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The Effects of a Problem Solving Model as an Alternative in the General Mathematics Curriculum

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THE EFFECTS OF A PROBLEM SOLVING MODEL

AS AN ALTERNATIVE IN THE GENERAL

MATHEMATICS CURRICULUM

by

Leo Edwards, Jr.

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Leo Edwards, Jr.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

The Effects of a Problem Solving Model

as an Alternative in the General

Mathematics Curriculum

by

Leo Edwards, Jr., Doctor of Education

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The purpose of this study was to evaluate an approved problem solving module as a model for use in the general mathematics curriculum, and assess its effectiveness in bringing about literacy in and a better attitude toward mathematics. Included in the module were topics on fundamental operations, fractions, decimals and percents, and problem solving. Specifically, the module focused on concerns of the National Assessment of Educational Progress (NAEP), that is the inability of students and young adults to use numbers skillfully enough to meet the demands of a modern society.

The sample used in this study consisted of eight intact classes--four control and four experimental groups. The groups were composed of students enrolled in ninth grade general mathematics from Logan

City School District, Cache County School District, and Box Elder County School District. The three districts are comprised of urban, semi-rural, and rural developments in northern Utah. The total of 210 students included 131 males and 79 females. There were 117 students in the experimental groups and 93 students in the control groups.

The diagrammatic problem solving module (composed of 20 lessons) was distributed among the teachers of the experimental groups for their perusal of content, lesson plans, and approach. When the teachers finished previewing the module, sessions were scheduled to accommodate each of their questions and concerns, and to cement the philosophy of the unit. Also, sessions were held with the control group teachers to delineate the kinds of experiences that would enhance equitable comparisons.

To facilitate the use of the Solomon Four-Group Design, a least squares analysis of variance (ANOVA) technique with unequal numbers of subjects per treatment was used to analyze the null hypotheses at the .05 level of significance. The ANOVA was used for both the Algebra Readiness Test and the Mathematics Attitude Scale.

On the basis of the findings of the five null hypotheses that were replicated in five different fashions, in regard to treatment effects, the following conclusions seem warranted.

1. There was a significantly more favorable attitude toward mathematics among students who were taught using the diagrammatic method.

2. There was a significantly more favorable performance among students on fundamental operations who were taught using the diagrammatic method.

3. There was a significantly more favorable performance among students on fractions who were taught using the diagrammatic method.

4. There was no difference in the performance among students on decimals and percents who were taught using the diagrammatic method and students who were taught using the traditional method.

5. There was a significantly favorable difference in the performance of students on problem solving who were taught using the diagrammatic method.

6. Sex differences were highly significant on one of the five replications, favoring the females. Inasmuch as the number of males far exceeded the number of females, a conclusion was not reached on sex differences.

7. Students who took both pretests and posttests, using the attitude scale, scored significantly higher than those who took only the posttest. Therefore it was concluded that pretesting served to raise scores on the attitude scale.

8. Students who took both pretests and posttests, using the readiness battery, did not perform significantly better than students who only took the posttest. Therefore, it was concluded that pretesting did not serve to enhance scores on the readiness battery.

(229 pages)

CHAPTER I

INTRODUCTION

In February, 1958, three national organizations (The National Council of Teachers of Mathematics, The Mathematics Association of America, and the American Mathematical Society) joined to form the School Mathematics Study Group (SMSG). The SMSG project financed by these national organizations was a necessary step in the establishment of a new curriculum in mathematics. However, some research studies and published articles on the development of secondary mathematics curriculum warned about "the formalism, excessive symbolism, and the experimental programs and commercial publications for elementary and secondary school mathematics."

