COORDINATED MOTION PLANNING AND OPTIMAL FORCE DISTRIBUTION FOR ROBOTS WITH MULTIPLE COOPERATING ARMS

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Abstract. This paper is concerned with the problems of inter-arm coordination which arise when multiple robot arms attach their end effectors to a single payload in order to perform a manipulation task. The approach taken is to let the arms cooperate with equal rights, and to exploit the redundancy present in the system for the minimization of a quadratic cost criterion.
The main elements of the proposed control system are generators for coordinated nominal motions and force/torque interactions with the payload, and an active compliance scheme capable of resolving kinematic conflicts and inconsistencies between motion and force/torque commands. The paper gives a detailed mathematical description of both the motion and the force/torque generators. The proposed active compliance scheme is presented in summary form.

Keywords. Robots; cooperating robot arms; force distribution; active compliance; hierarchical systems; process control; mathematical analysis.

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I. INTRODUCTION

The reader will be aware, out of personal experience, that precise handling of large and heavy objects is much more easily achieved with two arms than with one. We can learn from this observation that the performance features of robots could be improved, and their field of application widened, if they could be made to use their arms cooperatively in a similar way. The main problem to be solved on the way to such advanced manipulation systems is that of fast and precise inter-arm coordination. Bad coordination may cause damage to the payload or the arms themselves.

Up to now the problem of inter-arm coordination for multi-armed robots has not received much attention in the control literature. Some researchers have proposed (and partly demonstrated) master/slave systems as a solution to the problem. Nakano et al. /1/ use a position-controlled master arm together with a force-controlled slave which supports the motion enforced by the master. Alford and Belyeu /2/ have developed a predictive coordination method where both master and slave are position-controlled. The slave commands are modified on the basis of predicted position values for the next step. The goal of this control strategy is to keep the relative position of the two end-effectors constant. No account is taken of interactive forces between the arms, which may occur due to kinematic modelling errors. In 1975, Fuji and Kurono /3/ have introduced the first method where two arms cooperate with equal rights. Their approach, called the method of "virtual reference", is based on position error feedback and leads to compliant behaviour of the two position controlled arms. Ishida /4/ describes and compares two alternative control methods applied to two reference tasks called parallel transfer and rotational transfer. The first control method is the master/slave method of /1/. In the second one, both arms are force servoed and cooperate with equal rights. They receive their commands from a coordination controller which enforces tracking of the desired load trajectory with a prescribed dynamic behaviour in operational coordinates. Ishida states that the second method is superior (though slower) in the case of the rotational transfer task. The problem of force distribution between the cooperating arms, which has not been addressed in the publications mentioned so far, has been studied intensively by Orin and Oh /5/, who treat it as a linear programming problem in the joint torque space. The objective function penalizes energy consumption and terminal reaction force imbalance.

The objective of the work presented in our paper was to construct, as an extension and further elaboration of Mason's work /6/, a general theory of multi-arm cooperative manipulation which would, to the largest possible extent, incorporate the ideas introduced in the aforementioned publications. We have carried out our studies under the following general assumptions, guidelines and constraints:

- The theory shall be valid for any number of cooperating arms.
- Both the motion and the force/torque aspect of manipulation shall be represented by the theory.
- No dependence on special features of any specific robot arm. The theory shall be formulated in terms of "ideal effectors". The ideal effector is the most general abstraction of a manipulator arm: a fictitious device which can ideally execute force/torque or motion commands, or exhibit a specified compliance behaviour in all six spatial directions.
- We have assumed a rigid body load with a grapple interface for each arm. Grappling

is assumed to lock the end-effectors to the load so that they behave like one rigid body.
- In addition to the common internal sensors, each arm must be equipped with a sensor system to measure force/torque interaction with the load (usually a wrist sensor).
- The control system to be developed shall be modular in the sense that every robot shall be addressed by the coordination controller through the same "virtual robot interface" (i.e. from the coordination level, the robots shall all look the same).

