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**DIAGNOSING PRIMARY PUPILS' LEARNING OF THE
CONCEPT OF *AFTER* IN THE TOPIC *TIME* THROUGH
KNOWLEDGE STATES BY USING COGNITIVE DIAGNOSTIC
ASSESSMENT**

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ABSTRACT

Purpose – Knowledge state specifies pupils' mastery level and informs about their strength and weaknesses in the tested domain. This study attempted to diagnose primary pupils' learning the concept of *after* through their knowledge states by using Cognitive Diagnostic Assessment (CDA).

Methodology – This study employed a survey research design to gauge pupils' knowledge states for the concept of “finding the date after a specific number of days from a given date” [abbreviated as the concept of *after*]. Quantitative data from the pupils' pattern of response to the items in Cognitive Diagnostic Assessment (CDA) were collected and analyzed. Items in the CDA were designed by three experienced Mathematics Education researchers and content validated by a panel of seven expert primary mathematics teachers. It was then administered to 238 Grade Six pupils from 11 primary schools in Penang, Malaysia.

The pupils' item responses were interpreted into knowledge states and mastery levels.

Findings –The overall analysis showed that there were 18 knowledge states diagnosed in the concept of *after*. This large number of knowledge states indicated the specificity of pupils' mastery level and thus provided detailed information about their strengths and weaknesses in the concept of *after*. The findings of this study imply that primary pupils face different levels of difficulty when they are learning the topic of *Time*.

Significance – This method of diagnosing pupils' knowledge in terms of mastery level of each attribute tested is different from a conventional diagnostic test which provides only the final score for each pupil. By knowing the pupils' knowledge states, teachers can make use of this fine-grained information to enable them to carry out differentiated instructional planning and other remedial work more effectively. Pupils can also use this information to monitor their own learning by maintaining their strengths and overcoming their weaknesses to cope with their own studies.

Keywords: knowledge state, time, date, Cognitive Diagnostic Assessment, concept of *after*, mathematics

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INTRODUCTION

Time is one of the essential concepts to be learnt in school since its application is needed in everyday life. Daily activities revolve around the concept of time. Hence, in the school mathematics curriculum of many countries, including Malaysia, pupils are required to learn the topic of time from pre-school level. However, time is a complex and abstract topic which covers many aspects such as clock reading, conversion between units of time, calendar reading, calculation of duration and many others. Through informal interviews, several primary school teachers have voiced concern that their pupils were weak at mathematical tasks involving time, particularly in the calculation of duration and conversion of different units of time.

The following shows two examples of time-related problems that pupils have difficulty in solving:

- (1) Today is 26 November. If Linda is celebrating her birthday 10 days later, when will Linda's birthday be?
- (2) Zaid started an English language course on 27 February, 2014. He completed the course on 3 March, 2014. How many days was his course?

To solve the above problems, the pupils need to first consider whether the given starting date (i.e., 26 November or 27 February, 2014) should be included in the calculation of the total number of days. According to mathematics teachers, many pupils have difficulties in solving these problems, which involve the concept of *after*. The concept of *after* refers to “finding the date after a specific number of days from a given date”. When these pupils are asked, they are unable to pin-point exactly what they do not know and also what they already know.

Indeed, there are many possible causes of their errors. For example, for problem (1), the pupils who answer wrongly might not know that November has 30 days or that December is the month which comes after November. Likewise, for problem (2), the pupils might not know that the starting date should be counted as one day in the calculation of duration; some pupils might be unsure whether February in the year of 2014 has 28 or 29 days, thus resulting in wrong answers.

In such a situation, the teachers need an effective diagnostic tool that provides detailed information about which skills or knowledge the pupils have or have not mastered. However, as argued by Buhagiar (2007), the most conventional kinds of assessment are ‘unfriendly’ in four aspects, namely towards learning, curriculum, teacher and pupil. These conventional assessments are considered as learning and curriculum “unfriendly” because they tend to reveal only the contents that the learners have remembered rather than provide a comprehensive picture of the mastery level of their knowledge or skills. They are teacher “unfriendly” because the assessment focuses on rote-learning more than conceptual understanding. Consequently, teachers cannot use the score to track their pupils’ progress. Lastly, the assessment scores are considered pupil unfriendly because pupils are labelled as high or low performers based on their scores, rather than providing a meaningful analysis of the pupils’ learning progress. Nevertheless, this kind of diagnostic information may be gained through observing pupils’ knowledge states.

Hence, a larger study was conducted, which aimed to develop a Cognitive Diagnostic Assessment (CDA) tool for the topic of *Time* in primary school learning. This assessment tool was developed based on both experts’ (teachers’) and students’ perspectives. The

student sample of the study involved 1749 Year Six pupils from 17 primary schools in Penang, Malaysia, while the teacher sample involved seven experienced mathematics teachers with six to 31 years of teaching experience in primary schools.

For the purpose of this paper, only data obtained from 238 Grade Six pupils from 11 primary schools in Penang were analyzed. The main aim of this paper is to discuss how primary pupils' learning on the topic of *Time* can be diagnosed through their knowledge states by using CDA. This paper focuses on how the diagnosed knowledge states reflect pupils' learning in terms of mastery level for the concept of *Time* involving dates, particularly the concept of *after*.

LITERATURE REVIEW

CDA for the topic *Time*

Cognitive Diagnostic Assessment (CDA) is an educational test designed to measure examinees' cognitive process, learning, specific knowledge structures and skills for diagnostic purposes (Ketterlin-Geller & Yovanoff, 2009). In CDA, items are designed to diagnose and measure pupils' specific knowledge structures and processing skills for a particular topic (Leighton & Gierl, 2007a). Each item is characterized by specific cognitive attributes that measure domain-specific structural knowledge, skill and process.

