.K. factors have a negligible impact on the U.S. economy. But our aim differs from that of the global VAR applied by Georgiadis (2017), Fadejeva et al. (2017), and Fernández et al. (2017) and the factor-augmented VAR presented by Mumtaz and Surico (2009). Given the tight relationship between the U.S. and the U.K., we are concerned about measuring spillovers to the U.K. economy originating directly from the U.S. rather than spillovers to the U.K. from the U.S. mediating through other foreign countries. We thus develop a two-country DFM, rather than a global model, to provide policy recommendations for the U.K. as a dominant receiver of spillovers from the U.S.

We define the shortfall in the U.K. as the difference between the counterfactual and actual recoveries. To derive the counterfactual recovery for the U.K. series, we answer this question: what would the normal recovery path of the U.K. series have been if there was a normal recovery in the U.S.? We then measure the magnitude of the shortfall spillovers to the U.K. conditional on the real activity shortfall (RAS) in the U.S. We measure the U.S. RAS by subtracting the U.S. real activity factor (RAF) from the U.S. counterfactual RAF.

In contrast to the traditional impulse response function suggested by Sims (1980) and Pesaran and Shin (1998), where shocks hit the economy at a single point, the U.S. RAS in our model is a sequence of shocks that continuously hit the economy during the whole period of recovery after the 2007–09 financial crisis. The favourable feature of our model is that the counterfactual RAF, by construction, is consistent with the dynamics of other U.S. factors, which means incorporating the U.S. RAS does not affect the estimation of other U.S. factors. This feature enables us to directly measure the magnitude of the spillovers by estimating a reduced DFM rather than a structural DFM and, thus, our method would not be subject to the identification issues.

On this basis, we present a DFM which adopts both the U.S. and U.K. factors as follows:

$$X_t^{UK} = \Lambda_1^{US} F_{1,t}^{US} + \Lambda_{2:r}^{US} F_{2:r,t}^{US} + \Lambda_{1:q}^{UK} F_{1:q,t}^{UK} + e_t \tag{1.19}$$

In this setup, X_t^{UK} is a vector of U.K. series. $F_{1,t}^{US}$, $F_{2:r,t}^{US}$ and $F_{1:q,t}^{UK}$ denote the U.S. RAF, other U.S. factors, and U.K. factors orthogonal to the U.S. factors, respectively. Λ_1^{US}, $\Lambda_{2:r}^{US}$ and $\Lambda_{1:q}^{UK}$ are also factor loadings of the U.K. series on the U.S. RAF, other U.S. factors, and U.K. orthogonal factors, and e_t is a vector of idiosyncratic white noise error terms. According to Bai and Ng (2002), we consider $r = 4$ for U.S. factors and $q = 4$ for U.K. orthogonal factors. For better illustration, Eq. (1.20) shows the extended form of each matrix:

$$\begin{bmatrix} X_{1,t}^{UK} \\ X_{2,t}^{UK} \\ \cdots \\ X_{N,t}^{UK} \end{bmatrix} = \begin{bmatrix} \Lambda_{11}^{US} & \Lambda_{12}^{US} & \cdots & \Lambda_{1r}^{US} & \Lambda_{11}^{UK} & \cdots & \Lambda_{1q}^{UK} \\ \Lambda_{21}^{US} & \Lambda_{22}^{US} & \cdots & \Lambda_{2r}^{US} & \Lambda_{21}^{UK} & \cdots & \Lambda_{2q}^{UK} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ \Lambda_{N1}^{US} & \Lambda_{N2}^{US} & \cdots & \Lambda_{Nr}^{US} & \Lambda_{N1}^{UK} & \cdots & \Lambda_{Nq}^{UK} \end{bmatrix} \begin{bmatrix} F_{1,t}^{US} \\ F_{2,t}^{US} \\ \vdots \\ F_{r,t}^{US} \\ F_{1,t}^{UK} \\ \vdots \\ F_{q,t}^{UK} \end{bmatrix} + \begin{bmatrix} e_{1,t} \\ e_{2,t} \\ \vdots \\ e_{N,t} \end{bmatrix} \tag{1.20}$$

where $X_{i,t}^{UK}$ is the i^{th} U.K. series. $F_{j,t}^{US}$ and $F_{j,t}^{UK}$ are, respectively, U.S. factors and U.K. orthogonal factors. Comparably, Λ_{ij}^{US} and Λ_{ij}^{UK} are factor loadings capturing the relation between the i^{th} U.K. series and j^{th} U.S. factor as well as the i^{th} U.K. series and the j^{th} U.K. orthogonal factor. We assume that the factors evolve according to a VAR model with order P:

$$\begin{bmatrix} F_{1,t}^{US} \\ F_{2:r,t}^{US} \\ F_{1:q,t}^{UK} \end{bmatrix} = \phi(L) \begin{bmatrix} F_{1,t}^{US} \\ F_{2:r,t}^{US} \\ F_{1:q,t}^{UK} \end{bmatrix} + \eta_t \tag{1.21}$$

where η_t is a vector of serially uncorrelated white noise error terms, $\phi(L) = \phi^1 L + \cdots + \phi^p L^p$ is an $(r+q) \times (r+q)$ matrix polynomial, whose elements are scalar polynomials in the lag operator L. As shown in Eq. (1.22), ϕ^l is a matrix containing the autoregressive coefficients $\varphi_{i,j}^l$ that identify the consequence of a change in the j^{th} factor l periods ago on i^{th} factor at current time. For example, $\varphi_{5,1}^l$ measures the effect of a change in the first U.S. factor l periods ago ($F_{1,t-l}^{US}$) on the first U.K. factor at the current time ($F_{1,t}^{UK}$). Given that U.S. factors are block-exogenous to the U.K. factors, we impose that those elements in rows $i = 1, 2, \ldots, r$ are zero for columns that $r + 1 \leq j \leq r + q$.

$$\phi^l = \begin{bmatrix} \varphi_{1,1}^l & \varphi_{1,2}^l & \cdots & \varphi_{1,r}^l & \varphi_{1,r+1}^l & \cdots & \varphi_{1,r+q}^l \\ \varphi_{2,1}^l & \varphi_{2,2}^l & \cdots & \varphi_{2,r}^l & \varphi_{2,r+1}^l & \cdots & \varphi_{2,r+q}^l \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \varphi_{r,1}^l & \varphi_{r,2}^l & \cdots & \varphi_{r,r}^l & \varphi_{r,r+1}^l & \cdots & \varphi_{r,r+q}^l \\ \varphi_{r+1,1}^l & \varphi_{r+1,2}^l & \cdots & \varphi_{r+1,r}^l & \varphi_{r+1,r+1}^l & \cdots & \varphi_{r+1,r+q}^l \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \varphi_{r+q,1}^l & \varphi_{r+q,2}^l & \cdots & \varphi_{r+q,r}^l & \varphi_{r+q,r+1}^l & \cdots & \varphi_{r+q,r+q}^l \end{bmatrix} \tag{1.22}$$

1.3.3.2. Measuring the spillovers of real activity shortfall from the U.S.

In this section, we explain how to follow six steps to measure the spillovers of RAS from the U.S. to the U.K. First, the U.S. RAF ($F_{1,t}^{US}$) and other U.S. factors ($F_{2:r,t}^{US}$) are estimated by running PCA on locally demeaned U.S. data over the period 1960–2016. We construct the U.S. RAF by using PCA on 24 series in the three categories, including National Income and Product Accounts (NIPA), industrial production, and credit, along with 30 variables in the employment category. Second, we obtain the residuals of regressions of all the other 99 series on the first estimated factor (RAF). We need these residuals to estimate the other three U.S. factors using a second-step PCA on the estimated residuals.[21] In the third step, to obtain the residuals of regressions of U.K. series on the U.S. factors, which represent the part of the U.K. series not explained by the U.S. factors, all U.K. series are fitted to the U.S. factors:

$$X_t^{UK} = \Lambda_1^{US} F_{1,t}^{US} + \Lambda_{2:r}^{US} F_{2:r,t}^{US} + \chi_t^{UK} \tag{1.23}$$

where χ_t^{UK} is a vector of residuals that are orthogonal to the U.S. factors. Then, a third-step PCA on residuals is performed to derive U.K. orthogonal factors ($F_{1:q,t}^{UK}$).[22] The fourth step is estimating factor loadings Λ_1^{US}, $\Lambda_{2:r}^{US}$ and $\Lambda_{1:q}^{UK}$ by regressing U.K. series on U.S. RAF, other U.S. factors, and U.K. orthogonal factors, treating them as data over the period 1981–2016. Based on Eq. (1.24), we derive the common component of each U.K. series:

$$\hat{X}_t^{UK} = \hat{\Lambda}_1^{US} \hat{F}_{1,t}^{US} + \hat{\Lambda}_{2:r}^{US} \hat{F}_{2:r,t}^{US} + \hat{\Lambda}_{1:q}^{UK} \hat{F}_{1:q,t}^{UK} \tag{1.24}$$

for $t = 2009Q4, \dots, 2016Q2$, where $\hat{X}_t^{UK}$ is a vector of U.K. common components.

In the fifth step, the U.S. counterfactual RAF ($\tilde{F}_{1,t}^{US}$) is derived by running PCA on U.S. counterfactual series for 24 series in the NIPA, industrial production, and credit categories and U.S. actual series for 30 series in employment category. Based on Section 1.3.1, since a significant structural break in the factor loadings is identified as the determinant of the U.S. slow recovery, we estimate the U.S. counterfactual series as post-crisis fitted values by using the factor model given the 2009–16 recovery of U.S. factors under the assumption of the stability of factor loadings.[23]

In the sixth step, the post-crisis counterfactual recovery of the U.K. series is estimated conditional on the U.S. counterfactual RAF ($\tilde{F}_{1,t}^{US}$) presented below:

$$\tilde{X}_t^{UK} = \hat{\Lambda}_1^{US} \tilde{F}_{1,t}^{US} + \hat{\Lambda}_{2:r}^{US} \tilde{F}_{2:r,t}^{US} + \hat{\Lambda}_{1:q}^{UK} \tilde{F}_{1:q,t}^{UK} \tag{1.25}$$

for $t = 2009Q4, \dots, 2016Q2$, where $\tilde{X}_t^{UK}$ are U.K. counterfactual common components and $\tilde{F}_{1,t}^{US}$ stands for the U.S. counterfactual RAF. $\tilde{F}_{2:r,t}^{US}$ and $\tilde{F}_{1:q,t}^{UK}$ also denote other U.S. factors and U.K.

[21] All U.S. factors are cyclical factors since they are estimated by running PCA on locally demeaned series as we intend to remove the trace of trend growth from the U.S. factors in order to generate a counterfactual in the absence of a single determinant factor: a fall in output growth in the U.S. due to a structural break after the 2007–09 financial crisis.

[22] To satisfy the ceteris paribus condition, we do not remove the variation in trend growth of U.K. orthogonal series, and thus we let U.K. orthogonal factors capture the variation in the U.K. trend growth.

[23] We observe that structural break in the intercept, and factor loadings is remarkably stronger for 24 time series in the NIPA, industrial organization, and credit categories presented in Table 1.A.2. Likewise, a significant shortfall is observed in the series in these categories. This suggests that the recovery of the 24 series in these three categories is not consistent with the dynamics of factors estimated over the period 1960–2016. We therefore estimate the counterfactual recovery for these 24 series. Counterfactual recovery is defined as the post-crisis fitted values of each of these 24 series estimated by the DFM as in Eq. (1.8) over the period 1981–2016, under the assumption of stability of the intercept and factor loadings. For variables in employment and other categories, we include the actual series in PCA to check the ceteris paribus condition.

orthogonal factors that may differ from their counterparts in Eq. (1.24). Measuring the shortfall by subtracting a single series ($X_{i,t}^{UK}$) from its counterfactual ($\tilde{X}_{i,t}^{UK}$), the shortfall in the U.K. will be equal to:

$$U.K.shortfall_t = \left(\hat{\Lambda}_{i,1}^{US}\tilde{F}_{1,t}^{US} - \Lambda_{i,1}^{US}F_{1,t}^{US}\right) + \left(\hat{\Lambda}_{i,2:r}^{US}\tilde{F}_{2:r,t}^{US} - \Lambda_{i,2:r}^{US}F_{2:r,t}^{US}\right) + \left(\hat{\Lambda}_{i,1:q}^{UK}\tilde{F}_{1:q,t}^{UK} - \Lambda_{i,1:q}^{UK}F_{1:q,t}^{UK}\right) - e_{i,t} \qquad (1.26)$$

for $t = 2009Q4, \ldots, 2016Q2$. The open-economy model is robust to co-breaking, meaning that even if there is a structural break in the U.S., the break in parameters could be confined to the U.S. model and the parameters of the domestic country need not be subject to a similar break (Dees et al., 2007).[24] Therefore, if the nature of the macroeconomic relationship between the U.S. and the U.K. remains unchanged, we can reasonably assume the stability of parameters $\Lambda_{i,1}^{US}$, $\Lambda_{i,2:r}^{US}$ and $\Lambda_{i,1:q}^{UK}$, and simplify the shortfall as below:

$$U.K.shortfall_t = \hat{\Lambda}_{i,1}^{US}\left(\tilde{F}_{1,t}^{US} - F_{1,t}^{US}\right) + \hat{\Lambda}_{i,2:r}^{US}\left(\tilde{F}_{2:r,t}^{US} - F_{2:r,t}^{US}\right) + \hat{\Lambda}_{i,1:q}^{UK}\left(\tilde{F}_{1:q,t}^{UK} - F_{1:q,t}^{UK}\right) - e_{i,t} \qquad (1.27)$$

Since the counterfactual RAF is estimated based on the dynamics of U.S. factors in the absence of the structural break, it is consistent with the dynamics of other U.S. factors. As a result, incorporating the U.S. RAS, by construction, does not affect the estimation of other U.S. factors, and thus the second term in Eq. (1.27) is supposed to be zero.[25] Running PCA on 24 U.S. counterfactual series and all other actual series to estimate the U.S. counterfactual RAF and other U.S. factors supports this notion that other U.S. factors are invariant to the U.S. RAS.

Moreover, since U.K. orthogonal factors are part of the U.K. economy that is unrelated to the U.S. economy, the assumption stating that incorporating the U.S. RAF does not affect U.K. orthogonal factors is defensible. If this is the case, the third term is simply zero. However, since lags and leads of U.K. orthogonal factors may be affected by U.S. factors, they are not necessarily invariant to the U.S. RAS. Nonetheless, for the robustness test, by applying a VAR model specified in Eq. (1.21) and Eq. (1.22), we take the potential response of U.K. orthogonal factors to the U.S. RAS into account. Favourably, the construction of the U.S. RAS is such that the magnitude of the third term is small and does not change our findings. Indeed, we derive the same, or sometimes almost the same, shortfall for each series using this alternative model.

Conclusively, this study provides a robust approximation of the shortfall in the U.K. series emanated from the spillovers of U.S. real activity shortfall (RAS) as follows:

$$U.K.shortfall_t \cong \hat{\Lambda}_{i,1}^{US}\left(\tilde{F}_{1,t}^{US} - F_{1,t}^{US}\right) - e_{i,t} \qquad (1.28)$$

[24] Break tests suggest no structural break in the $\Lambda_{i,1}^{US}$, $\Lambda_{i,2:r}^{US}$ and $\Lambda_{i,1:q}^{UK}$. See Table 1.L.15 in Appendix 1.L.

[25] The generated counterfactual recovery for each of the 24 series in the NIPA, industrial production, and credit categories is estimated using a DFM, assuming that factor loadings are stable. Thus, as 24 counterfactual series are estimated based on U.S. factors, they are consistent with the other three U.S. factors. As a result, U.S. counterfactual RAF is in conformity with other U.S. factors. Indeed, as show in Figure 1.8, substitution of the 24 series by their 24 counterfactual series to estimate the counterfactual RAF does not affect the estimation of other U.S. factors.

for $t = 2009Q4, \dots, 2016Q4$. The shortfall is equal to a mean-zero error term if there was a normal recovery in the U.S., but it is equal to Eq. (1.28) if there is a shortfall in the U.S. If the magnitude of the spillovers of U.S. RAS is significant, the U.K. productivity puzzle is tightly associated with the U.S. slow recovery, which itself is reflected in the structural break in U.S. Okun's law and factor loadings. We will conclude that the U.K. productivity puzzle is considerably—if not entirely—attributable to the spillovers of the RAS from the U.S. To substantiate the causal effect, we remove the shortfall in the U.S. RAF by deriving U.S. factors by running two separate principal component analyses over the two regimes, before and after 2009. If this eliminates the shortfall, we document the causal effect of the U.S. slow recovery due to the structural break that occurred in 2009 on the U.K. productivity puzzle.

1.4. Results and discussion

Our findings document a significant shortfall in output per capita, TFP, and capital input in both countries. Based on the first method, the annual shortfall in output growth per capita is 1.32 and 0.83 percentage points per year in the U.S. and the U.K. We also identify a significant structural break in the parameters of Okun's law and the DFM that took place during the 2007−09 financial crisis in both the U.S. and U.K. As a result, the structural break during the financial crisis is identified as the main determinant of the U.S. slow recovery and the U.K. productivity puzzle.

The second method also yields a comparable annual shortfall of 1.23 and 1.07 percentage points per year in the U.S. and the U.K., which leads to a cumulative shortfall of more than 10 percentage points in output per capita in each of these two countries. The trend-cycle decomposition for output per capita identifies three distinct driving forces for the slow recovery. First, until the start of the financial crisis, trend growth for output per capita had been gradually declining, from higher rates of 3% in the 1960s to lower rates of 1.6% in 2007. Second, in the aftermath of the financial crisis, trend growth has fallen from around 1.6% in 2007 to a desperately low rate of less than 0.8% in 2016. Third, after the financial crisis, the growth of the cyclical component estimated by Okun's law and the DFM, on average, fell short of its counterpart during the previous recoveries by 0.44 and 0.06 percentage points per year in the U.S. and the U.K.[26] However, since the Great Recession was deep compared to three previous recessions, a faster recovery of the cyclical component was expected according to the plucking model. In particular, the recovery of the cyclical component in the U.S. was below expectations, which indicates a lack of bounce-back effect during the recovery after the financial crisis.

[26] The average of the shortfalls in cycle growth for the U.S. and the U.K. is the average of shortfalls estimated by two approaches, Okun's law and the DFM.

The growth-accounting gauges an average contribution of 0.6, 0.5, and 0.2 percentage points per year in TFP, capital input, and labour input, respectively, to the shortfall in U.S. output per capita. Likewise, on average, a shortfall in TFP and capital input accounts for 1.08 and 0.47 percentage points per year of the shortfall in U.K. output per capita, while labour input exhibits a surplus of 0.72 (shortfall of -0.72) percentage points per year.[27] This identifies the slowdown in TFP and capital input as the main contributors to the slow recovery.

Respecting the connection between the two phenomena, the slow recovery in the U.S. and the productivity puzzle in the U.K., we measured the shortfall spillovers from the U.S. to the U.K. The results document that the spillovers of the U.S. RAS explain 0.62 percentage points per year of the shortfall in the U.K.

1.4.1. The result of the first method: post-crisis estimation of output

In the first method, by comparing the post-crisis fitted output per capita with the actual output, we identify a structural break in the Okun's law and DFM parameters during the 2007–09 financial crisis as the determinant of the slow recovery in the U.S. and U.K. Considering the first approach, where the counterfactual is estimated by Okun's law, the top-left and top-right panels of Figures 1.3.1 and 1.4.1 illustrate the shortfall for the level and growth of output per capita. Tracking the gap between the counterfactual and actual, it is clear that the shortfall formed just after the recession trough in 2009Q3. Other panels plot the shortfall in the growth-accounting components of output per capita. Supporting the above results, columns (c) and (d) of Tables 1.1.1 and 1.2.1 report a shortfall of 1.32 and 0.82 percentage points per year, with a Wilcoxon p-values of 0.002 and 0.017, in U.S. and U.K. output per capita.

The growth-accounting presented in Table 1.1.1 implies that the shortfall in TFP, capital input, and labour input contributed 0.46, 0.48, and 0.37 percentage points per year to the shortfall in U.S. output per capita. Also, as reported in Table 1.2.1, the shortfall in TFP and capital input accounts for 1.07 and 0.46 percentage points per year of the shortfall in U.K. output per capita, while the insignificant Wilcoxon p-value of 0.977 for labour input indicates a surplus of 0.70 (a shortfall of -0.70) percentage points per year. The shortfall in TFP and capital input for both countries is evident in the middle panels of Figures 1.3.1 and 1.4.1. The finding that suggests a slowdown in TFP is consistent with that of Fernald et al. (2017) and Antolin-Diaz et al. (2017) for the U.S., and Patterson et al. (2016) and Oulton (2019) for the U.K. The result suggesting a sharp fall in the growth of capital

[27] The average of the shortfalls in growth-accounting components of output per capita is the average of the shortfalls in TFP, capital input, and labour input that are presented in Tables 1.1.1, 1.1.2, 1.3.1, and 1.3.2 for the U.S. and Tables 1.2.1, 1.2.2, 1.4.1, and 1.4.2 for the U.K., where each table has been derived from one of the two methods and two approaches applied in this study.

input is analogous to the result presented by Zhang (2019) for the U.S. and Van Ark (2016) for the U.K.

As seen in the eighth row of Table 1.1.1 and the bottom-right panel of Figure 1.3.1, a decline of 0.67 percentage points per year in LFP contributed to the U.S. slow recovery, whereas LFP has not contributed to the slow recovery in the U.K., as the shortfall reported in the eighth row of Table 1.2.1 is small and insignificant with a Wilcoxon p-value of 0.810. The alternative decomposition presented in rows 9–12 of Tables 1.1.1 and 1.2.1 implies similar results. One remarkable result is that the shortfall in output per hour is 0.87 and 1.70, with Wilcoxon p-values of 0.020 and 0.001, which indicates the productivity performance of the U.K. lags behind the U.S.

Turning to the second approach in which the counterfactual is estimated by the DFM, Figure 1.3.2 is the counterpart of Figure 1.3.1, and Table 1.1.2 is the counterpart of Table 1.1.1 for the U.S. Similarly, Figure 1.4.2 is the counterpart of Figure 1.4.1, and Table 1.2.2 is the counterpart of Table 1.2.1 for the U.K. The top and middle panels of these figures show that output per capita, TFP, and capital input all fall short of their common components in the U.S. and the U.K., which reaffirms the findings derived from the first approach. Also, column (c) of Tables 1.1.2 and 1.2.2 report a shortfall of 1.26 and 0.59 percentage points per year in output per capita, with significant Wilcoxon p-values of 0.001 and 0.033, respectively, in the U.S. and the U.K. Shortfalls in TFP, capital input, and labour input are 0.78, 0.37, and 0.12 for the U.S., and 0.64, 0.44, and -0.48 for the U.K., which are all consistent with their counterparts in Tables 1.1.1 and 1.2.1. The similarity of results implies that the finding of a significant shortfall is robust to the choice of the approaches for estimation of the counterfactual recovery.

1.4.1.1. The result of the structural break tests

We discuss the results of structural breaks presented in Figure 1.3.3 and Tables 1.1.3 and 1.1.4 for the U.S., and Figure 1.4.3 and Tables 1.2.3 and 1.2.4 for the U.K. We apply several tests, including Andrews and Ploberger (1994), Hansen (2000), and Bai and Perron (2003), which together document a sharp break in the parameters of Okun's law and the DFM in both countries during the financial crisis. The first three columns of Tables 1.1.3 and 1.2.3 present results of the Hansen (2000) method, which tests the joint break in the Okun's law intercept and coefficient. It identifies a significant structural break in U.S. output per capita and TFP in 2009Q1 with Quandt Likelihood Ratios (QLRs) of 23.1 and 11.4, respectively, and significant breaks in U.K. output per capita and TFP in 2008Q1 and 2007Q4 with QLRs of 39.2 and 37.0. This test also identifies a significant break in the U.S. and U.K. capital input in 2002Q1 and 2001Q2 with QLRs of 78.0 and 86.1.

The other columns of Tables 1.1.3 and 1.2.3, based on the Bai and Perron (2003) method that is able to identify multiple breaks, report the results of testing the presence of breaks in the Okun's law intercept and Okun's law coefficient separately. The middle two columns of Table 1.1.3 report a break in the Okun's intercept in 2006Q1 for U.S. output per capita and a break in 2012Q1 for TFP. As shown in Table 1.2.3, 2008Q1 is also selected as the date of the break in the Okun's intercept for U.K. output per capita, although the corresponding QLR statistics (5.2) is less than the 5% critical value (9.1). The last two columns of this test locate a significant break in the Okun's coefficient for U.S. output per capita, TFP, capital input, and labour input all occurred in 2009, with QLRs of 21.0, 10.6, 20.5, and 28.2, respectively. For the U.K., the coefficient break test identifies two breaks for U.K. output per capita that are less significant compared to the coefficient break in the U.S. These two breaks occurred in 2005Q4 with a QLR of 11.9 and 2009Q3 with a QLR of 9.4. It also finds two breaks for TFP (1984Q2 with a QLR of 10.3 and 2009Q3 with a QLR of 7.5).

Figures 1.3.3 and 1.4.3, based on the Andrews and Ploberger (1994) method, support the above results for the joint and individual break tests. The top panels identify joint structural breaks in output per capita and TFP in 2009 and 2008 with comparable numerical values for QLRs to those in the first three columns of Tables 1.1.3 and 1.2.3. The middle panels display a sequence of significant breaks in Okun's intercept from the late-1990s until the 2007–09 financial crisis, which spiked in 2010 and 2008 for the U.S. and the U.K. Since the Okun's intercept stands for trend growth, its instability is attributable to the unaccounted for time-variation in trend growth in the Okun's regression. The spike in the QLR statistics clearly corresponds to the trend deceleration (a fall in trend growth) during the financial crisis. The bottom panels document a single structural break in the Okun's coefficient in 2009 and 2008 for the U.S. and the U.K. The spike in the QLR statistics for the U.S. (23) is highly significant and associated with the sluggish cyclical recovery. These findings together are in line with the findings of instability of Okun's law during the Great Recession by Owyang and Sekhposyan (2012), Basu and Foley (2013), and Grant (2018).

Besides, based on Tables 1.1.4 and 1.2.4, the structural break in the intercept and cyclical factor loadings during the 2007–09 financial crisis is significant for output per capita and TFP for the U.S. and the U.K. In particular, both joint and individual break tests identify a break in the intercept and factor loadings around 2008, all with QLRs greater than 5% critical values. The results also support the occurrence of a significant break in the capital input in 2002 and 2001 with QLRs of 117.1 and 100.7 for the U.S. and U.K. capital input, respectively, which are close to the QLRs of 78.0 and 86.1 reported for the joint break in Okun's law. As a result, the break in capital input preceded those in output and TFP, suggesting the contribution of capital input to the slow recovery.

1.4.1.2. The result of allowing a time-varying intercept and a break in the coefficient

After establishing the extent of the shortfall and the significance of the breaks in the parameters of Okun's law and the DFM for output per capita and its growth-accounting components, we now report the results of columns (e), (f), and (g) of Tables 1.1.1 and 1.2.1, as well as their counterparts, Tables 1.1.2 and 1.2.2, to substantiate the causal effect of an individual structural break on the slow recovery.

Based on the column (e) of Table 1.1.1, allowing a time-varying Okun's intercept as explained in Eq. 1.4, reduces the shortfall in U.S. output per capita from 1.32 to 0.57 percentage points per year. This 0.75 percentage point reduction in the shortfall, therefore, is attributable to the instability of Okun's intercept related to the time-variation in trend growth, which appears as both a gradual decline in trend growth before the 2007–09 financial crisis and a fall in trend growth in the aftermath of the financial crisis, illustrated in the top panel of Figure 1.6. Despite the significant reduction, as it fails to eliminate the shortfall thoroughly, the remaining shortfall (0.57) is attributable to the instability of the Okun's coefficient. Indeed, based on column (f), allowing a structural break in the Okun's coefficient in 2009, substantially reduces the shortfall from 1.32 to 0.71 percentage points per year.[28] We thus infer the causal effect of the structural break in the Okun's coefficient on the slow recovery, which is related to the sluggish cyclical recovery. Finally, column (g) reports small and insignificant shortfalls when a structural break in both the intercept and coefficient is allowed for, implying that considering both breaks is necessary to eliminate the shortfall. As a result, both the time-variation in the Okun's intercept and the structural break in the Okun's coefficient are the determinants of the slow recovery in the U.S.

According to the column (e) of Table 1.2.1, allowing a time-varying intercept entirely reduces the shortfall in U.K. output per capita from 0.82 to an insignificant value of -0.07 percentage points per year. Alternatively, allowing a break in the Okun's intercept entirely eliminates the shortfall (column g). This suggests that the time-variation in trend growth is mainly in the form of a sharp fall during the financial crisis.[29] Furthermore, the bottom panel of Figure 1.6 demonstrates a sharp fall in trend growth from 1.8 to 0.6 percent right after the 2007–09 financial crisis. Hence, we maintain the causal effect of the structural break in the Okun's intercept (trend growth) on the slow recovery in the U.K.[30] Considering the second approach, columns (e), (f), and (g) of Tables 1.1.2 and 1.2.2 report the

[28] Since both the growth-accounting decomposition, derived in Appendix 1.I, and trend-cycle decomposition, derived by using Okun's law regression, preserve the additivity of the components, the elements in all tables are vertically and horizontally additive. The exception to this general property is column (e) of Tables 1.1.1, 1.1.2, 1.2.1, and 1.2.2, because it is estimated by Kalman's (1960) filter though elements in columns (e) still approximately preserve additivity.
[29] Allowing the structural break in the Okun's coefficient for the U.K. fails to reduce the shortfall (column f).
[30] Fitted output estimated by Okun's law for the U.K. neither explains the depth of the Great Recession nor the subsequent slow recovery. However, allowing a structural break in the Okun's law intercept enables the model to better capture both the depth of the Great Recession and the slow recovery.

shortfalls by relaxing the assumptions of stability of the intercept, stability of the cyclical factor loadings, and both, respectively, which are similar to the results discussed above.

Putting these all together, our results identify a structural change in the regime during the 2007–09 financial crisis as the determinant of the slow recovery. Therefore, in contrast to Fernald et al. (2017) and Antolin-Diaz et al. (2017), we underline the important role of the 2007–09 financial crisis in the subsequent slow recovery in the U.S. and the productivity puzzle in the U.K.

1.4.2. The result of the second method: trend-cycle decomposition

The second method, by using a trend-cycle decomposition that accommodates structural breaks in the Okun's coefficient and cyclical factor loadings, finds that (1) a decline in the growth of the trend (potential output); (2) an unprecedented deceleration of the trend during the 2007–09 financial crisis; and (3) an unusually sluggish recovery of the cycle following the financial crisis, together contributed to the slow recovery.

The right panels of Figure 1.5 plot the level of the trend component (potential output) for output per capita. They show a gradual slowdown in the trend that began in the late-1990s and, later on, a sudden slowdown in the trend following the financial crisis. Figure 1.6 better distinguishes between these two trend-related driving forces: (1) a decline in trend growth in the late-1990s, which was expected due to the pause in the technology boom; and (2) an unexpected fall in trend growth in the aftermath of the 2007–09 financial crisis. The top panel plots trend growth for U.S. output per capita, which has a periodic dynamic that consists of four episodes: (i) a high-growth period from the 1950s to the mid-1970s, when trend growth was above 2.5% per year; (ii) a medium-growth period from the mid-1970s to the mid-1990s, when trend growth was between 1.6% and 2% per year; (iii) another high-growth period from the mid-1990s to the mid-2000s, which was associated with the technology boom and bust; and (iv) a low-growth period due to the continued fall in trend growth from 1.6% per year in 2007 to an unprecedented rate of less than 0.8% per year. The bottom panel also clearly illustrates two forces for trend growth in the U.K. After a mild and gradual decline in trend growth from 2.5% per year in the 1980s to lower rates of 1.8% per year in 2007, there was a sharp fall in trend growth from 1.8% to a dreadfully low rate of 0.6% per year.

Figure 1.5 signifies the cycle-related driver of the slow recovery. It is clear that the recovery of the cyclical component (output gap) in the U.S. was not as strong as it should be consistent with the plucking model, though the Great Recession was deep compared to previous recessions. In addition, based on Figure 1.7, there is a persistent negative output gap, suggesting a sluggish recovery of the cyclical component because of the absence of a bounce-back effect. Considering that the growth of the cyclical component (output gap) is simply the product of the Okun's coefficient and the change in unemployment, this result is consistent with the finding of a significant structural break in the Okun's coefficient for the U.S. This is unsurprisingly the opposite of that concluded by Fernald et al. (2017) and Antolin-Diaz et al. (2017), who applied a trend-cycle decomposition in which the break in the Okun's coefficient and cyclical factor loadings are not accounted for.[31] The finding of a persistent negative output gap provides empirical evidence for other studies presented by Michau (2018), Anzoategui (2019), and Cerra et al. (2022), in which constrained demand causes a Keynesian secular stagnation, productivity slowdown, and hysteresis effects.

The first three columns of Tables 1.3.1 and 1.4.1 report a shortfall of 1.23 and 1.07 percentage points per year in U.S. and U.K. output per capita. The middle three columns show that the growth of the cyclical component during the recovery following the crisis fell short of its counterpart during the previous recoveries by 0.22 and 0.03 percentage points per year in the U.S. and the U.K. This shows that while we expected a stronger recovery compared to the three previous recoveries, there was a persistent negative output gap in the U.S. The last four columns also report the shortfall in the trend components. The annual shortfall in smooth trend for output per capita, TFP, and capital input is 0.79, 0.26, and 0.25 for the U.S., and 0.73, 0.62, and 0.46 for the U.K. Considering the second approach, the estimated shortfall in the growth of the cycle and the trend of output per capita are 0.65 and 0.39 for the U.S. and 0.09 and 0.47 for the U.K., confirming the results derived by the first approach.

Combining all the results, the trend-cycle decomposition supports the finding of the first method. We concur with the notion that the decline in trend growth, which had begun in the 1960s and continued after the technology boom in the late-1990s until the mid-2000s, is unrelated to the 2007–09 financial crisis. However other driving forces, such as a drop in trend growth from higher rates of 1.6% to unprecedented low rates of 0.8% in the U.S. and 0.6% in the U.K., as well as a sluggish recovery of the cyclical component in the U.S., occurred in the aftermath of the 2007–09 financial crisis. Hence, we state that the role of the 2007–09 financial crisis in the subsequent slow recovery is substantial and undeniable in both the U.S. and the U.K.

[31] The structural break tests demonstrate a sharp break in the Okun's coefficient and cyclical factor loadings. Hence, as explained in Section 1.3, the methods applied by Fernald et al. (2017) and Antolin-Diaz et al. (2017), in which the breaks are not accommodated, tend to overestimate the contribution of the trend and underestimate the contribution of the cycle to the slow recovery. Comparing the results of the Tables 1.L.11 and 1.L.12 in the Appendix 1.L with those in the Tables 1.3.1 and 1.4.1, confirms this argument. Indeed, our results contrast with those of Fernald et al. (2017) and Antolin-Diaz et al. (2017), who identify the decline in trend growth as the only determinant of the slow recovery and report a normal recovery of the cyclical component in the U.S.

1.4.3. Spillovers of real activity shortfall from the U.S. to the U.K.

The concomitant change in regimes in two countries during the 2007–09 financial crisis motivates us to investigate the connection between the slow recovery in the U.S. and the productivity puzzle in the U.K. Additionally, Okun's law and the DFM are not able to capture the depth of the recession and the subsequent slow recovery in the U.K. We, therefore, develop a DFM that adopts both the U.S. and the U.K. factors. In the end, since the U.K. economy is small relative to the U.S. economy, we measure the transmission of the U.S. RAS to the U.K. to explore an external driving force behind the U.K. productivity puzzle.

Figure 1.8 plots the U.S. RAF and the other three U.S. factors along with the U.S. counterfactual RAF and counterfactuals for the other three U.S. factors. The gap between the U.S. RAF and the U.S. counterfactual RAF in the bottom panel reflects the U.S. RAS. The resemblance between the other U.S. factors and their counterfactual counterparts in the bottom panel shows that incorporating the U.S. RAS does not change the other three U.S. factors. In addition, Figure 1.9 displays U.K. orthogonal factors and their counterfactuals derived using a forecast with a rolling origin which, for robustness check, incorporates the potential response of U.K. orthogonal factors to the U.S. RAS in the measurement of the shortfall transmission.

Based on the columns (c) and (d) of Table 1.5, the magnitude of the shortfall spillovers from the U.S. to U.K. output per capita is 0.62 percentage points per year with a Wilcoxon p-values of 0.023. The shortfall spillovers for the U.K. TFP and capital input are 0.44 and 0.35. Column (e) also reports the shortfall when the potential response of the other three U.S. factors to the U.S. RAS is considered. As seen in Figure 1.8, because there is no change in the other three U.S. factors in response to the U.S. RAS, column (e) measures almost the same shortfall for each series as reported in column (c). Finally, column (f) estimates the shortfall while the potential response of U.K. orthogonal factors to the U.S. RAS has been taken into account, which reports similar shortfalls as column (c) for each series.

1.4.4. Robustness tests

The similarity of results derived in Sections 1.4.1 and 1.4.2 implies that the findings are robust to the choice of the methods and approaches for estimation of the counterfactual. Additionally, by estimating alternative setups for each method and each approach, we substantiate the robustness of our results to parameter settings. Namely, Figures 1.K.1, 1.K.2, 1.K.4, and 1.K.5 in Appendix 1.K show that changing the number of leads and lags to $p = 0, 1, 2$ has no effect on the results. Tables 1.L.9 and 1.L.10 in Appendix 1.L also confirm the robustness of the results to the number of leads and lags in the trend-cycle decomposition based on Okun's law. Figures 1.K.3 and 1.K.6 indicate the results of the DFM are invariant to the estimation period. We additionally re-estimate the parameters of Okun's law and the DFM over the period of 1981–2009 and assume that the Okun's law parameters and cyclical factor loadings during the post-crisis recovery are equal to those before the 2007–09 financial crisis. Tables 1.L.5, 1.L.6, 1.L.7, and 1.L in Appendix 1.L indicate that a change in estimation period does not affect the results.

The results of the open-economy hierarchical DFM are also robust to the choice of estimation period, VAR order (P), and forecasting method. Tables 1.L.13 and 1.L.14 report similar results for the estimation periods of 1971–2016 and 1985–2016 to dispel any doubt about the stability of the cross-block factor loadings. Further, as explained in Section 1.3.3.2, we take the potential response of U.K. orthogonal factors to the U.S. RAS into account by forecasting the responses based on a VAR model with order $P = 4$, and rolling origin. As in Tables 1.L.15 and 1.L.16, we derive almost the same numerical values for shortfall spillovers even if we change the VAR order to $P = 1, 2, 3$. We find very similar shortfall spillovers when we derive the responses of U.K. orthogonal factors by using a forecast with a fixed origin.

1.5. Concluding remarks

Our findings document a significant output shortfall in the U.S. and the U.K. in the aftermath of the 2007–09 financial crisis. We also identify structural breaks in the Okun's law intercept, Okun's law coefficient, and cyclical factor loadings during the financial crisis as the determinants of the slow recovery. Thus, our results imply that a change in regime following the 2007–09 financial crisis, depending on model specifications, has appeared in different but related phenomena: a structural break in the parameters of Okun's law and the DFM; the slowdown in trend growth; the slow recovery; and the productivity puzzle.

According to a trend-cycle decomposition, we find three distinct driving forces of the slow recovery: (1) a gradual decline in the growth of potential output (trend component); (2) an unprecedented deceleration of potential output in the aftermath of the 2007−09 financial crisis; and (3) an unusually sluggish cyclical recovery (a persistent negative output gap) known as hysteresis effects. While the first driving force is unrelated to the financial crisis as it started in the 1960s, the other two forces are the consequences of the financial crisis as they formed during the financial crisis.

In addition, the growth-accounting decomposition recognises the slow growth in TFP and capital input as the main contributors to the slow recovery in the U.S. and the U.K. Finally, the open-economy hierarchical DFM establishes that spillovers of RAS from the U.S. to the U.K. account for 0.62, 0.44, and 0.35 percentage points per year of the annual shortfall in U.K. output per capita, TFP, and capital input.

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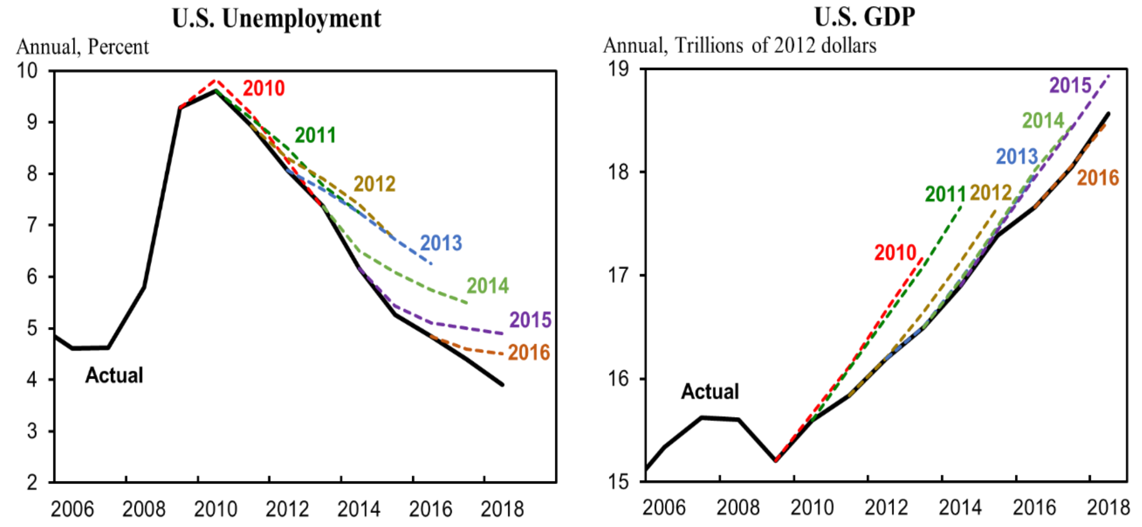
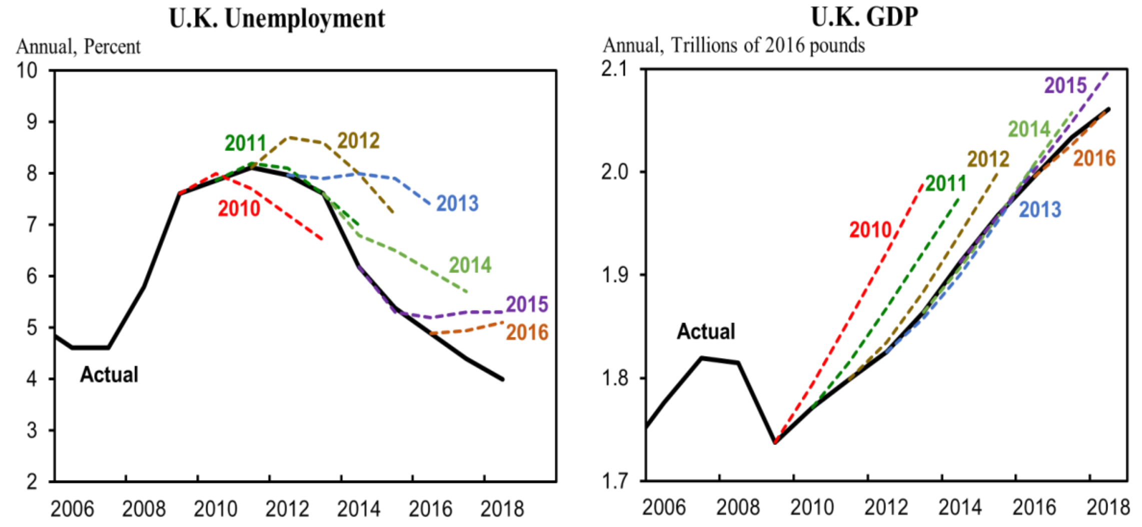
This study cites 85 sources, including journal articles and books.

Figures



Source: Updated from Fernald et al., 2017 using BLS and SPF

(a) Survey of professional forecasts for the United States



Sources: Office for Budget Responsibility and Office for National Statistics

(b) Office for budget responsibility forecasts for the United Kingdom

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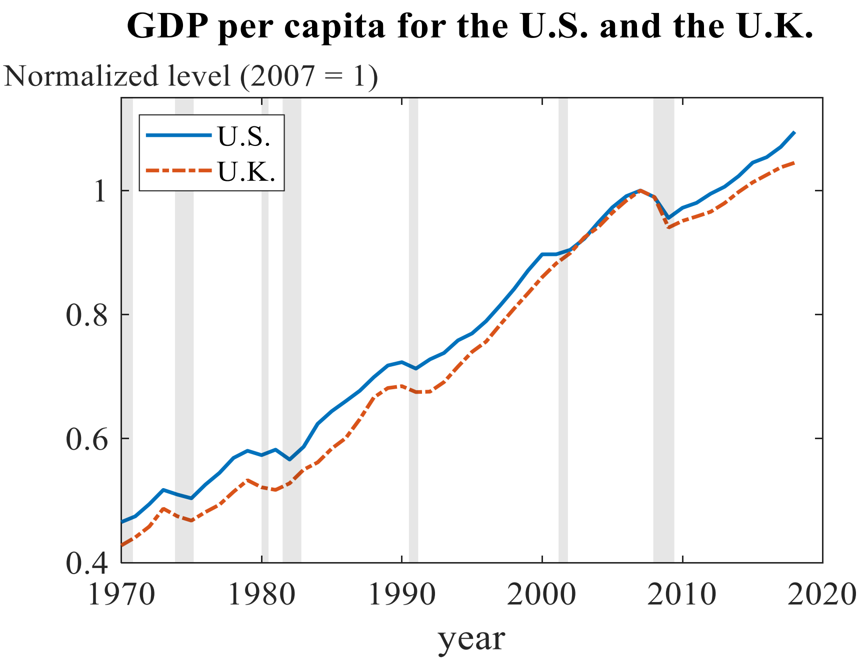
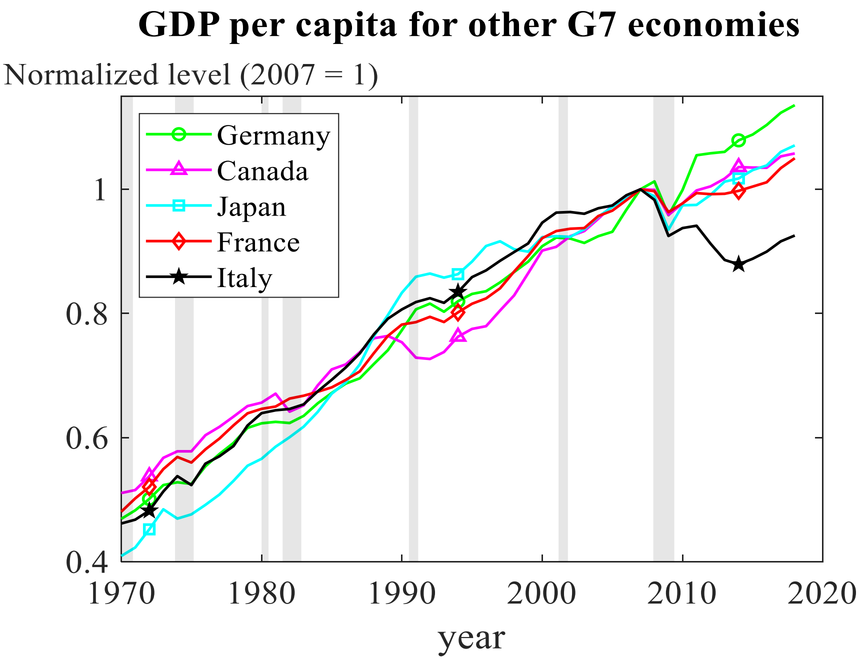
Figure 1.1: Actual and forecasts for the U.S. and the U.K. real activity variables

Notes: Professional forecasters repetitively over-predicted the GDP recovery while the unemployment rate was under-predicted. This implies that they were unaware of the slow recovery of output emanated from the structural break in Okun's Law in 2009.



(a) The U.S. and the U.K.



(b) Other G7 economies

Figure 1.2: Dynamics of the normalized GDP per capita for G7 economies

Notes:

(1) Outputs are normalized to be equal to one in 2007.

(2) Although the slowdown in growth for G7 economies started in the 2000s, the growth of output was acutely decelerated after the 2007–09 financial crisis. The fall in trend growth of output is sharper for the U.S., the U.K., France, and Italy and is milder for Germany, Canada, and Japan.

(3) The shaded areas are the NBER recession dates.

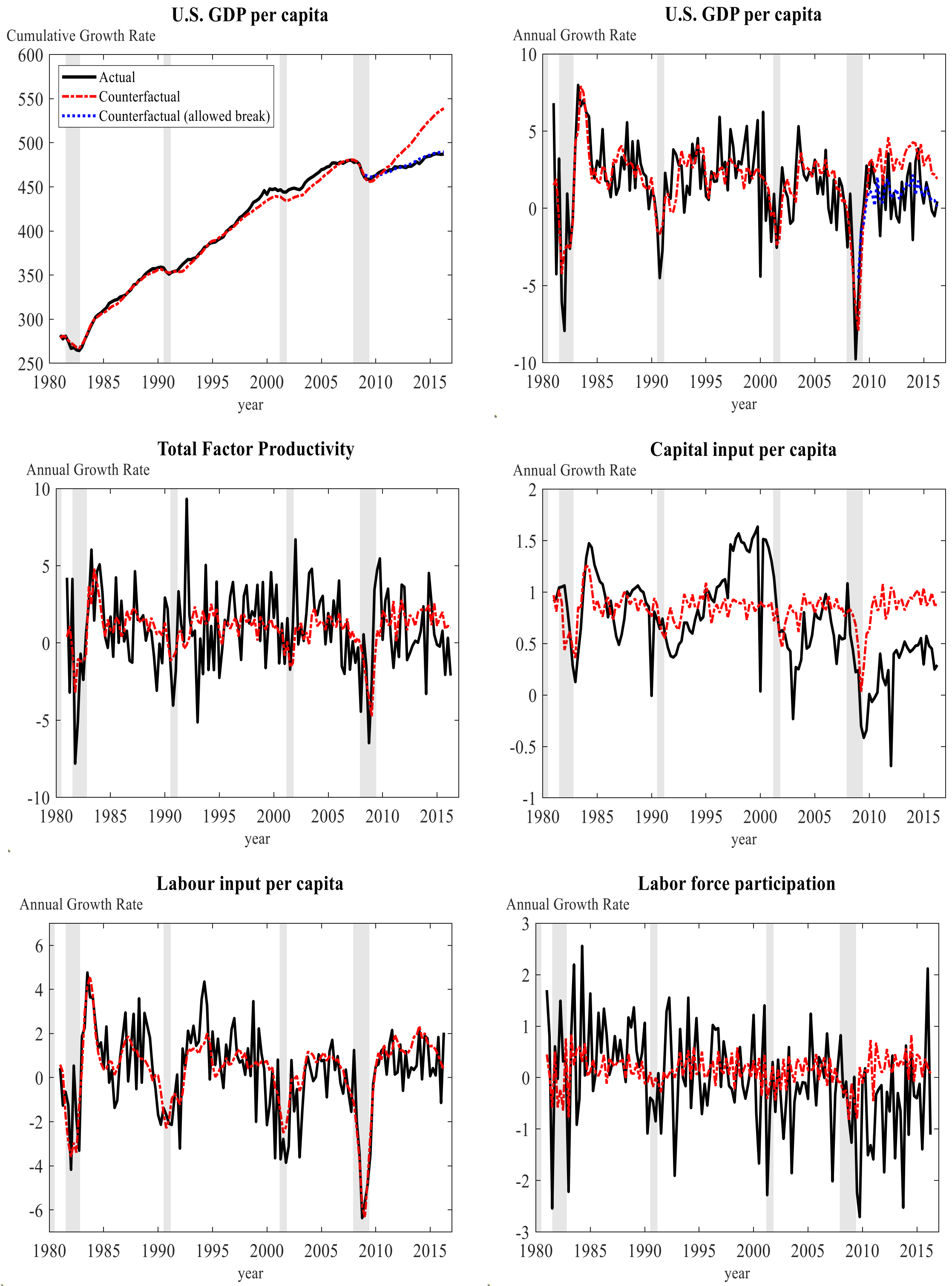


Figure 1.3.1: Shortfall of the U.S. post-crisis recovery estimated by Okun's law

Notes:

(1) The red dashed lines represent counterfactual recoveries. They are estimated by Okun's law with $p = 1$ over the period 1981–2009, assuming that the Okun's law parameters during the post-crisis recovery are equal to those before the 2007–09 financial crisis. Shortfalls are the difference between counterfactual and actual recoveries.
(2) The blue dotted line, depicted for the top panels, is the fitted output per capita estimated by Okun's law, which allows for a structural break in the Okun's law intercept and coefficient in 2009Q1 to eliminate the shortfall.
(3) Changing the number of leads and lags to $p = 0$ or $p = 2$ has no effect on the results (See Appendix 1.K).
(4) The shaded areas are the NBER recession dates.

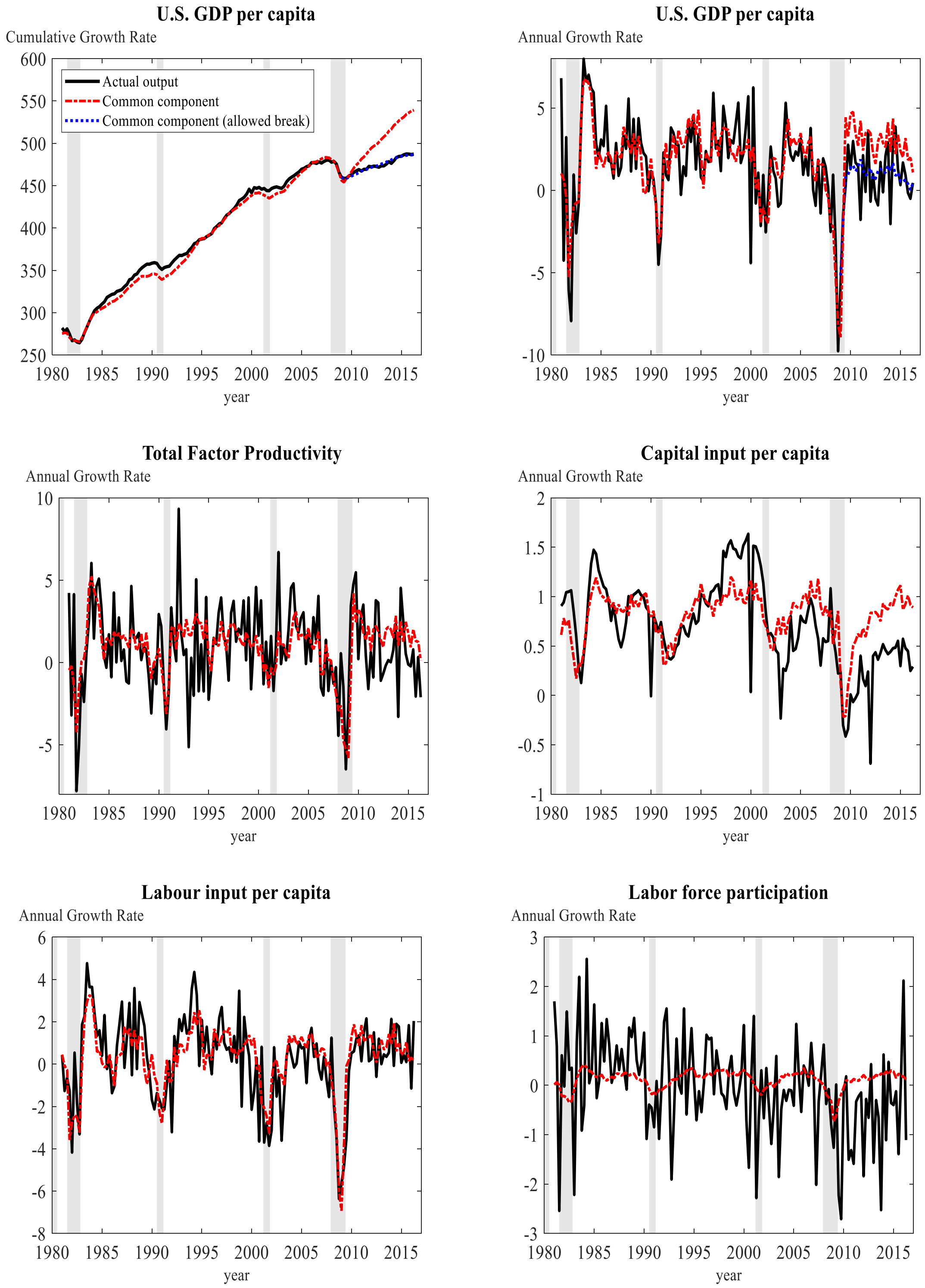


Figure 1.3.2: Shortfall of the U.S. post-crisis recovery estimated by the DFM

Notes:

(1) The red dashed lines represent counterfactual recoveries. They are estimated by the DFM over the sample period 1981–2009, assuming that the factor loadings during the post-crisis recovery are equal to those before the 2007–09 financial crisis. Shortfalls are the difference between counterfactual and actual recoveries.
(2) The blue dotted line, depicted for the top panels, is the fitted output per capita estimated by the DFM, which allows for a structural break in the intercept and cyclical factor loadings in 2009Q1 to eliminate the shortfall.
(3) The shaded areas are the NBER recession dates.

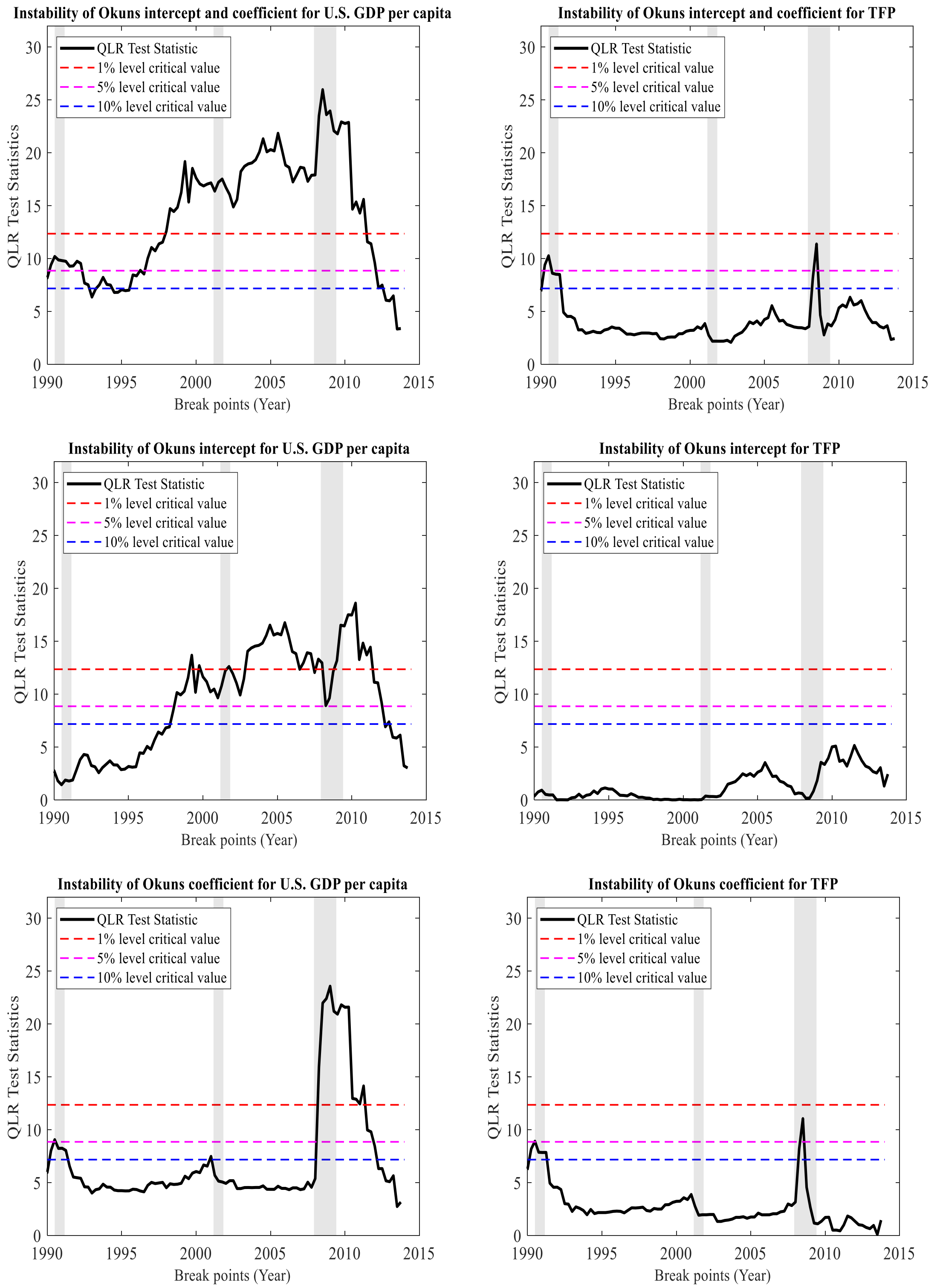


Figure 1.3.3: Structural break tests in Okun’s law for the U.S. GDP per capita and TFP

Notes:

(1) The Quandt Likelihood Ratio (QLR) is the supremum of F statistics. We test the joint break in the intercept and coefficient. We also run the test for a break in the Okun’s intercept and a break in the Okun’s coefficient separately.
(2) The largest OLR statistics for all of the break tests occurred around 2009.
(3) The second largest QLR statistics for a break in the Okun’s intercept took place around 2006.
(4) The shaded areas are the NBER recession dates.
(5) For additional break test results, see Tables 1.1.3 and 1.1.4.

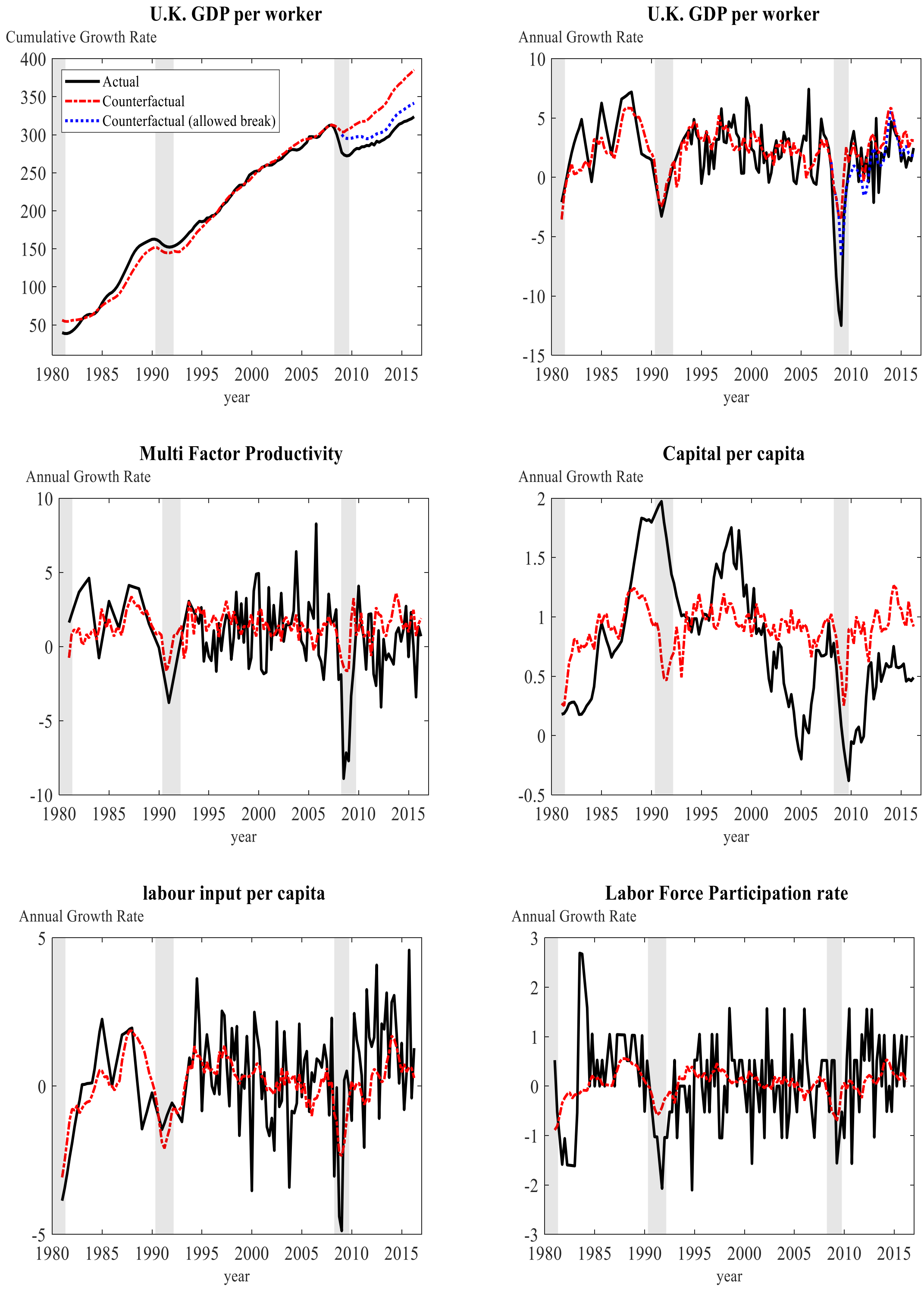


Figure 1.4.1: Shortfall of the U.K. post-crisis recovery estimated by Okun's law

Notes:

(1) The red dashed lines represent counterfactual recoveries. They are estimated by Okun's law with $p = 1$ over the period 1981–2009, assuming that the Okun's law parameters during the post-crisis recovery are equal to those before the 2007–09 financial crisis. Shortfalls are the difference between counterfactual and actual recoveries.
(2) Changing the number of leads and lags to $p = 0$ or $p = 2$ has no effect on the results (See Appendix 1.K).
(3) The shaded areas are the ECRI recession dates.

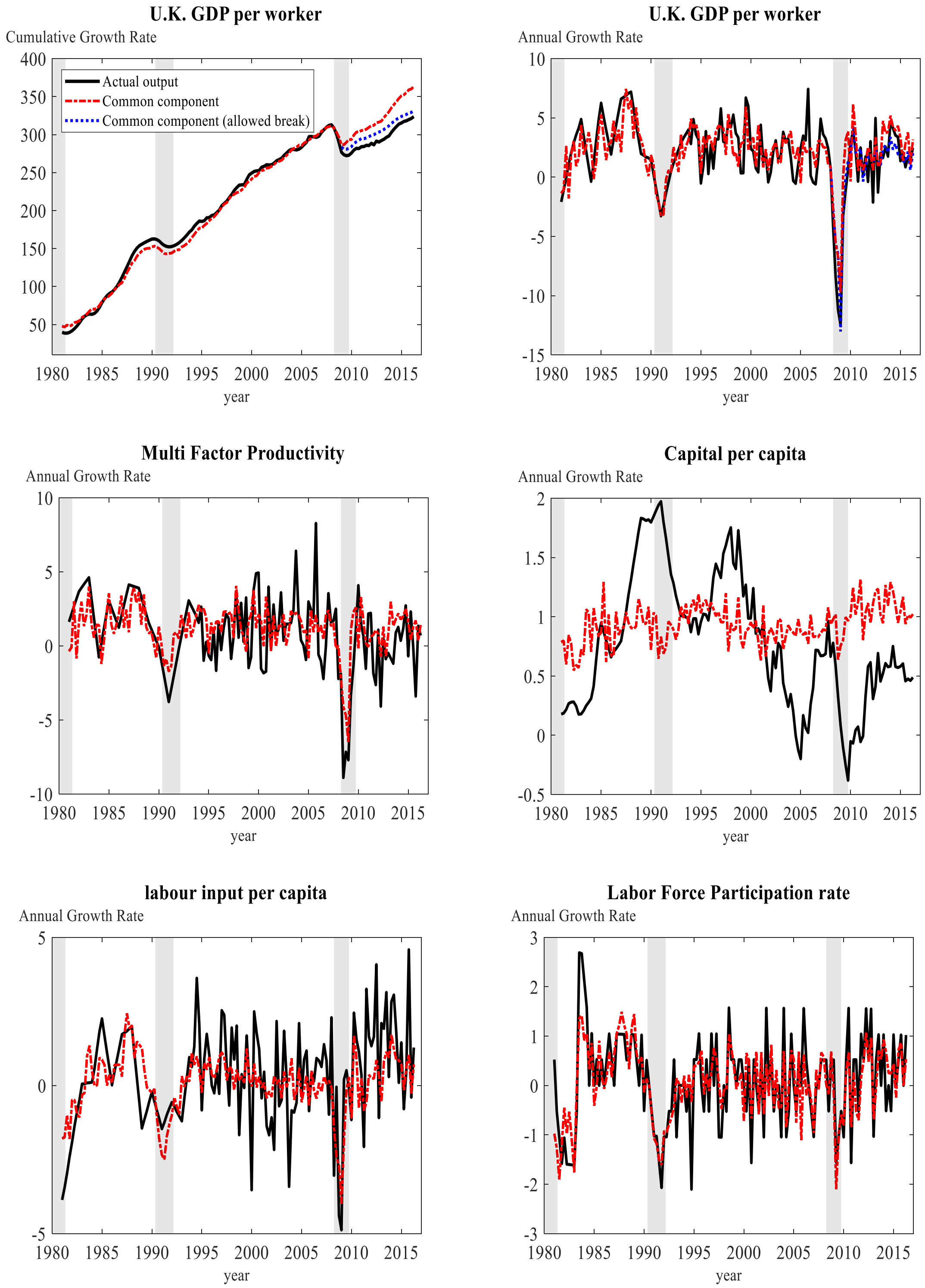


Figure 1.4.2: Shortfall of the U.K. post-crisis recovery estimated by the DFM

Notes:

(1) The red dashed lines represent counterfactual recoveries. They are estimated by the DFM over the sample period 1981–2009, assuming that the factor loadings during the post-crisis recovery are equal to those before the 2007–09 financial crisis. Shortfalls are the difference between counterfactual and actual recoveries.

(2) The shaded areas are the ECRI recession dates.

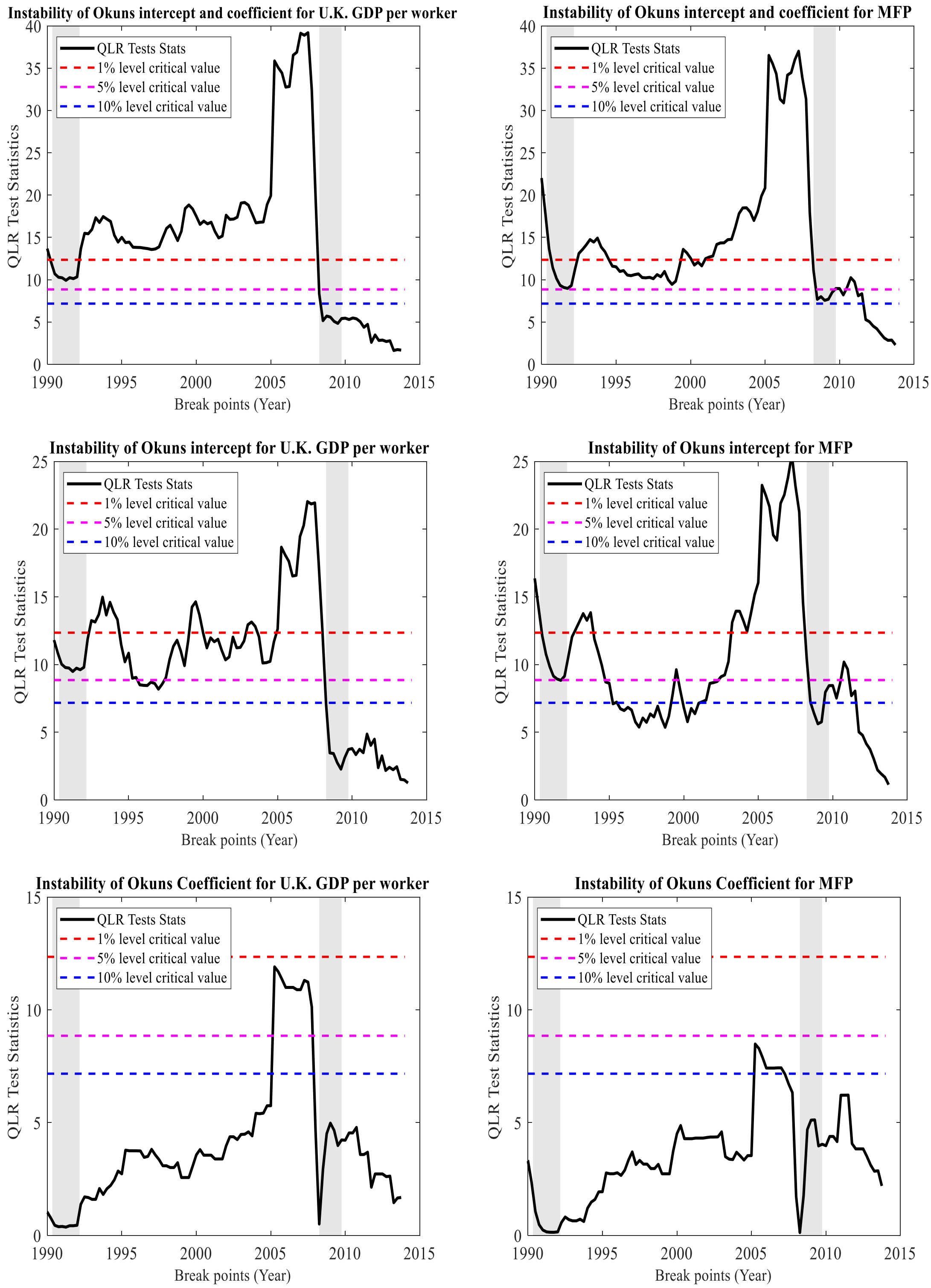
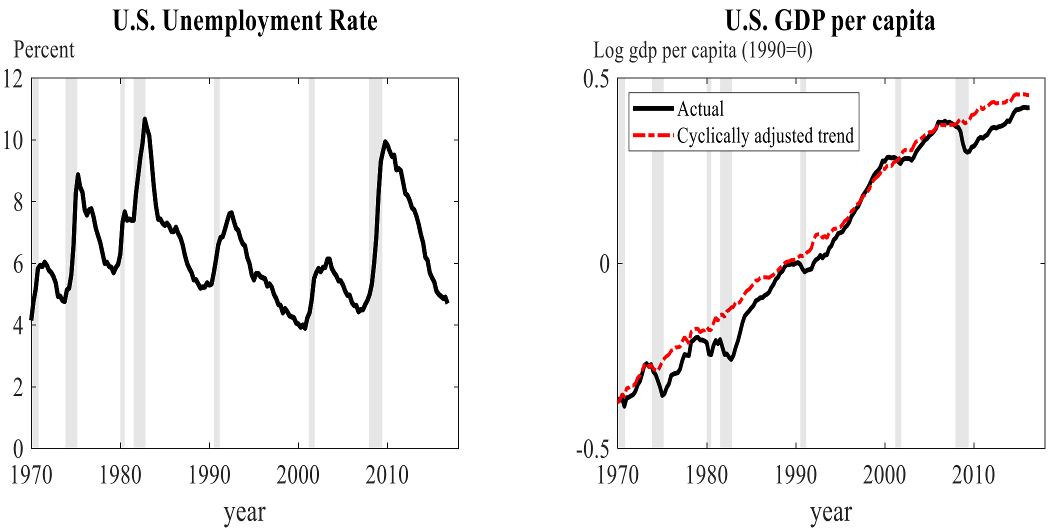


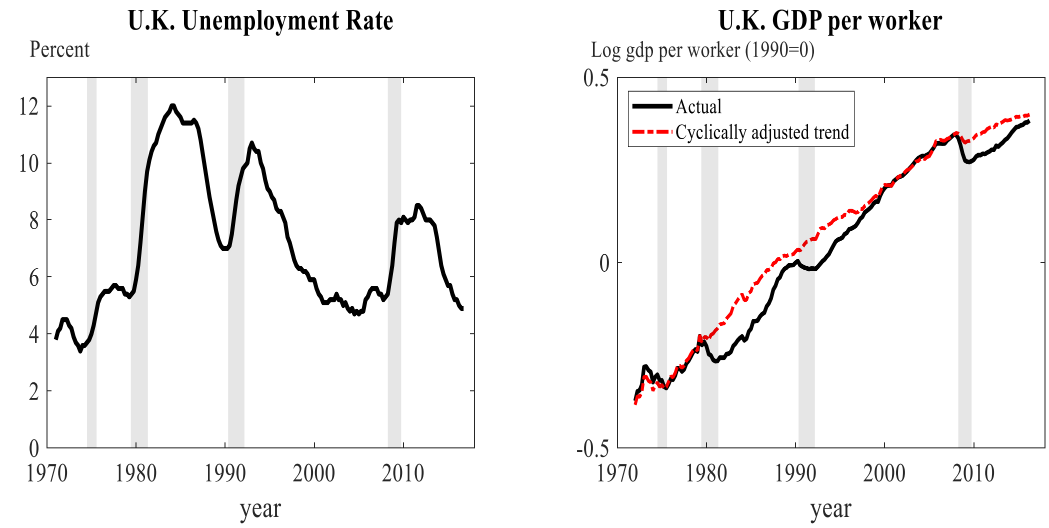
Figure 1.4.3: Structural break tests in Okun’s law for the U.K. GDP per worker and MFP

Notes:

(1) The Quandt Likelihood Ratio (QLR) is the supremum of F statistics. We test the joint break in the intercept and coefficient. We also run the test for a break in the Okun’s intercept and a break in the Okun’s coefficient separately.
(2) The largest OLR statistics for all of the break tests occurred between 2006 and 2008.
(3) The largest QLR statistics for a break in the Okun’s intercept took place in 2008.
(4) The shaded areas are the ECRI recession dates.
(5) For additional break test results, see Tables 1.2.3 and 1.2.4.



(a) U.S. unemployment and cyclically adjusted output

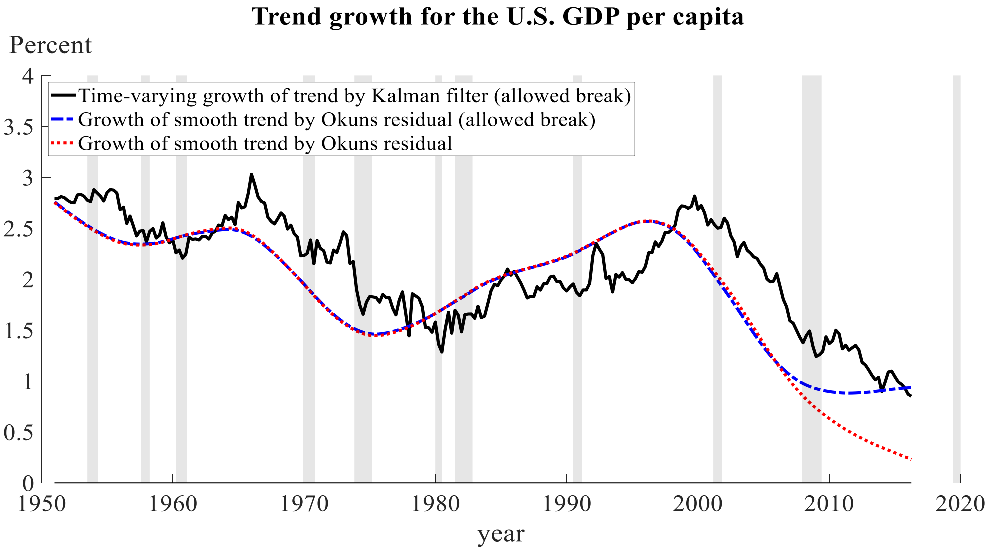


(b) U.K. unemployment and cyclically adjusted output

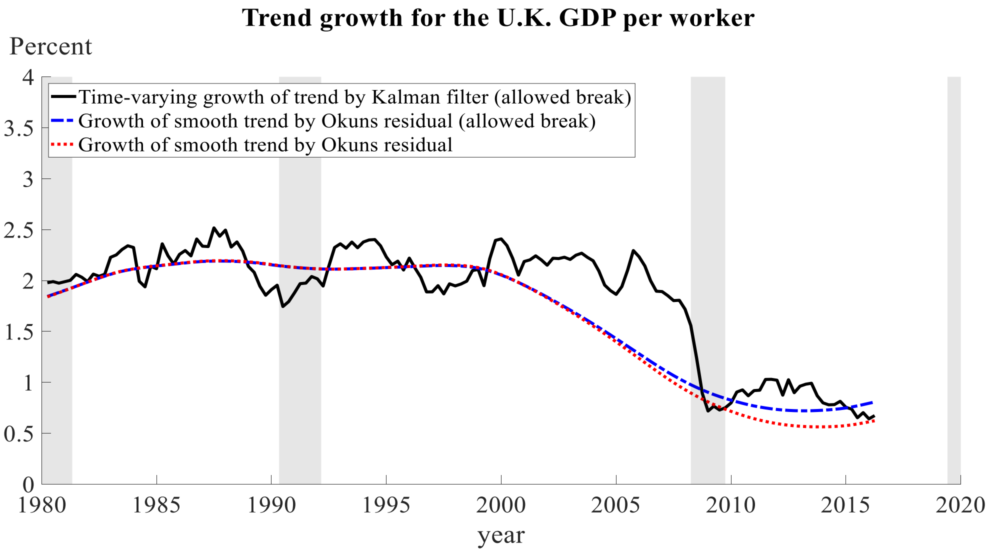
Figure 1.5: Output trend component estimated by a trend-cycle decomposition based on Okun's law

Notes:

(1) The right panels show the level (i.e., the cumulative growth rate) of the U.S. GDP per capita and the U.K. GDP per worker (population aged between 16 and 64), both normalized to be zero in 1990.
(2) The red dashed line is the cyclically adjusted trend estimated by Okun's law with $p = 1$ leads and lags over the sample period of 1971 to 2016, where a structural break in the Okun's coefficient in 2009Q1 is allowed for.
(3) Changing the number of leads and lags to $p = 0$ or $p = 2$ has no effect on the results (See Appendix 1.K).
(4) The shaded areas are the NBER and ECRI recession dates for the U.S. and the U.K, respectively.