II. A GENERIC CONTROL HIERARCHY FOR MULTI-ARMED ROBOTS

In the generic control hierarchy for multi-armed robots shown as fig. 1, three functional levels can be distinguished:

- On the intelligence level, the robot task is decomposed into a sequence of operations, and the required motion of the load and its required force/torque interaction with the environment are planned for each operation.
- On the dynamic control level, nominal motion and force/torque commands are derived for each arm. An active compliance scheme eliminates un-wanted counter-active forces exchanged between the arms and resolves possible conflicts(due to imperfections of the task model used in the planning process) between motion and force/torque commands. The corrected commands are then sent to the individual arm servo-controllers.
- The machine level comprises power electronics (or any other power source and conditioning unit) and the arms themselves.

This paper deals only with the dynamic control level, with emphasis on the generation of coordinated nominal motion and force/torque trajectories, and on the theoretical basis for the active compliance scheme.

III. GENERATION OF NOMINAL MOTION TRAJECTORIES

Given the load's geometry and its desired motion trajectory we can derive the nominal motion commands for each of the cooperating robot arms in an open loop process. The mathematical description of the coordinated motion generator will be derived in the following two paragraphs.

III.1 Task Geometry

For the purpose of generation of coordinated desired trajectories for the cooperating manipulator arms we consider the load as a rigid body with n grapple interfaces. As in /7/, the frame concept and homogeneous transformations will be used to describe the spatial relations between the n manipulator arms, the load, and an inertial reference frame referred to as the task frame. All relevant frames and transformations are shwown in fig. 2.

The following names will be used for the coordinate frames in fig. 2:

$(\vec{i}_R, \vec{j}_R, \vec{k}_R)$: (inertial) reference frame, task frame

$(\vec{i}_L, \vec{j}_L, \vec{k}_L)$: load frame

$(\vec{i}_C, \vec{j}_C, \vec{k}_C)$: load center frame (a coordinate system with its origin in the load's center of mass and its axes aligned with the load's principal axes)

$(\vec{i}_{Gi}, \vec{j}_{Gi}, \vec{k}_{Gi})$: grapple frame i

$(\vec{i}_{Bi}, \vec{j}_{Bi}, \vec{k}_{Bi})$: base frame i

$(\vec{i}_{Hi}, \vec{j}_{Hi}, \vec{k}_{Hi})$: hand frame i

The transformations G_i are constant because of the rigid body assumption. The transformations B_i are also constant as long as the bases of the arms are inertially fixed. The transformations L and H_i, being input and outputs of the motion trajectory generator of fig. 1, are changing in time.

III.2 Derivation of Kinematic Equations

The kinematic equations depend on how the load is grasped by the hands. the general from of the grasping condition reads:

$$\underline{L}(t)\ \underline{G}_i\ \underline{R}_i = \underline{B}_i\ \underline{H}_i(t); \quad i = \overline{1,n} \tag{1}$$

The transformation R_i, which closes the transformation graph of fig. 2, describes the relative position and orientation of hand and grapple interface in the grasped condition. Sometimes the contact between hand and grapple interface can be established in different ways, with different values of R. The choice may have important consequences for collision avoidance, but it doesn't make any difference for the considerations in this paper, where R can be treated as a constant, retaining the value it had upon closure of the hand.

Solving (1) for $\underline{H}_i(t)$ we obtain a set of equations describing the coordinated desired motion trajectories in terms of homogeneous transformations:

$$\underline{H}_i(t) = \underline{B}_i^{-1}\ \underline{L}(t)\ \underline{G}_i\ \underline{R}_i; \quad i = \overline{1,n} \tag{2}$$

The spatial relationship of hand and base of a manipulator can be specified more concisely by a vector of generalized coordinates

$$\underline{x}_i^t = (p_{1i}, p_{2i}, p_{3i}, \Psi_i, \Theta_i, \Phi_i) \tag{3}$$

where p_1, p_2, p_3 are the position coordinates, and Ψ, Θ, Φ three angles determining the orientation (pseudo-coordinates). With $\underline{x}_L(t)$ denoting the desired trajectory of the load in generalized coordinates we can formally write

$$\underline{x}_i(t) = \underline{f}_i(\underline{x}_L(t)); \quad i = \overline{1,n} \tag{4}$$

The bundle of functions $\underline{f}_i$ comprises (2) and the conversion from generalized coordinates to homogeneous transformations and vice versa. Concatenation of the n equations of (4) into one column vector yields

$$\underline{\xi}(t) = \underset{i=1}{\overset{n}{\mathrm{col}}}(\underline{x}_i(t)) = \underline{f}(\underline{x}_L(t)) \tag{5}$$

This is the function to be implemented in the motion trajectory generator block of fig. 1.