During this same period (1958-1959), a concerted effort was launched to shift the theory of curriculum of the time between the 1930s and 1950s (a mixture of mathematically meaningful and socially relevant theories) to a theory that depicted the "structure of the Discipline." It was conceived, to put it succinctly, that the way of "knowing" in a discipline is also a way of learning it. To learn mathematics one learns to think like a mathematician (Bruner, 1960). Recent analyses have shown, however, that the curriculum suffered and perhaps the interest, attitudes, and performance were victimized by the rigor that was instituted. Vincent J. Glennon, in Phi Delta Kappan (January 1976) states that the mean test scores on the mathematics part of the Scholastic Aptitude Test taken by many high school students have fallen steadily from 502 (on a range of 200 to 800) in 1962-1963 to 472 in 1974-1975. Moreover, Husen (1967) found that achievement was positively correlated with interest in mathematics at all levels in all twelve countries that he studied. Certain investigators have also found that the correlation between attitudes and achievement is frequently higher for mathematics than for school subjects with more verbal content. Furthermore, Neal (1969) pointed to the fact that not only do attitudes affect achievement in mathematics but achievement also affects attitudes.

Ford (1969), in an investigation of the public image of "new math," found that until 1965, the public seemed to have no serious misgivings about new mathematical programs. In that year, however, two articles reversed the trend. One, in Time (May 1969) said that Max Beberman, one of the leaders in the movement for mathematics curriculum change, had suggested that care be exercised not to sacrifice the ability to compute. The other article, in Newsweek (July 1968) suggested that perhaps just as many sins were being committed with the "new math" as had been committed with the old, and questioned whether or not there was justification in making a substitution.

In Learning (October 1973), it was reported that more often than not, the bewildered parent and the harried teacher viewed "new math" either as a collection of esoteric topics added to the traditional content or as an attempt to dramatically accelerate children through higher, more abstract forms of mathematics. Frequently, these beliefs were well founded. It was not uncommon to find programs offering Euclidean geometry or axiomatic algebra to children six to ten years of age. Such programs violated what is commonly known about the ability of children to do the formal logical thinking necessary to give such material mathematical meaning, lasting value and relevance (Jean Piaget, 1959).

In retrospect, it is obvious to many that the mathematician's influence was too great. Consider the statement of Carroll V. Newsom, mathematician, former president of New York University and constant student of trends in mathematical education. In an article published in The American Mathematical Monthly (October 1972, p. 879), Newson stated:

> ...the new elementary mathematics curricula developed in recent years for school and college are superb when analyzed with respect to their mathematical content. They were well designed to produce good mathematicians. I must confess my early satisfaction in regard to the programs. Now, however, we are learning that good mathematicians had <u>too free a hand</u> in the development of the programs (emphasis added).

Vincent J. Glennon (Phi Delta Kappan, 1976, p. 304) director of the Mathematics Education Center, University of Connecticut, further states that:

> When viewed exclusively as a body of subject matter, new math is alive and well in the sense that textbook programs are now more mathematically correct than they were in the 1940s and 1950s. However, it is of critical importance to the mental health of young children--those of the greatest ability to do school-type learning. Highly formalized programs are

of questionable appropriateness to the middle third of the children, and clearly inappropriate, perhaps even harmful, to the self-concepts of the children in the lowest third.

Most people will agree that many of the middle and all of the lowest third (lower-achievers and under-achievers) are channeled into general mathematics classrooms for their Carnegie unit of required secondary mathematics; more often than not many general mathematics curricula are built around repetitions of unresolved difficulties; "more of the same," and remedial programs. Johntz (1970), founder and director of SEED (Special Elementary Education for the Disadvantaged), states that remedial programs reinforce a student's feelings of failure and inferiority. But he considers abstract mathematics (not arithmetic) to be a pure, culture-free subject that allows all students to start with a fresh slate.

To incorporate a program of abstractions, the mathematician must be cautious and mindful of the proliferation of research that has accrued since 1964 regarding "abstractions." Abstractions, surely, at some level, must generate ideas which lead to problem solving and real world application. Woodby in Education U.S.A. (September 1973), said students should be involved in solving problems about their real world. He also believes that the new mathematics creates a higher literacy to favor the more capable student and makes less readable the content of mathematics for the less able student. To consider literacy is to consider yet another dimension of reading mathematics--comprehension of the mathematical ideas. Hater (1974) believes that the best place to learn the reading of mathematics is in the mathematics class. Thus, the student who is confronted with problem solving is also confronted with its by-products--reading and symbolism. It is widely accepted, that verbal learning is invariably rote (glib verbalism) unless preceded by recent non-verbal problem solving experiences (Gagne, 1961). More credence is given to the previous statements when one considers the first report of the National Assessment of Educational Progress (NAEP) on mathematical achievement as reported in Phi Delta Kappan (January 1976). The NAEP surveyed the attainments of 9-year olds, 13-year-olds, 17-year-olds, and young adults (ages 26-35). Some of the findings are:

