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Ergodic and non-ergodic variogram uncertainty in Agbabu bitumen field porosity data

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Abstract. A recent investigation estimated and modeled the variogram of the porosity attribute of the bitumen field located at Agbabu, south-west Nigeria. In spite of methodological interventions made during the estimation, the variogram estimates obtained are still uncertain due to preferential and limited sample data. This current research therefore assesses the uncertainty in the ergodic and non-ergodic variogram estimates. Multiple samples required for ergodic and non-ergodic variogram estimates have been drawn from simulated realizations of the porosity random field. Variogram estimates have been obtained (using a recently-formulated lag-cluster technique and conventional technique) for each sample. The sampling distributions of these estimates have been examined with histograms and Normal Q-Q plots. The estimates exhibit approximately Normal distributions. The uncertainty in the ergodic variogram increases from 1.4×10^{-7} at lag 16.7 m to 4.6×10^{-7} at lag 4601.4 m; thereafter, it appears to flatten out. For most lag distances, the uncertainty in the non-ergodic variogram is constant at about 1.8×10^{-7} ; it fluctuates at large lag distances. The uncertainty in non-ergodic estimates is observed to be lower than the uncertainty in ergodic estimates. Consequently, that the current well spacing and distribution in Agbabu field is deemed adequate in the context of variogram estimation.

Keywords: Ergodic; Non-ergodic; Variogram; Uncertainty; Agbabu field; Heavy oil and bitumen; Nigeria.

1. Introduction

Recent trends in the metrics of Nigeria's oil and gas sector bring to fore the necessity of giving attention to the unconventional hydrocarbon resources vastly present in the field. The scenario currently prevailing in the sector is that of stagnant reserves, declining production capacity, and rising domestic oil consumption [1]. Conversely, new conventional field discoveries are occurring less frequently in the matured Niger Delta oil province. It is therefore soothing to note that vast deposits of heavy oil and natural bitumen have been long-discovered in the Dahomey basin (Benin basin) south-western Nigeria. However, inconsistency in the estimates of volumes of hydrocarbon contained in these deposits as reported by several studies has partly discouraged serious commercial interests in the exploitation of the resources. Reported values range from 30 to 420 billion barrels of hydrocarbon [2] – [9]. Considering the official value of 43 billion barrels [5], the Nigerian heavy oil and natural bitumen deposits have the potential of doubling the nation's oil reserves if proven and booked as such. The inconsistency in the estimates of volumes of hydrocarbon contained in the deposits is attributable to the little or no consideration for spatial correlations of volumetric attributes in those studies aforementioned. With the exception of Falebita et al. [9], none of the studies has considered the spatial variability (heterogeneity) of key volumetric attributes of the deposits. The consideration for spatial correlation by Falebita et al. [9] was rather very superficial. For example, the various plots of estimated empirical variogram obtained in that attempt did not show any interpretable structure. The plots exhibited large fluctuations and show no stability. No effort to resolve or analyze



these issues was reported. The incorporation of spatial variability measures into volumetric resource assessment is in the realms of geostatistical reservoir characterization.

In recognition of the foregoing, recently, attention has been given to the geostatistical characterization of the heavy oil and natural bitumen field located north of Agbabu being part of the Dahomey basin. Mosobalaje et al. [10] presented detailed analyses and robust discussions on the descriptive statistics and probability distributions of porosity, depth-to-top and thickness attributes of the field. In advancing the geostatistical investigation, Mosobalaje et al. [11] estimated and modeled the spatial variability of these attributes. Specifically, the spatial variability was measured as variogram estimates and models. The authors reported various challenges faced in the variography of the attributes data. These include severe instability, apparent lack of interpretable spatial correlation structure, and geological inconsistency. Various methodological interventions were made in finding solutions to these challenges. Significantly, a recently formulated lag-cluster approach to variogram estimation [12] was applied. The lag-cluster approach is aided by a machine learning algorithm known as Density-based Spatial Clustering of Applications with Noise (DBSCAN). Ultimately, Mosobalaje et al. [11] presented an integrated three-dimensional anisotropic porosity variogram model. The improvements made possible by the application of the new approach notwithstanding, the variogram estimates so obtained are still necessarily uncertain due to preferential and limited sample data and the inherent measurement errors. This current research therefore focuses on the assessment of the inherent uncertainty in the ergodic and non-ergodic variogram estimates. This paper includes robust discussions on the comparison between the ergodic and non-ergodic uncertainty values obtained. The ultimate objective of this research is to interpret the difference between ergodic and non-ergodic uncertainty measures and thereby offer an assessment of the adequacy (or otherwise) of the well spacing and distribution in Agbabu field. Also, measures of variogram uncertainty obtained in this work would be available to be incorporated into the volumetric attributes uncertainty leading to a robust unbiased estimate of resource uncertainty. Such resource uncertainty is very important in technical, investment and management decisions.

In this work, a distinction is made between ergodic variogram and non-ergodic variogram. Ergodic variogram is that which is averaged over multiple realizations of the random process (variogram model and conditioning data) that generated the field. Non-ergodic variogram is that which is averaged over multiple sampling of a single realization of the random field. In recent developments, a single realization of a random field may be sampled multiple times by translating (shifting) the original sampling grid across the field [13], [14]. In essence, ergodic variogram is that of the random process that generated the field while non-ergodic variogram is that of the field itself; these two are not necessarily equal. The non-ergodic variogram is of more practical interests in geostatistical modeling because it better characterizes the variability inherent in the field [15]. Each of these variogram types has its associated uncertainty (fluctuations). Ergodic variogram uncertainty refers to the fluctuations of variogram estimates over multiple realizations of the random process underlying the field. On the other hand, non-ergodic variogram uncertainty refers to the fluctuations of variogram estimates over multiple sampling of a single realization of the field. The uncertainty in non-ergodic variogram estimates is entirely due to sampling (i.e. random variations across samples). Hence, uncertainty in non-ergodic variogram has been variously referred to as sampling variance, sampling error, sampling fluctuations or estimation variance. However, uncertainty in ergodic variogram encompasses fluctuations due to random variations of the random variables (over multiple realizations), in addition to sampling fluctuations. Hence, uncertainty in ergodic variogram has been variously referred to as random variance, random error, or fluctuation variance. From the foregoing, it is expected that non-ergodic variance be less than ergodic variance. When this expectation is met, it implies that the sample locations are well distributed over the entire field therefore

most variations are explained [16]. However, when ergodic and non-ergodic variances are near identical; it implies the sample locations are sparse; and significant parts of the field are unsampled. The unsampled parts therefore contain variations not explained and manifesting in sampling variance [16].

2. Field and Data Description

Agbabu field is part of the vast deposits of heavy oil and natural bitumen in the Dahomey Basin. The Dahomey basin is a coastal sedimentary basin that spans from Ghana-Ivory Coast border to western Nigeria. The presence of heavy oil and natural bitumen deposits in the eastern part of the Dahomey basin has been affirmed by several authors [2], [5], [8], [17] – [21]. Figure 1 is the geologic map of the outcrop sections of the deposits showing the Agbabu field. A comprehensive review of the geographical extent, geology, lithology and stratigraphy of the Dahomey basin and of the Agbabu field is presented by Mosobalaje et al. [10].

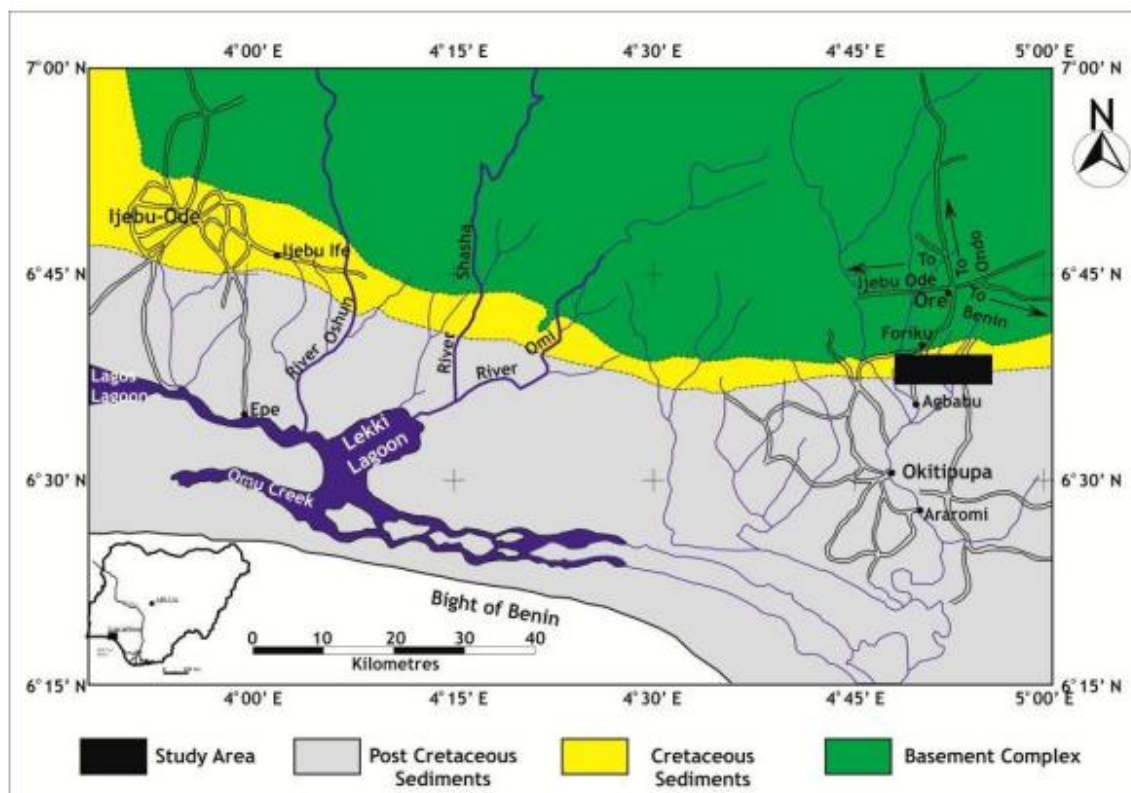


Figure 1: Map of the outcrop sections of the Nigerian bitumen deposits (Source: [9])