There are three elements to consider when designing the framework of a diagnostic assessment. They are: (1) conjecturing the interaction between the attributes assessed in a test with each other; (2) applying a reliable, accurate, and efficient statistical estimation method; and (3) conducting effective implementation procedures for real data analyses (Hartz & Roussos, 2008; Su, 2013). Ketterlin-Geller and Yovanoff (2009) pointed out that attributes can be identified by studying the knowledge, process and skills used in solving the task.

In this study, the term *attributes* is equivalent to basic knowledge, skills, or cognitive processes. The process of identifying the attributes for the topic *Time* started with conducting item analysis and reviewing mathematics textbooks. The researchers

then analyzed pupils' working steps in solving the given items and identified the attributes required in solving each item. Expert reviews and comments were also sought to define a list of attributes. Finally, a verbal protocol analysis was carried out. This involved the identification of attributes through think-aloud sessions with pupils while solving the given items. The panel of experts then arranged these attributes hierarchically based on their own knowledge and teaching experiences and generated an expected response pattern in Q matrix which specified pupils' expected response patterns. Subsequently, three items were developed for each attribute based on the attribute hierarchy.

Knowledge state in CDA

Knowledge state is a systematic way of expressing the mastery level of a test taker in the respective tested domain. Knowledge state informs pupils of their strengths and weaknesses based on a unique combination of attributes. While an attribute of a task is a description of "procedural and declarative knowledge" (Gierl, Wang, & Zhou, 2008, p. 3), it also refers to a skill or a process needed to successfully solve the given tasks. Building on the necessary skills or processes (i.e., attributes), a cognitive model (Leighton & Gierl, 2007b) is constructed to describe how a test taker solves standardized educational tasks at different levels of learning. The tested attributes in a cognitive model must be specific and fine-grained so that precise diagnostics inferences can be made based on the test performance of a pupil. This kind of assessment which serves both cognitive and diagnostic purposes is known as Cognitive Diagnostic Assessment [CDA].

The result of CDA, expressed in the form of *knowledge state*, provides detailed diagnostic information about a learner's learning process in terms of mastery or non-mastery of a particular skill or sub-skill. Knowledge state refers to patterns of attribute mastery (Cui, Leighton, & Zheng, 2006) which represents the pupil's unobserved latent variable. Tatsuoka (2009) maintains that it is impossible to directly observe the underlying knowledge and cognitive skills of the pupils and hence they are named latent variables. It is not an easy task to measure pupils' latent variables. Birenbaum, Kelly, and Tatsuoka (1993) diagnosed pupils' knowledge states in algebra. The pupils were classified into different knowledge states based on their

solution approaches. In the same year, Birenbaum and Tatsuoka (1993) also diagnosed pupils' state of knowledge in multiplication and division of quantities with exponents. They listed out 25 most frequent knowledge states of the pupils and suggested remediation routes for specific pupils to achieve all-mastery knowledge state.

Due to a lack of existing diagnostic tests, CDA has been retrofitted to existing proficiency tests. Two commonly used psychometric methods to model the hierarchical dependencies among attributes underlying examinees' problem solving on test items are Rule Space Method or RSM (Tatsuoka, 1985) and the Attribute Hierarchy Method or AHM (Leighton, Gierl, & Hunka, 2004). RSM has been widely applied to classify students into a dichotomous pattern of attribute mastery and non-mastery (i.e., knowledge states). There have been many applications of RSM in the past that used TIMSS released items to study pupils' knowledge states. Um, Dogan, Im, Tatsuoka and Corter (2003) used RSM to compare knowledge states of eighth graders from Korea, Czech and America using TIMSS-R 1999 released items. Similarly, Birenbaum, Tatsuoka and Yamada (2004) compared the knowledge states of eighth graders from the United States, Japan and Israel using TIMSS-Revised (TIMSS-R) items for the year 1999. In the same year, Tatsuoka, Corter, and Tatsuoka (2004) studied samples of 20 countries on the patterns of diagnosed specific mathematical contents and process skills using TIMSS-R items in 1999. Using RSM, Dogan and Tatsuoka (2008) analyzed the knowledge states of Turkish pupils' mathematics skills using TIMSS-R 1999 released items. Besides RSM, many other researchers (for example, Gierl, Alves, & Majeau 2010a, 2010b; Roberts & Gierl, 2010; Su, 2013) have also developed items and classified examinee profiles into different attribute patterns using the structured hierarchical AHM models.

Due to the paucity of literature measuring pupils' knowledge states for the topic of *Time* involving dates, this study intended to bridge the gap in the literature with an in-depth examination of pupils' learning of *Time* involving dates.

Past studies on the topic of Time

There have been a few studies (De Coster, 2004; Friedman, 1986; Godard & Labelle, 1998; Labrell, Mikaeloff, Perdry, & Dellatolas, 2016) on the learning of *Time*. Friedman (1986) indicated that

children start using their imagination to solve calendar-related tasks by the age of 10. Godard and Labelle's (1998) study on the learning of the conventional time system among 5 to 9 year-old children included the capacity to evaluate the interval of time between the present and some important events such as birthdays, and the capacity to make judgments of the relationship between two moments in time. They found that the three factors which influenced the learning of these examined concepts were: (1) the expansion of the conceptual span; (2) the ability to associate these concepts with mental images or personal experience; and (3) verbal mediation. In addition, De Coster (2004) further studied the acquisition of the concept of time and verified the three factors in Godard and Labelle (1998), and concluded that the construction of the concept *Time* and time representations arise from the contribution and interweaving of multiple contributions: cognitive, emotional, environmental and language. Apart from that, Labrell, Mikaeloff, Perdry, and Dellatolas (2016) researched among 6-11 year-old children and found that there was an association between numerical skills and time knowledge. Nevertheless, to the best knowledge of the authors, there is yet any literature examining the concept of *after* among primary pupils. Hence, this paper aims to contribute in this field by proposing an alternative form of diagnostic assessment that expresses pupils' knowledge states in learning *Time* involving dates, focusing on the concept of *after*.

