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Abstract

Interpolation of time series by the sum of exponents of the function of a complex variable makes it possible to achieve an approximation accuracy no worse than using regression analysis. Despite the fact that time series are interpolated by functions of a complex variable, the values of these functions under certain conditions are real numbers. To calculate the interpolating function, the author used standard procedures used in the MATHLAB software.

The absence of extremum points for exponents is the main advantage when using exponent sums for interpolation purposes compared to interpolation by polynomials. Numerical series characterizing the volume of exports and imports within the Visegrad four were approximated as a sum of sixteen exponents, which are functions of a complex variable.

Keywords

BRINKS model of international trade, functions of a complex variable, interpolation, economic and mathematical modeling, BRINKS model

Introduction

The globalization of international economic processes is the engine of the development of international trade exchange. The creation of an international trade organization, as well as various other intergovernmental and commercial organizations, has a beneficial effect on the exchange of services and goods.

International trade . it becomes a kind of production technology that allows you to produce spare parts and components in different countries and on different continents.

According to [1,2,3], all available countries participate in the process of international trade.

The economic crisis associated with the COVID-19 pandemic clearly demonstrates the interdependence of the world economy.

According to [4,5], routing and the volume of trade flows are important for the development of international trade.

In this regard, the construction of various economic and mathematical models describing international trade processes deserves attention.

One of such models is the linear model of international trade [6,7,8].

According to this model , there are budgets N countries that we will designate y_1, y_2, \dots, y_n .

Denote by A_{ij} part of the budget y_j , which it spends on purchasing goods and services from the country j .

A_{ij} can be represented as a square matrix:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \quad (1)$$

where a_{ij} – are fractional real numbers and

$$\sum_{i=1}^n a_{ij} = 1 , \text{ where } j = 1, 2, \dots, n \quad (2)$$

The economic meaning of the coefficient a_{ij} means the share of the budget that the i -th country spends on the purchase of goods and services of the j -th country

$$a_{ij} = \frac{y_{ij}}{y_j} \quad (3)$$

According to the linear model of international trade [4]

expenses N_i countries have equal rights to purchase services and goods on the international market :

$$y_i = a_{1i} * y_1 + a_{2i} * y_2 + \dots + a_{ni} * y_n \quad (4)$$

where $i = 1, 2, \dots, n$

Table 2. Coefficients C_{ij} defining the part of exports that the i -th country exports to country j .
Calculated by the authors, based on the data in Table 1 by solving equation (11)

Year	Czech Republic	Slovakia	Poland	Hungary
2020	1,390990747	-0,106091918	1,374595747	1,580743321
2019	1,411896198	0,189279923	1,186032271	1,337428423
2018	1,239798912	0,31339237	1,073700654	1,612624807
2017	1,268291741	0,113627008	1,30793229	1,552705712
2016	1,128879647	-0,030531617	1,53898458	1,79684679
2015	1,218362359	0,032975538	1,396675514	1,787239015
2014	1,464221995	0,113586689	0,345668709	1,73255559
2013	1,184125935	0,060583828	1,302409171	2,117036881
2012	1,140508321	-0,224681679	1,683440981	2,484506878
2011	1,199701001	-0,211988848	1,615686583	2,320812357

Table 3. Import volumes between the Visegrad Four countries (Czech Republic, Slovakia, Poland and Hungary) in 2010-2020 (US\$ Thousand) excluding reimport [10]

Year	Czech Republic	Slovakia	Poland	Hungary
2020	32984258	17319930	22235556	16251892
2019	33622740	19286320	22245752	17001778
2018	33600172	20264886	23168270	17032432
2017	30271892	17273464	20142908	15159672
2016	27570444	15522488	18821282	13116132
2015	28055532	15744729	17670184	13078844
2014	29927096	17390100	19210870	14885218
2013	28188180	17075256	18211044	14474826
2012	27299204	15857072	17086880	13764482
2011	28491534	16362288	18010224	14371400

Table 4. Coefficients b_{ij} defining the part of the import that the i -th country imports from the j -th country. Calculated by the authors based on the data in Table 1 by solving equation (12)