In the Agbabu area, sand/shale sequences deposited in the Afowo formation and in the lower parts of Araromi formation are bitumen-saturated. The bitumen-saturated sand deposits (tar sands) have been observed to occur in both Horizon X and Horizon Y. These two horizons are separated by an organic-rich

shale layer (oil shale). Adegoke et al. (1980) drilled forty (40) wells on the 17km² study area from which some 583 tar sand and oil shale core samples were obtained. The investigation proceeded to determine the weight percent bitumen and water saturations of each core sample as well as the depth-to-top and thickness of identified horizons in each well. Figure 2 shows the X-Y coordinates of the locations of the wells. Mosobalaje et al. [10] deployed basic principle of volumetric proportions to compute and generate reservoir porosity database from the existing Adegoke et al. [2] raw database. The descriptive analyses by Mosobalaje et al. [10] were conducted only on bituminous sand Horizons X and Y 443 data points; leaving out the shale layer. Consequent on the exclusion of certain spurious data points, only 408 data points from 33 wells were included in the analyses. Furthermore, exploratory data analysis conducted by Mosobalaje et al. [11] detected some spatial outlier pairs in the 408-points porosity database. These spatial outlier pairs were excluded from the estimation and modeling of porosity variogram. The resulting database containing 362 core porosity data is the subject of the uncertainty assessment reported in this paper.

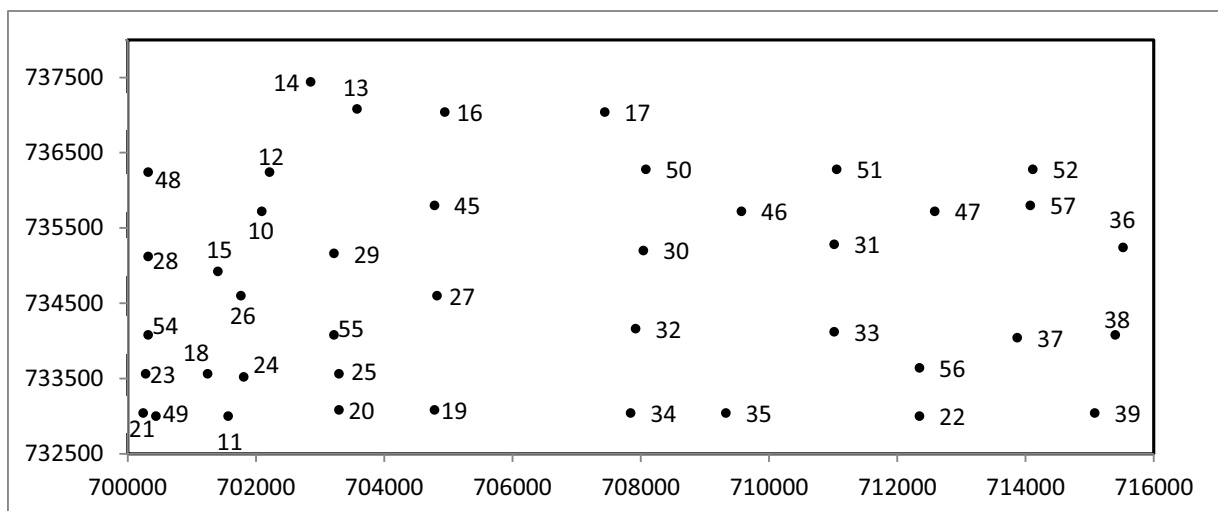


Figure 2: X-Y coordinates of well locations in Agbabu Field (Adapted from [9])

3. Random Field Simulation, Validation and Sampling

The methodology of obtaining repeated samples for ergodic variogram and associated uncertainty is straightforward. Several realizations of a dense grid of values of the random field are simulated using a plausible variogram model; each realization is sampled at sample points. For the non-ergodic case, the methodology of obtaining multiple samples (re-sampling) of a single realization is varied. A concise review of the evolution of the variants of the methodology and associated limitations/problem is presented by Mosobalaje [22]. Also, a new algorithm that overcomes the limitations and problems of previous methods has been recently formulated by Mosobalaje [22]. This algorithm is tagged Sampling Grid Shifting Algorithm (SGSA). The plausible 3-D variogram model for the porosity data has been presented by Mosobalaje et al [11]; it is here shown as Equation 1.

$$\gamma = 0.0024 + 0.0016 Sph_{a_{h_{max}}=3500, a_{h_{min}}=1500, a_{vert}=15}(\vec{h}) + 0.0005 Sph_{a_{h_{max}}=3500, a_{h_{min}}=1500, a_{vert}=\infty}(\vec{h}) - - - - - 1$$

In simulating the porosity random field, the Agbabu field was discretized into a 40×13×100 3-D grid. Gridblock dimensions in x-, y-, and z-axis are 400m, 400m, and 1m respectively. The gridblock indices of these 362 sample points have been obtained, both in engineering ordering (x, y, z) and natural ordering. These indices, coupled with the actual Eastings, Northings and Depths, make up the suite of spatial coordinates of the original sampling grid of this field. Upon implementing the SGSA on this field and the associated 362-point sampling grid, a total of 78 possible shiftings of the sampling grid were obtained. Accordingly, 78 realizations of the full-grid field were simulated for the purpose of obtaining the ergodic variogram and the associated uncertainty. The conditional simulation was by ordinary kriging in sequential Gaussian simulation (SGS) using the variogram model in Equation 1 and the 362 sample data as the conditioning data. Figure 3 is the histogram of the exhaustive (full-grid: 52,000 data points) simulated porosity data. Figure 4 is the empirical variogram (90° azimuth) plot of the exhaustive data with the input variogram curve superimposed. Both figures are for Realization 1; all other realizations generated plots similar to these. While Figure 3 validates the Gaussianity of the simulated porosity data; Figure 4 presents the input variogram reproducibility of the simulation.

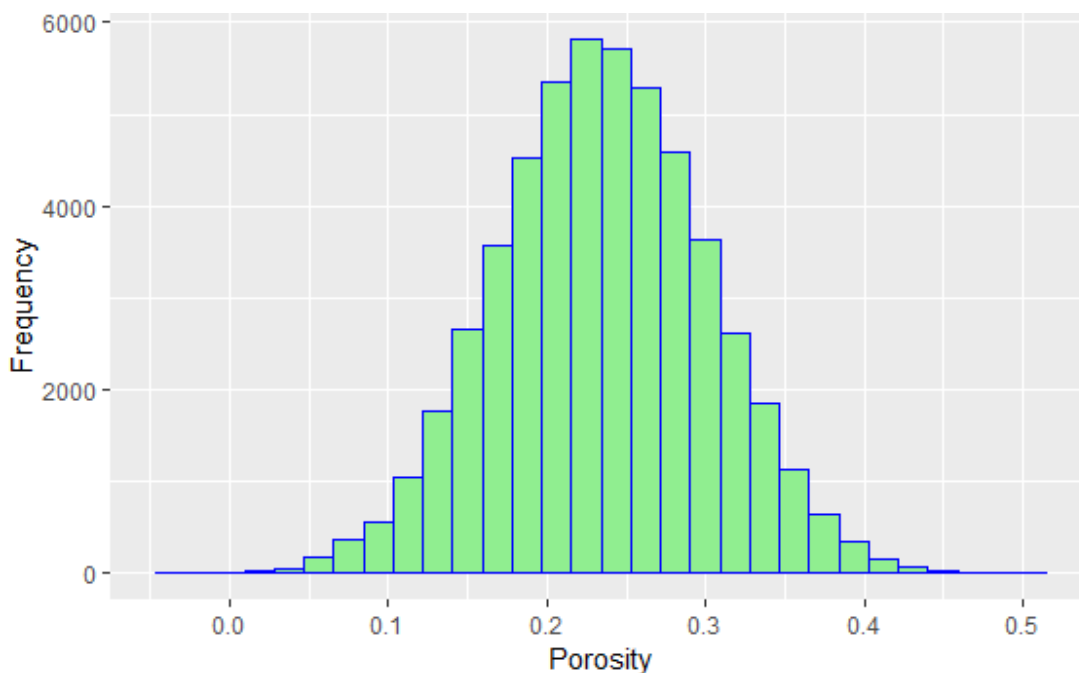


Figure 3: Histogram of the exhaustive simulated porosity data: Realization 1

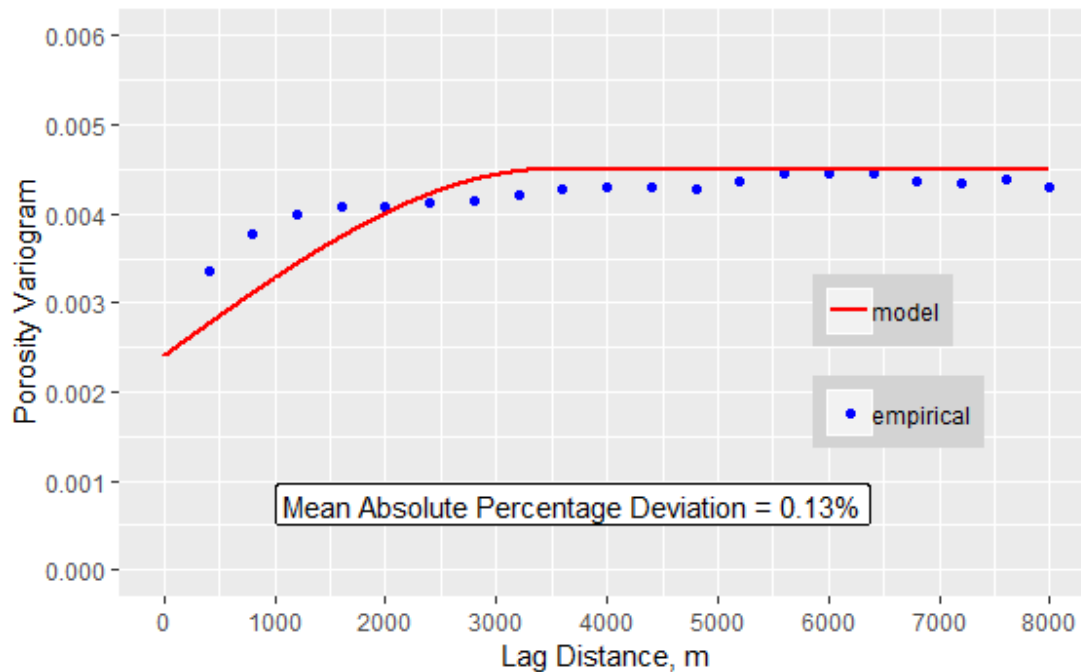


Figure 4: Directional (90°) empirical and model variogram of the exhaustive porosity data: Realization 1