(a) The growth of the U.S. potential output

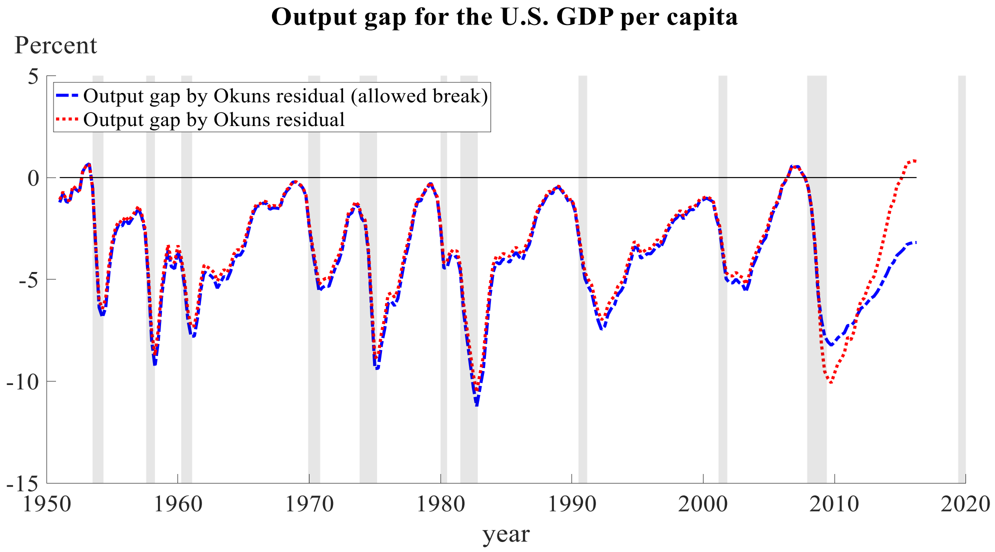


(b) The growth of the U.K. potential output

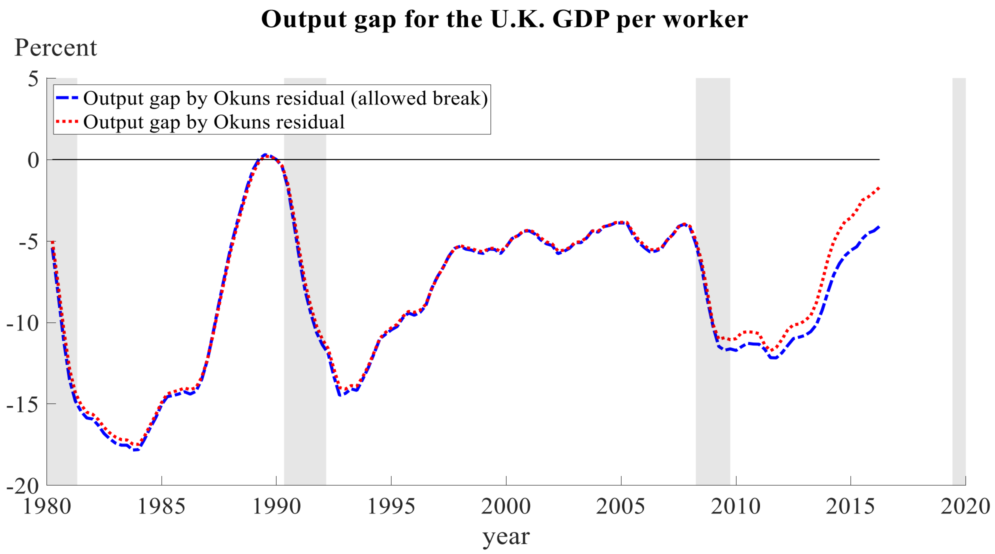
Figure 1.6: The annualized growth of potential output (trend component)

Notes:

(1) The black solid line is the Kalman's (1960) filter estimation of the growth of potential output by using Okun's law while a structural break in the Okun's coefficient in 2009Q1 is allowed for. The blue dashed line and the red dotted line are the growth of potential output (for GDP per capita) estimated as the Okun's law residuals with a break in the Okun's coefficient in 2009Q1, and without including a structural break, respectively. The estimation period is from 1951 to 2016 for the U.S. and from 1981 to 2016 for the U.K.
(2) We consider an Okun's law with $p = 1$ leads and lags, whereas changing the number of leads and lags to $p = 0$ or $p = 2$ has no effect on the results (See Appendix 1.K). The smooth trend is derived by passing the estimated residuals through a Tukey's bi-weight filter with a bandwidth of 40 quarters. The results are robust to the change in the bandwidth to 40, 80, or 100 quarters.
(3) The shaded areas are the NBER and ECRI recession dates for the U.S. and the U.K, respectively.



(a) The U.S. cyclical component

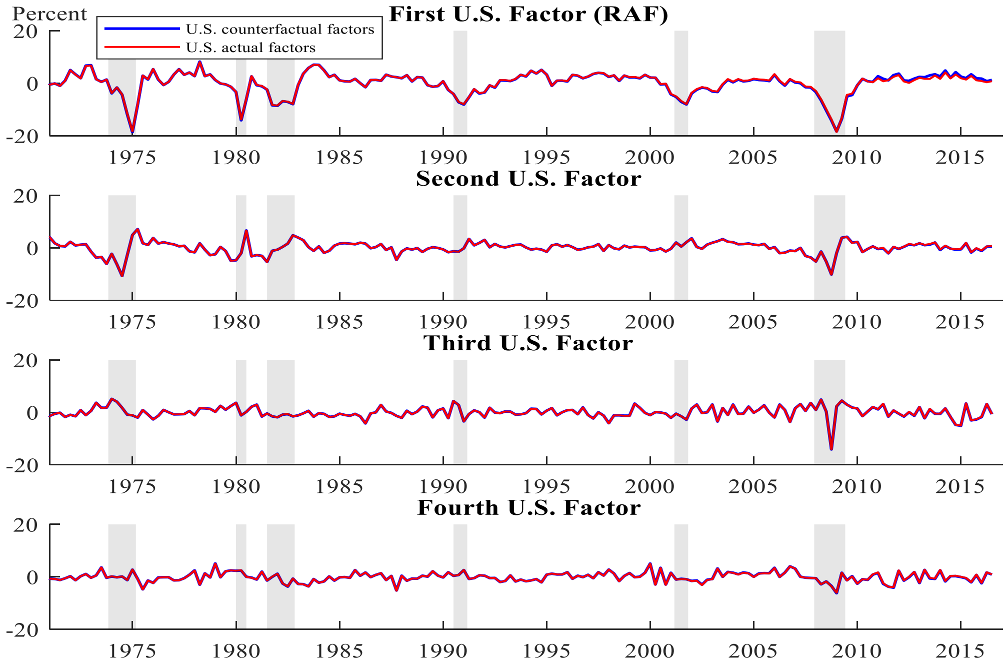


(b) The U.K. cyclical component

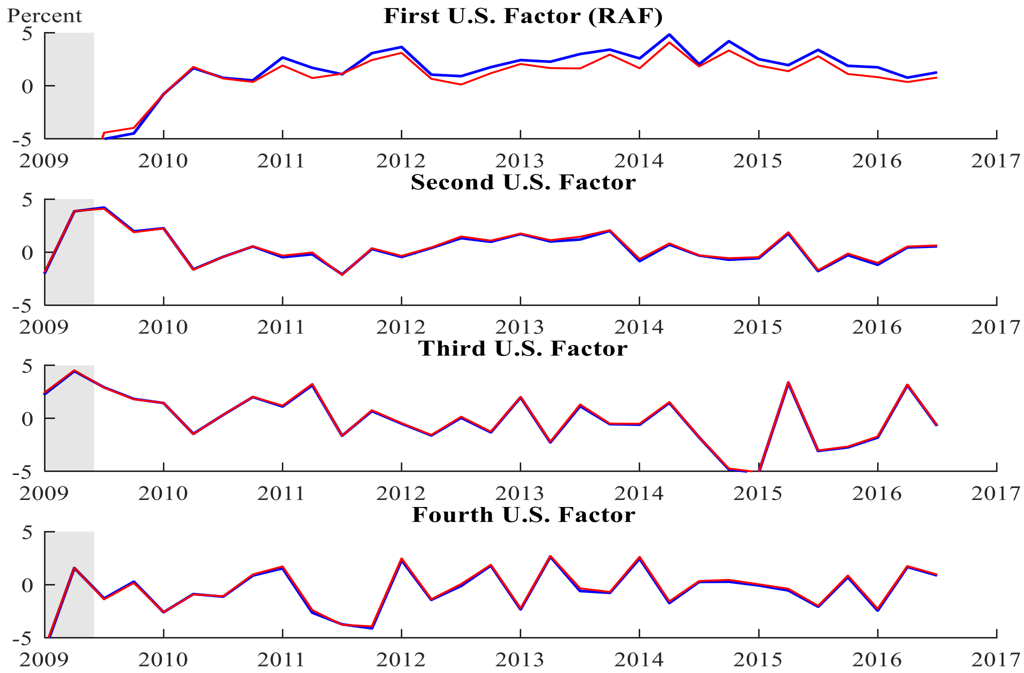
Figure 1.7: Output cyclical component estimated by a trend-cycle decomposition based on Okun's law

Notes:

(1) The blue dashed line and the red dotted line are the estimations of the output gap (for GDP per capita) using Okun's law regression with a break in the Okun's coefficient in 2009Q1 and without including a structural break, respectively.
(2) The estimation period is from 1951 to 2016 for the U.S. and from 1981 to 2016 for the U.K. We consider an Okun's law with $p = 1$ leads and lags and a Tukey's bi-weight filter with a bandwidth of 40 quarters. The results are robust to the choice of the number of leads and lags ($p = 0, 1, 2$) and the bandwidth (40, 60, 80, or 100 quarters). See Appendix 1.K for some results for other settings.
(3) The shaded areas are the NBER and ECRI recession dates for the U.S. and the U.K, respectively.



(a) U.S. factors in the whole sample

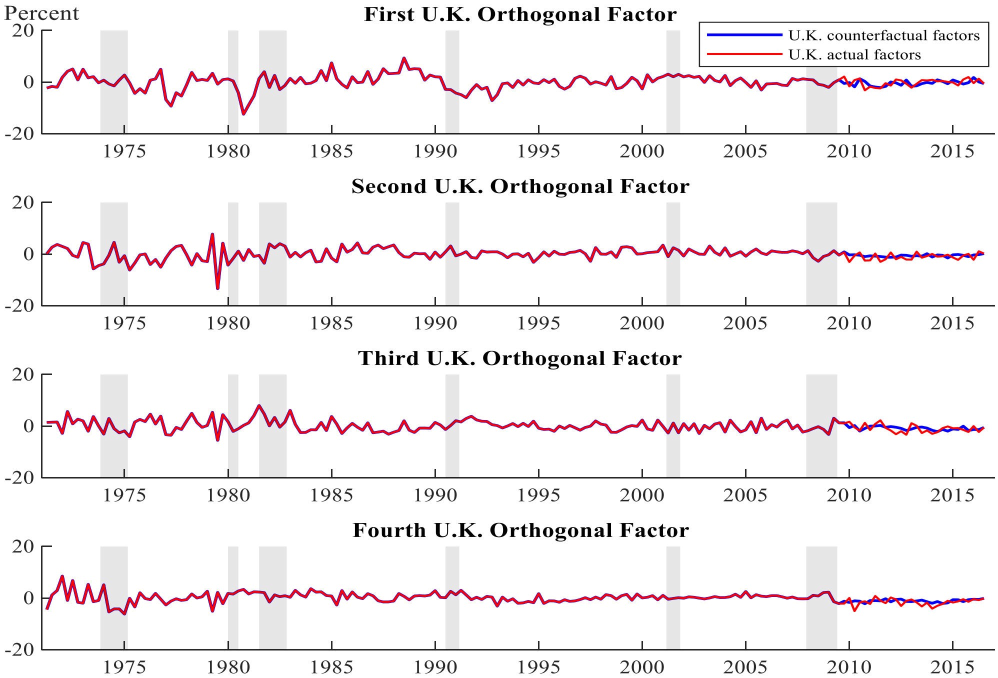


(b) U.S. factors after the Great Recession

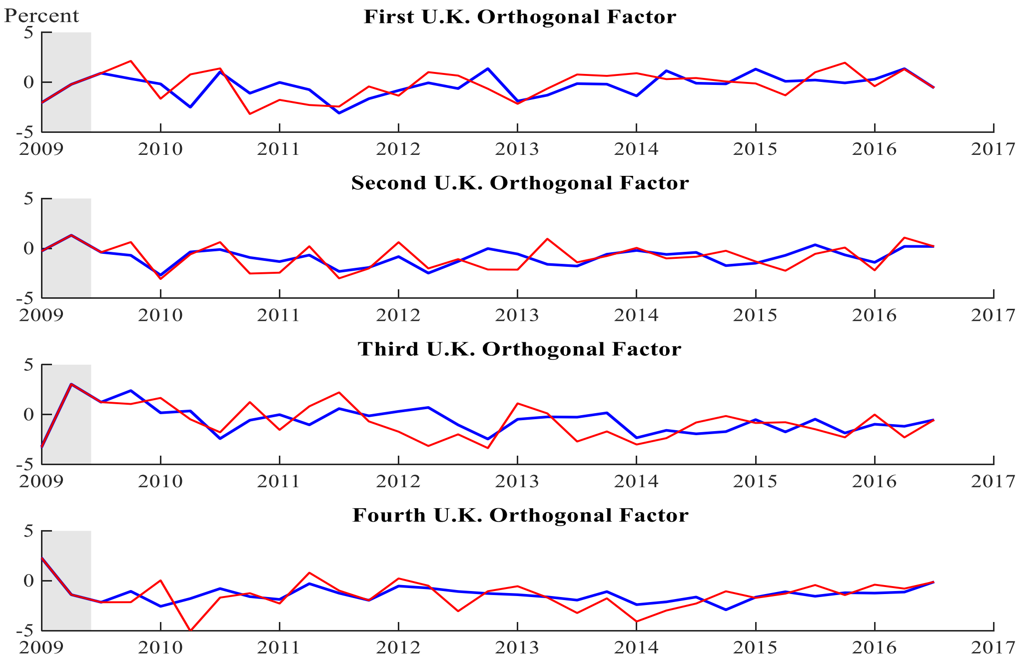
Figure 1.8: Comparison of U.S. factors and U.S. counterfactual factors

Notes:

(1) The red lines are U.S. actual factors and the blue lines are U.S. counterfactual factors estimated over the whole sample. The first U.S. factor is the U.S. real activity factor (RAF), whose counterfactual is named the U.S. counterfactual RAF. The U.S. real activity shortfall (RAS) is the difference between the U.S. RAF (the red line) and the U.S. counterfactual RAF (the blue line). The significant gap between them during the whole period of recovery, indicates that RAS is remarkable.
(2) The resemblance of the other three U.S. factors to their counterfactual counterparts shows that incorporating the U.S. RAS does not affect the estimation of other U.S. factors.



(a) U.K. orthogonal factors in the whole sample



(b) U.K. orthogonal factors after the Great Recession

Figure 1.9: Comparison of U.K. factors and U.K. counterfactual factors

Notes:

(1) The red lines are U.K. orthogonal factors. The blue lines are U.K. counterfactual factors estimated by using a one-step ahead forecast with a rolling origin based on a VAR with order $P = 4$ to incorporate the potential response of U.K. orthogonal factors to the U.S. RAS. See Figure 1.K.12 and Table 1.L.15 in Appendices 1.K and 1.L for forecasting with a fixed origin in 2009Q1.
(2) The magnitude of the product of U.K. orthogonal factor loadings and the gap between the red line and blues lines is unimportant and does not change the results. See Table 1.5 for details.

Tables

Table 1.1.1: Shortfall of the U.S. post-crisis recovery estimated by Okun's law

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Series	Actual Recovery	Counterfactual Recovery	Annual Shortfall	Wilcoxon p-value	Shortfall with a TV intercept	Shortfall with a break in coefficient	Shortfall with a break in intercept and coefficient
1. GDP/population	1.72	3.04	1.32	0.002	0.57	0.71	-0.05
2. TFP	0.89	1.35	0.46	0.156	0.33	0.09	-0.03
3. $\alpha \times$ Capital/population	0.24	0.72	0.48	0.000	-0.00	0.40	0.01
4. $(1-\alpha) \times$ LQ×hours/population	0.59	0.97	0.37	0.027	0.04	0.22	-0.03
5. Hours/population	0.63	1.08	0.45	0.023	0.01	0.29	-0.03
6. Hours/employed worker	0.24	0.15	-0.09	0.459	0.05	-0.07	-0.03
7. Employment Rate	0.68	0.68	-0.00	0.371	-0.00	0.01	-0.00
8. Labor force participation	-0.66	0.01	0.67	0.001	0.02	0.58	0.03
9. GDP / hour	1.09	1.96	0.87	0.020	0.47	0.42	-0.02
10. TFP / $(1-\alpha)$	1.44	2.04	0.61	0.190	0.61	0.06	-0.05
11. Capital/output $\times \alpha/(1-\alpha)$	-0.69	-0.48	0.21	0.432	-0.07	0.33	0.04
12. Labor quality	0.33	0.39	0.06	0.414	0.04	0.03	-0.01

Notes:

(1) Column (a) presents the average annualized growth rates of each series from 2009Q4 to the 2016Q2. Column (b) is the average of the annualized growth rates of the counterfactual recovery for each series in the same period (2009Q4–2016Q2). Units are annualized percent changes.

(2) Counterfactual recoveries are estimated by Okun's law with $p = 1$ leads and lags, assuming that the Okun's intercept and coefficient are stable. The generalized Okun's law coefficient, estimated over the sample from 1981Q1 to 2016Q2, is -1.51 with the standard deviation of 0.17.

(3) The average of the annual shortfalls over the current recovery, from 2009Q4 to 2016Q2, is shown in column (c). The shortfall in each quarter, as specified in Eq. (1.2), is simply the difference between the counterfactual and actual recovery. Column (d) reports the Wilcoxon signed rank test p-value, which examines whether the actual recovery falls short of the counterfactual recovery. Thus, a significant shortfall indicates a slow recovery.

(4) Column (e) represents the average of the annual shortfalls where the counterfactual is estimated by a model with a time-varying intercept. Tracking the time-varying Okun's intercept indicates a fall in the growth of the U.S. potential output after the 2007–09 financial crisis. The column (f) is the average of the annual shortfalls where the counterfactual is estimated by a model in which a structural break in the Okun's coefficient in 2009Q1 is allowed for.

(5) Column (g) is the average of annual shortfalls where a structural break in both Okun's intercept and coefficient in 2009Q1 is allowed for. To eliminate the shortfall entirely, it is necessary to consider the model with breaks in both the Okun's intercept and coefficient in 2009Q1. This date is consistent with the identified structural break in Okun's law in 2009Q1, which is presented in Table 1.1.3. Allowing for a break in other candidate break dates from 2007Q4 to 2008Q4, decreases the shortfall but does not eliminate it. Also, including a time-varying intercept and a break in the Okun's coefficient in 2009Q1 eliminates the shortfall.

(6) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.1.2: Shortfall of the U.S. post-crisis recovery estimated by the DFM

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Series	Actual Recovery	Counterfactual Recovery	Annual Shortfall	Wilcoxon p-value	Shortfall with a TV intercept	Shortfall with a break in factor loadings	Shortfall with a break in intercept and factor loadings
1. GDP/population	1.72	2.98	1.26	0.001	0.70	0.53	0.01
2. TFP	0.89	1.67	0.78	0.016	0.53	-0.01	0.02
3. $\alpha \times$ Capital/population	0.24	0.61	0.37	0.000	-0.00	0.50	-0.00
4. $(1-\alpha) \times$ LQ×hours/population	0.59	0.71	0.12	0.296	-0.06	0.05	-0.01
5. Hours/population	0.63	0.69	0.06	0.514	-0.14	-0.01	-0.01
6. Hours/employed worker	0.24	0.24	-0.00	0.696	0.05	-0.19	-0.00
7. Employment Rate	0.68	0.52	-0.16	0.941	0.01	-0.19	0.01
8. Labor force participation	-0.66	-0.12	0.54	0.003	0.07	0.63	0.01
9. GDP/hour	1.09	2.29	1.20	0.001	0.62	0.54	0.03
10. TFP/$(1-\alpha)$	1.44	2.53	1.09	0.029	0.96	-0.11	0.04
11. Capital/output $\times\ \alpha/(1-\alpha)$	-0.69	-0.62	0.06	0.532	-0.03	0.61	-0.01
12. Labor quality	0.33	0.38	0.05	0.486	0.05	0.04	-0.00

Notes:

(1) Column (a) presents the average annualized growth rates of each series from 2009Q4 to the 2016Q2. Column (b) is the average of the annualized growth rates of the counterfactual recovery for each series in the same period (2009Q4–2016Q2). Units are annualized percent changes.

(2) Counterfactual recoveries are estimated by the DFM, assuming that the intercept and cyclical factor loadings are stable. The sample for estimation of the parameters is 1981Q1 to 2016Q2.

(3) The average of the annual shortfalls over the current recovery, from 2009Q4 to 2016Q2, is shown in column (c). The shortfall in each quarter, as specified in Eq. (1.2), is simply the difference between the counterfactual and actual recovery. Column (d) reports the Wilcoxon signed rank test p-value, which examines whether the actual recovery falls short of the counterfactual recovery. Thus, a significant shortfall indicates a slow recovery.

(4) Column (e) represents the average of the annual shortfalls where the counterfactual is estimated by a model with a time-varying intercept. The column (f) is the average of the annual shortfalls, where the counterfactual is estimated by a model in which a structural break in the cyclical factor loadings in 2009Q1 is allowed for. Indeed, two sets of cyclical factors are estimated by using PCA over two regimes, and the post-crisis fitted series are calculated by the cyclical factor loadings and cyclical factors derived from the second regime from 2009Q1 to 2016Q2.

(5) Column (g) is the average of annual shortfalls where a structural break in both intercept and cyclical factor loadings in 2009Q1 is allowed for. To eliminate the shortfall entirely, it is necessary to consider the model with breaks in both the intercept and cyclical factor loadings in 2009Q1. This date is consistent with the identified structural break in the intercept and cyclical factor loadings between 2007 and 2009, which is presented in Table 1.1.4. Allowing for the break in other candidate break dates from 2007Q4 to 2008Q4 decreases the shortfall but does not eliminate it entirely. Also, including a time-varying intercept along with a break in the cyclical factor loadings in 2009Q1 eliminates the shortfall.

(6) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.1.3: Structural Breaks in the Okun's Law intercept and coefficient for the U.S.

	Joint Break Test Hansen (2000)			Intercept Break Test Bai and Perron (2003)		Coefficient Break Test Bai and Perron (2003)	
Series	Break Date	Andrews p-value	QLR (Sup F)	Break Date	QLR (Sup F)	Break Date	QLR (Sup F)
1. GDP/population	2009Q1	0.00	23.1	2006Q1	14.0	2009Q1	21.0
2. TFP	2009Q1	0.08	11.4	2012Q1	15.5	2009Q1	10.6
3. $\alpha \times$ Capital/population	2002Q1	0.00	78.0	2002Q1	22.1	2009Q4	20.5
4. $(1-\alpha) \times$ LQ×hours/population	1998Q4	0.01	15.6	1999Q3	24.8	2009Q3	28.2

Table 1.1.4: Structural Breaks in the intercept and cyclical factor loadings for the U.S.

	Joint Break Test Hansen (2000)			Intercept Break Test Bai and Perron (2003)		Factor Loadings Test Bai and Perron (2003)	
Series	Break Date	Andrews p-value	QLR (Sup F)	Break Date	QLR (Sup F)	Break Date	QLR (Sup F)
1. GDP/population	2006Q1	0.00	32.9	2009Q4	22.0	2007Q3*	38.7*
2. TFP	1982Q3	0.00	24.1	2009Q4	10.1	2007Q1*	37.5*
3. $\alpha \times$ Capital/population	2002Q2	0.00	117.1	2002Q2	73.5	2010Q1	20.4
4. $(1-\alpha) \times$ LQ×hours/population	1998Q4	0.02	21.0	1998Q4	21.4	2008Q4	10.1

* The reported QLR statistics with an asterisk, test the hypothesis of two breaks against a zero break. See note (4) for an explanation.

Notes for both Tables:

(1) The Quandt Likelihood Ratio (QLR) is the supremum of F statistics. The estimation period is 1981–2016.

(2) We use the Hansen (2000) method to test the joint break in parameters and the Bai and Perron (2003) method to test the break in parameters separately.

(3) Regarding Table 1.1.3, we test the joint break in the Okun's law intercept and coefficient, the results of which are presented in the first three columns. The 5% critical value for this test is 11.8. The other columns report the results of testing a break in the Okun's intercept and a break in the Okun's coefficient separately. The 5% critical value for this test is 9.1.

(4) Regarding Table 1.1.4, we test the joint break in the intercept and cyclical factor loadings, the results of which are presented in the first three columns. The 5% critical value for this test is 18.4. The other columns report the results of testing a break in the intercept and a break in the cyclical factor loadings separately with 5% critical values of 9.1 and 16.8, respectively. The factor loading break test, based on Bai and Perron (2003), identifies two breaks for GDP/population (1991Q1 with a QLR of 24.7 and 2007Q3 with a QLR of 38.7) and two breaks for TFP (1991Q1 with a QLR of 21.9 and 2007Q1 with a QLR of 37.5).

(5) The break dates for almost all of the break tests are around 2009. The confidence intervals for break in Okun's coefficient identified by Bai and Perron (2003) is 2008Q1-2011Q2.

(6) The QLR of the joint break test in the four U.S. cyclical factor loadings is significant.

(7) In Section 1.3.3, we constructed the U.S. RAF by using PCA on variables in the National Income and Product Accounts (NIPA) and industrial production categories. The other three U.S. factors are also constructed using PCA on variables in other categories (e.g., unemployment rate, price, interest rate, and money). We find that the structural break is much stronger in the factor loadings of the first U.S. cyclical factor and weaker in those of the other U.S. cyclical factors. This suggests that the recovery of the first factor, which explains the dynamics of variables in the NIPA and industrial production categories, is not consistent with the recovery of other factors, which capture the dynamics of variables in other categories.

Table 1.2.1: Shortfall of the U.K. post-crisis recovery estimated by Okun's law

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Series	Actual Recovery	Counterfactual Recovery	Annual Shortfall	Wilcoxon p-value	Shortfall with a TV intercept	Shortfall with a break in coefficient	Shortfall with a break in intercept and coefficient
1. GDP/population	1.80	2.62	0.82	0.017	-0.07	0.88	-0.17
2. TFP	0.18	1.24	1.07	0.006	-0.14	1.00	-0.14
3. $\alpha \times$ Capital/population	0.36	0.82	0.46	0.000	-0.00	0.44	0.01
4. $(1-\alpha) \times$ LQ×hours/population	1.26	0.56	-0.70	0.977	0.02	-0.55	-0.04
5. Hours/population	1.46	0.58	-0.88	0.976	-0.08	-0.46	-0.08
6. Hours/employed worker	0.75	-0.02	-0.77	0.959	-0.04	-0.34	-0.09
7. Employment Rate	0.43	0.45	0.01	0.450	0.03	0.02	-0.00
8. Labor force participation	0.28	0.15	-0.12	0.810	-0.02	-0.14	0.01
9. GDP/hour	0.34	2.04	1.70	0.001	0.13	1.34	-0.09
10. TFP/$(1-\alpha)$	0.28	2.01	1.74	0.006	-0.21	1.59	-0.23
11. Capital/output $\times\ \alpha/(1-\alpha)$	-0.47	-0.31	0.17	0.216	0.02	0.14	0.11
12. Labor quality	0.53	0.34	-0.19	0.929	0.07	-0.39	0.03

Notes:

(1) Column (a) presents the average annualized growth rates of each series from 2009Q4 to the 2016Q2. Column (b) is the average of the annualized growth rates of the counterfactual recovery for each series in the same period (2009Q4–2016Q2). Units are annualized percent changes.

(2) Counterfactual recoveries are estimated by Okun's law with $p = 1$ leads and lags, assuming that the Okun's intercept and coefficient are stable. The generalized Okun's law coefficient, estimated over the sample from 1981Q1 to 2016Q2, is -1.98 with the standard deviation of 0.47.

(3) The average of the annual shortfalls over the current recovery, from 2009Q4 to 2016Q2, is shown in column (c). The shortfall in each quarter, as specified in Eq. (1.2), is simply the difference between the counterfactual and actual recovery. Column (d) reports the Wilcoxon signed rank test p-value, which examines whether the actual recovery falls short of the counterfactual recovery. Thus, a significant shortfall indicates a slow recovery.

(4) Column (e) represents the average of the annual shortfalls where the counterfactual is estimated by a model with a time-varying intercept. In this model, the time-varying intercept is able to capture a potential break in the intercept and without any need for a break in the Okun's coefficient, the model eliminates the shortfall. The column (f) is the average of the annual shortfalls where the counterfactual is estimated by a model in which a structural break in the Okun's coefficient in 2009Q1 is allowed for. This model is not able to decrease the shortfall.

(5) Column (g) is the average of annual shortfalls where a structural break in both Okun's intercept and coefficient in 2009Q1 is allowed for. This model eliminates the shortfall. Nevertheless, to eliminate the shortfall it is enough to include a break in the intercept in 2009Q1 (without any need for a break in the cyclical factor loadings). This date is the same as the break date in the U.S. and is also close to the identified date (2008Q3) of the structural break in the Okun's law intercept, which is presented in Table 1.2.3. Allowing for the break in other candidate break dates from 2007Q4 to 2008Q4, decreases the shortfall but does not eliminate it. Also, as presented in column (e), including a time-varying intercept (without any need for a break in the Okun's coefficient) eliminates the shortfall. Tracking the time-varying Okun's intercept indicates a sharp fall in the growth of the U.K. potential output after the 2007−09 financial crisis.

(6) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP and capital input to the slow recovery in the U.K.

Table 1.2.2: Shortfall of the U.K. post-crisis recovery estimated by the DFM

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Series	Actual Recovery	Counterfactual Recovery	Annual Shortfall	Wilcoxon p-value	Shortfall with a TV intercept	Shortfall with a break in factor loadings	Shortfall with a break in intercept and factor loadings
1. GDP/population	1.80	2.39	0.59	0.033	0.09	0.69	0.01
2. TFP	0.18	0.82	0.64	0.026	-0.06	0.97	-0.05
3. $\alpha \times$ Capital/population	0.36	0.79	0.44	0.000	-0.00	0.47	0.01
4. $(1-\alpha) \times$ LQ×hours/population	1.26	0.78	-0.48	0.953	0.03	-0.75	0.05
5. Hours/population	1.46	0.80	-0.66	0.941	-0.03	-0.86	0.08
6. Hours/employed worker	0.75	0.21	-0.54	0.855	-0.01	-0.59	0.06
7. Employment Rate	0.43	0.38	-0.05	0.696	-0.01	-0.13	-0.00
8. Labor force participation	0.28	0.20	-0.07	0.828	0.03	-0.14	0.02
9. GDP/hour	0.34	1.59	1.25	0.004	0.22	1.55	-0.07
10. TFP/$(1-\alpha)$	0.28	1.34	1.06	0.023	0.01	1.54	-0.08
11. Capital/output $\times\, \alpha/(1-\alpha)$	-0.47	-0.20	0.27	0.156	0.04	0.30	0.01
12. Labor quality	0.53	0.46	-0.07	0.784	0.04	-0.28	0.01

Notes:

(1) Column (a) presents the average annualized growth rates of each series from 2009Q4 to the 2016Q2. Column (b) is the average of the annualized growth rates of the counterfactual recovery for each series in the same period (2009Q4–2016Q2). Units are annualized percent changes.

(2) Counterfactual recoveries are estimated by the DFM, assuming that the intercept and cyclical factor loadings are stable. The sample for estimation of the parameters is 1981Q1 to 2016Q2.

(3) The average of the annual shortfalls over the current recovery, from 2009Q4 to 2016Q2, is shown in column (c). The shortfall in each quarter, as specified in Eq. (1.2), is simply the difference between the counterfactual and actual recovery. Column (d) reports the Wilcoxon signed rank test p-value, which examines whether the actual recovery falls short of the counterfactual recovery. Thus, a significant shortfall indicates a slow recovery.

(4) Column (e) represents the average of the annual shortfalls where the counterfactual is estimated by a model with a time-varying intercept. In this model, the time-varying intercept is able to capture a potential break in the intercept and without any need for a break in the cyclical factor loadings, the model eliminates the shortfall. The column (f) is the average of the annual shortfalls, where the counterfactual is estimated by a model in which a structural break in the cyclical factor loadings in 2009Q1 is allowed for. Indeed, two sets of cyclical factors are estimated by using PCA over two regimes, and the post-crisis fitted series are calculated by the cyclical factor loadings and cyclical factors derived from the second regime from 2009Q1 to 2016Q2. This model is not able to decrease the shortfall.

(5) Column (g) is the average of annual shortfalls where a structural break in both intercept and cyclical factor loadings in 2009Q1 is allowed for. This model eliminates the shortfall. Nevertheless, to eliminate the shortfall it is enough to include a break in the intercept in 2009Q1 (without any need for a break in the cyclical factor loadings). This date is the same as the break date in the U.S. and is also close to the identified date (2008Q1) of the structural break in the intercept, which is presented in Table 1.2.4. Allowing for a break in other candidate break dates from 2007Q4 to 2008Q4, decreases the shortfall but does not eliminate it. Also, as presented in column (e), including a time-varying intercept (without any need for a break in the factor loadings) eliminates the shortfall.

(6) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP and capital input to the slow recovery in the U.K.

Table 1.2.3: Structural Breaks in the Okun's Law intercept and coefficient for the U.K.

	Joint Break Test Hansen (2000)			Intercept Break Test Bai and Perron (2003)		Coefficient Break Test Bai and Perron (2003)	
Series	Break Date	Andrews p-value	QLR (Sup F)	Break Date	QLR (Sup F)	Break Date	QLR (Sup F)
1. GDP/population	2008Q1	0.00	39.2	2008Q1*	5.2*	2009Q3*	9.4*
2. TFP	2007Q4	0.00	37.0	2007Q4*	4.2*	2009Q3*	7.5*
3. $\alpha \times$ Capital/population	2001Q2	0.00	86.1	2001Q2	2.9	1990Q2	2.8
4. $(1-\alpha) \times$ LQ×hours/population	2011Q2	0.03	14.1	2009Q1	14.9	1987Q1	8.9

* The reported QLR statistics with an asterisk, test the hypothesis of two breaks against a zero break. See note (3) for an explanation.

Table 1.2.4: Structural Breaks in the intercept and cyclical factor loadings for the U.K.

	Joint Break Test Hansen (2000)			Intercept Break Test Bai and Perron (2003)		Factor Loadings Test Bai and Perron (2003)	
Series	Break Date	Andrews p-value	QLR (Sup F)	Break Date	QLR (Sup F)	Break Date	QLR (Sup F)
1. GDP/population	2005Q4	0.00	29.3	2008Q1	17.3	2008Q3	46.4
2. TFP	1989Q4	0.02	22.1	2007Q4*	18.0*	2008Q3	30.3
3. $\alpha \times$ Capital/population	2001Q3	0.00	100.7	2001Q3	18.4	1990Q2	9.6
4. $(1-\alpha) \times$ LQ×hours/population	2004Q3	0.19	15.4	2004Q3	13.2	2008Q3	11.2

Notes for both Tables:

(1) The Quandt Likelihood Ratio (QLR) is the supremum of F statistics. The estimation period is 1981–2016.

(2) We use the Hansen (2000) method to test the joint break in parameters and the Bai and Perron (2003) method to test the break in parameters separately.

(3) Regarding Table 1.2.3, we test the joint break in the Okun's intercept and coefficient, the results of which are presented in the first three columns. The 5% critical value for this test is 11.8. The other columns report the results of testing a break in the Okun's intercept and a break in the Okun's coefficient separately with 5% critical values of 9.1. The coefficient break test, based on Bai and Perron (2003), identifies two breaks for GDP/population (2005Q4 with a QLR of 11.9 and 2009Q3 with a QLR of 9.4) and two breaks for TFP (1984Q2 with a QLR of 10.3 and 2009Q3 with a QLR of 7.5). Similarly, the intercept break test identifies two breaks for GDP/population (1988Q1 with a QLR of 8.7 and 2008Q1 with a QLR of 5.2) and two breaks for TFP (1988Q2 with a QLR of 4.5 and 2007Q4 with a QLR of 4.2).

(4) Regarding Table 1.2.4, we test the joint break in the intercept and cyclical factor loadings, the results of which are presented in the first three columns. The 5% critical value for this test is 18.4. The other columns report the results of testing a break in the intercept and a break in the cyclical factor loadings separately with 5% critical values of 9.1 and 16.8, respectively.

(5) The break dates for almost all of the break tests are between 2006 and 2009.

(6) The QLR of the joint break test in the four U.K. cyclical factor loadings is significant.

Table 1.3.1: Trend-cycle decomposition of the U.S. series based on Okun's law

	Historical Values			Cycle			Cyclically Adjusted Trend			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
Series	Current recovery	Three previous recoveries	Annual Shortfall	Current recovery	Three previous recoveries	Annual Shortfall	Current recovery	Three previous recoveries	Annual Shortfall	Annual Shortfall in smooth trend
1. GDP/population	1.72	2.95	1.23	0.72	0.94	0.22	1.00	2.01	1.00	0.79
2. TFP	0.89	1.40	0.51	0.39	0.38	-0.01	0.50	1.02	0.52	0.26
3. $\alpha \times$ Capital/population	0.24	0.85	0.61	-0.30	0.04	0.34	0.54	0.81	0.27	0.25
4. $(1-\alpha) \times$ LQ×hours/population	0.59	0.70	0.11	0.63	0.52	-0.10	-0.03	0.17	0.21	0.28
5. Hours/population	0.63	0.61	-0.02	1.02	0.84	-0.17	-0.38	-0.23	0.16	0.32
6. Hours/employed worker	0.24	0.00	-0.24	0.43	0.14	-0.29	-0.19	-0.14	0.05	-0.01
7. Employment Rate	0.68	0.41	-0.27	0.68	0.41	-0.27	0.00	0.00	-0.00	-0.00
8. Labor force participation	-0.66	0.15	0.81	-0.55	0.05	0.60	-0.11	0.10	0.21	0.22
9. GDP/hour	1.09	2.33	1.25	-0.30	0.10	0.40	1.39	2.23	0.85	0.47
10. TFP/$(1-\alpha)$	1.44	2.08	0.65	0.69	0.57	-0.12	0.75	1.51	0.76	0.38
11. Capital/output $\times\ \alpha/(1-\alpha)$	-0.69	-0.18	0.51	-0.98	-0.41	0.57	0.29	0.23	-0.06	-0.01
12. Labor quality	0.33	0.43	0.09	-0.01	-0.06	-0.05	0.34	0.49	0.15	0.10

Notes:

(1) Column (a) presents the average growth rate of each series over the current recovery from 2009Q4 to 2016Q2. Similarly, column (b) reports the average growth rate over the three previous recoveries (1983Q1–1990Q2, 1991Q2–2000Q4, and 2002Q1–2007Q3). Taking the three previous recoveries as a counterfactual, column (c) measures the annual shortfall, the difference between the counterfactual and actual recovery. Units are annualized percent changes.

(2) Columns (d) and (e) show the average growth rate of cyclical components during the current recovery and the three previous recoveries. Thus, column (f) measures the annual shortfall in the cyclical component. Likewise, columns (g) and (h) show the average growth rate of trend components during the current recovery and the three previous recoveries. Thus, column (i) measures the annual shortfall in the trend component. Additionally, column (j) measures the shortfall in the smooth trend.

(3) To estimate the cycle and cyclically adjusted trend, we use Okun's regression, explained in Eq. (1.15), where a structural break in the Okun's coefficient in 2009Q1 is allowed for. The sample for estimation of the parameters is 1981Q1 to 2016Q2. The growth of the cyclical component of each series is the product of the Okun's coefficient and the change in the unemployment rate. The cyclically adjusted trend is the residuals of the regression, and the smooth trend is derived by passing the estimated residuals through a Tukey's bi-weight filter with a bandwidth of 60 quarters.

(4) In the benchmark decomposition, we consider Okun's regression with $p = 2$. Changing the number of leads and lags to $p = 0$ or $p = 1$ has no notable effect on the results.

(5) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.3.2: Trend-cycle decomposition of the U.S. series based on DFM

	Historical Values			Cycle			Cyclically Adjusted Trend			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
Series	Current recovery	Three previous recoveries	Annual Shortfall	Current recovery	Three previous recoveries	Annual Shortfall	Current recovery	Three previous recoveries	Annual Shortfall	Annual Shortfall in smooth trend
1. GDP/population	1.72	2.95	1.23	0.43	1.09	0.65	1.28	1.86	0.57	0.39
2. TFP	0.89	1.40	0.51	0.04	0.61	0.57	0.85	0.79	-0.06	-0.28
3. $\alpha \times$ Capital/population	0.24	0.85	0.61	0.02	0.03	0.01	0.22	0.82	0.60	0.44
4. $(1-\alpha) \times$ LQ×hours/population	0.59	0.70	0.11	0.37	0.45	0.07	0.22	0.25	0.03	0.23
5. Hours/population	0.63	0.61	-0.02	0.70	0.75	0.05	-0.06	-0.13	-0.07	0.22
6. Hours/employed worker	0.24	0.00	-0.24	0.15	0.14	-0.01	0.09	-0.14	-0.24	-0.19
7. Employment Rate	0.68	0.41	-0.27	0.39	0.38	-0.01	0.29	0.03	-0.25	-0.15
8. Labor force participation	-0.66	0.15	0.81	0.00	0.05	0.05	-0.66	0.10	0.77	0.52
9. GDP/hour	1.09	2.33	1.25	-0.26	0.34	0.61	1.35	1.99	0.64	0.17
10. TFP/$(1-\alpha)$	1.44	2.08	0.65	0.06	0.91	0.85	1.38	1.17	-0.20	-0.49
11. Capital/output $\times \alpha/(1-\alpha)$	-0.69	-0.18	0.51	-0.21	-0.50	-0.28	-0.47	0.32	0.79	0.56
12. Labor quality	0.33	0.43	0.09	-0.11	-0.08	0.03	0.44	0.50	0.06	0.10

Notes:

(1) Column (a) presents the average growth rate of each series over the current recovery from 2009Q4 to 2016Q2. Similarly, column (b) reports the average growth rate over the three previous recoveries (1983Q1–1990Q2, 1991Q2–2000Q4, and 2002Q1–2007Q3). Taking the three previous recoveries as a counterfactual, column (c) measures the annual shortfall, the difference between the counterfactual and actual recovery. Units are annualized percent changes.

(2) Columns (d) and (e) show the average growth rate of cyclical components during the current recovery and the three previous recoveries. Thus, column (f) measures the annual shortfall in the cyclical component. Likewise, columns (g) and (h) show the average growth rate of trend components during the current recovery and the three previous recoveries. Thus, column (i) measures the annual shortfall in the trend component. Additionally, column (j) measures the shortfall in the smooth trend.

(3) To estimate the cycle and cyclically adjusted trend, we use regression, explained in Eq. (1.18), where a structural break in the factor loadings in 2009Q1 is allowed for. Indeed, two sets of cyclical factors are estimated by using PCA over two regimes, one before and another after the 2009Q1. The sample for estimation of the parameters is 1981Q1 to 2016Q2. The growth of the cyclical component of each series is the common cyclical component (linear regression of the cyclical factors). The cyclically adjusted trend is the residuals of this regression, and the smooth trend is derived by passing the estimated residuals through a Tukey's bi-weight filter with a bandwidth of 60 quarters.

(4) Since the structural break is much stronger in the factor loadings of the first U.S. cyclical factor, we also run Eq. (1.18) with only a structural break in the first cyclical factor loadings. This does not change the results.

(5) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.4.1: Trend-cycle decomposition of the U.K. series based on Okun's law

	Historical Values			Cycle			Cyclically Adjusted Trend			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
Series	Current recovery	Two previous recoveries	Annual Shortfall	Current recovery	Two previous recoveries	Annual Shortfall	Current recovery	Two previous recoveries	Annual Shortfall	Annual Shortfall in smooth trend
1. GDP/population	1.80	2.87	1.07	0.61	0.63	0.03	1.19	2.24	1.05	0.73
2. TFP	0.18	1.72	1.55	-0.04	0.33	0.37	0.22	1.40	1.18	0.62
3. $\alpha \times$ Capital/population	0.36	0.86	0.51	-0.02	0.04	0.05	0.37	0.82	0.45	0.46
4. $(1-\alpha) \times$ LQ×hours/population	1.26	0.28	-0.98	0.66	0.27	-0.39	0.60	0.02	-0.59	-0.35
5. Hours/population	1.46	0.25	-1.21	1.17	0.42	-0.75	0.29	-0.17	-0.46	-0.34
6. Hours/employed worker	0.75	-0.19	-0.93	0.61	0.02	-0.59	0.14	-0.21	-0.34	-0.20
7. Employment Rate	0.43	0.32	-0.11	0.43	0.32	-0.11	0.01	0.00	-0.00	-0.01
8. Labor force participation	0.28	0.12	-0.16	0.13	0.08	-0.05	0.15	0.04	-0.11	-0.13
9. GDP/hour	0.34	2.62	2.28	-0.56	0.22	0.78	0.90	2.41	1.50	1.07
10. TFP/$(1-\alpha)$	0.28	2.77	2.49	-0.07	0.55	0.62	0.34	2.22	1.87	0.99
11. Capital/output $\times \alpha/(1-\alpha)$	-0.47	-0.39	0.08	-0.36	-0.35	0.01	-0.11	-0.04	0.07	0.27
12. Labor quality	0.53	0.24	-0.29	-0.14	0.01	0.15	0.67	0.23	-0.44	-0.18

Notes:

(1) Column (a) presents the average growth rate of each series over the current recovery from 2009Q4 to 2016Q2. Similarly, column (b) reports the average growth rate over the two previous recoveries (1981Q3–1990Q1, and 1992Q2–2008Q1). Taking the two previous recoveries as a counterfactual, column (c) measures the annual shortfall, the difference between the counterfactual and actual recovery. Units are annualized percent changes.

(2) Columns (d) and (e) show the average growth rate of cyclical components during the current recovery and the two previous recoveries. Thus, column (f) measures the annual shortfall in the cyclical component. Likewise, columns (g) and (h) show the average growth rate of trend components during the current recovery and the two previous recoveries. Thus, column (i) measures the annual shortfall in the trend component. Additionally, column (j) measures the shortfall in the smooth trend.

(3) To estimate the cycle and cyclically adjusted trend, we use Okun's regression, explained in Eq. (1.15), where a structural break in the Okun's coefficient in 2009Q1 is allowed for. The sample for estimation of the parameters is 1981Q1 to 2016Q2. The growth of the cyclical component of each series is the product of the Okun's coefficient and the change in the unemployment rate. The cyclically adjusted trend is the residuals of the regression, and the smooth trend is derived by passing the estimated residuals through a Tukey's bi-weight filter with a bandwidth of 60 quarters.

(4) In the benchmark decomposition, we consider Okun's regression with $p = 2$. Changing the number of leads and lags to $p = 0$ or $p = 1$ has no notable effect on the results, suggesting the contribution of the trend to the slow recovery. However, for the cyclical component, the model with $p = 1$, presented in Appendix 1.L, suggests that the cyclical recovery in the U.K. was normal.

(5) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.4.2: Trend-cycle decomposition of the U.K. series based on DFM

	Historical Values			Cycle			Cyclically Adjusted Trend			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
Series	Current recovery	Two previous recoveries	Annual Shortfall	Current recovery	Two previous recoveries	Annual Shortfall	Current recovery	Two previous recoveries	Annual Shortfall	Annual Shortfall in smooth trend
1. GDP/population	1.80	2.87	1.07	0.64	0.72	0.09	1.16	2.14	0.98	0.47
2. TFP	0.18	1.72	1.55	0.33	0.48	0.15	-0.15	1.24	1.39	0.64
3. $\alpha \times$ Capital/population	0.36	0.86	0.51	0.03	-0.01	-0.04	0.33	0.87	0.54	0.48
4. $(1-\alpha) \times$ LQ×hours/population	1.26	0.28	-0.98	0.28	0.25	-0.03	0.98	0.03	-0.95	-0.64
5. Hours/population	1.46	0.25	-1.21	0.59	0.46	-0.13	0.86	-0.22	-1.08	-0.82
6. Hours/employed worker	0.75	-0.19	-0.93	0.31	0.10	-0.21	0.44	-0.29	-0.73	-0.58
7. Employment Rate	0.43	0.32	-0.11	0.22	0.26	0.04	0.21	0.06	-0.15	-0.11
8. Labor force participation	0.28	0.12	-0.16	0.06	0.10	0.04	0.22	0.01	-0.20	-0.13
9. GDP/hour	0.34	2.62	2.28	0.04	0.26	0.22	0.30	2.36	2.06	1.29
10. TFP/$(1-\alpha)$	0.28	2.77	2.49	0.51	0.79	0.27	-0.24	1.98	2.22	1.03
11. Capital/output $\times \alpha/(1-\alpha)$	-0.47	-0.39	0.08	-0.32	-0.47	-0.15	-0.15	0.08	0.23	0.44
12. Labor quality	0.53	0.24	-0.29	-0.15	-0.05	0.10	0.68	0.30	-0.39	-0.17

Notes:

(1) Column (a) presents the average growth rate of each series over the current recovery from 2009Q4 to 2016Q2. Similarly, column (b) reports the average growth rate over the two previous recoveries (1981Q3–1990Q1, and 1992Q2–2008Q1). Taking the two previous recoveries as a counterfactual, column (c) measures the annual shortfall, the difference between the counterfactual and actual recovery. Units are annualized percent changes.

(2) Columns (d) and (e) show the average growth rate of cyclical components during the current recovery and the two previous recoveries. Thus, column (f) measures the annual shortfall in the cyclical component. Likewise, columns (g) and (h) show the average growth rate of trend components during the current recovery and the two previous recoveries. Thus, column (i) measures the annual shortfall in the trend component. Additionally, column (j) measures the shortfall in the smooth trend.

(3) To estimate the cycle and cyclically adjusted trend, we use regression, explained in Eq. (1.18), where a structural break in the factor loadings in 2009Q1 is allowed for. Indeed, two sets of cyclical factors are estimated by using PCA over two regimes one before and another after the 2009Q1. The sample for estimation of the parameters is 1981Q1 to 2016Q2. The growth of the cyclical component of each series is the common cyclical component (linear regression of the cyclical factors). The cyclically adjusted trend is the residuals of this regression, and the smooth trend is derived by passing the estimated residuals through a Tukey's bi-weight filter with a bandwidth of 60 quarters.

(4) Since the structural break in the factor loadings of the U.K. cyclical factors is not as strong as that of the U.S. cyclical factors, we also run Eq. (1.18) without a structural break. This does not change the results.

(5) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.5: Shortfall of the U.K. post-crisis recovery estimated by a hierarchical DFM which adopts both U.S. and U.K. factors

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Series	Actual Recovery	Counterfactual Recovery	Annual Shortfall	Wilcoxon p-value	Annual Shortfall with change in other U.S. factors	Annual Shortfall with change in U.K. orthogonal factors	Annual Shortfall with a break in intercept and factor loadings
1. GDP/population	1.80	2.42	0.62	0.023	0.60	0.63	0.01
2. TFP	0.18	0.62	0.44	0.091	0.43	0.53	-0.00
3. $\alpha \times$ Capital/population	0.36	0.71	0.35	0.000	0.35	0.35	-0.00
4. $(1-\alpha) \times$ LQ×hours/population	1.26	1.09	-0.17	0.703	-0.17	-0.25	0.01
5. Hours/population	1.46	1.24	-0.21	0.612	-0.21	-0.33	0.02
6. Hours/employed worker	0.75	0.45	-0.30	0.646	-0.30	-0.27	0.03
7. Employment Rate	0.43	0.55	0.12	0.111	0.12	0.04	-0.00
8. Labor force participation	0.28	0.25	-0.03	0.711	-0.03	-0.10	-0.00
9. GDP/hour	0.34	1.17	0.83	0.015	0.80	0.96	-0.01
10. TFP/$(1-\alpha)$	0.28	1.03	0.76	0.077	0.73	0.88	-0.00
11. Capital/output $\times \alpha/(1-\alpha)$	-0.47	-0.36	0.11	0.379	0.12	0.11	-0.00
12. Labor quality	0.53	0.50	-0.03	0.695	-0.04	-0.03	-0.01

Notes:

(1) Column (a) presents the average annualized growth rates of U.K. series from 2009Q1 to the 2016Q2. Column (b) is the average of the annualized growth rates of the counterfactual recovery for each series in the same period (2009Q1–2016Q2). Units are annualized percent changes.

(2) Counterfactual recoveries are estimated by an open-economy hierarchical DFM, which adopts both U.S. factors and U.K. orthogonal factors. In particular, U.K. counterfactual recoveries are derived based on the U.S. counterfactual RAF, which is estimated by using PCA on the U.S. counterfactual series. These counterfactual series are fitted variables estimated by the DFM, where factor loadings are assumed to be stable. Thus, substituting the U.S. actual series in the NIPA, industrial production, and credit categories by faster counterfactual series means that the U.S. actual real activity factor is replaced by a faster counterfactual real activity factor. This gauges the impact of the structural break in the U.S. on the slow recovery in the U.K.

(3) The sample for estimation of the parameters is from 1981 to 2016 to be compatible with the other parts of this study. We assume that factor loadings across different blocks are stable. Tests for breaks in the cross-block factor loadings indicate a break in 1984Q2 and no break after it. We therefore estimate the parameters over alternative sample periods of 1971–2016 and 1985–2016, whose results are presented in Tables 1.L.13 and 1.L.14 in Appendix 1.L.

(4) The average of the annual shortfalls over the current recovery, from 2009Q1 to 2016Q2, is shown in column (c), which measures the magnitude of the output shortfall spillovers from the U.S. Column (d) reports the Wilcoxon signed rank test p-value, which examines whether the actual recovery falls short of the counterfactual recovery.

(5) Column (e) represents the average of the annual shortfalls where the potential change in other U.S. factors has been taken into account. Column (f) represents the average of the annual shortfalls where the potential response of U.K. orthogonal factors to the U.S. RAS has been accommodated by using a one-step ahead forecast with a rolling origin based on a VAR with order $P = 4$. In this setting, VAR model updates the forecasting origin. See Table 1.L.16 in Appendix 1.L, for forecast of U.K. factors with a fixed origin in 2009Q1.

(6) The column (g) is the average of annual shortfalls where a structural break in both intercept and cyclical factor loadings in 2009Q1 is allowed for. Since the shortfall in this setup is close to zero, the hierarchical model performs well in estimating the counterfactual which does not leave any shortfall.

(7) Rows 1-4, 5-8, and 9-12 each present a different growth-accounting decomposition. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

(8) The results are robust to the choice of the number of VAR order ($P = 0, 1, 2, 4$) and the estimation period. See Appendix 1.L for some results for other setups.

Supplementary Appendix to

Slow recovery of output after the 2007−09 financial crisis: U.S. shortfall spillovers and the U.K. productivity puzzle

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Appendix 1.A : Data

Table 1.A.1: Categories of quarterly time series used in the DFM for U.K.

N	Name of Category	Not included aggregate	Included Dis-aggregate	Total Number
1	National Income and Production Accounting	4	5	9
2	Industrial Production	1	6	7
3	Unemployment and Employment	5	10	15
4	Earning and Productivity	−	3	3
5	Retails and Trade Sales	−	2	2
6	Prices and house prices	1	7	8
7	Interest Rate	−	8	8
8	Money and Credit	−	7	7
9	Asset and Stock Prices	2	−	2
10	International and Exchange Rate	4	−	4
11	Miscellaneous	1	−	1
	Total	18	48	66

Note:

For U.S. data, see the online data appendix of Fernald et al. (2017).

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* Mohammad Dehghani is the corresponding author. This paper is entirely extracted from the first chapter of the PhD thesis of the main author, Mohammad Dehghani.

Table 1.A.2: List of the U.K. series and description of data used in the Dynamic Factor Model (sample used for variables is 1971Q1-2016Q4)

1. National Income and Production Accounting					
N	Name	Available Sample	T	F	Sources
1	Real GDP (2013 Prices in BP)	1955Q1-2016Q4	5	N	B of England
1.1	Consumption of goods and services (2013 Prices in BP)	1955Q1-2016Q4	5	Y	B of England
1.2	Investment (2013 Prices in BP)	1955Q1-2016Q4	5	Y	B of England
1.3	Government expenditure of goods and services (2013 Prices in BP)	1955Q1-2016Q4	5	Y	B of England
1.4	Export volume (2013 Prices in BP)	1955Q1-2016Q4	5	Y	B of England
1.5	Export volume (2013 Prices in BP)	1955Q1-2016Q4	5	Y	B of England
2. Industrial Production					
2-a	Industrial Production Index (2010 =100)	1955Q1-2016Q4	5	Y	OECD
2-b	Industrial Production Index (2012 =100)	1955Q1-2016Q4	5	N	B of England
2.1	Total Manufacturing Production Index (2010 =100)	1960Q1-2016Q4	5	Y	OECD
2.2	Manufacturing: Investment Goods Index (2010 =100)	1968Q1-2016Q4	5	Y	OECD
2.3	Manufacturing: Intermediate Goods Index (2010 =100)	1968Q1-2016Q4	5	Y	OECD
2.4	Total Construction Production Index (2010 =100)	1960Q1-2016Q4	5	Y	OECD
2.5	New housing starts Index (2010 =100)	1966Q1-2016Q3	5	Y	OECD
3. Unemployment and Employment (16-64)					
3.1	Employment absolute value (16-64)	1971Q1-2016Q4	5	N	ONS
3.1.1	Male Employment absolute value (16-64)	1971Q1-2016Q4	5	Y	ONS
3.1.2	Female Employment absolute value (16-64)	1971Q1-2016Q4	5	Y	ONS
3.2	Employment Rate (16-64)	1971Q1-2016Q4	2	N	ONS
3.2.1	Male Employment Rate (16-64)	1971Q1-2016Q4	2	Y	ONS
3.2.2	Female Employment Rate (16-64)	1971Q1-2016Q4	2	Y	ONS
3.2	Participation Rate (16-64)	1971Q1-2016Q4	2	N	ONS
3.3.1	Male Participation Rate (16-64)	1971Q1-2016Q4	2	Y	ONS
3.3.2	Female Participation Rate (16-64)	1971Q1-2016Q4	2	Y	ONS
3.4	Unemployment Rate (16-64)	1971Q1-2016Q4	2	N	ONS
3.4.1	Male Unemployment Rate (16-64)	1971Q1-2016Q4	2	Y	ONS
3.4.2	Female Unemployment Rate (16-64)	1971Q1-2016Q4	2	Y	ONS
3.5	Total Weekly Hours Worked for Total employed	1971Q1-2016Q4	5	N	ONS
3.5.1	Total Weekly Hours Worked for Male employed	1971Q1-2016Q4	5	Y	ONS
3.5.2	Total Weekly Hours Worked for Female employed	1971Q1-2016Q4	5	Y	ONS
3.6	Unfilled Job Vacancies	1960Q1-2016Q4	5	N	OECD
3. Unemployment and Employment (>16) not included in the model					
3.1-b	Employment absolute value (>16)**	1971Q1-2016Q4	5	N	ONS
3.1.1-b	Male Employment absolute value (>16)	1971Q1-2016Q4	5	N	ONS
3.1.2-b	Female Employment absolute value (>16)	1971Q1-2016Q4	2	N	ONS
3.2-b	Employment Rate (>16)	1971Q1-2016Q4	2	N	ONS
3.2.1-b	Male Employment Rate (>16)	1971Q1-2016Q4	2	N	ONS
3.2.2-b	Female Employment Rate (>16)	1971Q1-2016Q4	2	N	ONS
3.3-b	Participation Rate (>16)	1971Q1-2016Q4	2	N	ONS
3.3.1-b	Male Participation Rate (>16)	1971Q1-2016Q4	2	N	ONS
3.3.2-b	Female Participation Rate (>16)	1971Q1-2016Q4	2	N	ONS
3.4-b	Unemployment Rate (>16)	1971Q1-2016Q4	2	N	ONS
3.4.1-b	Male Unemployment Rate (>16)	1971Q1-2016Q4	2	N	ONS
3.4.2-b	Female Unemployment Rate (>16)	1971Q1-2016Q4	2	N	ONS
4. Earning and Productivity					
4.1	Real Compensation of Employees (2010 BP)	1960Q1-2016Q4	5	Y	OECD
4.2	Real Average Weekly Earnings (2010 BP)	1960Q1-2016Q4	5	Y	B of England
4.3	Real Average Weekly Earnings Index: Manufacturing	1963Q1-2016Q4	5	N	OECD

N	Name	Available Sample	T	F	Sources
4. Earning and Productivity (Continue of Table)					
4.4	Unit Labor Cost (2010 =100)	1971Q1-2016Q4	5	Y	OECD
4.5	Output per hour worked (2016 =100)	1971Q1-2016Q4	5	N	OECD
5. Retails and Trade Sales					
5.1	Total Retail Trade Index (2010 =100)	1957Q1-2016Q4	5	Y	OECD
5.2	Passenger Car Registration	1957Q1-2016Q4	5	Y	OECD
6. Prices and house prices					
6.1	Consumer Price Index of all items (2010 =100)	1955Q1-2016Q4	6	N	OECD
6.2	Consumer Retail Price Index of all items (2010 =100)	1960Q1-2016Q4	6	Y	OECD
6.3	Consumer Price Index excluding food and energy (2010 =100)	1971Q1-2016Q4	6	Y	OECD
6.4	Consumer Price Index: Food (2010 =100)	1960Q1-2016Q4	6	Y	OECD
6.5	Consumer Price Index: Energy (2010 =100)	1971Q1-2016Q4	6	Y	OECD
6.6	Producer Price Index of all items (2015 =100)	1960Q1-2016Q4	6	Y	OECD
6.7	Residential Property Price Index (2010 =100)	1968Q1-2016Q4	6	Y	OECD
6.8	GDP Implicit Price deflator	1960Q1-2016Q4	6	Y	OECD
6.9	Real GOLD Price (2012 USD per Ounce)	1968Q1-2016Q4	6	N	OECD
6.10	Real Crude Oil WTI Price (2012 USD per barrel)	1960Q1-2016Q4	6	N	FRED
7. Interest Rate					
7.1	Bank of England Policy Rate	1955Q1-2016Q4	2	Y	B of England
7.2	3-month London Interbank Offered Rate (LIBOR)	1971Q1-2016Q4	2	Y	B of England
7.3	Short-term commercial Paper Discount Rate	1955Q1-2016Q4	2	Y	B of England
7.4	Treasury Bill Interest Rate	1955Q1-2016Q4	2	Y	B of England
7.5	10-year Government Bond Yield	1960Q1-2016Q4	2	Y	B of England
7.6	Household Mortgage Rate	1955Q1-2016Q4	2	Y	B of England
7.7	Household Personal Loan Rate	1971Q1-2016Q4	2	Y	B of England
7.8	Banks Deposit Rate	1955Q1-2016Q4	2	Y	B of England
8. Real Money (discounted by GDP deflator) and Credit					
8.1	Total Notes and Coins in circulation (2010 BP)	1970Q1-2016Q4	5	N	B of England
8.2	M0 Money Base (2010 BP)	1955Q1-2016Q4	5	N	B of England
8.3	M1 Money Stock (2010 BP)	1955Q1-2016Q4	5	N	B of England
8.4	M4 Money Stock (2010 BP)	1955Q1-2016Q4	5	N	B of England
8.5	Total Credit to Private Sector by domestic Banks (2010 BP)	1963Q1-2016Q4	5	Y	B of England
8.6	Total Credit to Household (2010 BP)	1966Q1-2016Q4	5	Y	B of IS
8.7	Total Credit to Household (Percentage of GDP)	1966Q1-2016Q4	5	Y	B of IS
8.8	Total Credit to Private Non-Financial Sector (BP)	1963Q1-2016Q4	5	Y	B of IS
8.9	Total Credit to Private Non-Financial Sector (Percentage of GDP)	1963Q1-2016Q4	5	Y	B of IS
8.10	Total Debt Securities all maturities Government	1963Q1-2016Q4	5	Y	B of IS
8.11	International debt securities all maturities all users	1968Q1-2016Q4	5	Y	B of IS
9. Asset and Stock Prices					
9.1	Total Share Price for all shares (2010 = 100)	1960Q1-2016Q4	5	N	OECD
9.2	UK FTSE 100 share price index (2013=100)	1958Q1-2016Q4	5	N	OECD
10. International and Exchange Rates					
10.1	Real Narrow Exchange Rate for UK (2010 = 100)	1964Q1-2016Q4	5	N	B of IS
10.1	Real Narrow Exchange Rate for UK (2010 = 100)	1964Q1-2016Q4	5	N	B of IS
10.2	USD to GBP Exchange Rate	1960Q1-2016Q4	5	N	OECD
10.3	U.S. GDP Index (2015 = 100)	1960Q1-2016Q4	5	N	OECD
10.4	OECD countries GDP Index (2010 =1*)	1962Q1-2016Q4	5	N	OECD
10.5	U.S. Industrial Production Index (2012 = 100)	1955Q1-2016Q4	5	N	FRED
10.6	OECD Euro countries Industrial Production Index (2010 =1*)	1960Q1-2016Q4	5	N	OECD

11. Miscellaneous (Continue of Table)					
11.1	UK Population (16-64)	1971Q1-2016Q4	5	N	ONS

* See note (3).
** See note (4).

Note:

(1) The column labelled T indicates the transformation should be applied to each variable to make it stationary (1 = level, 2 = first difference, 3 = second difference, 4 = logarithm, 5 = first difference of logarithm, 6 = second difference of logarithm).

(2) The column labelled F indicates whether the series is included to estimate factors in the main model (Y = Yes, N = No). Note that, when disaggregate variables of a specific time series are available, in order to avoid double counting, the aggregate series is not included in the model.

(3) All indices with (2010 = 1) are converted to indices in percentage with (2010 = 100) by multiplying the series to 100.

(4) Two sets of series are collected for employment categories. The first set is variables for population aged between 16 and 64 and the second one is for population aged more than 16. Only the first set is included in the DFM.

(5) For the U.S. factor model, refer to the data appendix of Stock and Watson (2012) or Fernald et al. (2017).

Appendix 1.B: Seasonality adjustment

There are two main sources of seasonality. First, the seasonal effect, which is a cyclical pattern caused by changes in seasons (e.g., weather, school openings, and social events). The second one is the calendar effect, which refers to the change in the number of trading and working days per month or per quarter as a result of moving holidays such as Easter Monday and Good Friday.

To control for the aforementioned effects, we use seasonally adjusted time series. The source of the data used for many time series in this paper is seasonally adjusted by itself. For the small number of time series (price, interest rate, and credit categories) that are not seasonally adjusted by the source, the seasonal adjustment has been performed by the X11 feature in RATS software, which is also used by Fernald et al. (2017) on US data. The multiplicative method is selected for doing X11 seasonal adjustment. We perform seasonal adjustment without controlling for some minor events such as Easter, Labour Day, or Thanksgiving Day because of the difference in these days between the U.S. and the U.K.

We perform seasonal adjustment for series where the F-test shows seasonality. In the case of marginal seasonality in some of the series in the price category, we indeed conduct the seasonal adjustment. After performing adjustments, the F-test shows no seasonality in the adjusted series derived by X11 RATS. In addition, different tests, including ACF and Ljung–Box statistics on correlation between the residual terms, show that there is no significant autocorrelation between them. This indicates the validity of seasonal adjustment. Also, X13 ARIMA-SEATS designed by the U.S. Census Bureau can be used for seasonal adjustment, which is also popularly used by the Bank of England and Office of National Statistics. Seasonal adjustment by this package leads to very similar results and seasonally adjusted time series.

Table 1.B: Tests for seasonality adjustment for series that are not seasonally adjusted at source (Seasonality adjustment by X11 feature in RATS software)

N	Name	F-Ratio (p-value) before SA	Nonparametric KW* (p-value) before SA	F-Ratio (p-value) after SA
Seasonality Tests for 6. Prices Series				
6.1	Consumer Price Index of all items (2010 =100)	74.72 (0.00)	125.90 (0.00)	0.05 (0.98)
6.2	Consumer Retail Price Index of all items (2010 =100)	107.75 (0.00)	144.49 (0.00)	0.08 (0.97)
6.3	Consumer Price Index excluding food and energy (2010 =100)	80.29 (0.00)	127.76 (0.00)	0.12 (0.94)
6.4	Consumer Price Index: Food (2010 =100)	77.17 (0.00)	124.56 (0.00)	0.02 (0.99)
6.5	Consumer Price Index: Energy (2010 =100)	14.71 (0.00)	40.97 (0.00)	0.05 (0.98)
6.6	Producer Price Index of all items (2015 =100)	22.55 (0.00)	74.58 (0.00)	0.06 (0.98)
6.7	Residential Property Price Index (2010 =100)	83.94 (0.00)	112.19 (0.00)	0.01 (0.99)
6.8	GDP Implicit Price deflator**	2.5 (0.06)	13.89 (0.00)	0.33 (0.80)
6.9	Real GOLD Price (2012 USD per Ounce)**	1.97 (0.11)	6.12 (0.10)	0.23 (0.87)
6.10	Real Crude Oil WTI Price (2012 USD per barrel)**	3.98 (0.00)	23.60 (0.00)	0.07 (0.97)
Seasonality Tests for 7. Rates Series				
7.1	Bank of England Policy Rate	7.77 (0.00)	39.17 (0.00)	0.25 (0.86)
7.2	3-month London Interbank Offered Rate (LIBOR)	4.31 (0.00)	18.68 (0.00)	0.03 (0.99)
7.3	Short-term commercial Paper Discount Rate	5.36 (0.18)	19.74 (0.06)	0.18 (0.91)
7.4	Treasury Bill Interest Rate	2.76 (0.04)	12.33 (0.01)	0.08 (0.97)
7.5	10-year Government Bond Yield***	0.80 (0.49)	13.34 (0.00)	0.15 (0.94)
7.6	Household Mortgage Rate	8.64 (0.00)	24.91 (0.00)	0.20 (0.90)
7.7	Household Personal Loan Rate	3.51 (0.01)	12.74 (0.00)	0.16 (0.92)
7.8	Banks Deposit Rate	9.32 (0.00)	38.63 (0.00)	0.08 (0.97)
Seasonality Tests for selected 8. Credit Series				
8.6	Total Credit to Household (2010 BP)	8.30 (0.00)	53.45 (0.00)	0.01 (0.99)
8.7	Total Credit to Household (Percentage of GDP)	8.62 (0.00)	47.32 (0.00)	0.08 (0.97)
8.8	Total Credit to Private Non-Financial Sector (BP)	1.63 (0.18)	7.34 (0.06)	0.16 (0.92)
8.9	Total Credit to Private Non-Financial Sector (Per. of GDP)	1.68 (0.17)	6.11 (0.10)	0.02 (0.99)
8.10	Total Debt Securities all maturities Government	4.08 (0.00)	25.08 (0.00)	0.02 (0.99)
8.11	International debt securities all maturities all users	2.52 (0.05)	3.57 (0.31)	0.03 (0.98)
Seasonality Tests for selected 9. Asset Stocks Prices Series				
9.1	FTSE Total Share Price for all shares (2010 = 100)	15.36 (0.00)	48.39 (0.00)	0.04 (0.98)
Seasonality Tests for selected 10. International and Exchange Rate Series				
10.1	Real Narrow Exchange Rate for UK (2010 = 100)	11.66 (0.00)	28.71 (0.00)	0.30 (0.82)
10.2	USD to GBP Exchange Rate	0.55 (0.64)	1.84 (0.60)	0.14 (0.93)

Notes:

* KW stands for Kruskal−Wallis which is a non-parametric test.
** For the three series denoted by a double asterisk, X13 ARIMA-SEATS indicates no seasonality. Also, the statistics in X11 RATS are marginal. Thus, we consider the raw data for three series. Note that the GDP Implicit Price Deflator is seasonally adjusted at the source and there is no need for additional seasonal adjustment.
*** Based on F-test statistics of 0.80 derived by X13 ARIMA-SEATS there is no seasonality in 10-year government bonds. However, at 1% confidence level, the KW-test statistics of 13.34 indicates presence of a seasonality. Thus, we considered seasonally adjusted data for this variable.

Appendix 1.C: Pre-transforming nominal variables to real variables

Almost all of the series are real variables at their sources. However, for some of the variables that are nominal, for example, series in the money and credit category, we transform nominal series to real by adjusting for inflation trough dividing the series by implicit GDP deflator (a broad measure of price levels). Another way is to use the consumer price index or the producer price index, which is a narrower measure of inflation and the movements of prices and values in the economy.

In addition, in the estimation of cyclical factors by using PCA, the long-run mean of the real activity variables is removed by using Tukey's bi-weight filter with the bandwidth of 100 quarters. This treatment is recommended by Stock and Watson (2016) because of observing downward long-run trend growth in the real variables in the U.S. and U.K.[32]

[32] We use Tukey's bi-weight filter with $k_j = d\left(1-\left(\frac{j}{B}\right)^2\right)^2$ when $|j| \leq B$ and $k_j = 0$ otherwise. In this setup, B is the bandwidth and d is chosen such that $\kappa(1) = 1$.

Appendix 1.D: Business cycle dates

Table 1.D.1: Dates of the U.S. business cycles (peak and trough)

N	ECRI*	NBER**	Description
1	1957M8-1958M4	1957M8-1958M4	--
2	1960M4-1961M2	1960M4-1961M2	--
3	1969M12-1970M11	1969M12-1970M11	--
4	1973M11-1975M3	1973M11-1975M3	First Oil Crisis
5	1980M1-1980M7	1980M1-1980M7	Second Oil Crisis
6	1981M7-1982M11	1981M7-1982M11	Early 1980s recession
7	1990M7-1991M3	1990M7-1991M3	Early 1990s recession
8	2001M3-2001M11	2001M3-2001M11	Early 2000s recession
9	2007M12-2009M6	2007M12-2009M6	Global crisis and recession
10	2020M2-2009M4	2020M2-2020M4	COVID-19 recession

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Table 1.D.2: Dates of the U.K. business cycles (peak and trough)

N	ECRI*	NIESR**	Description
1	-	1951M3-1952M8	--
2	-	1955M12-1958M11	--
3	-	1961M3-1963M1	--
4	1974M9-1975M8	1973M1-1975M3	First Oil Crisis
5	1979M6-1981M5	1979M2-1982M4	Second Oil Crisis
6	-	1984M1-1984M3	--
7	-	1988M4-1992M2	Early 1990s recession
8	1990M5-1992M3	-	Early 1990s recession
9	2008M5-2010M1	-	Global crisis and recession
10	2019M10-2020M4	-	COVID-19 recession

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Appendix 1.E: Three different approaches to generate a counterfactual recovery

According to King and Zeng (2006), the ceteris paribus condition for a credible counterfactual recovery requires that the difference between actual and counterfactual recovery be confined to only a single factor in order to measure the pure impact of the specific factor. Finding such a counterfactual recovery that satisfies this condition is hard, if not impossible. In practice, therefore, to generate a credible counterfactual recovery when there are other confounding factors, we need to control for the impact of all of the unwanted factors that are different between actual and counterfactual recoveries.[33] Thus, the main novelty in the counterfactual experiment is to find a credible counterfactual recovery (as we have done in the first method) that satisfies the ceteris paribus condition. Alternatively, controlling the impact of unwanted confounding factors in generating the counterfactual (as we have done in the second method) is the challenge in counterfactual analysis.

In the first method of this study, the counterfactual recovery is defined as the post-crisis fitted output per capita. We follow two approaches to estimate the fitted output per capita: Okun's law and the dynamic factor model (DFM). In the first approach, fitted output per capita is estimated by Okun's law as a measure for counterfactual recovery. Thus, this method answers the question of what the counterfactual recovery of output per capita would have been given the 2009–16 recovery of the unemployment rate. In the second approach, for the robustness check, we use a DFM to generate the counterfactual recovery to explore the counterfactual recovery of output given the 2009–16 recovery of cyclical factors that capture the dynamics of so many variables included in the DFM.

In the second method, we use the retrospective data points to define the counterfactual recovery as the average growth of three previous recoveries. To measure the shortfall, we answer this question: what would the counterfactual recovery of output per capita have been, given the recovery path of output per capita in the three previous recoveries? We need to construct counterfactual recovery in a time frame exactly analogous to that of actual recovery, meaning that the time frame (start, end, and duration) of the counterfactual path should be juxtaposed to the actual path. In this experiment, as we are interested in measuring the impact of declining trend growth on the slow recovery, we impose an assumption of constant trend growth among different recovery periods. Because the depth of each recession is different from the others (Stock and Watson, 2012; Fernald, 2017),[34] we must control for the depth to satisfy the ceteris paribus condition. Indeed, controlling the depth of recessions (as an unwanted confounding factor) is essential to capture the pure impact of the decline in trend growth.

[33] In the literature of counterfactual analysis, terms such as aspect, factor, and intervention are used interchangeably.
[34] The deeper the recession, the stronger the recovery above normal, is the expression mentioned by Bordo and Haubrich (2016), and Fernald et al. (2017).

In order to control for different recession depths, we use a trend-cycle decomposition based on Okun's law in the first approach and a DFM in the second approach to derive the shortfall of the cyclically adjusted trend and evaluate the pure impact of declining trend growth on slow recovery. The shortcoming of Okun's law is that it controls the depth of the recession by considering a single variable (unemployment) as the benchmark for recovery of the cyclical component. Also, the stability of Okun's law during the current recovery has been called into question by empirical studies (Owyang and Sekhposyan, 2012; Basu and Foley, 2013; Grant, 2018). To address these issues, we use a DFM to control the depth of the recession. The advantage of using a DFM for controlling the depth of recessions is that it uses the dynamics of so many variables as proxies of the cyclical component.

In the third method, we consider output growth as the sum of trend growth and cycle growth. We use the prospective data points to define the counterfactual recovery as the average growth of the forecasted series. In fact, we use a forecast based on a VAR model to construct the counterfactual path. To measure the shortfall, we answer the question of what the counterfactual recovery of output per capita would have been given that the trend growth was constant after the Great Recession. Since we are interested in measuring the impact of declining trend growth on the slow recovery, we freeze trend growth at the trough to impose an assumption of constant trend growth during the whole period of recovery. We indeed generate the counterfactual path as the sum of the constant trend growth and the cycle growth forecasted from the 2009 trough. However, because increasing the forecast horizon leads to an increase in forecast error of the cyclical component (an unwanted confounding factor), there is a limit to the length of the forecast horizon.

The forecast horizon is supposed to be short enough to guarantee a small enough forecast error. To fulfil this requirement, denoting the number of quarters during recovery by q and the number of steps ahead forecast by h, we consider forecasting with a rolling origin to conduct $(q-h+1)$ counterfactual analyses. Each counterfactual answers this question: what would the counterfactual recovery of output per capita have been, given the constant trend growth at the forecasting origin? Since we expect the decline in trend growth to hold during the whole period of recovery, a change in the forecasting origin must not change the result of a significant shortfall. Thus, we expect to estimate a significant shortfall for all $(q-h+1)$ counterfactual experiments.

Table 1.E: Different definitions of shortfall based on actual and counterfactual paths

N	Actual	Counterfactual	Intervention factor (Assumption)	Unwanted confounding factor(s)	Controlling by	Caveats
1.1	Current recovery	Fitted output per capita estimated by Okun's law	Stability of Okun's intercept and coefficient	--	--	Unemployment is the benchmark.
1.2	Current recovery	Common component of output estimated by the DFM	Stability of intercept and cyclical factor loadings	--	--	--
2.1	Current recovery	Three previous recoveries	Constant trend	Different depth of the recessions	cyclical adjustment based on Okun's law	Uncertainty of cyclical adjustment due to instability of Okun's law. Unemployment is the benchmark.
2.2	Current recovery	Three previous recoveries	Constant trend	Different depth of the recessions	cyclical adjustment based on DFM	Uncertainty of cyclical adjustment due to instability of cyclical factor loadings.
3-1	Current recovery	Forecasted series from recession trough	Constant trend	Different depths of the recession	Start forecasting from the trough	Limit on the forecast horizon due to large error in forecasting cycle.
3-2	A sample of current recovery	A matched sample of Forecasted series from the same starting point with actual	Constant trend	Different states of the economy at forecasting origin	Start forecasting from the same point that actual is started	Forecasting by rolling a short-horizon window from the trough of the recession to the end of the recovery period.

Appendix 1.F: Derivation of the difference version of Okun's law

Okun's law explains the relationship between the deviation of output from its potential output and the deviation of unemployment from natural rate. Eq. (1.F.1) is known as the "gap version of Okun's law," which specifies the deviation of output (output gap) as a linear regression of deviation of the unemployment rate:

$$x_t - x_t^* = \beta(U_t - U_t^*) + \varepsilon_t \tag{1.F.1}$$

Where x_t is the log of output, x_t^* is the log of long-run output trend that stands for potential output, U_t is the unemployment rate, U_t^* is the natural rate of unemployment, β is a coefficient known as the Okun's coefficient, and e_t is an error term.

By taking the first difference of the gap version of Okun's law, we derive:

$$\Delta x_t - \Delta x_t^* = \beta(\Delta U_t - \Delta U_t^*) + \Delta\varepsilon_t \tag{1.F.2}$$

Or, equivalently

$$y_t - y_t^* = \beta(\Delta U_t - \Delta U_t^*) + e_t \tag{1.F.3}$$

Where y_t denotes the growth of the log of output, y_t^* is the growth of potential output, ΔU_t is the change in unemployment rate, ΔU_t^* is the change in natural rate of unemployment, and e_t is the new error term.

By rearranging the terms in Eq. (1.F.3), we derive the trend-cycle version of Okun's law as follows:

$$y_t = y_t^* + \beta(\Delta U_t - \Delta U_t^*) + e_t \tag{1.F.4}$$

Where y_t^* is a time-varying intercept which corresponds to the growth of potential output and $\beta(\Delta U_t - \Delta U_t^*)$ represents the cyclical component (output gap). Alternatively, we can rearrange Eq. (1.F.3) and derive Eq. (1.4) presented in Section 1.3.1, re-specified below:

$$y_t = \underbrace{(y_t^* - \beta\Delta U_t^*)}_{\alpha_t} + \beta(\Delta U_t) + e_t \tag{1.F.5}$$

Where α_t is a time-varying intercept. Eq. (1.F.4) and Eq. (1.F.5) are estimable by using their state-space form (see Appendix 1.G).

Assuming that the growth of potential output (y_t^*) is constant, the Okun's coefficient (β) is stable, and the natural rate of unemployment is constant ($\Delta U_t^* = 0$), we derive the difference version of Okun's law:

$$y_t = \alpha + \beta(\Delta U_t) + e_t \tag{1.F.6}$$

In order to identify the determinants of the slow recovery, we relax the first two above assumptions by estimating Eq. (1.4) as explained in the main text. Also, we relax the assumption that says the natural rate of unemployment is constant by estimating Eq. (1.F.4). We use reasonable measures of the natural rate of unemployment (ΔU_t^*) such as NAIRU or long-term unemployment rate. For the latter, we perform Tukey's bi-weight filter on data with different bandwidths of 60, 80, and 100 quarters. We find the same results, indicating the robustness of the significant output shortfall (See Appendix 1.K).[35]

[35] NAIRU is an abbreviation for the non-accelerating inflation rate of unemployment, estimated based on the Phillips curve by the congressional budget office (CBO).

Appendix 1.G: State-space model and time-varying parameters

Okun's law with time-varying parameters is represented in a state-space from in Eq. (1.G.5) and Eq. (1.G.6), which can be estimated by using Kalman's (1960) filter.[36] For simplicity, we rewrite Eq. (1.13) when $p = 0$, and $q = 0$, and β is time-varying:

$$c_t = \beta_t(\Delta U_t) + e_t \tag{1.G.1}$$

Eq. (1.12) will be transformed into a regression model with time-varying parameters. Assuming that parameters follow autoregressive of order one, we have the following system of equations:

$$y_t = \mu_t + \beta_t(\Delta U_t) + e_t,\ e_t \sim i.i.d.N(0, \sigma_z^2) \tag{1.G.2}$$

$$\mu_{t+1} = \kappa + \varphi\,\mu_t + \varepsilon_t,\ \varepsilon_t \sim i.i.d.N(0, \sigma_\varepsilon^2) \tag{1.G.3}$$

$$\beta_{t+1} = \lambda + \rho\,\beta_t + \eta_t,\ \eta_t \sim i.i.d.N(0, \sigma_\eta^2) \tag{1.G.4}$$

Where y_t is the growth of log of output, z_t, η_t, ε_t are mutually independent error terms. Then the state-space representation of Eq. (1.G.2) to Eq. (1.G.4) has the following form:

$$y_t = \begin{bmatrix} 1 & \Delta U_t \end{bmatrix} \begin{bmatrix} \mu_t \\ \beta_t \end{bmatrix} + e_t \tag{1.G.5}$$

$$\begin{bmatrix} \mu_{t+1} \\ \beta_{t+1} \end{bmatrix} = \begin{bmatrix} \kappa \\ \lambda \end{bmatrix} + \begin{bmatrix} \varphi & 0 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \mu_t \\ \beta_t \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} \tag{1.G.6}$$

Where μ_t and β_t are state variables, Eq. (1.G.5) is an observation or measurement equation, and Eq. (1.G.6) is the transition equation.