THEORETICAL FRAMEWORK

Since the main focus of this study was to diagnose how pupils learn the concept of time, the underpinning theory was schema theory (Burgin, 2005; Marshall, 1993; Skemp, 1986). Schema theory is an explanation of how pupils use their prior knowledge to solve current problems that they are facing (Skemp, 1986). Schema theory is "an approach to knowledge representation, organization, processing, and utilization" (Burgin, 2005, p. 2). Hence, schema has the same meaning as a conceptual structure (concept). A pupil's schema could be represented by his/her concept of understanding. Pupils' concept could be assessed by evaluating the content of a schema (Marshall, 1993). To assess the schema knowledge, the assessment should account for what knowledge is stored in the schema. There are four types of knowledge in the schema content: feature recognition,

constraint, planning/goal setting and execution. For example, a pupil will activate his or her existing schemas that he or she has constructed previously when solving similar tasks. The pupil will then decide whether the declarative facts in the activated schema correspond to the present task (i.e., feature recognition). After that, the pupil will examine if the declarative facts in the schema is sufficient to enable the schema to be used (i.e., constraint). The pupil must then decide whether he or she can use the procedural rules in the schema to achieve the goal of the task (i.e., planning or goal setting). Finally, the pupil shows the procedure to achieve the goal and solve the problem (i.e., execution).

Adopting the idea of Marshall (1993) on cognitive assessment, CDA diagnoses how fully developed the components of schema knowledge of an individual are. The components of schema refer to the elements stored in the schema. For example, in a schema that involves “finding the date after 10 days from a given date (e.g., 26 November, 2016)”, the components will include: element (1) November = 11th month of the year; element (2) November has 30 days; element (3) counting forward 10 days from 26 or adding 10 to 26, then regrouping 36 to 6 and element (4) December is the next month after November. Building on that, the components of schema are executed by the cognitive model and represent the result of pupils’ learning in knowledge state. Knowledge state is a systematic form of expressing explicitly the presence or absence of a specific schema component (as described in attribute) needed to solve a problem.

There are four defining characteristics of cognitive models for CDA: granularity, hierarchy of ordered attributes, measurability, and instructional relevancy (Gierl, Roberts, Alves, & Gotzmann, 2009). In this study, the cognitive model was used to assess the components of schema knowledge and to make it explicit as knowledge state because the characteristics of a cognitive model are similar to that of a schema.

An attribute in a cognitive model is a fine-grained, systematic description of the elements in a schema. The knowledge described in an attribute of a cognitive model is similar to the knowledge stored in a schema when executing feature recognition and constraint. The

process and skills illustrated in an attribute is similar to deciding which procedural rules to be used in planning or goal setting, and then executing it to solve the problem. Hence, the fine-grained characteristic of attributes is similar to that of the elements. Element in a schema is also fine-grained in a single schema component within its component network of schemas (Marshall, 1993).

Similar to the hierarchy of ordered attributes in a cognitive model, the schema is also hierarchically linked from basic elements to components of schema, and then developed into a network of schemas. The attributes in a cognitive model are ranked hierarchically from simple to more complex knowledge, process and skills. On the other hand, the linkage in schema is represented as nodes and links, with the lines connecting them representing the association among the facts and rules.

The measurability of a cognitive model is similar to the measurability of the schema. The items test an individual's nodes within a component of a particular schema by asking questions that call for knowledge only of that piece of information. The pupils' responses to the item are the basis for an estimate of the absence or presence of the corresponding node in the network. In a cognitive model, the skills are well specified, allowing the pupils to demonstrate their skill mastery through problem solving so that the test can accurately measure the application of the knowledge, skills, and processes possessed by them.

Both the cognitive model and schema are instructionally relevant. Since this study used experts' schemas as the basis in designing the items, the knowledge, processes and skills tested in the items of the CDA were relevant instructionally to the pupils. Hence, the relationship between the schema and cognitive model was used in interpreting the mastery level of pupils for the attributes and eventually presented as knowledge states of pupils.

CONCEPTUAL FRAMEWORK

Figure 1 shows the relationship between the four main constructs of the study, which are: schema, cognitive model, attributes and knowledge state.