In setting up the ergodic – non-ergodic variogram uncertainty comparisons, two scenarios were prepared in this study. For the ergodic scenario, each of the 78 simulated realizations of the field was sampled at the 362-point original sampling grid. For the non-ergodic scenario, a single realization (arbitrarily chosen) was sampled at each of the 78 shiftings of the 362-point sampling grid. In essence, on the one hand, there is a set of 78 repeated samples drawn from 78 realizations using one (the original) sampling grid. On the other hand, there is another set of 78 repeated samples drawn from a single realization but using 78 sampling grids. The 90° azimuth being the direction of major continuity is chosen for this uncertainty assessment.

4. Samples Variogram Estimates and Sampling Distributions

In estimating empirical variograms for the 78 sets of samples in both the ergodic and non-ergodic scenarios; the DBSCAN-aided lag-cluster approach developed by Mosobalaje et al. [12] was the obvious choice methodology. However, to assess the impact of this newly developed approach on uncertainty measures; the variogram estimates were obtained using both the DBSCAN-aided technique and the conventional lag interval technique. Figures 5 - 8 are plots of the ergodic and non-ergodic variogram estimates (and their means) generated using both techniques, for the 78 repeated samples in each case. In each plot, the input variogram model (Equation 1) is superimposed so as to check its inclusion in the range of estimates at each lag/cluster.

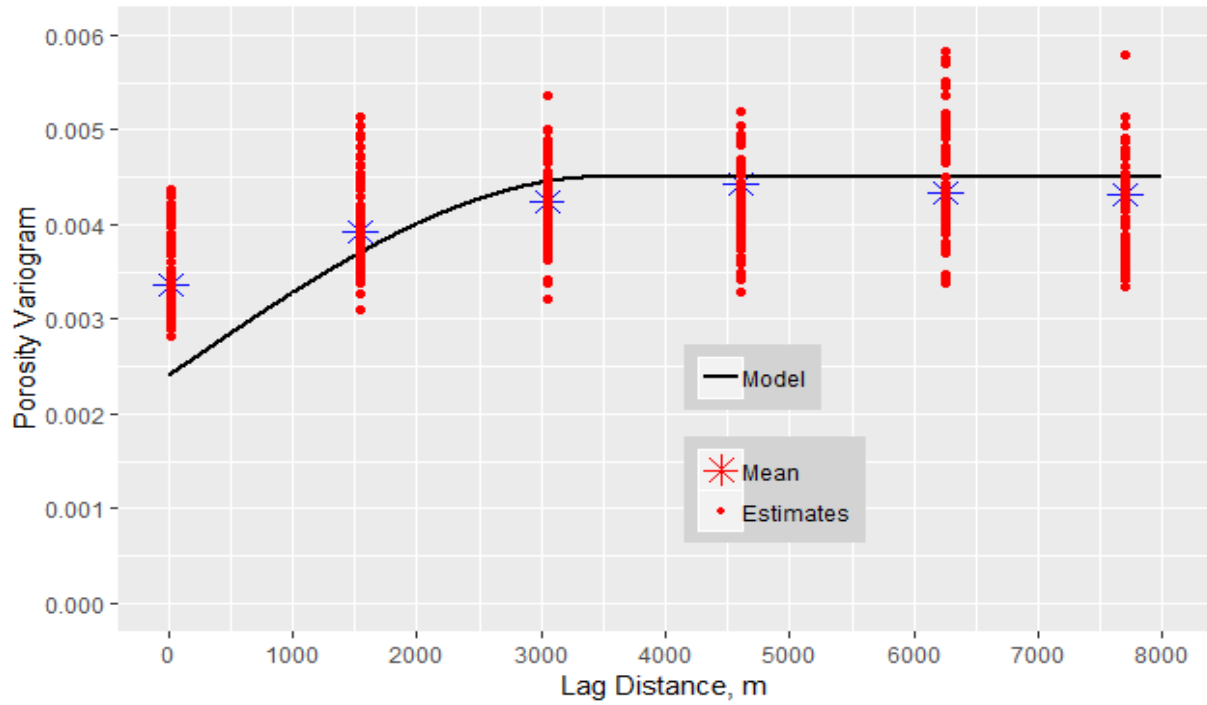


Figure 5: Ergodic estimates of the directional (90°) variogram of Agbabu porosity data – DBSCAN-aided technique

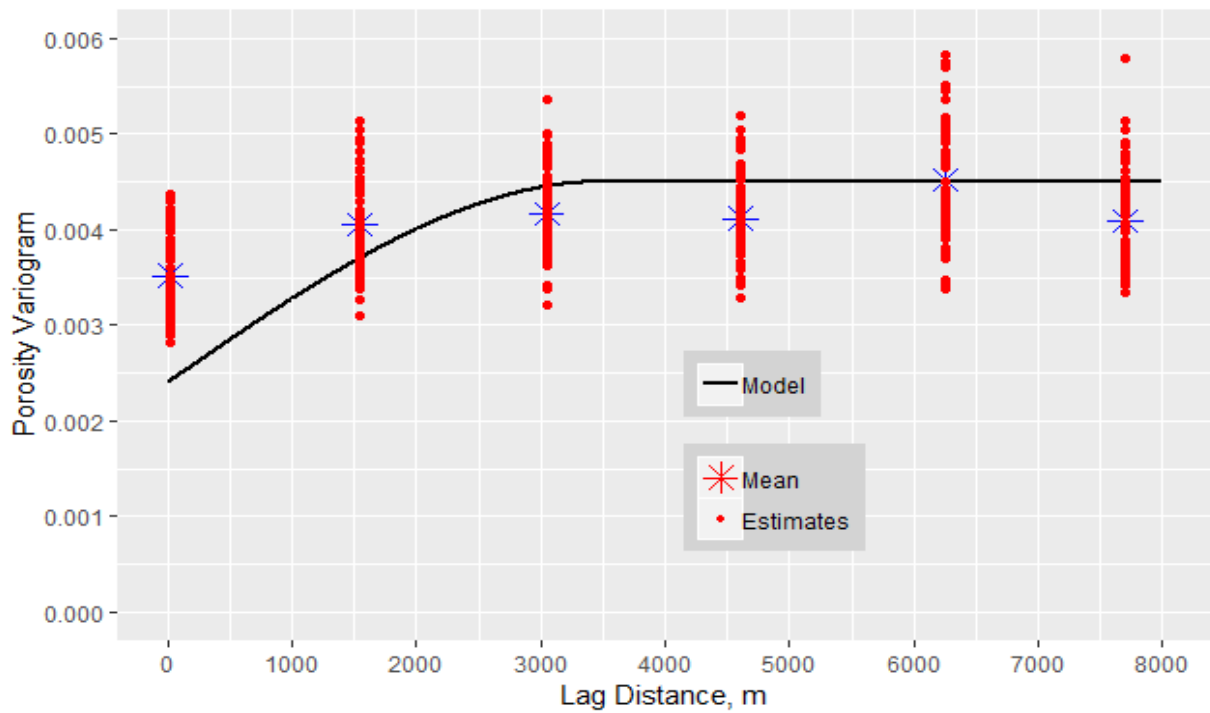


Figure 6: Non-ergodic estimates of the directional (90°) variogram of Agbabu porosity data – DBSCAN-aided technique

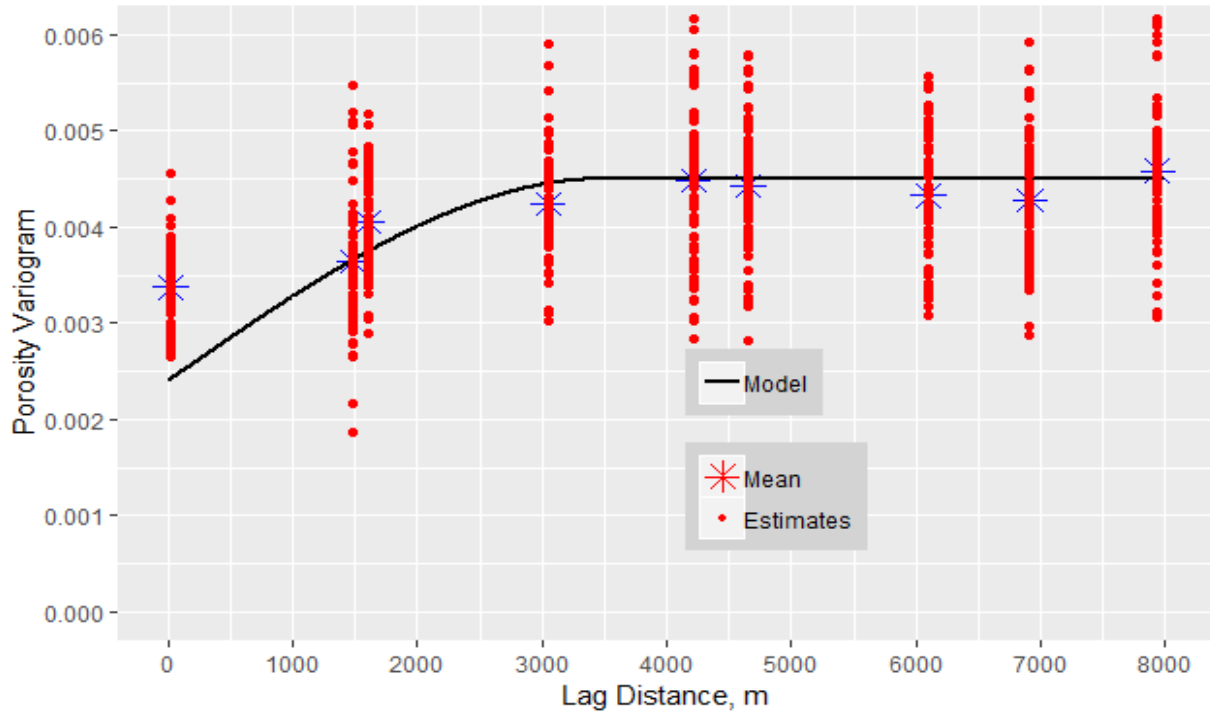


Figure 7: Ergodic estimates of the directional (90°) variogram of Agbabu porosity data – Conventional lag interval technique