Since structural break tests identify a single break for the Okun's coefficient and this coefficient is stable in each sub-regime, we do not need to estimate the model specified in Eq. (1.G.5) and Eq. (1.G.6). Instead, we consider a dummy variable (D_t) to capture the variation in the Okun's coefficient in the form of a structural break. Also, following (see, e.g., Clark, 1987, 1989; Crafts and Mills, 2017; Grant and Chan, 2017; Antolin-Diaz et al., 2017), we model the trend growth as a random walk process. This specification for the trend growth is equivalent to specifying the trend component as a random walk process with a stochastic drift. We, therefore, specify the state-space model in the following form:

$$y_t = \begin{bmatrix} 1 & \Delta U_t & \Delta U_t D_t \end{bmatrix} \begin{bmatrix} \mu_t \\ \beta \\ \delta \end{bmatrix} + e_t \tag{1.G.7}$$

$$[\mu_{t+1}] = [1][\mu_t] + [\varepsilon_t] \tag{1.G.8}$$

[36] See chapter 13 of Hamilton (1994), and chapter 3 of Kim and Nelson (1999) for Kalman filter.

Appendix 1.H: DFM with time-varying intercept

An alternative method to capture the declining trend growth in output (or output per capita) is to incorporate a time-varying intercept in the DFM as follows:

$$X_t = a_t + \Lambda F_t + e_t \tag{1.H.1}$$

Where a_t captures time-variation in the long-run trend of the variable of interest. The first strategy for estimation of Eq. (1.H.1) that has time-varying intercepts and cyclical factors is to use a Bayesian method presented by Antolin-Diaz et al. (2017). However, this method has two caveats: there is a limit on the number of factors considered since there are more parameters for estimation. In this sense, they have considered a single cyclical factor. They also presumed the stability of the cyclical factor loadings, which is not supported by the results of structural break tests in this study.

The second strategy is to apply Principal Component Analysis (PCA) on locally demeaned series rather than untransformed series that might contain the time-varying trend growth. As explained in Section 1.3.1.2, we follow a three-step estimation strategy. In the first step, based on Eq. (1.6), we estimate the cyclical factors by using PCA on all of the locally demeaned series. Then, according to Eq. (1.10), by treating the cyclical factors as data, we estimate the time-varying intercept, cyclical factor loadings, and the common component for each series. Eq. (1.H.2) and Eq. (1.H.3) specify the state-space model we use to estimate Eq. (1.10) by applying Kalman's (1960) filter:

$$X_{i,t} = \begin{bmatrix} 1 & F_{1,t} & \cdots & F_{r,t} & F_{1,t}D_t & \cdots & F_{r,t}D_t \end{bmatrix} \begin{bmatrix} \alpha_{i,t} \\ \Lambda_{i1} \\ \vdots \\ \Lambda_{ir} \\ \Delta_{i1} \\ \vdots \\ \Delta_{ir} \end{bmatrix} + e_{i,t} \tag{1.H.2}$$

$$\left[\alpha_{i,t+1}\right] = [1]\left[\alpha_{i,t}\right] + [\varepsilon_t] \tag{1.H.3}$$

Where Eq. (1.H.2) is an observation equation, Eq. (1.H.3) is the transition equation, and $\alpha_{i,t}$ is the state variable and stands for the time-varying trend growth. In this setup, $X_{i,t}$ denotes the i^{th} time series, Λ_{ij} is a cyclical factor loading associated with the j^{th} cyclical factor ($F_{j,t}$). In addition, Δ_{ij} captures the change in cyclical factor loading after the break, and D_t is a dummy variable that equals one after the structural break and zero otherwise. In the third step, we finally measure the shortfall by comparing the series with its estimated common component.

In the third strategy, we can use PCA on the series that are not locally demeaned in the initial step. However, in contrast to the cyclical factors in Eq. (1.6) to Eq. (1.11), the estimated factors in this way contain the variation in trend growth and capture the dynamics of the common trend and cycle together. As a result, unlike the second strategy, this alternative strategy is unable to distinguish between the contribution of the instability of the intercept related to a declining trend growth and the instability of the cyclical factor loadings related to the unusual sluggish recovery of the cycle.

This motivates us to apply PCA to the locally demeaned series to derive the cyclical factors rather than total factors. It is worth noting that the shortfall estimated by total factors that contain the dynamics of common trend growth (the third strategy) is very similar to the shortfall estimated by the second strategy.

Appendix 1.I: Growth-accounting decomposition

To establish the main contributors to the slow recovery, we use a growth-accounting decomposition. Considering the Cobb-Douglas production function, we decompose the growth of output per capita ($\frac{Y_t}{Pop_t}$) into the change in total factor productivity (TFP), capital input per capita, and labour input per capita as follows:[37]

$$\Delta log\left(\frac{Y_t}{Pop_t}\right) = \Delta logTFP_t + \alpha_t\, \Delta log\left(\frac{K_t}{Pop_t}\right) + (1-\alpha_t)\Delta log\left(\frac{LQ_t.Hours_t}{Pop_t}\right) \qquad (1.I.1)$$

Where TFP_t is the total factor productivity, α_t is the capital share of production, K_t represents the capital input, LQ_t is the labour quality that can be measured by wages, and $Hours_t$ is the total hours of work. By expanding $Hours_t$ in the form of Eq. (1.I.2), we can examine the contribution of the employment rate ($\frac{Employment_t}{LaborForce_t}$) as well as labour force participation ($\frac{LaborForce_t}{Pop_t}$) to the slow recovery:

$$\frac{Hours_t}{Pop_t} = \frac{Hours_t}{Employment_t}\frac{Employment_t}{LaborForce_t}\frac{LaborForce_t}{Pop_t} \qquad (1.I.2)$$

As an alternative decomposition, we can specify output per hour in the following form:

$$\Delta log\left(\frac{Y_t}{Hour_t}\right) = \frac{\Delta logTFP_t}{1-\alpha_t} + \frac{\alpha_t}{1-\alpha_t}\, \Delta log\left(\frac{K_t}{Y_t}\right) + \Delta log(LQ_t) \qquad (1.I.3)$$

By using the growth-accounting decompositions presented in Eq. (1.I.1) to Eq. (1.I.3), we can obtain the contribution of each element on the right hand side of the equations to the slow recovery of output per capita and output per hour.

[37] See the second part of Appendix 1.I, for derivation of Eq. (1.I.1) and Eq. (1.I.3).

Appendix 1.I (continued): Derivation of growth-accounting decomposition

To derive Eq. (1.I.1), consider the Cobb-Douglas production function:

$$Y_t = TFP_t {K_t}^{\alpha_t} L_t^{1-\alpha_t} \tag{1.I.4}$$

Where TFP_t is the total factor productivity, K_t is capital input, L_t is labour input, and α_t is the capital share of production. By taking the natural log and differencing of Eq. (1.I.4), we get:

$$\Delta logY_t = \Delta logTFP_t + \alpha_t \, \Delta logK_t + (1-\alpha_t)\Delta logL_t \tag{1.I.5}$$

We can decompose the labour input into the labour quality, average hours of work, and population in the following form:

$$\Delta logY_t = \Delta logTFP_t + \alpha_t \, \Delta logK_t + (1-\alpha_t)\Delta log\left(LQ_t.\frac{Hours_t}{Pop_t}.Pop_t\right) \tag{1.I.6}$$

Since the log of product is the sum of the logs, we re-write the third term on the right hand side of the Eq. (1.I.6) to get:

$$\Delta logY_t = \Delta logTFP_t + \alpha_t \, \Delta logK_t + (1-\alpha_t)\Delta log\left(LQ_t.\frac{Hours_t}{Pop_t}\right) + (1-\alpha_t)\Delta logPop_t \tag{1.I.7}$$

Finally, by rearranging the fourth term on the right hand side of the above, we derive Eq. (1.I.1) as the first growth-accounting decomposition used in this study, presented below:

$$\Delta log\left(\frac{Y_t}{Pop_t}\right) = \Delta logTFP_t + \alpha_t \, \Delta log\left(\frac{K_t}{Pop_t}\right) + (1-\alpha_t)\Delta log\left(\frac{LQ_t.Hours_t}{Pop_t}\right) \tag{1.I.8}$$

To derive Eq. (1.I.3), simplify Eq. (1.I.6) into an alternative form:

$$\Delta logY_t = \Delta logTFP_t + \alpha_t \, \Delta logK_t + (1-\alpha_t)\Delta log(LQ_t.Hours_t) \tag{1.I.9}$$

By rearranging the third term in the right hand side of the Eq. (1.I.9), and subtracting $\alpha_t \Delta logY_t$ from both hand side, we derive:

$$(1-\alpha_t)\Delta log\left(\frac{Y_t}{Hour_t}\right) = \Delta logTFP_t + \alpha_t \, \Delta log\left(\frac{K_t}{Y_t}\right) + (1-\alpha_t)\Delta log(LQ_t) \tag{1.I.10}$$

Finally, by dividing Eq. (1.I.10) by $(1-\alpha_t)$, we get Eq. (1.I.3) as the second growth-accounting decomposition used in this paper:

$$\Delta log\left(\frac{Y_t}{Hour_t}\right) = \frac{\Delta logTFP_t}{1-\alpha_t} + \frac{\alpha_t}{1-\alpha_t} \, \Delta log\left(\frac{K_t}{Y_t}\right) + \Delta log(LQ_t) \tag{1.I.11}$$

Appendix 1.J: Forecasting with a Dynamic Factor Model (DFM)

To capture the slow recovery, Fernald et al. (2017) use forecasting of the cyclical component for a long horizon of about 28 quarters. They consider output growth as the sum of trend growth and cycle growth. To generate the counterfactual recovery, they made an assumption that the trend growth after the trough of the recession is fixed and equal to its value at the trough (2009Q2). They forecasted the cycle growth by using a VAR with order of $P = 4$ with a fixed origin. Then, by comparing the counterfactual growth (the sum of fixed trend growth and forecasted cycle growth) with the actual growth within the whole recovery period from 2009Q3 to 2016Q2, they measure the shortfall and test for the slow recovery. In this setup, freezing the trend growth of a series at its trough value is an assumption made by the authors to check whether the derived shortfall, the difference between the counterfactual and actual recovery, is significant or not. If the shortfall is significant, the assumption of constant trend growth is incorrect, and thus, the trend growth is declining.

However, the above method does not satisfy the ceteris paribus condition. Because as the number of steps ahead forecast increases, the standard error of the forecast also increases, and thus, the forecast error becomes a confounding factor whose unavoidable effect interferes with the measurement of the shortfall. As a result, the shortfall in this method is highly sensitive to the length of the forecast horizon.[38]

To tackle this problem, we consider a forecast with a rolling origin starting from the recession trough (2009Q2) with a short forecast horizon to construct reliable counterfactual paths and compare each of them with its counterpart in actual paths. Because DFM outperforms small models in forecasting for both the U.S. and the U.K. (Eickmeier and Ziegler, 2008; Stock and Watson, 2016), we apply a DFM to forecast cycle growth of series in constructing counterfactual recovery.[39] The basic setup of this model is exactly the same as that explained in Section 1.3.1.2. All of the variables included in the factor model are locally demeaned by Tukey's bi-weight filter since the DFM is designed for forecasting the cyclical components. We call the locally demeaned series the cyclical component of the series.

We conduct the following procedure similar to the method applied by Fernald et al. (2017). First, we estimate the cyclical factors using principal component analysis (PCA) on locally demeaned time series over the whole sample from 1960Q1 to 2016Q4 for the U.S. and from 1971Q1 to 2016Q4 for the U.K. Let us represent the forecast origin by τ, and the forecast horizon by h. In this setting, we

[38] In particular, the estimated shortfall varies with the change in the forecast horizon from 1 to 24 (See Appendix 1.L). Meanwhile, nothing happened to the output trend growth during these 24 quarters that could shift the status of the recovery from the slow to the rapid or vice versa.

[39] We forecast the cycle growth of each series based on a DFM, and for the trend component, we assume that trend growth is constant over the whole period of recovery.

denote the estimated cyclical factors from 1984Q1 to forecast origin by $\hat{F}_t^{84-\tau}$. In a particular case, if we set $\tau = 2009Q2$ and $h = 28$, we derive a nested model presented by Fernald et al. (2017), which from a fixed forecast origin at the trough, forecasts the long period of recovery consists of 28 quarters by using the pre-crisis cyclical factors ($\hat{F}_t^{84-09}$). Second, to derive the matrix of factor loadings ($\hat{\Lambda}^{84-\tau}$), the cyclical factor loadings ($\Lambda_{ij}^{84-\tau}$) are estimated by regressing the cyclical component of each series on the cyclical factors ($\hat{F}_t^{84-\tau}$) for the sample from 1984Q1 to forecast origin.[40]

Third, treating cyclical factors as the data, we consider that the cyclical factors evolve according to a VAR model with order P:

$$\begin{bmatrix} F_{1,t} \\ F_{2,t} \\ \vdots \\ F_{r,t} \end{bmatrix} = \phi(L) \begin{bmatrix} F_{1,t} \\ F_{2,t} \\ \vdots \\ F_{r,t} \end{bmatrix} + \eta_t \tag{1.J.1}$$

where η_t is a vector of serially uncorrelated white noise error terms. In this setup, $\phi(L) = \phi^1 L + \cdots + \phi^p L^p$ is an $r \times r$ matrix polynomial, whose elements are scalar polynomials in the lag operator L. As shown in Eq. (1.J.2), ϕ^l is a matrix containing the autoregressive coefficients $\varphi_{i,j}^l$ that identify the consequence of a change in the j^{th} factor l periods ago on i^{th} factor at current time. We run this VAR model over the sample from 1984Q1 to the forecast origin τ to estimate the parameters. We thereby forecast the post-crisis cyclical factors after the forecast origin denote by $\hat{F}_{t|\tau}$.

$$\phi^l = \begin{bmatrix} \varphi_{1,1}^l & \varphi_{1,2}^l & \cdots & \varphi_{1,r}^l \\ \varphi_{2,1}^l & \varphi_{2,2}^l & \cdots & \varphi_{2,r}^l \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_{r,1}^l & \varphi_{r,2}^l & \cdots & \varphi_{r,r}^l \end{bmatrix} \tag{1.J.2}$$

Fourth, we forecast the post-crisis common cyclical component of the series according to the Eq. (1.J.3) stated below:

$$\hat{X}_{t|\tau} = \hat{\Lambda}^{84-09} \hat{F}_{t|\tau} \tag{1.J.3}$$

where $t = \tau + 1, \tau + 2, \ldots, \tau + h$ and $\hat{X}_{t|\tau}$ is the forecast of the cycle growth of series from the origin. For example, if we consider the forecast origin $\tau = 2010Q1$ and the forecast horizon $h = 8$, the forecast of cyclical factors and cycle growth are denoted by $\hat{F}_{t|\tau=2010\,Q1}$ and $\hat{X}_{t|\tau=2010Q1}$ for $t = 2010Q2, \ldots, 2010Q1$.

For simplicity of notation, we represent the forecast of the cycle growth for a series by $\hat{c}_{t|\tau}$ and the actual value of the cycle growth is denoted by c_t. In addition, $\bar{\mu}_{t|\tau}$ stands for the fixed trend growth

[40] In this setting, cyclical factor loadings capture the relationship between the cyclical component of each series and cyclical factors.

after the forecast origin, which is assumed to be equal to the trend growth at the origin (τ), and μ_t is the actual trend growth. We can find the shortfall as the difference between counterfactual growth ($\bar{\mu}_{t|\tau} + \hat{c}_{t|\tau}$) and actual growth ($y_t$) in the following form:

$$\underbrace{\left(\bar{\mu}_{t|\tau} + \hat{c}_{t|\tau}\right)}_{Counterfactual} - \underbrace{y_t}_{Actual} = \left(\bar{\mu}_{t|\tau} - \mu_t\right) + \left(\mu_t + \hat{c}_{t|\tau}\right) - y_t \tag{1.J.4}$$

Considering that $\hat{c}_{t|\tau} = c_t + \varepsilon_t$ where ε_t is the forecast error, Eq. (1.J.4) is equal to:

$$shortfall_t = \left(\bar{\mu}_{t|\tau} - \mu_t\right) + \varepsilon_t + \underbrace{(\mu_t + c_t) - y_t}_{z_t} \tag{1.J.5}$$

Eq. (1.J.5) is equal to white noise (z_t) conditional on (1) the accuracy of the assumption suggesting a constant trend growth ($\mu_t \approx \bar{\mu}_{t|\tau}$), and (2) the precision of forecasting the cycle growth ($\varepsilon_t \approx 0$). On the other hand, Eq. (1.J.5) is equal to Eq. (1.J.6) if one of the two conditions does not hold:

$$shortfall_t = \left(\bar{\mu}_{t|\tau} - \mu_t\right) + \varepsilon_t + z_t \tag{1.J.6}$$

where $t = \tau + 1, \tau + 2, \ldots, \tau + h$. If we set $\tau = 2009Q2$ and $h = 28$, we derive a nested model presented by Fernald et al. (2017). If the assumption of constant trend growth is accurate and the forecasting of the cycle growth is precise enough such that $\varepsilon_t \approx 0$, the above shortfalls must be mean-zero. However, if the shortfalls are not statistically different from zero, conditional on the precision of forecasting the cycle growth, the assumption of constant trend growth does not hold, meaning that the trend components of the series vary after the recession trough. To test whether the shortfalls corresponding to the origin τ are significant or not, we simply calculate the average of the shortfalls over h horizons as follows:

$$shortfall_{\tau+h|\tau} = \frac{1}{h}\sum_{t=\tau+1}^{\tau+h}\left[\left(\bar{\mu}_{t|\tau} + \hat{c}_{t|\tau}\right) - y_t\right] \tag{1.J.7}$$

This method, however, is reliable only if the error in forecasting the cycle growth is small ($\varepsilon_t \approx 0$). Normally, there should be a limit on the length of the forecast horizon (h) because as the number of steps ahead forecast increases, the forecast error also increases and ε_t will no longer be zero. Since Fernald et al. (2017) use this method for a long horizon of $h = 28$ quarters, their method is under question.[41]

To tackle this issue, we consider forecasting with a rolling origin and with a short forecast horizon to generate reliable counterfactual paths. Consider the number of quarters during the recovery is $q = 28$. By rolling the forecast origin from the trough (2009Q2) up to the h quarters before the end of

[41] They forecast the cycle component from $2009Q3$ to $2016Q2$ with a fixed origin at $\tau = 2009Q2$ for a long horizon of 28 quarters. They do not consider $\hat{c}_{t|\tau}$ in their notation as they assume the DFM performs fantastically well in forecasting cycle growth for a long horizon. This means that they have implicitly presumed that $\varepsilon_t \approx 0$ that is an unsubstantiated assumption.

the sample (2016Q2), we derive $(q-h+1)$ counterfactual paths. By comparing each counterfactual path with its actual path within the same horizon, we calculate $(q-h+1)$ averages of shortfalls based on Eq. (1.J.7). Finally, we take average of these $(q-h+1)$ averages of shortfalls:

$$shortfall = \frac{1}{q-h+1} \sum_{i=\tau}^{i=\tau+q-h} shortfall_{\tau+h|\tau} \tag{1.J.8}$$

In this setup, the forecast origin τ begins in 2009Q2 and ends in h quarters before 2016Q2. We choose several forecast horizons of $h = 1,\ 2,\ 3, 4, 8, 12$ for robustness check. Similar to Fernald et al. (2017), the trend is frozen at the forecast origin, but since the origin is updated on a rolling basis, the trend is also updated to use all the information available at the forecast origin. Since we expect a slowdown in the trend component, we examine whether the shortfall is significant or not. One can deem the significant shortfall as systematic forecast error, which means that persistently over-predicted counterfactual paths are caused by the time-varying declining trend growth. The results derived by this method, presented in the Appendix 1.L, support the contribution of the declining trend growth to the slow recovery in the U.S.

Appendix 1.K: Additional Figures

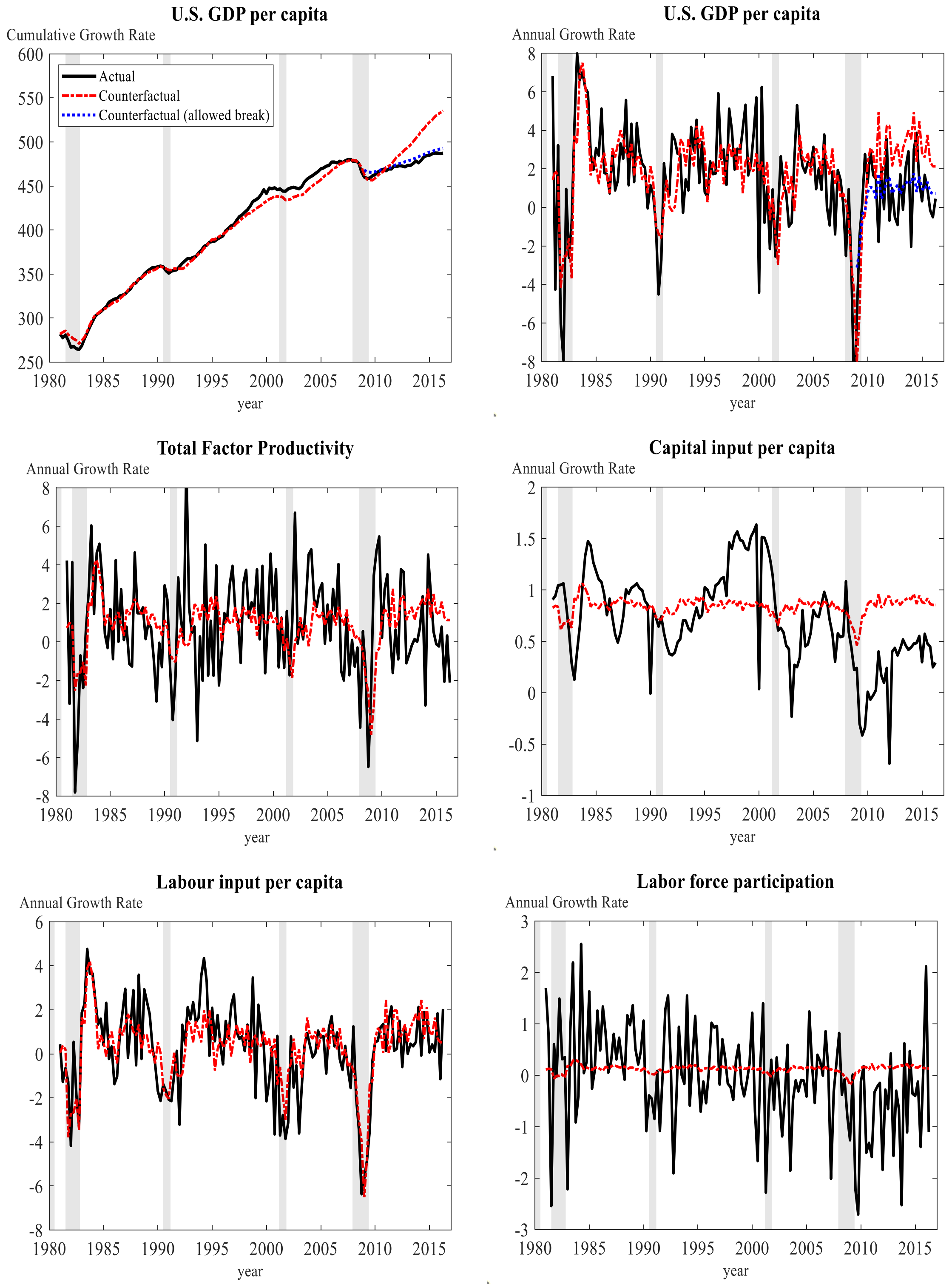


Figure 1.K.1: Shortfall of the U.S. post-crisis recovery estimated by Okun's law with $p = 0$

Notes:

(1) The red dashed lines represent counterfactual recoveries. They are estimated by Okun's law with $p = 0$ leads and lags over the period 1981–2009.

(2) The blue dotted line, depicted for the top panels, is the fitted output per capita estimated by Okun's law, which allows for a structural break in the Okun's law intercept and coefficient in 2009Q1 to eliminate the shortfall.

(3) The shaded areas are the NBER recession dates.

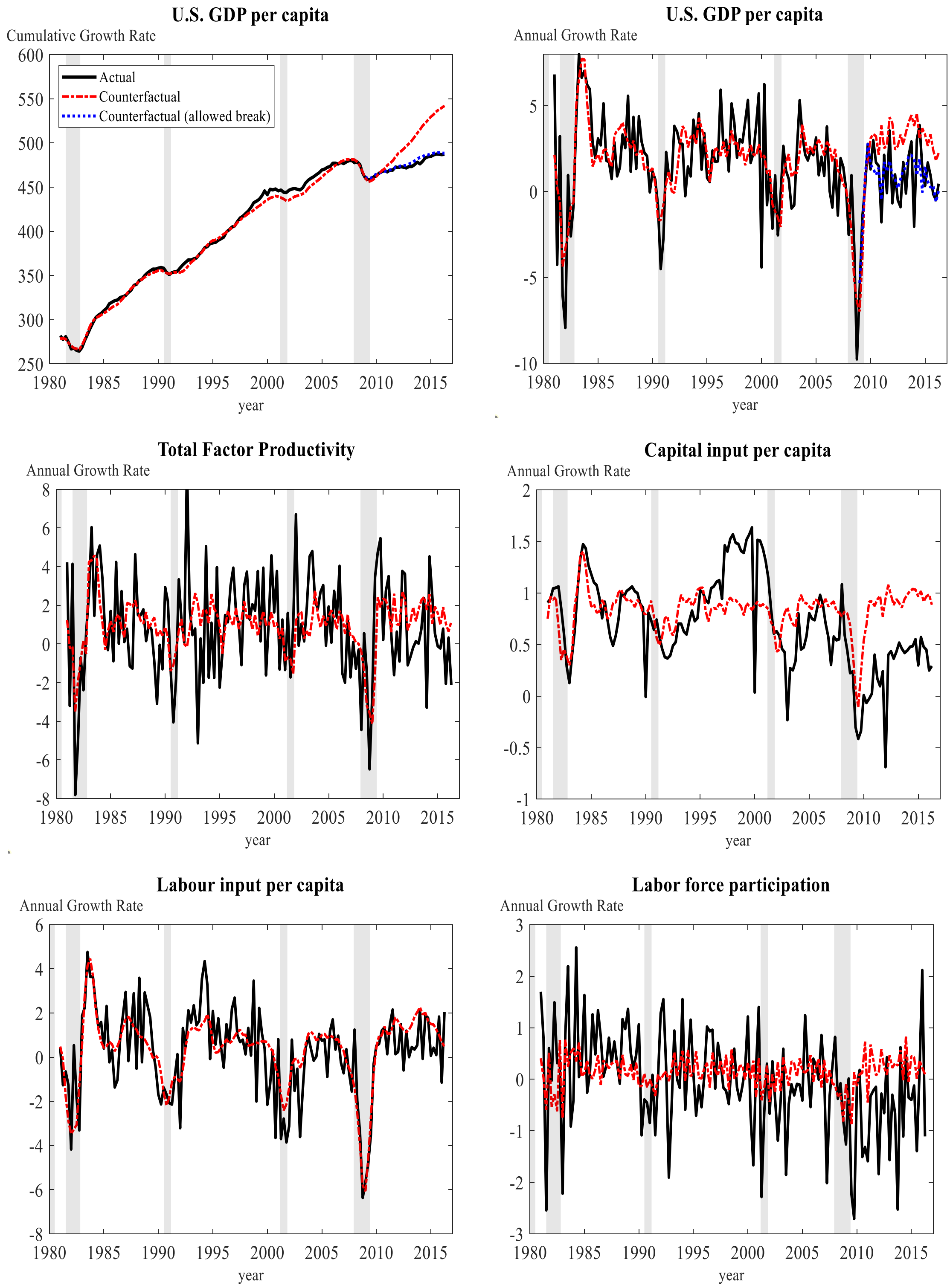


Figure 1.K.2: Shortfall of the U.S. post-crisis recovery estimated by Okun's law with $p = 2$

Notes:
(1) The red dashed lines represent counterfactual recoveries. They are estimated by Okun's law with $p = 2$ leads and lags over the period 1981–2009.
(2) The blue dotted line, depicted for the top panels, is the fitted output per capita estimated by Okun's law, which allows for a structural break in the Okun's law intercept and coefficient in 2009Q1 to eliminate the shortfall.
(3) The shaded areas are the NBER recession dates.

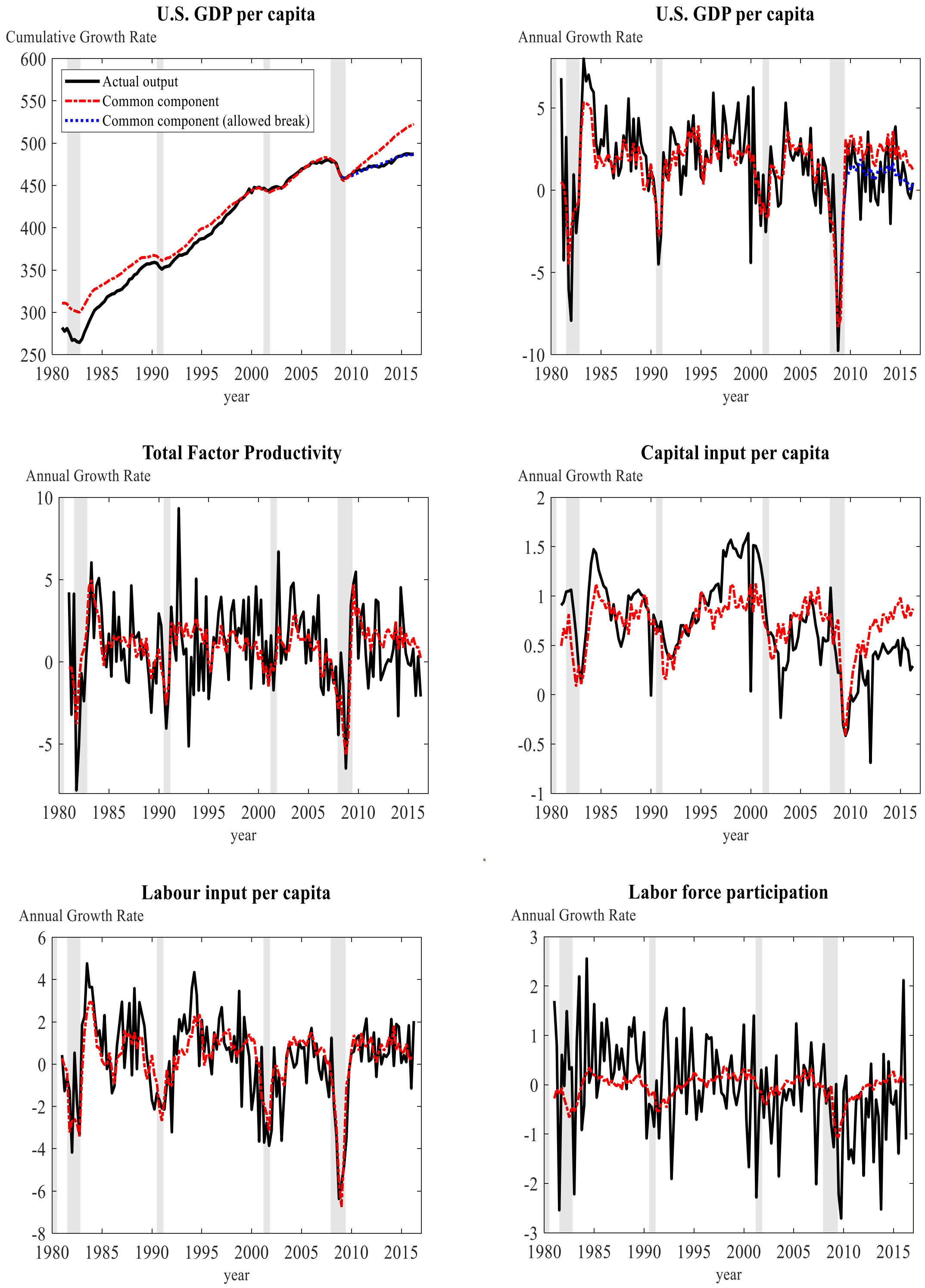


Figure 1.K.3: Shortfall of the U.S. post-crisis recovery estimated by the DFM over the sample 1981–2016

Notes:

(1) The red dashed lines represent counterfactual recoveries. They are estimated by the DFM over the sample period 1981–2016, assuming that the factor loadings are stable. Shortfalls are defined as the difference between actual and counterfactual recoveries.

(2) The blue dotted line, depicted for the top panels, is the fitted output per capita estimated by the DFM, which allows for a structural break in the intercept and cyclical factor loadings in 2009Q1 to eliminate the shortfall.

(3) The shaded areas are the NBER recession dates.

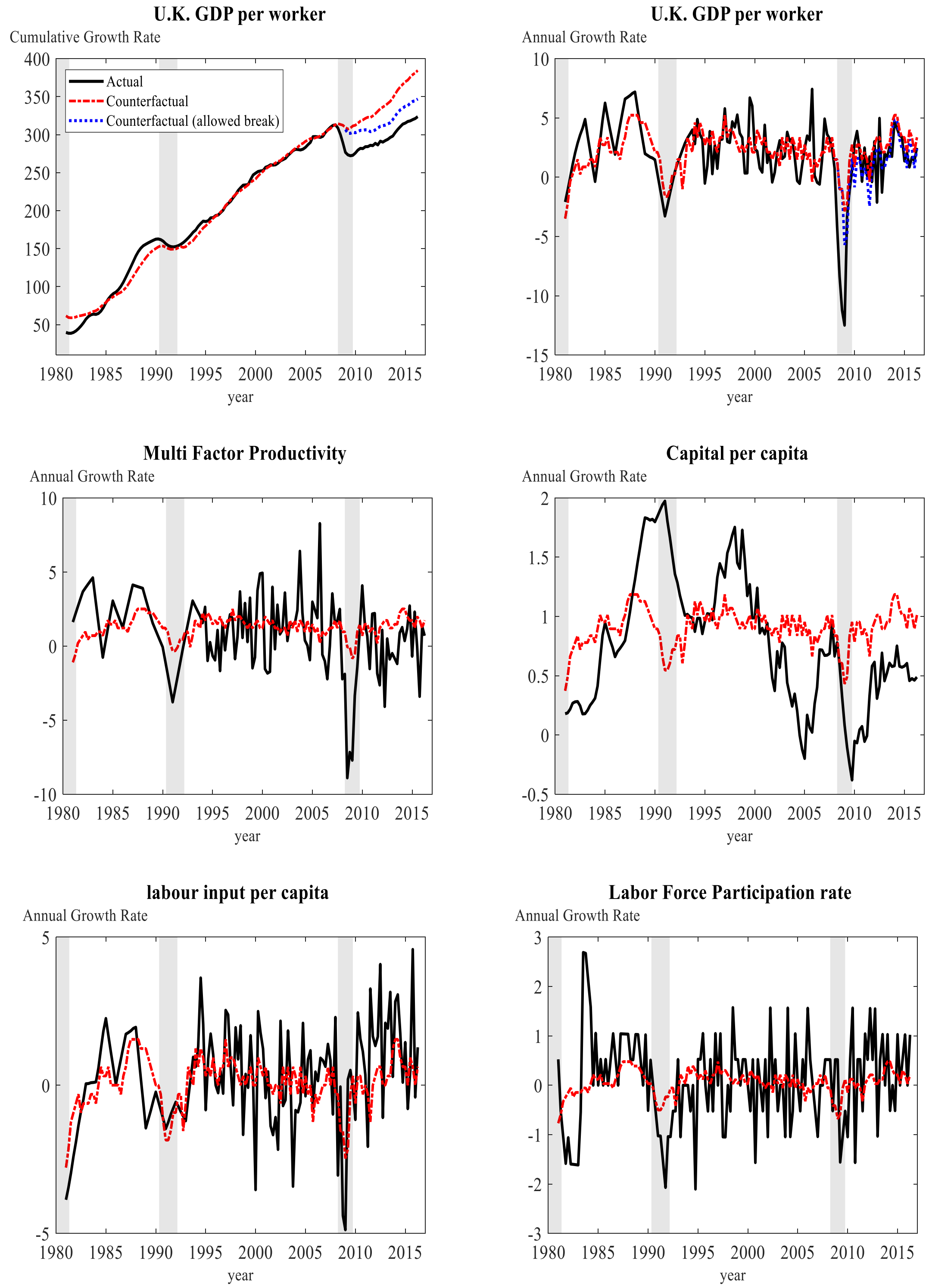


Figure 1.K.4: Shortfall of the U.K. post-crisis recovery estimated by Okun's law with $p = 0$

Notes:

(1) The red dashed lines represent counterfactual recoveries. They are estimated by Okun's law with $p = 0$ leads and lags over the period 1981–2009.

(2) The blue dotted line, depicted for the top panels, is the fitted output per capita estimated by Okun's law, which allows for a structural break in the Okun's law intercept and coefficient in 2009Q1 to eliminate the shortfall.

(3) The shaded areas are the ECRI recession dates.

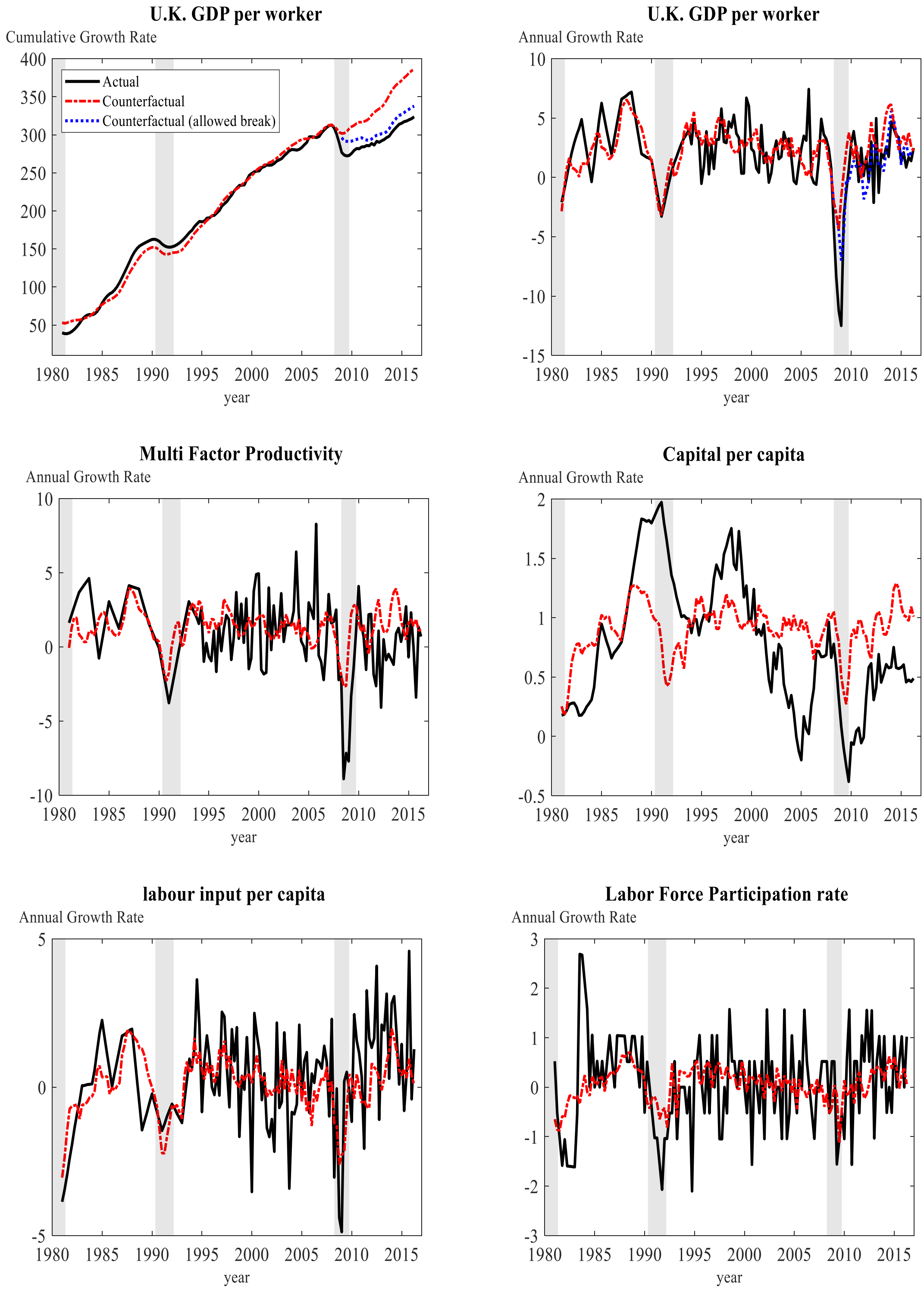


Figure 1.K.5: Shortfall of the U.K. post-crisis recovery estimated by Okun's law with $p = 2$

Notes:

(1) The red dashed lines represent counterfactual recoveries. They are estimated by Okun's law with $p = 2$ leads and lags over the period 1981–2009.

(2) The blue dotted line, depicted for the top panels, is the fitted output per capita estimated by Okun's law, which allows for a structural break in the Okun's law intercept and coefficient in 2009Q1 to eliminate the shortfall.

(3) The shaded areas are the ECRI recession dates.

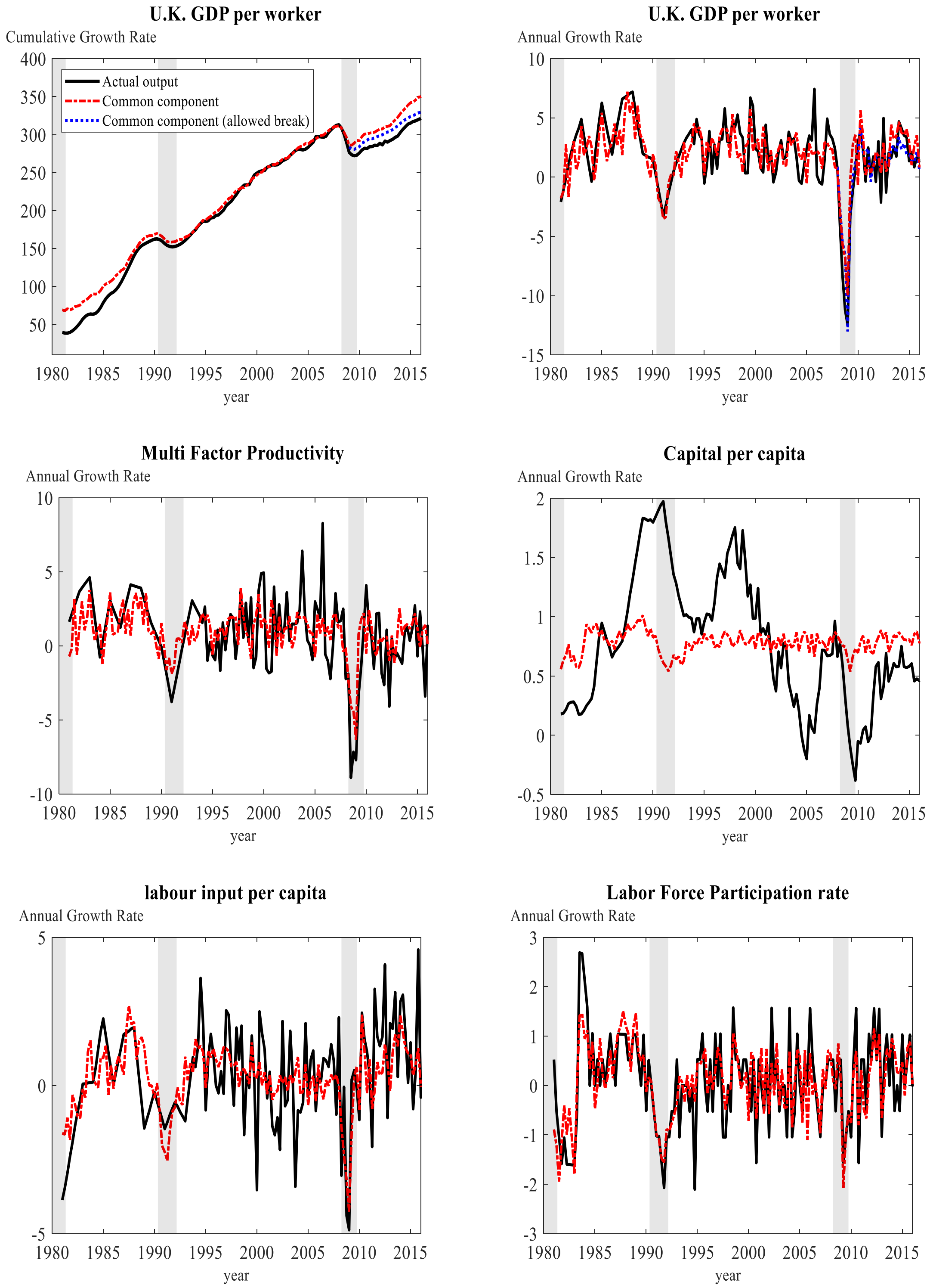


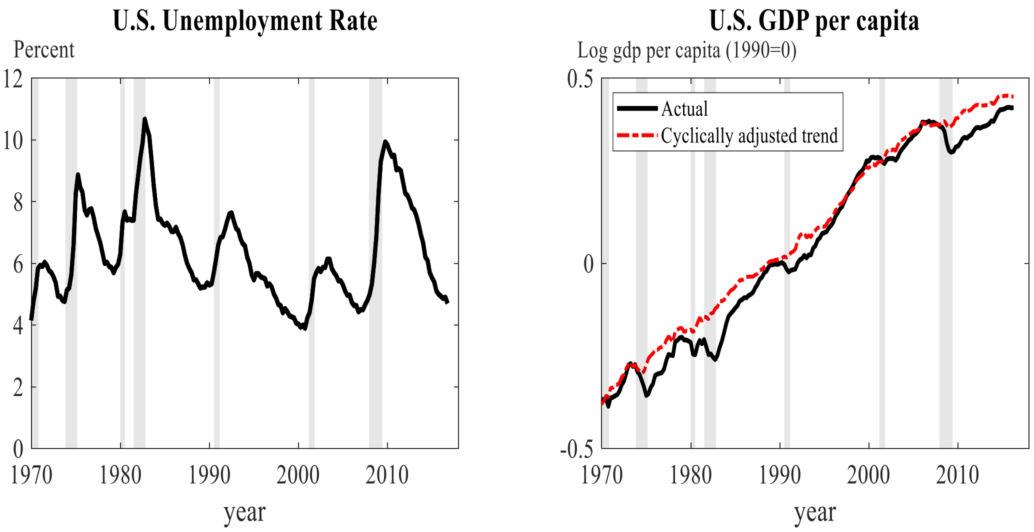
Figure 1.K.6: Shortfall of the U.K. post-crisis recovery estimated by the DFM over the sample 1981–2016

Notes:

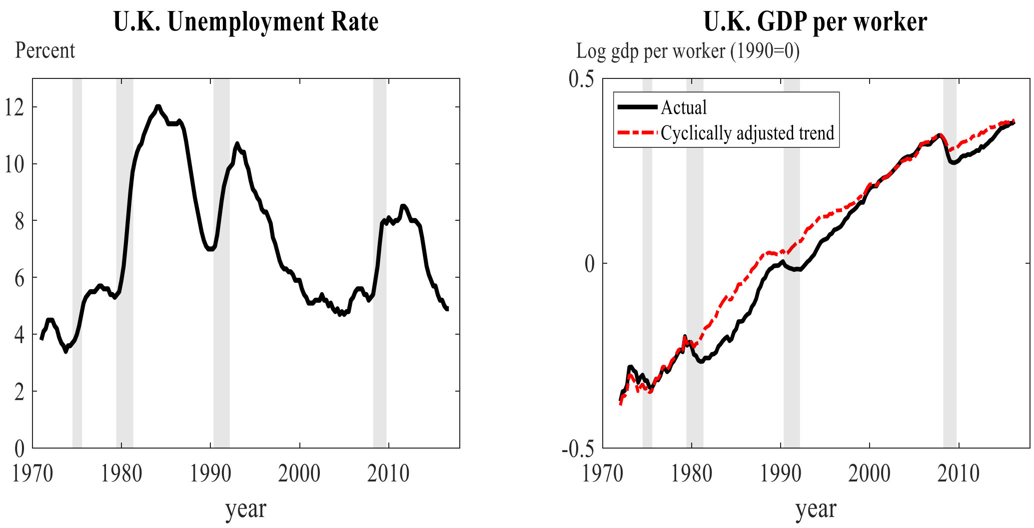
(1) The red dashed lines represent counterfactual recoveries. They are estimated by the DFM over the sample period 1981–2016, assuming that the factor loadings are stable. Shortfalls are defined as the difference between actual and counterfactual recoveries.

(2) The blue dotted line, depicted for the top panels, is the fitted output per capita estimated by the DFM, which allows for a structural break in the intercept and cyclical factor loadings in 2009Q1 to eliminate the shortfall.

(3) The shaded areas are the ECRI recession dates.



(a) U.S. unemployment and cyclically adjusted output

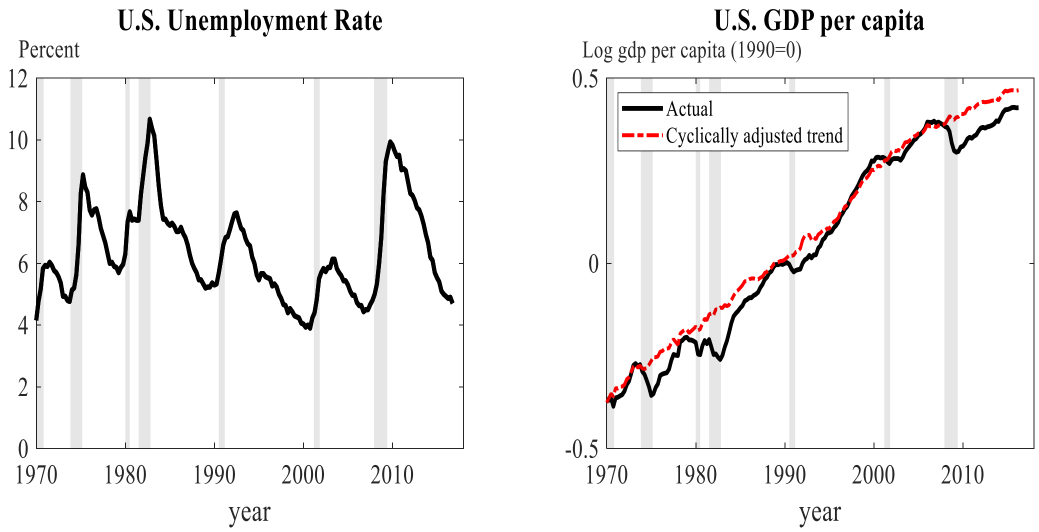


(b) U.K. unemployment and cyclically adjusted output

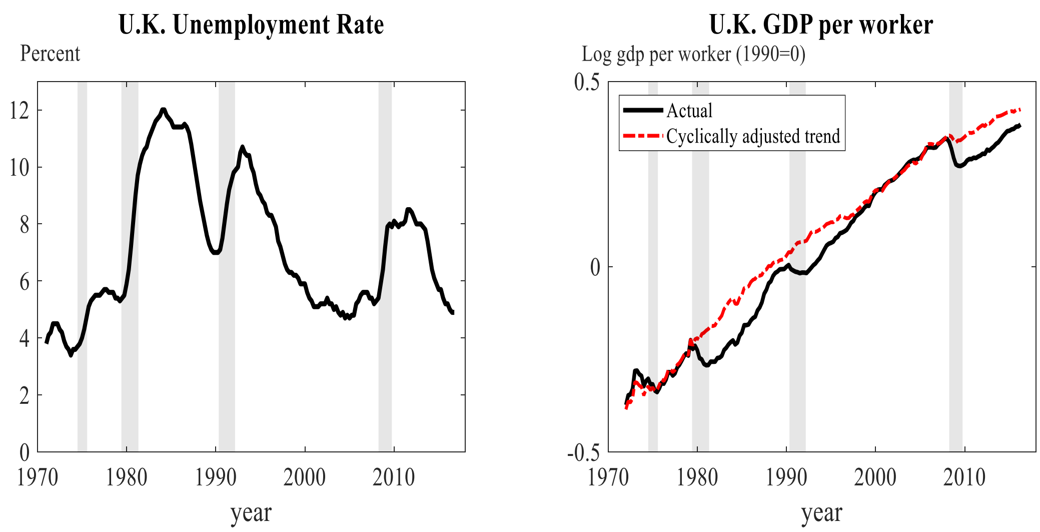
Figure 1.K.7: Output trend component estimated by a trend-cycle decomposition based on Okun's law

Notes:

(1) The right panels show the level (i.e., the cumulative growth rate) of the U.S. GDP per capita and the U.K. GDP per worker (population aged between 16 and 64), both normalized to be zero in 1990.
(2) The red dashed line is the cyclically adjusted trend estimated by Okun's law with $p = 0$ leads and lags over the sample period of 1971 to 2016, where a structural break in the Okun's coefficient in 2009Q1 is allowed for.
(3) The shaded areas are the NBER and ECRI recession dates for the U.S. and the U.K, respectively.



(a) U.S. unemployment and cyclically adjusted output

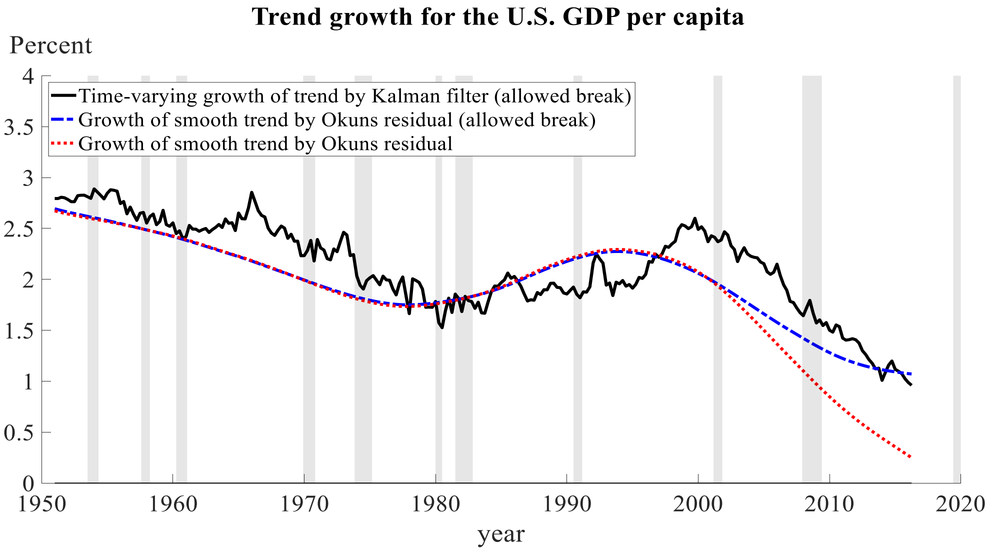


(b) U.K. unemployment and cyclically adjusted output

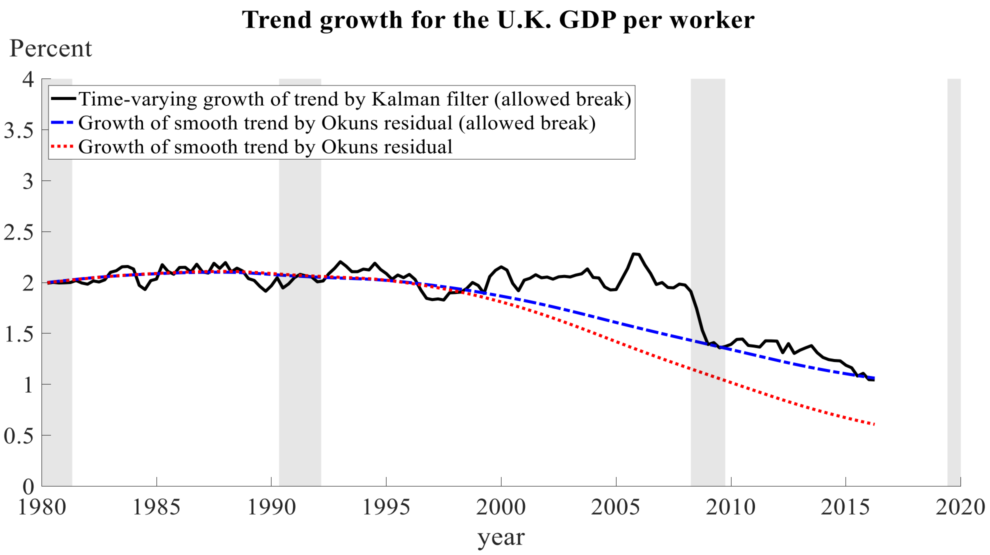
Figure 1.K.8: Output trend component estimated by a trend-cycle decomposition based on Okun's law

Notes:

(1) The right panels show the level (i.e., the cumulative growth rate) of the U.S. GDP per capita and the U.K. GDP per worker (population aged between 16 and 64), both normalized to be zero in 1990.
(2) The red dashed line is the cyclically adjusted trend estimated by Okun's law with $p = 2$ leads and lags over the sample period of 1971 to 2016, where a structural break in the Okun's coefficient in 2009Q1 is allowed for.
(3) The shaded areas are the NBER and ECRI recession dates for the U.S. and the U.K, respectively.



(a) The growth of the U.S. potential output

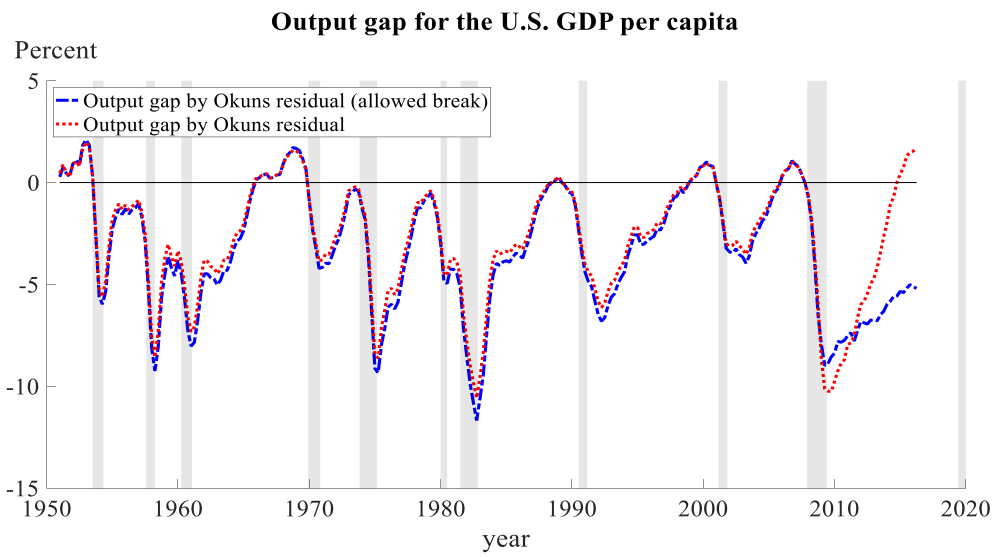


(b) The growth of the U.K. potential output

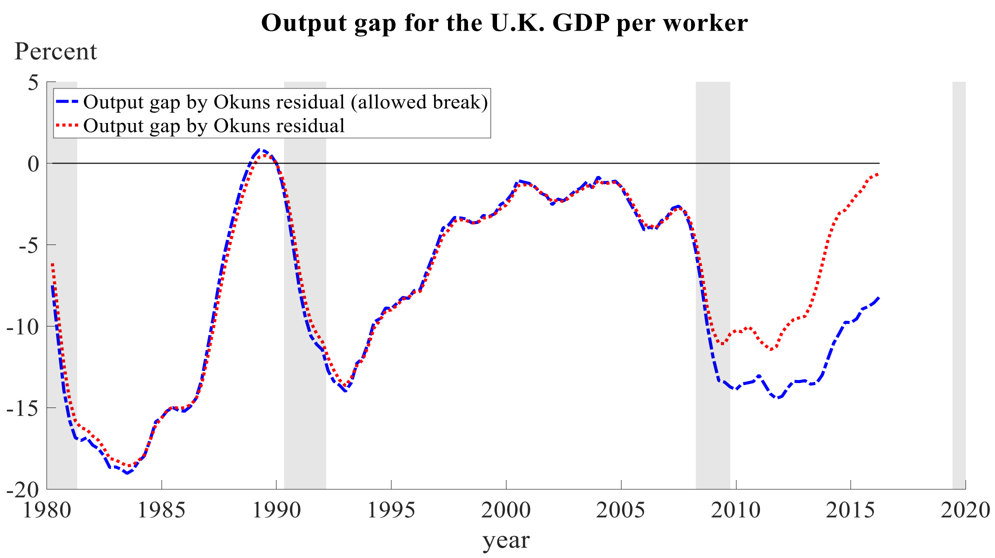
Figure 1.K.9: The annualized growth of potential output (trend component)

Notes:

(1) The black solid line is the Kalman's (1960) filter estimation of the growth of potential output by using Okun's law while a structural break in the Okun's coefficient in 2009Q1 is allowed for. The blue dashed line and the red dotted line are the growth of potential output (for GDP per capita) estimated as the Okun's law residuals with a break in the Okun's coefficient in 2009Q1, and without including a structural break, respectively.
(2) The estimation period is from 1951 to 2016 for the U.S. and from 1981 to 2016 for the U.K. We consider an Okun's law with $p = 2$ and a Tukey's bi-weight filter with a bandwidth of 60 quarters.
(3) The shaded areas are the NBER and ECRI recession dates for the U.S. and the U.K, respectively.



(a) The U.S. cyclical component



(b) The U.K. cyclical component

Figure 1.K.10: Output cyclical component estimated by a trend-cycle decomposition based on Okun's law

Notes:

(1) The blue dashed line and the red dotted line are the estimations of the output gap (for GDP per capita) using Okun's law regression with a break in the Okun's coefficient in 2009Q1 and without including a structural break, respectively.
(2) The estimation period is from 1951 to 2016 for the U.S. and from 1981 to 2016 for the U.K. We consider an Okun's law with $p = 2$ leads and lags and a Tukey's bi-weight filter with a bandwidth of 60 quarters.
(3) The shaded areas are the NBER and ECRI recession dates for the U.S. and the U.K, respectively.

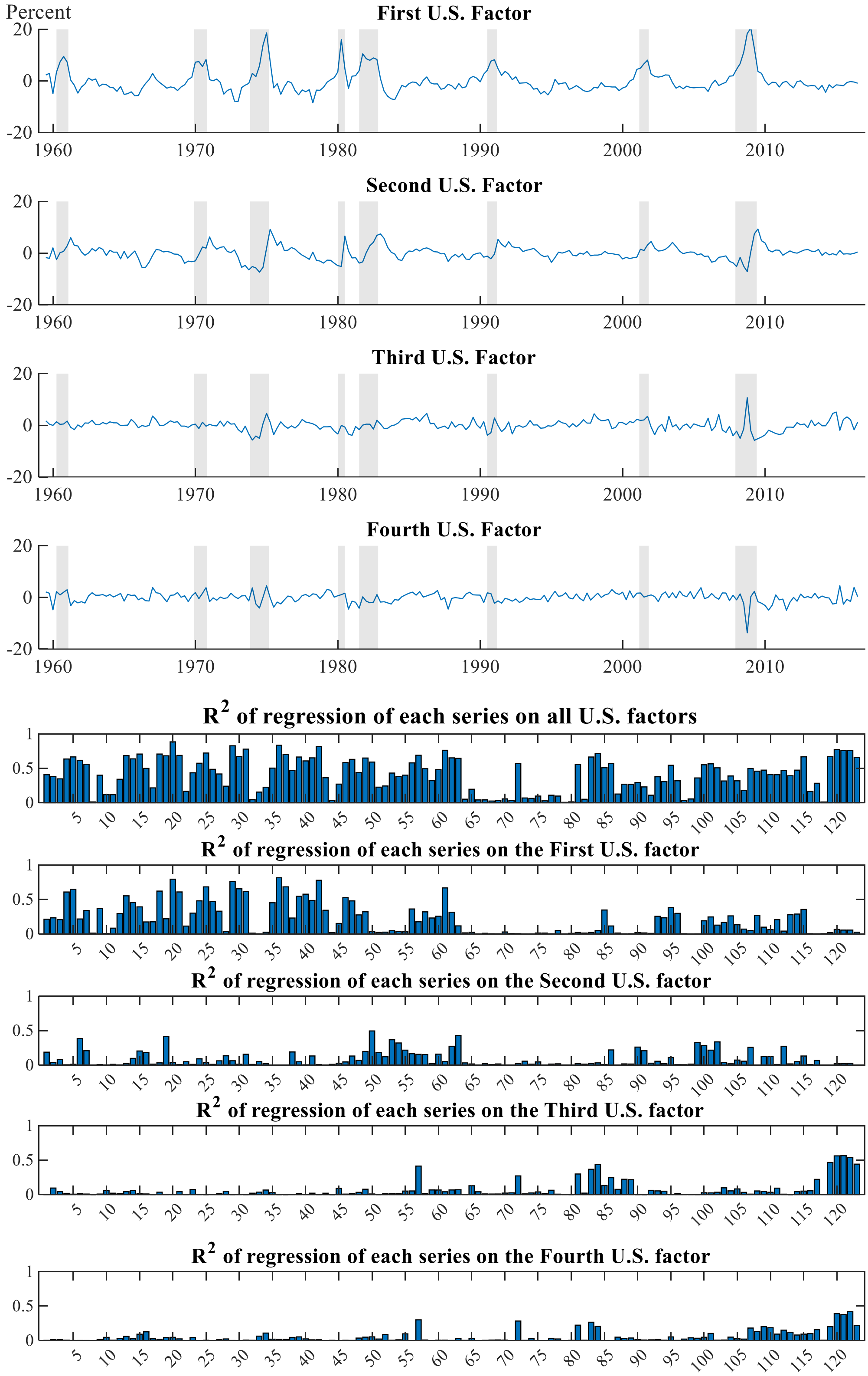


Figure 1.K.11: U.S. dynamics of cyclical factors and R^2 of each series on the U.S. cyclical factors

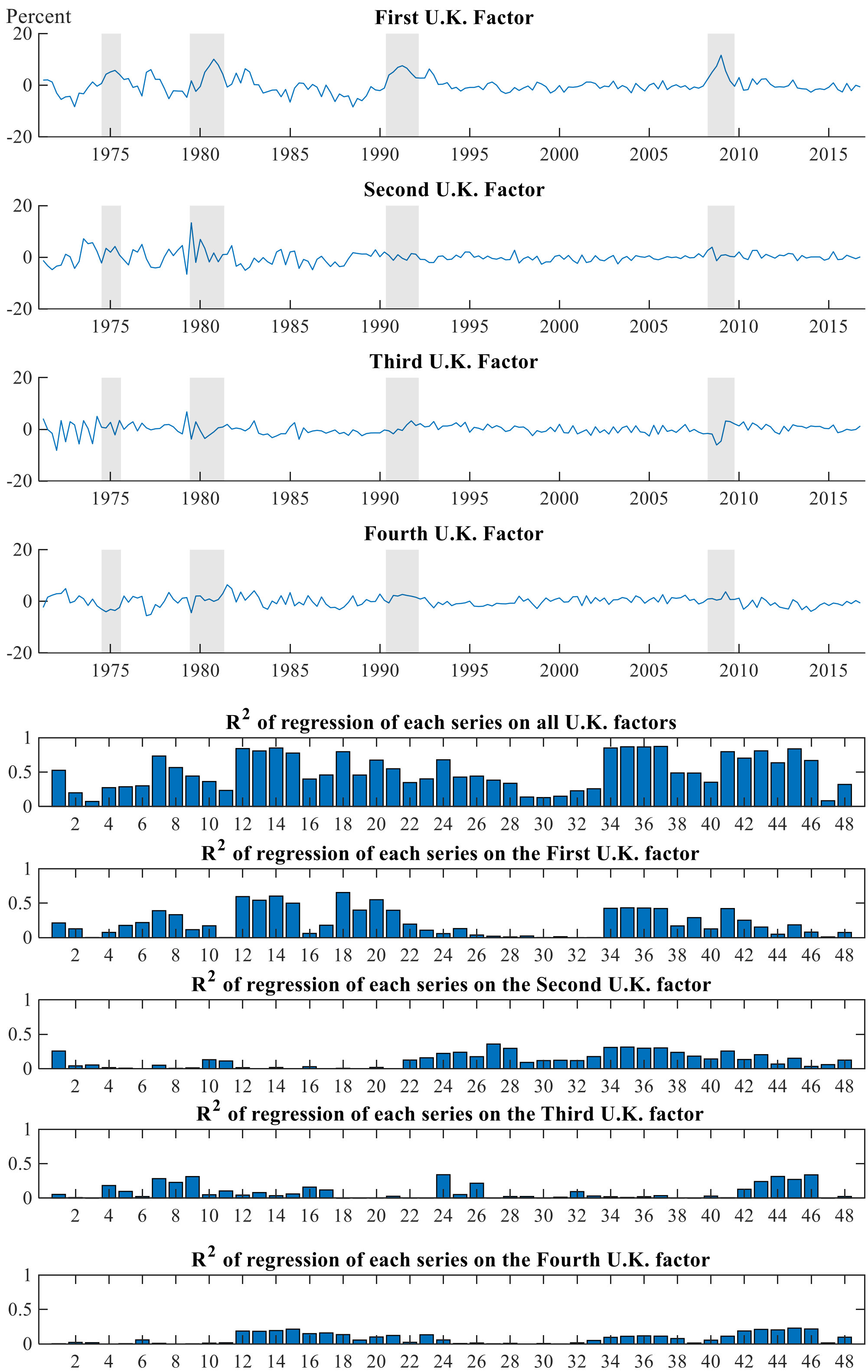


Figure 1.K.12: U.K. dynamics of cyclical factors and R^2 of each series on the U.K. cyclical factors

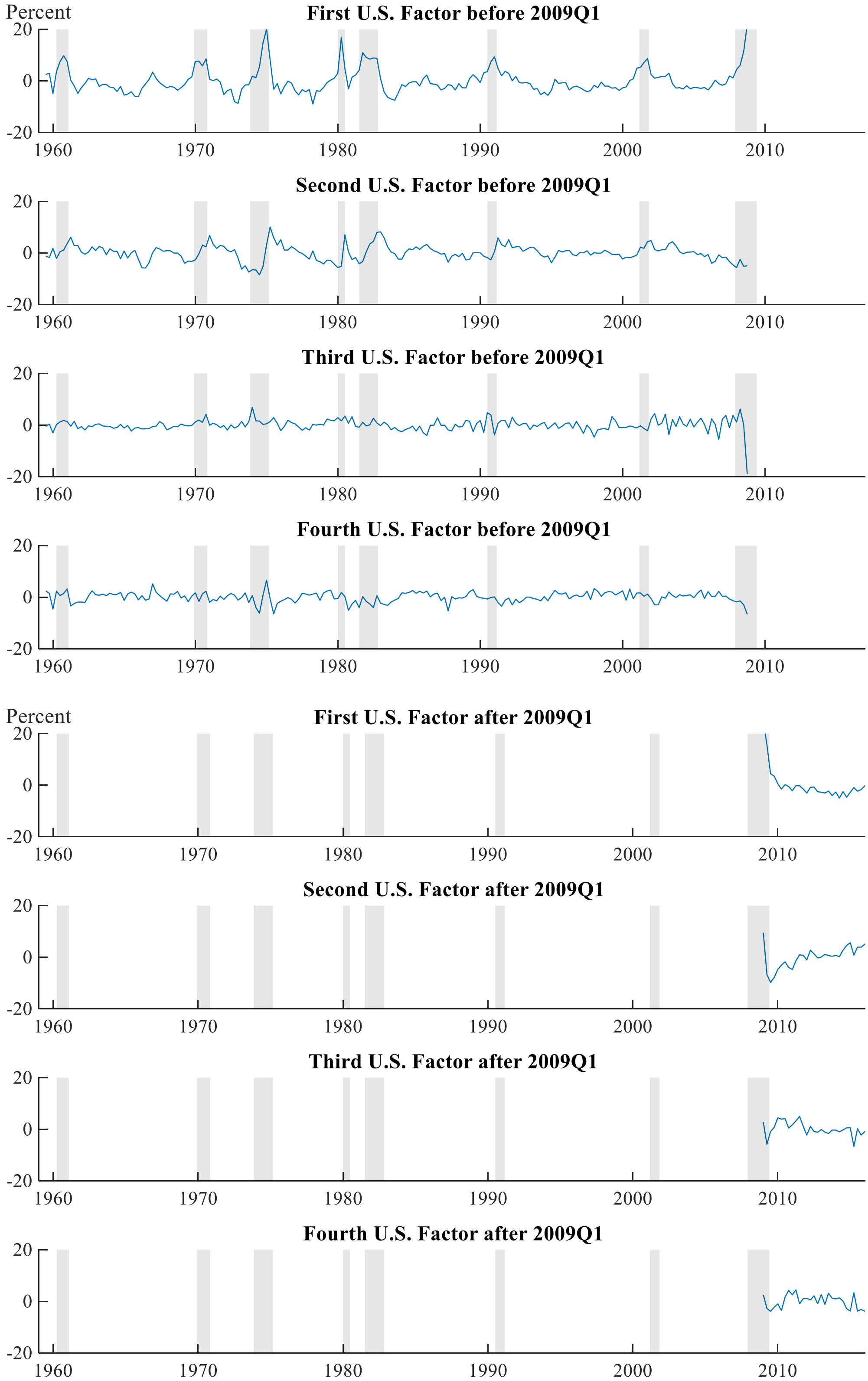


Figure 1.K.13: U.S. cyclical factors estimated within two regimes before and after the 2009Q1

Figure 1.K.14: U.K. cyclical factors estimated within two regimes before and after the 2009Q1

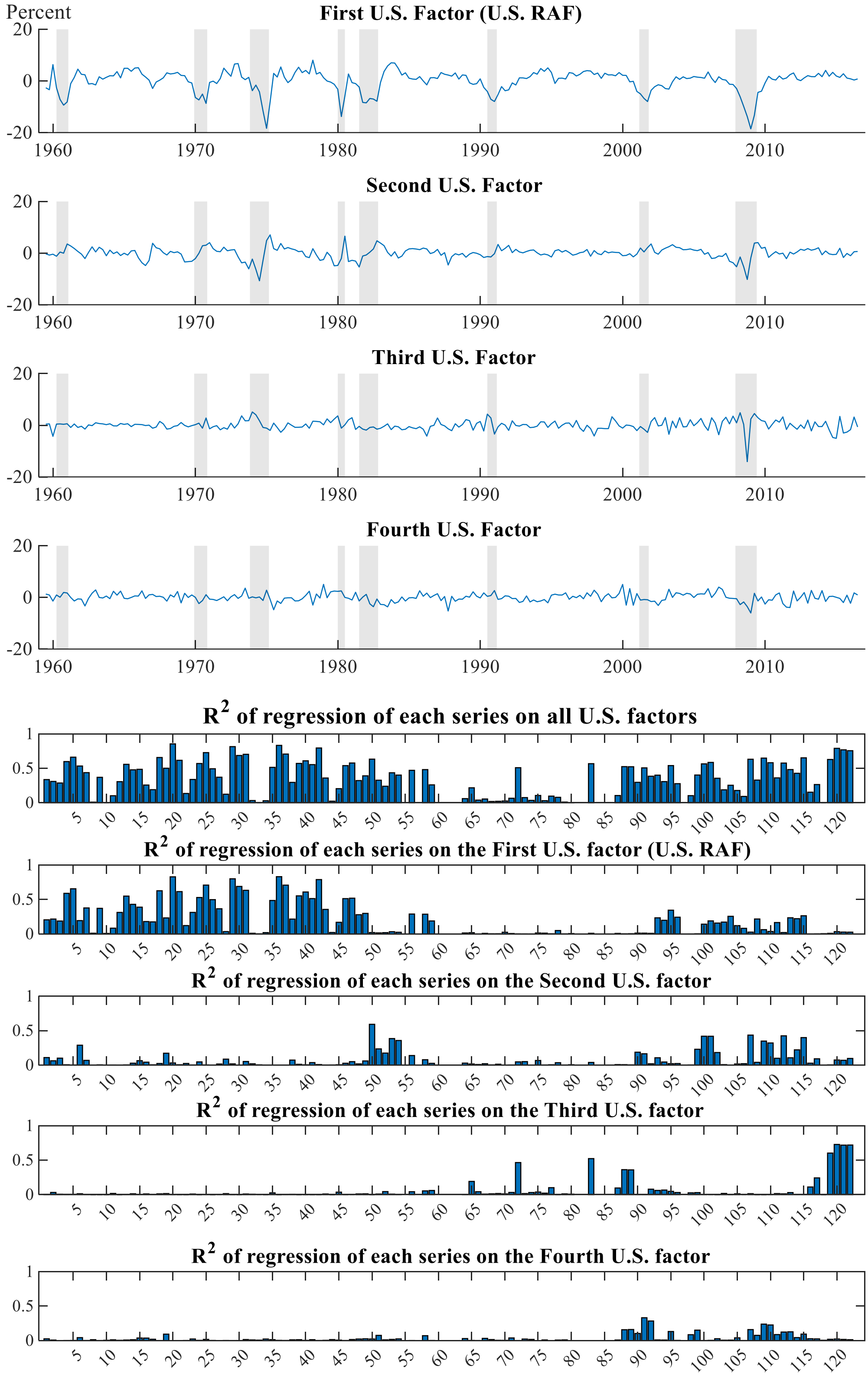


Figure 1.K.15: U.S. real activity factor (RAF), other three U.S. factors, and R^2 of each series on the U.S. cyclical factors

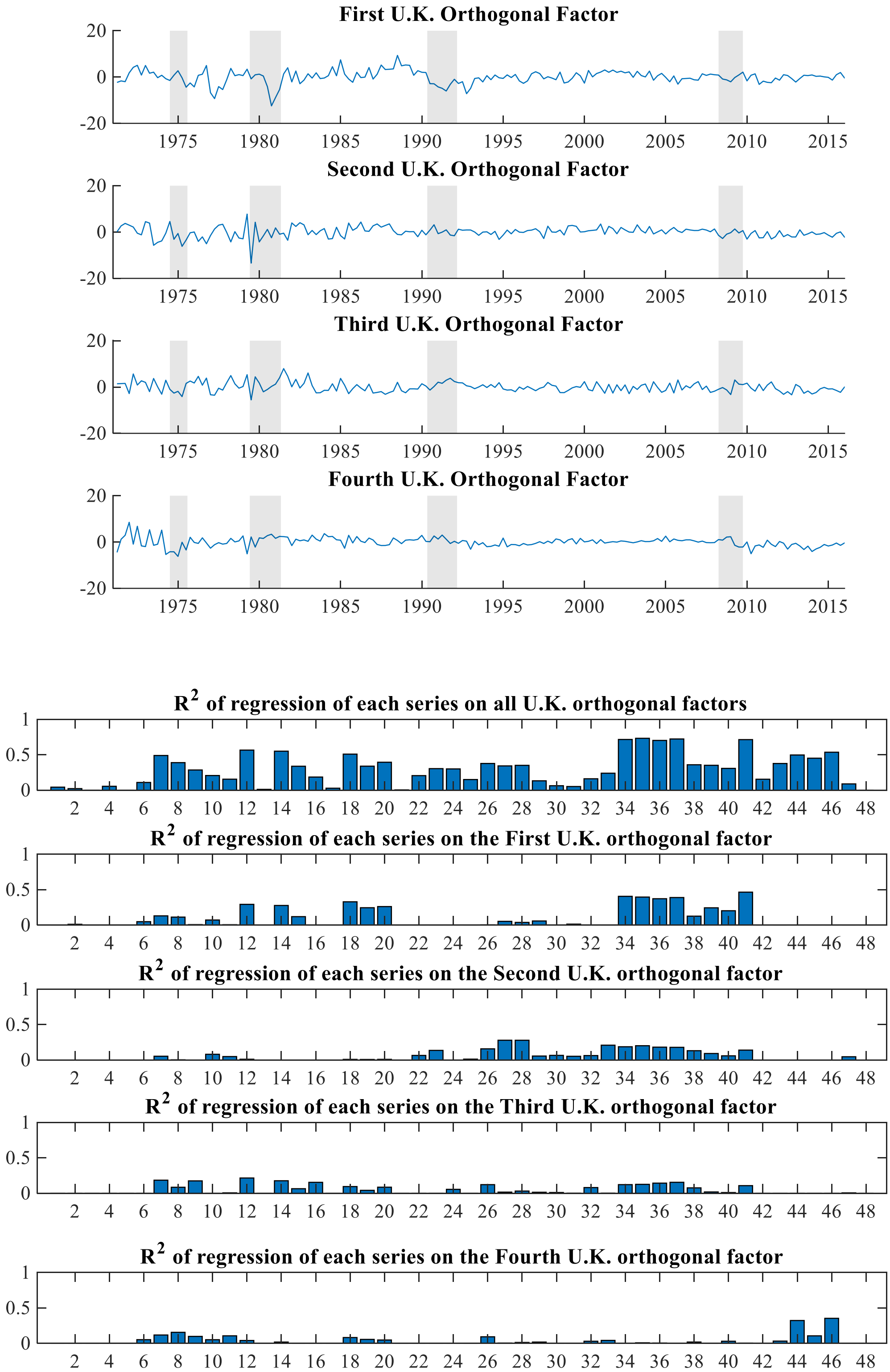
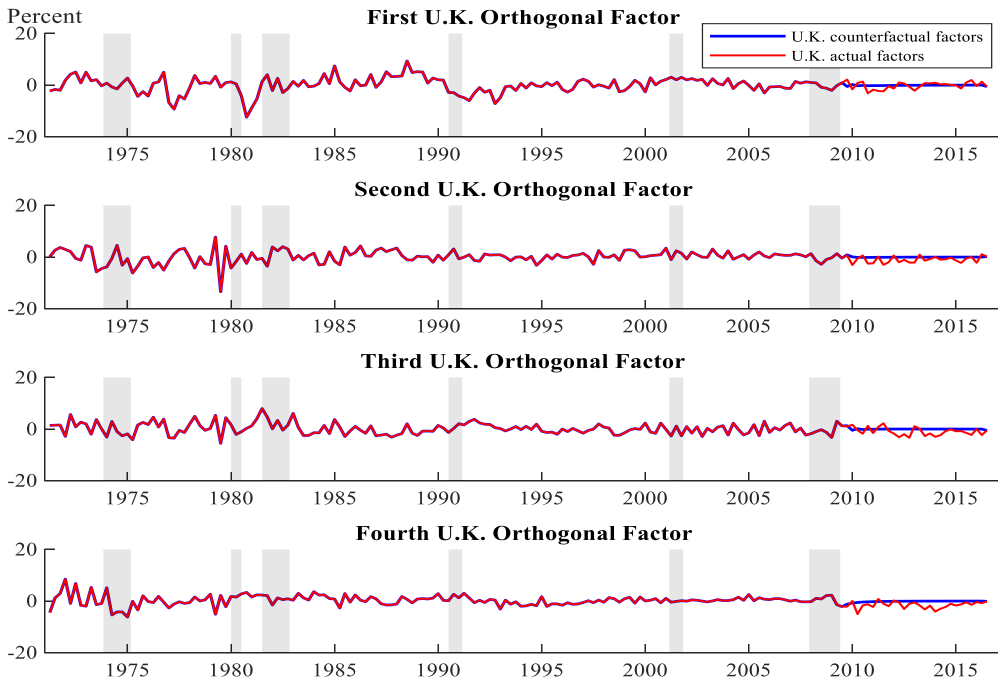
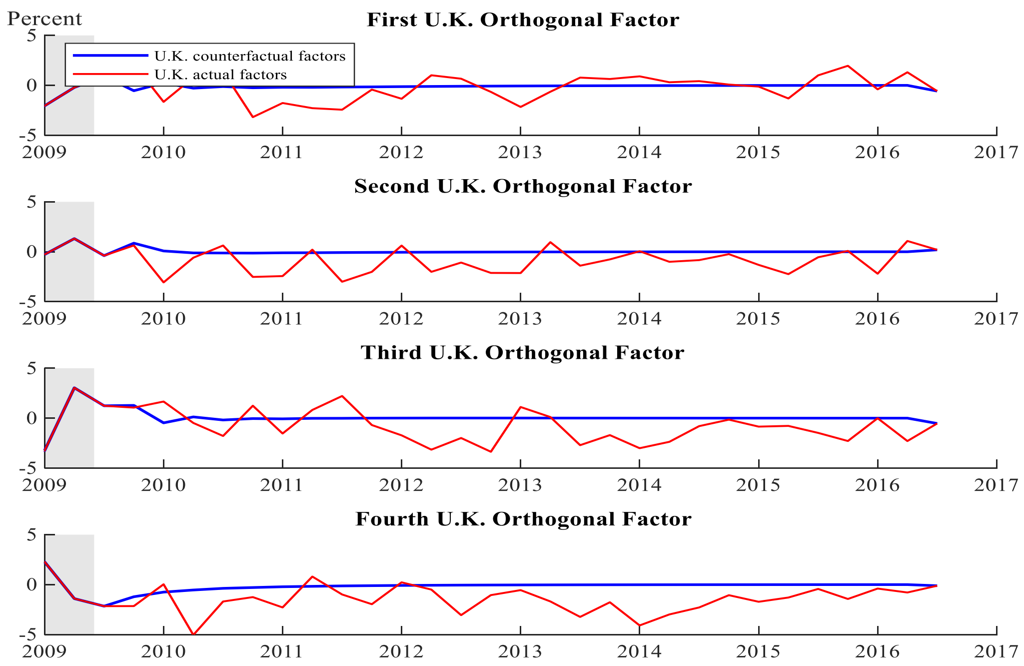


Figure 1.K.16: U.K. orthogonal factors, and R^2 of each series on U.K. orthogonal factors



(a) U.K. orthogonal factors in the whole sample



(b) U.K. orthogonal factors after the Great Recession

Figure 1.K.17: Comparison of U.K. factors and U.K. counterfactual factors

Notes:

(1) The red lines are U.K. orthogonal factors. The blue lines are U.K. counterfactual factors estimated by using a one-step ahead forecast with fixed origin in 2009Q1 based a VAR with order $P = 4$ to incorporate the potential response of U.K. orthogonal factors to the U.S. RAS.