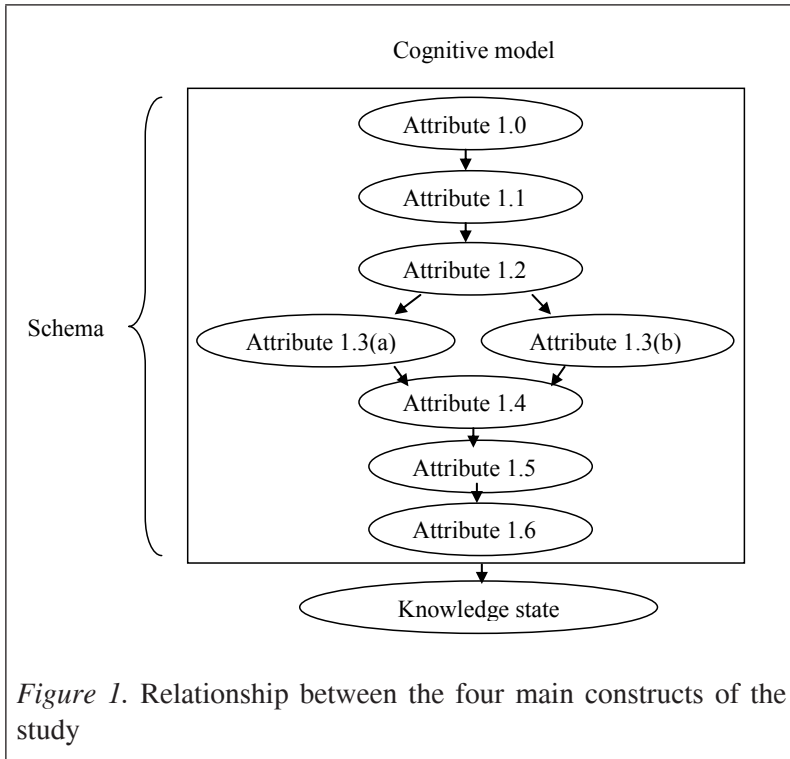


Figure 1. Relationship between the four main constructs of the study

In this study, the schema used by the panel of expert teachers when they responded to the mathematics items on the topic of *Time* served as the source to generate a cognitive model. A cognitive model is made up of attributes (i.e., knowledge, skills and processes used by the experts) which are hierarchically arranged from simple to complex. In this study, the pupils' responses to the items of each hierarchy of attributes in the cognitive model were analyzed and made explicit as the knowledge states of the pupils.

The mastery level of pupils on each hierarchy of knowledge was inferred through their responses to the items of each attribute. The responses of the pupils were analyzed using a psychometric method. The knowledge state of a pupil for the cognitive model was identified by using the result of the analysis, which specified the strengths and the weaknesses of the pupil in the attributes (components of schema) measured. Every individual pupil would have his or her own knowledge state in each cognitive model. In a way, knowledge state is an attempt at making the schemas of the pupils explicit, to inform about pupils' mastery level on a particular topic.

METHODOLOGY

This study employed a survey research design to gauge the pupils' knowledge states for the concept of 'after'. Quantitative data was collected by using the pupils' patterns of response in answering the CDA items.

Sample

The purposive sampling technique was used in selecting the sample. The targeted sample was chosen from four national schools (SK), three Chinese primary schools (SJKC) and four Tamil primary schools (SJKT) within Penang state. These 11 primary schools were average-performing primary schools which was ranked in Band 3 to Band 5 by the Malaysian Ministry of Education. For this study, only average performing schools were selected because the assessment was developed for diagnostic purposes.

According to the Performance Management and Delivery Unit [PEMANDU] (2010), Band 1 is the highest rank while Band 6 is the lowest rank for schools. Schools are evaluated and ranked according to their performance band based on a composite score. The composite score comprises the academic performance grade of the pupils (i.e., Grade Point Average of schools) in public examinations (such as the Primary School Performance Test [UPSR]) and the score of the school in the Standard for Quality Education in Malaysia [SQEM] (PEMANDU, 2010). SQEM is a self-evaluation tool which measures "vision and mission, organizational management, educational programme management and student accomplishment" (PEMANDU, 2010, p. 162).

For each sample school, only primary Year/Grade Six pupils were chosen to be administered the CDA. This was based on the assumption that pupils in Year Six would have been exposed to all the knowledge and skills required to answer the time-related items in this CDA.

Instrument

The instrument used was adopted from the larger study mentioned earlier, entitled "Developing a Two-Prong Cognitive Diagnostic

Assessment Model [CDAM] For Primary Mathematics Learning”. This instrument was developed based on the cognitive models designed by three researchers in Mathematics Education with a teaching experience ranging from 25 to 30 years, and content validated by a panel of seven expert primary mathematics teachers who have six to 31 years of teaching experience.

As stated in the Curriculum Standard and Assessment Document of Primary School Standard Curriculum (*Kurikulum Standard Sekolah Rendah* [KSSR]), pupils from Year One till Year Six were expected to equip themselves with knowledge and skills of *Time* which include the ability to:

- (1) Name the days and months and describe activities in line with the schooling days.
- (2) State time in hours and minutes.
- (3) State the relevance of the time.
- (4) Recognize the calendar and timetable in solving word problems.
- (5) Specify the relationship and convert between units of time involving:
 - (a) minutes and seconds;
 - (b) week and day;
 - (c) month and year;
 - (d) hours and minutes; and
 - (e) years, decades and centuries.
- (6) Calculate the duration between two given time in any unit by using the four basic operations.
- (7) The relationship between the 12-hours system and 24-hour system.
 - (i) Calculate the length of time in any unit of time.
 - (ii) Solve daily problems involving time, including time zone.

(Source. MOE, 2011a; 2011b; 2012; 2013b, 2014a; 2014b).

Based on the above KSSR curriculum specifications, past year UPSR examination questions and mathematics textbooks, an established CDA was developed following these five steps:

- Step 1: Identification of attributes;
- Step 2: Development of the attribute hierarchy or cognitive model;
- Step 3: Item design based on the attribute hierarchy;
- Step 4: Administration of assessment items on pupils; and
- Step 5: Statistical analysis on the collected data.

The established CDA [see details of CDA development in Lim, Sia, & Tan, in press] had 12 cognitive models. Models 1 to 8 focused on clock time while models 9 to 12 focused on calendar dates. The final CDA contained 126 items.

For this paper, our analysis focused only on one of the cognitive models, i.e., Cognitive Model 1 (CM 1) on the concept of *after*. As shown in Table 1, there were a total of eight attributes ranked in seven hierarchies of difficulty, from the most basic to the most complex. These attributes were measured by 24 items.

These items were initially constructed in the English language but later translated into Malay, Mandarin and Tamil languages. This was to ensure that test language would not be a barrier that interfered with the pupils' ability to answer these items. All the three versions were validated both in terms of content and language by the seven expert mathematics teachers, according to their respective expertise in their mother tongue, to ensure that all the items were equivalent linguistically across the three languages.