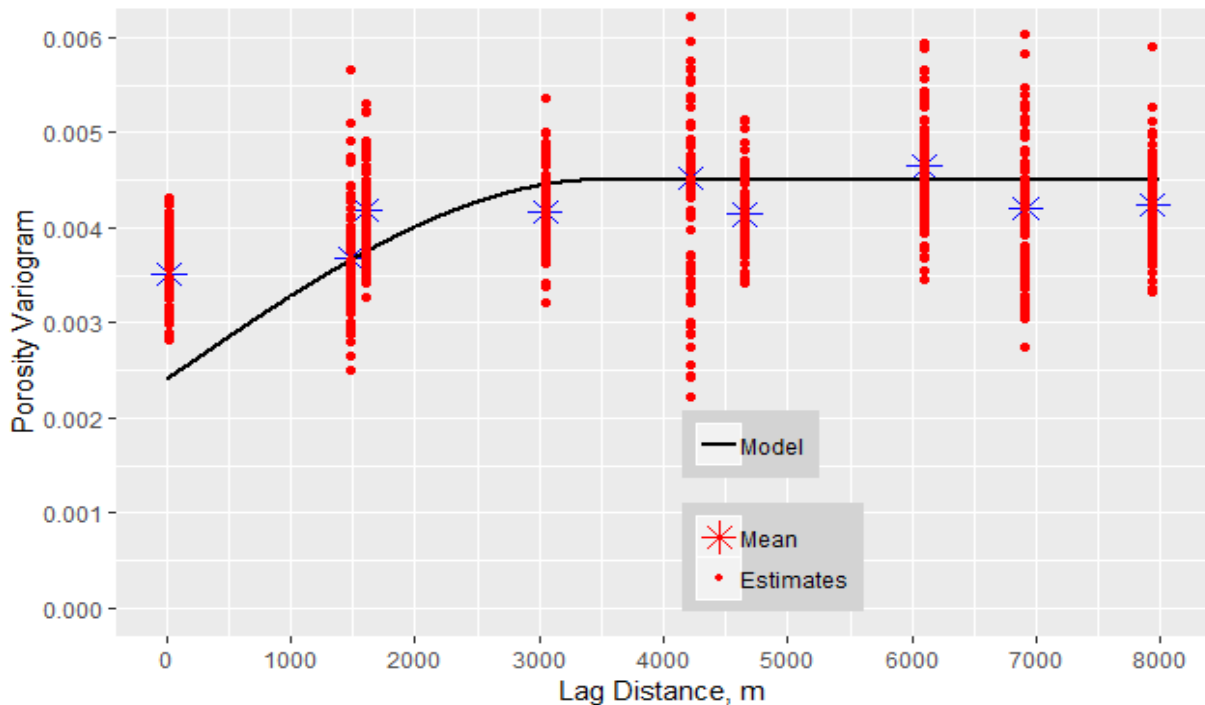


Figure 8: Non-ergodic estimates of the directional (90°) variogram of Agbabu porosity data – Conventional lag interval technique

Ahead of quantifying and discussing the uncertainty of the variogram estimates; the sampling distribution of the estimates at each cluster/lag is examined and discussed first. This becomes necessary since the uncertainty is essentially the variance of such cluster/lag variogram estimates. In doing this, histograms of the variogram estimates at each cluster/lag in all cases are generated. Figures 9 - 12 are cluster/lag histograms of the ergodic and non-ergodic variogram estimates generated using both techniques, for the 78 repeated samples in each case. As a preview into the uncertainty assessment, it is observed for the DBSCAN-aided estimates that the spread of values is less in the non-ergodic case than in the ergodic case. In all cases, the cluster/lag variogram estimates exhibit dome-shaped symmetric distributions. This observation fits well into results reported in existing literature. Three symmetric or near-symmetric distributions have been associated with variogram estimates at a given lag vector: Normal, Gamma and Chi-squared distributions. Cressie [23] has shown that the variogram estimator for a normally distributed variable is a linear sum of independent Chi-squared random variables. Ortiz and Deutsch [24] assumed an approximately Normal distribution; Marchant and Lark [16] and Khan and Deutsch [25] adopted Chi-squared distribution. Koushavand, Ortiz and Deutsch [26] assumed Gamma distribution. Derakhshan and Leuangthong [13] reported lag variogram estimates that are taken to be approximately normal. In articulating all these previous considerations, Rezvandehy [27] conducted a simulation study that considers the three distributions, and found the Chi-squared to be the best fit. For its simplicity and its status as a requirement in many statistical inferences, the Normal distribution is adopted in this work. In order to visually assess the adequacy of approximating these distributions as approximately normal, the corresponding Normal Q-Q plots are generated. A Normal Q-Q plot is a graphical measure of the plausibility of approximating the data as being normally distributed. It is created by plotting the quantiles

of the data against those of a theoretical normal distribution. If the data is approximately normally distributed, the scatter points will result in a fitted straight line. Typically, the fitted straight line passes through the first and third quartiles. The Normal Q-Q plots are presented in Figures 13 – 16. With the exception of tail values of the distributions, most of the scatter points in these figures fit into the straight lines. This justifies the approximation of the distribution of cluster/lag variogram estimates as Normal.

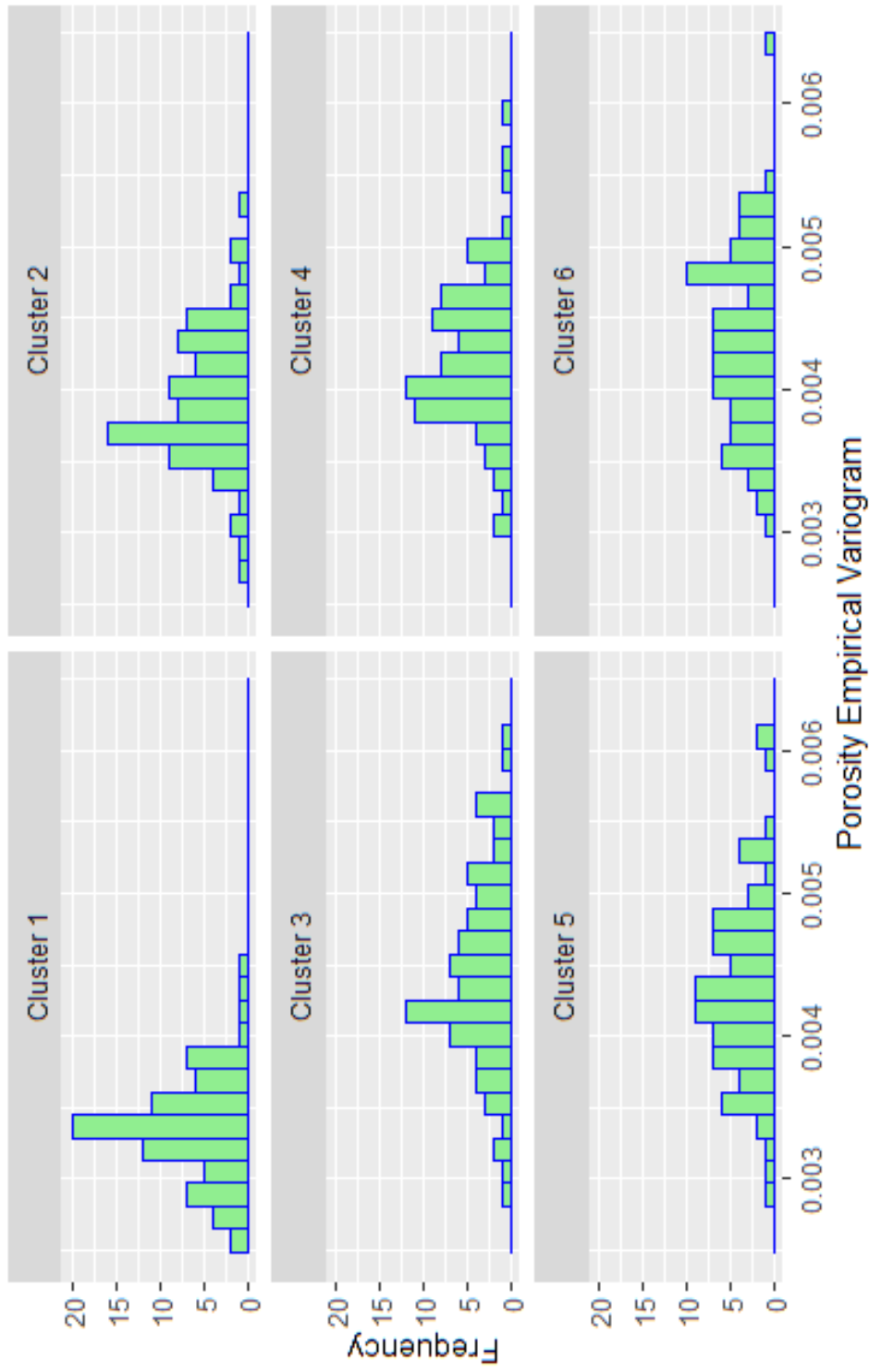


Figure 9: Cluster histograms of ergodic estimates of the directional (90°) variogram of Agbabu porosity data – DBSCAN-aided technique