Year	Czech Republic	Slovakia	Poland	Hungary
2020	0,566050957	1,878062106	1,281251389	0,560157909
2019	0,616711123	1,549487409	1,359243126	0,664623158
2018	0,799982199	1,575866333	1,230310798	0,396143905
2017	0,725630257	1,724362891	1,181757145	0,481012397
2016	0,652669817	1,933818219	0,955206042	0,689233416
2015	0,632029043	1,774862526	1,182067306	0,610550943
2014	0,654935167	1,395186173	1,9402279	1,460190014
2013	0,864726866	1,883214507	1,197776402	-0,027281972
2012	1,126833817	2,131827	1,173984575	-0,771427278
2011	0,95202884	2,181460421	1,078919935	-0,34892837

Within the framework of this study , additional conditions are accepted everywhere : countdown .

Table 5. Additional conditions accepted in the models

Year	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011
Number	1	2	3	4	5	6	7	8	9	10

We show that the numerical sequence

{24974282 , 25810272 , 27732344 , 24299106 , 22360564 , 21721962 , 23563600 , 22384086, 21788932 , 21992210 , 17279962}

The characteristic volume of exports of the Czech Republic within the Visegrad Four countries , excluding re-exports , can be interpolated by the function w_1

(13)

$$\begin{aligned}
 w_1 = & (-4.08520666114374e-10+4.14524148579626e- \\
 & 25i) * \exp((2.37338686394796+3.14159265358979i) * x) + (30001.1890805311- \\
 & 58388.8910331044i) * \exp((0.256866644324129+1.99228363957843i) * x) + (30001.1890805313+58388.89 \\
 & 10331044i) * \exp((0.256866644324129-1.99228363957843i) * x) + (- \\
 & 3111769.47283749+2.56063215910932e-09i) * \exp((-0.461564731555856+3.14159265358979i) * x) + (- \\
 & 442542.526573902-1516793.61163557i) * \exp((-0.0728507057041561+0.938319644384922i) * x) + (- \\
 & 442542.526573884+1516793.61163558i) * \exp((-0.0728507057041561- \\
 & 0.938319644384922i) * x) + (31416033.9221922-2.15365882545206e-08i) * \exp((- \\
 & 0.0460042164077144+0i) * x) + (-19130928.4093645+8.65172281004634e-08i) * \exp((- \\
 & 0.766940515696017+0i) * x) + (2.1137490078327e-06+2.67529322924004e- \\
 & 07i) * \exp((2.37338686394796+3.14159265358979i) * x) + (4.2957400446057e-09+2.00793646123045e- \\
 & 08i) * \exp((0.256866644324129+1.99228363957843i) * x) + (8.80158167740605e-09-1.57831435833431e- \\
 & 08i) * \exp((0.256866644324129-1.99228363957843i) * x) + (-2.04556583800745e-12-2.01901006255297e- \\
 & 12i) * \exp((-0.461564731555856+3.14159265358979i) * x) + (-2.32003592157845e-09+4.4704514321586e- \\
 & 09i) * \exp((-0.0728507057041561+0.938319644384922i) * x) + (3.42189363469204e-09- \\
 & 5.33199648983133e-09i) * \exp((-0.0728507057041561-0.938319644384922i) * x) + (8.62863063471562e- \\
 & 08+1.02767726570125e-08i) * \exp((-0.0460042164077144+0i) * x) + (-9.20175732715796e-13- \\
 & 2.31791880419569e-13i) * \exp((-0.766940515696017+0i) * x);
 \end{aligned}$$

where $i = \sqrt[2]{-1}$ here and further in the text the imaginary unit.