1. Only 40% of the 9-year olds could do this addition problem correctly

$$\begin{array}{r} \$\ 3.09 \\ 10.00 \\ 9.14 \\ \underline{5.10} \end{array}$$

2. Only 33% of the 13-year-olds could find the answer to the problem:

   "If John drives at an average speed of 50 miles per hour, how many hours will it take to drive 275 miles?"

3. Only 10% of the 17-year-olds could correctly calculate a taxi fare.

4. Only 1% of the 17-year-olds could balance a checkbook.

Purpose of Study

Having developed an approved learning module using a diagrammatic method, the writer of this study attempted to assess the effectiveness of this method (with associated materials) to determine the extent to which the level of literacy in the general mathematics curriculum for all students had been achieved. The second purpose of this study was to determine the extent to which the attitude of students toward mathematics improved as a result of heightened literacy. Specifically, the study focused on findings number one (operations with decimal notation), number two (problem solving), number three (real life situations involving fractions and whole numbers). It also attempted to set an awareness for number four (reconciling) of the NAEP findings stated above.

Procedure

In order to evaluate the effectiveness of the diagrammatic module in relation to the two above stated purposes, eight intact classes of general mathematics students were selected from four schools located in Cache and Box Elder Counties. Cache and Box Elder Counties are adjacent counties located in northern Utah. Each school that was selected was randomly assigned a control group and an experimental group. The sample consisted of 210 students.

The module included a discussion of techniques of illustrative diagramming which applies to nearly all kinds of elementary word problems. Transparencies keyed to various lesson plans with methods intended to enhance the reading skills involved, were given for each lesson.

Topics covered in the module included a treatment of symbolic language in mathematics, special problems (non-computational), inverse operations, reciprocals, fractions, decimals, and percents. Each topic was developed diagrammatically. The only prescription made was a very general one, that a Socratic rather than a purely lecture approach would be employed. Typically, the teacher led his students toward their own formulation of the day's concept by asking a series of carefully calculated questions. Sample questions were included in many of the lesson plans, but the teachers were cautioned to be alert and responsive enough to the students to take advantage of the opportunities they offer in original questions.

After the teacher and the students jointly developed a day's concept, the teacher circulated about the room to help individual students with classwork provided as part of each lesson. Also, students who finished the classwork early were encouraged to assist others. Following a discussion of portions of the classwork, the homework sheets were given to students in time for initial efforts to begin in the classroom.

Objectives

The following objectives were offered in support of the two previously stated purposes, which are believed realizable by teaching this particular diagrammatic module. The objectives were the development of:

1. A better attitude toward mathematics
2. A better understanding of problem solving
3. A better understanding of fractions
4. A better understanding of decimals
5. A better understanding of percents
6. A better understanding of the structures of mathematical language

Specifically, these objectives deal with the transfer of information learned from process and product to a more functioning symbolic daily life.

Statement of Hypotheses

In order to accomplish the above objectives, the following hypotheses were tested:

1. There is no difference in the attitudes among students who received the diagrammatic method and students who received the traditional method as measured by the Mathematics Attitude Scale.

2. There is no difference in the performance on fundamental operations among students who received the diagrammatic method and students who received the traditional method as measured by the Algebra Readiness Test.

3. There is no difference in the performance on fractions among students who received the diagrammatic method and students who received the traditional method as measured by the Algebra Readiness Test.

4. There is no difference in the performance on decimals and percents among students who received the diagrammatic method and students who received the traditional method as measured by the Algebra Readiness Test.

5. There is no difference in the performance on problem solving among students who received the diagrammatic method and students who received the traditional method as measured by the Algebra Readiness Test.