(2) The magnitude of the product of U.K. orthogonal factor loadings and the gap between the red line and blues lines is unimportant and does not change the results.

Appendix 1.L: Additional Tables

Table 1.L.1: Trend-Cycle Decomposition of the U.S. GDP per capita for different recoveries

Recessions	Recoveries	Historical Values	Cyclical component	Cyclically adjusted trend
1974Q3-1975Q3	1975Q4-1979Q4	3.49	1.68	1.82
1990Q2-1991Q1	1991Q2-1998Q1	3.15	0.69	2.47
2001Q1-2001Q4	2002Q1-2007Q2	1.88	0.41	1.48
2007Q3-2009Q2	2009Q3-2016Q2	1.72	0.70	1.02

Table 1.L.2: Trend-Cycle Decomposition of U.K. GDP per worker for different recoveries

Recessions	Recoveries	Historical Values	Cyclical component	Cyclically adjusted trend
1974Q3-1975Q3	1975Q4-1979Q2	2.68	-0.05	2.74
1979Q3-1981Q2	1981Q3-1988Q1	3.54	0.80	2.73
1990Q3-1992Q1	1992Q2-1998Q4	2.50	0.55	1.96
2008Q1-2009Q4	2009Q4-2016Q2	1.90	0.66	1.25

Notes:

(1) Units are annualized percent changes.

(2) The estimation period is from 1974Q3 to 2016Q2.

(3) Cyclically adjustment is done by Okun's Law with $p = 2$ leads and lags while the structural break in the Okun's coefficient is allowed for.

Table 1.L.3: Statistics to determine the optimal number of static factors for the U.S. dataset

Number of factors	Trace R^2	Marginal R^2	Bai-Ng criterion	Ahn-Horenstein ratio
1	0.215	0.215	-0.181	**2.472**
2	0.302	0.087	-0.237	1.422
3	0.364	0.061	-0.267	1.606
4	**0.402**	**0.038**	**-0.267**	1.263
5	0.432	0.030	-0.257	1.129
6	0.459	0.027	-0.244	1.064
7	0.484	0.025	-0.230	1.171

Table 1.L.4: Statistics to determine the optimal number of static factors for the U.K. dataset

Number of factors	Trace R^2	Marginal R^2	Bai-Ng criterion	Ahn-Horenstein ratio
1	0.218	0.218	-0.144	**1.805**
2	0.338	0.121	-0.209	1.402
3	0.425	0.086	-0.247	1.076
4	**0.505**	**0.08**	-0.294	1.453
5	0.56	0.055	**-0.31**	1.392
6	0.599	0.04	-0.302	1.234
7	0.631	0.032	-0.283	1.093

Notes:

(1) Marginal R^2 is the contribution of adding the $(r+1)^{th}$ factor to the average R^2 of the regressions of all series on all factors.

(2) Bai-Ng (2002) ICp2 is an information criterion. The minimum of the Bai-Ng criterion determines the optimal number of factors.

(3) Ahn-Horenstein (2013) ratio is the ratio of $(i+1)^{th}$ to i^{th} eigenvalues. The maximum of the Ahn-Horenstein ratio determines the optimal number of factors.

Table 1.L.5: Shortfall of the U.S. post-crisis recovery estimated by Okun's law over 1981–2009

	(a)	(b)	(c)	(d)	(e)	(f)
Series	Actual Recovery	Counterfactual Recovery	Annual Shortfall	Wilcoxon p-value	Shortfall with a break in coefficient	Shortfall with a break in intercept and coefficient
1. GDP/population	1.72	3.57	1.85	0.00	0.71	-0.05
2. TFP	0.89	1.59	0.70	0.05	0.09	-0.03
3. $\alpha \times$ Capital/population	0.24	0.87	0.63	0.00	0.40	0.01
4. $(1-\alpha) \times$ LQ×hours/population	0.59	1.11	0.52	0.01	0.22	-0.03
5. Hours/population	0.63	1.25	0.61	0.01	0.29	-0.03
6. Hours/employed worker	0.24	0.13	-0.11	0.60	-0.07	-0.03
7. Employment Rate	0.68	0.68	-0.00	0.39	0.01	-0.00
8. Labor force participation	-0.66	0.21	0.87	0.00	0.58	0.03
9. GDP/hour	1.09	2.33	1.24	0.00	0.42	-0.02
10. TFP/$(1-\alpha)$	1.44	2.38	0.94	0.08	0.06	-0.05
11. Capital/output $\times\, \alpha/(1-\alpha)$	-0.69	-0.47	0.22	0.42	0.33	0.04
12. Labor quality	0.33	0.42	0.08	0.38	0.03	-0.01

Notes:

(1) Column (a) presents the average annualized growth rates of each series from 2009Q4 to the 2016Q2. Column (b) is the average of the annualized growth rates of the counterfactual recovery for each series in the same period (2009Q4–2016Q2). Units are annualized percent changes.

(2) Counterfactual recoveries are estimated by Okun's law with $p = 1$ leads and lags, over the period 1981–2009, assuming that the Okun's law parameters during the post-crisis recovery are equal to those before the 2007–09 financial crisis.

(3) The average of the annual shortfalls over the current recovery, from 2009Q4 to 2016Q2, is shown in column (c). The shortfall in each quarter, as specified in Eq. (1.2), is simply the difference between the counterfactual and actual recovery. Column (d) reports the Wilcoxon signed rank test p-value, which examines whether the actual recovery falls short of the counterfactual recovery. Thus, a significant shortfall indicates a slow recovery.

(4) Column (e) is the average of the annual shortfalls where the counterfactual is estimated by a model in which a structural break in the Okun's coefficient in 2009Q1 is allowed for.

(5) Column (f) is the average of annual shortfalls where a structural break in both Okun's intercept and coefficient in 2009Q1 is allowed for. To eliminate the shortfall entirely, it is necessary to consider the model with breaks in both the Okun's intercept and coefficient in 2009Q1. This date is consistent with the identified structural break in Okun's law in 2009Q1, which is presented in Table 1.1.3. Allowing for a break in other candidate break dates from 2007Q4 to 2008Q4, decreases the shortfall but does not eliminate it.

(6) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.L.6: Shortfall of the U.S. post-crisis recovery estimated by the DFM over 1981–2009

	(a)	(b)	(c)	(d)	(e)	(f)
Series	Actual Recovery	Counterfactual Recovery	Annual Shortfall	Wilcoxon p-value	Shortfall with a break in factor loadings	Shortfall with a break in intercept and factor loadings
1. GDP/population	1.72	3.51	1.79	0.00	0.53	0.01
2. TFP	0.89	1.88	0.99	0.01	-0.01	0.02
3. $\alpha \times$ Capital/population	0.24	0.74	0.50	0.00	0.50	-0.00
4. $(1-\alpha) \times$ LQ×hours/population	0.59	0.88	0.29	0.13	0.05	-0.01
5. Hours/population	0.63	0.91	0.27	0.18	-0.01	-0.01
6. Hours/employed worker	0.24	0.27	0.03	0.65	-0.19	-0.00
7. Employment Rate	0.68	0.53	-0.15	0.93	-0.19	0.01
8. Labor force participation	-0.66	0.13	0.80	0.00	0.63	0.01
9. GDP/hour	1.09	2.60	1.51	0.00	0.54	0.03
10. TFP/$(1-\alpha)$	1.44	2.79	1.35	0.02	-0.11	0.04
11. Capital/output $\times \alpha/(1-\alpha)$	-0.69	-0.60	0.09	0.48	0.61	-0.01
12. Labor quality	0.33	0.41	0.08	0.49	0.04	-0.00

Notes:

(1) Column (a) presents the average annualized growth rates of each series from 2009Q4 to the 2016Q2. Column (b) is the average of the annualized growth rates of the counterfactual recovery for each series in the same period (2009Q4–2016Q2). Units are annualized percent changes.

(2) Counterfactual recoveries are estimated by the DFM, over the period 1981–2009, assuming that the intercept and cyclical factor loadings during the post-crisis recovery are equal to those before the 2007–09 financial crisis.

(3) The average of the annual shortfalls over the current recovery, from 2009Q4 to 2016Q2, is shown in column (c). The shortfall in each quarter, as specified in Eq. (1.2), is simply the difference between the counterfactual and actual recovery. Column (d) reports the Wilcoxon signed rank test p-value, which examines whether the actual recovery falls short of the counterfactual recovery. Thus, a significant shortfall indicates a slow recovery.

(4) Column (e) is the average of the annual shortfalls, where the counterfactual is estimated by a model in which a structural break in the cyclical factor loadings in 2009Q1 is allowed for. Indeed, two sets of cyclical factors are estimated by using PCA over two regimes, and the post-crisis fitted series are calculated by the cyclical factor loadings and cyclical factors derived from the second regime from 2009Q1 to 2016Q2.

(5) Column (f) is the average of annual shortfalls where a structural break in both intercept and cyclical factor loadings in 2009Q1 is allowed for. To eliminate the shortfall entirely, it is necessary to consider the model with breaks in both the intercept and cyclical factor loadings in 2009Q1. This date is consistent with the identified structural break in the intercept and cyclical factor loadings between 2007 and 2009, which is presented in Table 1.1.4. Allowing for the break in other candidate break dates from 2007Q4 to 2008Q4 decreases the shortfall but does not eliminate it entirely.

(6) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.L.7: Shortfall of the U.K. post-crisis recovery estimated by Okun's law over 1981–2009

	(a)	(b)	(c)	(d)	(e)	(f)
Series	Actual Recovery	Counterfactual Recovery	Annual Shortfall	Wilcoxon p-value	Shortfall with a break in coefficient	Shortfall with a break in intercept and coefficient
1. GDP/population	1.80	2.91	1.11	0.00	0.88	-0.17
2. TFP	0.18	1.63	1.45	0.00	1.00	-0.14
3. $\alpha \times$ Capital/population	0.36	0.94	0.59	0.00	0.44	0.01
4. $(1-\alpha) \times$ LQ×hours/population	1.26	0.34	-0.92	1.00	-0.55	-0.04
5. Hours/population	1.46	0.30	-1.16	0.99	-0.46	-0.08
6. Hours/employed worker	0.75	-0.27	-1.02	0.99	-0.34	-0.09
7. Employment Rate	0.43	0.45	0.02	0.45	0.02	-0.00
8. Labor force participation	0.28	0.12	-0.16	0.86	-0.14	0.01
9. GDP/hour	0.34	2.61	2.27	0.00	1.34	-0.09
10. TFP/$(1-\alpha)$	0.28	2.64	2.36	0.00	1.59	-0.23
11. Capital/output $\times\ \alpha/(1-\alpha)$	-0.47	-0.31	0.16	0.25	0.14	0.11
12. Labor quality	0.53	0.29	-0.24	0.93	-0.39	0.03

Notes:

(1) Column (a) presents the average annualized growth rates of each series from 2009Q4 to the 2016Q2. Column (b) is the average of the annualized growth rates of the counterfactual recovery for each series in the same period (2009Q4–2016Q2). Units are annualized percent changes.

(2) Counterfactual recoveries are estimated by Okun's law with $p = 1$ leads and lags, over the period 1981–2009, assuming that the Okun's law parameters during the post-crisis recovery are equal to those before the 2007–09 financial crisis.

(3) The average of the annual shortfalls over the current recovery, from 2009Q4 to 2016Q2, is shown in column (c). The shortfall in each quarter, as specified in Eq. (1.2), is simply the difference between the counterfactual and actual recovery. Column (d) reports the Wilcoxon signed rank test p-value, which examines whether the actual recovery falls short of the counterfactual recovery. Thus, a significant shortfall indicates a slow recovery.

(4) Column (e) is the average of the annual shortfalls where the counterfactual is estimated by a model in which a structural break in the Okun's coefficient in 2009Q1 is allowed for. This model is not able to decrease the shortfall.

(5) Column (f) is the average of annual shortfalls where a structural break in both Okun's intercept and coefficient in 2009Q1 is allowed for. This model eliminates the shortfall.

(6) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP and capital input to the slow recovery in the U.K.

Table 1.L.8: Shortfall of the U.K. post-crisis recovery estimated by the DFM over 1981–2009

	(a)	(b)	(c)	(d)	(f)	(g)
Series	Actual Recovery	Counterfactual Recovery	Annual Shortfall	Wilcoxon p-value	Shortfall with a break in factor loadings	Shortfall with a break in intercept and factor loadings
1. GDP/population	1.80	2.73	0.93	0.01	0.69	0.01
2. TFP	0.18	1.25	1.08	0.01	0.97	-0.05
3. $\alpha \times$ Capital/population	0.36	1.08	0.72	0.00	0.47	0.01
4. $(1-\alpha) \times$ LQ×hours/population	1.26	0.41	-0.85	1.00	-0.75	0.05
5. Hours/population	1.46	0.22	-1.24	1.00	-0.86	0.08
6. Hours/employed worker	0.75	-0.34	-1.09	1.00	-0.59	0.06
7. Employment Rate	0.43	0.38	-0.05	0.60	-0.13	-0.00
8. Labor force participation	0.28	0.18	-0.10	0.90	-0.14	0.02
9. GDP/hour	0.34	2.51	2.17	0.00	1.55	-0.07
10. TFP/$(1-\alpha)$	0.28	2.06	1.78	0.00	1.54	-0.08
11. Capital/output $\times\ \alpha/(1-\alpha)$	-0.47	-0.01	0.47	0.04	0.30	0.01
12. Labor quality	0.53	0.47	-0.06	0.76	-0.28	0.01

Notes:

(1) Column (a) presents the average annualized growth rates of each series from 2009Q4 to the 2016Q2. Column (b) is the average of the annualized growth rates of the counterfactual recovery for each series in the same period (2009Q4–2016Q2). Units are annualized percent changes.

(2) Counterfactual recoveries are estimated by the DFM, over the period 1981–2009, assuming that the intercept and cyclical factor loadings during the post-crisis recovery are equal to those before the 2007–09 financial crisis.

(3) The average of the annual shortfalls over the current recovery, from 2009Q4 to 2016Q2, is shown in column (c). The shortfall in each quarter, as specified in Eq. (1.2), is simply the difference between the counterfactual and actual recovery. Column (d) reports the Wilcoxon signed rank test p-value, which examines whether the actual recovery falls short of the counterfactual recovery. Thus, a significant shortfall indicates a slow recovery.

(4) Column (e) is the average of the annual shortfalls, where the counterfactual is estimated by a model in which a structural break in the cyclical factor loadings in 2009Q1 is allowed for. Indeed, two sets of cyclical factors are estimated by using PCA over two regimes, and the post-crisis fitted series are calculated by the cyclical factor loadings and cyclical factors derived from the second regime from 2009Q1 to 2016Q2. This model is not able to decrease the shortfall.

(5) Column (g) is the average of annual shortfalls where a structural break in both intercept and cyclical factor loadings in 2009Q1 is allowed for. This model eliminates the shortfall.

(6) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP and capital input to the slow recovery in the U.K.

Table 1.L.9: Trend-cycle decomposition of the U.S. series based on Okun's law

	Historical Values			Cycle			Cyclically Adjusted Trend			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
Series	Current recovery	Three previous recoveries	Annual Shortfall	Current recovery	Three previous recoveries	Annual Shortfall	Current recovery	Three previous recoveries	Annual Shortfall	Annual Shortfall in smooth trend
1. GDP/population	1.72	2.95	1.23	0.70	0.94	0.24	1.02	2.01	0.99	0.82
2. TFP	0.89	1.40	0.51	0.11	0.39	0.28	0.78	1.01	0.23	0.10
3. $\alpha \times$ Capital/population	0.24	0.85	0.61	-0.08	0.03	0.11	0.32	0.82	0.50	0.37
4. $(1-\alpha) \times$ LQ×hours/population	0.59	0.70	0.11	0.67	0.52	-0.15	-0.08	0.18	0.26	0.35
5. Hours/population	0.63	0.61	-0.02	1.19	0.83	-0.36	-0.56	-0.22	0.34	0.46
6. Hours/employed worker	0.24	0.00	-0.24	0.29	0.14	-0.15	-0.05	-0.14	-0.09	-0.05
7. Employment Rate	0.68	0.41	-0.27	0.69	0.41	-0.27	-0.01	0.00	0.01	-0.00
8. Labor force participation	-0.66	0.15	0.81	-0.04	0.05	0.09	-0.62	0.10	0.73	0.48
9. GDP/hour	1.09	2.33	1.25	-0.49	0.11	0.60	1.58	2.23	0.65	0.36
10. TFP/$(1-\alpha)$	1.44	2.08	0.65	0.19	0.59	0.39	1.24	1.50	0.25	0.09
11. Capital/output $\times \alpha/(1-\alpha)$	-0.69	-0.18	0.51	-0.55	-0.42	0.13	-0.14	0.24	0.37	0.20
12. Labor quality	0.33	0.43	0.09	-0.13	-0.06	0.07	0.47	0.49	0.02	0.07

Notes:

(1) Column (a) presents the average growth rate of each series over the current recovery from 2009Q4 to 2016Q2. Similarly, column (b) reports the average growth rate over the three previous recoveries (1983Q1–1990Q2, 1991Q2–2000Q4, and 2002Q1–2007Q3). Taking the three previous recoveries as a counterfactual, column (c) measures the annual shortfall, the difference between the counterfactual and actual recovery. Units are annualized percent changes.

(2) Columns (d) and (e) show the average growth rate of cyclical components during the current recovery and the three previous recoveries. Thus, column (f) measures the annual shortfall in the cyclical component. Likewise, columns (g) and (h) show the average growth rate of trend components during the current recovery and the three previous recoveries. Thus, column (i) measures the annual shortfall in the trend component. Additionally, column (j) measures the shortfall in the smooth trend.

(3) To estimate the cycle and cyclically adjusted trend, we use Okun's regression, explained in Eq. (1.15), where a structural break in the Okun's coefficient in 2009Q1 is allowed for. The sample for estimation of the parameters is 1981Q1 to 2016Q2. The growth of the cyclical component of each series is the product of the Okun's coefficient and the change in the unemployment rate. The cyclically adjusted trend is the residuals of the regression, and the smooth trend is derived by passing the estimated residuals through a Tukey's bi-weight filter with a bandwidth of 60 quarters.

(4) In this setup, we consider Okun's regression with $p = 1$.

(5) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.L.10: Trend-cycle decomposition of the U.K. series based on Okun's law

	Historical Values			Cycle			Cyclically Adjusted Trend			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
Series	Current recovery	Two previous recoveries	Annual Shortfall	Current recovery	Two previous recoveries	Annual Shortfall	Current recovery	Two previous recoveries	Annual Shortfall	Annual Shortfall in smooth trend
1. GDP/population	1.80	2.87	1.07	0.88	0.57	-0.31	0.91	2.30	1.38	0.95
2. TFP	0.18	1.72	1.55	0.30	0.27	-0.03	-0.12	1.45	1.57	0.90
3. $\alpha \times$ Capital/population	0.36	0.86	0.51	0.02	0.04	0.02	0.34	0.82	0.48	0.47
4. $(1-\alpha) \times$ LQ×hours/population	1.26	0.28	-0.98	0.57	0.26	-0.31	0.70	0.03	-0.67	-0.41
5. Hours/population	1.46	0.25	-1.21	1.12	0.40	-0.73	0.33	-0.15	-0.48	-0.36
6. Hours/employed worker	0.75	-0.19	-0.93	0.60	0.01	-0.59	0.14	-0.20	-0.34	-0.23
7. Employment Rate	0.43	0.32	-0.11	0.46	0.32	-0.15	-0.03	0.00	0.03	0.01
8. Labor force participation	0.28	0.12	-0.16	0.06	0.07	0.01	0.22	0.05	-0.17	-0.14
9. GDP/hour	0.34	2.62	2.28	-0.24	0.18	0.42	0.58	2.44	1.86	1.31
10. TFP/$(1-\alpha)$	0.28	2.77	2.49	0.47	0.46	-0.00	-0.19	2.30	2.50	1.43
11. Capital/output $\times \alpha/(1-\alpha)$	-0.47	-0.39	0.08	-0.47	-0.31	0.17	0.00	-0.09	-0.09	0.15
12. Labor quality	0.53	0.24	-0.29	-0.24	0.02	0.25	0.77	0.23	-0.54	-0.26

Notes:

(1) Column (a) presents the average growth rate of each series over the current recovery from 2009Q4 to 2016Q2. Similarly, column (b) reports the average growth rate over the two previous recoveries (1981Q3–1990Q1, and 1992Q2–2008Q1). Taking the two previous recoveries as a counterfactual, column (c) measures the annual shortfall, the difference between the counterfactual and actual recovery. Units are annualized percent changes.

(2) Columns (d) and (e) show the average growth rate of cyclical components during the current recovery and the two previous recoveries. Thus, column (f) measures the annual shortfall in the cyclical component. Likewise, columns (g) and (h) show the average growth rate of trend components during the current recovery and the two previous recoveries. Thus, column (i) measures the annual shortfall in the trend component. Additionally, column (j) measures the shortfall in the smooth trend.

(3) To estimate the cycle and cyclically adjusted trend, we use Okun's regression, explained in Eq. (1.15), where a structural break in the Okun's coefficient in 2009Q1 is allowed for. The sample for estimation of the parameters is 1981Q1 to 2016Q2. The growth of the cyclical component of each series is the product of the Okun's coefficient and the change in the unemployment rate. The cyclically adjusted trend is the residuals of the regression, and the smooth trend is derived by passing the estimated residuals through a Tukey's bi-weight filter with a bandwidth of 60 quarters.

(4) In the setup, we consider Okun's regression with $p = 1$.

(5) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.L.11: Trend-cycle decomposition of the U.S. series based on Okun's law (the structural break is not accommodated)

	Historical Values			cycles			Cyclically Adjusted Trend			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
Series	Current recovery	Three previous recoveries	Annual Shortfall	Current recovery	Three previous recoveries	Annual Shortfall	Current recovery	Three previous recoveries	Annual Shortfall	Annual Shortfall in smooth trend
1. GDP/population	1.72	2.95	1.23	1.50	0.79	-0.71	0.22	2.16	1.94	1.21
2. TFP	0.89	1.40	0.51	0.63	0.28	-0.35	0.26	1.12	0.87	0.39
3. $\alpha \times$ Capital/population	0.24	0.85	0.61	0.00	0.02	0.02	0.24	0.83	0.59	0.41
4. $(1-\alpha) \times$ LQ×hours/population	0.59	0.70	0.11	0.87	0.49	-0.38	-0.28	0.20	0.48	0.41
5. Hours/population	0.63	0.61	-0.02	1.40	0.82	-0.59	-0.77	-0.20	0.57	0.54
6. Hours/employed worker	0.24	0.00	-0.24	0.33	0.16	-0.17	-0.08	-0.16	-0.07	-0.06
7. Employment Rate	0.68	0.41	-0.27	0.68	0.42	-0.27	0.00	0.00	-0.00	-0.00
8. Labor force participation	-0.66	0.15	0.81	0.03	0.03	-0.00	-0.69	0.12	0.81	0.55
9. GDP/hour	1.09	2.33	1.25	0.09	-0.03	-0.12	0.99	2.36	1.37	0.67
10. TFP/$(1-\alpha)$	1.44	2.08	0.65	0.96	0.42	-0.54	0.48	1.66	1.19	0.53
11. Capital/output $\times\, \alpha/(1-\alpha)$	-0.69	-0.18	0.51	-0.77	-0.38	0.40	0.09	0.20	0.11	0.10
12. Labor quality	0.33	0.43	0.09	-0.09	-0.07	0.02	0.43	0.50	0.07	0.04

Notes:

(1) Column (a) presents the average growth rate of each series over the current recovery from 2009Q4 to 2016Q2. Similarly, column (b) reports the average growth rate over the three previous recoveries (1983Q1–1990Q2, 1991Q2–2000Q4, and 2002Q1–2007Q3). Taking the three previous recoveries as a counterfactual, column (c) measures the annual shortfall, the difference between the counterfactual and actual recovery. Units are annualized percent changes.

(2) Columns (d) and (e) show the average growth rate of cyclical components during the current recovery and the three previous recoveries. Thus, column (f) measures the annual shortfall in the cyclical component. Likewise, columns (g) and (h) show the average growth rate of trend components during the current recovery and the three previous recoveries. Thus, column (i) measures the annual shortfall in the trend component. Additionally, column (j) measures the shortfall in the smooth trend.

(3) To estimate the cycle and cyclically adjusted trend, we use Okun's regression, explained in Eq. (1.15), where a structural break in the Okun's coefficient is not allowed for. The sample for estimation of the parameters is 1981Q1 to 2016Q2. The growth of the cyclical component of each series is the product of the Okun's coefficient and the change in the unemployment rate. The cyclically adjusted trend is the residuals of the regression, and the smooth trend is derived by passing the estimated residuals through a Tukey's bi-weight filter with a bandwidth of 60 quarters.

(4) Since the model does not allow a break, it tends to overestimate the contribution of the trend and underestimate the contribution of the cycle to the slow recovery by attributing the break in the Okun's coefficient to the trend while it is attributable to the cyclical component.

(5) In this setup, we consider Okun's regression with $p = 2$.

(6) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.L.12: Trend-cycle decomposition of the U.K. series based on Okun's law (the structural break is not accommodated)

	Historical Values			Cycle			Cyclically Adjusted Trend			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
Series	Current recovery	Two previous recoveries	Annual Shortfall	Current recovery	Two previous recoveries	Annual Shortfall	Current recovery	Two previous recoveries	Annual Shortfall	Annual Shortfall in smooth trend
1. GDP/population	1.80	2.87	1.07	0.91	0.63	-0.28	0.89	2.24	1.36	1.00
2. TFP	0.18	1.72	1.55	0.47	0.30	-0.17	-0.29	1.43	1.71	1.02
3. $\alpha\times$ Capital/population	0.36	0.86	0.51	0.03	0.03	0.01	0.33	0.83	0.50	0.48
4. $(1-\alpha)\times$ LQ×hours/population	1.26	0.28	-0.98	0.42	0.29	-0.12	0.85	-0.01	-0.86	-0.50
5. Hours/population	1.46	0.25	-1.21	0.68	0.49	-0.19	0.78	-0.24	-1.02	-0.59
6. Hours/employed worker	0.75	-0.19	-0.93	0.13	0.09	-0.04	0.62	-0.28	-0.89	-0.46
7. Employment Rate	0.43	0.32	-0.11	0.45	0.32	-0.13	-0.02	0.00	0.02	0.01
8. Labor force participation	0.28	0.12	-0.16	0.10	0.08	-0.02	0.18	0.03	-0.14	-0.13
9. GDP/hour	0.34	2.62	2.28	0.23	0.14	-0.09	0.11	2.48	2.37	1.59
10. TFP/$(1-\alpha)$	0.28	2.77	2.49	0.78	0.50	-0.28	-0.50	2.27	2.77	1.65
11. Capital/output $\times\ \alpha/(1-\alpha)$	-0.47	-0.39	0.08	-0.54	-0.35	0.19	0.06	-0.04	-0.11	0.11
12. Labor quality	0.53	0.24	-0.29	-0.01	-0.02	-0.01	0.54	0.26	-0.28	-0.16

Notes:

(1) Column (a) presents the average growth rate of each series over the current recovery from 2009Q4 to 2016Q2. Similarly, column (b) reports the average growth rate over the two previous recoveries (1981Q3–1990Q1, and 1992Q2–2008Q1). Taking the two previous recoveries as a counterfactual, column (c) measures the annual shortfall, the difference between the counterfactual and actual recovery. Units are annualized percent changes.

(2) Columns (d) and (e) show the average growth rate of cyclical components during the current recovery and the two previous recoveries. Thus, column (f) measures the annual shortfall in the cyclical component. Likewise, columns (g) and (h) show the average growth rate of trend components during the current recovery and the two previous recoveries. Thus, column (i) measures the annual shortfall in the trend component. Additionally, column (j) measures the shortfall in the smooth trend.

(3) To estimate the cycle and cyclically adjusted trend, we use Okun's regression, explained in Eq. (1.15), where a structural break in the Okun's coefficient is not allowed for. The sample for estimation of the parameters is 1981Q1 to 2016Q2. The growth of the cyclical component of each series is the product of the Okun's coefficient and the change in the unemployment rate. The cyclically adjusted trend is the residuals of the regression, and the smooth trend is derived by passing the estimated residuals through a Tukey's bi-weight filter with a bandwidth of 60 quarters.

(4) Since the model does not allow a break, it tends to overestimate the contribution of the trend and underestimate the contribution of the cycle to the slow recovery by attributing the break in the Okun's coefficient to the trend while it is attributable to the cyclical component.

(5) In this setup, we consider Okun's regression with $p=2$.

(6) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.L.13: Shortfall of the U.K. post-crisis recovery estimated by a hierarchical DFM (1971 to 2016)

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Series	Actual Recovery	Counterfactual Recovery	Annual Shortfall	Wilcoxon p-value	Annual Shortfall with change in other U.S. factors	Annual Shortfall with change in U.K. orthogonal factors	Annual Shortfall with a break in intercept and factor loadings
1. GDP/population	1.80	2.40	0.60	0.04	0.57	0.66	0.01
2. TFP	0.18	0.78	0.61	0.06	0.58	0.77	0.00
3. $\alpha \times$ Capital/population	0.36	0.76	0.40	0.00	0.40	0.41	0.00
4. $(1-\alpha) \times$ LQ×hours/population	1.26	0.87	-0.40	0.91	-0.40	-0.52	0.01
5. Hours/population	1.46	0.96	-0.49	0.86	-0.50	-0.66	0.02
6. Hours/employed worker	0.75	0.15	-0.59	0.89	-0.60	-0.63	0.03
7. Employment Rate	0.43	0.50	0.07	0.22	0.06	-0.03	0.00
8. Labor force participation	0.28	0.31	0.03	0.44	0.04	-0.00	0.00
9. GDP/hour	0.34	1.44	1.10	0.01	1.07	1.32	-0.01
10. TFP/$(1-\alpha)$	0.28	1.29	1.02	0.05	0.97	1.27	0.00
11. Capital/output $\times \alpha/(1-\alpha)$	-0.47	-0.29	0.18	0.22	0.20	0.18	0.00
12. Labor quality	0.53	0.44	-0.09	0.82	-0.09	-0.11	-0.01

Notes:

(1) Column (a) presents the average annualized growth rates of U.K. series from 2009Q1 to the 2016Q2. Column (b) is the average of the annualized growth rates of the counterfactual recovery for each series in the same period (2009Q1–2016Q2). Units are annualized percent changes.

(2) Counterfactual recoveries are estimated by an open-economy hierarchical DFM, which adopts both U.S. factors and U.K. orthogonal factors. In particular, U.K. counterfactual recoveries are derived based on the U.S. counterfactual RAF, which is estimated by using PCA on the U.S. counterfactual series. These counterfactual series are fitted variables estimated by the DFM, where factor loadings are assumed to be stable. Thus, substituting the U.S. actual series in the NIPA, industrial production, and credit categories by faster counterfactual series means that the U.S. actual real activity factor is replaced by a faster counterfactual real activity factor. This gauges the impact of the structural break in the U.S. on the slow recovery in the U.K.

(3) The sample for estimation of the parameters is from 1971 to 2016.

(4) The average of the annual shortfalls over the current recovery, from 2009Q1 to 2016Q2, is shown in column (c), which measures the magnitude of the output shortfall spillovers from the U.S. Column (d) reports the Wilcoxon signed rank test p-value, which examines whether the actual recovery falls short of the counterfactual recovery.

(5) Column (e) represents the average of the annual shortfalls where the potential change in other U.S. factors has been taken into account. Column (f) represents the average of the annual shortfalls where the potential response of U.K. orthogonal factors to the U.S. RAS has been accommodated by using a VAR with order $P = 4$.

(6) The column (g) is the average of annual shortfalls where a structural break in both intercept and cyclical factor loadings in 2009Q1 is allowed for. Since the shortfall in this setup is close to zero, the hierarchical model performs well in estimating the counterfactual which does not leave any shortfall.

(7) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.L.14: Shortfall of the U.K. post-crisis recovery estimated by a hierarchical DFM (1985 to 2016)

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Series	Actual Recovery	Counterfactual Recovery	Annual Shortfall	Wilcoxon p-value	Annual Shortfall with change in other U.S. factors	Annual Shortfall with change in U.K. orthogonal factors	Annual Shortfall with a break in intercept and factor loadings
1. GDP/population	1.80	2.46	0.67	0.02	0.65	0.77	0.01
2. TFP	0.18	0.70	0.52	0.07	0.50	0.71	-0.00
3. $\alpha \times$ Capital/population	0.36	0.70	0.34	0.00	0.34	0.39	-0.00
4. $(1-\alpha) \times$ LQ×hours/population	1.26	1.07	-0.19	0.67	-0.19	-0.32	0.01
5. Hours/population	1.46	1.26	-0.20	0.58	-0.19	-0.34	0.02
6. Hours/employed worker	0.75	0.45	-0.30	0.67	-0.29	-0.29	0.03
7. Employment Rate	0.43	0.59	0.16	0.07	0.16	0.07	-0.00
8. Labor force participation	0.28	0.22	-0.06	0.73	-0.06	-0.12	-0.00
9. GDP/hour	0.34	1.21	0.87	0.01	0.84	1.11	-0.01
10. TFP/$(1-\alpha)$	0.28	1.15	0.87	0.05	0.84	1.18	-0.00
11. Capital/output $\times\, \alpha/(1-\alpha)$	-0.47	-0.41	0.07	0.48	0.08	0.08	-0.00
12. Labor quality	0.53	0.47	-0.06	0.78	-0.07	-0.14	-0.01

Notes:

(1) Column (a) presents the average annualized growth rates of U.K. series from 2009Q1 to the 2016Q2. Column (b) is the average of the annualized growth rates of the counterfactual recovery for each series in the same period (2009Q1–2016Q2). Units are annualized percent changes.

(2) Counterfactual recoveries are estimated by an open-economy hierarchical DFM, which adopts both U.S. factors and U.K. orthogonal factors. In particular, U.K. counterfactual recoveries are derived based on the U.S. counterfactual RAF, which is estimated by using PCA on the U.S. counterfactual series. These counterfactual series are fitted variables estimated by the DFM, where factor loadings are assumed to be stable. Thus, substituting the U.S. actual series in the NIPA, industrial production, and credit categories by faster counterfactual series means that the U.S. actual real activity factor is replaced by a faster counterfactual real activity factor. This gauges the impact of the structural break in the U.S. on the slow recovery in the U.K.

(3) The sample for estimation of the parameters is from 1985 to 2016. The start of the sample is selected 1985 because tests for breaks in the cross-block factor loadings indicate no break from 1984Q2 to 2016. The results of this table are very similar to those in Table 1.5.

(4) The average of the annual shortfalls over the current recovery, from 2009Q1 to 2016Q2, is shown in column (c), which measures the magnitude of the output shortfall spillovers from the U.S. Column (d) reports the Wilcoxon signed rank test p-value, which examines whether the actual recovery falls short of the counterfactual recovery.

(5) Column (e) represents the average of the annual shortfalls where the potential change in other U.S. factors has been taken into account. Column (f) represents the average of the annual shortfalls where the potential response of U.K. orthogonal factors to the U.S. RAS has been accommodated by using a VAR with order $P = 4$.

(6) The column (g) is the average of annual shortfalls where a structural break in both intercept and cyclical factor loadings in 2009Q1 is allowed for. Since the shortfall in this setup is close to zero, the hierarchical model performs well in estimating the counterfactual which does not leave any shortfall.

(7) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.L.15: Shortfall of the U.K. post-crisis recovery estimated by a hierarchical DFM

	Annual Shortfall without a VAR for U.K. orthogonal factors				Annual Shortfall with a VAR for U.K. orthogonal factors		
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Series	Actual Recovery	Counterfactual Recovery	Annual Shortfall	Wilcoxon p-value	VAR of order $P = 1$	VAR of order $P = 2$	VAR of order $P = 3$
1. GDP/population	1.80	2.42	0.62	0.02	0.82	0.65	0.57
2. TFP	0.18	0.62	0.44	0.09	0.84	0.63	0.52
3. $\alpha \times$ Capital/population	0.36	0.71	0.35	0.00	0.39	0.37	0.36
4. $(1-\alpha) \times$ LQ×hours/population	1.26	1.09	-0.17	0.70	-0.41	-0.34	-0.30
5. Hours/population	1.46	1.24	-0.21	0.61	-0.45	-0.40	-0.40
6. Hours/employed worker	0.75	0.45	-0.30	0.65	-0.44	-0.35	-0.30
7. Employment Rate	0.43	0.55	0.12	0.11	0.01	-0.01	-0.01
8. Labor force participation	0.28	0.25	-0.03	0.71	-0.03	-0.05	-0.09
9. GDP/hour	0.34	1.17	0.83	0.02	1.28	1.05	0.96
10. TFP/$(1-\alpha)$	0.28	1.03	0.76	0.08	1.39	1.04	0.87
11. Capital/output $\times \alpha/(1-\alpha)$	-0.47	-0.36	0.11	0.38	0.06	0.13	0.16
12. Labor quality	0.53	0.50	-0.03	0.70	-0.16	-0.11	-0.05

Notes:

(1) Column (a) presents the average annualized growth rates of U.K. series from 2009Q1 to the 2016Q2. Column (b) is the average of the annualized growth rates of the counterfactual recovery for each series in the same period (2009Q1–2016Q2). Units are annualized percent changes.

(2) Counterfactual recoveries are estimated by an open-economy hierarchical DFM, which adopts both U.S. factors and U.K. orthogonal factors. In particular, U.K. counterfactual recoveries are derived based on the U.S. counterfactual RAF, which is estimated by using PCA on the U.S. counterfactual series. These counterfactual series are fitted variables estimated by the DFM, where factor loadings are assumed to be stable. Thus, substituting the U.S. actual series in the NIPA, industrial production, and credit categories by faster counterfactual series means that the U.S. actual real activity factor is replaced by a faster counterfactual real activity factor. This gauges the impact of the structural break in the U.S. on the slow recovery in the U.K. The sample for estimation of the parameters is from 1981 to 2016.

(3) The average of the annual shortfalls over the current recovery, from 2009Q1 to 2016Q2, is shown in column (c), which measures the magnitude of the output shortfall spillovers from the U.S. Column (d) reports the Wilcoxon signed rank test p-value, which examines whether the actual recovery falls short of the counterfactual recovery.

(4) Columns (e), (f), and (g) represent the average of the annual shortfalls where the potential response of U.K. orthogonal factors to the U.S. RAS has been accommodated by using a one-step ahead forecast with a rolling origin based on a VAR with orders $P = 1, 2, 3$, respectively. In this setting, VAR model updates the forecasting origin.

(5) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.L.16: Shortfall of the U.K. post-crisis recovery estimated by a hierarchical DFM with a fixed origin

	Annual Shortfall without a VAR for U.K. orthogonal factors				Annual Shortfall with a VAR for U.K. orthogonal factors			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Series	Actual Recovery	Counterfactual Recovery	Annual Shortfall	Wilcoxon p-value	VAR of order $P = 1$	VAR of order $P = 2$	VAR of order $P = 3$	VAR of order $P = 4$
1. GDP/population	1.80	2.42	0.62	0.02	1.00	1.01	0.77	0.79
2. TFP	0.18	0.62	0.44	0.09	1.27	1.27	1.08	0.93
3. $\alpha \times$ Capital/population	0.36	0.71	0.35	0.00	0.42	0.42	0.39	0.37
4. $(1-\alpha) \times$ LQ×hours/population	1.26	1.09	-0.17	0.70	-0.68	-0.68	-0.70	-0.51
5. Hours/population	1.46	1.24	-0.21	0.61	-0.85	-0.84	-0.93	-0.68
6. Hours/employed worker	0.75	0.45	-0.30	0.65	-0.50	-0.46	-0.34	-0.26
7. Employment Rate	0.43	0.55	0.12	0.11	-0.15	-0.17	-0.29	-0.17
8. Labor force participation	0.28	0.25	-0.03	0.71	-0.19	-0.22	-0.30	-0.26
9. GDP/hour	0.34	1.17	0.83	0.02	1.85	1.85	1.70	1.47
10. TFP/$(1-\alpha)$	0.28	1.03	0.76	0.08	2.05	2.05	1.74	1.51
11. Capital/output $\times\ \alpha/(1-\alpha)$	-0.47	-0.36	0.11	0.38	-0.00	-0.01	0.10	0.06
12. Labor quality	0.53	0.50	-0.03	0.70	-0.19	-0.19	-0.13	-0.09

Notes:

(1) Column (a) presents the average annualized growth rates of U.K. series from 2009Q1 to the 2016Q2. Column (b) is the average of the annualized growth rates of the counterfactual recovery for each series in the same period (2009Q1–2016Q2). Units are annualized percent changes.

(2) Counterfactual recoveries are estimated by an open-economy hierarchical DFM, which adopts both U.S. factors and U.K. orthogonal factors. In particular, U.K. counterfactual recoveries are derived based on the U.S. counterfactual RAF, which is estimated by using PCA on the U.S. counterfactual series. These counterfactual series are fitted variables estimated by the DFM, where factor loadings are assumed to be stable. Thus, substituting the U.S. actual series in the NIPA, industrial production, and credit categories by faster counterfactual series means that the U.S. actual real activity factor is replaced by a faster counterfactual real activity factor. This gauges the impact of the structural break in the U.S. on the slow recovery in the U.K. The sample for estimation of the parameters is from 1981 to 2016.

(3) The average of the annual shortfalls over the current recovery, from 2009Q1 to 2016Q2, is shown in column (c), which measures the magnitude of the output shortfall spillovers from the U.S. Column (d) reports the Wilcoxon signed rank test p-value, which examines whether the actual recovery falls short of the counterfactual recovery.

(4) Columns (e), (f), and (g) represent the average of the annual shortfalls where the potential response of U.K. orthogonal factors to the U.S. RAS has been accommodated by using a forecast based on a VAR with orders $P = 1, 2, 3$, 4, respectively. In this setting, VAR model does not update the forecasting origin and it forecasts the U.K. factors with a fixed origin in 2009Q1.

(5) There are three types of growth-accounting practices. Rows 1-4, 5-8, and 9-12 each present a different growth-accounting. They capture the contribution of TFP, capital input, and labour force participation to the slow recovery in the U.S.

Table 1.L.17: Shortfall of the U.S. series using a 28-step ahead forecast with a fixed origin

Series	decomposed Cycle	Forecast of Cycle	Cycle Shortfall	Actual series	Forecast of series	Annual Shortfall	Standard Error
1. GDP/population	0.98	0.82	-0.15	1.72	1.99	0.27	0.09
2. TFP	0.33	0.61	0.28	0.89	1.40	0.52	0.09
3. $\alpha \times$ Capital/population	-0.14	-0.04	0.10	0.24	0.43	0.19	0.01
4. $(1-\alpha) \times$ LQ×hours/population	0.79	0.26	-0.53	0.59	0.15	-0.44	0.05
5. Hours/population	1.34	0.49	-0.85	0.63	-0.08	-0.72	0.06
6. Hours/employed worker	0.34	0.20	-0.14	0.24	0.08	-0.16	0.03
7. Employment Rate	0.68	0.26	-0.42	0.68	0.26	-0.42	0.02
8. Labor force participation	-0.19	0.00	0.19	-0.66	-0.27	0.40	0.03
9. GDP/hour	-0.36	0.34	0.70	1.09	2.07	0.98	0.08
10. TFP/$(1-\alpha)$	0.54	0.91	0.37	1.44	2.15	0.72	0.12
11. Capital/output $\times \alpha/(1-\alpha)$	-0.79	-0.48	0.31	-0.69	-0.43	0.26	0.03
12. Labor quality	-0.11	-0.09	0.02	0.33	0.34	0.01	0.04

Table 1.L.18: Shortfall of the U.S. series using a 14-step ahead forecast with a fixed origin

Series	decomposed Cycle	Forecast of Cycle	Cycle Shortfall	Actual series	Forecast of series	Annual Shortfall	Standard Error
1. GDP/population	1.06	2.07	1.01	1.96	3.23	1.27	0.30
2. TFP	0.80	2.11	1.31	1.48	2.91	1.43	0.34
3. $\alpha \times$ Capital/population	-0.38	-0.33	0.05	0.04	0.14	0.10	0.06
4. $(1-\alpha) \times$ LQ×hours/population	0.64	0.29	-0.35	0.44	0.18	-0.26	0.24
5. Hours/population	1.00	0.67	-0.33	0.29	0.10	-0.19	0.29
6. Hours/employed worker	0.64	0.65	0.02	0.53	0.54	0.01	0.15
7. Employment Rate	0.46	0.41	-0.05	0.46	0.41	-0.05	0.07
8. Labor force participation	-0.44	-0.12	0.32	-0.88	-0.38	0.50	0.14
9. GDP/hour	0.06	1.40	1.34	1.67	3.13	1.46	0.36
10. TFP/$(1-\alpha)$	1.31	3.15	1.84	2.38	4.39	2.02	0.50
11. Capital/output $\times \alpha/(1-\alpha)$	-1.24	-1.51	-0.27	-1.14	-1.46	-0.32	0.12
12. Labor quality	-0.01	-0.25	-0.23	0.43	0.19	-0.24	0.17

Notes:

(1) Table 1.L.16 replicates Table 4 of the study by Fernald et al. (2017). The counterfactuals are forecasted by the DFM assuming constant trend growth at origin. See Appendix 1.J for details. The forecast horizon covers the period from 2009Q2 to 2016Q2 (28 quarters).

(2) Table 1.L.17 decreases the forecast horizon to $h = 14$ quarters. The forecast horizon covers the period from 2009Q2 to 2012Q4 (14 quarters).

Table 1.L.19: Shortfall of the U.S. series using a 8-step ahead forecast with a rolling origin

Series	decomposed Cycle	Forecast of Cycle	Cycle Shortfall	Actual series	Forecast of series	Annual Shortfall
1. GDP/population	1.10	1.06	-0.04	1.85	2.22	0.37
2. TFP	0.26	0.71	0.45	0.82	1.50	0.68
3. $\alpha \times$ Capital/population	-0.10	-0.04	0.06	0.28	0.44	0.16
4. $(1-\alpha) \times$ LQ×hours/population	0.95	0.39	-0.55	0.75	0.28	-0.47
5. Hours/population	1.62	0.73	-0.89	0.92	0.16	-0.76
6. Hours/employed worker	0.33	0.26	-0.07	0.23	0.14	-0.09
7. Employment Rate	0.87	0.39	-0.48	0.87	0.39	-0.48
8. Labor force participation	-0.19	0.00	0.19	-0.66	-0.26	0.40
9. GDP/hour	-0.52	0.33	0.85	0.94	2.07	1.13
10. TFP/$(1-\alpha)$	0.44	1.06	0.62	1.34	2.30	0.96
11. Capital/output $\times \alpha/(1-\alpha)$	-0.82	-0.59	0.22	-0.71	-0.54	0.18
12. Labor quality	-0.14	-0.14	0.00	0.31	0.30	-0.01

Table 1.L.20: Shortfall of the U.S. series using a 4-step ahead forecast with a rolling origin

Series	decomposed Cycle	Forecast of Cycle	Cycle Shortfall	Actual series	Forecast of series	Annual Shortfall
1. GDP/population	1.11	0.97	-0.14	1.86	2.13	0.27
2. TFP	0.32	0.67	0.35	0.88	1.47	0.58
3. $\alpha \times$ Capital/population	-0.12	-0.06	0.06	0.27	0.42	0.15
4. $(1-\alpha) \times$ LQ×hours/population	0.91	0.36	-0.55	0.71	0.25	-0.46
5. Hours/population	1.57	0.67	-0.90	0.87	0.10	-0.77
6. Hours/employed worker	0.38	0.27	-0.10	0.28	0.16	-0.12
7. Employment Rate	0.82	0.35	-0.46	0.82	0.35	-0.46
8. Labor force participation	-0.18	-0.01	0.17	-0.65	-0.28	0.38
9. GDP/hour	-0.46	0.30	0.76	1.00	2.04	1.04
10. TFP/$(1-\alpha)$	0.53	1.01	0.48	1.44	2.25	0.81
11. Capital/output $\times \alpha/(1-\alpha)$	-0.84	-0.58	0.26	-0.74	-0.52	0.21
12. Labor quality	-0.15	-0.13	0.02	0.30	0.31	0.01

Notes:

(1) The counterfactuals are forecasted by the DFM assuming constant trend growth at origin. See Appendix 1.J for details. In Table 1.L.18, the forecast origin rolls from 2009Q2 to 2014Q2 with forecast horizon of 8 quarters.

(2) In Table 1.L.19, the forecast origin rolls from 2009Q2 to 2015Q2 with forecast horizon of 4 quarters.

Table 1.L.21: Shortfall of the U.S. series using a h-step ahead forecast with a rolling origin

Series/forecasting horizon (h)	1	2	3	4	8	12	16	20	24
1. GDP/population	0.37	0.29	0.30	0.27	0.37	0.58	0.57	0.47	0.36
2. TFP	0.54	0.55	0.58	0.58	0.68	0.73	0.65	0.50	0.44
3. $\alpha \times$ Capital/population	0.14	0.14	0.14	0.15	0.16	0.19	0.21	0.25	0.23
4. $(1-\alpha) \times$ LQ×hours/population	-0.31	-0.40	-0.42	-0.46	-0.47	-0.33	-0.28	-0.28	-0.31
5. Hours/population	-0.57	-0.67	-0.69	-0.77	-0.76	-0.53	-0.46	-0.47	-0.54
6. Hours/employed worker	-0.05	-0.11	-0.11	-0.12	-0.09	-0.04	-0.05	-0.13	-0.16
7. Employment Rate	-0.36	-0.39	-0.42	-0.46	-0.48	-0.41	-0.38	-0.36	-0.37
8. Labor force participation	0.34	0.37	0.37	0.38	0.40	0.43	0.45	0.47	0.43
9. GDP/hour	0.94	0.97	1.00	1.04	1.13	1.11	1.03	0.94	0.90
10. TFP/$(1-\alpha)$	0.76	0.77	0.81	0.81	0.96	1.05	0.92	0.70	0.61
11. Capital/output $\times \alpha/(1-\alpha)$	0.14	0.19	0.19	0.21	0.18	0.09	0.13	0.25	0.28
12. Labor quality	0.04	0.00	0.00	0.01	-0.01	-0.03	-0.03	-0.01	0.02

Note:

The counterfactuals are forecasted by the DFM assuming constant trend growth at origin. See Appendix 1.J for details. The forecast origin rolls from 2009Q2 with different forecast horizon of $h = 1, 2, 3, \ldots, 24$ quarters for robustness check.

Chapter 2

Friedman's Plucking Model and Okun's Law

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Abstract

We integrate Friedman's plucking model and the gap version of Okun's law by embedding output and the unemployment rate in a bivariate unobserved components model with Markov-switching to investigate their asymmetric co-fluctuations in the U.S. We demonstrate that the plucking property of unemployment, through stable Okun's law, transmits to output. Our model additionally deciphers two puzzling dilemmas in trend-cycle decomposition: First, by considering stochastic rather than deterministic trend growth, we identify an unprecedented deceleration in U.S. potential output in the aftermath of the 2007−09 financial crisis. Second, including unemployment as an auxiliary variable in the bivariate model yields a robust and insensitive estimation of parameters and components with an insignificant correlation between shocks to the trend and symmetric cyclical components, which we refer to as correlation irrelevance.

Keywords: Trend–Cycle Decomposition, Business Cycle Asymmetries, Friedman's Plucking Model, Okun's Law, Structural Break.

JEL Classification: C32, E32, F44, O47.

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2.1. Introduction

In contrast to the mainstream view in the business cycle literature, the plucking model proposed by Milton Friedman (1964, 1993) suggests an asymmetric cyclical component, meaning that output does not fluctuate around a long-run trend but instead is steeply plucked down below a ceiling known as the potential output during recessions and gradually returns toward this ceiling during recoveries.[1] This business cycle asymmetry is referred to as the plucking property or ceiling effect. Likewise, the U.S. unemployment rate does not fluctuate around the trend but is characterized by steep jumps above the natural rate of unemployment during recessions and gradual decrements to its natural level during recoveries. Few studies with statistical modelling have examined the business cycle asymmetry (see, e.g., Kim and Nelson, 1999a; Sinclair, 2010; Morley and Piger, 2012; Eo and Morley, 2022), despite its decisive implications for policymakers: stabilization policies may raise economic welfare by affecting the total average of output and the unemployment rate (DeLong et al., 1988).

Moreover, Okun’s law, first proposed by Arthur Okun (1962), is an empirical correlation between the U.S. output and the unemployment rate gaps. Although some studies state that Okun’s law has weakened over time or cast doubt on its stability during recessions (Gordon, 2010; Owyang and Sekhposyan, 2012; Grant, 2018), many other researchers conclude in favour of the stability of Okun’s law (Galí et al., 2012; Daly et al., 2014; Ball et al., 2017; Michail, 2019; among others). Hence, assuming stability of Okun’s law, fluctuations in output and the unemployment rate are synchronous and proportional; that is what we call “co-fluctuations.”

Considering Friedman’s plucking model and Okun’s law together, asymmetric fluctuations appear to be a common feature of both U.S. output and the unemployment rate. We refer to this phenomenon as “asymmetric co-fluctuations” of U.S. output and the unemployment rate. Since business cycle asymmetries are more pronounced in the unemployment rate than in output (Falk, 1986; Sichel, 1993; McKay and Reis, 2008; among others), it seems the plucking property transmits from unemployment to output. Recently, Ferraro (2018), Dupraz et al. (2019), and Ferraro and Fiori (2022), by developing an equilibrium business cycle model for the unemployment rate, document that the source of the plucking property is search frictions and nominal wage rigidity in the U.S. labour market, yet their

[1] In the symmetric business cycle jargon, contraction and expansion are two phases of business cycles. During contractions, output moves from a peak to a trough, and during expansions, output moves from a trough toward a peak. Conversely, in the asymmetric business cycle terminology, the words “recession” and “recovery” are preferred. Recessions are receding from potential output and recoveries are returning to potential output. In different studies, nevertheless, the terms “contraction” and “recession” as well as “expansion” and “recovery” have been used interchangeably to refer to the same phase of the business cycle. Besides, in some studies, a recovery is the phase in which an economy returns to its previous peak. Hence, an expansion itself consists of two phases: a recovery and a normal time, which are not accounted for in this study. In this study, we will use the terms recession and recovery to describe two phases of business cycles. The recovery phase in our study therefore corresponds to the whole period from the end of a recession to the start of a new recession.

models disregard two possibilities: (1) asymmetry in output and the transmission of the plucking property from unemployment to output; and (2) other potential sources of asymmetry, including the binding financial constraints proposed by Jensen et al. (2020), and the effect of uncertainty suggested by Caggiano et al. (2014), Jurado et al. (2015), and Ahmed et al. (2022).

As cited in Table 2.A.1 in Appendix 2.A, we review the literature of trend-cycle decomposition, namely for U.S. output and unemployment, to perceive four puzzling and influential specification aspects: (1) whether the shocks to the cyclical component are asymmetric or symmetric; (2) whether the unemployment rate should be included in a bivariate model or not; (3) whether the trend growth of U.S. output and other advanced economies is stochastic or deterministic; and (4) whether the correlation between shocks to the trend and cyclical components is zero or not. The specification of each of these four aspects might remarkably change the features and interpretation of the estimated trend and cyclical components. Hence, modelling the trend-cycle decomposition faces four dilemmas of choosing between the two alternative assumptions for each aspect that are discussed as follows.

First, almost all trend-cycle decompositions assume shocks to the cyclical component are symmetric (see, e. g., Beveridge and Nelson, 1981; Nelson and Plosser, 1982; Harvey, 1985; Clark, 1987, 1989; Morley et al., 2003; Perron and Wada, 2009; Grant and Chan, 2017a, 2017b; Grant, 2018, Kim and Chon, 2020). However, Kim and Nelson (1999a), Mills and Wang (2002), De Simone and Clarke (2007), Sinclair (2010), Morley and Piger (2012), and Eo and Morley (2022), by using an unobserved components (UC) model with a Markov-switching process similar to Hamilton (1989), conclude that output fluctuations in the U.S. and other advanced economies are asymmetric rather than symmetric, verifying Friedman’s plucking model that was previously documented by a few studies (Neftci, 1984; Goodwin and Sweeney, 1993; Sichel, 1993).

Second, most of the trend-cycle decompositions are univariate models that yield very different results depending on the choice of the other specification aspects. The approaches applied by Harvey (1985), Clark (1987), Kim and Nelson (1999a), Perron and Wada (2009), Sinclair (2010), Luo and Startz (2014), and Grant and Chan (2017a) bear a cyclical component that is large in amplitude, while the approaches employed by Beveridge and Nelson (1981), Nelson and Plosser (1982), Morley et al. (2003), Grant and Chan (2017b), Kim and Chon (2020), and Kim and Kim (2020) estimate a small and noisy cyclical component with little resemblance to the NBER chronology. Nevertheless, including the unemployment rate as an auxiliary within a bivariate UC model, which was proposed by Clark (1989), makes the features of the estimated output trend and cyclical components much less sensitive to the choice of the other specification aspects, in particular the correlation between shocks (Gonzalez-Astudillo and Roberts, 2022). In this respect, Fernald et al. (2017) and Grant (2018), based on the difference version and gap version of Okun’s law, respectively, applied the unemployment rate as a proxy for estimating the output cyclical component. Morley and Wong (2020) also substantiate that unemployment is an important variable for estimating the output gap.

Third, although many studies impose a constant-drift constraint on the trend component (see, e.g., Beveridge and Nelson, 1981; Morley et al., 2003; Sinclair, 2010; Grant, 2018), there is a plethora of evidence supporting the presence of a time-varying drift (trend growth) in output of the U.S. and other advanced economies (Antolin-Diaz et al., 2017; Fernald et al., 2017; Oulton, 2019; Dehghani et al., 2022). In fact, trend growth varies because of changes in productivity growth and technological progress. In this regard, some studies detect significant structural breaks in trend growth (Perron and Wada, 2009; Luo and Startz, 2014; Grant and Chan, 2017b; Eo and Morley, 2022), whereas others advocate that the drift (trend growth) is a stochastic process, namely a random walk (Clark, 1987; Grant and Chan, 2017a; Kim and Chon, 2020). The latter is preferred over the former because a trend component with a stochastic drift is more robust to potential misspecification when the actual process is characterized by discrete breaks (Antolin-Diaz et al., 2017). The random walk drift is flexible and can accommodate structural breaks (Grant and Chan, 2017a).

Fourth, there is a puzzling dilemma about the correlation between shocks to the trend and cyclical components. The Unobserved Components (UC) model, introduced by Harvey (1985) and Clark (1987), presumes orthogonality between shocks to the trend and cyclical components. By contrast, the Beveridge-Nelson (BN) model introduced by Beveridge and Nelson (1981) and the correlated UC model proposed by Morley et al. (2003) allow for correlation between shocks. Although the former is the restricted model, it yields intuitive results and suggests that the cyclical component is large in amplitude. Surprisingly, albeit the latter relaxes the orthogonality assumption, it leads to a counter-intuitive result and attributes most of the output fluctuations to the trend by estimating a very small and noisy cyclical component.

The reasons behind such misleading results derived by the correlated UC model are grounded in misspecification of two other aspects. First, according to Sinclair (2010), ignoring business cycle asymmetry underestimates the role of the cyclical component. Second, as attested by Clark (1989), including unemployment along with the output in a bivariate correlated UC model estimates a less significant correlation between shocks to the trend and cyclical components. Similarly, Gonzalez-Astudillo and Roberts (2022) demonstrate that once the unemployment rate is included in the model, the trend and cyclical components of the correlated UC model would be remarkably similar to those of the zero-correlated UC model because the estimated parameters are less sensitive to the potential pile-up issue related to the spurious correlation.[2]

[2] For a detailed discussion about the misspecification and the potential pile-up issue, see the methodology section.

In this study, we integrate Friedman's plucking model and Okun's law to investigate the asymmetric co-fluctuations of U.S. output and the unemployment rate. Related to the four specification aspects, we answer four questions. Regarding business cycle asymmetry, the first question is whether the fluctuations in output and unemployment are asymmetric or symmetric. Are both characterized by the plucking property? Are recessions deep and steep and subsequent recoveries commensurate with their depth and gradual? Regarding the stability of Okun's law, the second question is whether output and the unemployment rate co-fluctuate or not. By answering the first and second questions, we show that asymmetric fluctuations in output and the unemployment rate are synchronous and proportional, which affirms the asymmetric co-fluctuations. We also jointly estimate potential output as a ceiling (upper limit) for output and natural rate as a floor (lower limit) for unemployment. Furthermore, the statistical modelling of this study leads to disentangling the third and fourth puzzling specification aspects. We explore whether the drift (output trend growth) in the trend component is stochastic or deterministic. Finally, we examine if the correlation between the output trend shocks and symmetric cyclical shocks really matters in determining the estimated parameters and features of the trend and cyclical components.

Methodologically, we present a novel trend-cycle decomposition to incorporate Friedman's plucking model and Okun's law into a bivariate UC model with Markov-switching. In a preliminary step, we execute the univariate UC model of Clark (1987), the univariate correlated UC model proposed by Morley et al. (2003), the univariate UC model with Markov-switching as in Kim and Nelson (1999a) and Sinclair (2010), and the bivariate correlated UC model applied by Clark (1989) and Gonzalez-Astudillo and Roberts (2022) to replicate their results with updated data. For the benchmark model, we cast it into a bivariate state-space model with Markov-switching to jointly capture the plucking property in output and the unemployment rate. For hypothesis testing, we use a pairwise comparison of log likelihood values estimated for 39 alternative models.

The estimation results establish the presence of asymmetric co-fluctuations; indeed, grounded on the stable gap version of Okun's law, we find that output and the unemployment rate are synchronously and proportionally characterized by the plucking property. The related results are briefly presented as follows.

First, our model captures the plucking property in both economic indicators because the estimated coefficients of the plucking property and Okun's law are both substantial ($\pi_u = 0.70$ and $\beta = -1.45$ with standard errors of 0.06 and 0.12). Estimated gaps that are large in magnitude and often negative for output and positive for unemployment verify the ceiling effect: output seldom ascends above the ceiling (potential output) and, likewise, the unemployment rate seldom descends below the floor (natural rate). Further, the expected duration is about 3 quarters for recessions and 28 quarters for recoveries, implying short recessions and long recoveries. Overall, the co-fluctuations tend to be asymmetric in amplitude, speed, and duration, which supports the idea that deep, steep, and transitory recessions will be followed by commensurate, gradual, and permanent recoveries. These results are in accordance with the findings of Neftci (1984), Sichel (1993), Friedman (1993), Kim and Nelson (1999a), Sinclair (2010), Morley and Piger (2012), and Eo and Morley (2022), who have tested cycle asymmetry for output and the unemployment rate separately. It is also consistent with the theoretical works of Ferraro (2018), Dupraz et al. (2019), and Ferraro and Fiori (2022), who characterize the asymmetry in the unemployment rate. Our results, nonetheless, refute the empirical results of small and noisy cyclical components derived by Beveridge and Nelson (1981) , Nelson and Plosser (1982), Morley et al. (2003), Grant and Chan (2017b), Kim and Chon (2020), and Kim and Kim (2020), who view fluctuations as symmetric movements around a natural level.[3]

Second, we demonstrate that the gap version of Okun's law is stable. Accordingly, given that Ferraro (2018), Dupraz et al. (2019), and Ferraro and Fiori (2022) identify the labour market as the source of the plucking property, we conclude that the plucking property transmits from the unemployment rate to output. The result of the stability of Okun's law is in line with the findings of Ball et al. (2017) and Michail (2019), among many others, while it is in opposition to Berger et al. (2016) and Grant (2018), among a few others, who report instability of Okun's law.[4] The reason behind the results derived by the latter studies llies in their underlying assumptions suggesting that output cyclical component is symmetric and trend growth is deterministic. These assumptions, however, have been called into question by ample evidence pointing to asymmetry in the business cycle and a decline in trend growth of U.S. output. In fact, the unaccounted for asymmetry in the cycle and unaccounted for instability in trend growth are reflected in the Okun's law coefficient, bearing a misleading result about the instability of Okun's law. Our findings indeed indicate that Okun's law is satisfactorily stable whenever the model heeds the first and third specification aspects.

Third, consistent with the suggestion of Antolin-Diaz et al. (2017), we corroborate the presence of a stochastic trend growth in U.S. output. In this regard, similar to Grant and Chan (2017a), Fernald et al. (2017), Kim and Chon (2020), and Dehghani et al. (2022), we document a gradual decline in trend growth, which started in the 1960s. Further, we observe that this decline has been exacerbated by an

[3] Since our findings strongly favour Friedman's plucking model over other symmetric alternatives, we believe that an effective stabilization policy can raise the average output. This is in line with the opinions of Keynes (1936), Friedman (1964, 1993), and DeLong et al. (1988), and is in contrast to the implications of the real business cycle (RBC) introduced by Kydland and Prescott (1982) and Long and Plosser (1983), who see output fluctuations as a series of symmetric real technology shocks.

[4] The result of stability of Okun's law is also consistent with that of Sögner and Stiassny (2002), Daly et al. (2011), Galí et al. (2012), Daly et al. (2014), and Economou and Psarianos (2016), while it is in contrast to that of Owyang and Sekhposyan (2012), Basu and Foley (2013), and Valadkhani and Smyth (2015). In the methodology section, we explain why the result of instability of Okun's law is spurious and how it emanates from misspecification of other aspects.

unprecedented deceleration in U.S. potential output in the aftermath of the 2007−09 global financial crisis, which is consistent with the finding of a structural break around 2007 in trend growth by Luo and Startz (2014), Grant and Chan (2017b), Eo and Morley (2022), and Dehghani et al. (2022).[5]

Fourth, bivariate models that include both output and the unemployment rate are moderately robust to the assumption about the correlation between shocks to the trend and cyclical components. This means that cyclical components are large in amplitude no matter whether the correlation is assumed to be zero or not, which was previously reported by Clark (1989) and Gonzalez-Astudillo and Roberts (2022). Moreover, we substantiate that the asymmetric bivariate model, which encompasses both asymmetry and co-fluctuations, yields robust results with an insignificant correlation, which we refer to as correlation irrelevance.

To establish the robustness of our benchmark model, we also estimate several models with alternative specifications. In particular, a structural break is allowed to accommodate the potential instability of the Okun's law coefficient, the volatility of the shocks to the remaining cyclical component, output trend growth, and the drift in the unemployment trend. To determine the unknown break, we estimate the likelihood ratio statistics for a sequence of break dates rolling from 1960 to 2010, and compare their supremum with a reasonable threshold such as the critical values for the Quandt Likelihood Ratio (QLR) test. We conclude in favour of the stability of the gap version of Okun's law. The model also explains the great moderation, the decrease in the volatility of the output-specific cyclical shock after 1983Q1. For output trend growth, we observe a sequence of considerable breaks in trend growth that occurred repeatedly in every period from the mid-1960s to 2010. This accords with the idea that the trend growth in the U.S. is gradually declining over time, implying the presence of stochastic trend growth. Furthermore, the likelihood ratios are exceedingly remarkable during the 2000s, which spiked around the 2007−09 financial crisis, reaffirming a momentous deceleration in U.S. potential output. We also report a significant break in the drift term of the unemployment trend (natural rate of unemployment) that occurred in 1981Q1.

This study makes five contributions to the literature. First, the remarkable policy implications of business cycle asymmetries and the stability of Okun's law place a great deal of importance on the concept of asymmetric co-fluctuations of two macroeconomic indicators. This is the first study to simultaneously characterize the plucking property in U.S. output and the unemployment rate by integrating Friedman's plucking model and Okun's law. This allows us to capture the transmission of the plucking property from unemployment to output.

[5] Additionally, our results hint at an unusual persistency in the output gap following the 2007−09 financial crisis, which is in line with the findings of Fatás and Mihov (2013), Reinhart and Rogoff (2009, 2014), Bordo and Haubrich (2017), Michau (2018), and Dehghani et al. (2022). This underscores the influence of the financial crisis on U.S. output growth.

Second, in terms of methodology, we augment the univariate UC models with Markov switching, presented by Kim and Nelson (1999a), Sinclair (2010), and Eo and Morley (2022) by including the unemployment rate within a bivariate UC model. Besides, we augment the bivariate UC models of Clark (1989), Grant (2018), and Gonzalez-Astudillo and Roberts (2022) by incorporating a Markov-switching process into the model. Third, to the best of our knowledge, the bivariate state-space model with Markov-switching presented in this study has never been used before to explain the asymmetric co-fluctuations of any other two variables. Further, our model deciphers two puzzling dilemmas related to the third and fourth aspects of trend-cycle decomposition. We emphasize that trend growth in the U.S. is stochastic and the correlation between shocks to the trend and cyclical components is irrelevant.

Fifth, jointly estimating the trends of output and the unemployment rate while the plucking property is accounted for, offers a new measure for the natural rate as the lower limit of unemployment rate, which is linked to potential output as the upper limit of production. Based on Okun's law, when the unemployment rate is equal to the natural rate, the economy is at full employment and working at its full capacity, where actual output is equal to potential output, and so output gap is zero. Accordingly, since in our model, the natural rate is the unemployment rate at which the output gap is zero, we call it the Zero Output Gap Rate of Unemployment (ZOGRU). This new measure can be used as a reliable substitute for the Non-Accelerating Inflation Rate of Unemployment (NAIRU), whose estimation has been called into question by Mishkin and Estrella (2000) and Heimberger et al. (2017).[6] Finally, the estimated gap for economic activities is useful for a new estimation of Taylor rule (1993) with the aim of setting monetary policy interest rate.

The remainder of this paper reviews the literature on asymmetric business cycles and Okun's law in Section 2.2. Section 2.3 describes the data and explain the methodology, univariate models for output and unemployment, and the bivariate model. This section also justifies our choices in the benchmark model regarding four specification aspects of trend-cycle decomposition. Section 2.4 presents the results and discussions for the benchmark asymmetric bivariate model as well as alternative models, and we finally provide a conclusion in Section 2.5.

[6] The natural rate of unemployment, first proposed by Phelps (1967) and Friedman (1968), is the unemployment rate which would prevail in the absence of any cyclical fluctuations and is independent from temporary and seasonal fluctuations. In this regard, NAIRU measures the natural rate, according to the Phillips curve, by estimating the unemployment rate at which the inflationary acceleration is zero. On the other hand, ZOGRU measures the natural rate, according to Okun's law, by estimating the unemployment rate at which the output gap is zero. Since Okun's law is certainly more reliable than the Phillips curve (Ball et al., 2017), ZOGRU offers a better estimation of the natural rate of unemployment.

2.2. Literature review

This study is related to the three branches of the existing literature: the business cycle, Okun's law, and trend-cycle decomposition. In this section, we first describe the controversy between two schools of thought in the business cycle literature. We then turn to the disagreement on the stability of Okun's law. We argue that Okun's law is alive and the reason behind the result of its instability is model misspecification. Finally, the trend-cycle decomposition, four specification aspects, and our model are explained in the methodology section.

2.2.1. Business cycles: Friedman's plucking model versus real business cycles

There are two strands in the business cycle literature. The dominant strand supposes output fluctuates symmetrically around a trend known as the natural level. In this view, the peak of an expansion is above the trend meaning that the economy can produce more than its natural level, and at the trough of a recession, output is below its natural level. On the contrary, Friedman's plucking model (1993) suggests business cycle asymmetry by considering a ceiling of maximum feasible output referred to as the potential output determined by available resources. In this view, output cannot go above this ceiling and most of the time it is close to potential output except that occasionally it is plucked down by abrupt negative shocks during recessions. Then, during the subsequent recoveries, output tends to gradually return toward its potential output through a series of self-equilibrating forces known as the "bounce-back" effect. In this context, therefore, recessions and recoveries refer to periods of time when output recedes from and returns to its potential capacity.

Concerning the first strand, the Real Business Cycle (RBC) model, introduced by Kydland and Prescott (1982) and Long and Plosser (1983), is heavily built on the premise that output shocks are symmetric. Under this premise, the RBC identifies technological shocks as the main drivers of fluctuations. However, although technological shocks play a role, the RBC overlooks the role of adverse events such as wars, oil crises, financial crises, and the COVID-19 pandemic in shaping recessions and the responsive monetary policy in fostering the subsequent recoveries. In addition, most of the empirical literature on trend-cycle decomposition views fluctuations as symmetric movements around a natural level. Some examples include the UC models of Harvey (1985) and Clark (1987), the BN model of Beveridge and Nelson (1981), and the correlated UC model of Morley et al. (2003), among others. As a result, the RBC and trend-cycle decomposition, by imposing symmetry on business cycles, tend to ignore the importance of asymmetric shocks in explaining economic fluctuations.

On the contrary, the second strand that suggests business cycle asymmetry, has received less attention compared to the symmetric business cycle models. Informally, business cycle asymmetry has been observed by Mitchell (1927, p. 333) and Keynes (1936), who noted that recessions take place briefly and violently, whereas there are no such sharp turning points during expansions. Friedman (1964) also viewed output as bumping along the ceiling of maximum feasible output except that every now and then it is plucked down by cyclical contractions. Afterwards, output gradually returns to the ceiling potential. To provide supporting evidence for the plucking property, he documented that the correlation between the amplitude of recessions and recoveries is asymmetric: the amplitude of recessions is strongly correlated with the amplitude of succeeding expansions, while the correlation between the amplitude of expansions and the amplitude of the succeeding recessions is insignificant. After a long period of oblivion, Friedman (1993) reaffirmed the idea of the plucking model by observing an asymmetrical correlation pattern in the U.S. and other seven advanced economies.

The policy implications of these two perspectives are radically conflicting. Under the symmetric cycle assumption, stabilization policy does not raise the average of output; hence the welfare gain of the stabilization policy is negligible (Lucas, 1987; 2003). In line with the neoclassical view, RBC models also view output fluctuations as the Pareto optimal responses of the household to productivity shocks but not as welfare-reducing deviations from some ideal path (Kydland and Prescott, 1982; Long and Plosser, 1983). However, similar to Keynes (1936), who said that contractions are shorter but sharper than expansions, the plucking model states that deep, steep, and transitory recessions take place because of occasional adverse events, rather than self-generating cyclical processes, and the subsequent recoveries are commensurate with previous recession depth, gradual, and permanent. Therefore, in line with the New-Keynesians, stabilization policy aims to not only dampen the fluctuations but also raise the average level of output.

2.2.2. Business cycle asymmetries

Reviewing the limited literature on business cycle asymmetry, we distinguish four asymmetries: correlation, deepness, steepness, and duration asymmetries.[7] While these four types of asymmetries are explained separately in several studies to focus on one aspect of the general concept, they are tightly related to each other so that they together describe the same phenomenon, Friedman's plucking property.

The first asymmetry investigated by Friedman (1964, 1993) to support the plucking model is the correlation asymmetry: the amplitude of recessions is strongly correlated with the amplitude of succeeding expansions, but the amplitude of expansions is uncorrelated with the amplitude of the succeeding recessions. The correlation asymmetry is in accordance with the ceiling effect. When output is plucked down by negative shocks during the current recession, the depth of the recession

[7] See Table 2.A.2 in Appendix 2.A for a detailed explanation.

can vary depending on the severity of negative shocks. Thus, the amplitude of the previous expansion is unrelated to the amplitude of the current recession. Afterward, when the subsequent recovery starts, output cannot go above a ceiling named "potential output," so the amplitude of the subsequent expansion tends to be correlated with the amplitude of the current recession.

An alternative expression of this asymmetry states that the deeper the recessions, the stronger the subsequent recoveries, which is empirically supported by several studies for U.S. output (Friedman, 1993; Wynne and Balke, 1992; Beaudry and Koop, 1993; Fatás and Mihov, 2013). Goodwin and Sweeney (1993) and Fatás and Mihov (2013) also provide substantial support for the ceiling effect in the U.S. and other advanced economies. In addition, by analysing 26 episodes of business cycles beginning in 1882 and ending with the Great Recession, Bordo and Haubrich (2017) confirm that the recovery of output is stronger following those recessions that are deep and coincide with financial crises. Recently, Dupraz et al. (2019) present evidence that the U.S. unemployment rate displays a striking plucking property, meaning that the amplitude of recessions forecasts the amplitude of the subsequent recoveries but not vice versa.

The second and third asymmetries are deepness and steepness. Deepness signifies that recession troughs are deep whereas expansion peaks are small in magnitude, and steepness means that the recessions are steep (violent) and expansions are gradual (mild). To test these two asymmetries in a time series, Sichel (1993) suggests measuring the distributional asymmetry of the series itself and its difference. Output, exhibits deepness asymmetry if it displays negative skewness relative to the trend. Similarly, output exhibits steepness if its first difference displays negative skewness.[8] In this regard, Sichel (1993) and Goodwin and Sweeney (1993) document a significant negative skewness in the distribution of the de-trended output. Recently, Jensen et al. (2020), by comparing the skewness of the growth of real output before and after 1984, detect a more negative skewness for the U.S. and other advanced economies, suggesting a deepening business cycle asymmetry. They identify binding financial constraints during recessions as one of the sources of U.S. business cycle asymmetry and its evolution over time.

Further, Neftci (1984), Sichel (1993), and Dupraz et al. (2019) show that the unemployment rate distribution displays a remarkable positive skewness[9]. These results indicate that, during recessions, output falls deeply and the unemployment rate jumps sharply, whereas during recoveries, they both gradually return to their trend.

[8] Because the unemployment rate is a counter-cyclical time series, we define deepness and steepness asymmetries for it to be compatible with those of output. Precisely, the unemployment rate exhibits deepness asymmetry if it displays positive skewness and it exhibits steepness if its first difference displays positive skewness.

[9] Ramsey and Rothman (1996) also, by relating the concept of time reversibility to deepness and steepness, find deepness and steepness asymmetries in output and the unemployment of the U.S. and other advanced economies.

Finally, the duration asymmetry states that recessions are short and recoveries are long. Given that recessions are deep, clearly, the duration asymmetry is comparable to the steepness asymmetry. Regarding the duration asymmetry, for the first time, Neftci (1984) applied a Markov process to compare the transition probabilities between contraction and expansion states. He concluded that unemployment is characterized by sudden jumps during contraction and gradual decrements during expansion.[10]

Empirically, excluding some basic statistical evidence provided by the abovementioned studies, there are very few studies that have examined the plucking model by developing a rigorous econometric model. For the first time, Kim and Nelson (1999a) by developing a state-space model with Markov-switching, have investigated the asymmetric fluctuations in U.S. output and concluded in favour of Friedman's plucking model against the symmetric alternatives. Mills and Wang (2002) and De Simone and Clarke (2007) also provide international evidence on the validity of the plucking model. Later, Sinclair (2010), by incorporating correlation between shocks to the trend and cyclical components, demonstrates that ignoring business cycle asymmetry underestimates the amplitude of the cyclical component.

Theoretically, Ferraro (2018), Dupraz et al. (2019), and Ferraro and Fiori (2022) by developing an equilibrium business cycle model, suggest that the unemployment rate does not fluctuate around a trend but instead is characterized by steep jumps above the natural rate of unemployment during recessions. They conclude that search frictions and downward nominal wage rigidity in the U.S. labour market are the main sources of the asymmetry.[11] Beforehand, DeLong and Summer (1984), Falk (1986), Sichel (1993), and McKay and Reis (2008) documented that business cycle asymmetries (correlation, deepness and steepness) are more pronounced in the unemployment rate than in output, implying that the source of the plucking property is the U.S. labour market.

Considering all asymmetries and Friedman's plucking model together, fluctuations in output and the unemployment rate appear to be asymmetric in amplitude, speed, and duration, which means that the recessions are deep, steep, and transitory while subsequent recoveries are proportional to the previous depth, gradual, and permanent. Also, output seldom goes above the ceiling known as potential output. Likewise, the unemployment rate seldom goes below a floor called the natural rate.