Indeed

Table 6. Calculation of the sequence of export indicators of the Czech Republic within the framework of the countries of the Visegrad Four, excluding re-exports produced according to the formula (13)

x	Calculated by the formula $w_1(x)$	Tabular data [10]
1	24974281.9999774-2.87057908590126e-06i	24974282
2	25810272.0002435+3.08191105230516e-05i	25810272
3	27732343.997386-0.000330850073674354i	27732344
4	24299106.0280573+0.00355112243910283i	24299106
5	22360563.6988399-0.0381167173666179i	22360564
6	21721965.2325581+0.409132841824994i	21721962
7	23563565.3027425-4.39150238195704i	23563600
8	22384458.4294058+47.1369998704742i	22384086
9	21784934.4603261-505.953676783066i	21788932
10	22035118.3289182+5430.74704743642i	21992210

We show that the numerical sequence

{24742058 , 25384420 , 25629902 , 23243614 , 21826716 , 22116876 , 24263934 , 24198162 , 24326646 , 25070042 , 19832150}

The characteristic volume of exports of the Slovak Republic within the Visegrad Four countries, excluding re-exports, can be interpolated by the w_2 function

(14)

$$\begin{aligned}
 w_2 = & (-0.0163392544743643-6.39439418000371e-17i) \cdot \exp((1.31177175580263+0i) \cdot x) + (- \\
 & 871.49584034297-62005.4840072101i) \cdot \exp((0.260768417723383+2.0444228881267i) \cdot x) + (- \\
 & 871.495840342668+62005.4840072101i) \cdot \exp((0.260768417723383- \\
 & 2.0444228881267i) \cdot x) + (295124.69174797- \\
 & 610421.941162697i) \cdot \exp((0.0641231171789403+0.850641127714897i) \cdot x) + (295124.691747963+61042 \\
 & 1.941162701i) \cdot \exp((0.0641231171789403-0.850641127714897i) \cdot x) + (25843157.2412407- \\
 & 6.50896491283694e-08i) \cdot \exp((-0.0150413776553088+0i) \cdot x) + (7269370.31762103- \\
 & 3076820.54086012i) \cdot \exp((- \\
 & 0.989619053207588+2.36524261069861i) \cdot x) + (7269370.31762107+3076820.5408604i) \cdot \exp((- \\
 & 0.989619053207588-2.36524261069861i) \cdot x) + (-1.31411084768241e-06+3.58020080995357e- \\
 & 07i) \cdot \exp((1.31177175580263+0i) \cdot x) + (-2.11938362272338e-08-4.01689589259178e- \\
 & 08i) \cdot \exp((0.260768417723383+2.0444228881267i) \cdot x) + (-4.24097671136809e-10+3.42425975627777e- \\
 & 08i) \cdot \exp((0.260768417723383-2.0444228881267i) \cdot x) + (2.47373573637538e-08+4.13314202980139e- \\
 & 08i) \cdot \exp((0.0641231171789403+0.850641127714897i) \cdot x) + (-1.93915591637177e-09- \\
 & 5.41921666623471e-08i) \cdot \exp((0.0641231171789403-0.850641127714897i) \cdot x) + (-1.06629202284247e- \\
 & 06+5.14336394782463e-08i) \cdot \exp((-0.0150413776553088+0i) \cdot x) + (-1.88521075035221e- \\
 & 14+1.57179644641764e-14i) \cdot \exp((-0.989619053207588+2.36524261069861i) \cdot x) + (- \\
 & 2.52323942833382e-14-4.49844480449435e-15i) \cdot \exp((-0.989619053207588-2.36524261069861i) \cdot x);
 \end{aligned}$$

Really,

Table 7. Calculation of the sequence of export indicators of the Slovak Republic within the Visegrad Four countries, excluding re-exports produced according to the formula (14)

x	Calculated by the formula $w_2(x)$	Tabular data [10]
1	24742057.9999955+1.23112945677511e-06i	24742058
2	25384419.9999817+5.00199278679591e-06i	25384420
3	25629901.9999327+1.83440136362668e-05i	25629902
4	23243613.9997507+6.79584129900352e-05i	23243614
5	21826715.9990731+0.000252594733400035i	21826716
6	22116875.9965582+0.000937671575230591i	22116876
7	24263933.9872217+0.00348139607040852i	24263934
8	24198161.9525545+0.0129262959531739i	24198162
9	24326645.8238469+0.047991603194718i	24326646
10	25070041.3459912+0.178180086435988i	25070042