Definitions

Definitions of terms as used in this study are as follows:

1. Attitude toward mathematics: The predisposition or emotionalized feelings toward mathematics as measured by the Mathematics Attitude Scale.

2. Transfer: The degree to which a set of facts learned in one situation help or hinder the learning of another set of facts.
3. Mathematical literacy: The ability to read, write and use numbers skillfully enough to meet the demands of modern society.
4. Traditional method: Conventional teaching materials and methods employed by the teachers of the control groups.
5. Diagrammatic method: A specially-designed method (treatment) for the experimental groups, utilizing pictorial representations of mathematical sentences. That is, each word or phrase is translated into rectangular symbols, signs for operations and equality.
6. Module: A segment of the teaching episode employed in the use of the diagrammatic method.
7. Time: A measure of gain or loss in attainment from pretesting to posttesting.

CHAPTER II

REVIEW OF LITERATURE

The review of the literature consists of: (1) problem solving strategies, effects and implications, (2) the relationship of reading to problem solving and (3) the affective domain (attitude) and related measurements, as they relate to mathematics.

Problem Solving Strategies, Effects and Implications

Problem solving occupies a certral role in the teaching and learning of mathematics (Bassler, Beers, and Richardson, 1975). Unfortunately, only a relatively small amount of class time is devoted to discussing a method for solving verbal problems (Stilwell, 1967). In fact, Stillwell (1967) found that less than three percent of the time spent in problem solving activity was devoted to developing a method for solving problems. He also found that only seven percent of the problem solving activity was devoted to looking back at a problem or ahead to its implication. In relating these statements to the solution of verbal problems, it would seem that teachers avoid teaching a general strategy which may apply to many types of verbal problems but rather attempt to identify an algorithm which is limited in scope and applies to only one type of problem. This is also true of several current textbooks which restrict each section of a chapter entitled "verbal problems" to a specific type of problem (Bassler, Beers and Richardson, 1975). Perhaps, due to these difficulties, many teachers avoid presentation of units of verbal problems or place so little emphasis upon the solution of verbal problems (Van Engen, 1963). With so few hours of instructional time alloted to the study of problem solving, it seems reasonable that Manheim, (1961, p. 236) should state that

> the 'word problem' remains a generator of fear and frustraction for many students. ...If we ask our students, or ourselves, why such a problem is more difficult than a non-word problem, we are apt to find the difficulty attributed to 'the nonmathematical nature of the problem.'

The search of the literature revealed a number of suggestions to articulate how a teacher can assist a group of students to successfully solve verbal problems (Polya, 1957; Breslich, 1920; Greenholz, 1967; Dahmus, 1970; Sim, 1969, Berstein and Wells, 1969; Welchons, Kricherberger and Pearson, 1970; Troutman, et al. 1974). Yet many of these and other (Jacobson, 1969; Obrien and Shapiro, 1969; Schoenherr, 1968; Young, 1969) fail to delineate how pupil success is measured. Riedesel (1969), has suggested that when presenting problems and problem situations in the elementary school classroom, one should make use of drawings and diagrams as a technique to help pupils solve problems. Butler and Wren (1970) suggested that one method of overcoming problem solving difficulties is to "require the drawing and labeling of pertinent diagram." Trimble (1966) also favored the use of pictorial representations in a problem solving situation. He also suggested that "errors made on purpose" be used in order to capitalize on such errors during the classroom discussion. Still others have mentioned the use of errors in drawing diagrams and studies (Sherrill, 1970; Webb, 1974) with pre-service elementary teachers showed that those receiving a 10 item mathematical word problem test with accurate pictures accompanying each problem scored significantly higher than those receiving no pictures and those having inaccurate diagrams; the "no picture" group scored significantly higher than the "inaccurate pictures" group.