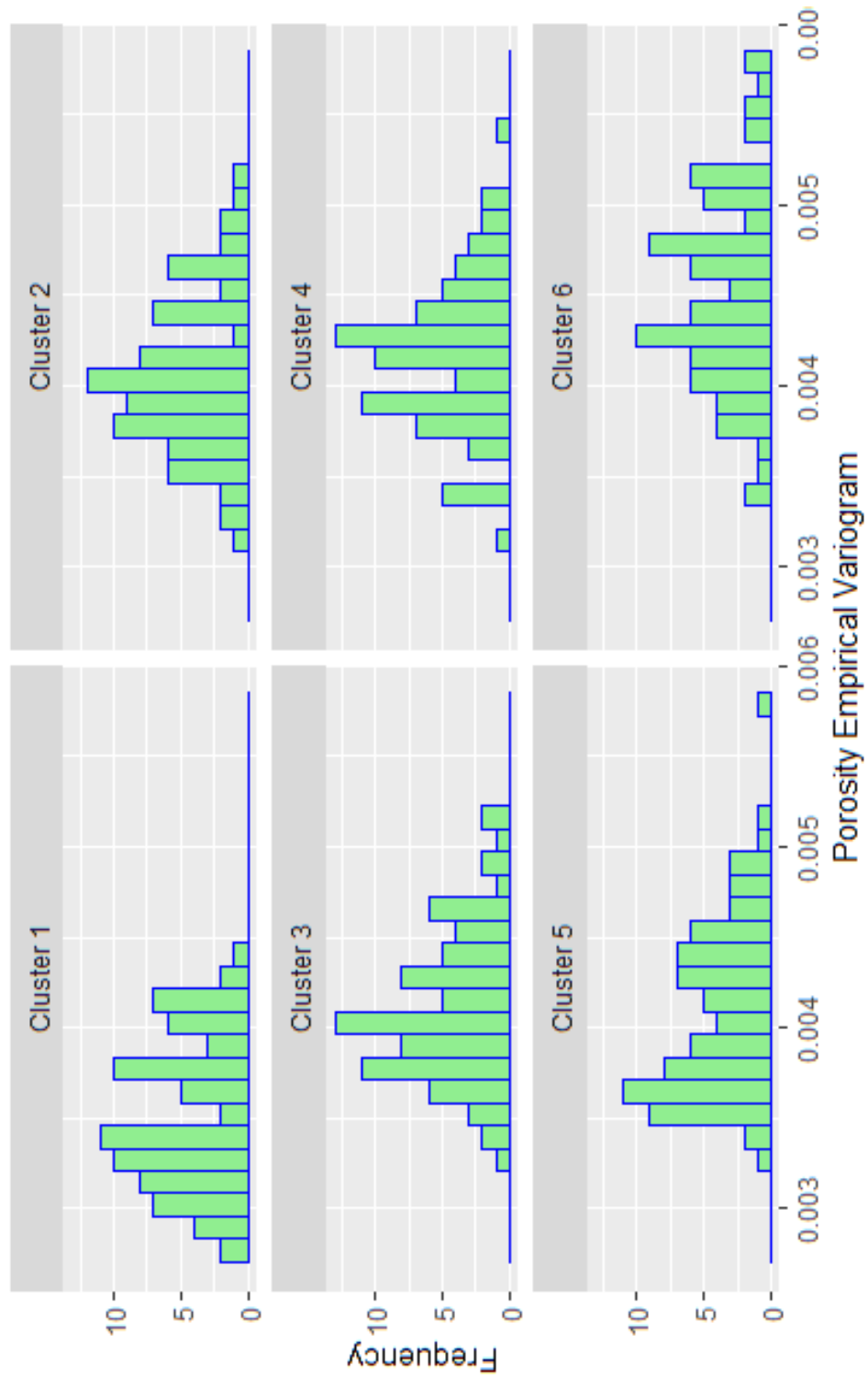


Figure 10: Cluster histograms of non-ergodic estimates of the directional (90°) variogram of Agbabu porosity data – DBSCAN-aided technique

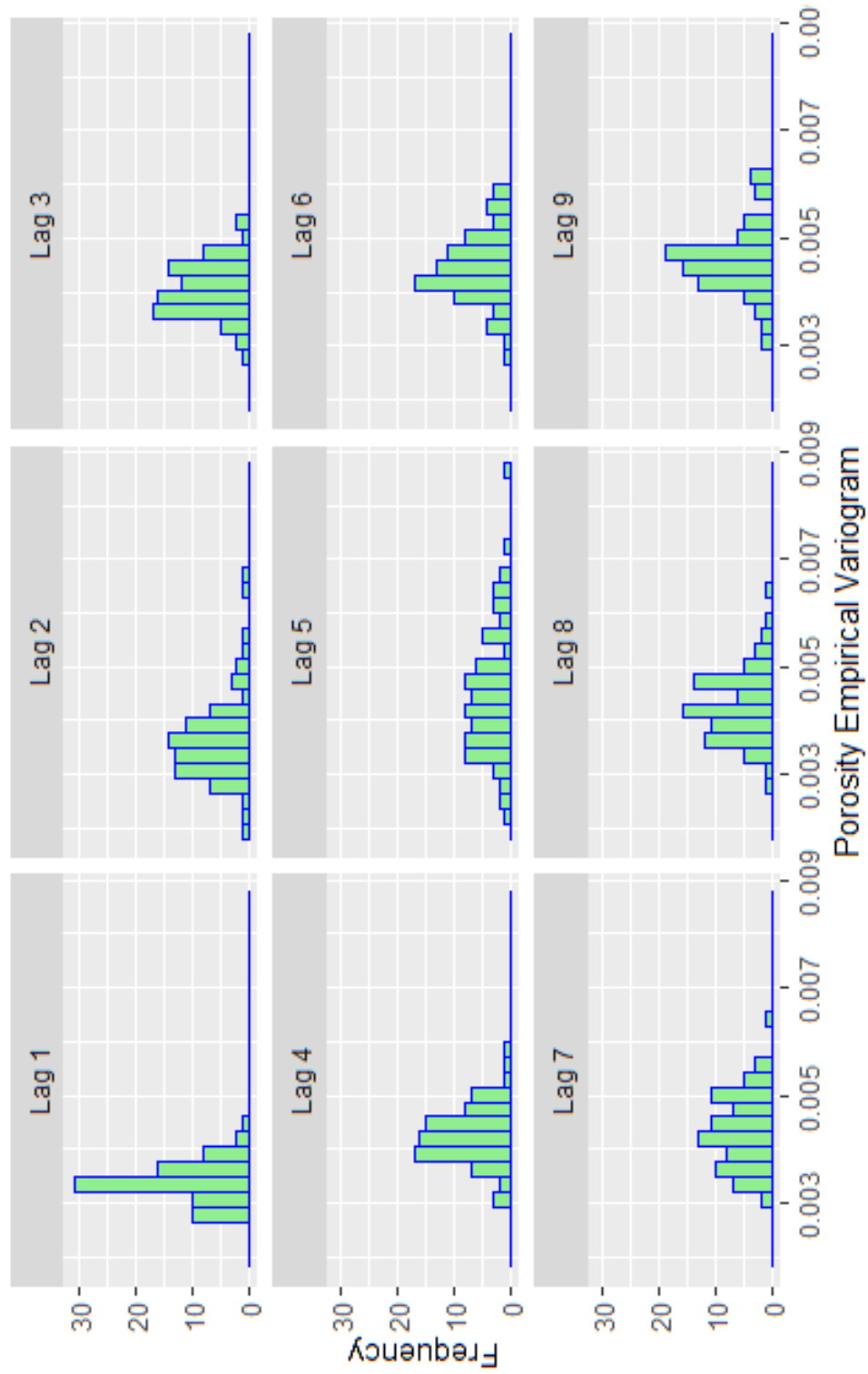


Figure 11: Lag histograms of ergodic estimates of the directional (90°) variogram of Agbabu porosity data – Conventional lag interval technique