We show that the numerical sequence

{ 24693178 , 25583234 , 25956312 , 22184332 , 19257960 , 19285376 , 21419132 , 20165664 , 18319448 , 19656612 }

The characteristic volume of Poland 's exports within the Visegrad Four countries , excluding re- exports , can be interpolated by the function w_3

(15)

$$w_3 = (-0.012886998343324 - 7.79479893450277e-17i) \cdot \exp((1.27731665299375 + 3.14159265358979i) \cdot x) + (-26148.542387492 - 85223.9515803332i) \cdot \exp((0.231446190867208 + 2.03327095764704i) \cdot x) + (-26148.542387492 + 85223.9515803333i) \cdot \exp((0.231446190867208 - 2.03327095764704i) \cdot x) + (-7209106.11790765 - 5944206.33302076i) \cdot \exp((-0.385637270252897 + 1.01242198548796i) \cdot x) + (-7209106.11790765 + 5944206.33302077i) \cdot \exp((-0.385637270252897 - 1.01242198548796i) \cdot x) + (31812974.6601666 - 1.21146516062585e-08i) \cdot \exp((-0.0563913787363134 + 0i) \cdot x) + (-25370571.0146267 + 3.59519234839984e-08i) \cdot \exp((-0.493145904907284 + 0i) \cdot x) + (-63190621.3021273 + 3.89941553995001e-07i) \cdot \exp((-2.03778226763692 + 3.14159265358979i) \cdot x) + (1.79970812380328e-07 - 1.16871478100056e-07i) \cdot \exp((1.27731665299375 + 3.14159265358979i) \cdot x) + (1.96599872536313e-08 - 9.43849496114752e-09i) \cdot \exp((0.231446190867208 + 2.03327095764704i) \cdot x) + (2.28539542656307e-08 + 1.00631003836365e-08i) \cdot \exp((0.231446190867208 - 2.03327095764704i) \cdot x) + (1.20816720408599e-11 + 1.31882418944944e-10i) \cdot \exp((-0.385637270252897 + 1.01242198548796i) \cdot x) + (1.44858199437027e-11 - 1.47993611011311e-10i) \cdot \exp((-0.385637270252897 - 1.01242198548796i) \cdot x) + (1.4359227834196e-07 + 4.39745723289906e-09i) \cdot \exp((-0.0563913787363134 + 0i) \cdot x) + (-1.43592513018963e-10 - 4.6107493199272e-12i) \cdot \exp((-0.493145904907284 + 0i) \cdot x) + (6.5732542893469e-21 + 2.82526139685141e-22i) \cdot \exp((-2.03778226763692 + 3.14159265358979i) \cdot x);$$

Really,

Table 8. Calculation of the sequence of Polish export indicators within the Visegrad Four countries, excluding re-exports produced according to the formula (15)

x	Calculated by the formula $w_3(x)$	Tabular data [10]
1	24693177.9999995+3.56217571072714e-07i	24693178
2	25583234.0000023-1.48230482760475e-06i	25583234
3	25956311.9999917+5.3932593125399e-06i	25956312
4	22184332.0000298-1.93581002133078e-05i	22184332
5	19257959.999893+6.94040332794587e-05i	19257960
6	19285376.0003834-0.00024893719319945i	19285376
7	21419131.9986249+0.000892934995135277i	21419132
8	20165664.004932-0.00320302842122929i	20165664
9	18319447.9823077+0.0114892908858138i	18319448
10	19656612.0634627-0.0412120913178581i	19656612

We show that the numerical sequence

{ 14382118 , 15378665 , 14747202 , 13120882 , 11585107 , 11425076 , 12166618 , 11201395 , 9572614 , 10516582}

Характеризующую объем экспорта Венгрии в рамках стран вышеградской четверки без учета реэкспорта можно интерполировать функцией w_4