IV. GENERATION OF NOMINAL FORCE/TORQUE TRAJECTORIES

Just as for the motion we can calculate nominal values for the force/torque interaction of the cooperating robot arms with the payload. The nominal force/torque to be applied has two components required for the desired force/torque interaction between load and environment, and for acceleration/deceleration of the load, respectively. Chapter IV.1 shows how the latter component can be determined from known or estimated load dynamics. Chapters IV.2 through IV.5 deal with force distribution, i.e. with the question how the required force/torque effort is to be shared by the robot arms.

IV.1 Load Dynamics

With the notation introduced in fig. 3, and using the laws of rigid body dynamics, the equations of motion of the load read

$$\sum_{i=1}^{n} \underline{f}_i + \underline{f}_{ext} = \dot{\underline{p}}_c \tag{6a}$$

$$\sum_{i=1}^{n} [\underline{t}_i + \underline{X}(\underline{r}_i - \underline{r}_c) \cdot \underline{f}_i] + \underline{t}_{ext} = \dot{\underline{l}}_c \tag{6b}$$

Explanations:

- $\underline{f}_i$, $\underline{t}_i$: force and torque vector applied to the load by arm i (line of action through the center of grapple frame i)
- $\underline{f}_{ext}$, $\underline{t}_{ext}$: external reaction force and torque vectors acting on the load (line of action through the center of mass c)
- $\underline{p}_c$, $\underline{l}_c$: linear and angular momentum of the load, both with respect to its center of mass
- $\underline{r}_i$, $\underline{r}_c$: position vectors from the inertial reference frame to the grapple interfaces and to the load's center of mass

All vectors are expressed with respect to the inertial reference frame. The "dot" operator performs differentiation by time, and the operator $\underline{X}$ builds up the skew-symmetric matrix

$$\underline{X}(\underline{r}) = \underline{X}((r_1, r_2, r_3)^t) = \begin{pmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{pmatrix} \tag{7}$$

which is used for the computation of the cross product when vectors are represented by the calculus of matrices:

$$\vec{t} = \vec{r} \times \vec{f} \quad \Longleftrightarrow \quad \underline{t} = \underline{X}(\underline{r}) \cdot \underline{f} \tag{8}$$

After arranging the forces $\underline{f}_i$ and torques $\underline{t}_i$ in one column vector

$$\underline{\phi} = \operatorname*{col}_{i=1}^{n} \begin{pmatrix} \underline{f}_i \\ \underline{t}_i \end{pmatrix} \tag{9}$$

we can re-write equations (6a,b) in the simple form

$$\underline{B}\,\underline{\phi} = \underline{h} \tag{10}$$

with $\underline{B} = \operatorname*{row}_{i=1}^{n} (\underline{B}_i)$

$$\underline{B}_i = \begin{pmatrix} \underline{I}_3 & \underline{O}_{3x3} \\ \underline{X}(\underline{r}_i - \underline{r}_c) & \underline{I}_3 \end{pmatrix} \; ; \; i = \overline{1,n}$$

$$\underline{h} = \begin{pmatrix} \dot{\underline{p}}_c - \underline{f}_{ext} \\ \dot{\underline{l}}_c - \underline{t}_{ext} \end{pmatrix}$$

($\underline{I}$: identity matrix, $\underline{O}$: null matrix)

The matrices $\underline{B}_i$ are all of order 6x6 and, consequently, $\underline{B}$ is a 6x6n matrix. It is easily seen that

$$\begin{aligned} \text{rank}\ (\underline{B}_i) &= 6 \quad ; \quad i = \overline{1,n} \\ \text{rank}\ (\underline{B}) &= 6 \end{aligned} \tag{11}$$

(10) is hence a system of 6 independent equations in the 6n unknown elements of $\underline{\phi}$.