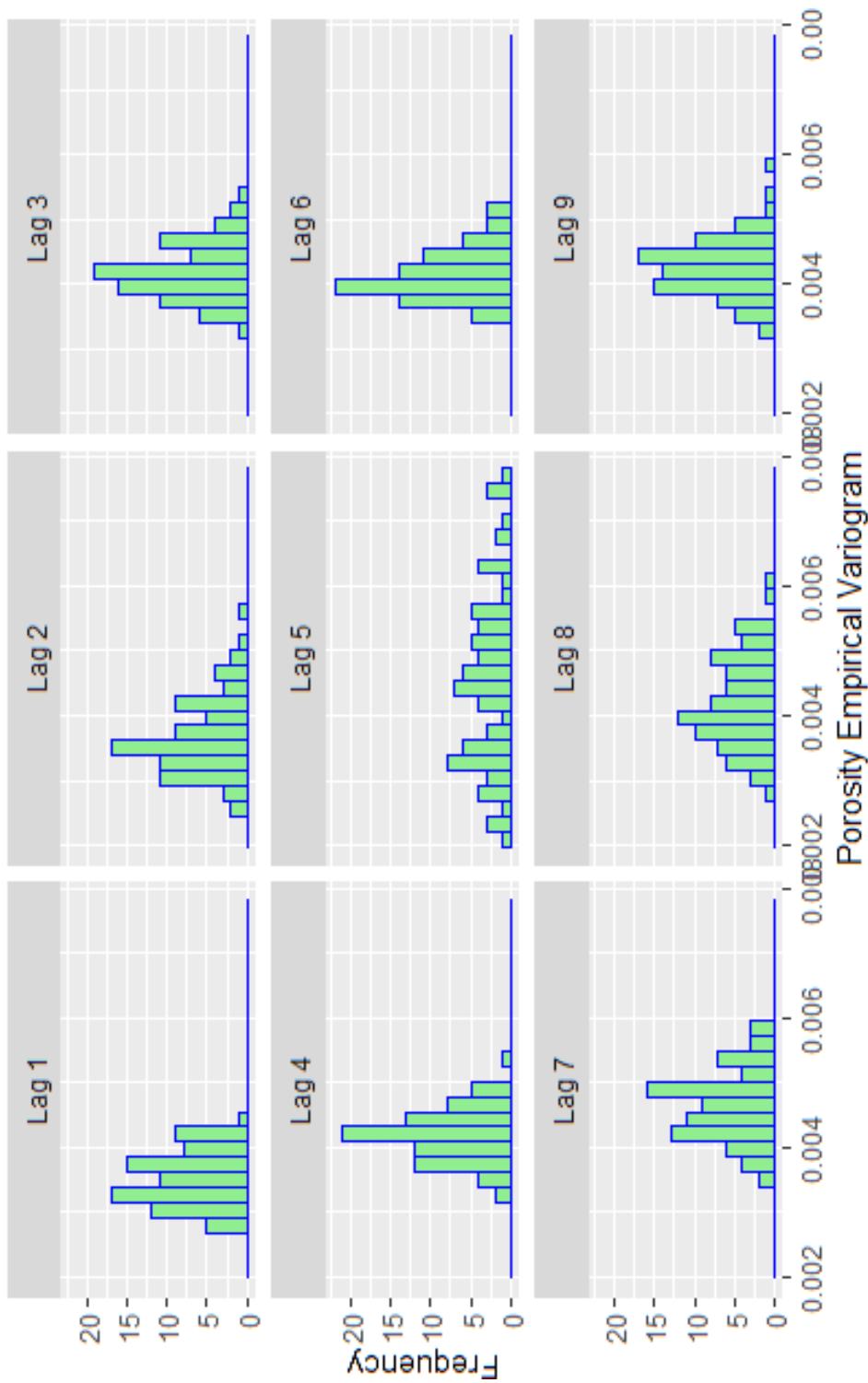


Figure 12: Lag histograms of non-ergodic estimates of the directional (90°) variogram of Agbabu porosity data – Conventional lag interval technique

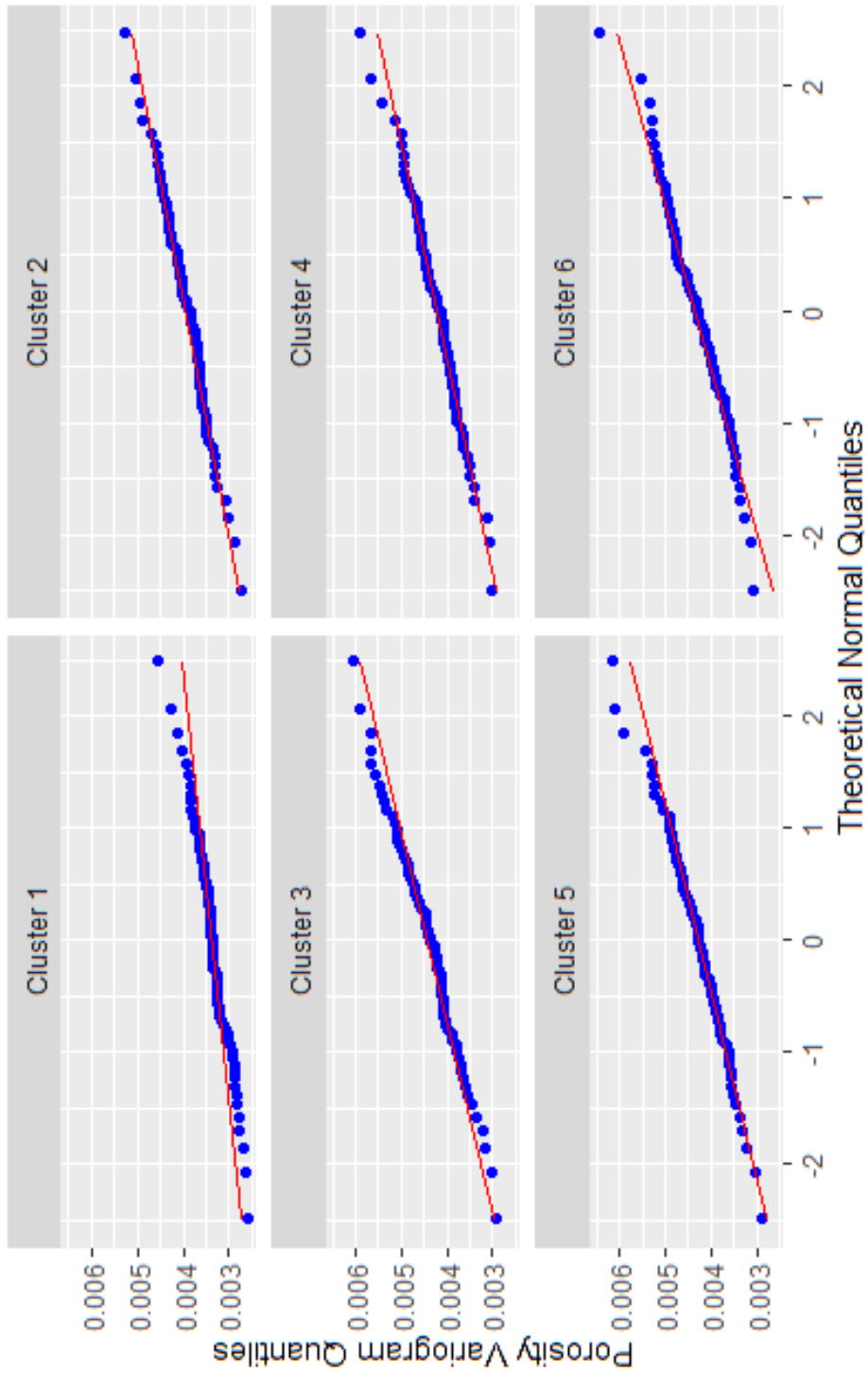


Figure 13: Normal Q-Q plot of ergodic estimates of the directional (90°) variogram of Aghabu porosity data – DBSCAN-aided technique

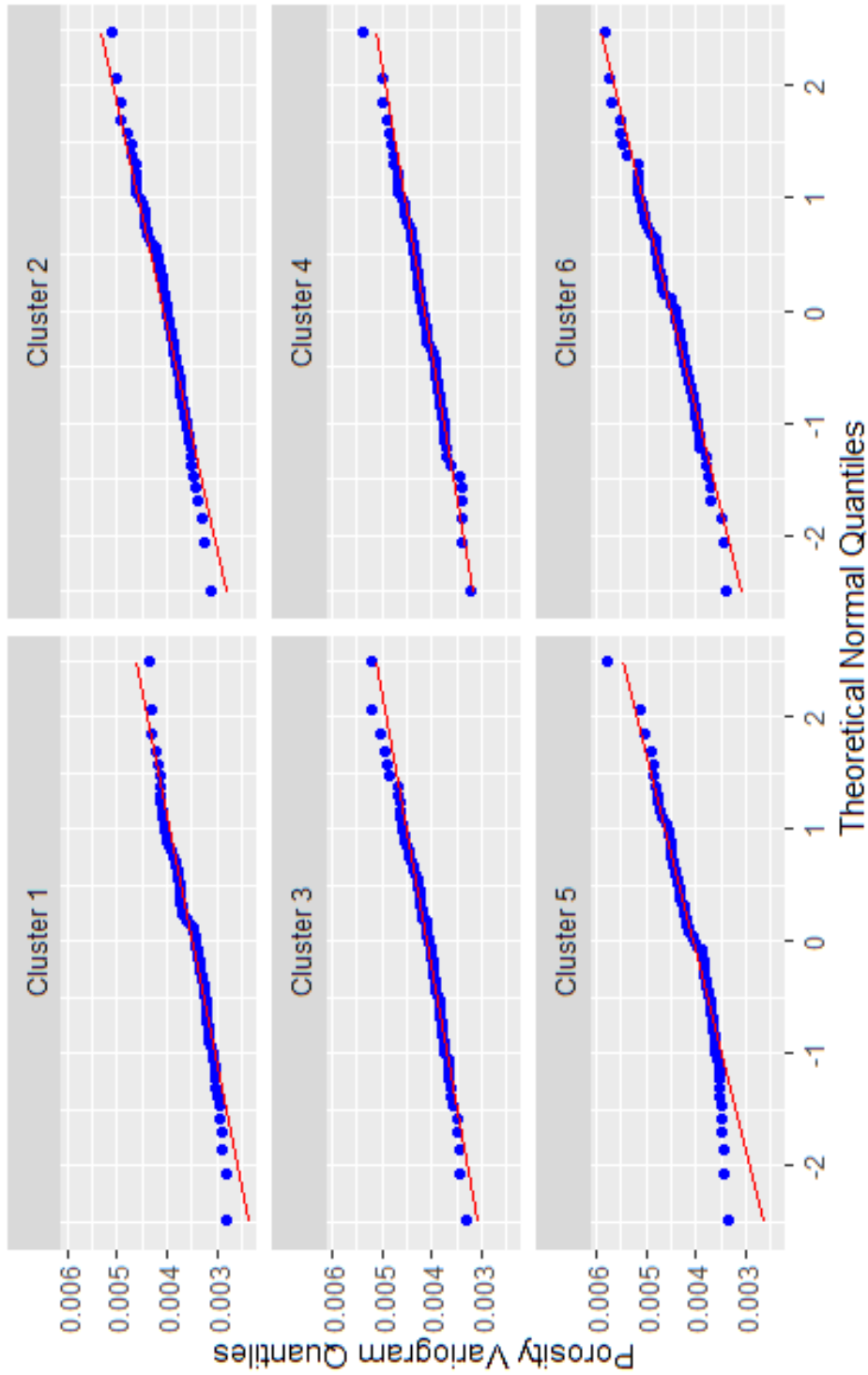


Figure 14: Normal Q-Q plot of non-ergodic estimates of the directional (90°) variogram of Aghabu porosity data – DBSCAN-aided technique