(16)

$$\begin{aligned}
 w_4 = & (-0.00237091908282981 - \\
 & 0.00237198737374214i) \cdot \exp((1.47101911100826 + 0.250677763547052i) \cdot x) + (- \\
 & 0.00237091908282981 + 0.00237198737374214i) \cdot \exp((1.47101911100826 - \\
 & 0.250677763547052i) \cdot x) + (2158.99064587232 + 5.63854303209847e- \\
 & 12i) \cdot \exp((0.399638046114549 + 3.14159265358979i) \cdot x) + (-16667.8033475479 - \\
 & 15975.4603916164i) \cdot \exp((0.306640687539077 + 2.09197105388936i) \cdot x) + (- \\
 & 16667.8033475479 + 15975.4603916164i) \cdot \exp((0.306640687539077 - \\
 & 2.09197105388936i) \cdot x) + (15914278.6699005 - 2.14154884275405e-11i) \cdot \exp((- \\
 & 0.0482520995521711 + 0i) \cdot x) + (-934108.777234004 + 235013.809382657i) \cdot \exp((- \\
 & 0.161136494422463 + 1.27980602149871i) \cdot x) + (-934108.777234005 - 235013.809382656i) \cdot \exp((- \\
 & 0.161136494422463 - 1.27980602149871i) \cdot x) + (7.18942827546879e-08 - 8.34380671290221e- \\
 & 08i) \cdot \exp((1.47101911100826 + 0.250677763547052i) \cdot x) + (1.05156582769469e-07 + 7.34620176152269e- \\
 & 08i) \cdot \exp((1.47101911100826 - 0.250677763547052i) \cdot x) + (1.69232957448109e-09 + 3.61036535807993e- \\
 & 10i) \cdot \exp((0.399638046114549 + 3.14159265358979i) \cdot x) + (7.33969903731478e-09 - 5.54508903979964e- \\
 & 09i) \cdot \exp((0.306640687539077 + 2.09197105388936i) \cdot x) + (7.70082680293972e-09 + 6.66565935879364e- \\
 & 09i) \cdot \exp((0.306640687539077 - 2.09197105388936i) \cdot x) + (-2.62728387565418e-08 + 6.12102492898845e- \\
 & 12i) \cdot \exp((-0.0482520995521711 + 0i) \cdot x) + (-1.31744009321392e-10 + 6.37972352158789e-10i) \cdot \exp((- \\
 & 0.161136494422463 + 1.27980602149871i) \cdot x) + (-2.29984061429097e-10 - 6.46585294021235e- \\
 & 10i) \cdot \exp((-0.161136494422463 - 1.27980602149871i) \cdot x);
 \end{aligned}$$

Really,

Table 8. Calculation of the sequence of Hungarian export indicators within the Visegrad Four countries produced according to the formula (15)

x	Calculated by the formula w_4	Tabular data [10]
1	14382118.0000009-7.85878919348787e-08i	14382118
2	15378665.0000043-4.68679136266502e-07i	15378665
3	14747202.0000196-2.47554053179781e-06i	14747202
4	13120882.0000818-1.20031820371228e-05i	13120882
5	11585107.0003195-5.43014877760587e-05i	11585107
6	11425076.0011466-0.000230523414378661i	11425076
7	12166618.0036131-0.000915421278099986i	12166618
8	11201395.0087462-0.00335198400559095i	11201395
9	9572614.00529346-0.0109235832971812i	9572614
10	10516581.8788668-0.0286074735826604i	10516582

Table 8. Calculation of export-import data for individual countries for 2009

	Calculated by the authors based on the functions w_1, w_2, w_3, w_4	Tabular data [11]
Czech exports within the Visegrad Four countries in 2010 excluding re-exports	$1.6819e+07 - 5.8292e+04i$	17279962
Slovakia's exports within the Visegrad Four countries in 2010, excluding re-exports	$1.9832e+07 + 6.6154e-01i$	19832150
Poland's exports within the Visegrad Four countries in 2010, excluding re-exports	$1.6639e+07 + 1.4783e-01i$	16639277
Hungary's exports within the Visegrad Four countries in 2010, excluding re-exports	$8.8661e+06 - 3.4259e-02i$	8866136