In other words, the problem of moving a load with more than one arm is dynamically underdetermined. The open question is, how the effort $\underline{h}$ necessary to move the load is to be shared by the arms. We shall take this fact as a chance to optimize the operation.

IV.2 Optimal Distribution of Generalized Forces

Let the optimal distribution of generalized forces among cooperating robot arms be governed by a quadratic cost function

$$c = \underline{\phi}^t \, \underline{C} \, \underline{\phi} \tag{12}$$

The 6nx6n matrix of cost coefficients ("cost matrix") is assumed to be symmetric and strictly positive definite, and thus regular.

The force distribution problem can now be formulated as the following constrained optimization problem:

> Find the vector of generalized forces $\underline{\phi}$ that minimizes criterion (12) and fulfils the constraints (10) (13)

This problem can be solved with the classical methods of constrained optimization.

Introducting the vector $\underline{\lambda}$ of Lagrangian multipliers we combine (10) and (12) and obtain the Lagrangian function

$$L = \underline{\phi}^t \, \underline{C} \, \underline{\phi} + \underline{\lambda}^t [\underline{B} \, \underline{\phi} - \underline{h}] \tag{14}$$

The necessary conditions for a constrained local extremum are

$$\left.\frac{\partial L}{\partial \underline{\phi}}\right|_{\substack{\underline{\phi}=\underline{\phi}^* \\ \underline{\lambda}=\underline{\lambda}^*}} = 2\, \underline{C}\, \underline{\phi}^* + \underline{B}^t \, \underline{\lambda}^* = \underline{O} \tag{15a}$$

$$\left.\frac{\partial L}{\partial \underline{\lambda}}\right|_{\substack{\underline{\phi}=\underline{\phi}^* \\ \underline{\lambda}=\underline{\lambda}^*}} = \underline{B}\, \underline{\phi}^* - \underline{h} = \underline{O} \tag{15b}$$

This is a system of 6n+6 linear equations for the 6n+6 unknown elements of $\underline{\phi}^*$ and $\underline{\lambda}^*$. Solving (15a) for $\underline{\phi}^*$

$$\underline{\phi}^* = -\frac{1}{2} \underline{C}^{-1} \underline{B}^t \underline{\lambda}^* \tag{16}$$

and substituting this into (15b) yields

$$\underline{B}\, \underline{C}^{-1} \underline{B}^t \underline{\lambda}^* + 2\underline{h} = \underline{O} \tag{17}$$

It can be shown that, with $\underline{C}$ strictly positive definite and rank $(\underline{B}) = 6$, $\underline{B}\, \underline{C}^{-1} \underline{B}^t$ is regular:

$$\text{rank}\ (\underline{B}\, \underline{C}^{-1} \underline{B}^t) = 6 \tag{18}$$

Therefore, (16) and (17) have one unique solution:

$$\underline{\lambda}^* = -2(\underline{B}\, \underline{C}^{-1} \underline{B}^t)^{-1} \cdot \underline{h} \tag{19a}$$

$$\underline{\phi}^* = \underline{D}\, \underline{h} \quad \text{with} \quad \underline{D} = \underline{C}^{-1} \underline{B}^t (\underline{B}\, \underline{C}^{-1} \underline{B}^t)^{-1} \tag{19b}$$

The 6nx6 matrix $\underline{D}$ is called the optimal force distribution matrix. An analysis of its rank shows

$$\text{rank}\ (\underline{D}) = \text{rank}(\underline{B}) = 6 \tag{20}$$

This follows from (19b) and the fact that both $\underline{C}^{-1}$ and $(\underline{B}\, \underline{C}^{-1} \underline{B}^t)$ are regular.

Although conditions (15a,b) were only <u>necessary</u> for the existence of a <u>local extremum</u>, it can be shown that $\underline{\phi}^*$ is indeed the global minimum of the constrained optimization problem. This is the case because the cost function is positive definite and the constraints are linear.