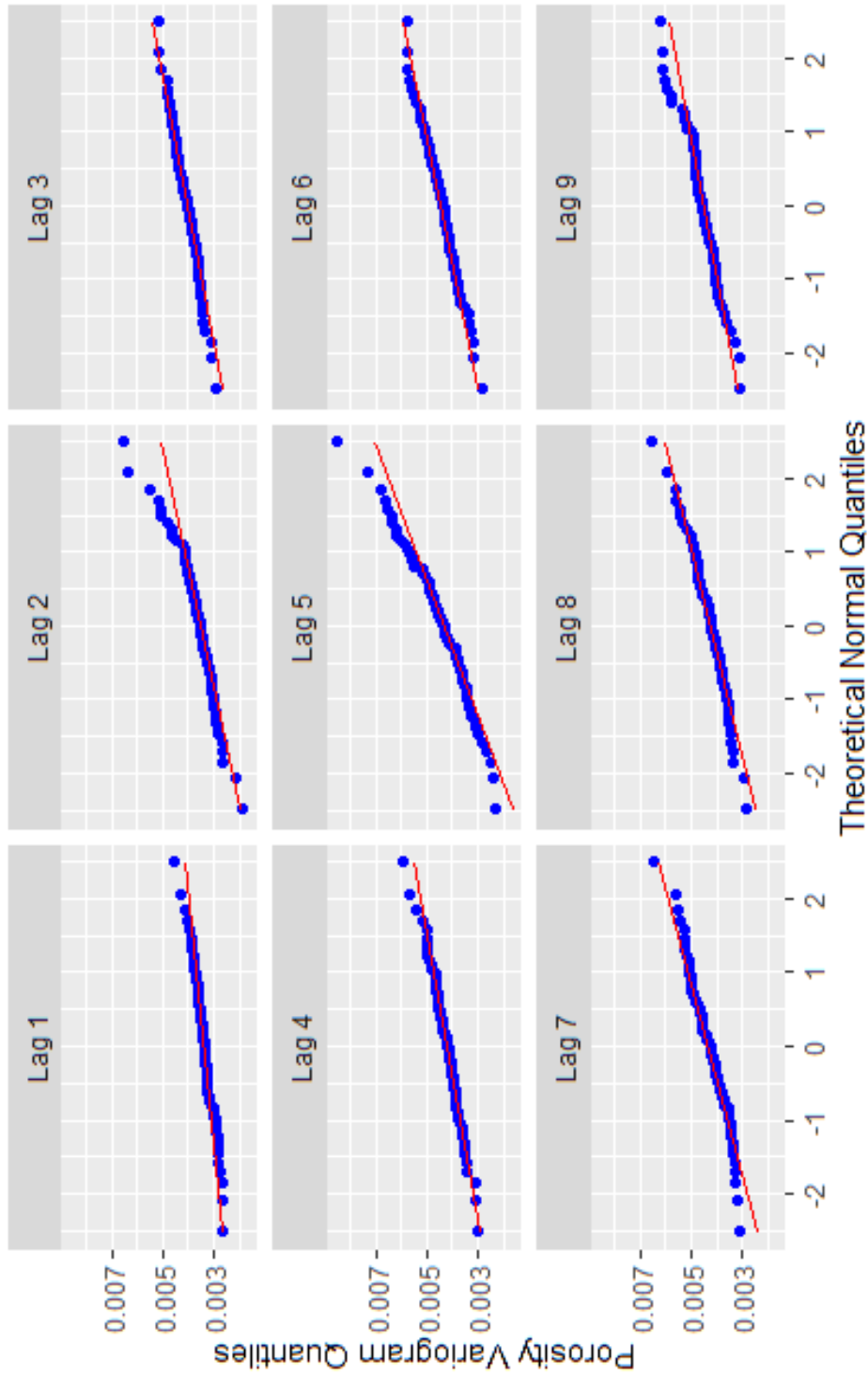


Figure 15: Normal Q-Q plot of ergodic estimates of the directional (90°) variogram of Aghabu porosity data – Conventional lag interval technique

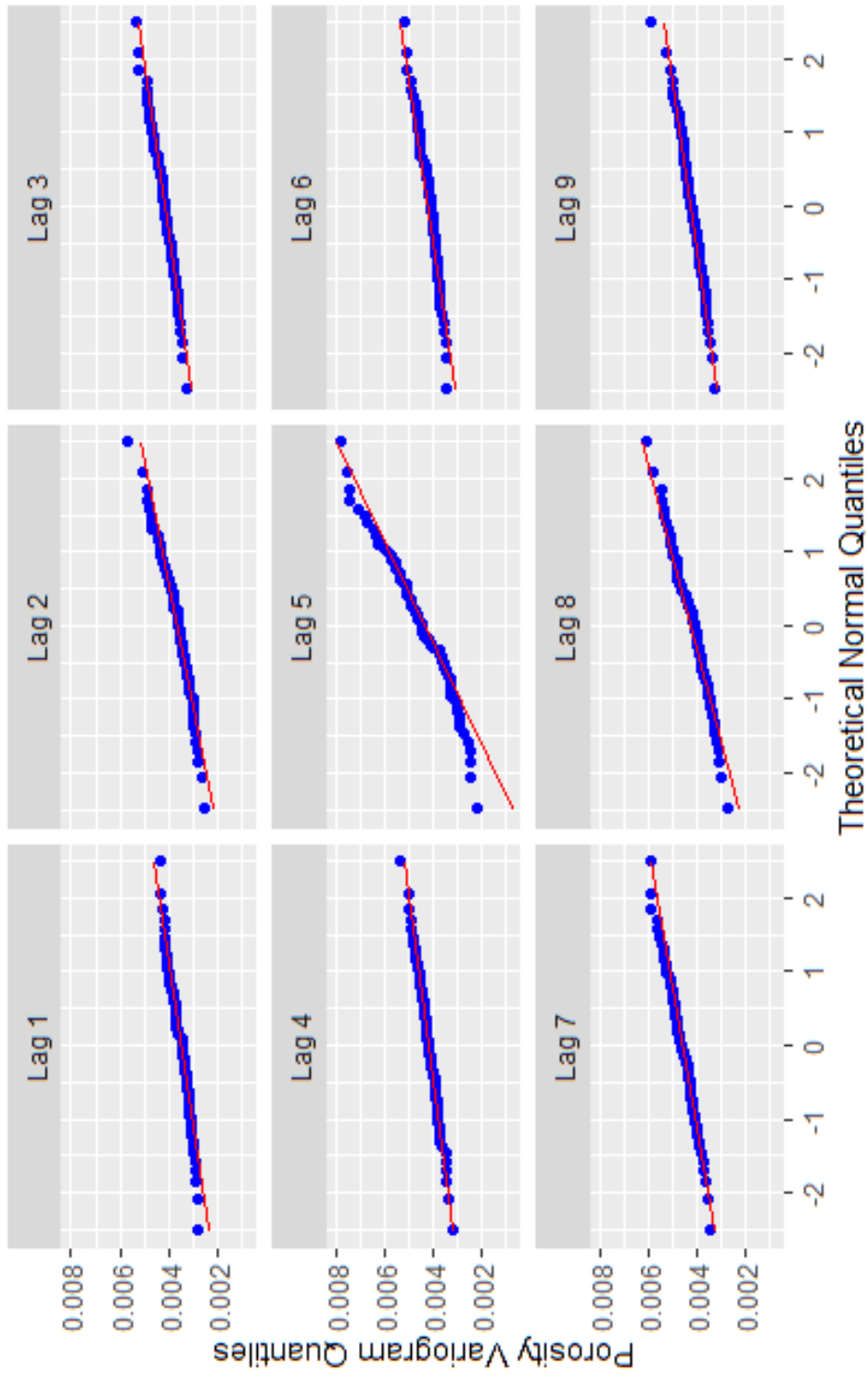


Figure 16: Normal Q-Q plot of non-ergodic estimates of the directional (90°) variogram of Aghabu porosity data – Conventional lag interval technique

5. Ergodic and Non-ergodic Variogram Uncertainty

Quantitatively, uncertainty in an unbiased estimator of an independent variable is measured as the variance of its sampling distribution. For correlated variables, uncertainty is measured as the variance-covariance matrix of the estimator. Estimates of ergodic variogram are expected to be independent since they are obtained from independent realizations of the random process. Conversely, non-ergodic estimates are correlated since they are obtained from a single realization. This thesis considers variance as the uncertainty measure for both ergodic and non-ergodic variogram estimates. The implication of this consideration is discussed later. In essence, the variance of the variogram is thus:

$$\sigma_{\hat{\gamma}(h)}^2(h) = \frac{1}{K} \sum_{i=1}^K [\hat{\gamma}_i(h) - \overline{\hat{\gamma}(h)}]^2 \quad \text{--- -- -- -- -- 2}$$

$\hat{\gamma}_i(h)$ is the ergodic/non-ergodic variogram estimate at the indicated lag vector h for a given realization or sampling grid shifting. $\overline{\hat{\gamma}(h)}$ is the average of the estimates. K is the number of realizations or number of possible shiftings of the sampling grid.

For comparison purpose, Figure 17 shows the variance of ergodic and non-ergodic lag variogram estimates obtained via the DBSCAN-aided technique.