Thus, we can write the BRINKS model of international trade between the Visegrad Four countries without taking into account re-export can be written in the following form

$$\begin{cases} b_{11} * Re(w_1(t)) + b_{12} * Re(w_2(t)) + b_{13} * Re(w_3(t)) + b_{14} * Re(w_4(t)) = Re(w_1(t)) \\ b_{21} * Re(w_1(t)) + b_{22} * Re(w_2(t)) + b_{23} * Re(w_3(t)) + b_{24} * Re(w_4(t)) = Re(w_2(t)) \\ b_{31} * Re(w_1(t)) + b_{32} * Re(w_2(t)) + b_{33} * Re(w_3(t)) + b_{34} * Re(w_4(t)) = Re(w_3(t)) \\ b_{41} * Re(w_1(t)) + b_{42} * Re(w_2(t)) + b_{43} * Re(w_3(t)) + b_{44} * Re(w_4(t)) = Re(w_4(t)) \end{cases} \quad (17)$$

где $b_{11} = b_{22} = b_{33} = b_{44} = 0$,

Discussion

The use of interpolation of time series by sums of exponents allows you to achieve a good result when constructing an interpolation function. Although time series are interpolated by functions of a complex variable, if the model (17) does not take into account the imaginary part of the numbers arising during the calculation w_i , модель может иметь практическое применение

Interpolation of time series by the sum of exponents of a function of a complex variable gives an approximation no worse than using regression analysis. To calculate the interpolating function, the author used standard procedures used in the MATLAB software.

The absence of extremum points for exponents is the main advantage when using exponent sums for interpolation purposes compared to interpolation by polynomials.

Conclusions

The use of complex variable functions for the interpolation of numerical series makes it possible to expand the capabilities of researchers to build more accurate economic and mathematical models describing socio-economic processes.

The use of the sum of exponents of the complex variable function as an interpolation function makes it possible to achieve the accuracy required for practical application, while using standard software packages.

References

1. H. Krejčí, M. Stárová, I. Hrbek, M. Navrátilová, M. Beranová, J. For. Sci. **65(6)**, 226-233 (2019). doi: 10.17221/138/2018-JFS
2. Moravec, L., Hinke, J., & Kaňka, S. (2018). VAT Gap Estimation - Czech Republic Case Study. *Politická ekonomie*, 66(4), 450-472. doi: 10.18267/j.polek.1212
3. Tousek, Z., Hinke, J., Malinska, B., & Prokop, M. (2021). The Performance Determinants of Trading Companies: A Stakeholder Perspective. *Journal of Competitiveness*, 13(2), 152–170. <https://doi.org/10.7441/joc.2021.02.09>
4. Ugurlu, Erginbay, and Irena Jindřichovská. 2022. "Effect of COVID-19 on International Trade among the Visegrad Countries" *Journal of Risk and Financial Management* 15, no. 2: 41. <https://doi.org/10.3390/jrfm15020041>

5. Galetska, Tetiana & Topishko, Natalia & Galetskyi, Sergii. (2022). GRAVITY MODEL OF INTERNATIONAL TRADE: ORIGIN AND MODERN APPROACHES. 1. 96-103. 10.25264/2311-5149-2022-27(55)-96-103.
6. Sahho, Nazhan & Khudair, Moneim. (2023). Reading in International Trade Reality, Importance and Indicators/Iraq as a Model During the Period from 2003-2019. Journal of Humanities and Social Studies. 1. 9-17.
7. Blahun, Ivan & Nadvirnianskyi, Yulian. (2022). The Model of Economic Analysis of International Trade Between Ukraine and the European Union. Business Inform. 11. 76-81. 10.32983/2222-4459-2022-11-76-81.
8. Anwar, Ahmad & Wicaksono, Agung. (2022). GRAVITY MODEL TO UNDERSTAND CHINA INTERNATIONAL TRADE: A REVIEW LITERATURE. 8. 77-84.
9. Leont'ev A.F. Riady eksponent . Glavnaja redakcia fiziko-vatematicheskoy literatury izdatelstva "Nauka" , 1976 , 536 p. (In Russian)
10. <https://wits.worldbank.org/CountryProfile/en/>