IV.3 Computational Simplification of the Optimal Force Distribution Law

It is generally not necessary to penalize cross terms containing products of generalized forces applied to the load by different arms. Hence we assume from here on that the cost matrix $\underline{C}$ has block diagonal form

$$\underline{C} = \operatorname*{diag}_{i=1}^{n} (\underline{C}_i) \tag{21}$$

with 6x6 matrices $\underline{C}_i$ stating the cost of forces and torques applied by arm i. The total cost of (12) is now obtained by adding up the individual cost components

$$c = \sum_{i=1}^{n} \underline{\phi}_i^t \, \underline{C}_i \, \underline{\phi}_i \tag{22}$$

$$\underline{\phi}_i^t = (\underline{f}_i^t, \underline{t}_i^t); \quad i = \overline{1,n}$$

Using (21) we can reduce the number of operations required to calculate the force distribution matrix $\underline{D}$:

$$\underline{C}^{-1}\underline{B}^t = \operatorname*{diag}_{i=1}^{n} (\underline{C}_i^{-1}) \cdot \operatorname*{col}_{i=1}^{n} (\underline{B}_i^t) = \operatorname*{col}_{i=1}^{n} (\underline{C}_i^{-1}\underline{B}_i^t)$$

$$\underline{B}\underline{C}^{-1}\underline{B}^t = \operatorname*{row}_{i=1}^{n}(\underline{B}_i) \cdot \operatorname*{col}_{i=1}^{n}(\underline{C}_i^{-1}\underline{B}_i^t) = \sum_{i=1}^{n} \underline{B}_i \underline{C}_i^{-1} \underline{B}_i^t$$

$$\Rightarrow \underline{D} = \operatorname*{col}_{i=1}^{n}(\underline{C}_i^{-1}\underline{B}_i^t)\Big(\sum_{i=1}^{n} \underline{B}_i C_i^{-1} \underline{B}_i^t\Big)^{-1} \tag{23}$$

Splitting $\underline{D}$ into 6x6 matrices $\underline{D}_i$ according to

$$\underline{D} = \operatorname*{col}_{i=1}^{n} (\underline{D}_i) \tag{24a}$$

puts the result into a slightly different from:

$$\underline{D}_i = \underline{C}_i^{-1} \, \underline{B}_i^t \Big(\sum_{j=1}^{n} \underline{B}_j \, \underline{C}_j^{-1} \, \underline{B}_j^t\Big)^{-1} \; ; \; \overline{i=1,n} \tag{24b}$$

$\underline{D}_i$ assigns a portion of the total forces and torques to be exerted on the load to arm number i:

$$\underline{\phi}_i = \underline{D}_i \, \underline{h} \; ; \; \overline{i=1,n} \tag{24c}$$

The advantage of (24b) over (19b) is twofold: There are fewer computations, and on a multiprocessor system the n equations of (24b) can be evaluated in parallel.

IV.4 Some Features of the Optimal Force Distribution Law

1) From (19b) we can draw the important conclusion that the nominal forces and torques are all zero in the absence of accelerations and external forces and torques on the load. This means that the forces distribution law eliminates all static internal forces in the load.

2) It further follows from (19b) that the optimal force vector $\underline{f}^*$ is always contained in a six-dimensional subspace $S^+ \subset R^{6n}$. This subspace will be referred to as the space of co-active forces. It is spanned by the six columns of the force distribution matrix $\underline{D}$. Co-active force vectors $\underline{f} \in S^+$ will be marked $\underline{f}^+$.

3) The orthogonal complement of S^+ is called the space of counter-active forces $S^- \subset R^{6n}$. Measured counter-active force vectors $\underline{f} \in S^-$ will be marked $\underline{f}^-$, accordingly.