[10] Note that duration asymmetry and duration dependence are different concepts. Duration asymmetry means that recessions are short and expansions are long. On the other hand, duration dependence states that a longer expansion period increases the likelihood of the occurrence of a recession. Diebold & Rudebusch (1990) find little evidence for duration dependence in the U.S., which supports the assumption that Markov transition probabilities are constant.

[11] Another theoretical work on business cycle asymmetry is a model of firm's investment choice conducted by Acemoglu and Scott (1997). This model attributes the high persistency in the output recovery to the intertemporal increasing returns to scale.

2.2.3. Okun's law

Regarding the second branch, Okun's law is an empirical relationship between output and the unemployment rate fluctuations, which is well-established for different countries (Ball et al., 2017). The gap version of Okun's law is the relationship between the deviation of output from potential output (output gap) and the deviation of the unemployment rate from its natural rate (unemployment gap), while the difference version of Okun's law explains the relationship between output growth and unemployment rate change.[12] Although some studies cast doubt on its stability during recessions and subsequent recoveries, specifically the Great Recession (see, e.g., Gordon, 2010; Owyang and Sekhposyan, 2012; Basu and Foley, 2013; Valadkhani and Smyth, 2015; Berger et al., 2016; Grant, 2018), the bulk of the literature conclude in favour of the stability of Okun's law in the U.S. as well as the U.K. and other advanced economies (Sögner and Stiassny, 2002; Daly et al., 2011; Galí et al., 2012; Daly et al., 2014; Economou and Psarianos, 2016; Ball et al., 2017; Michail, 2019). Indeed, the rumours around the death of Okun's law are greatly exaggerated, and the deviations from Okun's law are small and short- lived (Daly et al., 2014; Ball et al., 2017).

The literature provides enough evidence to support the stability of Okun's law, which confirms the co-fluctuations of U.S. output and the unemployment rate. Nevertheless, we argue that the reported instability of Okun's law by some studies is attributable to model misspecification, namely ignoring two features of U.S. output: the asymmetry in the cyclical component and the decline in trend growth.[13] If these two features are not accounted for, their traces will be revealed in the form of a spurious instability of Okun's law.[14]

Overall, considering two branches of the literature, Friedman's plucking model and Okun's law together, asymmetric fluctuations must be a common feature of both output and the unemployment rate, which is referred to as "asymmetric co-fluctuations" of U.S. output and the unemployment rate.

[12] By taking the difference of the gap version of Okun's law, assuming that the growth of potential output (output trend growth) is constant, and change in natural rate of unemployment is zero, one can obtain the difference version of Okun's law.

[13] Asymmetric fluctuations in output is documented by many studies (see, Neftci, 1984; Hamilton, 1989; Sichel, 1993; Kim and Nelson, 1999a; Sinclair, 2010; Fatás and Mihov, 2013; Bordo and Haubrich, 2017; among others). Regarding the second feature, Antolin-Diaz et al. (2017), Fernald et al. (2017), Grant and Chan (2017a), and Dehghani et al. (2022), observe a decline in the U.S. trend growth.

[14] For a detailed discussion, see the methodology section.

2.3. Data and methodology

This study makes use of the data of two economic indicators available in FRED (Federal Reserve Economic Data), including the seasonally adjusted real gross domestic product (GDPC1) and the unemployment rate for people aged 16 and over (UNRATE). The quarterly sample period runs from 1948Q1 to 2019Q4, though we will extend the data until 2022Q1 to explore the COVID-19 recession. We use the natural log of quarterly real GDP multiplied by 100 and the quarterly unemployment rate as two observed series in the model. We calculate the quarterly unemployment rate as the average of the unemployment rates of the three months within the corresponding quarter. For example, the rate of unemployment for the first quarter is the average of the unemployment rates for January, February, and March. Alternatively, to control for the lead-lag effect between output and the unemployment rate, we calculate the leading quarterly unemployment rate by finding the average of three months, two of which are within the same quarter and the other one is in the subsequent quarter. As an illustration, the leading unemployment rate for the first quarter is calculated as the average of the unemployment rates for February, March, and April. We additionally apply the benchmark model to U.S. real GDP per capita and U.K. real GDP.

In an initial step described in Section 2.3.1, we design a univariate UC model with Markov-switching in the spirit of the plucking model presented by Kim and Nelson (1999a), Sinclair (2010) and Morley and Piger (2012), where the asymmetry is modelled by embedding a Markov-switching process into the cyclical component. We apply this model, separately, to U.S. output and the unemployment rate to gain insight into the asymmetric fluctuations of these two indicators. For pairwise comparison, we impose the plucking coefficient to be zero, in order to estimate several nested models, including the univariate uncorrelated UC model of Clark (1987), the univariate correlated UC model of Morley et al. (2003), the univariate UC model with a break in trend growth proposed by Perron and Wada (2009), and the univariate correlated UC model with a break in trend growth presented by Grant and Chan (2017b).

In the benchmark model, discussed in Section 2.3.2, we incorporate Friedman's plucking model and the gap version of Okun's law into an asymmetric bivariate model to examine the asymmetric co-fluctuations. To model the asymmetry, we include a Markov-switching process in the unemployment cyclical component with the aim of capturing the plucking property. To model the co-fluctuations, we apply a gap version of Okun's law, where the unemployment rate is placed on the right-hand side, with the intention of capturing the transmission of the plucking property from the unemployment rate to U.S. output. For pairwise comparison, we estimate several models, including the benchmark asymmetric bivariate model as well as the symmetric bivariate model applied by Clark (1989) and Gonzalez-Astudillo and Roberts (2022), by restricting the plucking coefficient to be zero.

In this study, considering different combinations of four specification aspects, we estimate fourteen univariate models for output, four univariate models for the unemployment rate, and twenty-one bivariate models. The detailed specifications of each model are presented in Tables 2.B and 2.C in Appendices 2.B and 2.C. We cast each of the above models in a state-space form to estimate models using Kalman's (1960) filter. In symmetric models, we use the maximum likelihood method. For asymmetric models in the presence of the Markov-switching process proposed by Hamilton (1989), we use Kim's (1994) approximate maximum likelihood method to make the Kalman filter operable.[15] We finally test the benchmark model against other alternatives by reporting coefficients and their standard errors, deriving likelihood ratios based on pairwise comparisons, and running residuals diagnoses.

We explore the robustness of the benchmark model by estimating models with structural breaks in parameters. To determine the unknown break date, we first truncate the first and last ten years (15%) of the sample. Then, we sequentially estimate the log likelihood values for a sequence of potential break dates that roll from 1960Q1 to 2010Q1 within the whole sample from 1950 to 2020. We then compute the likelihood ratio values by comparing the estimated log likelihood values for unrestricted model with the value of the restricted model. We finally compare the supremum of likelihood ratios with a reasonable threshold such as the Quandt Likelihood Ratio (QLR) critical values, presented in Andrews (1993), to identify structural breaks.[16]

2.3.1. Justification of model specifications

In this section, by reviewing the current literature on the trend-cycle decomposition, we justify four specification aspects selected in the benchmark model of this study: (1) the cyclical component is asymmetric; (2) the unemployment rate is included within a bivariate model in light of the stability of Okun's law; (3) the trend growth is stochastic; and (4) the correlation between shocks to the trend and the cyclical component is irrelevant. We will also discuss the consequences of neglecting these aspects.

First, the mainstream literature of both empirical and theoretical macroeconomics ignored business cycle asymmetries despite having been documented by Neftci (1984), Sichel (1993), Goodwin and

[15] For more explanation, especially the state-space representation of models, see Appendices 2.B and 2.C. For estimation methods, initial values for parameters and state variables, see Appendices 2.D and 2.E. See chapters 3-5 of Kim and Nelson (1999b) and chapters 13 and 22 of Hamilton (1994) for details on the estimation method.

[16] Since we take the supremum of the log likelihood ratio values, the critical values to test for the structural break are considerably larger than those of the general likelihood ratio test. We therefore use QLR critical values. In addition, in the presence of Markov-switching, because we apply an approximate maximum likelihood method, the critical values for estimated likelihood ratios appear to be different from the QLR statistics. Hence, to have a reliable threshold for identifying remarkable breaks, we use very conservative critical values, such as 0.1% likelihood ratio critical value and 1% QLR critical value.

Sweeney (1993), and Kim and Nelson (1999a). We, thus, accommodate asymmetries by including a Markov-switching process in the cyclical component, which is sufficient to capture all four types of asymmetries. A significant plucking coefficient with the addition of estimating output gaps that are often negative and rarely positive confirms the ceiling effect and the plucking property. In addition, estimating deep output gaps, short recessions, and long recoveries implies deepness, steepness, and duration asymmetries.

Second, to model the co-fluctuations of output and the unemployment rate, we use the gap version of Okun's law, which states a linear relationship between the output gap and the unemployment rate gap. Depending on the researcher's purpose, some studies put the unemployment rate on the left (see, e.g., Sögner and Stiassny, 2002; Gordon, 2010; Owyang and Sekhposyan, 2012; Berger et al., 2016; Ball et al., 2017), and some others put the unemployment rate on the right side of Okun's law (see, e.g., Daly et al., 2014; Valadkhani and Smyth, 2015; Economou and Psarianos, 2016; Fernald et al., 2017; Grant, 2018; Dehghani et al., 2022). In this study, we place unemployment on the right side of Okun's law, which has three advantages. First, because the U.S. labour market is identified as the source of the plucking property (Ferraro, 2018; Dupraz et al., 2019; Ferraro and Fiori, 2022), our model can capture the transmission of the plucking property from the U.S. unemployment rate to output. Second, because the time-variation in the natural rate of unemployment is mild (Fernald et al., 2017), it properly measures the cyclical fluctuations. Third, including the unemployment rate as an auxiliary variable in the bivariate UC model makes the features of the estimated output trend and cyclical components much less sensitive to the choice of the other specification aspects and more robust to the potential pile-up issue (Clark, 1989; Gonzalez-Astudillo and Roberts, 2022). As a result, we include unemployment in the bivariate model with Markov-switching to facilitate capturing the asymmetric fluctuations and the plucking property in output.

Including the unemployment rate in the bivariate model is grounded on the stability of the gap version of Okun's law. As explained in the literature review, all but a few studies support the stability of Okun's law. We argue that those few studies that suggest instability of Okun's law are subject to caveats of misspecification. Their results are artifacts of neglecting two striking features of U.S. output related to the first and third specification aspects: the asymmetry in the cyclical component and the decline in trend growth.

Concerning time-variation in trend growth, the gap or difference version of Okun's law applied by Owyang and Sekhposyan (2012), Basu and Foley (2013), and Grant (2018) imposes a deterministic trend growth. This assumption is not innocuous because there is evidence implying a gradual decline in the drift (trend growth) in the U.S. and other advanced economies. Consequently, if the stochastic drift is the true model and the deterministic drift is the false model, it is highly likely that the declining drift (trend growth), which is not accounted for, reflects itself in the form of a spurious instability in the Okun's law coefficient.[17]

Concerning the asymmetry in the cyclical component, the model presented by Berger et al. (2016), although it allows for stochastic trend growth, imposes a symmetric business cycle. Since there is evidence for the plucking property, this model fails to precisely gauge the depth of recessions. As a result, it finds that while an Okun's law coefficient is properly constant during normal times, it drops and bounces back during recessions. Consequently, the unaccounted for bounce-back effect in the output cyclical component is reflected in the Okun's coefficient. Not surprisingly, they have reported that Okun's coefficient tends to quickly return to the historical average during normal times. The instability of Okun's law reported by Valadkhani and Smyth (2015) appears to be spurious because of a combination of model misspecifications.[18]

Conclusively, Okun's law is alive and stable (Daly et al., 2014; Ball et al., 2017), and the finding of instability in Okun's law emanates from two restrictive assumptions. To address these shortcomings, it is necessary to allow for both asymmetric fluctuations and stochastic trend growth in the U.S. With these two specification aspects being accommodated, the stable Okun's law captures the synchronous and proportional fluctuations of output and the unemployment rate.

Third, in the benchmark model, we specify a stochastic trend growth in the form of a random walk to accommodate both the gradual decline and structural breaks in trend growth because a random walk is robust to misspecification and capable of accommodating potential breaks (Antolin-Diaz et al., 2017). This specification also copes with the problem of spurious instability in the Okun's law coefficient, which is explained above. We alternatively model the trend growth in the form of a non-stochastic drift with a structural break for the robustness test.

Fourth, the correlated UC model of Morley et al. (2003) estimates a close-to-unity correlation and bears economically unimportant and noisy cyclical components. This model is subject to the caveat

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[17] Grant (2018) reports contradictory results about the stability of the Okun's law coefficient. Although some results support a substantial time-variation in the Okun's coefficient before the onset of the Great Recession, she indicates that the probability of a shift in the Okun's coefficient is significant only during and after the Great Recession. Given that she has not accommodated the time-variation in trend growth, this result is actually analogous to our results confirming adequate stability of the gap version of Okun's law, whereas there is an unprecedented trend deceleration in the aftermath of the 2007−09 financial crisis.

[18] The model proposed by Valadkhani and Smyth (2015) has three misspecifications. First, the stochastic output trend growth is not accommodated as the trend is estimated by Hodrick and Prescott (1977) filter. Second, the plucking property is not included to identify the depth of the recession precisely. Third, they impose a single Markov-switching process to explain the regime-switching in both Okun's law coefficient and output shock volatility, yet no evidence supports this idea that Okun's law and volatility have the same regime-switching timing and dynamics. As a result, it is not surprising to find a structural break in 1982, which coincides with the great moderation. In fact, the regime-dependence of the volatility prevails over the stability of Okun's law: a significant change in the volatility of output shock after 1982 is the dominant feature related to the regime-switching that enforces the Markov process to switch accordingly.

of spurious correlation, which is attributable to misspecification of the two other aspects and the pile-up problem. Regarding the former, Sinclair (2010) suggests that imposing symmetry on the business cycle underestimates the magnitude of cyclical components. However, as said by Clark (1989) and Gonzalez-Astudillo and Roberts (2022), by including the unemployment rate in the bivariate model, the correlated UC model bears remarkably similar trend and cyclical components to those of the zero-correlated UC model. Moreover, the correlated UC model suffers from the pile-up problem, which arises due to the model misspecification. By using a Monte Carlo simulation, Basistha (2007) and Gonzalez-Astudillo and Roberts (2022) find that the univariate trend-cycle decomposition suffers from the pile-up problem, meaning that the estimated correlation tends to pile-up toward positive or negative unity even though the true correlation parameter is zero. Iwata and Li (2015) also say that the ARIMA model proposed by Morley et al. (2003) cannot rule out the possibility that actual U.S. output is generated by the uncorrelated UC model. Wada (2012) evinces that the perfect correlation derived in the correlated UC model is artificially created due to the additional restriction imposing that the variance-covariance matrix of the shocks should be positive-semi-definite.[19]

Altogether, the bivariate model in which the asymmetry is accommodated and the unemployment rate is also incorporated, would be robust to the caveat of spurious correlation. As a result, in the benchmark model, we assume the orthogonality of all shocks. To inspect the correlation irrelevance, we also estimate alternative models by allowing for correlation between each pair of shocks. We show that the estimated correlations are insignificant and small and do not change the features of the estimated trend and cyclical components.

2.3.2. The univariate model

In line with the literature of univariate trend-cycle decomposition, consider Eq. (2.1), where a single variable of interest, either output or the unemployment rate, is decomposed into a trend and a cyclical component:

$$z_t = z_t^* + z_t^c \tag{2.1}$$

where the observed series is denoted by z_t. Accordingly, z_t^* and z_t^c are unobserved trend and cyclical components. If we intend to decompose the log level of output, z_t^* and z_t^c play the roles of potential output and the output gap; and if we decompose the unemployment rate, z_t^* and z_t^c play the roles of the natural rate of unemployment and the unemployment gap, respectively.

[19] Kim and Kim (2020) find another pile-up problem related to the variance of the trend component. They state that the estimated variance of the shocks to the trend component using maximum likelihood tends to pile-up toward zero, although the true parameter is not zero.

2.3.2.1. The trend component

We consider that the trend component is a random walk process with a drift:

$$z_t^* = \mu_{t-1} + z_{t-1}^* + \varepsilon_{z^*,t} \tag{2.2}$$

where μ_t is the drift (trend growth) that might be time-varying due to changes in productivity growth and technological progress, and $\varepsilon_{z^*,t}$ is the trend shock. In this study, we assume that all of the shocks (also called innovations or disturbances) are white noise and normally distributed. For the trend shock, for example, we have $\varepsilon_{z^*,t} \sim N(0, \sigma_{z^*}^2)$. Concerning U.S. output, since a decline in trend growth is noted by Grant and Chan (2017a), Antolin-Diaz et al. (2017), Fernald et al. (2017), and Dehghani et al. (2022), we specify output trend growth as a random walk:

$$\mu_t = \mu_{t-1} + \varepsilon_{\mu,t} \tag{2.3.a}$$

where $\varepsilon_{\mu,t} \sim N(0, \sigma_\mu^2)$ stands for the shock to the trend growth and it is uncorrelated with other shocks. Alternatively, following Perron and Wada (2009), and Grant and Chan (2017b), we can model the trend growth as a non-stochastic drift with a structural break:

$$\mu_t = \gamma + \delta \mathbb{1}_t (t \geq T_\mu) \tag{2.3.b}$$

In this set up, $\mathbb{1}_t$ is an indicator function that takes the value of one after the break date (T_μ), and zero otherwise. Hence, γ is the trend growth before the break and $(\gamma + \delta)$ is the trend growth after the break. Comparing two competing specifications for U.S. output in Eq. (2.3.a) and Eq. (2.3.b), we choose the former as the benchmark because stochastic drift enables the model to capture both the gradual decline in trend growth and the potential break. This is in consistent with Antolin-Diaz et al. (2017), who prefer random walk over structural break because it is more robust to misspecification.

Regarding the U.S. unemployment rate, the natural rate has no considerable time-variation. In fact, since the 1950s, NAIRU has smoothly increased from 3% to 6% until the 1980s; and since then, it has been decreasing to the level of 3% until now.[20] As a result, we adapt Eq. (2.3.c) to control for the structural break in the drift term of the natural rate of unemployment as follows:

$$\mu_t = \eta + \theta \mathbb{1}_t (t \geq T_\mu) \tag{2.3.c}$$

In this set up, $\mathbb{1}_t$ is an indicator function that takes the value of one after the break date (T_μ), and zero otherwise. η is the drift before and $(\eta + \theta)$ is the drift after the break date.

[20] The natural rate of unemployment can be affected by structural change related to demographics and regulations (Ball and Mankiew, 2002; Arnold, 2008). For example, the estimated NAIRU by the Congressional Budget Office (CBO) has smoothly declined since the 1980s, reflecting that the U.S. baby boom tapered off.

2.3.2.2. The cyclical component

To allow asymmetric fluctuations, we consider that shocks to the cyclical component are a mixture of asymmetric and symmetric shocks. To accommodate the asymmetric shocks, we incorporate an unobservable, first-order, and two-state Markov switching variable into the cycle and to allow for the possible high persistence of the cyclical component, we model the cycle as an AR(2) process:

$$z_t^c = \pi_z S_t + \varphi_1 z_{t-1}^c + \varphi_2 z_{t-2}^c + \varepsilon_{z^c,t} \tag{2.4}$$

where φ_1 and φ_2 are coefficients of the AR(2) process and their sum ($\varphi_1 + \varphi_2$) is expected to be less than one. In Eq. (2.4), π_z is the amplitude of the asymmetric shocks (plucking coefficient) and S_t identifies the state of the economy: $S_t = 0$ during normal times, and $S_t = 1$ during recessions. A significant π_z, which is expected to be negative for output and positive for the unemployment rate, confirms Friedman's plucking property. The state of the economy will be determined endogenously as it evolves according to the first-order Markov-switching process proposed by Hamilton (1989):

$$\Pr[S_t = 1 | S_{t-1} = 1] = p \tag{2.5}$$

$$\Pr[S_t = 0 | S_{t-1} = 0] = q \tag{2.6}$$

In this approach, p and q determine the transition probabilities. p is the probability of staying in the recession, and thus, $(1-p)$ is the probability of transitioning from the recession to the normal state. Similarly, q is the probability of staying in the normal state, and thus, $(1-q)$ is the probability of transitioning from the normal state to the recession state. In Eq. (2.4), $\varepsilon_{z^c,t} \sim N(0, \sigma_{z^c}^2)$ is the usual symmetric shock to the cyclical component. For robustness tests, $\sigma_{z^c}^2$ is allowed to be different before and after the great moderation:

$$\sigma_{z^c}^2 = \sigma_{z^c,0}^2 \mathbb{1}_t(t \le T_\sigma) + \sigma_{z^c,1}^2 \mathbb{1}_t(t \ge T_\sigma) \tag{2.7}$$

In Eq. (2.7), $\mathbb{1}_t$ is another indicator function to capture the potential break in the variance of shocks to the output cyclical component. The variance is equal to $\sigma_{z^c,0}^2$ before the break and is $\sigma_{z^c,1}^2$ after the break date (T_σ).

2.3.2.3. The variance-covariance matrix of shocks

Finally, the variance-covariance matrix of shocks is represented in Eq. (2.8):

$$\begin{bmatrix} \varepsilon_{z^*,t} \\ \varepsilon_{\mu,t} \\ \varepsilon_{z^c,t} \end{bmatrix} \sim N\left(\mathbf{0}_{3\times1}, \begin{bmatrix} \sigma_{z^*}^2 & 0 & \rho_{z^*,z^c}\sigma_{z^*}\sigma_{z^c} \\ 0 & \sigma_\mu^2 & 0 \\ \rho_{z^*,z^c}\sigma_{z^*}\sigma_{z^c} & 0 & \sigma_{z^c}^2 \end{bmatrix}\right) \tag{2.8}$$

Concerning correlation in the benchmark model, we maintain the assumption that all shocks are uncorrelated. For robustness tests, we allow for correlation between shocks to the output trend and cyclical components (ρ_{z^*,z^c}) to address the possibility of non-zero correlation, which is suggested by

Morley et al. (2003) and Sinclair (2010). On the other hand, since the natural rate represents the structural unemployment rate that exists independently of all temporary and seasonal fluctuations (Friedman, 1968; Phelps, 1968; Heimberger et al., 2017), we suppose the correlation between shocks to the unemployment trend and the cyclical component is zero. Nevertheless, we allow for this correlation in the robustness tests, where the estimation result shows that this correlation is insignificant.[21]

In the rest of this paper, to distinguish between the components of output and the unemployment rate, we denote the observed series, unobserved trend and cyclical components for output by x_t, x_t^* and x_t^c, and we denote those for the unemployment rate by u_t, u_t^* and u_t^c.

2.3.3. The bivariate model: Friedman's Plucking Model and Okun's Law

In the bivariate model specified in Eq. (2.9) and Eq. (2.10), we decompose each of the output and the unemployment rate into a trend and a cyclical component:

$$x_t = x_t^* + x_t^c \tag{2.9}$$

$$u_t = u_t^* + u_t^c \tag{2.10}$$

where x_t is the log of output and u_t is the unemployment rate. x_t^* and x_t^c are unobserved trend and cyclical components of output that play the roles of potential output and output gap. Similarly, u_t^* and u_t^c are unobserved trend and cyclical components of the unemployment rate, which play the roles of the natural rate and unemployment gap.

2.3.3.1. The trend components of output and the unemployment rate

We model the output trend as a random walk process with a stochastic drift:

$$x_t^* = \mu_{t-1} + x_{t-1}^* + \varepsilon_{x^*,t} \tag{2.11}$$

where $\varepsilon_{x^*,t} \sim N(0, \sigma_{x^*}^2)$ is the output trend shock and is assumed to be white noise and normally distributed, along with other shocks. μ_t stands for the time-varying drift (trend growth). Given the flexibility of random walk in capturing both the gradual decline and structural breaks in trend growth (Antolin-Diaz et al., 2017), we allow the drift (trend growth) to evolve according to a random walk process to let the data to speak for itself:

$$\mu_t = \mu_{t-1} + \varepsilon_{\mu,t} \tag{2.12.a}$$

This specification is compliant with Clark (1987), Grant and Chan (2017a), Fernald et al. (2017), and Kim and Chon (2020). In Eq. (2.12.a), $\varepsilon_{\mu,t} \sim N(0, \sigma_\mu^2)$ is white noise and uncorrelated with $\varepsilon_{x^*,t}$.

[21] For more justification, see Section 2.3.3.4.

Although we favour the above specification, alternatively, similar to Perron and Wada (2009) and Grant and Chan (2017b), we consider a non-stochastic drift with a structural break:

$$\mu_t = \gamma + \delta \mathbb{1}_t(t \geq T_\mu) \tag{2.12.b}$$

where $\mathbb{1}_t$ is an indicator function for the break date, T_μ, γ is output trend growth before the break date, and $(\gamma + \delta)$ is the trend growth after the break date.

Since the natural rate of unemployment increased from the 1950s to the 1980s and thereafter has been decreasing, the unemployment trend is modelled as a random walk with a drift that allows for a break:

$$u_t^* = \eta + \theta \mathbb{1}_t(t \geq T_{u^*}) + u_{t-1}^* + \varepsilon_{u^*,t} \tag{2.13}$$

Under this equation, $\mathbb{1}_t$ is an indicator that takes one after the break date (T_{u^*}) and zero otherwise. η is the drift before and $(\eta + \theta)$ is the drift after the break date. Finally, $\varepsilon_{u^*,t} \sim N(0, \sigma_{u^*}^2)$ is the shock to the natural rate of unemployment. Since changes in the natural rate relate to demographics and labour market regulations, $\varepsilon_{u^*,t}$ is essentially independent of cyclical fluctuations (Friedman, 1968; Phelps, 1968; Heimberger et al., 2017).

2.3.3.2. The cyclical component of unemployment rate

Since the asymmetric fluctuations are more distinguishable in the unemployment rate (DeLong and Summer, 1984; Falk, 1986; Sichel, 1993; McKay and Reis, 2008), and also the U.S. labour market is identified as the source of the plucking property (Ferraro, 2018; Dupraz et al., 2019; Ferraro and Fiori, 2022), we embed a Markov switching variable into the cyclical component of unemployment, rather than output, as follows:

$$u_t^c = \pi_u S_t + \varphi_1 u_{t-1}^c + \varphi_2 u_{t-2}^c + \varepsilon_{u^c,t} \tag{2.14}$$

where π_u is the amplitude of the asymmetric shock (plucking coefficient), and $\varepsilon_{u^c,t} \sim N(0, \sigma_{u^c}^2)$ is a symmetric cyclical shock to the unemployment rate. Since unemployment is counter-cyclical, we expect that π_u to be positive. The state of the economy (S_t) is zero during normal times and one during recessions and evolves according to the first-order Markov-switching process as in Hamilton (1989), specified in Eq. (2.5) and Eq. (2.6). In addition, φ_1 and φ_2 are coefficients of the AR(2) process, which allows for high persistence in the unemployment rate.

2.3.3.3. Okun's law and the cyclical component of output

After characterizing the asymmetric fluctuations of unemployment, we now capture the transmission of the plucking property from the unemployment rate to output, in accordance with the gap version of Okun's law:

$$x_t^c = \beta u_t^c + \varepsilon_{x^c,t} \tag{2.15}$$

where β is the Okun's coefficient and captures the co-fluctuations of output and the unemployment rate, and $\varepsilon_{x^c,t}$ is the Okun's law residuals or remaining cyclical component modelled as follows:

$$\varepsilon_{x^c,t} = \pi_x S_t + \psi \varepsilon_{x^c,t-1} + \xi_{x^c,t} \tag{2.16}$$

In Eq. (2.16), π_x is the output-specific plucking coefficient that measures the part of the plucking property in output that is not explained by the plucking property in the unemployment rate. And ψ is an autoregressive coefficient to control any persistency in the Okun's residuals. If the plucking property in output is sourced from the plucking property in unemployment, it is expected that the remaining plucking property in the Okun's residuals are negligible. Overall, a significant and positive π_u and a significant and negative β, together, confirm the asymmetric co-fluctuations of U.S. output and the unemployment rate, which means that output and the unemployment rate are synchronously and proportionally characterized by the plucking property. Furthermore, $\xi_{x^c,t} \sim N(0, \sigma_{x^c}^2)$ is the shock to the remaining cyclical component (Okun's law residuals). For robustness checks, we allow $\xi_{x^c,t}$ to have different variances before and after the great moderation:

$$\sigma_{x^c}^2 = \sigma_{x^c,0}^2 \mathbb{1}_t(t \leq T_\sigma) + \sigma_{x^c,1}^2 \mathbb{1}_t(t \geq T_\sigma) \tag{2.17}$$

where, $\mathbb{1}_t$ is an indicator function so that the variance before the break date is equal to $\sigma_{x^c,0}^2$ and after the break date is $\sigma_{x^c,1}^2$. We also allow for a structural break in the Okun's coefficient to address the concern about the stability of Okun's law.

2.3.3.4. Variance-covariance matrix of shocks

Considering five shocks to the components in Eq. (2.9) to Eq. (2.17), the variance-covariance matrix of shocks is:

$$\begin{bmatrix} \varepsilon_{x^*,t} \\ \varepsilon_{u^c,t} \\ \varepsilon_{\mu,t} \\ \varepsilon_{u^*,t} \\ \xi_{x^c,t} \end{bmatrix} \sim N\left(\mathbf{0}_{5\times1}, \begin{bmatrix} \sigma_{x^*}^2 & \rho_{x^*,u^c}\sigma_{x^*}\sigma_{u^c} & 0 & 0 & \rho_{x^*,x^c}\sigma_{x^*}\sigma_{x^c} \\ \rho_{x^*,u^c}\sigma_{x^*}\sigma_{u^c} & \sigma_{u^c}^2 & 0 & 0 & \rho_{u^c,x^c}\sigma_{u^c}\sigma_{x^c} \\ 0 & 0 & \sigma_\mu^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{u^*}^2 & 0 \\ \rho_{x^*,x^c}\sigma_{x^*}\sigma_{x^c} & \rho_{u^c,x^c}\sigma_{u^c}\sigma_{x^c} & 0 & 0 & \sigma_{x^c}^2 \end{bmatrix}\right) \tag{2.18}$$

where ρ_{x^*,u^c} is the correlation between shocks to the output trend and the symmetric cyclical component, ρ_{x^*,x^c} is the correlation between shocks to the output trend and remaining cyclical component, and ρ_{u^c,x^c} is the correlation between shocks to the symmetric cyclical component and remaining cyclical component. In the benchmark model, we assume all correlations are zero. Yet, for robustness tests, we relax the zero-correlation assumptions. Favourably, the results show that these correlations are insignificant, confirming that our model handles the issues related to the spurious correlation.

In bivariate models, it is presumed that the shock to the unemployment trend ($\varepsilon_{u^*,t}$) is uncorrelated with all other shocks. This assumption is reasonable based on three rationales. First, by definition, the natural rate of unemployment is the portion of unemployment that is unrelated to cyclical fluctuations (Friedman, 1968; Phelps, 1968; Heimberger et al., 2017). Second, it is also common in the empirical literature to specify the trend in the unemployment rate as a random walk, with its shocks assumed to be uncorrelated with other shocks (Clarke, 1989; Heimberger, 2017; Gordon, 1997; Semmler and Zhang, 2006; Watson, 2014; Gonzalez-Astudillo and Roberts, 2022; Grant, 2018; among others). Third, Gonzalez-Astudillo and Roberts (2022), by using a likelihood ratio test and information criterion, demonstrate that this assumption is innocuous.[22]

2.4. Results and discussion

We estimate twenty-one bivariate models, fourteen univariate models for output, and four univariate models for unemployment. We denote each model with an identifier and a descriptor. The descriptor consists of five parts, four of which are related to one of the specification aspects of output. For instance, the identifier of the benchmark model is (1.a) and its descriptor is A-Bi-RW-SB-UC, which means that the model (1.a) is **A**symmetric and **Bi**variate. The trend growth of output is specified as a **R**andom **W**alk, the natural rate of unemployment has a **S**tructural **B**reak, and finally, the model is **U**n**C**orrelated as the correlation between shocks is assumed to be zero. The list of identifiers and descriptors of all models is presented in Tables 2.B and 2.C in Appendices 2.B and 2.C.

In the next subsection, we will discuss the results of the bivariate models presented in Figures 2.1, 2.2, and 2.3 and Table 2.1. We compare the results of the benchmark model in column (1.a) with those of its symmetric counterpart reported in column (4.a). Regarding the third specification aspect, we present the outcomes of the two asymmetric bivariate models, one in column (2.a), which accounts for a break in output trend growth, and another one in column (3.a), which imposes constant trend growth. Concerning the fourth aspect, we execute models (1.b), (2.b), and (3.b) that are correlated versions of models (1.a), (2.a), and (3.a), respectively.

[22] In addition, the estimation results of both asymmetric and symmetric univariate UC models for unemployment show that the correlation between shocks to the unemployment trend and the cyclical component is insignificant.

In other subsections, we cover the results of structural break tests for the benchmark model. We also present the findings of univariate models for output and univariate models for unemployment. In the end, by extending the estimation period up to 2022Q1 and using a dummy variable, we explore the COVID-19 recession as an epitome of the plucking property. We additionally report the results for two additional series: U.S. output per capita and U.K. output.

2.4.1. The results of the bivariate models

The results of the benchmark version of the asymmetric bivariate model substantiate the asymmetric co-fluctuations of U.S. output and the unemployment rate. The Okun's law coefficient is $\beta = -1.45$ with a standard error of 0.12, implying that a 1% gap in unemployment is accompanied by a 1.45% gap in output. This corroborates their co-fluctuations, which means fluctuations in output and the unemployment rate are indeed synchronous and proportional. Furthermore, these co-fluctuations are asymmetric because the estimated plucking coefficient is $\pi_u = 0.70$ with a standard error of 0.06, and the product of two coefficients ($\beta \times \pi_u = -1.01$) gauges the plucking property in output. Given that the labour market is identified as the source of the plucking property (Ferraro, 2018; Dupraz et al., 2019; Ferraro and Fiori, 2022), our findings capture the transmission of the plucking property from the unemployment rate to output, yet we do not rule out the role of other exacerbating factors, such as binding financial constraints during recessions, as mentioned by Jensen et al. (2020).

The benchmark model corroborates the plucking property and ceiling effect, suggested by Friedman (1993), in both economic indicators. The top-right panel of Figure 2.1 displays that estimated output gaps are deep, often negative, and rarely positive; and likewise, the bottom-right panel illustrates that unemployment gaps are large in magnitude, often positive, and rarely negative. Further, the estimated transition probability reported in column (1.a) of Table 2.1 is low for recessions ($p = 0.660$) and high for recoveries ($q = 0.965$); thus, the expected duration is around 3 quarters for recessions and 28 quarters for recoveries.[23] Furthermore, the sum of autoregressive coefficients estimated for the cyclical component ($\varphi_1 + \varphi_2$) is 0.93 for model (1.a) and 0.95 for model (4.a), suggesting a relatively persistent cyclical component and gradual recoveries. Overall, we highlight that the co-fluctuations of U.S. output and the unemployment rate are asymmetric in amplitude, speed, and duration, which implies that deep, steep, and transitory recessions will be followed by commensurate, gradual, and permanent recoveries.

We summarize the findings regarding the four specification aspects of trend-cycle decomposition. First, comparing the log likelihood value of -11.9 for the asymmetric bivariate model in column (1.a)

[23] The expected duration of recessionary and recovery states can be derived by formulas $\frac{1}{1-p}$ and $\frac{1}{1-q}$, respectively, where p is the probability of staying in the recession and q is the probability of staying in the normal state.

of Table 2.1 with the value of -57.5 for its symmetric counterpart reported in column (4.a) yields a likelihood ratio of 91.2, which is substantially greater than the critical value of 10.8 for a conservative 0.1% significance level.[24] This remarkable plucking coefficient documents that shocks to the cyclical component are asymmetric. This finding holds true for alternative asymmetric bivariate models such as (2.a) and (3.a), and accords with the outcome of univariate models presented by Kim and Nelson (1999a), Sinclair (2010), and recently Eo and Morley (2022), who report the presence of asymmetric fluctuations in U.S. output.[25]

Second, the resemblance of the results obtained from the benchmark model presented in Figure 2.1 to those of its correlated counterpart in Figure 2.F.1 in Appendix 2.F, affirms that the asymmetric bivariate model yields unobserved components with robust features no matter whether the correlation is involved in the model or not. In particular, independent from the assumption about the correlation between shocks, embedding both the plucking property (asymmetry) and Okun's law (co-fluctuation) in the model ensures that cyclical components have substantial amplitude. This refutes the empirical results of Beveridge and Nelson (1981), Nelson and Plosser (1982), Morley et al. (2003), Grant and Chan (2017b), Kim and Chon (2020), and Kim and Kim (2020), and the theoretical works of Kydland and Prescott (1982) and Long and Plosser (1983), whose unreasonable assumptions about one of the specification aspects (asymmetry and correlation) produce a result suggesting that the variation in output is almost entirely dominated by variation in the supply-related trend, and that the demand-related cyclical component is small and noisy.

Since our benchmark model is grounded on the stability of Okun's law, we investigate the possibility of instability in Okun's law. We compare the log likelihood value of model (8), which accounts for a structural break in the Okun's law coefficient, with the log likelihood value of the benchmark model (1.a), where the Okun's law coefficient is assumed to be stable. The top-left panel of Figure 2.3 plots the corresponding likelihood ratio values for a sequence of breaks in the Okun's law coefficient rolling from 1960 to 2010, conditioned on the other aspects of the benchmark model (asymmetric cyclical component and stochastic trend growth). The likelihood ratio values in the sample are fairly less than any reasonable threshold, such as QLR critical values of 8.9 and 7.2 for 5% and even 10%

[24] In the presence of a Markov-switching process, testing hypotheses based on the likelihood ratio statistics is non-standard as the nuisance parameter is not identified under the null hypothesis, and consequently the asymptotic distribution of the likelihood ratio test is unknown and does not follow the standard χ^2 distribution. Few papers have proposed theoretically questionable and computationally burdensome simulation-based or bootstrap-based methods to test for Markov-switching that are operable for simple models (see, e.g., Hansen, 1992; Garcia, 1998; Di Sanzo, 2009). Because of the large dimension of our models and the 39 different models estimated in this study, we maintain the use of the non-standard likelihood ratio test. Further, exceptionally large likelihood ratios derived for testing asymmetry in this study leave very little doubt, if not no doubt, that co-fluctuations are asymmetric.

[25] By comparing the log likelihood values of -7.4 and -19.9 for asymmetric models (2.a) and (3.a) reported in Table 2.1 with values of -52.6 and -62.5 for their symmetric counterparts (5.a) and (6.a), we favour the asymmetric models over symmetric models because the corresponding likelihood ratios of 90.4 and 85.2 are exceedingly greater than the critical value of 10.8 for a 0.1% significance level.

significance levels. We therefore rule out the possibility of instability in Okun's law, which accords with Daly et al. (2014), Ball et al. (2017), and Michail (2019). As a result, as long as the asymmetry in the cyclical component and stochastic drift in the trend component are accommodated, as is the case in our benchmark model, the Okun's law coefficient is satisfactorily stable.

Third, the results of the model (1.a) and the alternative model (2.a) provide convincing evidence for time-variation in output trend growth in the U.S. in the form of both a gradual decline, which began in the 1960s, and a sharp structural break following the 2007–09 financial crisis. The middle-left panel of Figure 2.1 plots the dynamics of the trend growth estimated by the benchmark model. It is clear that annual trend growth had gradually declined from about 4% in the 1960s to 2.6% in the mid-2000s, and then it sharply fell from 2.6% to an unprecedented rate of 1.2% in the aftermath of the 2007–09 financial crisis. Moreover, the top-right panel of Figure 2.1 exhibits a lack of bounce-back effect in the initial stage of the recovery following the 2007−09 financial crisis compared to that of previous recoveries, indicating an unusually persistent output gap following the financial crisis. Therefore, we underscore the role of financial crisis in both the drop in trend growth and the sluggish cycle recovery.

For additional evidence, we first test for a known structural break in trend growth in 2009Q3 that corresponds to the end of the Great Recession, by comparing the log likelihood value of -7.4 reported in column (2.a) of Table 2.1 for the model with a structural break in trend growth in 2009Q3 with a log likelihood of -19.9 in column (3.a) for the model with constant trend growth. The corresponding likelihood ratio value of 25.0 is greater than the critical value of 10.8 for a 0.1% significance level, confirming the occurrence of a structural break in 2009. We also explore an unknown break in trend growth by rolling the break date in the central 70% of the sample. The middle-left panel of Figure 2.3 plots the corresponding likelihood ratio values for a sequence of breaks from 1960 to 2010. We clearly discern two distinct episodes. First, from the mid-1960s to the mid-1990s, structural breaks are repeatedly significant in every period. These breaks are comparably moderate, which hints at the gradual decline in trend growth in this episode. Second, after the mid-1990s, the likelihood ratio statistics are striking and they peak substantially twice in a row in 2006 and 2010, suggesting an unprecedented deceleration in the U.S. potential output. This deceleration (negative acceleration) of U.S. potential output is shown in the top-right panel of Figure 2.2. The top-left panel also flags up an unusual shortfall of 1.3 percentage points per year following the financial crisis by comparing the actual trend growth with long-run extrapolations from 1990 and 2009 as two counterfactuals.

Comparing competing specifications for trend growth, the bottom-left panel of Figure 2.3 implies that the random walk performs better than almost all alternative models with a structural break date before 2000 and is also fairly close to the best models with a selected break date near the 2007–09 financial crisis. We additionally compare the log likelihood of -11.9 in column (1.a) of Table 2.1 for the model with stochastic drift with a log likelihood value of -19.9 in column (3.a) for model with constant trend growth. The improvement in log likelihood accords with the estimate of $\sigma_\mu = 0.03$ for the standard deviation of shocks to trend growth with a standard error of 0.01.[26] Conclusively, random walk is capable of accommodating both the gradual decline and structural breaks in trend growth, as previously suggested by Antolin-Diaz et al. (2017), Grant and Chan (2017a), and Kim and Chon (2020).

Fourth, including both the plucking property (asymmetry) and Okun's law (co-fluctuation) in the asymmetric bivariate model makes the model robust to the assumption about the correlation between the trend and symmetric cyclical shocks (ρ_{x^*,u^c}). We compare the log likelihood value of -11.9 for the benchmark model in column (1.a) with the value of -11.4 reported for its correlated counterpart in column (1.b) of Table 2.1. We accept the null hypothesis of zero-correlation since the likelihood ratio of 1.0 is less than the critical values of 3.84 and even 2.71 for 5% and 10% significance levels. This finding remains unchanged for other asymmetric bivariate models (2.a) and (3.a). Besides, according to the results presented in Table 2.G.1 in Appendix 2.G, other two correlations (ρ_{x^*,x^c} and ρ_{u^c,x^c}) are insignificant with likelihood ratios close to 0.[27] On this basis, correlations between shocks are entirely insignificant and irrelevant in asymmetric bivariate models, and unsurprisingly, the features of the trend and cyclical components are insensitive to the assumption about correlation. By contrast, the correlation between shocks is significant and affects the estimation of other parameters to some degree in the asymmetric univariate model and symmetric bivariate model, although incorporating one of the above two specification aspects helps to alleviate the sensitivity of the results to the correlation between shocks.[28] As a result, we conclude that both aspects need to be taken into account in order to generate a reliable trend-cycle decomposition.

Lastly, the top-left and bottom-left panels of Figure 2.1 depict potential output as a ceiling for output and the natural rate of unemployment (ZOGRU) as a floor for the unemployment rate, which are

[26] We also compare the log likelihood of -11.9 for model (1.a) with the values of -20.5 for model (3'.a), which is fully nested in model (1.a) and its estimation is presented in Table 2.G.1 in Appendix 2.G. The corresponding likelihood ratio is 17.2.

[27] For the first correlation (ρ_{x^*,u^c}), comparing the log likelihood values of -11.9, -7.4, and -19.9 for uncorrelated models (1.a), (2.a), and (3.a) with values of -11.4, -7.2, and -18.8 for their correlated counterpart models (1.b), (2.b), and (3.b), respectively, yields likelihood ratios of 1.0, 0.4, and 2.2. For the second correlation (ρ_{x^*,x^c}), the likelihood ratios are 0.0, 0.0, and 1.4, which are derived by comparing the above log likelihood values for uncorrelated models (1.a), (2.a), (3.a) with values of -11.9, -7.4, and -19.2 for correlated models (1.c), (2.c), and (3.c). Regarding the third correlation (ρ_{u^c,x^c}), we compare the log likelihood value of -11.9 for the uncorrelated model with values of -11.9 for the correlated model (1.d) to ensure this likelihood ratio is also 0.0. Finally, testing for all three correlations jointly, through model (1.e), indicates a small likelihood ratio of 2.0, which is far smaller than the critical value of 7.81 for a 5% significance level with three restrictions.

[28] See Sections 2.4.2 and 2.4.4 for detailed results about correlation in symmetric bivariate and asymmetric univariate models.

estimated jointly. We observe an increase in ZOGRU from 3% in the 1950s to 6% in the 1980s, and then a gradual decrease to the levels lower than 4% until now. Because the estimated standard deviation of shocks to the natural rate of unemployment is negligible ($\sigma_{u^*} = 0.0004$), the variation in ZOGRU is completely explained by estimating two constant drift terms.

2.4.2. Supplementary results of the bivariate models

In our model, estimating $\beta = -1.45$ and $\pi_u = 0.70$ along with gaps in output and unemployment with large amplitude captures the transmission of the plucking property from the unemployment rate to output. To be fully substantiated, the middle-left panel of Figure 2.2 shows the leftover plucking property in the Okun's residuals, whose depth is small in amplitude. This implies that the plucking property in output is mainly sourced from the plucking property in unemployment. The middle-right panel indicates controlling the 1-month lead-lag effect between output and unemployment entirely removes the small leftover plucking property in output. Therefore, the remaining plucking property in the benchmark model is attributable to the lead-lag effect between output and unemployment. Correspondingly, while the output-specific plucking coefficient is notable in the benchmark model ($\pi_x = -1.01$), as reported in Table 2.G.3 in Appendix 2.G, it is negligible in the model with the 1-month lead-lag effect controlled ($\pi_x = -0.46$), and finally is zero in the model with the 2-month or 3-month lead-lag effect controlled ($\pi_x = -0.000$). As a result, considering that unemployment lags behind output for only one or two months, we conclude that their co-fluctuations are sufficiently synchronous. For routine diagnostic tests on error terms in the benchmark model, the bottom panels of Figure 2.2 displays that shocks to the Okun's residuals are zero-mean noise, and also their autocorrelation functions are fast decaying.

We have so far discussed the main findings of the asymmetric bivariate model. Let us inspect the results of symmetric bivariate models from the lens of the third and fourth aspects. By comparing the log likelihood value of -52.6 for the symmetric bivariate model (5.a) with a break in trend growth with a log likelihood of -62.5 for the model (6.a) with constant trend growth, we find a likelihood ratio of 19.8 greater than the critical value of 10.8 for a 0.1% significance level, reconfirming the structural break in 2009.

For correlation between shocks to the trend and remaining cyclical components, we compare the log likelihood values of -57.5, -52.6, and -62.5 for the uncorrelated models (4.a), (5.a), and (6.a) with values of -49.1, -46.2, and -54.4 for the correlated counterpart models (4.b), (5.b), and (6.b). We reject the null hypothesis of zero-correlation because the likelihood ratios of 17.0, 12.8, and 16.2 are greater than the critical value of 10.8 for a 0.1% significance level. Conclusively, the correlation is not completely irrelevant in the symmetric bivariate model and might affect the estimation of other parameters, namely Okun's coefficient, to some degree.

2.4.3. Structural breaks and robustness tests

We explore the robustness of our findings by estimating alternative models, each of which accounts for a structural break in one of the following parameters: the Okun's law coefficient, the volatility of shocks to the remaining cyclical component, output trend growth, and the drift in the unemployment trend. The panels in Figure 2.3 plot the likelihood ratios for a sequence of breaks, rolling from 1960 to 2010, in one of the above parameters. Based on the top-left panel, we dismiss the possibility of instability in the gap version of Okun's law as the likelihood ratio values are comparably less than the critical value of 7.2 for 10% significance levels. The top-right panel reports that a break in the volatility of shocks to the remaining cyclical component occurred in 1983. This date for the break in volatility is close to the break date of 1982 derived by Eo and Morley (2022). The estimation result of model (7) in Table 2.G.1 in Appendix 2.G shows that, analogous to the great moderation, the volatility has decreased from $\sigma^2_{x^c,0} = 0.55$ to $\sigma^2_{x^c,1} = 0.04$. In this model, the estimation of other parameters is almost the same as that of the benchmark model.

Regarding the instability in trend growth, as discussed before, the middle-left panel identifies two sources of instability: a gradual decline that began in the 1960s and a structural break in trend growth in 2009, which are considered by a random walk drift in the benchmark model (1.a) and a break in 2009 in model (2.a). The resemblance of the results in columns (1.a) and (2.a) of Table 2.1, along with the bottom panels of Figure 2.3, which compare the log likelihoods of these competing models, reaffirms the competence of the random walk drift to accommodate structural breaks. Finally, the middle-right panel reports a significant break in the drift term of the unemployment trend (ZOGRU) in 1981Q1, which is accommodated in the benchmark model.

Moreover, our results related to each of the four specification aspects are robust to the choice of the other aspects. First, the finding of asymmetry in business cycles stands up in models (1.a), (2.a), and (3.a), no matter what the specification for trend growth is and also remains unchanged for correlated models (1.b), (1.c), (1.d), (1.e), (2.b), (2.c), (3.b), (3.c), independent from the assumption about correlation. In the next subsections, we will show that the plucking coefficients in univariate models estimated separately for output and unemployment are also significant. Thus, the result supporting the plucking property and asymmetric fluctuations is independent from the other three specification aspects. Second, estimating an analogous Okun's law coefficient in almost all models substantiates the co-fluctuations of output and the unemployment rate. Regardless of the specifications for trend growth and correlation, the estimated Okun's coefficient is around -1.4 for most asymmetric models and is around -1.7 for most symmetric models; yet it is -1.04 and -1.27 for correlated models (3.b) and (3.6) where the trend growth is assumed to be constant. Overall, all bivariate models improve the robustness and estimate cyclical components that are large in amplitude regardless of the assumptions about the asymmetry, trend growth, and the correlation between shocks.

Third, our finding concerning the time-variation in trend growth in the form of a gradual decline and a break in 2009 is strong because it is true for asymmetric models (1.a) and (2.a) as well as symmetric models (4.a) and (5.a). It also holds unchanged for correlated models (1.b), (2.b), (4.b), and (5.b). Additionally, the univariate model for output supports the time-variation in trend growth and rejects the hypothesis of constant trend growth, which will be discussed in the next section. Finally, we find that correlation irrelevance is specific to the asymmetric bivariate UC model: both asymmetry and co-fluctuation are needed for robust trend-cycle decomposition, although each of the asymmetric univariate UC model and the symmetric bivariate UC model alone helps to mitigate the sensitivity of the model to the assumption about correlation.

2.4.4. Results of the univariate models for output

We first uncover the consequence of misspecifications in the symmetric univariate models. We then discuss the results of the univariate models presented in Table 2.2 through pairwise comparison of asymmetric univariate models (1.a), (2.a), and (3.a) with their symmetric counterparts (4.a), (5.a), and (6.a). We also implement the correlated versions of the above models, denoted by (1.b), (2.b), and (3.b), to inspect the correlation irrelevance in asymmetric univariate models.

To show that counter-intuitive and sensitive features of the estimated trend and cyclical components are the consequence of misspecifications of the third and fourth aspects, we briefly discuss the results of symmetric univariate models (4.a), (5.a), and (6.a). Figure 2.4 illustrates the trend and cyclical components estimated by model (6.a) on the left and its correlated counterpart (6.b) that replicates the results derived by Morley et al. (2003) on the right panels, where in both models the trend growth is assumed to be deterministic (constant). Such an unreasonable assumption leads to a misleading cyclical component that exhibits a downward leftover trend in the bottom-left panel and an unusually small amplitude in the bottom-right panel. As a result, the features of components in a model with constant trend growth are dubious and sensitive to the assumption about the correlation. Figure 2.5 shows that relaxing the assumption of constant trend growth yields more intuitive and less sensitive results. The cyclical components of model (4.a) with stochastic trend growth shown in the upper-left panel and model (5.a) with a break in trend growth shown in the bottom-left panel do not contain a noticeable leftover trend as with model (6.a). Further, although the cyclical components of correlated models (4.b) and (5.b) illustrated in the top-right and bottom-right panels have considerably smaller amplitude compared to their uncorrelated counterparts, they are not diminutive and noisy like model (6.a).

We now turn to the asymmetric univariate model. The estimated plucking coefficient is $\pi_z = -1.67$ with a standard error of 0.19. By comparing the log likelihood value of -336.9 for the asymmetric univariate model (1.a) with the value of -354.4 for its symmetric counterpart in column (4.a) of Table 2.2 and obtaining a likelihood ratio of 35.0, which is greater than the critical value of 10.8 for a 0.1% significance level, we confirm the asymmetric fluctuations of U.S. output. This finding stands up in asymmetric univariate models (2.a) and (3.a) and accords with the results of asymmetric bivariate models in this study and univariate models in earlier studies (e.g., Kim and Nelson, 1999a; Sinclair, 2010).[29] The transition probabilities reported in column (1.a) are $p = 0.644$ for recessions and $q = 0.956$ for recoveries, which are close to those in the asymmetric bivariate model. Likewise, the expected duration is around 3 quarters for recessions and 23 quarters for recoveries. The top panels of Figure 2.6 plot estimated potential output and output gaps that are deep and negative, confirming that output fluctuations are asymmetric in amplitude, speed, and duration. The sum of autoregressive coefficients for the cyclical component ($\varphi_1 + \varphi_2$) is 0.76 for model (1.a) and is 0.91 for model (4.a), showing sensitivity of the estimated cyclical persistency to the assumption about the asymmetry in the univariate model.

To shed light on the time-variation in trend growth, we compare the log likelihoods of -336.9 and -334.7 for models (1.a) and (2.a) with the value of -341.0 for model (3.a), all presented in Table 2.2. We reject the hypothesis of constant trend growth in favour of the occurrence of a break in 2009 as the corresponding likelihood ratio of 12.6 is greater than the 0.1% critical value of 10.8.[30] In this regard, Figure 2.F.3 in Appendix 2.F reveals the consequence of imposing a constant trend growth in model (3.a). An unaccounted for break in trend growth is reflected in the cyclical component as the plucking probability is forced to stay at one, which brings a paradoxical result: a permanent output gap in the transitory component. However, once a time-varying trend growth is accommodated in models (1.a) and (2.a), as shown in top-right panel of Figure 2.6 as well as Figures 2.F.4 and 2.F.5 in Appendix 2.F, the features of the estimated cyclical component are sensible and similar to the results of asymmetric bivariate models. As additional evidence, the left panel of Figure 2.F.6 illustrates two clusters of repeatedly highly significant breaks, one in the 1970s and another in the 2000s, with multiple local peaks in 1973, 1978, 2000, and 2006, which supports the use of random walk drift to characterize the dynamics of output trend growth in the U.S.

[29] By comparing the log likelihood values of -334.7 and -341.0 for asymmetric models (2.a) and (3.a) in Table 2.2 with values of -352.6 and -357.1 for their symmetric counterparts (5.a) and (6.a), we obtain corresponding likelihood ratios of 35.8 and 32.2 that are greater than the critical value of 10.8 for a 0.1% significance level.

[30] Note that the log likelihood for model (3'.a), which is presented in Table 2.G.2 in Appendix 2.G and is fully nested in model (1.a) is -340.9, almost the same -340.0 for model (3.a). Considering symmetric models, comparing the log likelihood value of -352.6 for model (5.a) with a break in trend growth with a log likelihood of -357.1 for model (6.a) with constant trend growth, reaffirms the result of a structural break in 2009 because the likelihood ratio of 9.0 is greater than the critical value of 6.63 for a 1% significance level.

Concerning the correlation between shocks, we derive the likelihood ratios of 8.6, 10.8, and 19.0 by comparing the log likelihoods of -336.9, -334.7, and -341.0 for the uncorrelated asymmetric models (1.a), (2.a), and (3.a) with values of -332.6, -329.3, and -331.5 for their correlated counterpart models (1.b), (2.b), and (3.b). The correlation is therefore significant in the asymmetric univariate model, although its consequence on the features of components is milder compared to the symmetric univariate model.[31] Combining the results of asymmetric univariate models and symmetric bivariate models, we conclude that correlation irrelevance can be achieved only through accounting for both the asymmetry and co-fluctuations.

2.4.5. Results of the univariate models for unemployment

The results in Table 2.3 and Figure 2.6 are derived from the asymmetric univariate model applied to the unemployment rate. We present these results with a focus on four specification aspects. First, the estimated plucking coefficient in the asymmetric univariate model is $\pi_z = 0.75$ with a standard error of 0.06, which is close to $\pi_u = 0.70$ in the asymmetric bivariate model. We report a likelihood ratio of 63.6 by comparing the log likelihood of -4.9 for the asymmetric univariate model (1.a) with the value of -36.7 for its symmetric counterpart (2.a), confirming the asymmetry in the fluctuations of U.S. unemployment.

Second, transition probabilities reported in column (1.a) of Table 2.3 are $p = 0.63$ for recessions and $q = 0.97$ for recoveries, which are close to those in the asymmetric univariate model for output and the asymmetric bivariate model. Comparing the middle-left and middle-right panels of Figure 2.6 implies a striking resemblance between the plucking probabilities in univariate models estimated separately for output and unemployment. The synchronous and proportional fluctuations of output and the unemployment rate are also depicted in the top-right and bottom-right panels, which suggests the co-fluctuations. By comparing the likelihood ratio of 63.6 for asymmetry in unemployment with the value of 35.0 for asymmetry in output obtained in the previous subsection, we support the notion that the plucking property is more pronounced in unemployment (Sichel, 1993; Dupraz et al., 2019; Ferraro and Fiori, 2022) and appears to be the source of the plucking property in output. The sum of autoregressive coefficients ($\varphi_1 + \varphi_2$) for the unemployment cyclical component (0.94) is greater than that for the output cyclical component (0.76), implying more persistency in unemployment.

[31] Similarities of the trend and cyclical components in Figure 2.6 for the uncorrelated asymmetric model to those in Figure 2.F.5 in Appendix 2.F for the correlated asymmetric model shows that embedding the asymmetry reduces the model sensitivity to the assumption about the correlation. In addition, the trend and cyclical components of the uncorrelated and correlated models in the left and right panels of Figure 2.F.4 in Appendix 2.F are also very similar.

Third, as shown in the bottom-left panel of Figure 2.6 and top panels of Figure 2.F.7 in Appendix 2.F, there is a very mild increase in the natural rate of unemployment from the 1950s to the 1980s, and then a mild reduction until now. The variation in unemployment is then mainly attributable to the variation in its cyclical component, which introduces the unemployment rate as a reliable and straightforward proxy for estimation of the cyclical component.

Fourth, both asymmetric and symmetric univariate models for unemployment are robust to the assumption about the correlation between shocks to the trend and symmetric cyclical components. By comparing the log likelihood values of -4.9 and -36.7 for uncorrelated models (1.a) and (2.a) with the values of -4.9 and -36.4 for their correlated counterparts (1.b) and (2.b), we strongly accept the null hypothesis of zero-correlation because both likelihood ratios of 0.0 and 0.8 are negligible. This is consistent with the definition of the natural rate of unemployment that represents the structural unemployment rate that exists independently of all cyclical fluctuations (Friedman, 1968; Phelps, 1968; Heimberger et al., 2017).

Overall, unemployment inherently encompasses the plucking property and correlation irrelevance. These desirable features introduce this indicator as an eligible auxiliary to be included in a bivariate model to facilitate the trend-cycle decomposition of output.

2.4.6. Exploring the COVID-19 recession

The COVID-19 recession exemplifies the plucking property because, by imposing severe constraints on the economy, it led to a spare capacity and a deep output gap in the U.S., U.K., and many other countries. A sharp jump in the unemployment rate from 3.8% in 2020Q1 to 13% in 2020Q2 and a proportional steep pluck-down in output identify the COVID-19 recession as the deepest and shortest recession among post-World War II recessions.

Because the COVID-19 recession is deeper, steeper, and shorter than other recessions, Eq. (2.14) that considers a single plucking coefficient (π_u) for all recessions cannot account the unprecedented jump in unemployment rate and fall in output during the COVID-19. To deal with this issue, we follow a simple approach by adding a dummy variable for the COVID-19 pandemic and estimating the secondary plucking coefficient ($\pi_{u,COVID}$) as follows:

$$u_t^c = \pi_u S_t + \pi_{u,COVID} \times \mathbb{1}_t(T_{COVID-start} \leq t \leq T_{COVID-end}) + \varphi_1 u_{t-1}^c + \varphi_2 u_{t-2}^c + \varepsilon_{u^c,t} \qquad (2.19)$$

where $\mathbb{1}_t$ is an indicator function that takes the value of one during the COVID-19 pandemic and zero otherwise. We set the start of the COVID-19 ($T_{COVID-start}$) at 2020Q2 and the end ($T_{COVID-end}$) at 2021Q1, with unemployment remaining above 6%. In Eq. (2.19), $\pi_{u,COVID}$ captures the excessive plucking property during the COVID-19 recession.[32] Figure 2.7 presents the estimated components that are akin to the result of the main model. By extending the data up to 2022Q1, the model measures 8.5% and 11% gaps in unemployment and output. Based on Table 2.G.4 in Appendix 2.G that reports the parameters, the COVID-19 plucking coefficient ($\pi_{u,COVID} = 1.36$) is remarkable since the depth of the COVID-19 recession is greater than those of previous recessions. The estimated plucking coefficient (π_u) is also larger compared to that of the benchmark model because the greater depth and shorter duration of the COVID-19 recession requires a larger coefficient to be explained by a Markov-switching process. Overall, the excessive plucking property during the COVID-19 recession is captured by both an increase in π_u and an estimation of a significant $\pi_{u,COVID-19}$.

2.4.7. Results for U.S. output per capita and U.K. output

In addition to U.S. real output, we apply the asymmetric bivariate model to two other macroeconomic time series: (1) U.S. output per capita to show that annual trend growth of U.S. output per capita in the aftermath of the financial crisis is lower than 1%; and (2) U.K. output to provide international evidence for asymmetric co-fluctuations, whose results are presented in Table 2.G.5 in Appendix 2.G. For U.S. output per capita, we find the same Okun's and plucking coefficients ($\beta = -1.45$ and $\pi_u = 0.70$), and transition probabilities ($p = 0.66$ and $q = 0.96$) as reported for output. Based on Figure 2.F.8 in Appendix 2.F, the components have also very similar features as derived for output. The middle-left panel shows a disappointing annual trend growth of 0.7% for U.S. output per capita. For U.K. output, the estimated Okun's and the plucking coefficients are $\beta = -1.53$ and $\pi_u = 0.23$, and the transition probabilities are $p = 0.88$ and $q = 0.97$, indicating that although the amplitude of the U.K. plucking property is milder, its recession duration is longer.

2.4.8. Research limitations

This study focused on examining the asymmetric co-fluctuations and the transmission of the plucking property from unemployment to output. As a result, this study is silent on the other potential sources of asymmetry related to uncertainty and financial frictions; such as borrowing constraints, liquidity shortages, credit crunches, and banking agency problems. Further, we impose a constant plucking coefficient for all recessions, although the depth of each recession differs from the others. The effect of this assumption, however, is moderate since the model has enough flexibility to adjust the duration of the state of the economy for a recession to capture its special depth.

[32] A more suitable approach to modelling the plucking property during the COVID-19 recession is to embed two independent Markov-switching processes, one for all previous recessions and another for the COVID-19 recession, whose depth is around double the average of previous recessions. We leave this for future study since this model is outside the scope of this study.

2.5. Concluding remarks

By embedding output and the unemployment rate in a bivariate state-space model with a Markov-switching process, we integrate Friedman's plucking model and Okun's law. Estimating substantial plucking and Okun's law coefficients ($\pi_u = 0.70$ and $\beta = -1.45$) establishes the asymmetric co-fluctuations, which states that fluctuations in output and the unemployment rate are synchronously and proportionally characterized by the plucking property.

Our model sheds light on four specification aspects of trend-cycle decomposition. First, the plucking property and ceiling effect are remarkable in both indicators so that gaps are large in magnitude and often negative for output and positive for unemployment. By capturing business cycle asymmetries, our empirical findings indicate that recessions are deep, steep, and transitory and will be followed by commensurate, gradual, and permanent recoveries. We also document that the gap version of Okun's law is stable as long as the asymmetry in the cyclical component and stochastic drift in the trend component are accommodated. As a result, since the labour market is the source of the plucking property, we capture the transmission of the plucking property from unemployment to output. Third, by specifying the trend growth (drift) as a random walk, we report a gradual decline in trend growth, which started in the 1960s, as well as an unprecedented deceleration in U.S. potential output in the aftermath of the 2007−09 financial crisis. Fourth, the asymmetric bivariate model that encompasses both Friedman's plucking property (asymmetry) and Okun's law (co-fluctuation) yields robust results with an insignificant correlation, which we refer to as correlation irrelevance.

Further, by jointly estimating the trends of output and the unemployment rate and accounting for the plucking property in both indicators, our model provides a substitute for the Non-Accelerating Inflation Rate of Unemployment (NAIRU) to measure the natural rate of unemployment. We call this new measure the Zero Output Gap Rate of Unemployment (ZOGRU), the unemployment rate at which the output gap is zero.

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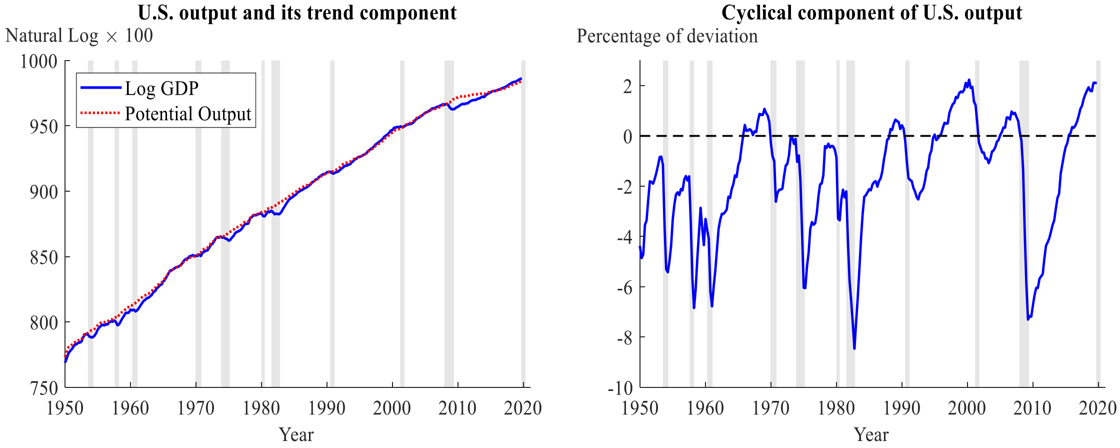
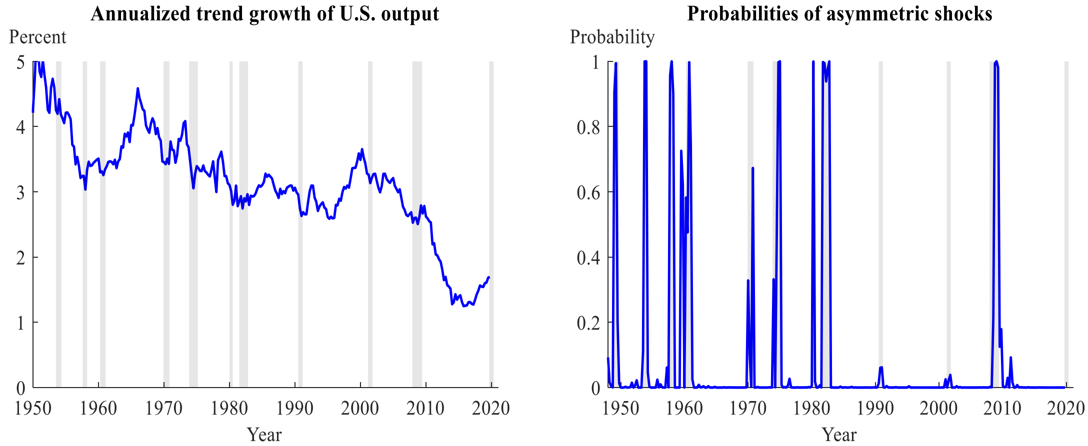
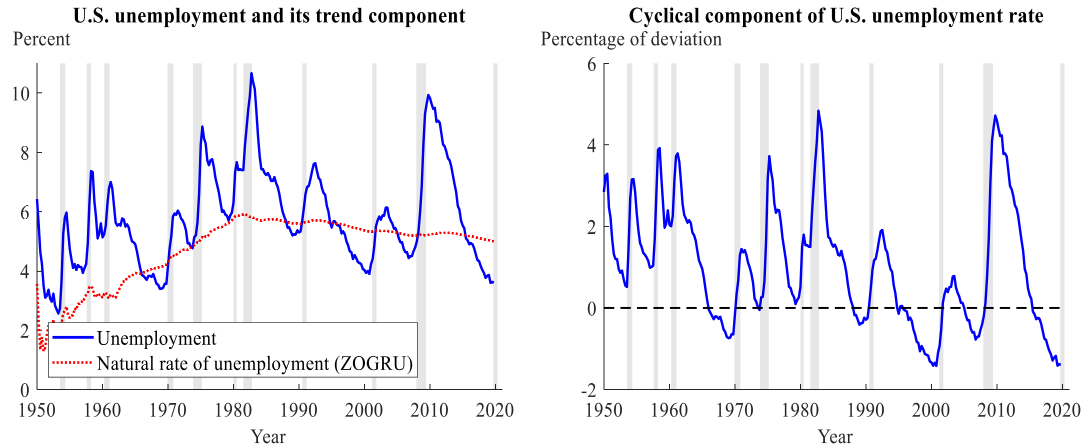
Figures



(a) Potential output (trend) and output gap (cyclical component)



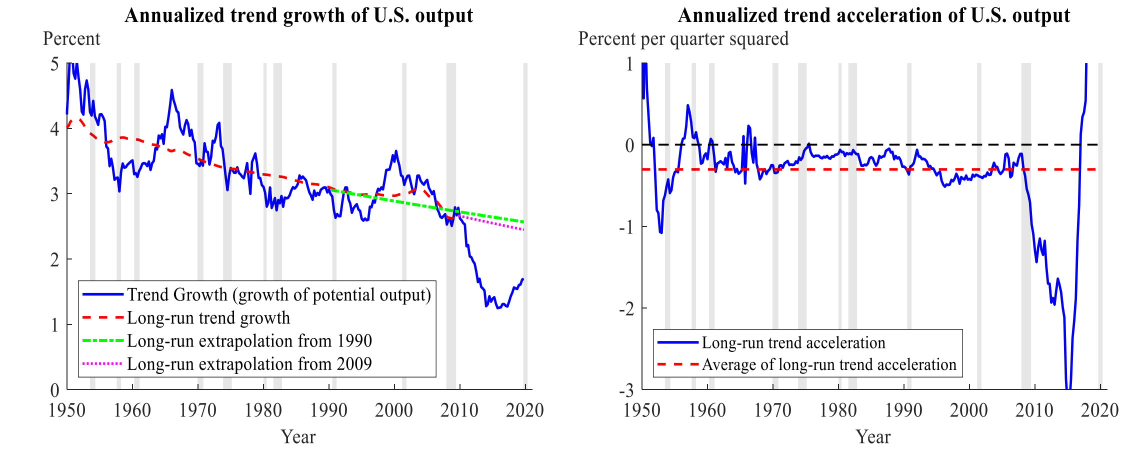
(b) Trend growth of output and the plucking probabilities for bivariate model



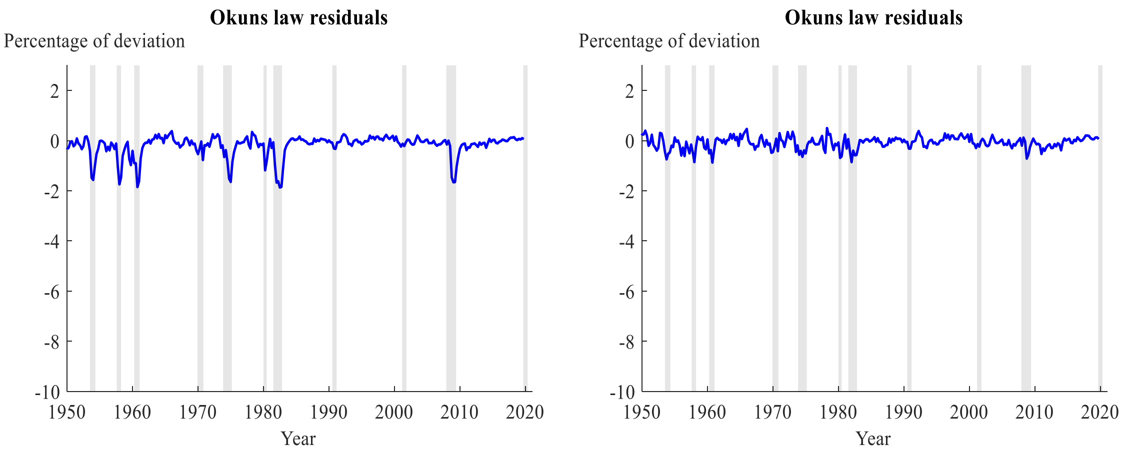
(c) Natural rate of unemployment (trend) and unemployment gap (cyclical component)

Figure 2.1: Results of the asymmetric bivariate model

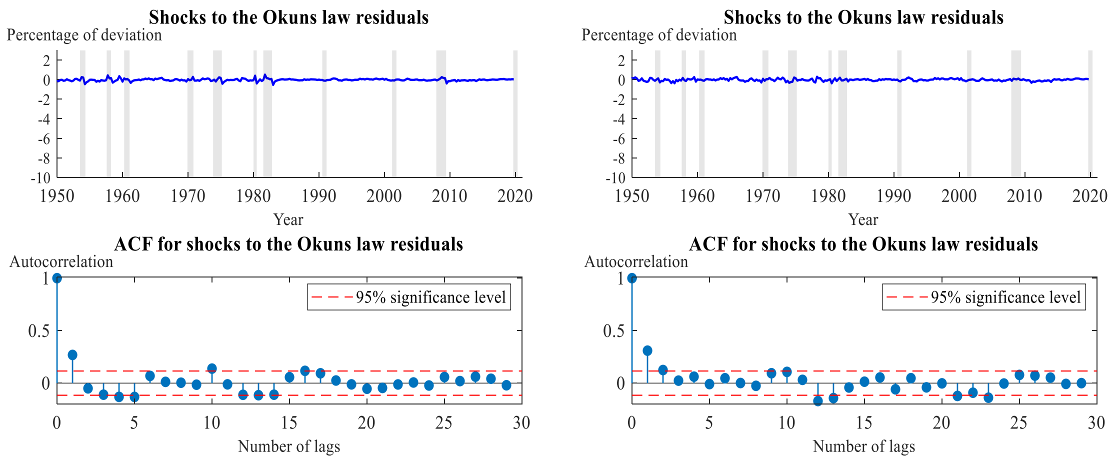
Notes:
(1) All panels plot the results of the benchmark model (Asymmetric-Bivariate-RW-SB-UC).
(2) The top panels plot potential output and the output gap, and the middle-left panel plots the trend growth of output.
(3) The middle-right panel plots the plucking probabilities, which are estimated for both output and unemployment jointly.
(4) The bottom panels plot the trend and gap for unemployment.
(5) The shaded areas are the NBER recession dates. See Table 2.A.3 in Appendix 2.A for details.



(a) Trend growth and trend acceleration of U.S. output



(b) Okun's law residuals without (left) and with (right) the 1-month lead-lag effect controlled



(c) Diagnostic of shocks to the Okun's residuals for model without (left) and with (right) the 1-month lead-lag effect controlled

Figure 2.2: Supplementary results of the asymmetric bivariate model

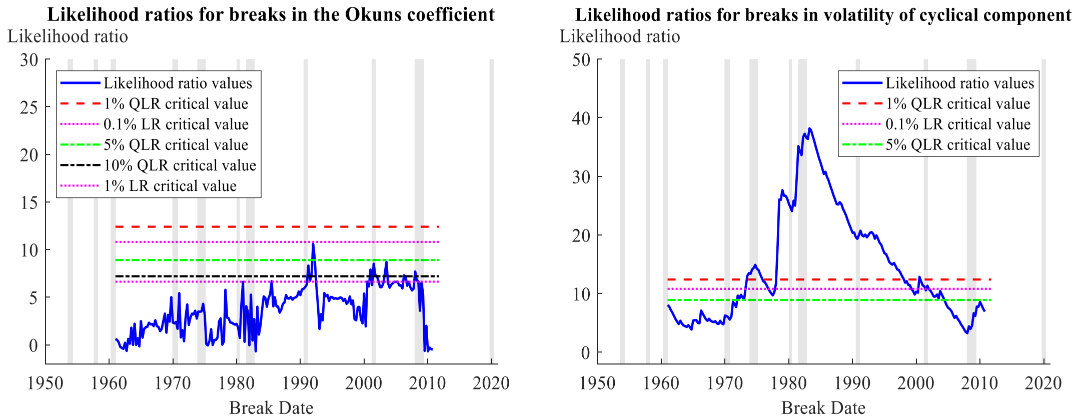
Notes:

(1) The top panels plot trend growth and trend acceleration (growth of trend growth) of U.S. output.

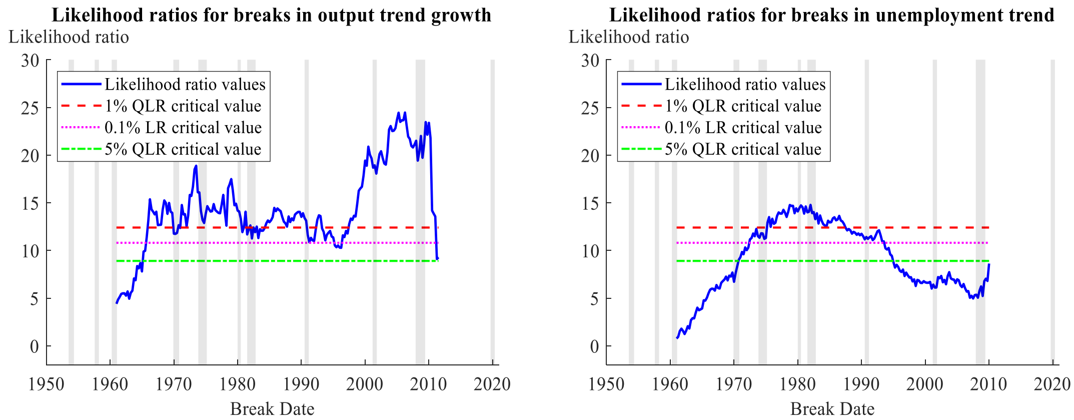
(2) The middle panel plots the Okun's law residuals. In the left panel, although some part of the plucking property remains in the residuals because it is not explained by Okun's law, the depth of the residuals is small in amplitude. The right panel shows controlling the 1-month lead-lag effect between output and unemployment entirely removes the remaining plucking property. This supports that the plucking property in output is mainly sourced from the plucking property in the unemployment rate.

(3) The shocks to the Okuns law residuals are zero-mean noise and its autocorrelation function decays very fast.

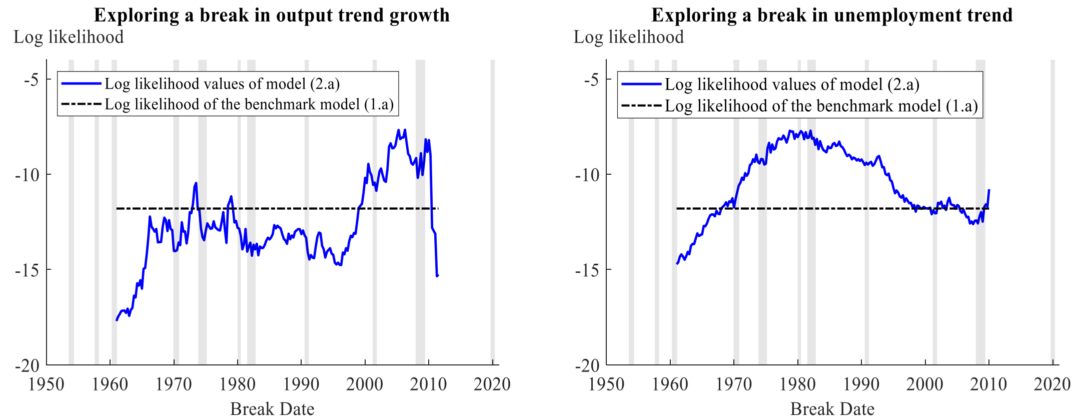
(4) The shaded areas are the NBER recession dates.



(a) Likelihood ratios at different break dates for Okun's law coefficient (left) and volatility (right)



(b) Likelihood ratios at different break dates for trend growth (left) and the unemployment trend (right)



(c) Log likelihood values of models (1.a) and (2.a)

Figure 2.3: Exploring structural breaks in the parameters

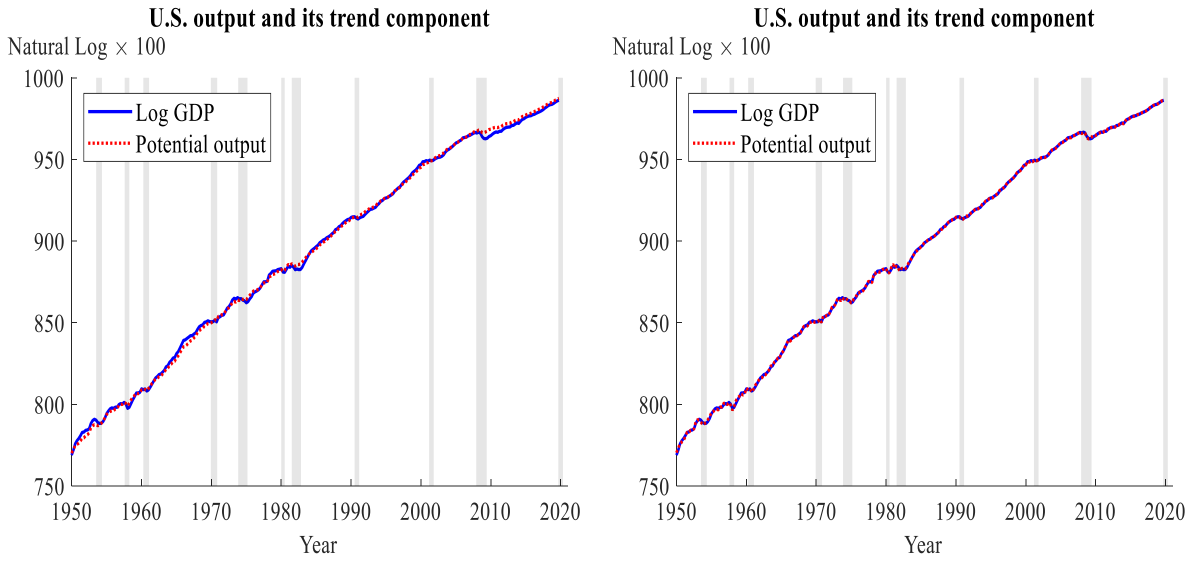
Notes:

(1) All panels plot the log likelihood values for a sequence of breaks rolling from 1960 to 2010.