As shown in Table 1, there were a total of eight attributes i.e., A1.0 to A1.6, ranked in seven hierarchies and measured by a total of 24 items in this cognitive model. The attribute A1.0 ranked the lowest while the attribute A1.6 ranked the highest. This ranking implied that the attribute A1.0 was the most basic attribute which formed the foundation for the topic of *Time* involving dates. This attribute was also the prerequisite to all the other attributes. In brief, the hierarchy indicated that the lower ranking attributes were always fundamental and prerequisite to the attributes of higher ranking.

For example, items 1, 2 and 3 measured solely the attribute A1.0 while items 4, 5 and 6 measured both attributes A1.0 and A1.1. Pupils were expected to answer correctly the items 1, 2 and 3 before

they could answer items 4, 5 and 6 correctly. This was because items 4, 5, and 6 are more difficult than items 1, 2 and 3. Similarly, pupils were expected to be able to answer items 1 to 15 and show mastery in attributes A1.0, A1.1, A1.2, A1.3 (a) as well as A1.3 (b) before they would be able to answer items 16, 17 and 18 which measured the attribute A1.4.

Attributes A1.3 (a) and A1.3 (b) ranked the same in this attribute hierarchy. Even though the concept of “the starting month of 30 days” and “the starting month of 31 days” were two different concepts, they were equivalent to each other in terms of content complexity and level of difficulty. Hence, these two attributes ranked in the same hierarchy in this cognitive model.

Table 1

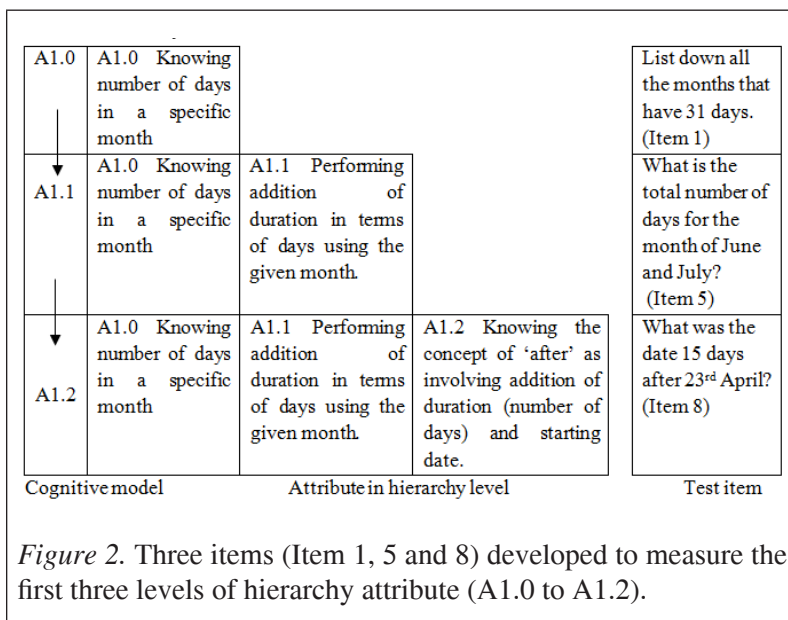
Description of attributes, hierarchy and the corresponding number of items in this cognitive model.

A	Description of attribute	Attribute Hierarchy								Item no.
A1.0	Knowing number of days in a specific month	A1.0								1,2,3
A1.1	Performing addition of duration in terms of days using the given month	A1.0	A1.1							4,5,6
A1.2	Knowing the concept of 'after' as involving addition of duration (number of days) and starting date.	A1.0	A1.1	A1.2						7,8,9
A1.3 (a)	Applying the concept of 'after' as addition; involving within two consecutive months for starting month with 30 days (for regrouping of 30 days into 1 month)	A1.0	A1.1	A1.2						10,11, 12
A1.3 (b)	Applying the concept of 'after' as addition; involving within two consecutive months for starting month with 31 days (for regrouping of 31 days into 1 month)	A1.0	A1.1	A1.2	A1.3 (a)	A1.3 (b)				13,14,15
A1.4	Applying the concept of 'after' as addition; involving more than two consecutive months with correct sequence of regrouping the number of days into one month	A1.0	A1.1	A1.2	A1.3 (a)	A1.3 (b)	A1.4			16,17,18
A1.5	Transforming word problem into mathematical operation to find the ending date of an event.	A1.0	A1.1	A1.2	A1.3 (a)	A1.3 (b)	A1.4	A1.5		19,20,21
A1.6	Expressing the final answer accurately as ending date for word problems	A1.0	A1.1	A1.2	A1.3 (a)	A1.3 (b)	A1.4	A1.5	A1.6	22,23,24

A=Attribute

To provide a clearer picture on how the hierarchy of the attributes in a cognitive model relates to the items, Figure 2 illustrates three items (Items 1, 5 and 8) developed to measure the first three levels of hierarchy attribute (A1.0 to A1.2). The attribute A1.0 is the first level of attribute hierarchy for the cognitive model and measured by item 1. The second level of attribute hierarchy is A1.1. Both A1.0 and A1.1 are measured by item 5. Subsequently, the third level of attribute hierarchy is A1.2. The three attributes, i.e., A1.0, A1.1 and

A1.2 are measured by Item 8. Refer to Lim, Sia, Chew, Kor, and Tan (2017) to know more about the items in each level of attribute hierarchy.



Analysis of data

To obtain the knowledge states, the pupils' responses towards the 24 items were analyzed using a psychometric method known as Attribute Hierarchy Method [AHM] (Leighton et al., 2004). AHM is a psychometric procedure which classifies pupils' responses towards the test items into different knowledge states associated with specific attributes in a cognitive model. Before obtaining the knowledge state of each pupil, their responses were calculated as attribute mastery probabilities using Artificial Neural Network (ANN) found in the MathLab software, 2012(b) version.