4) It is interesting to analyze the relation between the spaces of co-active and counter-active forces, and the space S_r which is spanned by the rigid body motion modes of the system, i.e. the admissible combinations of arm motions under the constraints imposed by the interconnection via a rigid load, which is free to move in all six degrees of freedom. It can be shown that the rigid body motion modes of the system are given by the columns of the matrix $\underline{B}^t$. On the other hand, we can see from (19b) that $\underline{C}^{-1} \cdot \underline{B}^t$ spans the same space as $\underline{D}$, which means that its columns may be used as an alternative basis of the space of co-active forces. In the special case of $\underline{C}$ being a multiple of the identity matrix, the space of co-active forces will coincide with the space of admissible rigid body motions:

$$\underline{C} = a \cdot \underline{I} \, (a > 0) \rightarrow S^+ = S_r \tag{25}$$

In general, S^+ and S_r are different, but never orthogonal, because it follows from (19b) that $\underline{B} \, \underline{D} = \underline{I}$, i.e. $\underline{D}$ is a pseudo-inverse of $\underline{B}$.

5) The optimization criterion (12) can be formulated in coordinate frames other than the inertial reference frame, i.e. costs can be assigned to force components along the axes of some other coordinate frame. An obvious possibility would be the load frame. It is also possible to specify the cost of forces/torques exerted by each arm in the respective robot base frames. The resulting force distribution will be different in each case, but it is always possible to transform the problem back to the inertial space so that the force distribution law retains the same form and everything said about it remains valid.

6) It is sometimes opportune to transform the force distribution law to another coordinate system, where the calculation of the force distribution matrix may become much simpler. This is the case, for example, when both the cost function and the force distribution law are formulated in load coordinates.

Example:

The following example of optimal force distribution in a very simple manipulation task will help to visualize the relations discussed in points 2-4 above. The task is illustrated by fig. 4a:

A total force h parallel to the X-axis has to be applied to a rigid body. The task is shared by two ideal effectors capable to exert forces f_1 and f_2 along the X-axis.

In this case the constraints (10) read

$$(1,1) \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = h \qquad \underline{B} = (1,1) \qquad \underline{f} = (f_1, f_2)^t$$

We choose a cost matrix

$$\underline{C} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

and calculate the force distribution matrix according to (19b):

$$\underline{D} = \begin{pmatrix} 1 & 0 \\ 0 & 0.5 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \left[(1,1) \begin{pmatrix} 1 & 0 \\ 0 & 0.5 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right]^{-1}$$

$$\underline{D} = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix} \cdot (1.5)^{-1} = \frac{1}{3} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

The only rigid motion mode of the system is

$$V_1 = V_2 = V \rightarrow \underline{V} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} V = \underline{B}^t \underline{V}$$

with V : velocity of the load

V_1: velocity of effector 1

V_2: velocity of effector 2

Fig. 4b shows the subspaces S^+, S^- and S_r for this case, and the splitting up of a measured force vector $\underline{f}$ into co-active and counter-active components.

IV.5 The General Linear Force Distribution Law

It is not necessary to view the force distribution law (19b) as the result of an optimal approach. If we had just postulated a linear force distribution law, we could have written down the first part of equation (19b) right away, with the force distribution matrix $\underline{D}$ free, but subject to the constraint constituted by the equations of motion of the load (10):

$$\underline{B}\ \underline{\phi} = \underline{B}\ \underline{D}\ \underline{h} \overset{!}{=} \underline{h} \quad \text{for arbitrary } \underline{h}$$
$$\Rightarrow \underline{B}\ \underline{D} = \underline{I} \tag{26}$$

This constraint is difficult to fulfil, however, if $\underline{D}$ is to be specified directly. (Remember that $\underline{B}$ was defined in (10) as a function of $\underline{r}_i - \underline{r}_c$. It varies, therefore, with the orientation of the load, except if the force distribution law is specified in load coordinates, in which case the transformation of the constraint (26) to the same coordinate system yields ${}^L\underline{B} \cdot {}^L\underline{D} = \underline{I}$, with constant ${}^L\underline{B}$.) The optimal approach, on the other hand, has the advantage that the constraint (26) is fulfilled automatically by the force distribution matrix calculated according to equation (19b).

We have not studied non-linear force distribution laws, as we think they are too difficult to handle analytically and therefore not suitable for our purposes.

V. SUMMARY OF THE ACTIVE COMPLIANCE SCHEME

The active compliance law that we propose for cooperating manipulator arms distinguishes internal compliance and external compliance. The role of internal compliance is to nullify any counter-active forces exchanged between the arms. It has no effect on the contact between load and environment. External compliance, on the other hand, is used to control contact forces between load and environment. The two compliance laws can be specified separately and then unified.