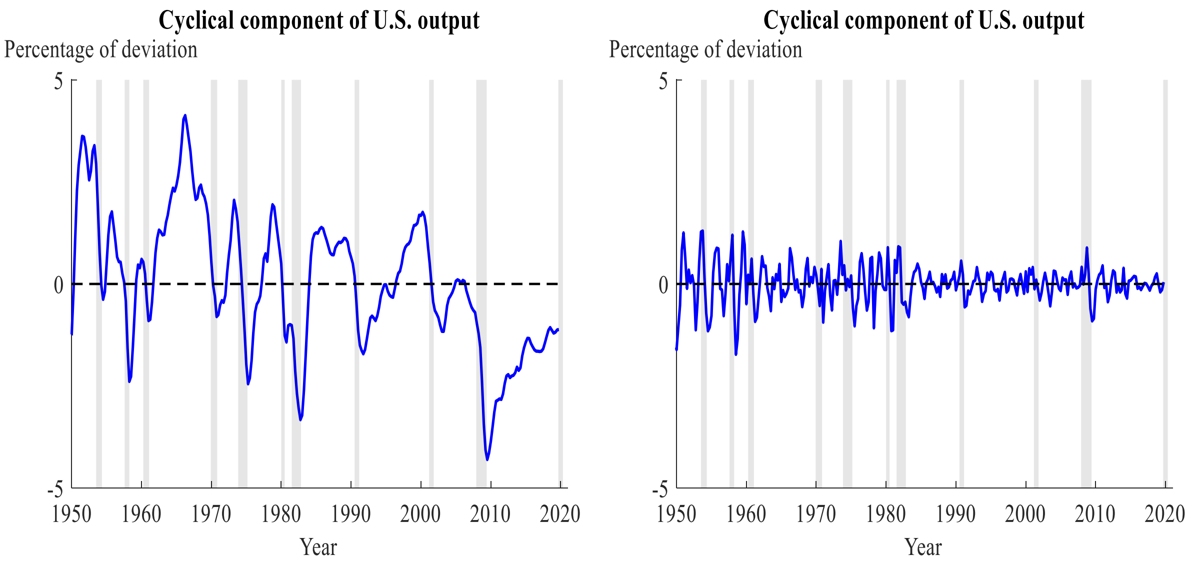
(2) The top-left panel plots the likelihood ratios for breaks in Okun's law, given the setup of the benchmark model (1.a). Since likelihood ratios are less than 5% QLR critical values, we rule out instability of Okun's law. In addition, likelihood ratios are even less than a 1% LR critical value of 6.63, which itself is less than the suitable critical value for the supremum of likelihood ratio among a sequence of breaks. The top-right panel plots the likelihood ratios for the breaks in volatility of shocks to the remaining cyclical component on different dates.

(3) The middle panels plot likelihood ratio test statistics. In the middle-left panel, likelihood ratio compares the log likelihood value of model (2.a) with a break in trend growth with that of its counterpart model (3.a) with constant trend growth. The middle-right panel plots the likelihood ratios testing for breaks in unemployment trend on different dates against a constant trend. The bottom-left panel shows the log likelihood values to detect a break in trend growth, conditioned on the setup of the model (2.a). The bottom-right panel shows the log likelihood values to detect the break in the unemployment trend, conditioned on a break in output trend growth in 2009. In both panels, the black dashed line represents the log likelihood of the benchmark model (1.a), which specifies the trend growth as a random walk.

(4) The shaded areas are the NBER recession dates.



(a) The trend component of uncorrelated (left) and correlated (right) models



(b) The cyclical component of uncorrelated (left) and correlated (right) models

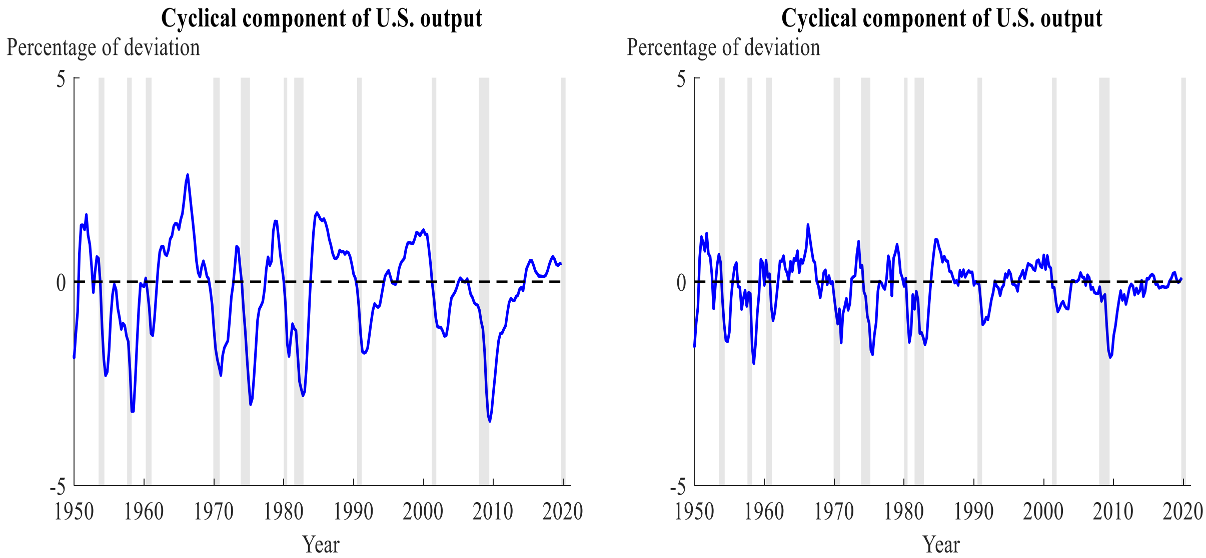
Figure 2.4: Results of the symmetric univariate models with constant trend growth for U.S. output

Notes:

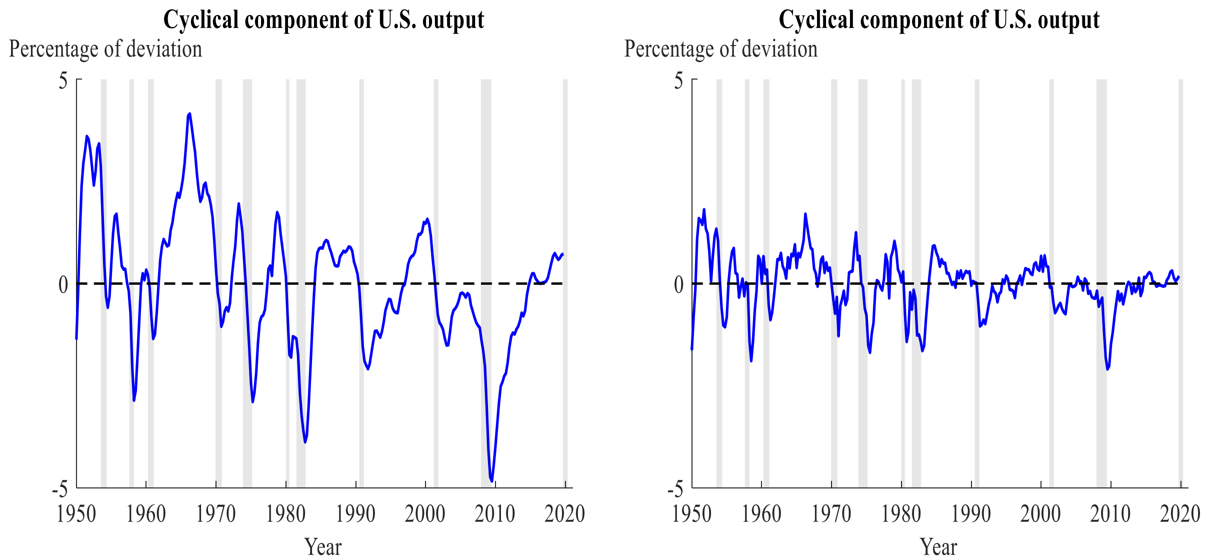
(1) The left panels plot the results of the symmetric model with constant trend growth, where shocks to the trend and cyclical components are uncorrelated (Symmetric-Univariate-Con-UC).

(2) The right panels plot the results of the symmetric model with constant trend growth, where shocks to the trend and cyclical components are correlated (Symmetric-Univariate-Con-C), which replicates those of Morley et al. (2003).

(3) The shaded areas are the NBER recession dates.



(a) The cyclical component of uncorrelated (left) and correlated (right) models, with stochastic trend growth



(b) The cyclical component of uncorrelated (left) and correlated (right) models, with a break in trend growth

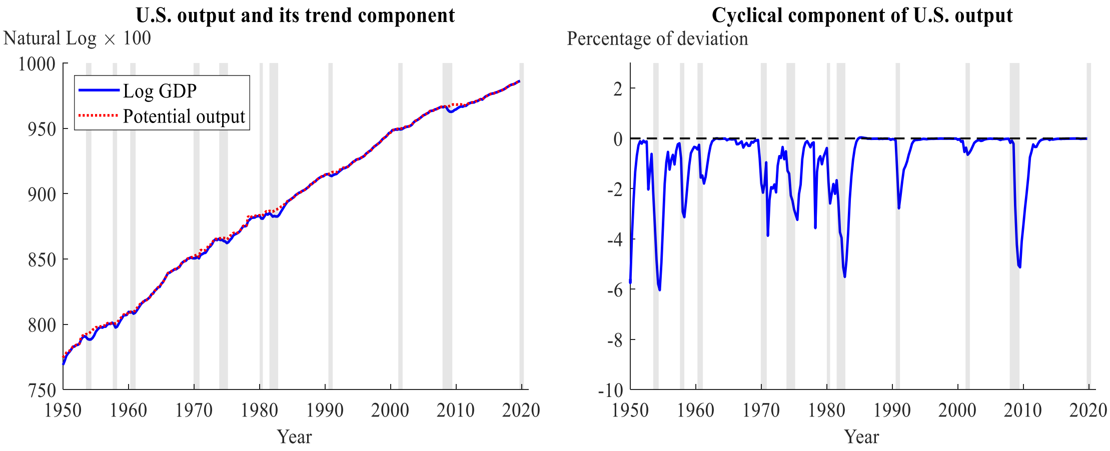
Figure 2.5. Results of the symmetric univariate models for U.S. output

Notes:

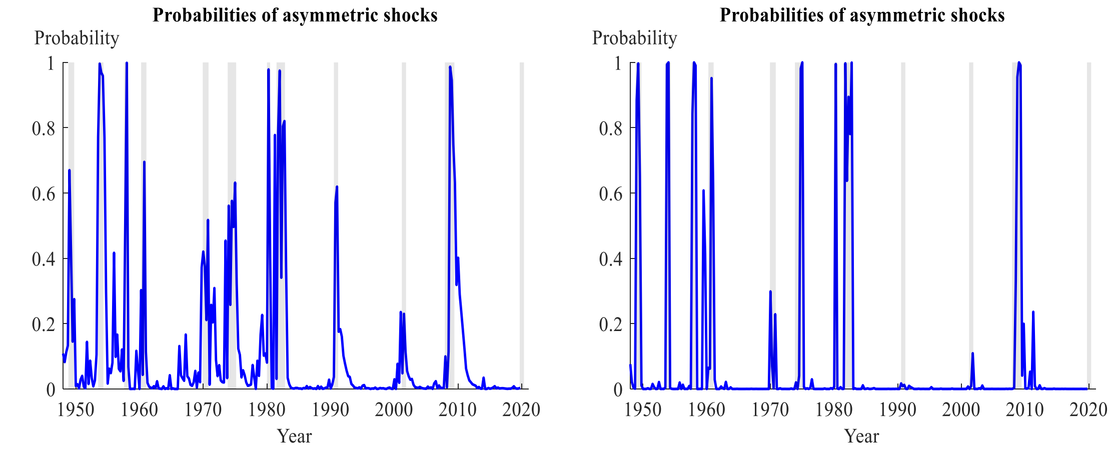
(1) The top panels are the results of the model with stochastic (random walk) trend growth where shocks to the trend and cyclical components are uncorrelated (Symmetric-Univariate-RW-UC) in the left panel and correlated (Symmetric-Univariate-RW-C) in the right panel. The former replicates the work of Clark (1987), whose setting will be applied in the benchmark asymmetric bivariate model.

(2) The bottom panels are the results of the model with a structural break in trend growth in 2009, where shocks to the trend and cyclical components are uncorrelated (Symmetric-Univariate-SB-UC) in the left and correlated in the right panels (Symmetric-Univariate-SB-C). They are similar to those of Perron and Wada (2009) and Grant and Chan (2017b) with a break in 2009.

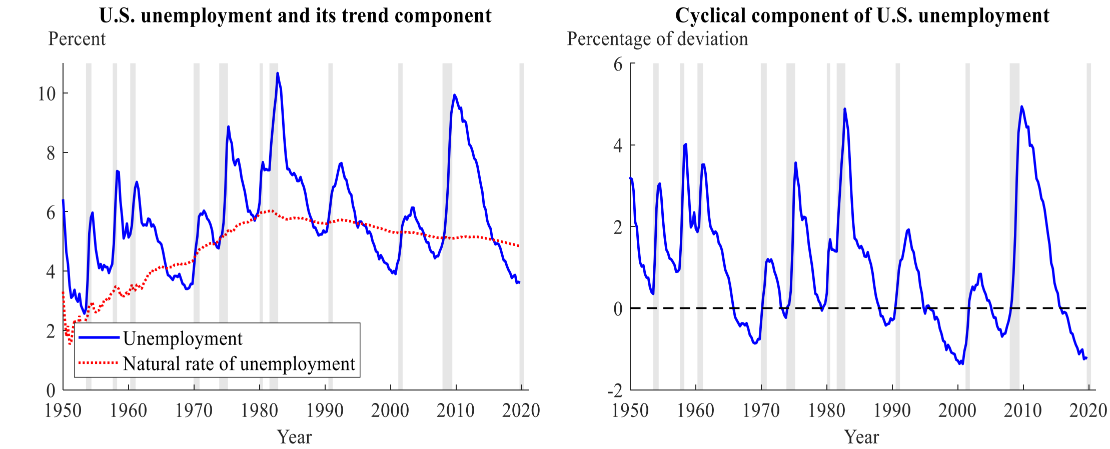
(3) The shaded areas are the NBER recession dates.



(a) Potential output (trend) and output gap (cyclical component)



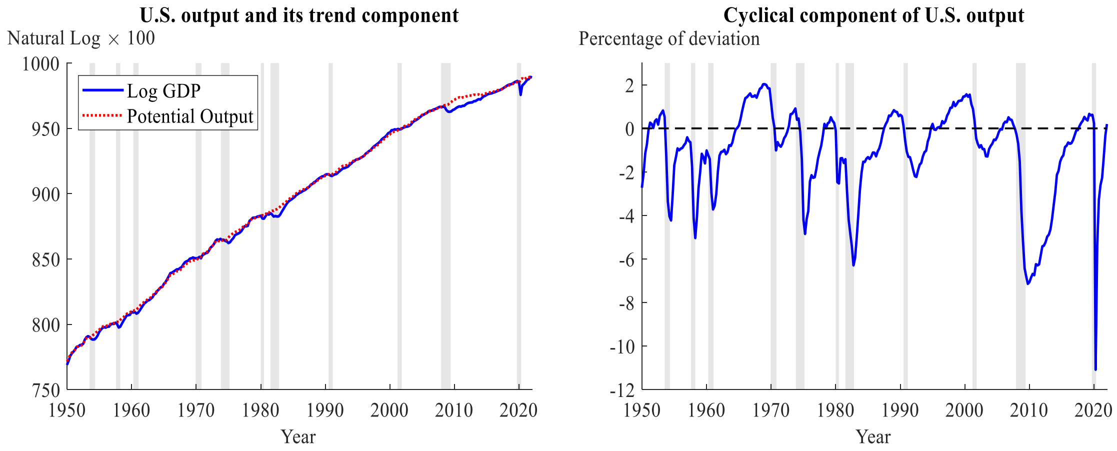
(b) Plucking probabilities for output (left) and unemployment (right)



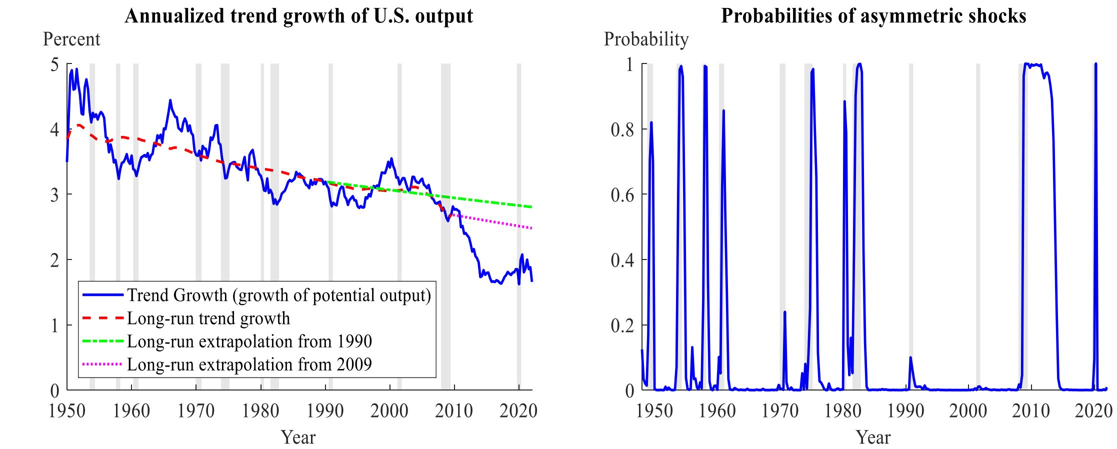
(c) Natural rate of unemployment (trend) and unemployment gap (cyclical components)

Figure 2.6: Comparing the results of the asymmetric univariate models for output and unemployment

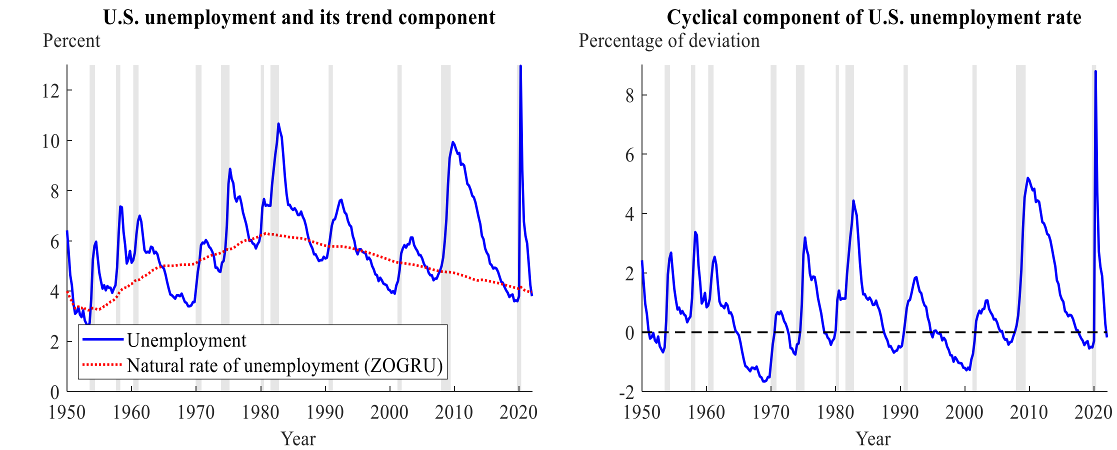
Notes:
(1) The top panels plot the results of the asymmetric univariate model for output with a stochastic (random walk) trend growth where shocks to the trend and cyclical components are uncorrelated (Asymmetric-Univariate-RW-UC). This model replicates the work of Kim and Nelson (1999a). For the correlated versions of the above models, see Figure 2.F.5 in Appendix 2.F.
(2) The middle panels plot the plucking probabilities for output and unemployment estimated separately.
(3) The bottom panels plot the results of the asymmetric univariate model for unemployment with a break in unemployment where shocks to the trend and cyclical components are uncorrelated (Asymmetric-Univariate-SB-UC).
(4) The shaded areas are the NBER recession dates.



(a) Potential output (trend) and output gap (cyclical component)



(b) Trend growth of output and the plucking probabilities for bivariate model



(c) Natural rate of unemployment (trend) and unemployment gap (cyclical component)

Figure 2.7: Results of the asymmetric bivariate model, including the COVID-19 recession

Notes:

(1) All panels plot the results of the benchmark model (Asymmetric-Bivariate-RW-SB-UC), which is estimated based on Eq. (2.19) for the period of 1948Q1 to 2022Q1 to include the COVID-19 recession.

(2) The top panels plot potential output and the output gap, and the middle-left panel plots the trend growth of output.

(3) The middle-right panel plots the plucking probabilities, which are estimated for both output and unemployment jointly.

(4) The bottom panels plot the trend and gap for unemployment.

(5) The shaded areas are the NBER recession dates.

Tables

Table 2.1: Estimated parameters of the bivariate models

Models	(1.a)	(1.b)	(2.a)	(2.b)	(3.a)	(4.a)	(5.a)	(6.a)
Parameters	A-Bi-RW-SB-UC	A-Bi-RW-SB-C1	A-Bi-SB-SB-UC	A-Bi-SB-SB-C1	A-Bi-Con-SB-UC	S-Bi-RW-SB-UC	S-Bi-SB-SB-UC	S-Bi-Con-SB-UC
σ_{x^*}	0.44 (0.09)	0.39 (0.10)	0.51 (0.06)	0.51 (0.06)	0.62 (0.02)	0.56 (0.11)	0.54 (0.21)	0.65 (0.03)
σ_{u^c}	0.21 (0.01)	0.19 (0.02)	0.21 (0.01)	0.21 (0.01)	0.21 (0.01)	0.27 (0.01)	0.27 (0.01)	0.27 (0.01)
σ_{μ}	0.03 (0.01)	0.03 (0.01)	–	–	–	0.02 (0.01)	–	–
σ_{u^*}	0.00 (0.02)	0.07 (0.07)	0.00 (0.03)	0.00 (0.02)	0.00 (0.01)	0.02 (0.04)	0.02 (0.05)	0.02 (0.05)
σ_{x^c}	0.33 (0.08)	0.37 (0.08)	0.25 (0.08)	0.26 (0.09)	0.00 (0.11)	0.25 (0.18)	0.31 (0.35)	0.05 (0.05)
γ	T-V	T-V	0.83 (0.03)	0.82 (0.03)	0.75 (0.03)	T-V	0.82 (0.04)	0.75 (0.04)
δ	T-V	T-V	-0.49 (0.08)	-0.48 (0.08)	–	T-V	-0.49 (0.09)	–
η	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)
θ	-0.04 (0.01)	-0.03 (0.01)	-0.04 (0.01)	-0.04 (0.01)	-0.04 (0.01)	-0.03 (0.01)	-0.04 (0.01)	-0.04 (0.01)
φ_1	1.38 (0.04)	1.39 (0.06)	1.36 (0.04)	1.36 (0.04)	1.36 (0.04)	1.60 (0.04)	1.60 (0.04)	1.60 (0.04)
φ_2	-0.45 (0.04)	-0.45 (0.06)	-0.43 (0.04)	-0.42 (0.04)	-0.43 (0.04)	-0.65 (0.04)	-0.65 (0.04)	-0.65 (0.04)
π_u	0.70 (0.05)	0.70 (0.06)	0.69 (0.05)	0.70 (0.05)	0.69 (0.05)	–	–	–
p	0.66 (0.09)	0.66 (0.09)	0.67 (0.09)	0.67 (0.09)	0.66 (0.09)	–	–	–
q	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	–	–	–
β	-1.45 (0.11)	-1.33 (0.17)	-1.44 (0.13)	-1.36 (0.19)	-1.34 (0.14)	-1.73 (0.10)	-1.78 (0.11)	-1.72 (0.11)
π_x	-1.01 (0.15)	-1.04 (0.18)	-1.05 (0.15)	-1.09 (0.17)	-1.04 (0.17)	–	–	–
ψ_x	0.49 (0.10)	0.54 (0.12)	0.50 (0.11)	0.53 (0.12)	0.54 (0.10)	0.56 (0.24)	0.73 (0.33)	0.77 (0.16)
ρ_{x^*,u^c}	–	-0.26 (0.27)	–	-0.06 (0.11)	–	–	–	–
ρ_{x^*,x^c}	–	–	–	–	–	–	–	–
ρ_{u^c,x^c}	–	–	–	–	–	–	–	–
Log likelihood	-11.9	-11.4	-7.4	-7.2	-19.9	-57.5	-52.6	-62.5

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are reported in parenthesis.
*** Numerical values for parameters denoted by 0.00 are respectively 0.0004 for model (1.a), 0.00001 for model (2.a), 0.0002 for model (2.b), and 0.0001 and 0.001 for model (3.a).

Notes:
(1) The estimation period runs from 1948Q1 to 2019Q4. We estimate twenty-one bivariate models on the basis of choices on the four specification aspects. We denote each model with a term consisting of five parts, four of which is related to one of the specification aspects of output and the other denotes the specification of the unemployment trend. For example, A-Bi-RW-SB-UC represents an Asymmetric-Bivariate-Random Walk-Structural Break-Uncorrelated model. For this model, the shocks are asymmetric, there are two variables (output and the unemployment rate), the trend growth is assumed to be a random walk (stochastic), the drift in unemployment trend has a structural break, and the correlation between shocks to the trend and cyclical components is assumed to be zero. For the list of models and their specifications, see Table 2.C in Appendix 2.C. The results of the other ten models are presented in Table 2.G.1 in Appendix 2.G.

Table 2.1: The notes Continue

(2) For all models, the structural break in the drift of the unemployment rate trend (natural rate of unemployment) in 1981Q1 is accounted for. The break date is identified based on likelihood ratio statistics estimated for a sequence of breaks from 1960 to 2010, which spiked around 1981, as shown in the middle-right panel of Figure 2.3. For models (2.a), (2.b), and (5.a), the structural break in trend growth in 2009Q3 is accounted for. The break date is determined based on likelihood ratio statistics estimated for a sequence of breaks from 1960 to 2010, which spiked around the 2007−09 financial crisis, as shown in the middle-left panel of Figure 2.3.

(3) By pairwise comparison of the log likelihood values of -11.9, -7.4, and -19.9 reported for asymmetric models (1.a), (2.a), and (3.a) with values of -57.5, -52.6, and -62.5 for their symmetric counterpart models (4.a), (5.a), and (6.a), respectively, we favour the asymmetric models over symmetric models. The corresponding likelihood ratios of 91.2, 90.4, and 85.2 are all exceedingly greater than the critical values of 10.8 for a 0.1% significance level. The comparison between likelihood ratios of -11.4 for the asymmetric correlated model (1.b) and -49.1 for the symmetric correlated model (4.b), which is presented in Table 2.G.1 in Appendix 2.G, bears a likelihood ratio of 75.4, indicating that including the correlation in the model does not change the result.

(4) We compare the log likelihood values of -7.4 and -52.6 reported for models (2.a) and (5.a) with values of -19.9 and -62.5 for their counterpart models (3.a) and (6.a), where the trend growth is assumed to be constant. We reject the null hypothesis of constant drift in the symmetric and asymmetric bivariate models because the likelihood ratios of 25.0 for the asymmetric and 19.8 for the symmetric models are greater than the critical value of 10.8 for 0.1% significance level. Regarding the models with stochastic trend growth, the log likelihood values of -11.9 and -57.5 for models (1.a) and (4.a) are respectively close to values of -7.4 and -52.6 for their counterpart models with a structural break in trend growth in 2009, which maximizes the approximate log likelihood with respect to the break date. In addition, by comparing the log likelihood values of -11.9 and -57.5 for models (1.a) and (3.a) with values of -19.9 and -62.5 for models (3.a) and (6.a) with constant trend growth, we observe a considerable improvement in the log likelihood value. We also report the log likelihood of -20.5 for a model named (3′.a), which is presented in Table 2.G.1 in Appendix 2.G and is fully nested in model (1.a). We therefore favour the model with stochastic drift over the model with constant drift. Thus, the random walk is fairly capable of accommodating unknown breaks in trend growth and competing with the best model selected among models with structural breaks.

(5) We relax the assumption of zero correlation between shocks to the output trend and the symmetric cyclical component (ρ_{x^*,u^c}) by estimating models (1.b) and (2.b). By comparing the log likelihood values of -11.9 and -7.4 for uncorrelated models (1.a) and (2.a) with values of -11.4 and -7.2 for their correlated counterpart models (1.b) and (2.b), respectively, we accept the null hypothesis of zero-correlation in the asymmetric bivariate model. Indeed, the likelihood ratio values of 1.0 for model (1) and 0.4 for model (2) are less than critical values of 3.84 and 2.71 for 5% and even 10% significance levels. As shown in Table 2.G.1 in Appendix 2.G, we estimate other correlations between shocks to the output trend and the remaining cyclical component (ρ_{x^*,x^c}) in model (1.c), and between shocks to the symmetric cyclical component and the remaining cyclical component (ρ_{u^c,x^c}) placed in the model (1.d). Comparing the likelihood values of model (1.a) with those of models (1.c) and (1.d) indicates that both correlations are zero, with log likelihood ratios of 0.0. Finally, to test whether all three correlations are jointly significant, we compare the log likelihood value of -11.9 for model (1.a) with the value of -10.9 for model (1.e), where all three correlations are allowed for. Since the corresponding likelihood ratio of 2.0 for three restrictions is less than critical values of 7.81 and 6.25 for 5% and even 10% significance levels, we accept the null hypothesis of zero-correlations. Overall, relaxing the zero-correlation assumption does not change the estimated parameters, confirming that the correlation is irrelevant in the asymmetric bivariate model.

Table 2.2: Estimated parameters of the univariate models for output

Models	(1.a)	(1.b)	(2.a)	(2.b)	(3.a)	(4.a)	(5.a)	(6.a)
Parameters	A-Uni-RW-UC	A-Uni-RW-C	A-Uni-SB-UC	A-Uni-SB-C	A-Uni-Con-UC	S-Uni-RW-UC	S-Uni-SB-UC	S-Uni-Con-UC
σ_{z^*}	0.63 (0.03)	1.06 (0.13)	0.68 (0.03)	1.04 (0.11)	0.70 (0.03)	0.60 (0.07)	0.62 (0.07)	0.67 (0.06)
σ_{z^c}	0.00 (0.15)	0.64 (0.16)	0.002 (0.17)	0.62 (0.16)	0.001 (0.08)	0.50 (0.09)	0.50 (0.08)	0.45 (0.07)
σ_μ	0.06 (0.02)	0.01 (0.01)	–	–	–	0.02 (0.01)	–	–
γ	T-V	T-V	0.82 (0.04)	0.85 (0.06)	0.77 (0.04)	T-V	0.82 (0.04)	0.77 (0.04)
δ	T-V	T-V	-0.38 (0.11)	-0.40 (0.17)	–	T-V	-0.35 (0.13)	–
φ_1	1.11 (0.08)	1.14 (0.07)	1.10 (0.07)	1.09 (0.07)	1.05 (0.09)	1.58 (0.10)	1.57 (0.08)	1.61 (0.11)
φ_2	-0.35 (0.08)	-0.40 (0.07)	-0.28 (0.07)	-0.35 (0.07)	-0.39 (0.09)	-0.67 (0.09)	-0.65 (0.08)	-0.69 (0.07)
π_z	-1.67 (0.19)	-1.88 (0.27)	-1.71 (0.20)	-1.82 (0.20)	-1.70 (0.26)	–	–	–
p	0.64 (0.06)	0.60 (0.09)	0.64 (0.08)	0.62 (0.09)	0.91 (0.02)	–	–	–
q	0.95 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.97 (0.01)	–	–	–
ρ_{z^*,z^c}	–	-0.88 (0.05)	–	-0.85 (0.06)	–	–	–	–
Log likelihood	-336.9	-332.6	-334.7	-329.3	-341.0	-354.4	-352.6	-357.1

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are reported in parenthesis.

Notes:

(1) The estimation period runs from 1948Q1 to 2019Q4. We estimate fourteen univariate models on the basis of choices on the four specification aspects. We denote each model with a term consisting of four parts, each of which is related to one of the specification aspects. For example, A-Uni-RW-UC means Asymmetric-Univariate-Random Walk-Uncorrelated. Indeed, in this model, shocks are asymmetric, there is only one variable, the trend growth is assumed to be a random walk (stochastic), and the correlation between shocks to the trend and cyclical components is assumed to be zero. For the list of models and specifications, see Table 2.B in Appendix 2.B. The results of the other models are presented in Table 2.G.2 in Appendix 2.G.

(2) In models (2.a), (2.b), and (5.a), a break in trend growth in 2009Q3 is accounted for. The break date is determined based on likelihood ratio statistics estimated for a sequence of break dates from 1960 to 2010, which spiked around the 2007−09 financial crisis, as shown in the middle-left panel of Figure 2.3 and in the left panel of Figure 2.F.6 in Appendix 2.F.

(3) By pairwise comparison of the log likelihood values of -336.9, -334.7, and -341.0 reported for asymmetric models (1.a), (2.a), and (3.a) with values of -354.4, -352.6, and -357.1 for their symmetric counterparts (4.a), (5.a), and (6.a), respectively, we favour the asymmetric models over symmetric models. The corresponding likelihood ratios of 35.0, 35.8, and 32.2 are all considerably greater than the critical value of 10.8 for a 0.1% significance level.

(4) We compare the log likelihood values of -334.7 and -352.6 reported for models (2.a) and (5.a) with values of -341.0 and -357.1 for their counterpart models (3.a) and (6.a), where the trend growth is assumed to be constant. We reject the null hypothesis of constant drift in the symmetric and asymmetric univariate models because the likelihood ratios of 12.6 for the asymmetric and 9.0 for the symmetric models are greater than the critical value of 6.63 for a 1% significance level. Regarding the models with stochastic trend growth, the log likelihood values of -336.9 and -354.4 for models (1.a) and (4.a) are close to those of their counterpart models with a structural break in trend growth in 2009, which maximizes the log likelihood with respect to the break date. In addition, by comparing the log likelihood values of -336.9 and -354.4 for models (1.a) and (4.a) with values of -341.0 and -357.1 for models (3.a) and (6.a) with constant trend growth, we observe a considerable improvement in log likelihood. We additionally report the log likelihood of -340.9 for a model named (3′.a), which is presented in Table 2.G.2 in Appendix 2.G and is fully nested in model (1.a). We therefore favour the model with stochastic drift over the model with constant drift. As a result, the random walk is indeed capable of accommodating unknown breaks in trend growth and competing with the best model among models with structural breaks.

(5) By comparing the log likelihood values of the uncorrelated and correlated versions of each of the models (1) and (2), we reject the null hypothesis of zero-correlation in the asymmetric univariate model because the likelihood ratio values of 8.6 for model (1) and 10.8 for model (2) are greater than the 1% critical value of 6.63. Although the correlation is significant, the change in the estimation of other parameters and features of the trend and cyclical components is mild when the business cycle asymmetry is accounted for. For example, the trend and cyclical components shown in Figure 2.6 for the uncorrelated asymmetric model are similar to those in Figure 2.F.5 in Appendix 2.F for the correlated asymmetric model. Likewise, the trend and cyclical components of the uncorrelated and correlated models in the left and right panels of Figure 2.F.4 in Appendix 2.F are very similar.

Table 2.3: Estimated parameters of the univariate models for unemployment

Models	(1.a)	(1.b)	(2.a)	(2.b)
Parameters	A-Uni-SB-UC	A-Uni-SB-C	S-Uni-SB-UC	S-Uni-SB-C
σ_{z^*}	0.001 (0.02)	0.001 (0.03)	0.001 (0.02)	0.03 (0.05)
σ_{z^c}	0.21 (0.009)	0.21 (0.02)	0.27 (0.01)	0.28 (0.04)
η	0.03 (0.006)	0.03 (0.006)	0.02 (0.008)	0.02 (0.009)
θ	-0.03 (0.009)	-0.03 (0.009)	-0.03 (0.01)	-0.03 (0.01)
φ_1	1.38 (0.04)	1.38 (0.04)	1.60 (0.04)	1.58 (0.06)
φ_2	-0.44 (0.04)	-0.44 (0.04)	-0.65 (0.04)	-0.64 (0.07)
π_z	0.75 (0.06)	0.75 (0.06)	–	–
p	0.63 (0.11)	0.63 (0.11)	–	–
q	0.97 (0.01)	0.97 (0.01)	–	–
ρ_{z^*,z^c}	–	0.58 (10.75)	–	-0.60 (0.80)
Log likelihood	-4.9	-4.9	-36.7	-36.4

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are reported in parenthesis.

Notes:

(1) The estimation period runs from 1948Q1 to 2019Q4. We estimate four bivariate models for unemployment. We denote each model with a term consisting of four parts, each of which is related to each specification aspect. For example, A-Uni-SB-UC means Asymmetric-Univariate-Structural Break-Uncorrelated. Indeed, in this model, shocks are asymmetric, there is one variable, the drift in unemployment trend has a break, and the correlation between shocks to the trend and cyclical components is assumed to be zero. For the list of models and specifications, see Table 2.B in Appendix 2.B.

(2) For all models, the structural break in the drift of the unemployment rate trend (natural rate of unemployment) in 1981Q1 is accounted for. The break date is identified based on likelihood ratio statistics estimated for a sequence of breaks from 1960 to 2020, which spiked around 1981, as shown in the middle-right panel of Figure 2.3 and Figure 2.F.6 in Appendix 2.F.

(3) By pairwise comparison of log likelihood values of -4.9 and -4.9 reported for two asymmetric models (1.a) and (1.b) with values of -36.7 and -36.4 reported for their symmetric counterpart models (2.a) and (2.b), we favour the asymmetric models over symmetric models. The corresponding likelihood ratios of 63.6 and 63.0 are exceedingly greater than the critical value of 10.8 for a 0.1% significance level.

(4) By comparing the log likelihood of -4.9 reported for the asymmetric uncorrelated model (1.a) with the value of -4.9 for its correlated counterpart (1.b), we accept the null hypothesis of zero-correlation because the likelihood ratio is 0.0.

(5) By comparing the log likelihood of -36.7 reported for the symmetric uncorrelated model (2.a) with the value of -36.4 for its correlated counterpart (2.b), we accept the null hypothesis of zero-correlation because the negligible likelihood ratio of 0.8 is less than the critical values of 3.84 and even 2.71 for 5% and 10% significance levels.

Supplementary Appendix to

Friedman's Plucking Model and Okun's Law

Mohammad Dehghani[†,*] Sungjun Cho[†] Stuart Hyde[†]

Appendix 2.A: Literature summary tables

Table 2.A.1: Four specification aspects of the literature of trend-cycle decomposition

Authors	Four underlying assumptions and potential caveats	
Bivariate Models		
Asymmetric		
This study	1. Asymmetric 2. Bivariate	3. Stochastic trend growth 4. Correlation irrelevance
Symmetric		
Owyang and Sekhposyan (2012), Grant (2018)	1. Symmetric 2. Bivariate	3. Constant trend growth 4. Uncorrelated shocks
Berger et al. (2016), Fernald et al. (2017), Dehghani et al. (2022)	1. Symmetric 2. Bivariate	3. Stochastic trend growth 4. Uncorrelated shocks
Clark (1989), Gonzalez-Astudillo and Roberts (2022)	1. Symmetric 2. Bivariate	3. Stochastic trend growth 4. Correlated shocks
Univariate Models		
Asymmetric		
Kim and Nelson (1999a), Mills and Wang (2002), De Simone and Clarke (2007), Morley and Piger (2012)	1. Asymmetric 2. Univariate	3. Stochastic trend growth 4. Uncorrelated shocks
Eo and Morley (2022)	1. Asymmetric 2. Univariate	3. Structural break in trend growth 4. Uncorrelated shocks
Sinclair (2010)	1. Asymmetric 2. Univariate	3. Constant trend growth 4. Correlated shocks
Symmetric		
Beveridge and Nelson (1981), Nelson and Plosser (1982), Morley et al. (2003)	1. Symmetric 2. Univariate	3. Constant trend growth 4. Correlated shocks
Perron and Wada (2009), Grant and Chan (2017b)	1. Symmetric 2. Univariate	3. Structural break in trend growth 4. Uncorrelated shocks
Luo and Startz (2014)	1. Symmetric 2. Univariate	3. Structural break in trend growth 4. Correlated shocks
Harvey (1985), Clark (1987), Grant and Chan (2017a)	1. Symmetric 2. Univariate	3. Stochastic trend growth 4. Uncorrelated shocks
Kim and Chon (2020), Kim and Kim (2020)	1. Symmetric 2. Univariate	3. Stochastic trend growth 4. Correlated shocks

Note:
(1) The first aspect specifies whether the shocks to the cyclical component are asymmetric or symmetric. The second aspect specifies whether the unemployment rate is added to the model or not. This study does not review other studies that run bivariate models with other variables such as inflation or cyclical factors. The third aspect specifies whether the trend growth is stochastic or deterministic. For deterministic trend growth, there are two cases: constant trend growth or a structural break in trend growth. The fourth aspect specifies whether the correlation between the shocks to the trend and cyclical components is assumed to be zero or not.
(2) In some studies, different setups for one or two of the specification aspects have been used. In the above table, we refer to their benchmark model.

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Appendix 2.A continued: Literature summary tables

Table 2.A.2: Four types of business cycle asymmetries

Definition and tests	Related literature
Correlation asymmetry (ceiling effect)	
The amplitude of recessions is strongly correlated with the amplitude of succeeding expansions, but the amplitude of expansions is uncorrelated with the amplitude of succeeding recessions.	Friedman (1964, 1993), Wynne and Balke (1992), Beaudry and Koop (1993), Goodwin and Sweeney (1993), Fatás and Mihov (2013), Bordo and Haubrich (2017), Dupraz et al. (2019).
Deepness asymmetry	
● Recession troughs are deep and expansion peaks are small in magnitude. ● Output displays a negative skewness relative to the trend. ● The unemployment rate displays a positive skewness.	Neftci (1984), DeLong and Summer (1984), Sichel (1993), Goodwin and Sweeney (1993).
Steepness asymmetry	
● Recessions are steep (violent) and expansions are gradual (mild). ● Output growth (first difference) displays a negative skewness. ● Unemployment growth (first difference) displays a positive skewness.	DeLong and Summer (1984), Falk (1986), Sichel (1993), McKay and Reis (2008), Jensen et al. (2020).
Duration asymmetry	
Recessions are short and recoveries are long.	Neftci (1984)

Note:

(1) The output gap skewness of -0.93 -0.4, and unemployment gap skewness of +0.75 and 0.94 reported in Figure 2.F.2 in Appendix 2.F, provides preliminary evidence for asymmetries in output and unemployment.

(2) The Markov-switching process has the potential to capture all types of asymmetries: A significant plucking coefficient with the addition of estimating output gaps that are often negative and rarely positive confirms the ceiling effect (correlation asymmetry). Estimating deep output gaps with a short expected duration for recessions and a long expected duration for recoveries implies deepness, steepness, and duration asymmetries.

(3) Besides the asymmetries explained here, other studies have defined alternative asymmetries. McQueen and Thorley (1993) explore the sharpness symmetry, which means the peaks are sharp and the troughs are round for the unemployment rate. Another classification suggests two asymmetries: asymmetry around the vertical line, and asymmetry around the horizontal line. In this sense, correlation and deepness are asymmetries around the horizontal line and steepness and duration are asymmetries around the vertical line.

Appendix 2.A continued: Business cycle dates

Table 2.A.3: Dates of the U.S. Business Cycles (Peak and Trough)

N	ECRI*	NBER**	Description
1	1957M8-1958M4	1957M8-1958M4	--
2	1960M4-1961M2	1960M4-1961M2	--
3	1969M12-1970M11	1969M12-1970M11	--
4	1973M11-1975M3	1973M11-1975M3	First Oil Crisis
5	1980M1-1980M7	1980M1-1980M7	Second Oil Crisis
6	1981M7-1982M11	1981M7-1982M11	Early 1980s recession
7	1990M7-1991M3	1990M7-1991M3	Early 1990s recession
8	2001M3-2001M11	2001M3-2001M11	Early 2000s recession
9	2007M12-2009M6	2007M12-2009M6	Global crisis and recession
10	2020M2-2020M4	2020M2-2020M4	COVID-19 recession

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Table 2.A.4: Dates of the U.K. Business Cycles (Peak and Trough)

N	ECRI*	NIESR**	Description
1	-	1951M3-1952M8	--
2	-	1955M12-1958M11	--
3	-	1961M3-1963M1	--
4	1974M9-1975M8	1973M1-1975M3	First Oil Crisis
5	1979M6-1981M5	1979M2-1982M4	Second Oil Crisis
6	-	1984M1-1984M3	--
7	-	1988M4-1992M2	Early 1990s recession
8	1990M5-1992M3	-	Early 1990s recession
9	2008M5-2010M1	-	Global crisis and recession
10	2019M10-2020M4	-	COVID-19 recession

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Appendix 2.B: Univariate state-space model with Markov-switching

For the univariate model specified in Eq. (2.1) to Eq. (2.8), we estimate the output trend and cyclical components by casting the univariate model in a state-space form. In this setting, we consider a stochastic trend growth that moves according to a random walk. The observation equation, the transition equation, and variance covariance matrix of error terms are as follows:

$$[z_t] = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_t^* \\ z_t^c \\ z_{t-1}^c \\ \mu_t \end{bmatrix} + [0] \tag{2.B.1}$$

$$\begin{bmatrix} z_t^* \\ z_t^c \\ z_{t-1}^c \\ \mu_t \end{bmatrix} = \begin{bmatrix} 0 \\ \pi_z S_t \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \varphi_1 & \varphi_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_{t-1}^* \\ z_{t-1}^c \\ z_{t-2}^c \\ \mu_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{z^*,t} \\ \varepsilon_{z^c,t} \\ 0 \\ \varepsilon_{\mu,t} \end{bmatrix} \tag{2.B.2}$$

$$\begin{bmatrix} \varepsilon_{z^*,t} \\ \varepsilon_{z^c,t} \\ 0 \\ \varepsilon_{\mu,t} \end{bmatrix} \sim N(\mathbf{0}_{4\times1}, \begin{bmatrix} \sigma_{z^*}^2 & \rho\sigma_{z^*}\sigma_{z^c} & 0 & 0 \\ \rho\sigma_{z^*}\sigma_{z^c} & \sigma_{z^c}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\mu^2 \end{bmatrix} \tag{2.B.3}$$

In the above model, we consider natural log GDP multiplied by 100 as the observed series (z_t). Because of the presence of Markov-switching, we need to use Kim's (1944) approximate maximum likelihood method to estimate the model. To test for asymmetry, we derive the restricted symmetric model by imposing $\pi_z = 0$ on the unrestricted asymmetric model. We then estimate this nested model by using Kalman's (1960) filter.

In an alternative specification, we model the trend growth as a non-stochastic drift with a structural break. Indeed, we specify the trend growth equation in the form of $\mu_t = \gamma + \delta\mathbb{1}_t(t \geq T_\mu)$, where $\mathbb{1}_t$ is equal to one after the break date (T_μ) date and zero otherwise. The state-space representation of this model is:

$$[z_t] = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_t^* \\ z_t^c \\ z_{t-1}^c \end{bmatrix} + [0] \tag{2.B.4}$$

$$\begin{bmatrix} z_t^* \\ z_t^c \\ z_{t-1}^c \end{bmatrix} = \begin{bmatrix} \gamma + \delta\mathbb{1}_t(t \geq T_\mu) \\ \pi_z S_t \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \varphi_1 & \varphi_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_{t-1}^* \\ z_{t-1}^c \\ z_{t-2}^c \end{bmatrix} + \begin{bmatrix} \varepsilon_{z^*,t} \\ \varepsilon_{z^c,t} \\ 0 \end{bmatrix} \tag{2.B.5}$$

$$\begin{bmatrix} \varepsilon_{z^*,t} \\ \varepsilon_{z^c,t} \\ 0 \end{bmatrix} \sim N(\mathbf{0}_{3\times1}, \begin{bmatrix} \sigma_{z^*}^2 & \rho\sigma_{z^*}\sigma_{z^c} & 0 \\ \rho\sigma_{z^*}\sigma_{z^c} & \sigma_{z^c}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.B.6}$$

We also apply the univariate model specified in Eq. (2.1) to Eq. (2.8) to estimate the unemployment rate trend and cyclical components. The state-space form applied on the unemployment rate is:

$$[z_t] = [1 \quad 1 \quad 0]\begin{bmatrix} z_t^* \\ z_t^c \\ z_{t-1}^c \end{bmatrix} + [0] \tag{2.B.7}$$

$$\begin{bmatrix} z_t^* \\ z_t^c \\ z_{t-1}^c \end{bmatrix} = \begin{bmatrix} \eta + \theta \mathbb{1}_t(t \geq T_\mu)) \\ \pi_z S_t \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \varphi_1 & \varphi_2 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} z_{t-1}^* \\ z_{t-1}^c \\ z_{t-2}^c \end{bmatrix} + \begin{bmatrix} \varepsilon_{z^*,t} \\ \varepsilon_{z^c,t} \\ 0 \end{bmatrix} \tag{2.B.8}$$

$$\begin{bmatrix} \varepsilon_{z^*,t} \\ \varepsilon_{z^c,t} \\ 0 \end{bmatrix} \sim N(\mathbf{0}_{3\times 1}, \begin{bmatrix} \sigma_{z^*}^2 & \rho\sigma_{z^*}\sigma_{z^c} & 0 \\ \rho\sigma_{z^*}\sigma_{z^c} & \sigma_{z^c}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.B.9}$$

In the above model, we consider unemployment as the observed series (z_t). For natural rate of unemployment, to control for the potential break in the drift, we consider the drift term specified as $\eta + \theta \mathbb{1}_t(t \geq T_\mu)$, where $\mathbb{1}_t$ is equal to one after the break date (T_μ). Note that we don't impose a specific time in the break, and we find the break by exploring the supremum of the log likelihood ratios calculated for a sequence of breaks rolling from 1960 to 2010. We find that the break date is around 1981.

Table 2.B: Specification of 18 univariate models for output and unemployment

Model name	Tables and Figures	Related model in the literature
Univariate models for output		
Model (1.a): Asymmetric-Univariate-RW-UC	Table 2.2, Figure 2.6	(Kim and Nelson, 1999a)
Model (1.b): Asymmetric-Univariate-RW-C (ρ_{z^*,z^c})	Table 2.2, Figure 2.F.5	--
Model (2.a): Asymmetric-Univariate-SB-UC	Table 2.2, Figure 2.F.4	--
Model (2.b): Asymmetric-Univariate-SB-C (ρ_{z^*,z^c})	Table 2.2, Figure 2.F.4	--
Model (3.a): Asymmetric-Univariate-Con-UC (3 state variables)	Table 2.2, Figure 2.F.3	--
Model (3′.a): Asymmetric-Univariate-Con-UC (4 state variables)	Table 2.G.2	--
Model (3.b): Asymmetric-Univariate-Con-C (ρ_{z^*,z^c})	Table 2.G.2, Figure 2.F.3	(Sinclair, 2010)
Model (4.a): Symmetric-Univariate-RW-UC	Table 2.2, Figure 2.5	(Clark, 1987)
Model (4.b): Symmetric-Univariate-RW-C (ρ_{z^*,z^c})	Table 2.G.2, Figure 2.5	--
Model (5.a): Symmetric-Univariate-SB-UC	Table 2.2, Figure 2.5	(Perron and Wada, 2009)
Model (5.b): Symmetric-Univariate-SB-C (ρ_{z^*,z^c})	Table 2.G.2, Figure 2.5	--
Model (6.a): Symmetric-Univariate-Con-UC	Table 2.2, Figure 2.4	--
Model (6.b): Symmetric-Univariate-Con-C (ρ_{z^*,z^c})	Table 2.G.2, Figure 2.4	(Morley, 2003)
Model 7: Asymmetric-Univariate-RW-UC-SB	Figure 2.3	--
Univariate models for unemployment		
Model (1.a): Asymmetric-Univariate-SB-UC	Table 2.3, Figure 2.6	--
Model (1.b): Asymmetric-Univariate-SB-C (ρ_{z^*,z^c})	Table 2.3, Figure 2.F.5	--
Model (2.a): Symmetric-Univariate-SB-UC	Table 2.3, Figure 2.F.7	--
Model (2.b): Symmetric-Univariate-SB-C (ρ_{z^*,z^c})	Table 2.3, Figure 2.F.7	--

Notes:

(1) We estimate fourteen univariate models for output and four univariate models for unemployment. We denote each model with an identifier and a descriptor. The descriptor consists of four parts, each of which is related to each specification aspect. The first part determines whether the model is asymmetric or symmetric, and the second part re-emphasises that the model is univariate. The third part hints at the specification of output trend growth (stochastic, structural break, or constant). Regarding the unemployment rate, the third part shows that we take a structural break in drift term of the natural rate of unemployment into account. The last part indicates whether the model is uncorrelated or correlated. For example, Model (1.a), which is denoted by Asymmetric-Univariate-RW-UC, means the model is asymmetric and univariate. The trend growth is assumed to be a random walk (stochastic) and the correlation between shocks to the trend and cyclical components is assumed to be zero.

(2) The additional part of Model 7 (SB in the end) expresses a structural break in the variance of the cyclical component to explain the change in variances before and after the great moderation. This model is used to find the break in residual volatility.

(3) We present the results of the bold models in Tables 2.2 and 2.3, and the rest are presented in Table 2.G.2 in Appendix 2.G.

Appendix 2.C: Bivariate state-space model with Markov-switching

We cast the bivariate model explained in Eq. (2.9) to Eq. (2.18) in a state-space form. The observation equation, the transition equation, and variance covariance matrix of error terms are as follows:

$$\begin{bmatrix} x_t \\ u_t \end{bmatrix} = \begin{bmatrix} 1 & \beta & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_t^* \\ u_t^c \\ u_{t-1}^c \\ \mu_t \\ u_t^* \\ \varepsilon_{x^c,t} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2.C.1}$$

$$\begin{bmatrix} x_t^* \\ u_t^c \\ u_{t-1}^c \\ \mu_t \\ u_t^* \\ \varepsilon_{x^c,t} \end{bmatrix} = \begin{bmatrix} 0 \\ \pi_u S_t \\ 0 \\ 0 \\ \eta + \theta \mathbb{1}_t(t \geq T_{u^*}) \\ \pi_x S_t \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \varphi_1 & \varphi_2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi \end{bmatrix} \begin{bmatrix} x_{t-1}^* \\ u_{t-1}^c \\ u_{t-2}^c \\ \mu_{t-1} \\ u_{t-1}^* \\ \varepsilon_{x^c,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{x^*,t} \\ \varepsilon_{u^c,t} \\ 0 \\ \varepsilon_{\mu,t} \\ \varepsilon_{u^*,t} \\ \xi_{x^c,t} \end{bmatrix} \tag{2.C.2}$$

$$\begin{bmatrix} \varepsilon_{x^*,t} \\ \varepsilon_{u^c,t} \\ 0 \\ \varepsilon_{\mu,t} \\ \varepsilon_{u^*,t} \\ \xi_{x^c,t} \end{bmatrix} \sim N\left(\mathbf{0}_{6\times1}, \begin{bmatrix} \sigma_{x^*}^2 & \rho_{x^*,u^c}\sigma_{x^*}\sigma_{u^c} & 0 & 0 & 0 & \rho_{x^*,x^c}\sigma_{x^*}\sigma_{x^c} \\ \rho_{x^*,u^c}\sigma_{x^*}\sigma_{u^c} & \sigma_{u^c}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\mu^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{u^*}^2 & 0 \\ \rho_{x^*,x^c}\sigma_{x^*}\sigma_{x^c} & 0 & 0 & 0 & 0 & \sigma_{x^c}^2 \end{bmatrix}\right) \tag{2.C.3}$$

In the above model, we consider natural log GDP multiplied by 100 and the unemployment rate as the observed series (x_t and u_t). In this model, we use Kim's (1944) approximate maximum likelihood method to estimate the model. To test for asymmetry, we estimate the restricted symmetric model, in which the coefficient of the asymmetric shock (π_x) is assumed to be zero. We estimate this nested model by using Kalman's (1960) filter.

There are more than one state-space representation for the bivariate model. In addition to the above representation, we can consider a measurement error for Okun's law and cast the bivariate model in the state-space form, explained below.

$$\begin{bmatrix} x_t \\ u_t \end{bmatrix} = \begin{bmatrix} 1 & \beta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_t^* \\ u_t^c \\ u_{t-1}^c \\ \mu_t \\ u_t^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{x^c,t} \\ 0 \end{bmatrix} \tag{2.C.4}$$

$$\begin{bmatrix} x_t^* \\ u_t^c \\ u_{t-1}^c \\ \mu_t \\ u_t^* \end{bmatrix} = \begin{bmatrix} 0 \\ \pi_u S_t \\ 0 \\ 0 \\ \eta + \theta \mathbb{1}_t(t \geq T_{u^*}) \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & \varphi_1 & \varphi_2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{t-1}^* \\ u_{t-1}^c \\ u_{t-2}^c \\ \mu_{t-1} \\ u_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{x^*,t} \\ \varepsilon_{u^c,t} \\ 0 \\ \varepsilon_{\mu,t} \\ \varepsilon_{u^*,t} \end{bmatrix} \tag{2.C.5}$$

$$\begin{bmatrix} \varepsilon_{x^c,t} \\ 0 \end{bmatrix} \sim N\left(\mathbf{0}_{2\times1}, \begin{bmatrix} \sigma_{x^c}^2 & 0 \\ 0 & 0 \end{bmatrix}\right) \tag{2.C.6}$$

$$\begin{bmatrix} \varepsilon_{x^*,t} \\ \varepsilon_{x^c,t} \\ 0 \\ \varepsilon_{\mu,t} \\ \varepsilon_{u^*,t} \end{bmatrix} \sim N\left(\mathbf{0}_{5\times1}, \begin{bmatrix} \sigma_{x^*}^2 & \rho_{x^*,u^c}\sigma_{x^*}\sigma_{u^c} & 0 & 0 & 0 \\ \rho_{x^*,u^c}\sigma_{x^*}\sigma_{u^c} & \sigma_{u^c}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\mu}^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{u^*}^2 \end{bmatrix}\right) \tag{2.C.7}$$

In this study, we estimate the first representation. The second one also estimates very similar parameters and concludes the same results.

Table 2.C: Specification of 21 bivariate models

Model name	Tables and Figures	Related model in the literature
Bivariate models		
Model (1.a): Asymmetric-Bivariate-RW-SB-UC	Table 2.1, Figures 2.1, 2.2	--
Model (1.b): Asymmetric-Bivariate-RW-SB-C1 (ρ_{x^*,u^c})	Table 2.1, Figures 2.F.1	--
Model (1.c): Asymmetric-Bivariate-RW-SB-C2 (ρ_{x^*,x^c})	Table 2.G.1	--
Model (1.d): Asymmetric-Bivariate-RW-SB-C3 (ρ_{u^c,x^c})	Table 2.G.1	--
Model (1.e): Asymmetric-Bivariate-RW-SB-C123 (ρ_{x^*,u^c}, ρ_{x^*,x^c}, ρ_{u^c,x^c})	Table 2.G.1	--
Model (2.a): Asymmetric-Bivariate-SB-SB-UC	Table 2.1	--
Model (2.b): Asymmetric-Bivariate-SB-SB-C1 (ρ_{x^*,u^c})	Table 2.G.1	--
Model (2.c): Asymmetric-Bivariate-SB-SB-C2 (ρ_{x^*,x^c})	Table 2.G.1	--
Model (3.a): Asymmetric-Bivariate-Con-SB-UC (5 state variable)	Table 2.1	--
Model (3'.a): Asymmetric-Bivariate-Con-SB-UC (6 state variables)	Table 2.G.1	--
Model (3.b): Asymmetric-Bivariate-Con-SB-C1 (ρ_{x^*,u^c})	Table 2.G.1	--
Model (3.c): Asymmetric-Bivariate-Con-SB-C2 (ρ_{x^*,x^c})	Table 2.G.1	--
Model (4.a): Symmetric-Bivariate-RW-SB-UC	Table 2.1	(Clark, 1989)
Model (4.b): Symmetric-Bivariate-RW-SB-C1 (ρ_{x^*,u^c})	Table 2.G.1	(Gonzalez-Astudillo and Roberts, 2022)
Model (4.c): Symmetric-Bivariate-RW-SB-C2 (ρ_{x^*,x^c})	Table 2.G.1	--
Model (5.a): Symmetric-Bivariate-SB-SB-UC	Table 2.1	--
Model (5.b): Symmetric-Bivariate-SB-SB-C1 (ρ_{x^*,u^c})	Table 2.G.1	
Model (6.a): Symmetric-Bivariate-Con-SB-UC	Table 2.1	--
Model (6.b): Symmetric-Bivariate-Con-SB-C1 (ρ_{x^*,u^c})	Table 2.G.1	
Model 7: Asymmetric-Bivariate-RW-SB-UC-SB	Figures 2.3	--
Model 8: Asymmetric-Bivariate (SB)-RW-SB-UC	Figures 2.3	--

Notes:

(1) We estimate twenty-one different bivariate models on the basis of different choices on the four specification aspects. Accordingly, we denote each model with an identifier and a descriptor. The descriptor consists of five parts, four of which are related to each specification aspect. The first part determines whether the model is asymmetric or symmetric, and the second part shows whether the model is univariate or bivariate. The third part hints at the specification of output trend growth (stochastic, structural break, or constant). The fourth part shows that we take a structural break in drift of the natural rate of unemployment into account. The last part indicates whether the model is uncorrelated or correlated. For example, Model (1.a), which is the benchmark and is denoted by Asymmetric-Bivariate-RW-SB-UC, means the model is asymmetric and bivariate. The trend growth is assumed to be a random walk (stochastic), and the natural rate has a structural break. The correlation between shocks to the trend and cyclical components is also assumed to be zero.

(2) The additional part of Model 7 (SB in the end) expresses a structural break in the volatility of shocks to the remaining cyclical component. This model is used to find a break in variances of the Okun's residual before and after the great moderation.

(3) The second part of Model 8 (the first SB) expresses a structural break in the Okun's law coefficient. This model is used to investigate instability in Okun's law, particularly during the global financial crisis.

(4) We present the results of the bolded models in Table 2.1, and the rest are presented in Table 2.G.1 in Appendix 2.G.

Appendix 2.D: Approximate maximum likelihood and constraints

For asymmetric models in the presence of the Markov-switching process of Hamilton (1989), we use Kim's (1994) approximate maximum likelihood method to make the Kalman’s (1960) filter operable. For more explanation, see chapters 4 and 5 of Kim and Nelson’s (1999b) book and chapters 13 and 22 of Hamilton (1994). For symmetric models, we use the maximum likelihood method, performed by using the Kalman filter as explained in chapters 2 and 3 of Kim and Nelson (1999b) and chapter 5 of Hamilton (1994).

We need to impose a set of constraints on parameters, which are explained thoroughly in the second part of Appendix 2.D. We consider a set of initial values for parameters as well as state variables. For the former, the initial values for parameters are presented in Tables 2.E.1, 2.E.2, and 2.E.3 in Appendix 2.E. For the latter, we use the first observation for trend components, zero for cyclical components, and 3.2% for annual trend growth to determine the prior values for state variables. The prior variances of state variables are set to be 10. The results are robust to changes in prior values of state variables and their variances. For example, we can use a wilder guess by setting the variances of state variables equal to 1000, which bears the same estimation for parameters.

To find the data and replicating code in Matlab, R, and Python for this paper, please see my website at this address: https://sites.google.com/view/mohammaddehghani. For details about the method and parameter constraints, see the comments in the main function as well as the likelihood function and the transformation function in the Matlab code.

Appendix 2.D (continued): Parameters constraints

We employ a numerical optimization procedure to maximize the approximate log likelihood function subject to a set of constraints. We need to impose these constraints on some of the parameters, namely coefficients, probabilities, and standard deviations of shocks. To this end, we account for constraints by using a transformation function, $T(\omega)$, which transforms a vector of unconstrained parameters $\omega = [\omega_1, \dots, \omega_{20}]'$ to a vector of constrained parameters $\Omega = [\Omega_1, \dots, \Omega_{20}]'$ presented below:

$$\Omega = [\sigma_{x^*}, \sigma_{u^c}, \sigma_\mu, \sigma_{u^*}, \sigma_{x^c}, \gamma, \delta, \eta, \theta, \varphi_1, \varphi_2, \pi_u, p, q, \beta, \pi_x, \psi_x, \rho_{x^*,u^c}, \rho_{x^*,x^c}, \rho_{u^c,x^c}]' \tag{2.D.1}$$

where $\Omega = T(\omega)$ is the vector of parameters of interest that we want to estimate and $T(\omega)$ is a vector function, whose elements are continuous transformation functions $T_i(\omega)$ for $i = 1, \dots, 20$. We know that performing unconstrained optimization with respect to ω is equivalent to performing constrained optimization with respect to Ω. We therefore adopt an unconstrained optimization with respect to the vector ω, where the objective (approximate log likelihood) function is considered as a function of the transformation function. We define each element of the transformation function as follows:

First, for coefficients and standard deviations of shocks that should be positive, we use an exponential transformation suggested by Kim and Nelson (1999b). For example,

$$\sigma_{x^*} = exp(\omega_1) \tag{2.D.2}$$

where σ_{x^*} is the standard deviation (square root of variance) of the shock to the trend component of output and is assumed to be positive. Similarly, for other standard deviations, including σ_{u^c}, σ_μ, σ_{u^*}, and σ_{x^c} and for coefficients γ, η, and π_u that are expected to be positive and for other coefficients δ, θ, β, and π_x that are expected to be negative, we apply an exponential transformation. For example, $\pi_u = exp(\omega_{12})$ ensures a positive plucking coefficient for the unemployment rate and $\beta = -exp(\omega_{15})$ ensures a negative Okun's law coefficient.

Second, to have transition probabilities in the $[0\ 1]$ interval, we exert the following transformations:

$$p = \frac{exp(\omega_{13})}{1 + exp(\omega_{13})} \quad \text{and} \quad q = \frac{exp(\omega_{14})}{1 + exp(\omega_{14})} \tag{2.D.3}$$

Third, for the coefficient of the autoregressive process of order one, we use Eq. (2.D.4):

$$\psi_x = \frac{\omega_{17}}{1 + |\omega_{17}|} \tag{2.D.4}$$

Clearly, ψ_x lies in the stationary region since $-1 < \psi_x < 1$. For coefficients of the autoregressive process of order two, we need to set the values of φ_1 and φ_2 within the stationary region that means the roots of the lag polynomial $(1 - \varphi_1 L - \varphi_2 L^2 = 0)$ must lie outside the unit circle. In this sense, we use the following transformations proposed by Morley et al. (2003):

$$\varphi_1 = 2\kappa_1 \quad \text{and} \quad \varphi_2 = -(\kappa_1^2 + \kappa_2) \tag{2.D.5.a}$$

where κ_1 and κ_2 are determined below:

$$\kappa_1 = \frac{\omega_{10}}{1 + |\omega_{10}|} \quad \text{and} \quad \kappa_2 = \frac{(1 - |\kappa_1|) \times \omega_{11}}{1 + |\omega_{11}|} + |\kappa_1| - \kappa_1^2 \tag{2.D.6.a}$$

For these two coefficients of the autoregressive process, we can take two alternative transformations proposed by Kim and Nelson (1999b):

$$\varphi_1 = \kappa_1 + \kappa_2 \quad \text{and} \quad \varphi_2 = \kappa_1 \times \kappa_2 \tag{2.D.5.b}$$

where κ_1 and κ_2 are determined below:

$$\kappa_1 = \frac{\omega_{10}}{1 + |\omega_{10}|} \quad \text{and} \quad \kappa_2 = \frac{\omega_{11}}{1 + |\omega_{11}|} \tag{2.D.6.b}$$

However, these transformations impose a further restriction that the roots of the autoregressive polynomial are real numbers.

Fourth, for correlation coefficients, we consider Eq. (2.D.7):

$$\rho_{x^*,u^c} = \frac{\omega_{18}}{1 + |\omega_{18}|} \tag{2.D.7}$$

where ρ_{x^*,u^c} is the correlation between shocks and clearly satisfies the condition $-1 < \rho_{x^*,u^c} < 1$.

Finally, for correlated models, we can use alternative constraints for standard deviations of shocks to components and their correlation. In this setting, we use a Cholesky factorization similar to Hamilton (1994**)** and Morley et al. (2003), which is presented as follows:

$$\sigma_{x^*} = \sqrt{[P'P]_{1,1}} \text{ and } \sigma_{u^c} = \sqrt{[P'P]_{2,2}} \text{ and } \rho_{x^*,u^c} = \frac{[P'P]_{1,2}}{\sqrt{[P'P]_{1,1}} \times \sqrt{[P'P]_{2,2}}} \tag{2.D.8}$$

In Eq. (2.D.8), $[P'P]_{i,j}$ is the element on the row i and column j of the symmetric positive definite matrix denoted by $[P'P]_{2\times2}$, which is known as the Cholesky factorization. To derive the elements of $[P'P]_{2\times2}$, we first need to construct the Cholesky factor ($P_{2\times2}$), a lower triangular matrix with positive diagonal elements $P_{1,1} = exp(\omega_1)$ and $P_{2,2} = exp(\omega_2)$ and an off-diagonal element of $P_{1,2} = \omega_{18}$.

It is worth noting that the results of this study are robust to the choice of the transformation functions when there are two alternatives. In addition, the plucking property (asymmetry) and Okun's law (co-fluctuations) are two pronounced features of U.S. macroeconomics so that excluding each or both of their corresponding constraints ($\pi_u > 0$ and $\beta < 0$) does not change the estimated parameters.

Appendix 2.E: Tables of initial values for parameters

Table 2.E.1: Initial values (after-transformation) for the parameters of the bivariate model

	(1.a)	(1.b)	(2.a)	(2.b)	(3.a)	(4.a)	(5.a)	(6.a)
Parameters	A-Bi-RW-SB-UC	A-Bi-RW-SB-C1	A-Bi-SB-SB-UC	A-Bi-SB-SB-C1	A-Bi-Con-SB-UC	S-Bi-RW-SB-UC	S-Bi-SB-SB-UC	S-Bi-Con-SB-UC
σ_{x^*}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
σ_{u^c}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
σ_{μ}	0.50	0.50	–	–	–	0.50	–	–
σ_{u^*}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
σ_{x^c}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
γ	T-V	T-V	0.80	0.80	0.80	T-V	0.80	0.80
δ	T-V	T-V	-0.30	-0.30	–	T-V	-0.30	–
η	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
θ	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03
φ_1	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
φ_2	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4
π_u	1.8	1.8	1.8	1.8	1.8	–	–	–
p	0.70	0.70	0.70	0.70	0.70	–	–	–
q	0.90	0.90	0.90	0.90	0.90	–	–	–
β	-1.50	-1.50	-1.50	-1.50	-1.50	-1.50	-1.50	-1.50
π_x	-1.8	-1.8	-1.8	-1.8	-1.8	–	–	–
ψ_x	0.50	0.50	0.50	0.50	0.50	0.50	0.70*	0.75*
ρ_{x^*,u^c}	–	0.50	–	0.50	–	–	–	–
ρ_{x^*,x^c}	–	–	–	–	–	–	–	–
ρ_{u^c,x^c}	–	–	–	–	–	–	–	–

Notes:

(1) The results of the benchmark model as well as other models are unbelievably robust to the choice of the initial values for each parameter. Therefore, we use the same initial values for almost all models. Alternatively, to provide a hierarchical method to find initial values, we propose to estimate the simplest model (symmetric univariate model) first and keep the estimated parameters to use as a best guess for the initial values for less restricted models.

(2) We set a few initial values for a few models different from those in other models, to avoid deriving imaginary standard errors in one or two parameters. These initial values are denoted by asterisks.

Table 2.E.2: Initial values (after-transformation) for the parameters of the univariate model for output

Models	(1.a)	(1.b)	(2.a)	(2.b)	(3.a)	(4.a)	(5.a)	(6.a)
Parameters	A-Uni-RW-UC	A-Uni-RW-C	A-Uni-SB-UC	A-Uni-SB-C	A-Uni-Con-UC	S-Uni-RW-UC	S-Uni-SB-UC	S-Uni-Con-UC
σ_{z^*}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
σ_{z^c}	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
σ_μ	0.50	0.50	–	–	–	0.50	–	–
γ	T-V	T-V	0.75	0.75	0.75	T-V	0.75	0.75
δ	T-V	T-V	-0.30	-0.30	–	T-V	-0.30	–
φ_1	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
φ_2	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4
π_z	-1.80	-1.80	-1.80	-1.80	-1.80	–	–	–
p	0.70	0.70	0.70	0.70	0.70	–	–	–
q	0.90	0.90	0.90	0.90	0.90	–	–	–
ρ_{z^*,z^c}	–	0.50	–	0.50	–	–	–	–

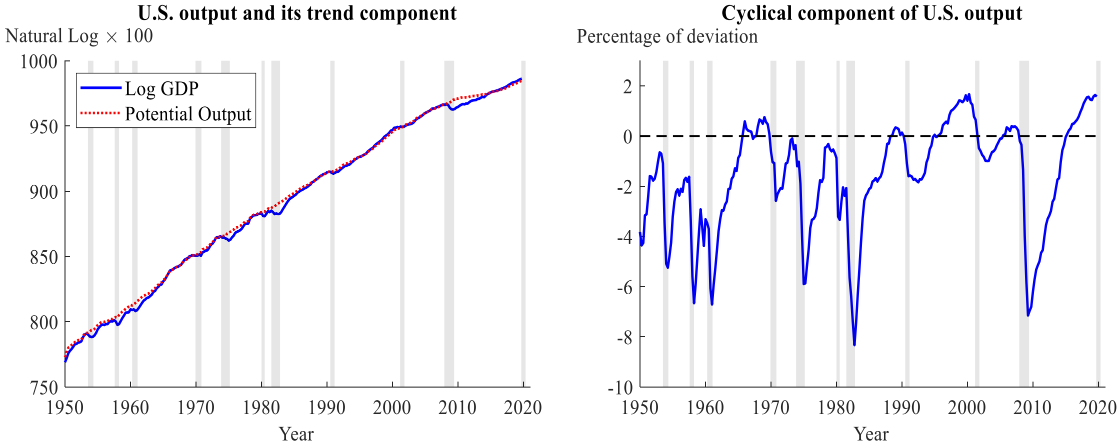
Notes:
(1) The results of all models are robust to the choice of the initial values for each parameter.
(2) We use the same initial values for all models that are almost the same as the initial values for the bivariate model.

Table 2.E.3: Initial values (after-transformation) for the parameters of the univariate model for unemployment

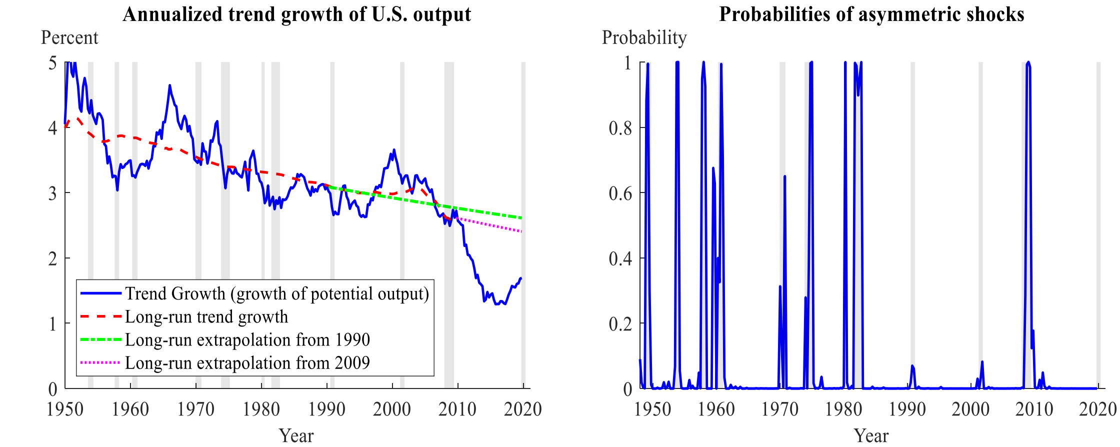
Models	(1.a)	(1.b)	(2.a)	(2.b)
Parameters	A-Uni-SB-UC	A-Uni-SB-C	S-Uni-SB-UC	S-Uni-SB-C
σ_{z^*}	0.50	0.50	0.50	0.50
σ_{z^c}	0.50	0.50	0.50	0.50
η	0.03	0.03	0.03	0.03
θ	-0.03	-0.03	-0.03	-0.03
φ_1	1.2	1.2	1.2	1.2
φ_2	-0.4	-0.4	-0.4	-0.4
π_z	1.8	1.8	–	–
p	0.70	0.70	–	–
q	0.90	0.90	–	–
ρ_{z^*,z^c}	–	0.65*	–	0.20*

Notes:
(1) The results of all models are robust to the choice of the initial values for each parameter.
(2) We use the same initial values for all models that are almost the same as the initial values for the bivariate model. For correlations in model (1.b) and (2.b), we use different initial values to avoid deriving imaginary standard errors in one or two parameters. These initial values are denoted by asterisks.

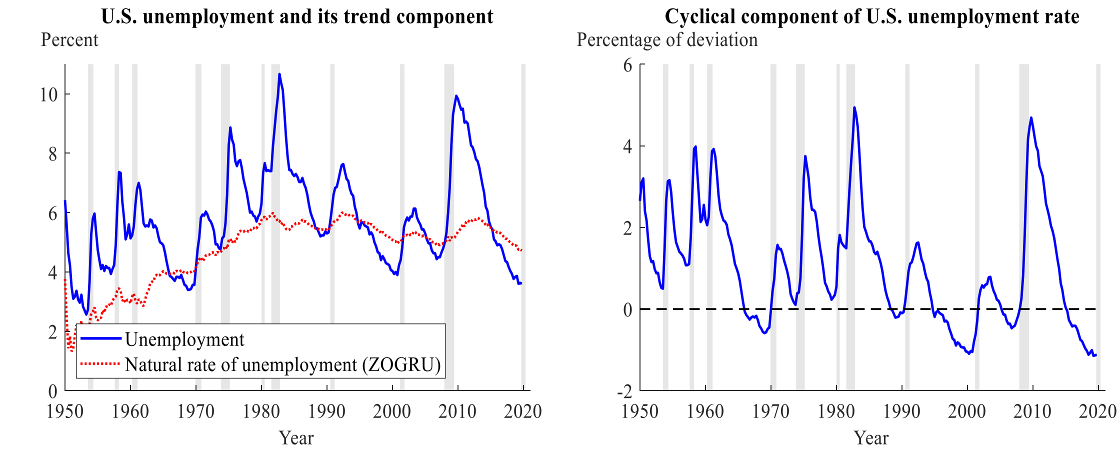
Appendix 2.F: Additional Figures



(a) Potential and gap for GDP per capita



(b) Trend growth of output per capita and the plucking probabilities for bivariate model



(c) Natural rate of unemployment (trend) and unemployment gap (cyclical component)

Figure 2.F.1: Results of the correlated version of the asymmetric bivariate model

Notes:

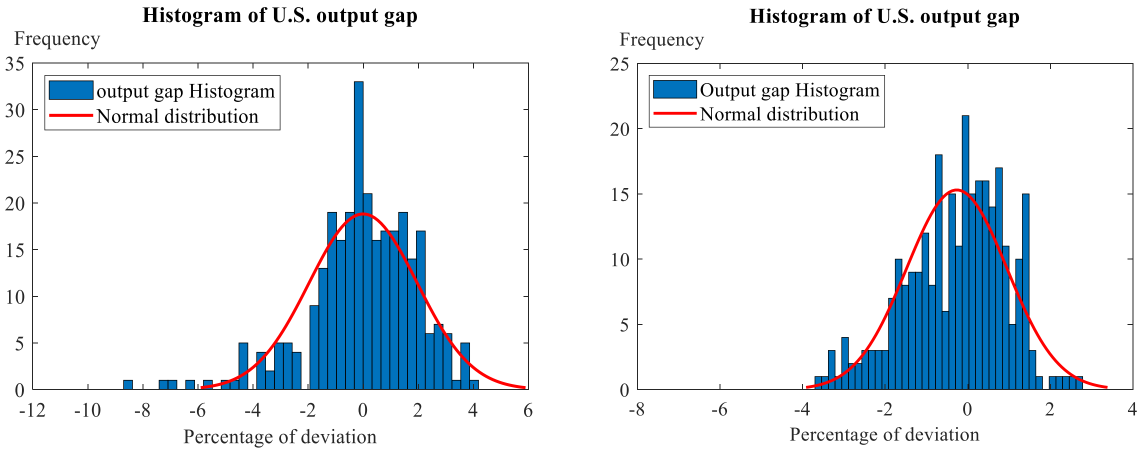
(1) All panels plot the results of the correlated version of the benchmark model (Asymmetric-Bivariate-RW-SB-C1).

(2) The top panels plot potential output and the output gap, and the middle-left panel plots the trend growth of output.

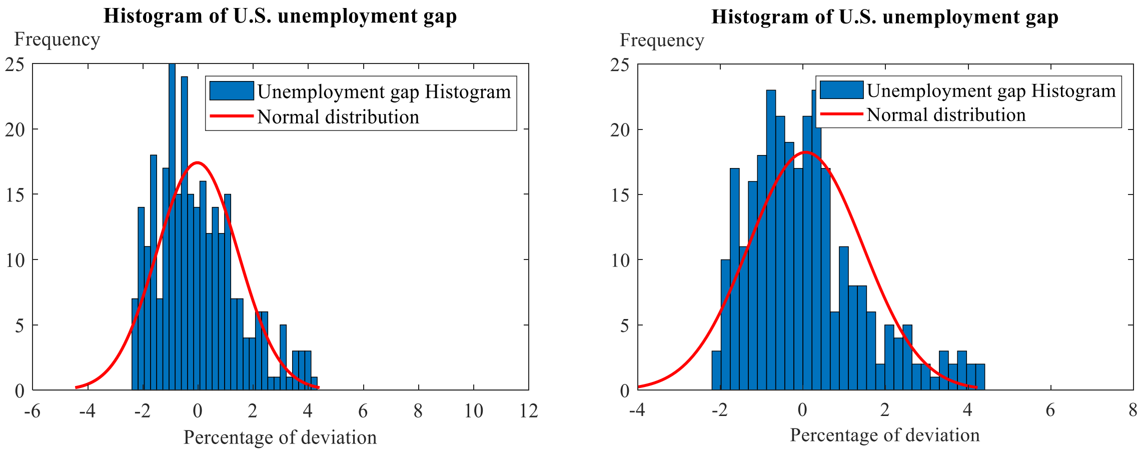
(3) The middle-right panel plots the plucking probabilities, which are estimated for both output and unemployment jointly.

(4) The bottom panels plot the trend and gap for unemployment.

(5) The shaded areas are the NBER recession dates.