Table 2 shows the cut-off value for an attribute mastery probability and its corresponding value in a knowledge state. These cut-off values were adopted from the current reporting standard for the Acceptable Standard and the Standard of Excellence used by Alberta Education (Alves, 2011). Hence, depending on the cut-off value, each attribute mastery probability can be classified into three levels of mastery:

(a) non-mastery (<0.5); (b) inconsistent-mastery (0.5-0.8); and (c) mastery (>0.8). Subsequently, these mastery levels were represented as “0”; “1/2” and “1” in the knowledge state respectively.

Table 2

Cut-off value of attribute mastery probability used for classification and corresponding value represented in knowledge state.

Cut-off value of attribute mastery probability (Calculated by ANN)	Classification of attribute mastery probability	Value represented in knowledge state
<0.5	Non-mastery	0
0.5-0.8	Inconsistent-mastery	½
>0.8	Mastery	1

FINDINGS

To illustrate how we can diagnose pupils’ learning through the knowledge state, Table 3 displays a sample of 15 pupils’ responses to each of the 24 items, and their knowledge states calculated by ANN, on the concept of *after* in the topic of *Time*, involving dates.

Table 3

Knowledge state of a sample of 15 pupils’ on the concept of after.

ID	Item Number																								TS	Knowledge State
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24		
K056	0	0	1	0	1	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	1½000000
K057	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	22	11111111	
K058	1	1	1	0	1	0	1	1	1	1	0	1	1	0	1	0	0	0	0	0	0	0	0	11	111½0000	
K059	1	0	1	1	1	0	1	1	1	1	0	0	1	0	1	1	0	0	0	0	0	0	0	10	111½0000	
K060	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	11½00000	
K061	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	11000000	
K062	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1	1	0	0	1	1	1	1	1	20	1111111½	
K063	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	00000000	
K064	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	00000000	
K065	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	4	½½½00000	
K066	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	24	11111111	
K067	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	00000000	
K068	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	0	0	20	11111110	
K069	0	0	1	1	0	1	1	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	9	11100000	
K070	0	0	1	0	1	0	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	8	11110000	

ID= Identity number TS= Total score

As shown in Table 3, both pupils K057 and K066 showed mastery in all the eight attributes, (i.e. A1.0 to A1.6) and thus obtained a knowledge state of “11111111”. In contrast, three pupils: K063, K064 and K067 had a knowledge state of “00000000”, indicating non-mastery of all the eight attributes. Meanwhile, K065 showed inconsistent mastery in the first four attributes, i.e., (A1.0 to A1.4) and non-mastery in the last four attributes, and thus obtained a knowledge state of “½ ½ ½ ½ 0000”. However, K070 was an interesting case to note. This pupil was diagnosed with a knowledge state of “11110000”, denoting that he/she has mastery of the first four attributes but non-mastery for the last four attributes. Nevertheless, more detailed analysis of this case showed that this pupil actually failed to answer two out of the three items on the first level of the attribute hierarchy (that is, only item 3 was correctly answered). Likewise, he or she answered correctly only two items (items 10 and 11) for the fourth level and one item (item 15) on the fifth level of the attribute hierarchy. So, how could this case be explained? As mentioned in the data analysis, before obtaining the knowledge state of this pupil, his responses were calculated as attribute mastery probabilities using Artificial Neural Network (ANN). Based on the cut-off value of the attribute mastery probability, this pupil’s score was considered as mastery because he/she was assumed to have mastered the most basic attribute before mastering the more complex ones. As such, when K070 could answer correctly items on the fourth and the fifth levels of attribute hierarchy, he or she was assumed to have mastered the first three basic attributes, even though he or she did not answer correctly all items in the first three attributes. In other words, a pupil does not need to answer every item correctly to gain a knowledge state of “1” for the attributes in the lower hierarchy. This is because he or she might have been careless in answering some items, but having mastered the more basic skills and processes, he or she was able to answer the more difficult items.

To illustrate another case, K065 was diagnosed with inconsistent mastery in the first four attributes with a knowledge state of “½ ½ ½ ½ 0000”. Table 4 displays K065’s responses to 24 items, and knowledge state and mastery level for each attribute.

Table 4

Interpretation of a pupil's item responses into knowledge state and mastery level.

K065																							
Item response	0	0	0	0	0	0	0	0	0	0	0	0	0	0									
Attribute mastery probability	0.52				0.71				0.77			0.50		0.00		0.02		0.00		0.00			
Knowledge state	½				½				½			½		0		0		0		0		0	
Mastery level	Incon-sistent Mastery-ry				Incon-sistent Mastery-ry				Incon-sistent Mastery-ry			Incon-sistent Mastery-ry		Non-Maste-ry		Non-Maste-ry		Non-Maste-ry		Non-Maste-ry		Non-Maste-ry	

As shown in Table 4, even though pupil K065 failed to answer the first six items (basic knowledge on the first and second attribute hierarchy) correctly, he or she was able to answer correctly all the three items in the third attribute hierarchy and one item in the fourth attribute hierarchy. Hence, the ANN processed and calculated the attribute mastery probability as 0.52, 0.71, 0.77 and 0.50 for the first, second, third and fourth attributes respectively. These attribute mastery probabilities were then classified as '½' in the knowledge state and interpreted as "inconsistent mastery" for the first four attributes, indicating that this pupil had not yet mastered all the four attributes completely. Therefore, appropriate remedial measures were needed for this pupil to improve on all these eight attributes.

Besides analyzing the knowledge state of individual pupils, we also conducted an overall analysis on the mastery level of the total sample (N=238) for the concept of 'after'. Table 5 presents the 18 knowledge states (KS) diagnosed and labelled as KS1 to KS18. These 18 knowledge states were arranged in increasing mastery level from non-mastery of all attributes "00000000" to mastery of all attributes "11111111".

As observed in Table 5, out of the total number of 238 pupils who answered the items, the knowledge state KS1 or "00000000" had the highest frequency percentage. There were 45 pupils (18.91%) diagnosed with this knowledge state, inferring that this number of pupils had not mastered any of the eight attributes measured for concept of *after*.