The external compliance law is formulated in terms of linear and angular velocities of the load, and of generalized contact forces with the environment, just as if there were only one manipulator arm involved. It is a direct application of Mason's method /6/, which doesn't need further discussions here.

The internal compliance law is formulated in terms of the combined terminal force/torque and generalized velocity vectors of the n arms. In the ideal domain*, the artificial constraints read:

$$\underline{A}^t\ \underline{\phi} = \underline{O} \tag{27}$$

where the matrix $\underline{A}$ forms a basis of S^-, the space of counter-active forces. The orthogonality of S^- and S^+ (for which the force distribution matrix $\underline{D}$ forms a basis) yields

$$\underline{D}^t\ \underline{A} = \underline{O} \tag{28}$$

Equations (27) and (28) show that the force distribution law determines the characteristic directions for the internal compliance law. These two functions are therefore very closely interrelated in our control approach for cooperating robot arms.

A unified compliance law is obtained by transformation of the external compliance law from load level to the level of the n ideal effectors. It can be shown that the combined internal and external, natural and artificial constraints are consistent and non-singular, which means that the task is fully and unambiguously specified in the ideal domain.

Our opinion concerning the realization of the active compliance scheme is, that the only methods having some practical importance at present are those sketched by Mason in /6/: the generalized spring and the generalized damper method, or a combination of the two. Other implementation methods may become feasible in the near future, however.

The mathematical relations describing the generalized spring and generalized damper methods for manipulator arms with position or rate servo may be inverted, for example, and applied to force-servoed arms. The best performance would probably be obtained with arm control systems implementing an adjustable end-effector compliance behaviour on the joint control level.

VI. CONCLUSIONS

Our study of the problems of inter-arm coordination in cooperative manipulator tasks has yielded a generic theory of coordinated motion planning and inter-arm distribution of generalized forces. This theory has been shown to be applicable as a basis for automatic control of multi-armed robots. Its strength lies in the fact that it is entirely formulated in the cartesian task space and hence, in principle, applicable to any combination of different manipulator arms, independently of their particular kinematic structures. This means, on the other hand, that quite a high computational burden for kinematic transformations is placed on the individual arm controllers. The performance requirements concerning dynamic accuracy are also very high (at least for high speeds of motion), so that the application of the control method will lie some way in the future. We are presently preparing a low-speed implementation of the control method for two laboratory arms in order to give a first demonstration of its capabilities.

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* The notion of the "ideal domain" has been introduced by Mason in /6/.

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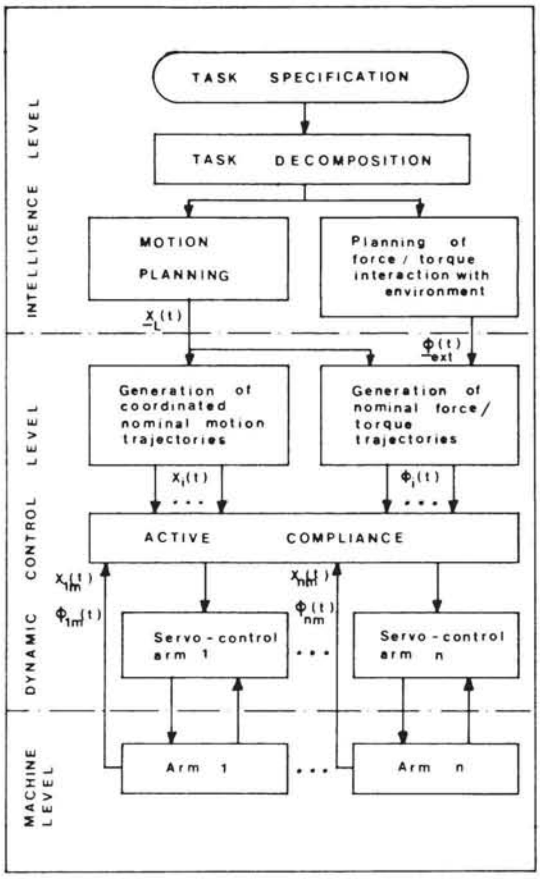