Figure 17: Uncertainty (Variance) in the directional (90°) variogram of Agbabu porosity data – DBSCAN-aided technique

The uncertainty in the ergodic variogram increases with lag distance up to about 4600m and appears to flatten out thereafter. For most of the lag distance, the uncertainty in the non-ergodic variogram is constant; it appears to fluctuate at large lag distances. In both cases, the increased uncertainty at large lag

distances is attributable to the fewer number of data pairs separated by such lag distances. A somewhat similar trend is reported by Derakhshan and Leuangthong [13]. It is noteworthy that within the range of spatial correlation; the uncertainty in non-ergodic variogram is stable while uncertainty in ergodic increases. Variogram estimation and modeling within the range of correlation are of significant importance in geostatistical modeling.

With the exception of the first cluster; the variance of the non-ergodic variogram is less than that of the ergodic variogram. The lower uncertainty in the non-ergodic case is of significant implications in the assessment of the adequacy of the present well spacing and distribution in the Agbabu field. The following discussion follows notions expressed by Marchant and Lark [16]. The source of fluctuations underlying the respective uncertainty types (ergodic and non-ergodic) is key factor in rationalizing this trend. On the one hand, uncertainty in non-ergodic variogram estimates is entirely due to sampling fluctuations (sampling error). On the other hand, uncertainty in ergodic variogram encompasses fluctuations due to random variations of the random variables (over multiple realizations), in addition to sampling fluctuations. It is therefore expected that non-ergodic variance be less than ergodic variance, particularly when sample locations are well distributed over the entire field and most variations are explained. It is on the merit of the lower non-ergodic uncertainty, as observed in Figure 17, that this research adjudges the well spacing in Agbabu field to be adequate and well distributed. This conclusion is one of the significant contributions of this research and achieves the ultimate research objective.

The implication of the fact that the non-ergodic variogram estimates are correlated is now discussed. Here, the results obtained in this work are discussed in the context of the two notions expressed by Pardo-Iguzquiza and Dowd [28] and Marchant and Lark [16]. The two notions are somewhat opposing. Marchant and Lark [16] noted that the correlation between the non-ergodic variogram estimates causes an underestimation thereof; more so when data points are re-used across shiftings of the sampling grid. Conversely, in Pardo-Iguzquiza and Dowd's [28] variance-covariance matrix expression, the re-use of data points leads to increased uncertainty. The expression accounts for multiple use of a data point by using an effective number of independent pairs which is less than the actual number of pairs. The greater uncertainty observed is due to the reduced number of pairs. The SGSA-generated repeated samples used in the computation of the non-ergodic variogram in this work certainly have some data points used multiple times across the shiftings. The lower non-ergodic uncertainty observed in this work could therefore be rationalized in line with the combined effects of correlation and data reuse. The correlation effect could have been responsible for the observed lower non-ergodic uncertainty. Also, the use of only independent number of pairs might increase the non-ergodic uncertainty. Resolving these concerns is being recommended as the objective of future further investigations.

Figure 18 shows the variance of ergodic and non-ergodic lag variogram estimates obtained via the conventional lag interval technique. A comparison between Figure 17 (uncertainty with DBSCAN-aided technique) and Figure 18 (uncertainty with conventional technique) highlights the advantage of the newly-formulated DBSCAN-aided technique. In Figure 17, the non-ergodic uncertainty is markedly lower; whereas, in Figure 18; both ergodic and non-ergodic uncertainties appear to be identical. As discussed by Mosobalaje et al. [12], the conventional technique disregards the 'clusteredness' of sample points in the random field. This disregard clearly affects both ergodic and non-ergodic estimates, resulting in higher uncertainty for both. However, the disregard affects the non-ergodic estimates more since it is a property of the random field. This greater influence on the non-ergodic estimates might have acted to shield the reduced-uncertainty effect inherent in the non-ergodic estimates. This reinforces the earlier recommendation of the lag-cluster approach instead of the conventional lag-interval approach, for irregularly sampled fields.

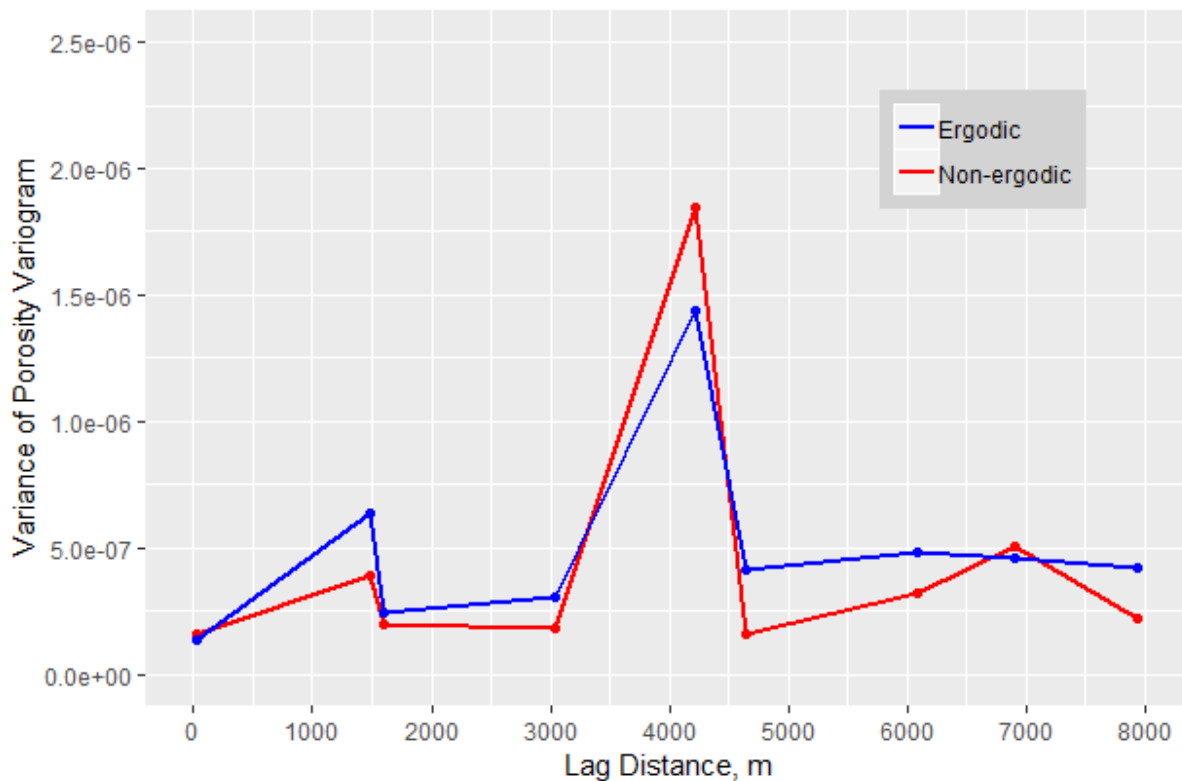


Figure 18: Uncertainty (Variance) in the directional (90°) variogram of Agbabu porosity data – Conventional lag interval technique

6. Summary and Conclusions

An existing plausible 3-D porosity variogram model as well as the 362-point porosity sample has been used to simulate full-grid realizations of the Agbabu field porosity random field. Also, the SGSA has here been applied to the discretized grid to obtain 78 total possible shiftings of the grid in the field. Accordingly, 78 repeated samples drawn from 78 realizations using one (the original) sampling grid were obtained for the ergodic uncertainty assessment. Also, another set of 78 repeated samples drawn from a single realization but using 78 sampling grids were obtained for the non-ergodic uncertainty. Variogram estimates have been obtained (using both the DBSCAN-aided and conventional techniques) for each of the 78 repeated samples in both ergodic and non-ergodic cases. The sampling distributions of these estimates for a given cluster/lag have been examined with histogram plots and Normal Q-Q plots of the estimates. The uncertainty (variance) in ergodic and non-ergodic variogram estimates has been obtained. Also, the implication of the fact that the non-ergodic variogram estimates are correlated has been discussed in the context of data points reuse. The results obtained in this work warrant the following conclusions.

- Variogram estimates obtained (using both the DBSCAN-aided and conventional techniques) for both ergodic and non-ergodic cases exhibit approximately Normal at all clusters/lags.

- For the DBSCAN-aided variogram estimates, the uncertainty in non-ergodic variogram estimates is markedly lower than the uncertainty in ergodic estimates.
- The current well spacing and distribution in Agbabu field is adjudged adequate in the context of variogram estimation.
- For the variogram estimates obtained by the conventional lag interval approach, both ergodic and non-ergodic uncertainties are identical. The neglect of cluster configurations in the conventional approach has shielded the reduced-uncertainty effect inherent in the non-ergodic estimates.

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Nomenclature

ABBREVIATIONS

3D	Three Dimensional
DBSCAN	Density-based Spatial Clustering of Applications with Noise
OPEC	Organization of Petroleum Exporting Countries
Q-Q	Quantile-quantile
SGS	Sequential gaussian simulation
SGSA	Sampling Grid Shifting Algorithm

SYMBOLS

∞	Infinity
a	Variogram correlation range parameter
$a_{h_{max}}$	Major horizontal correlation range
$a_{h_{min}}$	Minor horizontal correlation range
a_{vert}	Vertical correlation range
h	Lag distance, Average lag distance
\vec{h}	Lag vector
K	Number of random field realization or of possible sampling grid shiftings
N	Number of data points
$N(\vec{h}), n(\vec{h})$	Number of available pairs for lag vector \vec{h}

GREEK SYMBOLS

$\sigma_{\hat{\gamma}(h)}^2$	Variance of empirical variogram for lag vector \vec{h}
Σ	Summation
γ	Theoretical Variogram, Variogram model
$\hat{\gamma}$	Empirical Variogram

$\overline{\hat{\gamma}_i(h)}$

Average of empirical variogram estimates at a given lag

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