(b) The histogram of U.S. output gap estimated by Tukey's bi-weight filter (left) and UC model (right)



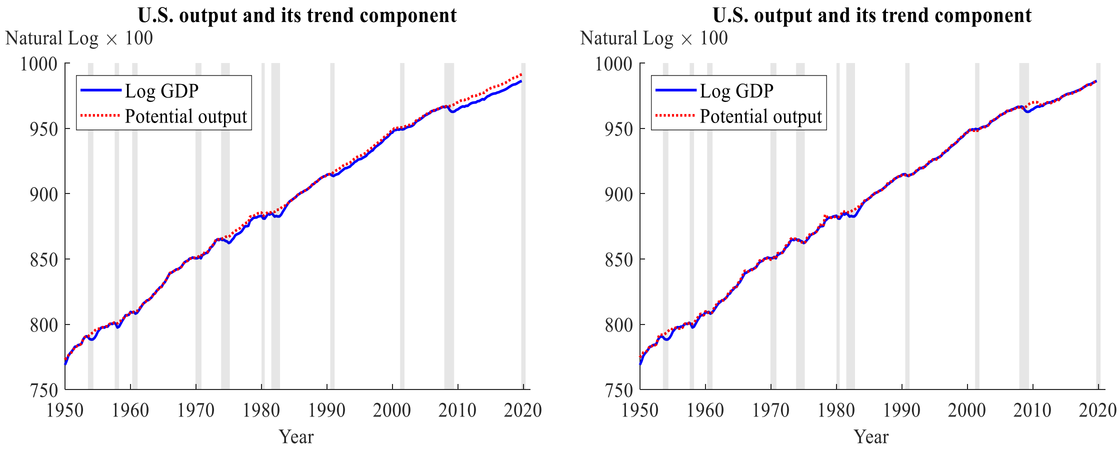
(c) The histogram of U.S. unemployment gap by Tukey's bi-weight filter (left) and UC model (right)

Figure 2.F.2: Histogram of output gap and unemployment gap.

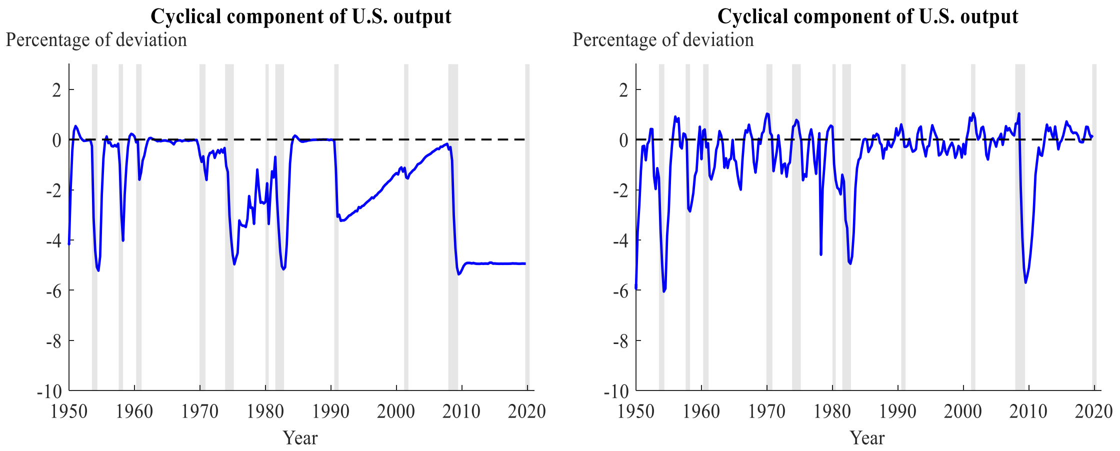
Notes:

(1) The output gap skewness is -0.93 for the left and -0.41 for the right panel.

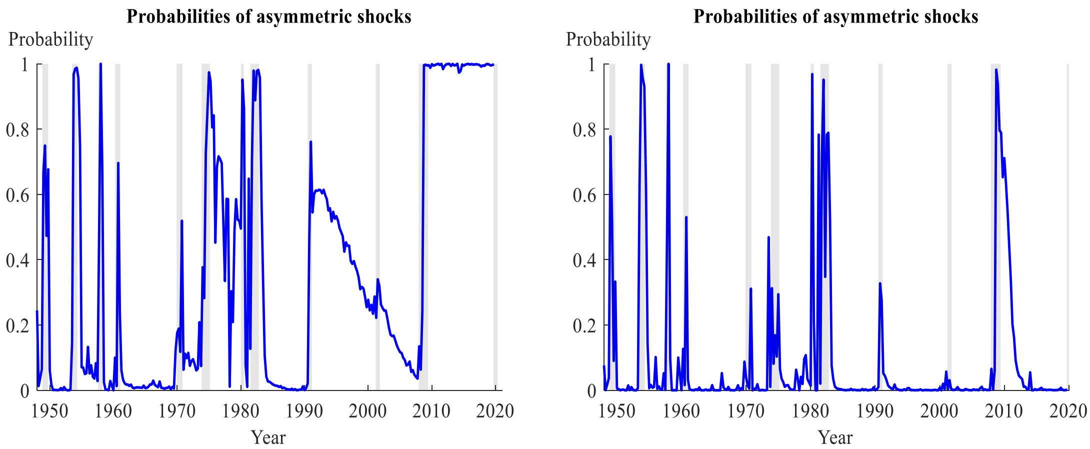
(2) The unemployment gap skewness is +0.75 for the left and +0.94 for the right panel.



(a) The trend component



(b) Output gap (cyclical component)



(c) Plucking probabilities

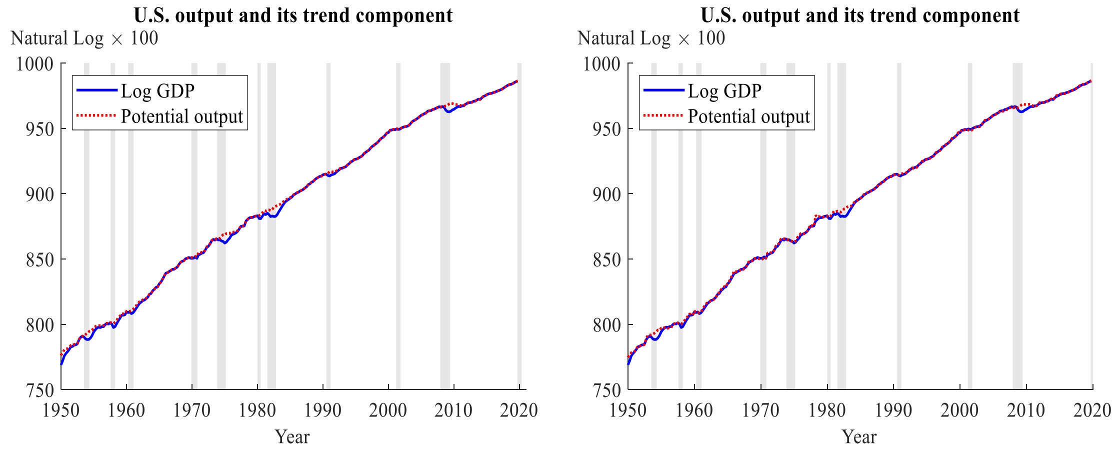
Figure 2.F.3: Results of the asymmetric univariate models with constant trend growth

Notes:

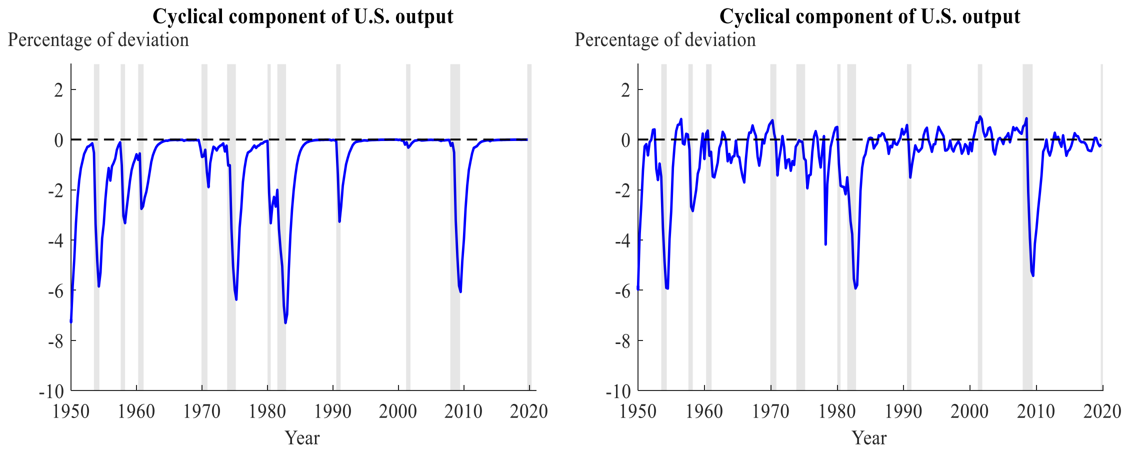
(1) The left panels plot the results of the asymmetric model with uncorrelated shocks.

(2) The right panels plot the results of the asymmetric model with correlated shocks, which replicates the work of Sinclair (2010)

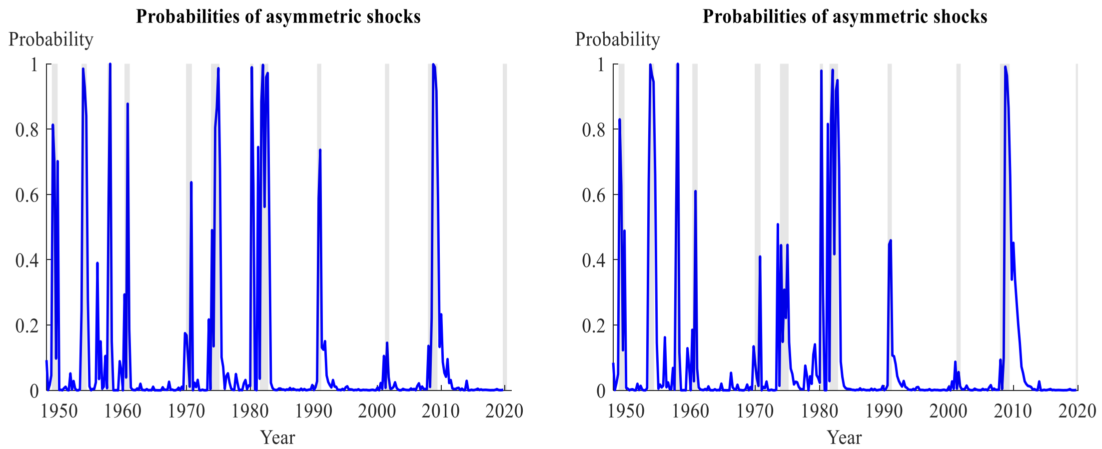
(3) The shaded areas are the NBER recession dates.



(a) The trend component



(b) Output gap (cyclical component)



(c) Plucking probabilities

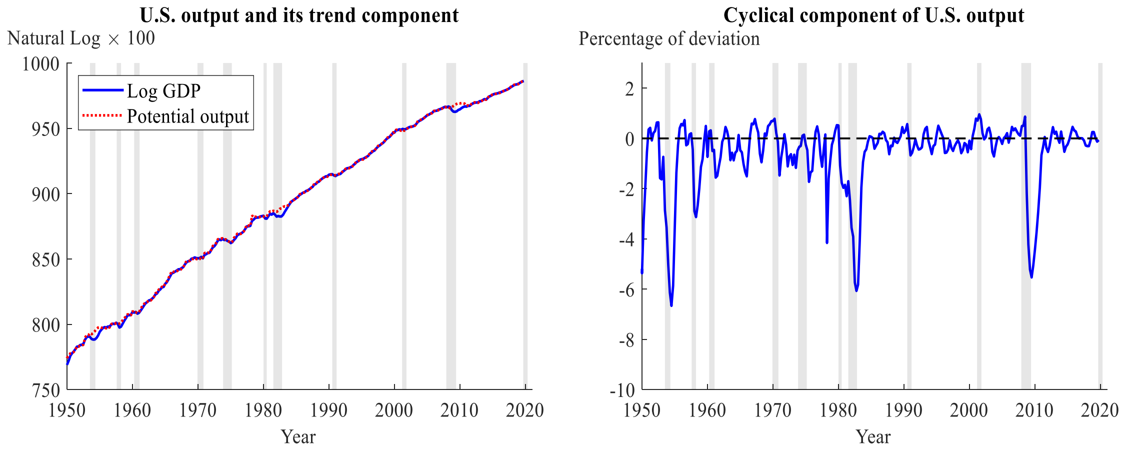
Figure 2.F.4: Results of the asymmetric univariate models with a break in trend growth

Notes:

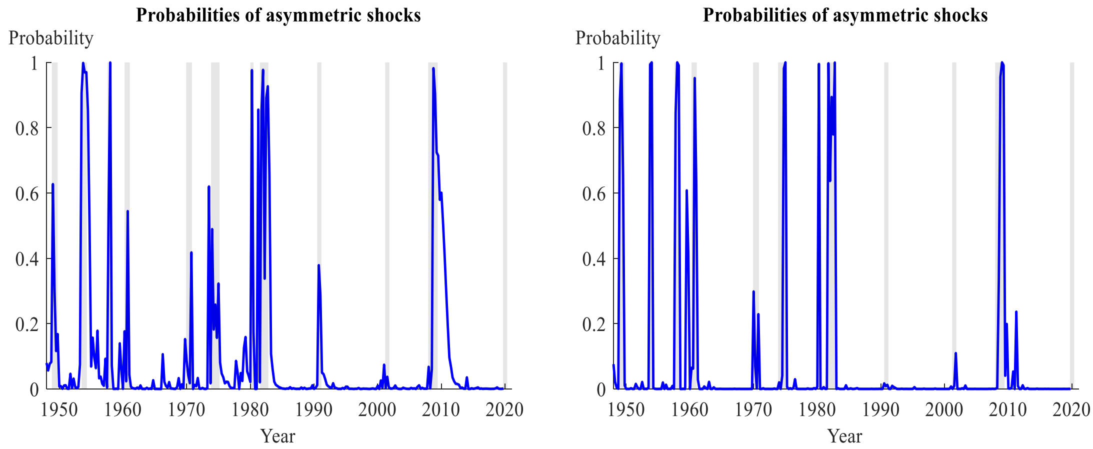
(1) The left panels plot the results of the asymmetric model with uncorrelated shocks.

(2) The right panels plot the results of the asymmetric model with correlated shocks.

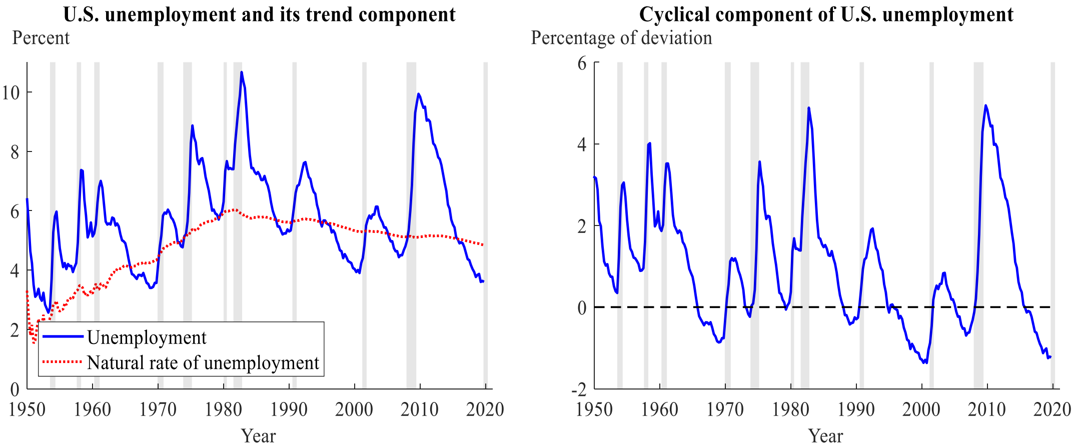
(3) The shaded areas are the NBER recession dates.



(a) Potential output (trend) and output gap (cyclical component)



(b) Plucking probabilities for output (left) and unemployment (right)



(c) Natural rate of unemployment (trend) and the unemployment gap (cyclical components)

Figure 2.F.5: Comparing results of the asymmetric univariate models for output and unemployment (correlated model)

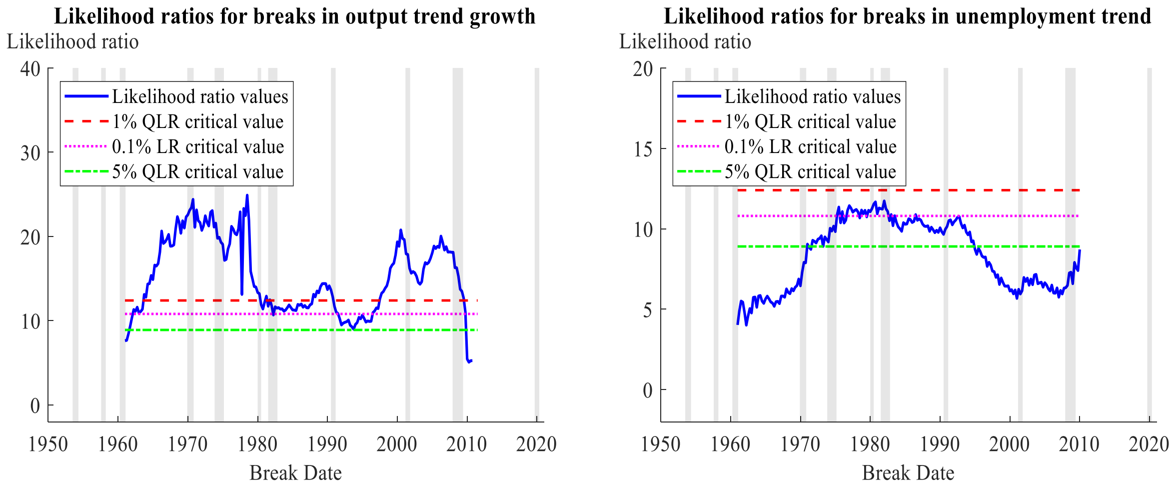
Notes:

(1) The top panels plot the results of the asymmetric univariate model for output with a stochastic (random walk) trend growth, where shocks to the trend and cyclical components are correlated (Asymmetric-Univariate-RW-C).

(2) The middle panels plot the plucking probabilities for output and unemployment estimated in two separate models.

(3) The bottom panels plot the results of the asymmetric univariate model for unemployment with a break in unemployment, where shocks to the trend and cyclical components are correlated (Asymmetric-Univariate-SB-C).

(4) The shaded areas are the NBER recession dates.



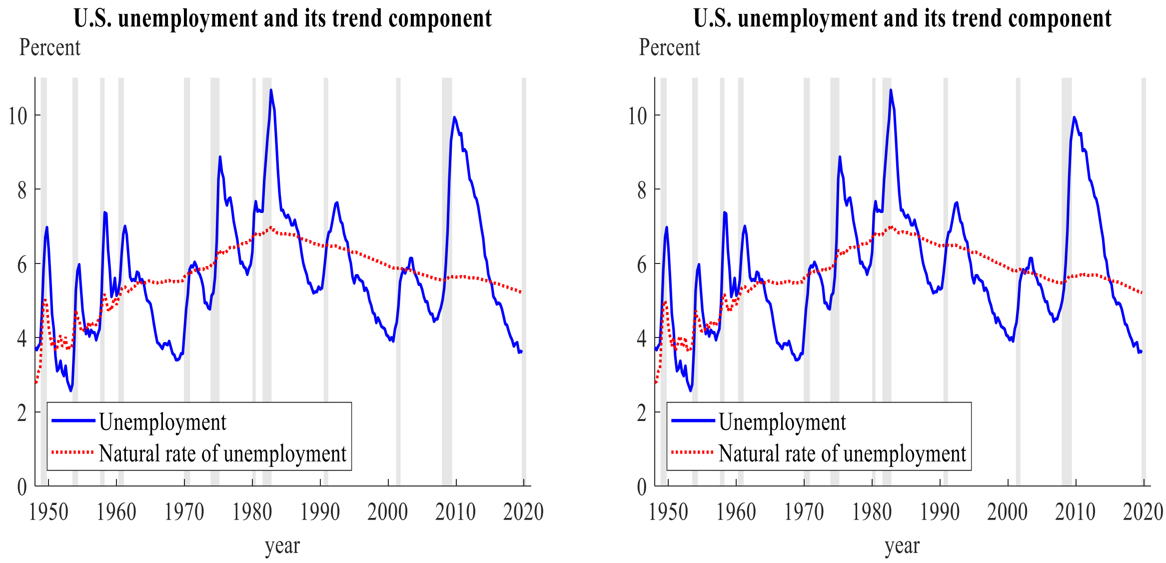
Likelihood ratios at different break dates for trend growth (left) and the unemployment trend (right)

Figure 2.F.6: Exploring structural breaks in the parameters for asymmetric univariate models

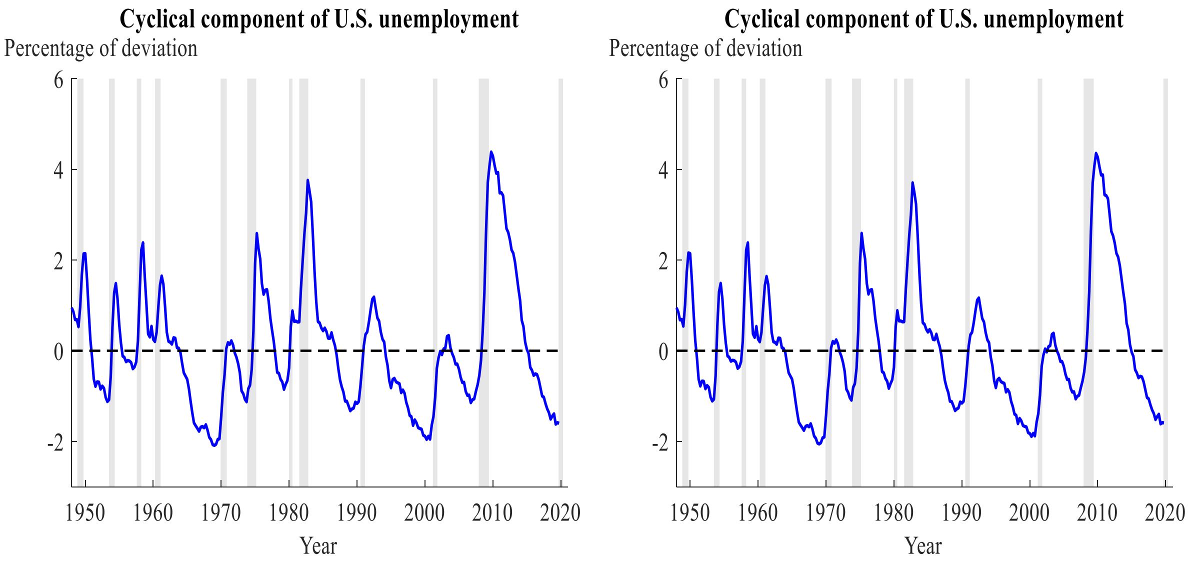
Notes:

(1) Both panels plot likelihood ratio values for a sequence of breaks rolling from 1960 to 2010. In the left panel, likelihood ratio compares the log likelihood value of univariate model (2.a) with a break in trend growth with that of its counterpart univariate model (3.a) with constant trend growth. The right panel plots the likelihood ratios testing for breaks in unemployment trend on different dates against a constant trend for univariate model.

(2) The shaded areas are the NBER recession dates.



(a) The trend component of uncorrelated (left) and correlated (right) models



(b) The cyclical component of uncorrelated (left) and correlated (right) models

Figure 2.F.7: Results of the symmetric univariate model for U.S. unemployment

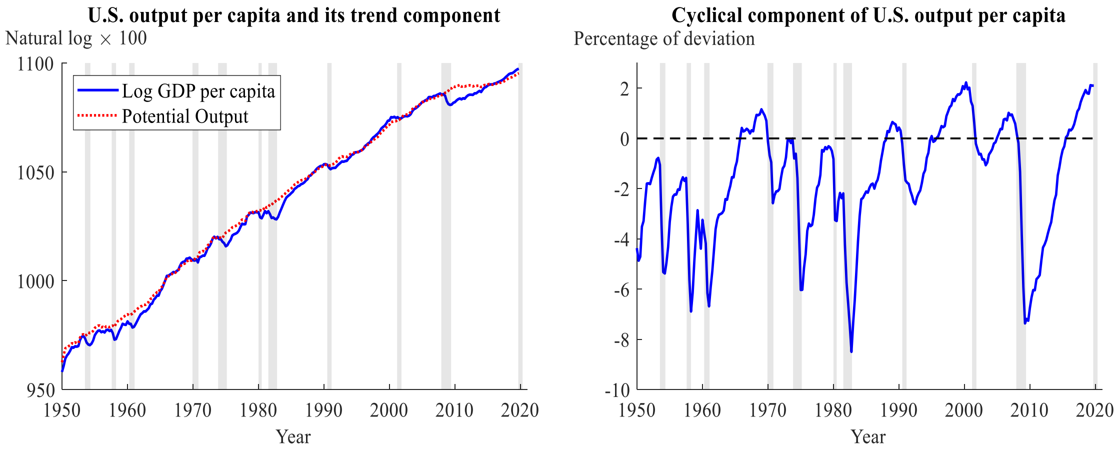
Notes:

(1) The left panels plot the results of the model with a structural break in the drift in unemployment rate trend (natural rate) and uncorrelated shocks to the trend and cyclical components (Symmetric-Univariate-SB-UC). This setting will be applied to the benchmark asymmetric bivariate model.

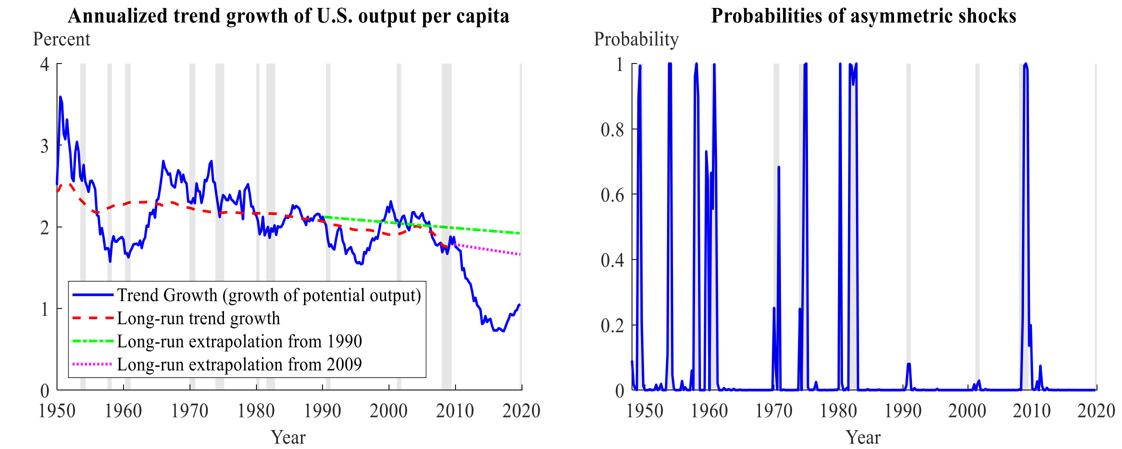
(2) The right panels plot the results of the model with a structural break in drift in unemployment rate trend (natural rate) and correlated shocks to the trend and cyclical components (Symmetric-Univariate-SB-C).

(3) The resemblance of the components of uncorrelated and correlated models indicates an insignificant (-0.60 with a standard error of 0.80) and irrelevant correlation for the unemployment rate.

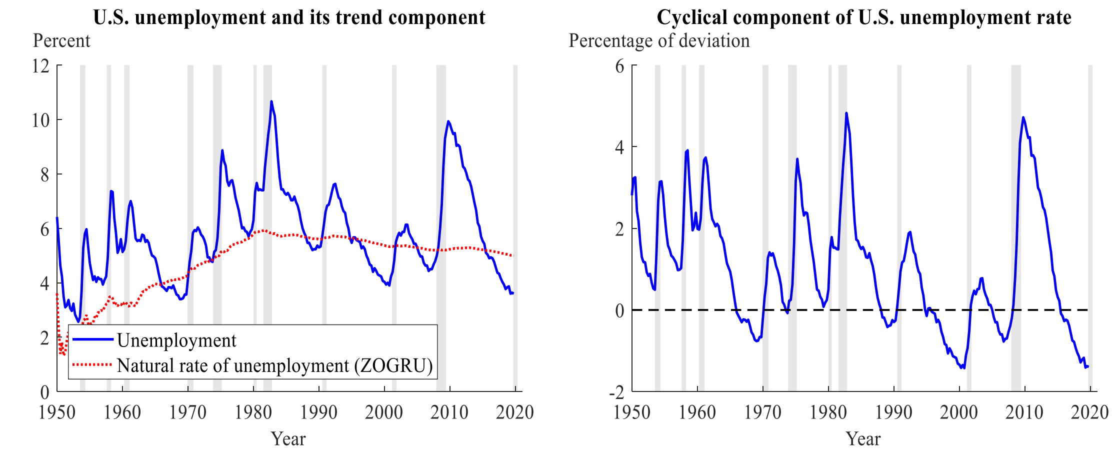
(4) The shaded areas are the NBER recession dates.



(a) Potential and gap for GDP per capita



(b) Trend growth of output per capita and the plucking probabilities for bivariate model



(c) Natural rate of unemployment (trend) and unemployment gap (cyclical component)

Figure 2.F.8: Results of the asymmetric bivariate model for GDP per capita

Notes:

(1) All panels plot the results of the benchmark model (Asymmetric-Bivariate-RW-SB-UC), applied to the U.S. GDP per capita.

(2) The top panels plot the potential and gap for GDP per capita, and the middle-left panel plots the trend growth of output per capita.

(3) The middle-right panel plots the plucking probabilities, which are estimated for both output and unemployment jointly.

(4) The bottom panels plot the trend and gap for unemployment.

(5) The shaded areas are the NBER recession dates.

Appendix 2.G: Additional Tables

Table 2.G.1 (Continue of Table 2.1): Estimated parameters of the bivariate models

Models	(1.c)	(1.d)	(1.e)	(2.c)	(3’.a)	(3.b)	(3.c)
Parameters	A-Bi-RW-SB-C2	A-Bi-RW-SB-C3	A-Bi-RW-SB-C123	A-Bi-SB-SB-C2	A-Bi-Con-SB-UC	A-Bi-Con-SB-C1	A-Bi-Con-SB-C2
σ_{x^*}	0.44 (0.09)	0.43 (0.10)	0.44 (0.10)	0.52 (0.06)	0.62 (0.02)	0.63 (0.03)	0.70 (0.09)
σ_{u^c}	0.21 (0.01)	0.21 (0.01)	0.21 (0.01)	0.21 (0.01)	0.21 (0.01)	0.21 (0.01)	0.21 (0.01)
σ_{μ}	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	–	–	–	–
σ_{u^*}	0.00 (0.02)	0.00 (0.03)	0.00 (0.03)	0.00 (0.02)	0.00 (0.01)	0.00 (0.03)	0.00 (0.02)
σ_{x^c}	0.18 (2.50)	0.35 (0.09)	0.20 (1.25)	0.28 (1.57)	0.00 (0.11)	0.00 (0.14)	0.13 (0.51)
γ	T-V	T-V	T-V	0.83 (0.03)	–	0.76 (0.03)	0.76 (0.04)
δ	T-V	T-V	T-V	-0.50 (0.08)	–	–	–
η	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.04 (0.01)
θ	-0.04 (0.01)	-0.04 (0.01)	-0.04 (0.01)	-0.04 (0.01)	-0.04 (0.01)	-0.04 (0.01)	-0.04 (0.01)
φ_1	1.38 (0.04)	1.38 (0.04)	1.37 (0.04)	1.36 (0.04)	1.37 (0.04)	1.35 (0.04)	1.37 (0.04)
φ_2	-0.44 (0.04)	-0.44 (0.04)	-0.44 (0.04)	-0.43 (0.04)	-0.44 (0.04)	-0.42 (0.04)	-0.44 (0.04)
π_u	0.70 (0.06)	0.70 (0.06)	0.70 (0.06)	0.70 (0.06)	0.70 (0.06)	0.71 (0.06)	0.70 (0.05)
p	0.66 (0.09)	0.66 (0.09)	0.66 (0.09)	0.67 (0.09)	0.67 (0.09)	0.66 (0.09)	0.66 (0.09)
q	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)
β	-1.44 (0.11)	-1.42 (0.14)	-1.30 (0.17)	-1.44 (0.13)	-1.34 (0.14)	-1.04 (0.29)	-1.29 (0.15)
π_x	-1.01 (0.15)	-1.01 (0.16)	-1.07 (0.17)	-1.05 (0.16)	-1.04 (0.17)	-1.20 (0.25)	-1.05 (0.17)
ψ_x	0.49 (0.10)	0.51 (0.12)	0.52 (0.12)	0.51 (0.11)	0.55 (0.10)	0.66 (0.12)	0.61 (0.11)
ρ_{x^*,u^c}	–	–	-0.24 (0.18)	–	–	-0.16 (0.12)	–
ρ_{x^*,x^c}	0.66 (16.6)	–	-0.65 (7.97)	-0.08 (3.59)	–	–	-0.69 (1.69)
ρ_{u^c,x^c}	–	-0.04 (0.12)	-0.25 (1.67)	–	–	–	–
Log likelihood	-11.9	-11.9	-10.9	-7.4	-20.5	-18.8	-19.2

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are reported in parenthesis.
*** Numerical values for parameters denoted by 0.00 are 0.0001 for model (1.c), 0.0002 for model (1.d) 0.0005 for model (1.e), 0.001 for model (2.c), .000001 and 0.001 for model (3’.a), 0.0002 and 0.001 for model (3.b), and 0.0006 for model (3.c).

Notes:

(1) The estimation period runs from 1948Q1 to 2019Q4.

(2) See Table 2.5 for the main results and explanations.

(3) For all models, the structural break in the drift of the unemployment rate trend in 1981Q1 is accounted for. For models (2.b) and (5.c), the structural break in trend growth in 2009Q3 is accounted for.

(4) Models (1.c), (2.c), and (3.c) are counterparts of uncorrelated asymmetric models (1.a), (2.a), and (3.a), respectively, which are presented in Table 2.5. By comparing the log likelihood values of -11.9, -7.4, and -19.9 for the uncorrelated models with values of -11.9, -7.4, and -19.2 for the correlated models, we accept the null hypothesis of zero-correlation because likelihood ratios of 0.0, 0.0, and 1.4 are negligible or zero.

(5) Model (3’.a) is another version of model (3.a) with almost identical estimation of parameters. Since in the former, we treat the drift term (constant trend growth) as a state variable and in the latter, the drift term is estimated as a parameter; model (3’.a) is fully nested in model (1.a), but model (3.a) is not.

Table 2.G.1 continued: Estimated parameters of the bivariate models

Models	(4.b)	(4.c)	(5.b)	(6.b)	(7)	(8)
Parameters	S-Bi-RW-SB-C1	S-Bi-RW-SB-C2	S-Bi-SB-SB-C1	S-Bi-Con-SB-C1	A-Bi-RW-SB-UC-SB	A-Bi (SB)-RW-SB-UC
σ_{x^*}	0.55 (0.07)	0.57 (0.13)	0.56 (0.06)	0.64 (0.04)	0.42 (0.20)	0.44 (0.09)
σ_{u^c}	0.23 (0.02)	0.27 (0.01)	0.23 (0.02)	0.22 (0.01)	0.21 (0.01)	0.21 (0.01)
σ_{μ}	0.02 (0.01)	0.02 (0.01)	–	–	0.03 (0.02)	0.03 (0.01)
σ_{u^*}	0.14 (0.02)	0.03 (0.04)	0.14 (0.02)	0.14 (0.02)	0.00 (0.03)	0.00 (0.02)
σ_{x^c}	0.21 (0.14)	0.27 (0.01)	0.18 (0.13)	0.06 (0.18)	0.550 (0.13) 0.039 (1.50)	0.34 (0.08)
γ	T-V	T-V	0.83 (0.04)	0.78 (0.04)	T-V	T-V
δ	T-V	T-V	-0.43 (0.10)	–	T-V	T-V
η	0.02 (0.02)	0.02 (0.01)	0.02 (0.02)	0.02 (0.02)	0.03 (0.01)	0.03 (0.01)
θ	-0.03 (0.02)	-0.03 (0.01)	-0.03 (0.02)	-0.05 (0.02)	-0.03 (0.01)	-0.04 (0.01)
φ_1	1.71 (0.04)	1.61 (0.04)	1.71 (0.04)	1.70 (0.04)	1.40 (0.06)	1.38 (0.04)
φ_2	-0.74 (0.04)	-0.67 (0.05)	-0.74 (0.04)	-0.73 (0.04)	-0.47 (0.05)	-0.44 (0.04)
π_u	–	–	–	–	0.70 (0.06)	0.70 (0.06)
p	–	–	–	–	0.64 (0.11)	0.66 (0.09)
q	–	–	–	–	0.96 (0.01)	0.96 (0.01)
β	-1.38 (0.15)	-1.74 (0.10)	-1.42 (0.16)	-1.27 (0.17)	-1.33 (0.13)	-1.45 (0.12) 0.03 (0.15)
π_x	–	–	–	–	-1.07 (0.20)	-1.00 (0.16)
ψ_x	0.12 (0.67)	0.55 (0.48)	0.06 (0.80)	0.51 (0.74)	0.58 (0.13)	0.49 (0.11)
ρ_{x^*,u^c}	-0.65 (0.14)	–	-0.63 (0.15)	-0.65 (0.13)	–	–
ρ_{x^*,x^c}	–	-0.07 (0.52)	–	–	–	–
ρ_{u^c,x^c}	–	–	–	–	–	–
Log likelihood	-49.1	-57.5	-46.2	-54.4	8.7	-11.9

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are reported in parenthesis.
*** Numerical values for parameters denoted by 0.00 are 0.0006 for model (7), and 0.0002 for model (8).

Notes continued:

(6) Models (4.b), (5.b), and (6.b) are the counterparts of uncorrelated symmetric models (4.a), (5.a), and (6.a), respectively, which are presented in Table 2.1. By comparing the log likelihood values of -57.5, -52.6, and -62.5 for the uncorrelated models with values of -49.1, -46.2, and -54.4 for the correlated models, we reject the null hypothesis of zero-correlation between shocks to the trend and cyclical components because likelihood ratios of 17.0, 12.8, and 16.2 are greater than the 0.1% critical value of 6.63. In contrast, the correlation between shocks to the trend and remaining cyclical components is zero since the corresponding likelihood ratio is 0.0.

(7) Models (1.c), (1.d), and (1.e) are the counterparts of uncorrelated asymmetric model (1.a), which is presented in Table 2.1. By comparing the log likelihood values of -11.9 for the uncorrelated model with values of -11.9, -11.9 and -10.9 for the correlated models, we accept the null hypothesis of zero-correlation because likelihood ratios of 0.0, 0.0, and 2.0 are zero or negligible and less than the 1% critical value of 6.63. The last one, tests for all three correlations jointly.

(8) In model (7), the estimated volatility of shocks to the cyclical component before and after 1983 is different. In model (8), a structural break in the Okun's law coefficient is allowed for.

Table 2.G.2 (Continue of Table 2.2): Estimated parameters of the univariate models for output

Models	(3’.a)	(3.b)	(4.b)	(5.b)	(6.b)	(7)
Parameters	A-Uni-Con-UC	A-Uni-Con-C	S-Uni-RW-C	S-Uni-SB-C	S-Uni-Con-C	A-Uni-Con-C-SB
σ_{z^*}	0.70 (0.03)	1.13 (0.12)	0.93 (0.19)	0.99 (0.19)	1.18 (0.15)	0.40 (0.03)
σ_{z^c}	0.001 (0.08)	0.70 (0.16)	0.80 (0.20)	0.86 (0.22)	0.79 (0.25)	0.950 (0.08) 0.003 (0.29)
σ_{μ}	–	–	0.02 (0.01)	–	–	0.05 (0.02)
γ	–	0.78 (0.06)	T-V	0.83 (0.06)	0.78 (0.07)	–
δ	–	–	T-V	-0.31 (0.17)	–	–
φ_1	1.05 (0.09)	1.09 (0.07)	1.44 (0.10)	1.44 (0.10)	1.24 (0.20)	1.36 (0.10)
φ_2	-0.40 (0.09)	-0.36 (0.07)	-0.57 (0.07)	-0.56 (0.09)	-0.63 (0.17)	-0.45 (0.08)
π_z	-1.76 (0.22)	-1.88 (0.23)	–	–	–	-1.21 (0.24)
p	0.91 (0.03)	0.61 (0.09)	–	–	–	0.40 (0.26)
q	0.97 (0.01)	0.96 (0.01)	–	–	–	0.97 (0.02)
ρ_{z^*,z^c}	–	-0.88 (0.04)	-0.71 (0.18)	-0.76 (0.14)	-0.92 (0.08)	–
Log likelihood	-340.9	-331.5	-353.2	-350.5	-349.4	-305.4

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are reported in parenthesis.

Notes:

(1) The estimation period runs from 1948Q1 to 2019Q4.

(2) See Table 2.3 for the main results and explanations.

(3) For model (5.b), the structural break in trend growth in 2009Q3 is accounted for.

(4) Model (3’.a) is another version of model (3.a). It is clear that their estimations for parameters are very similar. The former treats the drift term (constant trend growth) as a state variable, and the latter estimates the drift as a parameter. Hence, model (3’.a) is fully nested in model (1.a), but model (3.a) is not.

(5) Model (4.b) is the counterpart of uncorrelated model (4.a), which is presented in Table 2.3. By comparing the log likelihood of the uncorrelated model (-354.4) with that of the correlated model (-353.2), we accept the null hypothesis of zero-correlation because the likelihood ratio of 2.4 is less than the 1% critical value of 6.63.

(6) Model (5.b) is the counterpart of uncorrelated model (5.a), which is presented in Table 2.3. By comparing the log likelihood of the uncorrelated model (-352.6) with that of the correlated model (-350.5), we accept the null hypothesis of zero-correlation because the likelihood ratio of 4.2 is less than the 1% critical value of 6.63.

(7) In model (7), the estimated volatility of shocks to the cyclical component before and after 1983 is different.

Table 2.G.3: Estimated parameters of the asymmetric bivariate model, with controlled lead-lag effect

Models	(1.a)	1-month	2-month	3-month
Parameters	A-Bi-RW-SB-UC	A-Bi-RW-SB-UC	A-Bi-RW-SB-UC	A-Bi-RW-SB-UC
σ_{x^*}	0.44 (0.08)	0.34 (0.11)	0.36 (0.09)	0.33 (0.14)
σ_{u^c}	0.21 (0.01)	0.17 (0.01)	0.16 (0.01)	0.15 (0.01)
σ_{μ}	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)	0.03 (0.01)
σ_{u^*}	0.00 (0.02)	0.11 (0.01)	0.12 (0.01)	0.13 (0.01)
σ_{x^c}	0.33 (0.08)	0.38 (0.08)	0.35 (0.07)	0.44 (0.09)
γ	T-V	T-V	T-V	T-V
δ	T-V	T-V	T-V	T-V
η	0.03 (0.01)	0.02 (0.01)	0.03 (0.01)	0.03 (0.01)
θ	-0.03 (0.01)	-0.03 (0.01)	-0.03 (0.01)	-0.03 (0.01)
φ_1	1.37 (0.04)	1.47 (0.04)	1.48 (0.04)	1.47 (0.04)
φ_2	-0.44 (0.04)	-0.53 (0.04)	-0.55 (0.04)	-0.54 (0.04)
π_u	0.69 (0.05)	0.66 (0.06)	0.62 (0.05)	0.65 (0.06)
p	0.65 (0.09)	0.57 (0.13)	0.61 (0.11)	0.61 (0.11)
q	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)
β	-1.44 (0.11)	-1.86 (0.13)	-1.96 (0.10)	-1.91 (0.10)
π_x	-1.01 (0.15)	-0.46 (0.18)	-0.000 (0.001)	-0.000 (0.001)
ψ_x	0.49 (0.10)	0.39 (0.20)	0.12 (0.21)	0.28 (0.20)
Log likelihood	-11.9	-22.3	-23.3	-38.0

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are reported in parenthesis.

Notes:

(1) The estimation period runs from 1948Q1 to 2019Q4.

(2) All columns present the results of the benchmark model (1.a) with different observed series for unemployment. The first column is the model without controlling the lead-lag effect between output and the unemployment rate. In this model, we simply calculate the quarterly unemployment rate as the average of the rates of the three months within the corresponding quarter. On the other hand, columns two, three, and four present the estimation results when the 1-month, 2-month, and 3-month lead-lag effect is accounted for. For example, we use 1-month leading unemployment as the data for the observed series to estimate the 1-month model. We calculate the 1-month leading rate for each quarter by taking an average of three months, two of which are within the same quarter, and the other one is in the subsequent quarter.

(3) The output-specific plucking coefficient (π_x) is significant in the benchmark model. This suggests that some minor part of the plucking property is not explained by Okun's law. However, it is clear that this remaining plucking property is related to the lead-lag effect between output and the unemployment rate. Because by controlling the 1-month lead-lag effect, the coefficient will be less significant, and finally, by controlling the 2-month lead-lag effect, the output-specific plucking coefficient will be zero.

(4) Log likelihood values are not comparable since the data inputs for unemployment are different.

Table 2.G.4: Estimated parameters of the bivariate model, including the COVID-19 recession

Models	(1.a)
Parameters	A-Bi-RW-SB-UC
σ_{x^*}	0.60 (0.03)
σ_{u^c}	0.56 (0.02)
σ_{μ}	0.02 (0.01)
σ_{u^*}	0.00 (0.01)
σ_{x^c}	0.00 (0.04)
γ	T-V
δ	T-V
η	0.03 (0.01)
θ	-0.04 (0.01)
φ_1	0.70 (0.05)
φ_2	0.06 (0.05)
π_u	1.24 (0.01)
p	0.76 (0.07)
q	0.96 (0.01)
β	-1.21 (0.06)
π_x	-1.33 (0.15)
ψ_x	-0.17 (0.15)
$\pi_{u,Covid-19}$	1.36 (0.03)
ρ_{x^*,u^c}	–
ρ_{x^*,x^c}	–
ρ_{u^c,x^c}	–
Log likelihood	-301.8

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are reported in parenthesis.

Notes:

(1) The estimation period runs from 1948Q1 to 2022Q1. The table reports the results of the benchmark model (Asymmetric-Bivariate-RW-SB-UC), which accounts for the COVID-19 recession by using Eq. (2.19) that includes a dummy for the COVID-19 recession.

(2) The COVID-19 plucking coefficient ($\pi_{u,COVID}$) is significant since the depth of the Covid-19 recession is greater than those of previous recessions. Additionally, the estimated plucking coefficient (π_u) is larger compared to that of the benchmark model because the greater depth of the COVID-19 recession requires a larger coefficient to be explained.

(3) The log likelihood value of this extended model is not comparable with that of the benchmark model because the data inputs for output and the unemployment rate are different from those of the benchmark model.

Table 2.G.5: Estimated parameters of the bivariate model, for U.S. output per capita and U.K. output

Models	(1.a) for U.S. GDP per capita	(1.a) for U.K. GDP
Parameters	A-Bi-RW-SB-UC	A-Bi-RW-SB-UC
σ_{x^*}	0.50 (0.09)	0.75 (0.13)
σ_{u^c}	0.21 (0.01)	0.10 (0.01)
σ_{μ}	0.02 (0.01)	0.02 (0.01)
σ_{u^*}	0.00 (0.02)	0.08 (0.01)
σ_{x^c}	0.29 (0.10)	0.35 (0.22)
γ	T-V	T-V
δ	T-V	T-V
η	0.03 (0.01)	0.04 (0.01)
θ	-0.04 (0.01)	-0.05 (0.02)
φ_1	1.38 (0.05)	1.62 (0.07)
φ_2	-0.44 (0.04)	-0.65 (0.06)
π_u	0.70 (0.05)	0.23 (0.05)
p	0.66 (0.09)	0.88 (0.07)
q	0.96 (0.01)	0.97 (0.02)
β	-1.45 (0.12)	-1.53 (0.35)
π_x	-1.02 (0.16)	-0.77 (0.52)
ψ_x	0.48 (0.11)	0.60 (0.28)
ρ_{x^*,u^c}	–	–
ρ_{x^*,x^c}	–	–
ρ_{u^c,x^c}	–	–
Log likelihood	-13.4	28.2

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are reported in parenthesis.

Note:

The estimation period runs from 1948Q1 to 2019Q4 for U.S. GDP per capita and spans from 1955Q1 to 2019Q4 for the U.K. GDP.

Chapter 3

Asymmetric Fads, Inefficient Plunges, and Efficient Market Hypothesis

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Abstract

I propose the concept of inefficient plunges to characterize the asymmetric deviation of market prices from efficient prices with the aim of examining the efficient market hypothesis. To measure market inefficiency, I present an asymmetric Fads model, which allows for both inefficient plunges in the transitory component and a switching variance in the permanent component by embedding a Markov-switching process in an unobserved components model. Applying the model to the S&P 500 and the FTSE 250 confirms that inefficient plunges are deep, steep, and transient. Market inefficiency is a regime-dependent and asymmetric phenomenon, meaning that even though the U.S. and U.K. stock markets are adequately efficient during normal times, they are far below efficient prices during crises. Moreover, the asymmetric Fads model is entirely consistent with asymmetric volatility.

Keywords: Inefficient plunges, Efficient Market Hypothesis, Adaptive Market Hypothesis, Rational Bubbles, Negative bubbles, Unobserved Components Model.

JEL Classification: C32, C58, G14, G41.

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3.1. Introduction

I present an asymmetric Fads model to gauge the level of stock market inefficiency over time, which is defined as the deviation of the market price from the efficient price. This deviation appears to be regime-dependent and asymmetric because price deviations during downturns have no mirror image during upturns. Indeed, although the efficient price drops during a crash because of expectations of lower future cash flows, the actual market price tends to overreact and drop more deeply than the efficient price, so that the excessive drop in price creates a gap between the market and efficient prices.[1] While recent literature refers to these gaps as "negative bubbles," I denominate these gaps "inefficient plunges," to establish regime-dependence and asymmetry in market inefficiency.[2] In this regard, I show that downside deviations during crises tend to be deep and steep, whereas upside deviations during non-crisis periods are negligible.

This study lies at the crossroads of four branches of the finance and asset pricing literature, including the Efficient Market Hypothesis (EMH), the Adaptive Market Hypothesis (AMH), Rational Bubbles, and the Fads model. Each of these branches is explained here briefly and discussed thoroughly in the literature review.

First, the EMH, proposed by Samuelson (1965) and Fama (1970), argues that since market prices reflect all available information, future prices are unpredictable, and so market prices must follow a random walk process with a drift, which is referred to as the Random Walk Hypothesis (RWH). This claim entirely depends on the Rational Expectations Hypothesis (REH), introduced by Muth (1961) and later popularized by Lucas (1978), which states that all investors have rational expectations.

Second, contrary to the REH, behavioural finance and economics suggest that a sizeable portion of investors are not always rational (Simon, 1955; Arrow, 1982). As a result, the market cannot always be efficient (Russell and Thaler, 1985). Considering this disagreement, the AMH aims to reconcile the EMH with behavioural finance. Proposed by Lo (2004, 2019), the AMH describes a framework in which rationality and irrationality forces coexist, and investors are not unboundedly rational. This framework implies that market inefficiency is not constant but instead evolves over time. In this regard, a growing empirical literature highlights time-variation in market inefficiency, particularly

[1] To put it simply, I suggest that the efficient price that evolves as a random walk with stochastic drift cannot explain the whole drop in the actual market price.

[2] To achieve two aims, I prefer the term "inefficient plunges" over similar words such as negative bubbles, fear bubbles, panic selling, and crashes. First, in this study, the term "inefficient plunges" means an excessive price drop during crashes, where the actual market price negatively deviates from the efficient price. Second, although little attention has been paid to the possibility of negative bubbles, recent studies have been investigating this new concept (see, e.g., Cao et al., 2016; Acharya and Naqvi, 2019; Emery, 2021), which is very close to the concept of inefficient plunges. They both refer to the negative deviation of market prices from counterfactual prices, whether rational or fundamental. To avoid confusion between positive and negative bubbles, I suggest using speculative bubbles when the deviation is positive and inefficient plunges when the deviation is negative.

during crises (see, e.g., Lim et al., 2008; Lim and Brook, 2011; Anagnostidis et al., 2016; Ito et al., 2016; Hill and Motegi, 2019).

Third, Blanchard and Watson (1983) and Diba and Grossman (1988) present a model of “Rational Bubbles” to rationalize the formation of speculative bubbles. According to them, the occurrence of speculative bubbles is entirely consistent with the REH and EMH. As a result, if this is the case, market prices that might contain positive bubbles are efficient and hence must follow a random walk process with drift. However, as suggested by Emery (2021), rational bubbles ignore the possibility of negative bubbles during crises, which is one of the main culprits of market inefficiency.

Fourth, the conventional Fads model, adopted by Shiller et al. (1984), Summers (1986), Fama and French (1988), and Poterba and Summers (1988), is a trend-cycle decomposition aiming to examine the EMH and RWH by capturing the possibility of market price deviations from the efficient price, which measures the fundamental. This model, however, confounds positive and negative bubbles since it rules out two possibilities: (1) the Fads model ignores the possibility of presence of positive bubbles inside the permanent component. However, the permanent component (efficient price) might exceed the fundamental by the size of a positive bubble fueled by speculative activities during the boom phase of the bubble; and (2) the Fads model dismisses the asymmetry in the deviation of market prices from efficient prices. This shortcoming is partially addressed by Turner et al. (1989) and Kim and Kim (1996), who employed a Markov-switching process to accommodate the asymmetry.

Concerning the four mentioned branches of literature, I answer four questions. The first question is whether the market price is efficient or not. Does the market price follow a random walk with a drift? Based on the existing literature, there is some agreement about the answer to this question: the market is not always efficient and market inefficiency evolves over time (see, e.g., Ito et al., 2016; Noda, 2016; Hill and Motegi, 2019). However, there are still some unresolved questions about the level and dynamics of market inefficiency. The second question thus concerns the extent to which the market is inefficient and whether the level of market inefficiency is regime-dependent or not. Regarding the inadequacy of rational bubbles and the Fads model to explain negative bubbles, the third question explores whether the deviation of market prices from efficient prices, which measures inefficiency, is asymmetric or not. Finally, to capture the asymmetric deviations, I augment the conventional Fads model by embedding a Markov-switching process to estimate inefficient plunges in the transitory component, while the same Markov-switching process accommodates a switching variance in the permanent component.

For trend-cycle decomposition, I developed an Unobserved Components (UC) model to decompose the market price into its permanent and transitory components, which reflect the efficient price and the inefficient plunges, respectively. I specify the permanent component as a random walk process with stochastic drift to represent efficient prices that might contain positive bubbles.[3] The transitory component is modelled as an autoregressive of order two. In addition, to account for the asymmetry, I include inefficient plunges in the transitory component accompanied by a switching variance in the permanent component by embedding a single Markov-switching process in the UC model. Finally, by estimating deviations of market prices from efficient prices, I provide a new measure for market inefficiency during high-variance periods, which I refer to as inefficient plunges (also called negative bubbles).

By applying the model to the monthly inflation-adjusted S&P 500 and FTSE 250, I substantiate the asymmetry in the form of inefficient plunges accompanied by concomitant high-variance states. The estimated plunging coefficient is highly significant, and the switching variance of the shock to the permanent component during crisis periods is greater than its counterpart during normal times. The expected duration is short for crises and long for non-crisis periods. Accordingly, inefficient plunges are deep and transient; they reach a notable depth of 15% and even 20% during crises and last for around 4 months, and thereafter the corresponding gaps shrink and finally disappear within a couple of additional months. Considering a threshold of 10% for market inefficiency, I conclude that 12.7% and 13.9% of the time, which corresponds to the crisis periods, the magnitude of inefficient plunges is 10% or more in the S&P 500 and FTSE 250. As a result, I highlight that the deviations of the U.S. and U.K. market prices from efficient prices, which measure the level of market inefficiency, are regime-dependent and asymmetric. By running alternative models and using the daily and weekly S&P 500 and FTSE 250, I also confirm that these results is robust to changes in model specifications and frequency.

I discuss the findings of this study in four categories, each related to one of the four branches in the literature. First, the deep inefficient plunges estimated in the transitory component refute the accuracy of the EMH and REH, suggested by Muth (1961), Samuelson (1965), Fama (1970), Lucas (1978), and Durusu-Ciftci et al. (2019). Since inefficient plunges are deep and significant only during crises, market inefficiency is not constant but instead is regime-dependent, meaning that although the S&P 500 and the FTSE 250 are adequately efficient during normal times, they are below efficient prices during crises. This also corroborates predictability, implying that it is possible to outperform the market, but it entirely depends on the extent of market inefficiency.

Second, by establishing the regime-dependence of market inefficiency, I conclude in favour of the AMH of Lo (2004, 2019) against the EMH of Fama (1970). This notion is consistent with the finding

[3] Throughout this study, the permanent component, which is modelled as a random walk process with stochastic drift, represents the efficient price that might contain positive bubbles. There are four reasons for selecting this treatment, which are explained in the methodology section. Additionally, the terms “efficient price” and “rational price” can be used interchangeably since the EMH relies heavily on the REH.

of time-variation in market inefficiency by Campbell et al. (1998), Ito and Sugiyama (2009), Lim et al. (2008), Anagnostidis et al. (2016), Ito et al. (2016), Noda (2016), Hill and Motegi (2019), Le Tran and Leirvik (2019), and Mattera and Di Sciorio (2022). Since inefficient prices survive the arbitrage process for a considerable duration of 4.4 and 3.3 months for the U.S. and U.K. stock markets, this finding is in line with Simon (1955), Arrow (1982), and Russell and Thaler (1985), who mention that the market is not always efficient.

Third, consistent with the model of rational positive bubbles, a time-variation in the long-run return for the S&P 500 is observed. In particular, the long-run return reached 15% in the late-1990s, which was related to the technology boom and dot.com bubble. Given the presence of deep market plunges, I document the possibility of negative bubbles, where market prices are lower than efficient prices. This finding supports the results of a few studies by Yuan (2005), Yan et al. (2012), Cao et al. (2016), and Emery (2021), who attribute negative bubbles to the withdrawal of uninformed investors and/or binding borrowing constraints.

Fourth, in opposition to the conventional Fads model, I state that inefficient plunges, estimated as an asymmetric transitory component, are the foremost determinant of market inefficiency. Moreover, the asymmetric Fads model, by allowing inefficient plunges accompanied by a switching variance, explains the following stylized facts about the stock market. (1) Estimating inefficient plunges with a depth of more than 15%, along with a short plunging duration, is in accordance with the asymmetric return distribution suggested by Campbell and Hentschel (1992) and Adrian and Rosenberg (2008). (2) Inefficient plunges that are accompanied by high-variance states are consistent with asymmetric volatility, stating that the onset of episodes of high volatility coincides with that of large negative returns (see, e.g., Nelson, 1991; Hong and Stein, 2003; Jones et al., 2004; Avramov et al., 2006). (3) A transient asymmetric shock that lasts for about 4 months, on average, tends to be followed by a moderately gradual rebound that takes a couple of additional months to fill the price gap. The result suggesting a transient Fads component is consistent with that of Kim and Kim (1996), and the finding of the gradual rebound implies that the market prices exhibit episodes of high volatility following the onset of inefficient plunges, a phenomenon known as volatility clustering (see, e.g., Engle, 1982, 2004; Bollerslev, 1986).

This study makes two contributions to the literature. First, it repurposes the UC model with Markov-switching to establish a new concept, inefficient plunges, with the aim of examining the EMH. To the best of my belief, this is the first study to characterize the asymmetric deviation of market prices from efficient prices, by modelling in levels rather than in differences, to allow for inefficient plunges and a concomitant switching variance. As a result, this model is able to capture multiple phenomena, including market inefficiency, inefficient plunges (also known as negative bubbles), and asymmetric volatility. Second, the existing empirical literature on testing the EMH is incompetent to identify the nature of the observed time-variation in market inefficiency. By contrast, this study demonstrates that market inefficiency is a regime-dependent and asymmetric phenomenon. On this basis, my model pinpoints the level of inefficiency at any given moment, in contrast to previous works, which only provide an approximation of market inefficiency by performing correlation or random walk tests based on a rolling window analysis.[4]

Aside from the above contributions, diagnosing the symptom of inefficient plunges proposes three potential competing drivers for market inefficiency: market irrationality, market unawareness, and financial frictions. The first one sheds some light on the confrontation of forces of rationality and irrationality. During normal times, the rationality force predominates over irrationality, so that the plunging component is close to zero. Conversely, during crises, the “animal spirits” proposed by Keynes (1936) cause irrationality to prevail over rationality, resulting in a panic-driven plunge, which is consistent with the Over-Reaction Hypothesis (ORH) advocated by De Bondt and Thaler (1985). Afterwards, once the panic recedes, rationality will dominate again and the market price will revert to the efficient level. This attributes market inefficiency to market irrationality, whose magnitude depends on the proportion of irrational investors in the market. The second driver, however, attributes market inefficiency to the withdrawal of rational but uninformed investors (see, e.g., Emery, 2021). Finally, the third driver blames the binding financial constraints during crises for market inefficiency (see, e.g., Yuan, 2005).

Concerning the limitations, this study is unable to favor one of the above narratives over the other. Besides, since this study aims to examine the effect of negative bubbles on market inefficiency, it remains silent about another culprit of market inefficiency during the boom phase of speculative bubbles: greed and overconfidence.[5]

The remainder of this paper is organized as follows: Section 3.2 reviews the literature on the Efficient Market Hypothesis (EMH), the Adaptive Market Hypothesis (AMH), rational bubbles, and the Fads model. Section 3.3 describes the data and methodology and justifies the model specification. Section 3.4 presents the results and robustness tests, and finally, Section 3.5 provides the conclusion.

3.2. Literature Review

This study integrates the perspectives of four branches in the literature of finance and asset pricing, which are thoroughly discussed in this section. First, the EMH states that market prices reflect all available public information (Samuelson, 1965; Fama, 1970); thus, prices are purely unpredictable

[4] See Appendix 3.B for shortcomings in the existing empirical literature on examining the EMH.

[5] Positive bubbles are rationalized by the model of rational bubbles. Also, efficient prices that might contain bubbles are well characterized by a random walk with a drift. However, taking a conservative approach, I conclude that the market is inefficient at least 12% of the time that corresponds to crises, and I remain neutral about potential market inefficiency due to positive bubbles during some episodes within non-crisis periods.

and no investor can outperform the market. As a result, the future price movement depends only on the newly released information. If this is the case, mathematically, the market prices must follow a random walk process with a drift, what is referred to as the Random Walk Hypothesis (RWH), or weak form of the EMH.

Empirically, despite many studies that explored the RWH for different stock markets by using unit root, correlation, or variance ratio tests, Durusu-Ciftci et al. (2019) note a lack of agreement on stock market efficiency.[6] One reason for such a conflict is that market efficiency is treated as an absolute all-or-nothing measure, while market efficiency is fuzzy rather than a binary phenomenon (Campbell et al., 1998). Further, a part of the literature examined the RWH under the dubious assumption that market efficiency remains constant over time (Lim and Brook, 2011).

Theoretically, EMH relies on the REH, originally introduced by Muth (1961) and later popularized by Lucas (1978). REH states that since all investors have rational expectations, the market price implied by the investors' behaviour is essentially the same as the price predicted by the economic model. On the contrary, behavioural finance and economics advocate that, due to behavioural biases, a sizeable portion of investors in the market are not always rational (Simon, 1955; Kahneman and Tversky, 1979; Arrow, 1982; among others); thus, the market is not always efficient (Russell and Thaler, 1985; Lo, 2004).[7] For instance, De Bondt and Thaler (1985) present the ORH, implying that investors are not rational in the sense that they overreact to news. They advocate predictability because extreme movements in stock prices will be followed by subsequent price movements in the opposite direction. To be brief, based on behavioural finance, the main culprits of market inefficiency can be attributed to behavioural biases such as panic and overreaction during crises or greed and overconfidence during bubbles.

Second, first introduced by Farmer and Lo (1999) and later formalized by Lo (2004, 2019), the AMH tries to reconcile the EMH with behavioural finance and economics by establishing a framework in which rationality and irrationality forces coexist. In this framework, as introduced by Simon (1955), investors are not unboundedly rational and tend to find a satisfactory heuristic solution. Since the solution of the old regime is not suitable for the new one, investors must adapt their heuristic solution when the regime changes. This adaptation involves trial and error and, inevitably, behavioural biases in some investors that constitute an irrationality force, which tends to turn the price away from the rational level. Inversely, the rest of the investors, who are rational and take advantage of arbitrage opportunities, form a rationality force that tends to bring the price back to a rational level.

[6] By using a panel unit root test for 33 stock indices, they themselves conclude in favour of the RWH. This result is under question because they assumed that the level of market efficiency is constant.

[7] Also, Grossman and Stiglitz (1980) argue that a perfectly efficient market is impossible because traders don't have any incentive to acquire costly information unless there are profit-making arbitrage opportunities.

The AMH prompts the question of to what extent the irrationality force is stronger than the rationality force and how long the inefficient price survives the arbitrage process. Does the deviation disappear quickly or persist for a while? The EMH and behavioural economics respond to these questions in two opposite directions: the former maintains that irrationality forces are negligible and inefficient prices disappear immediately, but the latter says that irrationality forces are substantial and inefficient prices persist for a considerable period. The EMH proponents state that even if individual irrationality exists, its effect on the market price is negligible as irrational investors account for a small proportion of the market, and arbitrageurs immediately bring the price back to the rational level.[8] This argument is, however, rejected by Russell and Thaler (1985), who conclude that the presence of some rational agents is not sufficient to guarantee the existence of a rational expectations equilibrium. Further, Lo (2004) and Kindleberger (1989) presented anecdotal examples of speculative bubbles, panic during market crashes, and manias suggesting a prolonged deviation from the efficient price, which cast doubt on market rationality at the aggregate level.

Lo (2004) argues that the composition of the market is changing over time because investors with different attitudes and levels of irrationality are entering and quitting the market. Hence, the level of market irrationality and, consequently, market inefficiency is not constant but instead evolves over time. In this regard, a rapidly growing literature supports the AMH and casts doubt on the EMH. For example, Lim et al. (2008), Anagnostidis et al. (2016), and Ito et al. (2016) demonstrate that financial crises adversely affect the level of efficiency in the Eurozone, Asian, and U.S. stock markets. This is consistent with the suggestion of Lim and Brook (2011), to allow for time-variation in the level of market inefficiency. Similarly, Ito and Sugiyama (2009) and Hill and Motegi (2019), by applying a rolling window analysis, document a time-variation in market inefficiency in the U.S. and the U.K. stock markets during financial crises.[9] In line with the above work, Noda (2016), Le Tran and Leirvik (2019), Mattera and Di Sciorio (2022), construct measures for time-varying market inefficiency where all methods are based on overlapping or non-overlapping rolling windows to estimate the sub-sample autocorrelation.

Third, speculative bubbles are often defined as the positive deviation from the fundamental price that is followed by a burst. The Tulip Mania in 1637, the dot.com bubble in the late-1990s, and the real estate bubble in 2005 are examples of notorious bubbles. In this context, Blanchard and Watson (1983), Tirole (1985), and Diba and Grossman (1988) present “rational bubbles,” as implied by the

[8] The EMH presents another counterargument, which states that even if most investors are irrational, a few rational investors can make lots of money and eventually take over all the market. Thus, rational behaviour will be pervasive in the market. Nevertheless, Arrow (1982) rebutted this reasoning, pointing out that the irrationality of everyone else does not guarantee the benefits of one rational investor in the short term because the value of a security depends on other people’s opinions.

[9] As additional evidence, Eraker (2008), Kim et al. (2011) and Urquhart and McGroarty (2016) reject the EMH in favour of the AMH by documenting a time-varying return predictability in different stock markets.

name, a model that attempts to rationalize the formation of speculative bubbles.[10] Based on rational bubbles, speculative bubbles occur even if all investors have rational expectations and know that the bubble will eventually burst. Indeed, the market price rationally deviates from its fundamental value when the bubble is expected to grow further.[11]

Most of the literature on speculative bubbles, by imposing positive bubbles, implicitly rules out the possibility of negative bubbles.[12] On this basis, Blanchard and Watson (1983), Tirole (1985), Diba and Grossman (1988), Adam & Szafarz (1992), Lux and Sornette (2002), Barlevy (2007), and Basse (2021), among others, affirm the rationality of positive bubbles. Hence, if this is the case, the market price that contains positive bubbles is still efficient. As a result, market inefficiency is mostly attributable to negative bubbles during crises.

Negative bubbles that are negative deviations of market prices from efficient prices, are indications of market inefficiency and predictability.[13] In this vein, Yan et al. (2012), by applying a model of rational expectation bubbles proposed by Johansen et al. (2000) to sub-windows of the S&P 500, and Goetzmann and Kim (2018), by analyzing 101 global stocks, identify a pattern of crash-and-rebound: extremely large drops lead to negative bubbles and are typically followed by strong rebounds. The return following a crash is, on average, 10% higher than normal times. This 10% excessive rise during the rebound is the mirror image of the excessive drop (negative bubbles) during the crash.

Fourth, the Fads model is a trend-cycle decomposition to examine the possibility of deviation of the price from its fundamental caused by noisy traders who, based on fashions, fads, and sentiments, bid the price away from the fundamental (Shiller et al., 1984; Summers, 1986; Fama and French, 1988; Poterba and Summers, 1988). Regarding the specification in this model, the permanent component that represents the fundamental price is characterized as a random walk with drift, and the transitory

[10] Consider a simple example in which the bubble either survives by growing at a rate higher than the interest rate with probability θ or bursts with probability $(1-\theta)$. Clearly, the conventional risk-return tradeoff holds in this example because an investor who tolerates the risk of bursting the bubble can be compensated with a return higher than the interest rate.

[11] Tirole (1982) had initially rejected rational bubbles by taking the market restrictions, e.g., finite life and finite number of investors, into account. He discussed that when the number of investors is finite and the initial winners have exited and left a negative-sum game, no investor participates in the bubble. Later, Tirole (1985), by using an overlapping generation model, confirmed that, under some conditions, rational bubbles occur, which is not inconsistent with rationality.

[12] Few studies allow for both positive and negative bubbles, where asset prices might deviate from their fundamentals in either a positive or negative direction (see, e.g., by Lux, 1995; Shiller, 2000).

[13] A few studies have explored negative bubbles, where asset prices are lower than their fundamentals. In a recent study, following Akerlof's (1970) adverse selection model, Emery (2021) presents a theoretical model for financial markets, where uninformed investors withdraw from the market and cause a negative bubble. Similarly, Barlevy and Veronesi (2003) state that, under some conditions, a small decline in price leads uninformed agents to withdraw from the market and causes a crash, whose amplitude depends on the number of uninformed investors. Yuan (2005) also documents that asymmetric price movements arise endogenously from rational but uninformed investors and binding borrowing constraints. In another work, Cao et al. (2016) show that when the short-sale constraint is binding and investors are strictly risk averse, a negative bubble can arise. Finally, Acharya and Naqvi (2019), by taking banking agency problems into account, show that negative bubbles exist in response to tight monetary policy.

component, also known as the Fads component, is specified as an autoregressive process of order one or two to allow for potential mean-reversion. A close-to-unity autoregressive coefficient suggests that the Fads component is transitory but persistent, which supports predictability and rejects the EMH. Indeed, the market price deviates from the fundamental price and slowly returns to it, which induces autocorrelation in returns and enables investors to make a predictable profit.

The Fads model does not distinguish between positive and negative bubbles, and so is subject to two caveats. First, it imposes the transversality condition (the expected resale price is zero) to rule out the possibility of positive bubbles. On this basis, the fundamental price is assumed to be equal to the efficient price, while there is a plethora of studies stating that positive bubbles are not only possible but also rational.[14] In fact, although rational bubbles and the Fads component are two distinct features (Camerer, 1989), the Fads model tends to confuse them. Since the U.S. and other stock markets have traces of both features (Schaller and Van Norden, 2002), unaccounted for positive bubbles inevitably confound the measurement of the Fads component and fundamental price. Second, the Fads model neglects the asymmetry in the deviation of market prices from efficient prices and does not identify negative bubbles, whereas there is ample evidence for asymmetric deviation from efficiency. As a result, the Fads model is unable to capture the main determinant of market inefficiency as it overlooks the fact that large price movements tend to be downward rather than upwards (Yuan, 2005). To address this shortcoming, Turner et al. (1989) and Kim and Kim (1996) utilized a Markov-switching process to accommodate the asymmetry, whose methods are explained further in the next section.

Finally, since the asymmetric Fads model is consistent with the stock market's stylized facts, I review three regularities as follows. (1) The asymmetric return distribution means that returns are negatively skewed; downturns tend to be deeper and steeper than upturns (Campbell and Hentschel, 1992; Hong and Stein, 2003; Adrian and Rosenberg, 2008). (2) Asymmetric volatility indicates that periods of high volatility correspond to periods of negative returns (Nelson, 1991; Glosten et al., 1993; Jones et al., 2004; Avramov et al. 2006).[15] (3) And according to volatility clustering, large fluctuations in prices tend to be followed by further large fluctuations (Engle, 1982, 2004; Bollerslev, 1986), which occur more frequently during downturns (Ning et al., 2015).

[14] The literature on rational bubbles and the Fads model developed concurrently between 1983 and 1988. Thus, it is no wonder that each branch didn't accommodate both rational bubbles and the Fads component. For example, Summers (1986), by downgrading the suggestion made by Blanchard and Watson (1983) on the relationship between speculative bubbles and the Fads component, stated that he is not interested in including bubbles as they are large deviations from the fundamental. He, therefore, imposed the transversality condition to rule out positive bubbles.

[15] The leverage effect (see, e.g., Black, 1976; Christie, 1982; Bekaert and Wu, 2000; Bollerslev et al., 2006) and volatility feedback (see, e.g., Bekeart and Wu, 2000; Barucci et al., 2003; Carr and Wu, 2017) are two of the more well-known drivers of asymmetric volatility.

3.3. Data and Methodology

I use data of the monthly S&P 500 index from 1948M1 to 2022M6 and the monthly FTSE 250 index from 1986M1 to 2022M6, obtained from the Bloomberg. I apply models to indices without dividends reinvested, yet the results are not sensitive to the choice of dividend reinvestment. I adjust monthly indices for inflation by dividing them by the monthly consumer price index for the U.S. and the U.K. For robustness tests, I also use indices at daily and weekly frequencies.

Regarding the trend-cycle decomposition, the bulk of studies in finance and economics have applied different versions of the symmetric UC model of Harvey (1985) and Clark (1987). The conventional Fads model, notably, is a symmetric UC model that rules out the plausible possibility of asymmetric price deviations from efficiency. Meanwhile, very few studies in economics use the asymmetric UC model to explain business cycle asymmetries (see, e.g., Kim and Nelson, 1999a; and Dehghani et al., 2022), and even fewer studies in finance adopt the asymmetric UC model to examine asymmetric volatility (Turner et al., 1989), and transient Fads (Kim and Kim, 1996). With this in mind, I augment the Fads model to address its shortcomings and, more importantly, to repurpose the asymmetric UC model to examine the EMH.

I decompose the market price into permanent and transitory components that represent the efficient price and price deviation from efficiency, respectively. Regarding the permanent component, since rational bubbles are consistent with rational expectations, efficient prices that might contain positive bubbles are still supposed to follow a random walk process with drift. I therefore keep the positive bubble along with the fundamental price in the permanent component and place negative bubbles (inefficient plunges) in the transitory component.[16] In this setting, the transitory component is specified as the sum of inefficient plunges and an autoregressive process of order two to account for potential persistency during the price rebound. To accommodate the asymmetric price deviation, I include inefficient plunges in the transitory component and a concomitant switching variance in the

[16] According to the present value model and the law of iterated expectations, the efficient price is the sum of two components: the fundamental price, which is the sum of discounted expected dividend payments; and the speculative bubble, which is the expected resale price assumed to be positive or zero to rule out negative bubbles. There are four rationales for keeping the positive bubble inside the permanent component. First, the specification of the efficient price (the sum of the fundamental and the positive bubble) is given as a random walk with drift, whereas there is no clear specification for each separately. Second, based on Blanchard and Watson (1983), there is no reason to support the independence of the fundamental and the positive bubble. Indeed, both the fundamental and the positive bubble increase during the bubble formation, and drop together during the burst. As a result, decomposing them is exceedingly challenging, if not impossible. Third, Camerer (1989) warns against the confusion of rational bubbles with the Fads component. Thus, to avoid conflating the positive and negative bubbles in the Fads (transitory) component, my model places the positive bubbles along with the fundamental inside the permanent component to characterize efficient prices and places the negative bubbles in the transitory components to characterize inefficient plunges. Finally, the aim of this study is to examine the effect of negative bubbles (inefficient plunges) rather than positive bubbles on market inefficiency. So I circumvent the decomposition of the positive bubble and fundamental components and let these two stay together inside the permanent component.

permanent component by employing a Markov-switching process similar to Hamilton (1989). For estimation, in the style of Kim and Nelson (1999a), explained in Appendices 3.C and 3.D, I cast the model in a state-space form with Markov-switching to estimate it by using Kim's (1994) approximate maximum likelihood method. Finally, I test the model against symmetric alternatives by pairwise comparisons of the likelihood ratio values of competing models.

Let me briefly review two studies tried to address the Fads model's shortcomings. Turner et al. (1989) allow the asymmetry by including a Markov-switching process for both the mean and variance of returns; and Kim and Kim (1996) allow for the asymmetry by including two independent Markov-switching processes, each of which accounts for one of the switching variances of shocks to the permanent and transitory components. These two studies are thus methodologically close to this study; however, my model differs from theirs in three ways. First, I specify the model in levels rather than in differences to examine the EMH by measuring the deviation of market prices from efficient prices, while their models are incompetent to test the EMH as they are in differences. Second, I allow for asymmetry by including inefficient plunges in the transitory component and a switching variance in the permanent component, while the model in each of these two studies has a missing component. The model proposed by Turner et al. (1989) embeds a Markov-switching variable for both the mean and the variance; however, it ignores the transitory component, which is essential to characterize the price deviation from and reversion to the efficient price. Although Kim and Kim (1996) incorporate the transitory component, they do not allow Markov-switching for the mean of this component and rule out the possibility of asymmetric price deviations from efficinet prices. In addition, Turner et al. (1989) assume that the mean return within each regime is constant, and Kim and Kim (1996) impose a more restrictive assumption that the mean return is constant across all regimes.[17]

3.3.1. The Asymmetric Fads Model

Within a univariate model of trend-cycle decomposition, consider Eq. (3.1), where the actual market price is decomposed into a permanent and a transitory component:

$$p_t = p_t^r + p_t^i \tag{3.1}$$

where the natural log of prices are observed series and denoted by p_t. Accordingly, p_t^r and p_t^i are unobserved permanent and transitory components playing the roles of efficient prices and inefficient plunges. Considering the possibility of speculative bubbles, the efficient price might contain positive bubbles, and hence the efficient price is not necessarily equal to the fundamental price. As a result, the permanent component in this study characterizes the efficient price, rather than the fundamental.

[17] See Appendix 3.B for other shortcomings of these two studies.

3.3.1.1. The permanent component (efficient prices)

Given that speculative bubbles are rationalized by Blanchard and Watson (1983) and Diba and Grossman (1988), the efficient price must follow a random walk pricess with a drift. Hence, similar to Fama and French (1988), Poterba and Summers (1988), and Eraker (2008), I specify the permanent component as a random walk process with a drift:

$$p_t^r = \mu_{t-1} + p_{t-1}^r + \varepsilon_{p^r,t} \tag{3.2}$$

where $\varepsilon_{p^r,t} \sim N(0, \sigma_{p^r}^2)$ is the shock to the permanent component. To accommodate variance changes across two regimes, I allow Markov-switching variance for the shock to the permanent component that is explained further in Section 3.3.1.3. In this setting, μ_t is the drift term that represents the long-run return and might be time-varying. As the long-run episodes of bull and bear markets are observed in the U.S. stock markets due to changes in expectations about future cash flows, I specify the long-run return as a random walk:

$$\mu_t = \mu_{t-1} + \varepsilon_{\mu,t} \tag{3.3.a}$$

where $\varepsilon_{\mu,t} \sim N(0, \sigma_{\mu}^2)$ is the shock to the long-run return and is assumed to be uncorrelated with $\varepsilon_{p^r,t}$. Alternatively, following Summers (1986), Poterba and Summers (1988), and Turner et al. (1989), I can model the long-run return as a constant:

$$\mu_t = \mu \tag{3.3.b}$$

Considering Eq. (3.3.a) and (3.3.b) as two specifications, I run both as two versions of the benchmark model. However, since a stochastic drift enables the model to capture the time-variation in the long-run returns and identify the episodes corresponding to speculative (positive) bubbles, and because a random walk is more robust to misspecifications and provides more flexibility (Antolin-Diaz et al., 2017), I advocate the model with a random walk drift.

3.3.1.2. The transitory component (inefficient plunges)

Similar to Kim and Kim (1996), the transitory component comprises an autoregressive process of order two with coefficients φ_1 and φ_2. However, to accommodate the asymmetric price deviation, I consider that shocks to the transitory component are a mixture of asymmetric and symmetric shocks. Thus, I incorporate a first-order Markov-switching process into the transitory component, which I also call the plunging component:

$$p_t^i = \pi_i S_t + \varphi_1\, p_{t-1}^i + \varphi_2\, p_{t-2}^i + \varepsilon_{p^i,t} \tag{3.4}$$

where π_i is the inefficient plunge coefficient that measures the amplitude of inefficient plunges and is expected to be negative and significant to corroborate the plunging property. The state of the stock market is denoted by S_t, an unobservable indicator that distinguishes between crisis periods when

$S_t = 1$, and normal times when $S_t = 0$. This indicator will be determined endogenously as it evolves according to the Markov-switching process as in Hamilton (1989):

$$\Pr[S_t = 1 | S_{t-1} = 1] = p \tag{3.5}$$

$$\Pr[S_t = 0 | S_{t-1} = 0] = q \tag{3.6}$$

In this approach, p and q determine the transition probabilities. p is the probability of staying in the crisis, and thus, $(1 - p)$ is the probability of transitioning from a crisis to the normal state. Similarly, q is the probability of staying in the normal state, and thus, $(1 - q)$ is the probability of transitioning from a normal state to the crisis state. The term $\pi_i S_t$, therefore, characterizes the excessive drop in price (inefficient plunge) during crashes, which is similar to the negative bubble in the model of Yan et al. (2012), adopted from the model of bubbles and crashes presented by Johansen et al. (2000).