Table 5

Frequency and percentage for each knowledge state diagnosed in cognitive model which measures the concept of 'after'.

Label	Knowledge state	Frequency (%)
KS1	00000000	45 (18.91)
KS2	½ 0000000	4 (1.68)
KS3	½½ 000000	2 (0.84)
KS4	½ ½ ½ ½ 0000	1 (0.42)
KS5	1½ 000000	1 (0.42)
KS6	11000000	24 (10.09)
KS7	11 ½ 00000	20 (8.40)
KS8	11100000	8 (3.36)
KS9	111 ½ 0000	5 (2.10)

(continued)

Label	Knowledge state	Frequency (%)
KS10	11110000	20 (8.40)
KS11	1111 ½ 000	5 (2.10)
KS12	11111000	27 (11.35)
KS13	11111 ½ 00	2 (0.84)
KS14	11111100	9 (3.78)
KS15	111111 ½ 0	3 (1.26)
KS16	11111110	19 (7.98)
KS17	1111111 ½	3 (1.26)
KS18	11111111	40 (16.81)
Total		238 (100)

The second highest frequency percentage was KS18 or “11111111” with 40 pupils (16.81%) diagnosed with this knowledge state. Contrary to KS1, pupils with KS18 mastered all the eight attributes. However, when comparing the two percentages, KS18 was only 2.1% less than the percentage of KS1. This result implied that there were nearly equal numbers of pupils who mastered and did not master all the attributes for the concept of *after*.

However, there were two knowledge states with the lowest frequency. These knowledge states were: KS 4 or “½ ½ ½ ½ 0000” and KS 5 or “1½ 000000” with 0.42% of pupils respectively. These two knowledge states were uncommon and there was only one pupil diagnosed with each of these knowledge states. This result inferred that inconsistency or incomplete mastery of the attributes was uncommon among the pupils.

In this cognitive model, the last two attributes were: A1.5: “Transforming word problems into mathematical operations to find the ending date of an event” and A1.6: “Expressing the final answer accurately as ending date for word problems”. The analysis showed that 72.69% of the total sample did not master these two attributes. Again, this result implied that about three quarter of the sample were still weak in solving word problems involving finding the ending date of an event.

DISCUSSION

The findings of this study show that it is possible to diagnose primary pupils’ learning of the topic *time* through their knowledge states

from two aspects. One is diagnosing at an individual level while the other is as a group.

For diagnosis at an individual level, the knowledge state of each pupil provides details of his/her mastery of specific attributes. For instance, in this study, reading pupils' knowledge states based on the concept of *after*, we can diagnose specifically which level of attribute hierarchy this pupil has or has not mastered.

The pupil's knowledge state contains the mastery level of each tested attribute in the cognitive model. These mastery levels in knowledge states reflect the presence or absence of that component in a pupil's schema for the cognitive model. For example, a pupil's mastery of the first attributes, i.e., A1.0 "Knowing number of days in a specific month" tested in the CDA, denotes that in the pupil's schema, there is the knowledge of number of days in the specific month tested by the items. On the contrary, failing to master this attribute indicates that there was absence of that particular component in the pupil's schema for the concept of *after*. This result may aid pupils in improving on these non-mastered components of schema.

Taking K070 as an example, he or she mastered the first four attributes: A1.0 to A1.3(a), indicating that he or she: knew the number of days contained in a specific month (attribute A1.0); was able to perform addition of duration in terms of days based on the given month (attribute A1.1); knew the concept of *after* as involving addition of duration (number of days) and the starting date (attribute A1.2); and was able to apply the concept of *after* to solve word problems that involve calculating duration within two consecutive months for starting month with 30 days (for regrouping of 30 days into 1 month). However, he or she could not solve items measuring a higher level of attribute hierarchy (such as item 13 and above). Further analysis of this pupil's response pattern in answering items 13 (see Figure 3), shows that K070 had made a mistake by regrouping August as 30 days in a month (see Item 2 in Figure 3).

Through carefully designed items in the CDA, we can diagnose the pupil's unobserved latent variables (Tatsuoka, 2009) (such as K070's misconception) based on his or her response pattern towards the items or patterns of attribute mastery (Cui et al., 2006) in the form of knowledge states.

1) List down all the months that have 31 days.
 1) Senaraikan bulan-bulan yang mempunyai 31 hari.
 Januari Mac Mei Julai Oktober 6

2) List down all the months that have 30 days ONLY.
 2) Senaraikan bulan-bulan yang mempunyai HANYA 30 hari sahaja.
 April, Jun, Ogos, September, Disember

13)

After 49 days
 Selepas 49 hari

What was the date after 49 days?
 Apakah tarikh 49 hari selepas tarikh tersebut?

$$\begin{array}{r} 29 \\ + 49 \\ \hline 78 \\ - 30 \\ \hline 48 \end{array}$$

48
 18 0905 #

Figure 3. K070's response toward item 1 and 2 and item 13.

Besides providing fine-grained information, the knowledge state for each pupil is specific and dynamic. As in Table 3, we can observe that both pupils K060 and K069 obtained the same total score of "9" but they had different knowledge states: "11 1/2 00000" and "11100000" respectively. This result indicates the specificity of the knowledge state in informing the mastery level of each pupil. The knowledge state is individualized as a slight difference in the pupils' response pattern will yield different knowledge states. Hence, two pupils with the same total score may not have the same knowledge state, indicating individual pupils have different areas that need improvement. This is the strength of this CDA over the traditional diagnostic assessment which yields only the final total score and is unable to highlight an individual's areas of improvement.