Fig.1: Generic control hierarchy for multi-armed robots

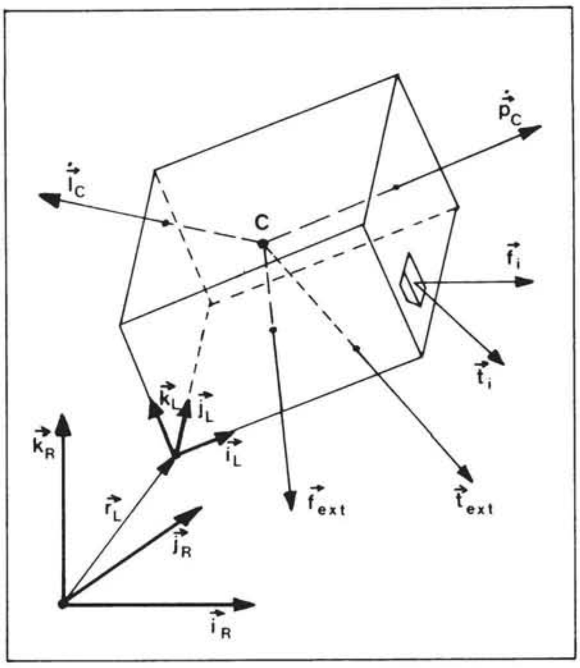


Fig.3: Forces and torques acting upon the load and resulting changes in linear and angular momentum

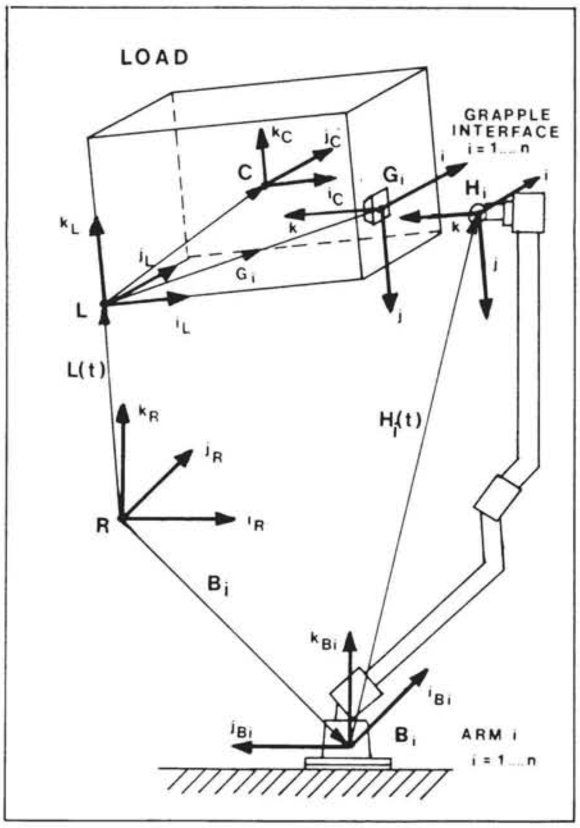
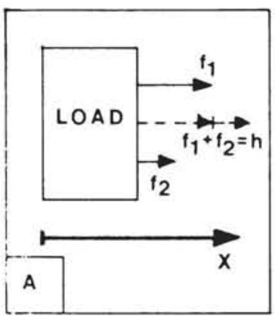


Fig.2: Task geometry: Coordinate frames and transformations



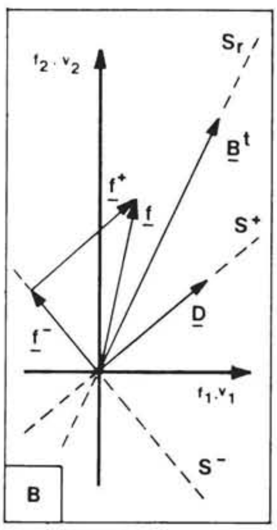


Fig.4:
a) Example of one-dimensional task to be shared by two effectors
b) Characteristic directions in the f_1-f_2 plane