3.3.1.3. The variance-covariance matrix of shocks

In this model, I maintain the assumption that all shocks (also known as innovations) are white noise, normally distributed and also uncorrelated with each other. Regarding the variance of the shock to the permanent component, I follow the approach of Turner et al. (1989) and Kim and Kim (1996), who allowed Markov-switching variance. In this sense, $\varepsilon_{p^r,t} \sim N(0, \sigma^2_{p^r})$ is allowed to have different variances during the crisis and normal periods:

$$\sigma^2_{p^r} = \sigma^2_{p^r,0}(1 - S_t) + \sigma^2_{p^r,1}(S_t) \tag{3.7}$$

where $\sigma^2_{p^r,1}$ and $\sigma^2_{p^r,0}$ are variances of the shock to the permanent component during the crisis and normal times. Given that asymmetric volatility suggests a concomitant occurrence of price fall and volatility jump, I use a single Markov-switching process for both inefficient plunges in the transitory component and the variance of the shock to the permanent component. Besides, $\varepsilon_{p^i,t} \sim N(0, \sigma^2_{p^i})$ is a typical symmetric shock to the transitory component, whose variance is assumed to be constant in the benchmark model because it is expected that the plunging coefficient in Eq. (3.4) accommodates the asymmetry in the price deviations. For robustness tests, however, I consider that this variance is also regime-dependent as follows:

$$\sigma^2_{p^i} = \sigma^2_{p^i,0}(1 - S_t) + \sigma^2_{p^i,1}(S_t) \tag{3.8}$$

where $\sigma^2_{p^i,1}$ and $\sigma^2_{p^i,0}$ are variances during the crisis and normal times. Lastly, given the assumptions I made for the three shocks in this model, the variance-covariance matrix of shocks is:

$$\begin{bmatrix} \varepsilon_{p^r,t} \\ \varepsilon_{p^i,t} \\ \varepsilon_{\mu,t} \end{bmatrix} \sim N\left(\mathbf{0}_{3\times1}, \begin{bmatrix} \sigma^2_{p^r} & 0 & 0 \\ 0 & \sigma^2_{p^i} & 0 \\ 0 & 0 & \sigma^2_{\mu} \end{bmatrix}\right) \tag{3.9}$$

3.4. Results and discussion

I estimated thirteen models to test for asymmetry by using a set of pairwise comparisons of estimated log likelihoods. As shown in the first and second rows of Tables 3.1 and 3.2 and explained in Table 3.C in Appendix 3.C, each model is denoted by an identifier and a descriptor. The descriptor consists of two parts. The first part expresses the specification of the model regarding the asymmetry in the transitory component and the asymmetry in the variance of shocks and the second part determines whether the drift term (long-run return) is modelled by a random walk or is assumed to be constant. As an illustration, the identifier of one of the benchmark models is (1.a) and its descriptor is denoted by "A (PT-VP)-RW" that means this model allows for the **A**symmetry by including both inefficient **P**lunges in the **T**ransitory component and a switching **V**ariance in the **P**ermanent component. In this model, the long-run return is specified in the form of a **R**andom **W**alk process. Besides, the descriptor "A (PT)-Con" shows that model (2.b) accommodates the **A**symmetry by considering only inefficient **P**lunges in the **T**ransitory component and the asymmetry in the variance is unaccounted for, whereas the descriptor "A (VP)-Con" suggests that model (3.b) accommodates the **A**symmetry by considering a switching **V**ariance in the **P**ermanent component but does not allow the asymmetry in the transitory component. These models also impose a **Con**stant long-run return.

Tables 3.1 and 3.2 report parameters and log likelihoods estimated by nine models for the S&P 500 and FTSE 250. The results of the asymmetric Fads model substantiate inefficient plunges since the plunging coefficients are $\pi_i = -7.19$ for the U.S. and $\pi_i = -10.66$ for the U.K. stock markets, with small standard errors of 1.08 and 2.14, respectively. The switching variance of the shock to the permanent component is $\sigma^2_{p^r,1} = 5.59^2$ for the U.S. and $\sigma^2_{p^r,1} = 8.30^2$ for the U.K. stock markets during crisis periods, significantly greater than their counterpart values of $\sigma^2_{p^r,0} = 2.46^2$ for the U.S. and $\sigma^2_{p^r,0} = 3.72^2$ for the U.K. during normal periods.

In the case of the S&P 500, the transition probabilities reported in column (1.a) of Table 3.1 is low ($p = 0.773$) for crisis periods and high ($q = 0.956$) for normal periods; thus, the expected duration is short (4.4 months) for crisis periods and long (23 months) for normal times.[18] For the FTSE 250, column (1.a) of Table 3.2 reports that transition probabilities are $p = 0.693$ and $q = 0.967$; hence, the expected duration is short (3.3 months) for crisis periods and long (31 months) for normal times. Furthermore, the sum of autoregressive coefficients of the cyclical component ($\varphi_1 + \varphi_2$) is 0.74 for the U.S. market and 0.75 for the U.K. market, suggesting a relatively transient plunging component.

[18] The expected duration of crisis and non-crisis (normal) states can be derived by formulas $\frac{1}{1-p}$ and $\frac{1}{1-q}$, respectively, where p is the probability of staying in the crisis and q is the probability of staying in the normal state.

Figures 3.1 and 3.2 plot the permanent and transitory components of these stock markets. The top-left panels demonstrate that the gap between market prices and efficient prices is negligible during normal times but is noticeable during downturns. The top-right panels, by plotting this gap, show that inefficient plunges are deep, almost always negative, and transient as the gap between actual and efficient prices tends to be filled quickly. In this sense, the depth of inefficient plunges reaches 15% and even 20% during crises, which typically coincide with NBER and ECRI recession dates for the S&P 500 and FTSE 250, respectively. A plunge lasts for 4.4 months in the U.S. stock market and 3.3 months in the U.K. stock market, on average, and thereafter the corresponding gap shrinks and finally disappears within around 4 additional months. The magnitude of inefficient plunges in the S&P 500 and FTSE 250 during 12.7% and 13.9% of their sample size, respectively, exceeds a threshold of 10%. On this basis, by taking a neutral position about market inefficiency related to positive bubbles, I follow a conservative approach and report that the U.S. and U.K. stock markets are inefficient at least 12% of the time. In addition, the bottom-right panels plot the probabilities of inefficient plunges accompanied by concomitant high-variance states that provide a measure of market inefficiency as a percentage at each moment.

Overall, results are in line with the AMH of Lo (2004) and in opposition to the EMH of Fama (1970). I highlight that deviations of U.S. and U.K. market prices from efficient prices, which measure the level of market inefficiency, are regime-dependent and asymmetric. This supports the ORH proposed by De Bondt and Thaler (1985), as well as the findings of the rolling window methods presented by Ito and Sugiyama (2009), Ito et al. (2016), Le Tran and Leirvik (2019), and Hill and Motegi (2019), who suggest that U.S. and U.K. stock markets are not efficient during crises. Furthermore, substantial inefficient plunges in the transitory component that are synchronous with jumps in variance in the permanent component during crisis periods suggest that excessive price drops and volatility jumps occur concomitantly, hinting at the asymmetric volatility suggested by Turner et al. (1989), Nelson (1991), Jones et al. (2004), and Avramov et al. (2006).

In the next subsection, to test for the asymmetry in the form of inefficient plunges in the transitory component and/or switching variance in the permanent component, I use pairwise comparisons of log likelihood values estimated for different alternative models.[19]

[19] In the presence of a Markov-switching process, testing hypotheses based on the likelihood ratio statistics is non-standard as the nuisance parameter is not identified under the null hypothesis, and the asymptotic distribution of the likelihood ratio test does not follow the standard χ^2 distribution. To tackle this issue, few papers have proposed theoretically questionable and computationally burdensome simulation-based or bootstrap-based methods to test for Markov-switching that are operable for simple models (see, e.g., Hansen, 1992; Garcia, 1998; Di Sanzo, 2009). In this study, I maintain the use of the non-standard likelihood ratio test because likelihood ratios derived for testing asymmetry are large and leave very little doubt that deviations are asymmetric. In addition, since I estimate thirteen different models for each stock market, using a simulation-based method for testing Markov-switching would be time-consuming.

3.4.1. Inefficient plunges in the transitory component

To test if the plunging coefficient is significant ($\pi_i \ngeq 0$), I compare the log likelihood values for two models (1.a) and (1.b), which account for the asymmetry by allowing both inefficient plunges in the transitory component and a switching variance in the permanent component, with the log likelihoods for nested models (3.a) and (3.b), which account for the asymmetry by allowing only a switching variance in the permanent component. For the S&P 500, by comparing log likelihoods of -2470.8 and -2469.7 for benchmark models in columns (1.a) and (1.b) of Table 3.1 with values of -2494.1 and -2491.7 for models (3.a) and (3.b), I report likelihood ratios of 46.6 and 44.0. For the FTSE 250, I compare the log likelihood values of -1261.5 and -1258.6 for columns (1.a) and (1.b) in Table 3.2 with values of -1274.0 and -1271.5 for nested models (3.a) and (3.b) and report likelihood ratios of 25.0 and 25.8. Since all likelihood ratios are substantially greater than the critical value of 10.8 for a conservative 0.1% significance level, estimation results highlight that deviation of market prices from efficient prices, which measures market inefficiency, is regime-dependent and asymmetric in both stock markets.

Moreover, comparing log likelihoods of -2483.6 and -2480.9 for models (2.a) and (2.b) with values of -2534.7 and -2532.2 for symmetric models (4.a) and (4.b) presented in Table 3.1 for the S&P 500; and likewise comparing log likelihoods of -1275.0 and -1272.2 for models (2.a) and (2.b) with values of -1310.9 and -1306.0 for symmetric models (4.a) and (4.b) shown in Table 3.2 for the FTSE 250 supports the presence of asymmetry in the form of inefficient plunges in the transitory component. The likelihood ratios are 127.8 and 125.0 for the U.S. and 71.8 and 67.6 for the U.K. stock markets, which are all extraordinarily greater than the 0.1% critical value of 10.8.

3.4.2. Switching variance in the permanent component

To test if the switching variance differs in each regime ($\sigma^2_{p^r,1} \neq \sigma^2_{p^r,0}$), I compared the log likelihood values of benchmark models (1.a) and (1.b) with those of their nested models (2.a) and (2.b), which account for the asymmetry by allowing only inefficient plunges in the transitory component. I accept the hypothesis of switching variance by deriving likelihood ratios of 25.6 and 22.4 for the S&P 500, and 27.0 and 27.2 for the FTSE 250, all greater than the 0.1% critical value of 10.8. Similarly, the likelihood ratios of 81.2 and 81.0 for the S&P 500 and 73.8 and 69 for the FTSE 250, derived by comparing log likelihoods for models (3.a) and (3.b) with those reported for symmetric models (4.a) and (4.b), support the presence of switching variance in the permanent component. If you compare the likelihood ratio values of the above pairwise comparisons in this subsection (e.g., comparing 25.6 with 81.2, or comparing 27.0 with 73.8), you notice that the likelihood ratios for the same hypothesis (testing for switching variance) are smaller for the pairwise comparison of models in which the asymmetry is accounted for by including inefficient plunges. This identifies inefficient plunges as a competing specification for switching variance to explain the asymmetry.

To differentiate two sources of asymmetry (inefficient plunges and switching variance), I compare models (2.a) and (2.b) with their competing models (3.a) and (3.b). Although these models are non-nested, comparing the log likelihood values of these models hints that including inefficient plunges outperforms switching variance in maximizing the log likelihood function by more than 10 units (equivalent to a likelihood ratio of 20) for the S&P 500. Performing the same comparison for the FTSE 250, I notice that these two competing models perform equivalently. This highlights the importance of including both inefficient plunges and a switching variance to characterize the stock market dynamics.

3.4.3. Time-varying long-run return

To shed light on the time-variation in long-run return, I compare the log likelihood values of -2470.8 for model (1.a) in Table 3.1 with the value of -2471.7 for the nested model named (1′.b), which is presented in Table 3.G.1 in Appendix 3.G. Since the corresponding likelihood ratio of 1.8 is less than the critical value of 2.71 for a 10% significance level, this test suggests that a random walk with a deterministic drift is enough to capture the dynamics of the efficient price in the U.S. stock market. I reach the same conclusion for the U.K. stock market.

However, for the S&P 500, the estimation of $\sigma_\mu = 0.050$ for the standard deviation of the shock to the long-run return, with a standard error of 0.026, along with the bottom-left panel of Figure 3.1, implies a considerable time-variation in the long-run return. While the annual average of inflation-adjusted long-run returns is 4.8%, the long-run return wanders away from its average in harmony with long-run periods of bull and bear markets in the U.S. Indeed, the long-run return gradually declined from about 10% in the 1960s to -5% in the 1970s and early-1980s, which corresponds to the episodes of high Fed fund rates and Volcker mandate. Then it returned to rates close to the average in the late-1980s and early-1990s. Afterward, the return soared to 15% in the late-1990s, which was related to the technology boom and dot.com bubble. And, with the exception of the low rates in the aftermath of the 2007–09 financial crisis, the long-run return has been between 4% and 10% from the early-2000s until now. The long-run return of the FTSE 250, on the other hand, does not exhibit a noticeable periodicity, which might be because of its smaller sample.

3.4.4. Robustness tests

The results derived based on the asymmetric Fads model are robust to the choice of the model setups. First, as explained in subsections 3.4.1 and 3.4.2, regardless of the specifications for long-run return, the plunging coefficient and switching variance are both significant for models (1.a) and (1.b).

Second, the benchmark model imposes a constant variance for the shock to the transitory component. Based on Eq. (3.8), I relax this assumption by including a switching variance for this shock in models (1.c) and (1.d). Comparing log likelihood values for models (1.a) and (1.c) confirms that this variance is not switching as the corresponding likelihood ratios are 0.2 for both stock markets. In addition, I performed additional pairwise comparisons of the log likelihoods for models (1.c) and (1.d), where the asymmetry is accounted for by allowing inefficient plunges in the transitory component and two switching variances of shocks to the permanent and transitory components, with their counterpart values for models (3.c) and (3.d), in which the asymmetry is accounted for by allowing two switching variances of shocks to the permanent and transitory components. The likelihood ratios of 47.0 and 43.8 for the S&P 500 and 17.6 and 14.6 for the FTSE 250 are all greater than the critical value of 10.8 for a 0.1% significance level, reaffirming the results for the benchmark models. Also, comparing log likelihoods of –2494.1 and -2491.7 for models (3.a) and (3.b) with values of -2494.2 and -2491.6 for models (3.c) and (3.d) shows that switching variance in the transitory component is not significant when switching variance is allowed in the permanent component.

Regarding the order of the autoregressive process in the transitory component, for robustness tests, I estimate models with an autoregressive process of order one, which bear almost the same results as those in models with order two.[20] Furthermore, when the log likelihood of model (1.a) with an autoregressive process of order two is compared to the log likelihood of its counterpart model with an autoregressive process of order one, the likelihood ratios of 15.0 for the S&P 500 and 11.6 for the FTSE 250, favour the model with an autoregressive process of order two.

The assumption of imposing a single Markov-switching process to explain both inefficient plunges in the transitory component and a switching variance in the permanent component may be criticized as being restrictive. This assumption, however, seems innocuous considering the evidence presented in Figure 3.F.1 in Appendix 3.F, which juxtapose the probabilities of inefficient plunges implied by model (2.a) and the probabilities of high-variance states implied by model (3.a) for the S&P 500 and FTSE 250 in the top and bottom panels. The probabilities of inefficient plunges and high-variance states are similar, suggesting that inefficient plunges and switches in variance occur concomitantly, although jumps in variance are more persistent than inefficient plunges.

Finally, the results remain unchanged for higher data frequencies. By applying all models to the daily and weekly S&P 500 and FTSE 250, I confirm the frequency-independence of the main finding in this study: inefficient plunges in the transitory component are accompanied by a switching volatility in the permanent component. These results are presented in Tables 3.G.3 and 3.G.4 in Appendix 3.G.

[20] To avoid redundancy, these results, along with results of applying models to indices with dividends reinvested, are not reported in this paper but are available upon request.

3.5. Concluding remarks

I define the new concept of inefficient plunges as negative deviations of market prices from efficient prices to measure the level of market inefficiency. To establish regime-dependence and asymmetry in market inefficiency, I proposed an asymmetric Fads model, in which both inefficient plunges in the transitory component and a concomitant switching variance in the permanent component are allowed for by employing a Markov-switching process.

By applying the benchmark model to the monthly S&P 500 and the FTSE 250, I report substantial plunging coefficients for the U.S. and the U.K. stock markets. The switching variance of the shock to the permanent component during crisis periods is considerably greater than its value during normal periods in both stock markets. The expected duration is relatively short (4.4 months for the U.S. and 3.3 months for the U.S.) for crisis periods and long (23 months for the U.S. and 31 months for the U.K.) for normal times.

Since the estimated inefficient plunges are deep, almost always negative, and transient, I conclude that market inefficiency is regime-dependent and asymmetric, meaning that although the U.S. and U.K. stock markets are adequately efficient during normal times, they are far below efficient prices during crises. These results support the AMH against the EMH and also confirm the possibility of negative bubbles. Since market inefficiency appears mainly, if not only, during crises, inefficient plunges are recognized as the main, if not the only, determinant of market inefficiency. Finally, the results are robust to changes in model specifications and frequencies. Applying the model to the daily and weekly data reaffirms the finding of asymmetry in the form of inefficient plunges in the transitory component and a concomitant switching variance in the permanent component.

The model presented in this study is competent to examine the asymmetry and inefficient plunges in other markets, such as futures, options, and currency markets. Besides, the finding of asymmetry in stock markets as underlying assets calls for further research to explore the asymmetry in call and put option pricing. Finally, developing a model to incorporate both positive bubbles and negative bubbles (inefficient plunges) is an interesting topic for further research.

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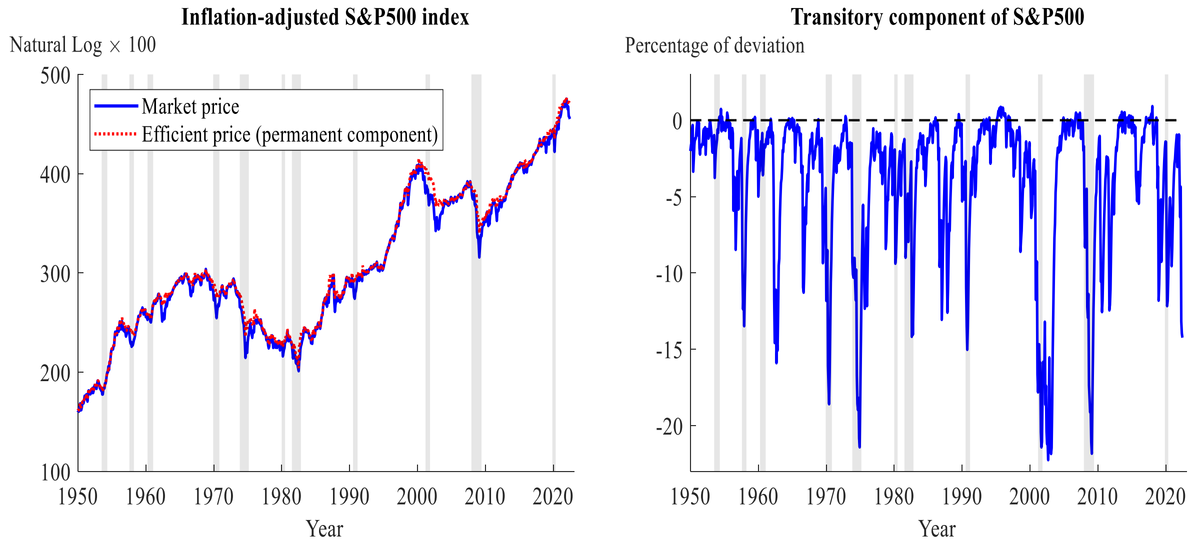
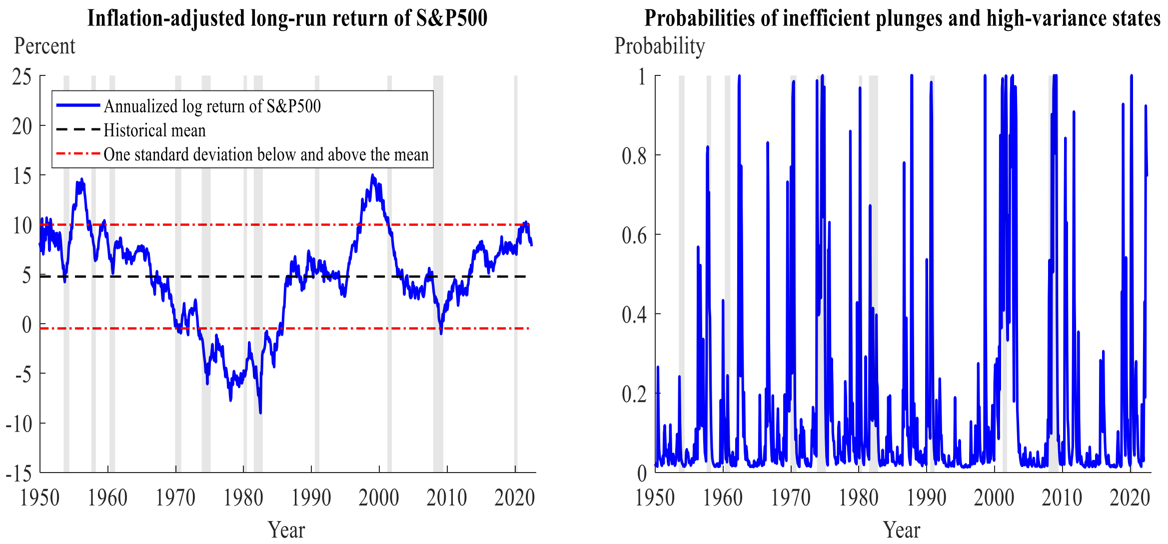
Figures



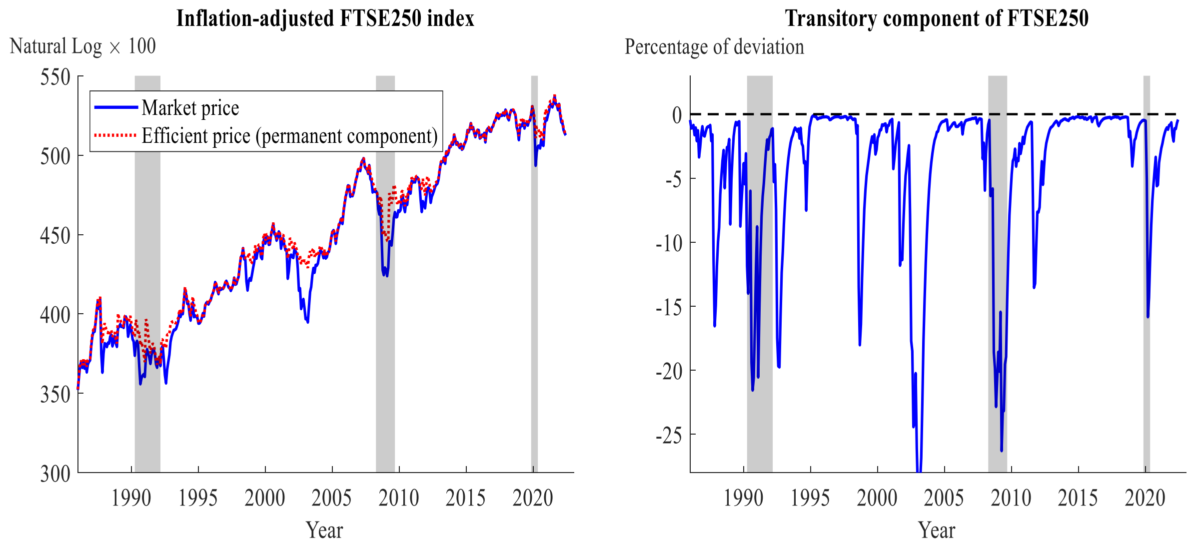
(a) Estimated efficient price (left) and inefficient plunges (right)



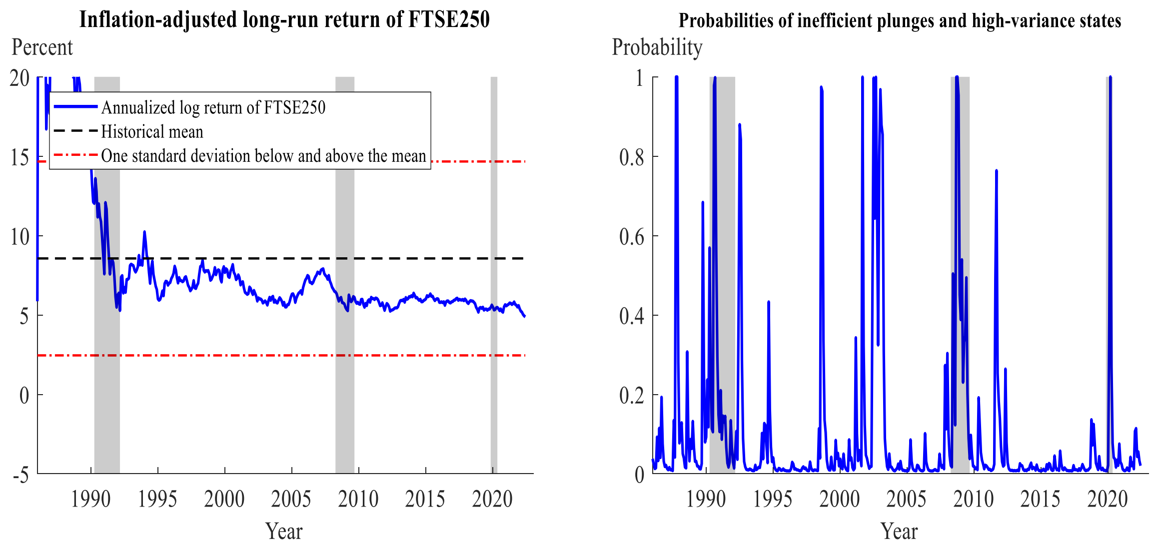
(b) Long-run return (left) and probabilities of asymmetric deviations (right)

Figure 3.1: The results of the asymmetric Fads model for the S&P 500

Notes:
(1) All panels plot the results of the benchmark model A (PT-VP)-RW.
(2) The top panels plot permanent and transitory components.
(3) The bottom-left panel plots trend growth of price (long-run return).
(4) The bottom-right panel plots the plunging probabilities.
(5) The shaded areas are the NBER recession dates. See Table 3.A.1 in Appendix 3.A for details.



(a) Estimated efficient price (left) and inefficient plunges (right)



(b) Long-run return (left) and probabilities of asymmetric deviations (right)

Figure 3.2: The results of the asymmetric Fads model for the FTSE 250

Notes:
(1) All panels plot the results of the benchmark model A (PT-VP)-RW.
(2) The top panels plot permanent and transitory components.
(3) The bottom-left panel plots trend growth of price (long-run return).
(4) The bottom-right panel plots the plunging probabilities.
(5) The shaded areas are the ECRI recession dates. See Table 3.A.2 in Appendix 3.A for details.

Tables

Table 3.1: Estimated parameters of different models for the S&P 500

Models	(1.a)	(1.b)	(1.c)	(2.a)	(2.b)	(3.a)	(3.b)	(4.a)	(4.b)
Parameters	A (PT-VP)-RW	A (PT-VP)-Con	A (PT-VPT)-RW	A (PT)-RW	A (PT)-Con	A (VP)-RW	A (VP)-Con	S-RW	S-Con
$\sigma_{p^r,0}$	2.46 (0.41)	2.76 (0.30)	2.48 (0.40)	3.45 (0.10)	3.45 (0.10)	2.91 (0.22)	2.90 (0.27)	4.21 (0.10)	4.21 (0.10)
$\sigma_{p^r,1}$	5.59 (0.48)	5.80 (0.45)	5.15 (1.03)	–	–	5.42 (0.34)	5.47 (0.37)	–	–
$\sigma_{p^i,0}$	1.77 (0.45)	1.37 (0.43)	1.76 (0.44)	0.10 (1.00 i)	0.21 (1.02 i)	0.24 (1.04)	0.35 (1.49)	0.09 (0.66)	0.17 (0.77)
$\sigma_{p^i,1}$	–	–	2.72 (1.64)	–	–	–	–	–	–
σ_μ	0.05 (0.03)	–	0.05 (0.03)	0.00 (0.02)	–	0.00 (0.02)	–	0.00 (0.01)	–
μ	T-V	0.41 (0.11)	T-V	T-V	0.37 (0.12)	T-V	0.54 (0.13)	T-V	0.34 (0.14)
φ_1	0.57 (0.09)	0.53 (0.10)	0.58 (0.09)	0.68 (0.06)	0.68 (0.06)	0.69 (0.68)	0.55 (0.76)	0.38 (3.08)	0.80 (0.95)
φ_2	0.16 (0.07)	0.19 (0.08)	0.16 (0.07)	0.09 (0.06)	0.09 (0.06)	0.14 (1.08)	0.30 (0.86)	0.30 (2.40)	-0.03 (1.73)
π_p	-7.19 (1.08)	-7.00 (1.06)	-7.04 (1.11)	-10.51 (0.72)	-10.53 (0.73)	–	–	–	–
p	0.77 (0.08)	0.78 (0.08)	0.76 (0.09)	0.50 (0.08)	0.50 (0.08)	0.95 (0.02)	0.94 (0.03)	–	–
q	0.96 (0.01)	0.95 (0.01)	0.95 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.01)	0.96 (0.02)	–	–
Log likelihood	-2470.8	-2469.7	-2470.7	-2483.6	-2480.9	-2494.1	-2491.7	-2534.7	-2532.2

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are in parenthesis. Those with the letter i for models (2.a) and (2.b) are imaginary numbers.
*** Numerical values for parameters denoted by 0.00 are respectively 0.0002 for model (2.a), 0.001 for model (3.a), and 0.002 for model (4.a).

Notes:

(1) The estimation period runs from 1948M1 to 2022M6. I estimate 13 models, each of which is denoted by an identifier and a descriptor to express the specification of the asymmetry by including inefficient plunges in the transitory component and/or a switching variance in the permanent component. The second part shows the specification of the long-run return. See the first paragraph of Section 3.4 and Table 3.C in Appendix 3.C for further explanation.

(2) To test for asymmetry in the transitory component, I compare the log likelihood values for models (1.a) and (1.b), in which the asymmetry is accounted for by including both inefficient plunges in the transitory component and a switching variance in the permanent component, with the values for models (3.a) and (3.b), where the asymmetry is accounted for by including only a switching variance in the permanent component. A pairwise comparison of the log likelihoods of -2470.8 and -2469.7 reported for models (1.a) and (1.b) with values of -2494.1 and -2491.7 for models (3.a) and (3.b), respectively, strongly favours the asymmetric Fads models over asymmetric variance models. The corresponding likelihood ratios of 46.6 and 44.0 are substantially greater than the critical value of 10.8 for a 0.1% significance level.

(3) I also assess the asymmetry in the transitory component by comparing two competing models. Models (2.a) and (2.b) allow for the asymmetry by including only inefficient plunges in the transitory component, while models (3.a) and (3.b) allow for the asymmetry by including only a switching variance in the permanent component. Although these models are non-nested, by comparing the log likelihood values of -2483.6 and -2480.9 reported for models (2.a) and (2.b) with values of -2494.1 and -2491.7 for models (3.a) and (3.b), I fairly support the asymmetric Fads models over asymmetric variance models. Just to provide a criterion, the likelihood ratios are 21.0 and 21.6, greater than the 0.1% critical value of 10.8.

(4) Additionally, comparing log likelihood values for each version of asymmetric models (1.a), (2.a), and (3.a) with those of the symmetric model (4.a) supports the presence of the asymmetry in the form of both inefficient plunges in the transitory component and a switching variance in the permanent component. The corresponding likelihood ratios of 127.8, 102.2, and 81.2 are all substantially greater than the critical value of 10.8 for a 0.1% significance level.

(5) In model (1.c), I relax the assumption of constant variance for the shock to the transitory component by including a switching variance for this shock based on Eq. (3.8). The result shows that the variance of the shock to the transitory component is not switching as the corresponding likelihood ratios is 0.2.

(6) Since the log likelihood values of each model are very close to those of its counterpart model with constant long-run return, I cannot reject the assumption of constant long-run return. However, the bottom-left panel of Figure 3.2 shows considerable variation in long-run return over time.

Table 3.2: Estimated parameters of different models for the FTSE 250

Models	(1.a)	(1.b)	(1.c)	(2.a)	(2.b)	(3.a)	(3.b)	(4.a)	(4.b)
Parameters	A (PT-VP)-RW	A (PT-VP)-Con	A (PT-VPT)-RW	A (PT)-RW	A (PT)-Con	A (VP)-RW	A (VP)-Con	S-RW	S-Con
$\sigma_{p^r,0}$	3.72 (0.21)	3.71 (0.19)	3.73 (0.17)	3.89 (0.66)	3.68 (0.62)	3.51 (0.51)	3.39 (0.46)	0.12 (0.85)	0.4 (0.34)
$\sigma_{p^r,1}$	8.30 (1.14)	8.23 (1.12)	8.30 (1.14)	–	–	11.36 (2.06)	11.43 (2.14)	–	–
$\sigma_{p^i,0}$	0.29 (1.43)	0.10 (2.67)	0.01 (1.55)	1.30 (1.69)	1.74 (1.14)	1.46 (1.04)	1.71 (0.75)	5.04 (0.17)	5.03 (0.17)
$\sigma_{p^i,1}$	–	–	0.13 (2.84)	–	–	–	–	–	–
σ_μ	0.00 (0.03)	–	0.00 (0.03)	0.00 (0.02)	–	0.01 (0.04)	–	0.00 (0.01)	–
μ	T-V	0.37 (0.20)	T-V	T-V	0.34 (0.17)	T-V	0.73 (0.19)	T-V	0.39 (0.03)
φ_1	0.61 (0.11)	0.61 (0.11)	0.61 (0.11)	0.69 (0.10)	0.70 (0.10)	1.04 (0.20)	1.04 (0.16)	1.10 (0.05)	1.10 (0.05)
φ_2	0.14 (0.10)	0.14 (0.10)	0.14 (0.10)	0.06 (0.09)	0.05 (0.09)	-0.24 (0.20)	-0.24 (0.16)	-0.16 (0.05)	-0.16 (0.05)
π_p	-10.66 (2.14)	-10.73 (2.12)	-10.64 (2.14)	-15.22 (1.44)	-15.01 (1.35)	–	–	–	–
p	0.70 (0.10)	0.70 (0.10)	0.70 (0.10)	0.65 (0.10)	0.66 (0.10)	0.60 (0.16)	0.59 (0.16)	–	–
q	0.97 (0.01)	0.97 (0.01)	0.97 (0.01)	0.98 (0.01)	0.98 (0.01)	0.95 (0.02)	0.95 (0.02)	–	–
Log likelihood	-1261.5	-1258.6	-1261.4	-1275.0	-1272.2	-1274.0	-1271.5	-1310.9	-1306.0

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are in parenthesis.
*** Numerical values for parameters denoted by 0.00 are respectively 0.0002 for model (1.a), 0.0001 for model (1.c), 0.002 for model (2.a), and 0.000001 for model (4.a).

Notes:

(1) The estimation period runs from 1986M1 to 2022M6. See the first paragraph of Section 3.4 and Table 3.C in Appendix 3.C for further explanation.

(2) To test for asymmetry in the transitory component, I compare the log likelihood values for models (1.a) and (1.b), in which the asymmetry is accounted for by including both inefficient plunges in the transitory component and a switching variance in the permanent component, with the values for models (3.a) and (3.b), where the asymmetry is accounted for by including only a switching variance in the permanent component. A pairwise comparison of the log likelihoods of -1261.5 and -1258.6 reported for models (1.a) and (1.b) with values of -1274.0 and -1271.5 for models (3.a) and (3.b), respectively, strongly favours the asymmetric Fads models over asymmetric variance models. The corresponding likelihood ratios of 25.0 and 25.8 are substantially greater than the critical value of 10.8 for a 0.1% significance level.

(3) I also assess the asymmetry in the transitory component by comparing two competing models. Models (2.a) and (2.b) allow for the asymmetry by including only inefficient plunges in the transitory component, while models (3.a) and (3.b) allow for the asymmetry by including only a switching variance in the permanent component. Although these models are non-nested, by comparing the log likelihood very similar values of -1275.0 and -1272.2 reported for models (2.a) and (2.b) with values of -1274.0 and -1271.5 for models (3.a) and (3.b), I highlight the equal importance of including both inefficient plunges and a switching variance.

(4) Additionally, comparing log likelihood values for each version of asymmetric models (1.a), (2.a), and (3.a) with those of the symmetric model (4.a) supports the presence of the asymmetry in the form of both inefficient plunges in the transitory component and a switching variance in the permanent component. The corresponding likelihood ratios of 98.8, 71.8, and 73.8 are all substantially greater than the critical value of 10.8 for a 0.1% significance level.

(5) In model (1.c), I relax the assumption of constant variance for the shock to the transitory component by including a switching variance for this shock based on Eq. (3.8). The result shows that the variance of the shock to the transitory component is not switching as the corresponding likelihood ratios is 0.2.

(6) Since the log likelihood values of each model are very close to those of its counterpart model with constant long-run return, I cannot reject the assumption of constant long-run return.

Supplementary Appendix to

Asymmetric Fads, Inefficient Plunges, and the Efficient Market Hypothesis

Mohammad Dehghani[†,*]

Appendix 3.A: Business cycle dates

Table 3.A.1: Dates of the U.S. Business Cycles (Peak and Trough)

N	ECRI*	NBER**	Description
1	1957M8-1958M4	1957M8-1958M4	--
2	1960M4-1961M2	1960M4-1961M2	--
3	1969M12-1970M11	1969M12-1970M11	--
4	1973M11-1975M3	1973M11-1975M3	First Oil Crisis
5	1980M1-1980M7	1980M1-1980M7	Second Oil Crisis
6	1981M7-1982M11	1981M7-1982M11	Early 1980s recession
7	1990M7-1991M3	1990M7-1991M3	Early 1990s recession
8	2001M3-2001M11	2001M3-2001M11	Early 2000s recession
9	2007M12-2009M6	2007M12-2009M6	Global crisis and recession
10	2020M2-2020M4	2020M2-2020M4	COVID-19 recession

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Table 3.A.2: Dates of the U.K. Business Cycles (Peak and Trough)

N	ECRI*	NIESR**	Description
1	-	1951M3-1952M8	--
2	-	1955M12-1958M11	--
3	-	1961M3-1963M1	--
4	1974M9-1975M8	1973M1-1975M3	First Oil Crisis
5	1979M6-1981M5	1979M2-1982M4	Second Oil Crisis
6	-	1984M1-1984M3	--
7	-	1988M4-1992M2	Early 1990s recession
8	1990M5-1992M3	-	Early 1990s recession
9	2008M5-2010M1	-	Global crisis and recession
10	2019M10-2020M4	-	COVID-19 recession

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[*] Corresponding author. For data and code, see the website: https://sites.google.com/view/mohammaddehghani.

Appendix 3.B: Shortcomings of existing empirical literature

3.B.1 (Shortcomings of empirical literature to examine the EMH):

The methods used by existing studies are subject to two caveats: First, they do not spot the level of market inefficiency at each moment. They provide only an approximation of the level of market inefficiency since they use methods such as rolling window, sub-window analysis, etc. Second, their estimation relies on subjective decisions on the length of the window, number of lags, normalization, etc. Hence, although these studies find some evidence that market inefficiency is more pronounced during financial crises, they are not well designed to capture such a regime-dependent phenomenon directly and precisely at each moment. Some of these studies are presented by Lim et al. (2008), Ito and Sugiyama (2009), Kim et al. (2011), Anagnostidis et al. (2016), Ito et al. (2016), Noda (2016), Hill and Motegi, 2019), Le Tran and Leirvik (2019), and Mattera and Di Sciorio (2022), among many others.

3.B.2 (Shortcomings of empirical literature to explain the asymmetry):

In the methodology section, I mentioned that each of the studies conducted by Turner et al. (1989) and Kim and Kim (1996), which allow for the asymmetry, has an important missing component. The model proposed by Turner et al. (1989) ignores the transitory component, and the model presented by Kim and Kim (1996) ignores Markov-switching for the mean of the transitory component. Let me explain the consequences of these two models' insufficient features.

First, Turner et al. (1989) examine a guess, which states that the return during high-variance states must be higher than during low-variance states because investors need to be compensated with a higher return for taking a higher level of risk during crises. They refer to this guess as the risk premium. Their estimation results, however, reject the risk premium conjecture and instead support the inverse statement: the return during high-variance states is lower than the return during low-variance states, which is later denominated as asymmetric volatility. The main reason for their incorrect guess is ignoring the transitory component, which leads to messing up the order of events in their modelling approach. Indeed, during crisis periods when the price falls, the return is clearly negative and the volatility jumps sharply, which implies asymmetric volatility. In these high-variance states, due to the high level of risk, investors agree to buy only at a lower price in the hopes of achieving a higher future return. Hence, when volatility is high, the future return (and not the current return) must be higher to compensate investors for taking a higher risk due to higher volatility.

Second, Kim and Kim (1996) allow for the presence of a transitory (also known as Fads) component to examine whether the transitory component is short-lived or not. They adopt a Markov-switching variance model with two independent Markov-switching processes, one of which accounts for the switching variance of the shock to the permanent component and the other accounts for the switching variance of the shock to the transitory components. Hence, they allow Markov-switching for the variance but not for the mean in the transitory component. Thus, this approach ignores the asymmetry in the most important information in any stock market: the direction of the price and the sign of the return. As a result, their model does not identify the sign of the return during high-variance and low-variance states, whereas according to asymmetric volatility, we expect a negative return during high-variance states and a positive return during low-variance states. Nevertheless, the estimated transitory component presented in Figure 3 by Kim and Kim (1996) suggests that the transitory component is asymmetric as it is almost always negative.

Appendix 3.C: Univariate state-space model with Markov-switching

I use the univariate model specified in Eq. (3.1) to Eq. (3.9) to estimate the permanent and transitory components that represents the efficient price and inefficient plunges respectively. I cast the model in a state-space form. In this set up, I consider a stochastic long-run return that evolves based on a random walk process. The observation equation, the transition equation, and variance covariance matrix of error terms are as follows:

$$[p_t] = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_t^r \\ p_t^i \\ p_{t-1}^i \\ \mu_t \end{bmatrix} + [0] \tag{3.C.1}$$

$$\begin{bmatrix} p_t^r \\ p_t^i \\ p_{t-1}^i \\ \mu_t \end{bmatrix} = \begin{bmatrix} 0 \\ \pi_i S_t \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \varphi_1 & \varphi_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{t-1}^r \\ p_{t-1}^i \\ p_{t-2}^i \\ \mu_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{p^r,t} \\ \varepsilon_{p^i,t} \\ 0 \\ \varepsilon_{\mu,t} \end{bmatrix} \tag{3.C.2}$$

$$\begin{bmatrix} \varepsilon_{p^r,t} \\ \varepsilon_{p^i,t} \\ 0 \\ \varepsilon_{\mu,t} \end{bmatrix} \sim N\left(\mathbf{0}_{4\times 1}, \begin{bmatrix} \sigma_{p^r}^2 & 0 & 0 & 0 \\ 0 & \sigma_{p^i}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\mu^2 \end{bmatrix}\right) \tag{3.C.3}$$

In the above model, I consider natural log price multiplied by 100 as the observed series (p_t). Because of the presence of Markov-switching, I use Kim's (1944) approximate maximum likelihood method to estimate the model. To test for asymmetry, I derive the restricted symmetric model by imposing $\pi_i = 0$ on the unrestricted asymmetric model to estimate the nested model by using Kalman's (1960) filter.

Table 3.C: Specification of 13 models for stock market index

Model name	Tables and Figures
Model (1.a): Asymmetric (PT-VP)-RW	Tables 3.1, 3.2, Figures 3.1, 3.2
Model (1.b): Asymmetric (PT-VP)-Con (3 states variables)	Tables 3.1, 3.2
Model (1'.b): Asymmetric (PT-VP)-Con (4 state variables)	Tables 3.G.1, 3.G.2
Model (1.c): Asymmetric (PT-VPT)-RW	Tables 3.1, 3.2
Model (1.d): Asymmetric (PT-VPT)-Con	Tables 3.G.1, 3.G.2
Model (2.a): Asymmetric (PT)-RW	Tables 3.1, 3.2
Model (2.b): Asymmetric (PT)-Con	Tables 3.1, 3.2
Model (3.a): Asymmetric (VP)-RW	Tables 3.1, 3.2
Model (3.b): Asymmetric (VP)-Con	Tables 3.1, 3.2
Model (3.c): Asymmetric (VPT)-RW	Tables 3.G.1, 3.G.2
Model (3.d): Asymmetric (VPT)-Con	Tables 3.G.1, 3.G.2
Model (4.a): Symmetric-RW	Tables 3.1, 3.2
Model (4.b): Symmetric-Con	Tables 3.1, 3.2

Notes:

(1) I estimate thirteen univariate models. I denote each model with an identifier and a descriptor. The descriptor consists of four parts. The first part expresses the specification of the model for the asymmetry in the transitory component as well as the asymmetry in the variance of shocks. The second part states whether the long-run return is modelled by a random walk or is assumed to be constant. For example, the identifier of the benchmark model is (1.a) and its descriptor is "A (PT-VP)-RW", which accounts for the **A**symmetry by including both **P**lunges in the **T**ransitory component and a switching **V**ariance in the **P**ermanent component. The long-run return is also specified as a **R**andom **W**alk in this model.

(2) We present the results of the bold models applied to the S&P 500 and FTSE 250 in Tables 3.1 and 3.2, and the rest are presented in Table 3.G.1 and 3.G.2 in Appendix 3.G.

(3) The benchmark model in this study is similar to the model presented by Kim and Nelson (1999a) and Dehghani et al. (2022), which was applied to the U.S. GDP in the economics literature.

(4) The benchmark model in this study is closest to the models presented by Turner et al. (1989) and Kim and Kim (1996), but with two important differences. First, while my model is competent to capture the asymmetric deviations from efficiency because it is specified in levels, other models are incapable of exploring the EMH as they are specified in differences. Second, Turner et al. (1989) ignore the Fads (transitory) component, and Kim and Kim (1996) ignore the switching mean component. These shortcomings are addressed in the benchmark model by including both inefficient plunges in the transitory component and switching variance in the permanent component.

(5) The conventional Fads model is similar to symmetric models (4.a) and (4.b).

Appendix 3.D: Approximate maximum likelihood and constraints

For asymmetric models in the presence of the Markov-switching process of Hamilton (1989), I use Kim's (1994) approximate maximum likelihood method to make the Kalman's (1960) filter operable. For more explanation, see chapters 4 and 5 of Kim and Nelson's (1999b) book and chapters 13 and 22 of Hamilton (1994). For symmetric models, I use the maximum likelihood method, performed by using the Kalman filter as explained in chapters 2 and 3 of Kim and Nelson (1999b) and chapter 5 of Hamilton (1994).

I need to impose a set of constraints on parameters, which are explained thoroughly in the second part of Appendix 3.D. I consider initial values for parameters as well as state variables. For the former, the initial values for parameters are presented in Tables 3.E.1 and 3.E.2 in Appendix 3.E. For the latter, I have taken the first observation for the permanent component, zero for the transitory component, and 4.8% for annual long-run return to determine the prior values for state variables. The prior variances of state variables are set to be 10. The results are robust to changes in prior values of state variables and their variances. For example, I used a wilder guess by setting the variances of state variables equal to 1000, which bears the same estimation for parameters.

To find the data and replicating code in Matlab, R, and Python for this paper, please see my website at this address: https://sites.google.com/view/mohammaddehghani. For details about the method and parameter constraints, see the comments in the main function as well as the likelihood function and the transformation function in the Matlab code.

Appendix 3.D (continued): Parameters constraints

I employ a numerical optimization procedure to maximize the approximate log likelihood function subject to a set of constraints. Hence, I impose constraints on some of the coefficients, probabilities, and standard deviations of shocks. To this end, I account for constraints by using a transformation function, $T(\omega)$, which transforms a vector of unconstrained parameters $\omega = [\omega_1, \dots, \omega_{10}]'$ to a vector of constrained parameters $\Omega = [\Omega_1, \dots, \Omega_{10}]'$ presented below:

$$\Omega = [\sigma_{p^r,0}, \sigma_{p^r,1}, \sigma_{p^i}, \sigma_\mu, \mu, \varphi_1, \varphi_2, \pi_i, p, q]' \tag{3.D.1}$$

where $\Omega = T(\omega)$ is the vector containing parameters and $T(\omega)$ is a vector function, whose elements are transformation functions $T_i(\omega)$ for $i = 1, \dots, 10$. Since performing unconstrained optimization with respect to ω is equivalent to performing constrained optimization with respect to Ω, I adopt an unconstrained optimization with respect to the vector ω, where the objective (approximate log likelihood) function is considered as a function of the transformation function. I define each element of the transformation function as follows:

First, for coefficients and standard deviations of shocks that should be positive, we use an exponential transformation suggested by Kim and Nelson (1999b). For example,

$$\sigma_{p^r,0} = exp(\omega_1) \tag{3.D.2}$$

In Eq. (3.D.2), $\sigma_{p^r,0}$ is the standard deviation (square root of variance) of the shock to the permanent component during normal times, which must be positive. Similarly, for other standard deviations, including $\sigma_{p^r,1}$, σ_{p^i}, and σ_μ that are positive and for the coefficient π_i that is expected to be negative, I apply an exponential transformation. For example, $\pi_i = -exp(\omega_{12})$ ensures a negative plunging coefficient.

Second, to have transition probabilities in the $[0\ 1]$ interval, we exert the following transformations:

$$p = \frac{exp(\omega_{13})}{1 + exp(\omega_{13})} \quad \text{and} \quad q = \frac{exp(\omega_{14})}{1 + exp(\omega_{14})} \tag{3.D.3}$$

Third, for coefficients of the autoregressive process of order two, we need to set the values of φ_1 and φ_2 within the stationary region that means the roots of the lag polynomial $(1 - \varphi_1 L - \varphi_2 L^2 = 0)$ must lie outside the unit circle. In this sense, I apply the transformations proposed by Morley et al. (2003):

$$\varphi_1 = 2\kappa_1 \quad \text{and} \quad \varphi_2 = -(\kappa_1^2 + \kappa_2) \tag{3.D.4.a}$$

where κ_1 and κ_2 are determined as follows:

$$\kappa_1 = \frac{\omega_{10}}{1+|\omega_{10}|} \quad \text{and} \quad \kappa_2 = \frac{(1-|\kappa_1|)\times\omega_{11}}{1+|\omega_{11}|} + |\kappa_1| - \kappa_1^2 \tag{3.D.5.a}$$

For these two coefficients of the autoregressive process, one can take two alternative transformations proposed by Kim and Nelson (1999b):

$$\varphi_1 = \kappa_1 + \kappa_2 \quad \text{and} \quad \varphi_2 = \kappa_1 \times \kappa_2 \tag{3.D.4.b}$$

where κ_1 and κ_2 are determined below:

$$\kappa_1 = \frac{\omega_{10}}{1+|\omega_{10}|} \quad \text{and} \quad \kappa_2 = \frac{\omega_{11}}{1+|\omega_{11}|} \tag{3.D.5.b}$$

However, these two transformations impose a further restriction that the roots of the autoregressive polynomial are real numbers.

It is worth noting that inefficient plunges (negative bubbles) do exist no matter whether the constraint on the plunging coefficient is imposed or not. Indeed, the phenomenon of inefficient plunges is such a pronounced feature of the U.S. and U.K. stock markets that excluding its corresponding constraints (π_i<0) does not change the estimated parameters.

Appendix 3.E: Tables of initial values for parameters

Table 3.E.1: Initial values (after-transformation) for the model parameters used for the S&P 500

Models	(1.a)	(1.b)	(1.c)	(2.a)	(2.b)	(3.a)	(3.b)	(4.a)	(4.b)
Parameters	A (PT-VP)-RW	A (PT-VP)-Con	A (PT-VPT)-RW	A (PT)-RW	A (PT)-Con	A (VP)-RW	A (VP)-Con	S-RW	S-Con
$\sigma_{p^r,0}$	0.50	0.50	0.50	0.50	0.50	0.75	0.50	0.50	0.50
$\sigma_{p^r,1}$	0.50	0.50	0.50	–	–	2.00	2.00	–	–
$\sigma_{p^i,0}$	0.50	0.50	0.50	0.50	0.50	0.75	0.50	0.50	0.50
$\sigma_{p^i,1}$	–	–	0.50	–	–	–	–	–	–
σ_{μ}	0.50	–	0.50	0.50	–	0.50	–	0.50	–
μ	T-V	0.75	T-V	T-V	0.75	T-V	0.75	T-V	0.75
φ_1	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
φ_2	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4
π_p	-1.8	-2.1	-2.1	-2.1	-2.1	–	–	–	–
p	0.70	0.60	0.60	0.60	0.60	0.60	0.60	–	–
q	0.90	0.90	0.90	0.90	0.90	0.90	0.90	–	–

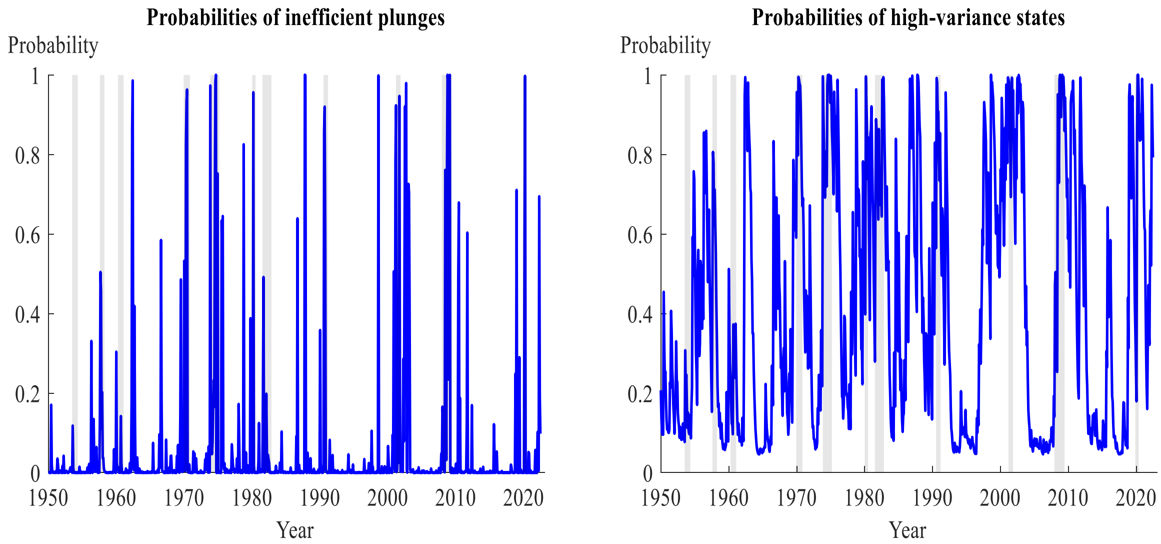
Notes:
(1) The results of all models are robust to the choice of the initial values for each parameter.
(2) I use the same initial values for all models. However, for models (3.a) and (3.b), I select higher initial values for variance during crisis (2.00^2).

Table 3.E.2: Initial values (after-transformation) for the model parameters used for the FTSE 250

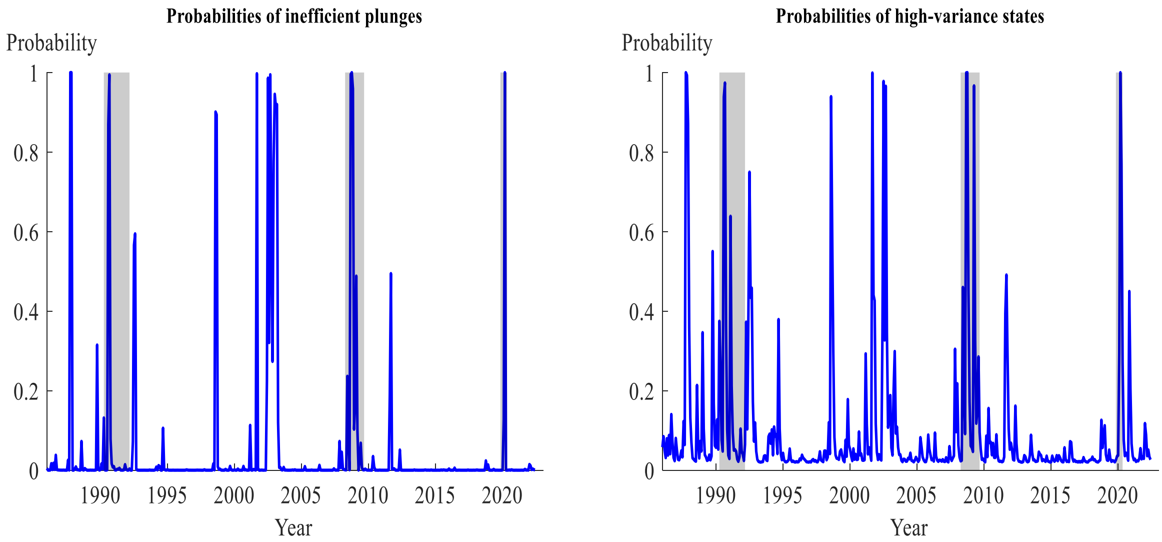
Models	(1.a)	(1.b)	(1.c)	(2.a)	(2.b)	(3.a)	(3.b)	(4.a)	(4.b)
Parameters	A (PT-VP)-RW	A (PT-VP)-Con	A (PT-VPT)-RW	A (PT)-RW	A (PT)-Con	A (VP)-RW	A (VP)-Con	S-RW	S-Con
$\sigma_{p^r,0}$	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.55	0.75
$\sigma_{p^r,1}$	0.75	0.75	0.75	–	–	1.50	1.50	–	–
$\sigma_{p^i,0}$	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
$\sigma_{p^i,1}$	–	–	0.75	–	–	–	–	–	–
σ_{μ}	0.75	–	0.75	0.75	–	0.75	–	0.75	–
μ	T-V	0.75	T-V	T-V	0.75	T-V	0.75	T-V	0.75
φ_1	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
φ_2	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4
π_p	-2.1	-2.1	-2.1	-2.1	-2.1	–	–	–	–
p	0.60	0.60	0.60	0.60	0.60	0.60	0.60	–	–
q	0.90	0.90	0.90	0.90	0.90	0.90	0.90	–	–

Notes:
(1) The results of all models are robust to the choice of the initial values for each parameter.
(2) I use the same initial values for all models that are almost the same as the initial values for the S&P 500. However, for the FTSE 250, I selected higher initial values for variances of shocks (0.75^2) for the FTSE 250 compared to the S&P 500 (0.50^2). For models (3.a) and (3.b), I select higher initial values for variance during crisis (1.50^2).
(3) For transition probability p, I use 0.6, rather than 0.7, because the estimated value for some models is 0.7. I also estimated all models with the initial values of 0.7 for probability p, while all the estimated parameters are identical.
(4) To avoid estimating unknown standard errors in model (4.a), I set different initial value for the variance of the shock to the permanent component (0.55^2).

Appendix 3.F: Additional Figures



(a) Probabilities estimated separately for model (2.a) in the left and for model (3.a) in the right (S&P 500)



(b) Probabilities estimated separately for model (2.a) in the left and for model (3.a) in the right (FTSE 250)

Figure 3.F.1: Synchrony of probabilities for asymmetric deviations and asymmetric variance

Notes:

(1) The top-left and bottom-left panels plot probabilities of asymmetric deviation in model (2.a), which allows for only inefficient plunges in the transitory component.

(2) The top-right and bottom-right panels plot probabilities of asymmetric variance in model (3.a), which allows for only switching variance in the permanent component.

(3) The shaded areas are the NBER and ECRI recession dates in the top and bottom panels, respectively. See Tables 3.A.1 and 3.A.2 in Appendix 3.A for details.

Appendix 3.G: Additional Tables

Table 3.G.1 (Continue of Table 3.1): Estimated parameters of different models for the S&P 500

Models	(1'.b)	(1.c)	(1.d)	(3.c)	(3.d)
Parameters	A (PT-VPT)-RW	A (PT-VPT)-RW	A (PT-VPT)-Con	A (VPT)-RW	A (VPT)-Con
$\sigma_{p^r,0}$	2.75 (0.30)	2.48 (0.40)	2.76 (0.30)	2.77 (0.33)	2.82 (0.33)
$\sigma_{p^r,1}$	5.80 (0.45)	5.15 (1.03)	5.78 (0.92)	5.38 (0.36)	5.42 (0.35)
$\sigma_{p^i,0}$	1.39 (0.43)	1.75 (0.44)	1.37 (0.45)	0.83 (0.71)	0.67 (0.93)
$\sigma_{p^i,1}$	–	2.72 (1.63)	1.43 (2.95)	0.60 (0.98)	0.36 (0.80)
σ_μ	–	0.05 (0.03)	–	0.00 (0.02)	–
μ	T-V	T-V	0.41 (0.11)	T-V	0.54 (0.13)
φ_1	0.53 (0.09)	0.58 (0.09)	0.53 (0.10)	0.60 (0.22)	0.49 (0.39)
φ_2	0.19 (0.08)	0.16 (0.06)	0.19 (0.08)	0.10 (0.26)	0.22 (0.30)
π_p	-6.98 (1.07)	-7.03 (1.11)	-7.00 (1.16)	–	–
p	0.78 (0.07)	0.76 (0.08)	0.78 (0.08)	0.95 (0.02)	0.95 (0.02)
q	0.96 (0.01)	0.95 (0.01)	0.95 (0.01)	0.96 (0.01)	0.96 (0.01)
Log likelihood	-2471.7	-2470.7	-2469.7	-2494.2	-2491.6

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are reported in parenthesis.
*** Standard errors with the letter i for models (2.a) and (2.b) are imaginary numbers.

Notes:

(1) The estimation period runs from 1948M1 to 2022M6. See Table 3.1 for the main results and explanations.

(2) The term "VPT" means that the switching variance is allowed for two variances; one for the shock to the permanent component and another one for the shock to the transitory component. Hence, model (1.c), with descriptor A (PT-VPT)-RW, accounts for the asymmetry by including inefficient plunges in the transitory component, one switching variance in the permanent component, and another switching variance in the transitory component.

(3) Model (1'.b) is another version of model (1.b) with similar estimation of parameters. Since in the former, we treat the drift term (constant long-run return) as a state variable and in the latter, the drift term is estimated as a parameter, model (1'.b) is fully nested in model (1.a), but model (1.b) is not. Comparing log likelihood of -2470.8 and -2471.7 yields a likelihood ratio of 1.8, less than the critical value of 2.71 for a 10% significance level. This suggests that a random walk with a deterministic drift is sufficient to capture the dynamics of the efficient price.

(4) To test for asymmetry in the form of inefficient plunges, I compare the log likelihood values for models (1.c) and (1.d), in which the asymmetry is accounted for by including both inefficient plunges in the transitory component and two switching variances for shocks to the permanent and transitory components, with the values for models (3.c) and (3.d), where the asymmetry is accommodated only by including two switching variances for shocks to the permanent and transitory components. A pairwise comparison of the log likelihoods of -2470.7 and -2469.7 reported for models (1.c) and (1.d) with values of -2494.2 and -2491.6 for models (3.c) and (3.d), respectively, strongly favours the asymmetric Fads models over asymmetric variance models. The corresponding likelihood ratios of 47.0 and 43.8 are substantially greater than the critical value of 10.8 for a 0.1% significance level. In addition, comparing the log likelihood values of -2470.8 and -2469.7 for models (1.a) and (1.b) in Table 3.1, with the values of -2470.7 and -2469.7 for models (1.c) and (1.d) bears likelihood ratios of 0.2 and 0.0. Further, the likelihood ratios of 0.2 and 0.2 are derived by comparing models (3.a) and (3.b) with models (3.c) and (3.d), confirming that the variance of the shock to the transitory component is not switching.

Table 3.G.2 (Continue of Table 3.2): Estimated parameters of different models for the FTSE 250

Models	(1′.b)	(1.c)	(1.d)	(3.c)	(3.d)
Parameters	A (PT-VPT)-RW	A (PT-VPT)-RW	A (PT-VPT)-Con	A (VPT)-RW	A (VPT)-Con
$\sigma_{p^r,0}$	3.72 (0.19)	3.73 (0.17)	3.70 (0.36)	3.68 (0.13)	0.17 (0.59)
$\sigma_{p^r,1}$	8.29 (1.13)	8.30 (1.14)	8.23 (1.13)	0.17 (2.98)	0.00 (0.03)
$\sigma_{p^i,0}$	0.19 (1.50)	0.01 (1.55)	0.40 (2.55)	0.28 (2.00 i)	3.79 (0.22)
$\sigma_{p^i,1}$	–	0.13 (2.84)	0.27 (3.03)	9.74 (1.47)	10.72 (1.92)
σ_{μ}	–	0.00 (0.03)	–	0.00 (0.02 i)	–
μ	–	T-V	0.37 (0.20)	T-V	0.44 (0.02)
φ_1	0.61 (0.11)	0.61 (0.11)	0.62 (0.12)	1.18 (0.10)	1.06 (0.05)
φ_2	0.14 (0.10)	0.14 (0.10)	0.14 (0.11)	-0.24 (0.09)	-0.10 (0.05)
π_p	-10.64 (2.14)	-10.64 (2.14)	-10.74 (2.12)	–	–
p	0.70 (0.10)	0.70 (0.10)	0.70 (0.10)	0.73 (0.14)	0.66 (0.16)
q	0.97 (0.01)	0.97 (0.01)	0.97 (0.01)	0.95 (0.02)	0.96 (0.02)
Log likelihood	-1260.5	-1261.4	-1258.6	-1270.2	-1265.9

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are reported in parenthesis. Those with the letter i for models (3.c) are imaginary numbers.
*** Numerical values for parameters denoted by 0.00 are respectively 0.0001 for model (1.c), 0.000001for model (3.c), and 0.000001 for model (3.d).

Notes:

(1) The estimation period runs from 1948M1 to 2022M6. See Table 3.2 for the main results and explanations.

(2) The term “VPT” means that the switching variance is allowed for two variances; one for the shock to the permanent component and another one for the shock to the transitory component. Hence, model (1.c), with descriptor A (PT-VPT)-RW, accounts for the asymmetry by including inefficient plunges in the transitory component, one switching variance in the permanent component, and another switching variance in the transitory component.

(3) Model (1′.b) is another version of model (1.b) with similar estimation of parameters. Since in the former, we treat the drift term (constant long-run return) as a state variable and in the latter, the drift is estimated as a parameter, model (1′.b) is fully nested in model (1.a), but model (1.b) is not. Comparing log likelihood of -1261.5 and -1260.5 yields a likelihood ratio of 2.0, less than the critical value of 2.71 for a 10% significance level. This suggests that a random walk with a deterministic drift is sufficient to capture the dynamics of the efficient price.

(4) To test for asymmetry in the form of inefficient plunges, I compare the log likelihood values for models (1.c) and (1.d), in which the asymmetry is accounted for by including both inefficient plunges in the transitory component and two switching variances for shocks to the permanent and transitory components, with the values for models (3.c) and (3.d), where the asymmetry is accommodated only by including two switching variances for shocks to the permanent and transitory components. A pairwise comparison of the log likelihoods of -1261.4 and -1258.6 reported for models (1.c) and (1.d) with values of -1270.2 and -1265.9 for models (3.c) and (3.d), respectively, strongly favours the asymmetric Fads models over asymmetric variance models. The corresponding likelihood ratios of 17.6 and 14.6 are greater than the critical value of 10.8 for a 0.1% significance level. In addition, comparing the log likelihood values of -1261.5 and -1258.6 for models (1.a) and (1.b) in Table 3.2, with the values of -1261.4 and -1258.6 for models (1.c) and (1.d) bears likelihood ratios of 0.2 and 0.0, confirming that the variance of the shock to the transitory component is not switching. However, the likelihood ratios of 7.6 and 11.2, derived by comparing models (3.a) and (3.b) with models (3.c) and (3.d), suggest that when inefficient plunges are not accounted for, the variance of the shock to the transitory component is switching.

Table 3.G.3: Estimated parameters of different models for the weekly and daily S&P 500

Models	(1.a) weekly	(1.b) weekly	(1.a) daily	(1.b) daily
Parameters	A (PT-VP)-RW	A (PT-VP)-Con	A (PT-VP)-RW	A (PT-VP)-Con
$\sigma_{p^r,0}$	1.38 (0.03)	1.38 (0.03)	0.50 (0.02)	0.69 (0.00)
$\sigma_{p^r,1}$	3.39 (0.10)	3.40 (0.10)	1.65 (0.03)	1.98 (0.04)
$\sigma_{p^i,0}$	0.19 (0.10)	0.18 (0.03)	0.34 (0.03)	0.00 (0.08)
$\sigma_{p^i,1}$	–	–	–	–
σ_μ	0.00 (0.00)	–	0.00 (0.00)	–
μ	T-V	0.18 (0.03)	T-V	0.06 (0.01)
φ_1	0.19 (0.08)	0.19 (0.08)	1.14 (0.03)	0.13 (0.05)
φ_2	0.49 (0.09)	0.48 (0.09)	-0.24 (0.03)	0.41 (0.07)
π_p	-2.54 (0.26)	-2.58 (0.26)	-0.37 (0.04)	-1.52 (0.13)
p	0.90 (0.01)	0.90 (0.01)	0.95 (0.01)	0.95 (0.01)
q	0.97 (0.01)	0.97 (0.01)	0.98 (0.01)	0.98 (0.01)
Log likelihood	-7900.0	-7895.2	-22897.0	-22892.3

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are in parenthesis. Those with the letter i for models (2.a) and (2.b) are imaginary numbers.
*** Numerical values for parameters denoted by 0.00 are respectively 0.0003 for model (1.a) weekly, and 0.0007 for model (1.a) daily.

Notes:

(1) The estimation period runs from 1948W1 to 2022W22 and from 1948D1 to 2022D104 for daily data.

(2) For weekly data, the log likelihood values of models (3.a) and (3.b) are -7943.3 and -7940.6. Hence, the likelihood ratio for testing inefficient plunges is 86.6 for model (1.a) and 90.8 for model (1.b).

(3) For daily data, the log likelihood values of models (3.a) and (3.b) are -22953.0 and -22948.6. Hence, the likelihood ratio for testing inefficient plunges is 112.0 for model (1.a) and 112.6 for model (1.b).

Table 3.G.4: Estimated parameters of different models for the weekly and daily FTSE 250

Models	(1.a) weekly	(1.b) weekly	(1.a) daily	(1.b) daily
Parameters	A (PT-VP)-RW	A (PT-VP)-Con	A (PT-VP)-RW	A (PT-VP)-Con
$\sigma_{p^r,0}$	1.69 (0.04)	1.69 (0.04)	0.36 (0.09)	0.45 (0.02)
$\sigma_{p^r,1}$	4.87 (0.34)	4.86 (0.34)	1.60 (0.04)	1.63 (0.03)
$\sigma_{p^i,0}$	0.01 (0.10)	0.02 (0.10)	0.42 (0.08)	0.31 (0.04)
$\sigma_{p^i,1}$	–	–	–	–
σ_{μ}	0.00 (0.00)	–	0.00 (0.00)	–
μ	T-V	0.19 (0.04)	T-V	0.04 (0.01)
φ_1	0.11 (0.11)	0.11 (0.11)	1.26 (0.09)	1.42 (0.07)
φ_2	0.39 (0.12)	0.39 (0.12)	-0.27 (0.09)	-0.43 (0.07)
π_p	-3.64 (0.56)	-3.63 (0.56)	-0.21 (0.04)	-0.13 (0.03)
p	0.86 (0.03)	0.86 (0.04)	0.94 (0.01)	0.94 (0.01)
q	0.98 (0.01)	0.98 (0.01)	0.98 (0.01)	0.98 (0.01)
Log likelihood	-4043.6	-4038.3	-10758.0	-10763.0

* T-V means that the model considers a time-varying state variable for the corresponding parameter.
** Standard errors of the estimated parameters are in parenthesis. Those with the letter i for models (2.a) and (2.b) are imaginary numbers.
*** Numerical values for parameters denoted by 0.00 are respectively 0.00005 for model (1.a) weekly, 0.002 for model (1.a) daily.

Notes:

(1) The estimation period runs from 1986W1 to 2022W22 for weekly data and from 1986D1 to 2022D104 for daily data.

(2) For weekly data, the log likelihood values of models (3.a) and (3.b) are -4052.2 and -4057.3. Hence, the likelihood ratio for testing inefficient plunges is 17.2 for model (1.a) and 38 for model (1.b).

(3) For daily data, the log likelihood values of models (3.a) and (3.b) are -10775.0 and -10771.0. Hence, the likelihood ratio for testing inefficient plunges is 34.0 for model (1.a) and 16 for model (1.b).

Chapter 4: Thesis conclusions

This thesis analyzes the dynamics of output, the unemployment rate, total factor productivity, capital input, and stock market indices in the U.S. and U.K. economies during and in the aftermath of many episodes of financial crises and economic recessions, starting in 1950 and ending with the COVID-19 recession. This research unravels three phenomena: (1) the U.S. slow recovery and the U.K. productivity puzzle by identifying a structural break in the parameters of Okun's law and dynamic factor model (DFM); (2) asymmetric co-fluctuations of U.S. output and the unemployment rate by integrating Friedman's plucking model and Okun's law; and (3) regime-dependence and asymmetry in market inefficiency through defining and estimating the concept of inefficient plunges. The main findings of the three studies, each related to one of the above phenomena, are explained below.

In the first study, we investigate why output in the U.S. and the U.K. has recovered slowly following the 2007−09 global financial crisis despite a comparably rapid recovery in the unemployment rate. By attributing the above mismatch to the identified structural break in the parameters of the empirical relationship between output and unemployment, known as Okun's law, we substantiate a change in regime in the aftermath of the 2007−09 financial crisis, revealed in the form of a break in the Okun's intercept, Okun's coefficient, and cyclical factor loadings. To capture the slow recovery, we measure the shortfall by comparing the actual recovery with two counterfactuals, each derived by one of the two methods. In the first method, the counterfactual is the post-crisis fitted output estimated by two approaches: Okun's law and a DFM. This method documents a significant shortfall of 1.32 and 0.83 percentage points per year in the U.S. and the U.K. The second method, which compares the trend and cyclical components of the recovery following the Great Recession with their counterparts in the three previous recoveries, yields a comparable annual shortfall of 1.23 and 1.07 percentage points per year in the U.S. and the U.K., which leads to a cumulative shortfall of more than 10 percentage points in output per capita in each of these two countries.

By using a trend-cycle decomposition, we report three distinct driving forces of the slow recovery. The first driver is a declining trend growth that started in the 1960s and corresponds to the finding of a gradual slowdown in potential output by Antolin-Diaz et al. (2017) and Fernald et al. (2017). The second reason is an unprecedented trend deceleration, which refers to the unusual slowdown in U.S. potential output that was initiated during the 2007−09 financial crisis. Identifying this driver accords with Patterson et al. (2016), Van Ark and Jäger (2017), and Oulton (2019). The third driving force is an unusually sluggish recovery of the U.S. output gap that emanates from the constrained demand and hysteresis effects suggested by Fatás and Mihov (2013) and Cerra et al. (2022).

In the second part of the first study, we develop a two-country DFM to measure the magnitude of shortfall spillovers from the U.S. to the U.K. by answering a simple question: what would the normal recovery of the U.K. output have been if there was a normal recovery in the U.S.? We report that the magnitude of the shortfall spillovers from the U.S. to the U.K. for output per capita is 0.62 percentage points per year. This result underscores that a major part of the U.K. productivity puzzle is inevitable since the U.K., as a small country, is a receiver of spillovers of real activity shortfall from the U.S. and thus is vulnerable to the long-term productivity slowdown in the U.S. This informs policymakers about the need to shift the U.K. economy away from a mostly bilateral economic relationship toward a more multilateral economic relationship.

The second study integrates two empirical phenomena into a bivariate unobserved components (UC) model with Markov-switching. First, Milton Friedman (1964, 1993) proposed a plucking model in which U.S. output tends to pluck down sharply below a ceiling (also known as potential output) during recessions and gradually returns toward the ceiling during recoveries, a phenomenon we call asymmetric fluctuations. Second, Arthur Okun (1962) established the empirical relationship between the U.S. output gap and the unemployment rate gap, which is known as Okun's law, a phenomenon we call co-fluctuation. In this context, the second study integrates Friedman's plucking model and Okun's law to capture the asymmetric co-fluctuation of output and unemployment, which means that asymmetry must be a common feature of both U.S. output and the unemployment rate.

Estimating significant plucking and Okun's law coefficients ($\pi_u = 0.70$ and $\beta = -1.45$) with small standard error establishes the asymmetric co-fluctuations: output and the unemployment rate in the U.S. are synchronously and proportionally characterized by the plucking property. The likelihood ratio to test for asymmetry is 91.2, which is extremely greater than the critical value of 10.8 for a conservative 0.1% significance level. Hence, consistent with the results of Kim and Nelson (1999) and Eo and Morley (2022), our empirical findings favour Friedman's plucking model over symmetric alternatives. The estimated expected duration is about 3 quarters for recessions and 28 quarters for recoveries, supporting the idea that recessions are deep, steep, and transitory and will be followed by commensurate, gradual, and permanent recoveries. Further, consistent with the findings of Ball et al. (2017) and Michail (2019), the gap version of Okun's law is sufficiently stable. Accordingly, given that Ferraro (2018), Dupraz et al. (2019), and Ferraro and Fiori (2022) identify the U.S. labour market as the source of the plucking property, we suggest the transmission of the plucking property from the unemployment rate to output.

Moreover, consistent with the suggestion of Antolin-Diaz et al. (2017), we corroborate the presence of a stochastic trend growth in U.S. output. In this regard, in line with Fernald et al. (2017) and Grant and Chan (2017), we document a decline in trend growth, which began in the 1960s. Additionally, we observe that this decline has been exacerbated by an unprecedented deceleration in U.S. potential output. This finding reaffirms the finding of a structural break during the 2007−09 financial crisis reported by the first study of this study, and is also in line with the finding of Eo and Morley (2022). Also, we state that our asymmetric bivariate model, through accounting for both the asymmetry and co-fluctuations, yields robust results with an insignificant correlation, which we refer to as correlation irrelevance. Lastly, our model provides a new measure for the natural rate of unemployment. We call it the Zero Output Gap Rate of Unemployment (ZOGRU) as it estimates the unemployment rate at which the output gap is zero.

In the third study, I define the concept of inefficient plunges as negative deviations of market prices from efficient prices, which measure the level of market inefficiency. So tracking the evolution of inefficient plunges over time addresses an unresolved question about the efficient market hypothesis (EMH): Is the level of market inefficiency regime-dependent and asymmetric? With this in mind, by repurposing the asymmetric UC model and augmenting the conventional Fads model, I estimate inefficient plunges with the aim of examining the EMH. The asymmetric Fads model decomposes the market price into its permanent and transitory components, which respectively represent efficient prices and inefficient plunges. To account for the asymmetric price deviation, I include inefficient plunges in the transitory component along with a concomitant switching variance in the permanent component by embedding a Markov-switching process as in Hamilton (1989) into the UC model.

By applying the model to the inflation-adjusted S&P500 and FTSE100 indices at daily, weekly, and monthly frequencies, I establish that the deviation of market prices from efficient prices is regime-dependent and asymmetric. More precisely, the transitory component is large in amplitude and short in duration, implying that inefficient plunges are deep and steep. Hence, market inefficiency is not constant but instead is a regime-dependent and asymmetric phenomenon, meaning that although the U.S. and U.K. stock markets are adequately efficient during normal times, they are considerably below the efficient price during crises. This result is consistent with the findings of time-variation in market inefficiency by Hill and Motegi (2019) and Mattera and Di Sciorio (2022).

I conservatively conclude that the U.S. and U.K. stock markets are inefficient for at least 12% of the time that corresponds to crisis periods. Hence, this study supports the AMH of Lo (2004) against the EMH of Fama (1970). This is consistent with the over-reaction hypothesis suggested by De Bondt and Thaler (1985), who attribute the overreaction to behavioural biases. Finally, this study supports the possibility of negative bubbles proposed by Cao et al. (2016), Acharya and Naqvi (2019), and Emery (2021), who attribute negative deviations of market prices from fundamental prices to the withdrawal of uninformed investors and/or binding borrowing constraints.

4.1. Research limitations

In the first study, we demonstrate that the difference version of Okun's law is not reliable for trend-cycle decomposition because of the instability of its parameters in the form of a time-variation in the intercept and a structural break in the coefficient. To circumvent this issue, within a framework of counterfactual analysis, we take account of instability in the difference version of Okun's law as the intervention factor to capture the output shortfall in the U.S. and the U.K. However, as the issue is still unresolved, it is essential to investigate conditions under which a trend-cycle decomposition is reliable. To this end, in the second paper, we devise a stable gap version of Okun's law by paying regard to four specification aspects, such as time-variation in trend growth and asymmetry in the cyclical component.

The second study imposes a constant plucking coefficient for all recessions, though the depth of each recession differs from the others. The consequence of this assumption is of no great concern because the Markov-switching model has enough flexibility to adjust the duration of the state of the economy for a given recession to capture its special depth. Regarding the scope of this study, we focused on establishing the transmission of the plucking property from the unemployment rate, which is sourced from labour market frictions, to U.S. output. Thus, we leave other potential sources of asymmetry associated with financial frictions for future research. For example, the role of borrowing constraints, liquidity shortages, credit crunches, and banking agency problems in the formation of asymmetries is worth investigating.

The model of the third study employs a Markov-switching process for both inefficient plunges in the transitory component and the switching variance in the permanent component. Although considering a single Markov-switching process may be seen as restrictive, this treatment is supported by evidence juxtaposing the probabilities of inefficient plunges and those of high-variance states. Inefficient plunges and jumps in variance are indeed synchronous. Also, since this study aims to examine the effect of inefficient plunges (negative bubbles) on market inefficiency, it remains silent about another culprit of market inefficiency during the boom phase of positive bubbles: greed and overconfidence.

Finally, a methodological limitation of studies that apply models with Markov-switching is that the likelihood ratio statistic, which is used to test for asymmetry, is non-standard. We still maintain the use of the non-standard likelihood ratio test in this thesis for two reasons. First, this limitation seems minor in this study since the extraordinarily large likelihood ratios derived by pairwise comparisons leave very little doubt, if not no doubt, that economic and financial fluctuations are asymmetric. Second, running simulation-based tests is not only theoretically questionable but computationally burdensome. The latter is the case in this thesis, given the large dimension and number of models applied in the second and third studies.

4.2. Implications for future research

The severity of the slow recovery following the 2007−09 global financial crisis in the G7 economies varies across countries, as shown in Figure 1.2. For example, the slow recovery is much worse in Italy than in Germany. With this in mind, future research may examine and compare the magnitude of shortfall spillovers from the U.S. to other economies by following the method we used to measure the shortfall spillovers from the U.S. to the U.K. The key idea worth thinking about is measuring the idiosyncratic exposure of each country to the U.S. economy to identify factors that make a country more vulnerable to the long-term productivity slowdown in the U.S.

The second study provides both academics and empirics with a reliable trend-cycle decomposition by proposing a bivariate state-space model with Markov-switching that accounts for asymmetry, time-variation in trend growth, and correlation between shocks. The third study calls for further research to explore the asymmetry in other financial markets, such as futures, options, and currency markets. Namely, the asymmetry in stock markets as underlying assets necessitates considering the asymmetry in call and put option pricing. Besides, developing a model to incorporate both positive bubbles and negative bubbles (inefficient plunges) is an interesting topic.

Establishing the interlinkage between the two asymmetries captured in the second and third studies is of great interest. The second study attributes the asymmetry in the real economy to search frictions and nominal wage rigidities in the U.S. labour market, while the third study attributes the asymmetry in financial markets to behavioural biases, e.g., panic and overreaction. Given the synchrony of these two asymmetries, the idea that comes to mind is to develop a bivariate model to characterize the connection between these asymmetries. To what extent are the causes of asymmetry in the real economy and the asymmetry in the financial markets related to each other?

Lastly, let me explain the methodological implications of this thesis. For counterfactual analysis, in contrast to the existing literature in which counterfactuals are retrospective or prospective data, we suggest designing a counterfactual based on fitted values. On this basis, in the first study, we identify the intervention factor as a structural break in the relationship between two variables, output and unemployment. Future studies on other topics may use this approach to counterfactual analysis by investigating a structural break in the empirical relationship between other sets of variables. In the second and third studies, we specify the model in levels rather than in differences to characterize phenomena related to asymmetry, such as the plucking property, inefficient plunges, and asymmetric volatility. Additionally, this approach enables the model to circumvent the issues related to unknown order of integration and potential over-differencing.

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