In this study, the attributes were designed to evaluate the knowledge, skills and process needed to solve items measuring the concept of

after. The basic attributes focus on measuring pupils' understanding of days and months in a year. Subsequently, the level of difficulty and content complexity of attributes increased, from evaluating pupil's ability to perform addition of days and months to solving word problems for the concept of *after* (see Table 1). This way of arranging the attributes was based on the assumption that pupils should master the basic knowledge and skills before mastering the more complex ones. Hence, for pupil K065 who answered correctly items measuring the 4th level of attributes but failed to answer items of the basic levels, all the first four attributes would be interpreted as "inconsistent-mastery", indicating this pupil had not completely mastered all the attributes, and thus needed improvement (refer to Table 4). This finer way of classifying pupils has an advantage over other studies (see Hartz & Roussos, 2008; Lee & Sawaki, 2009) which have classified pupils dichotomously into mastery and non-mastery only. The advantage of classifying the "inconsistent-mastery" is that more specific remedial work could be given to help this pupil.

For diagnosis at the group level, the display of knowledge states for a sample of pupils (e.g., Table 5) provides an overview of pupil performance in that particular cognitive model. In this case, the analysis showed that 73% of the total sample (N=238) had not mastered the last two attributes, which involved solving word problems. This result implied that the majority of these pupils were still weak in problem solving skills. Therefore, teachers could make appropriate adjustments to carry out differentiated instruction and other instructional strategies to cater for the different needs of the pupils in solving word problems.

However, there are two limitations in this study. One is that pupils' knowledge state is dynamic and ever-changing as they constantly receive different learning inputs and methods. Hence, the diagnosed knowledge state of pupils is only valid at the point the test was taken. Therefore, giving the diagnostic feedback to a specific pupil in a timely manner is extremely important, so that the given information is valid to be used to help the pupil and his or her teachers. However, this is linked to the second limitation, which is time factor. As the CDA was administered and analysed manually in this study, this process was time consuming. Hence, it is quite impossible to provide immediate diagnostic feedback which may cause the result

to be invalid. Therefore, we propose that future studies should focus on making the CDA online to help overcome these two limitations.

IMPLICATIONS

There are two major implications from this study:

Implications for mathematics teachers and pupils

The findings of this study showed that there were a total of 18 knowledge states diagnosed among the 238 pupils. This result indicated that primary pupils faced different levels of difficulties when they were learning the topic of *Time* involving dates. This way of diagnosis, in the form of knowledge states, provided detailed information about the pupils' strengths and weaknesses in the specific attribute in the topic of *Time* involving dates. This information provided multilevel feedback which can be used for various purposes by mathematics teachers and pupils.

By knowing the pupils' knowledge states, mathematics teachers could evaluate their instructional effectiveness. Teachers could know if the desired learning objectives have been achieved by inspecting the mastery level of attributes in pupils' knowledge states. Teachers could do remedial work based on the hierarchy of the non-mastered attributes. The hierarchical attributes, which are clearly defined, contain the knowledge, process and skills which not only guide teachers in designing remedial work but also emphasise the need to address basic knowledge before complex concepts.

Conversely, pupils could also use this assessment results to monitor their own progress in the tested topic. Through their knowledge state, pupils could maintain their strengths and take proper measures to overcome their deficiency. In knowing their mastery level in the fine-grained attributes in a cognitive model, pupils are informed on the specific knowledge, skills and process that they still need improvement in. This information enables pupils to study effectively and plan properly to improve on specific areas that they are weak at. Finally, the result can help to design future test items in the topic. By adopting the attributes in the cognitive model in this study, teachers are able to modify the items accordingly to diagnose the pupils for the same concept, or even extend to another similar topic.

Implications for school textbook writers

One of the findings in this study was that pupils were still weak at solving word problems for the topic of *Time* involving dates. Pupils' failure to solve word problems which are routine problems in textbooks hinted that they might not have learned well from the examples and items presented in the existing school textbooks. Textbook writers might wish to include a greater variety of word problems to provide a richer context for pupils to learn the mathematical concepts (Greer, 1997).

CONCLUSION

In elementary mathematics, the concept of *Time* remains a difficult and abstract topic. Several studies (such as Burny, 2012; McGuire, 2007; Pérez-Sedano, 2015) have shown that many pupils face difficulty in learning the topic of *Time* in their primary mathematics curriculum. Yet, effective methods of diagnosing primary pupils' strengths and weaknesses to inform their learning on this topic are still lacking. Hence, the findings of this study contribute to the literature by attempting to provide a useful diagnostic feedback on the primary pupils' learning of *Time*.

In this paper, we have described how primary pupils' learning on the topic of *Time* can be diagnosed through their knowledge states by using CDA. The CDA was developed based on a cognitive model which consisted of hierarchically arranged attributes. The cognitive model was designed by mathematics experts to provide an interpretive framework for developing test items to evaluate pupils' test performance. Then, it was used for interpreting diagnostic results so that pupils' test performance can be linked to specific cognitive inference (Gierl et al., 2010).

In this study, the primary pupils' diagnostic feedback for the topic of *Time* involving dates was expressed explicitly in the form of knowledge states, showing the mastery level of each tested attribute. This way of diagnosing pupils' knowledge is different from the normal diagnostic test which provides only the final score for each pupil. The use of the cognitive model which consists of hierarchically ranked attributes, from basic to complex, has provided a systematic

basis in measuring pupils' underlying knowledge and cognitive skills. The information obtained from the CDA would help pupils to manage their progress and cope with their own weaknesses. At the same time, teachers could use the comprehensive diagnostic feedback in designing suitable remedial work in order to help pupils with specific needs.

Lastly, as studies that use CDA in pupils' learning for the topic of *Time* are still scarce, this alternative method of diagnosis based on knowledge states should be viewed positively and remain a potential diagnostic assessment. To overcome the two limitations of time and timely feedback, further studies are recommended, including developing an online version of CDA.

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