The number of studies in the area is sparse in relationship to the secondary school involvement, but there have been some noteworthy accomplishments made. One of the earliest studies to consider relative effectiveness of two strategies for teaching verbal problems was conducted by Clark and Vincent (1925). In this study, the step method was compared with the graphical method for solving word problems. In the step method the students were given five (5) steps to complete in solving the problems, they were (1) read the problem, (2) select what was wanted, (3) determine what facts were given, (4) find necessary relationships, and operations for the correct solution and (5) solve the problem. The graphical method directs the pupil to determine what is to be found in the problem, what it depends on, what each of these dependents in turn depend upon, and so on, until the essential facts and relationships in the problem have been identified. These facts and relations are then expressed as a type of schematic diagram which indicates a graphical solution to the problem. In testing the relative effectiveness of these two strategies, Clark and Vincent found that use of the graphical method is preferable when assessment is made immediately following instruction, while the step method appears to be preferable when longer term effects are assessed. To fully appreciate the results of this study, perhaps the date of the study should be kept in mind. In another study conducted by Kinsella (1951), the step method was tested against the use of no formal teaching method. His findings were that the student's success in selecting the correct process was not dependent on the prior success with any certain step or combination of steps, and that the step method might lower the level of performance on the solution of the whole problem.

Three approaches of teaching verbal problems to elementary school children were tested by Wilson (1967). One approach was the wanted-given method where the student was to find the wanted from the given. The second approach was the action sequence method where the students obtain the necessary operation by recognizing the action that occurred in the problem situation. The third approach, which was used as a control group, allowed students to solve problems in any manner with no specific type of instruction. The test results in this study favored the wanted-given approach (Bassler, Beers, and Richardson, 1972).

In a similar study (Jerman, 1971) the effects of the Modified Wanted-Given Program was compared to the effects of the Productive Thinking Program. The Modified Wanted-Given Program is an experimental sequence which emphasizes the structure of arithmetical problem, while the Productive Thinking Program is a commercially available sequence which develops general problem-solving skills and contains no mathematics when compared to the Wanted-Given Program. Using analysis of covariance, no significant differences were found between the two methods of instruction.

More recently the step method of solving verbal problems has been tested against the translation method for solving verbal problems (Richardson, 1975). The approach used in the translation method is to convert all English statements into one or more mathematical statements. When translating, the order of the statements should be the same as in the verbal problem. No operation should be performed before the translation is complete. One should read slowly and translate each item and fact before reading further. Once translation is complete, the symbolic statements are used to solve the problem. Instructional discussions must focus on the representation of a variable by a □ or a letter; replacing variables by values, once the value is known; and identification and meaning by key words and phrases. The test results of this study are: (1) instruction by teachers knowledgeable of verbal problem solving strategies resulted in significant pupil growth in verbal problem solving scores after a relatively small number of instructional periods, (2) the step method and the translation method for solving verbal problems are about equally effective in producing pupil achievement and (3) significant retention effects were shown for both methods.

Cohen and Johnson (1967) as reported by Bassler, Beers and Richardson (1972), claim that when a student acquires the ability to translate accurately from a certain description of a physical situation to an appropriate mathematical sentence, it will allow him to cope with a large number of problems in an orderly and logical fashion. Apparently this ability to translate any situation into some form of mathematical symbolism is the most useful tool in problem solving. Manheim (1961) claims that the reason for transforming expressions from the non-mathematical language to mathematical language is to have greater manipulative facility.

Yet another study (Bassler, et al. 1972) tested the relative effects of two diametrically opposing approaches in problem solving. The two approaches were the Polya Method (1957) and the DPPC Method (Dahmus, 1970). The Polya Method requires the student to read and understand the problem; to plan for a solution of the problem, which includes identifying the unknown, identifying relations and operations to be performed, and drawing analogies to similar problems which may have been previously solved; to carry out the plan; and, finally to examine the obtained solution. The second strategy, the DPPC Method--the letters of which stand for the words, direct, pure, piecemeal, and complete, is in essence based upon a translation of the verbal statement into mathematical symbolism. Following translation, the mathematical symbols are used to find the solution of the problem. The null hypothesis was not rejected on the problem solution criterion for the two treatments and this would suggest that any differences in this assessment were due to chance factors and the two groups performed equally well.

Although some of the findings sometimes conflict on problem solving, it is clear from the review that no matter what the method, presentations in any approach can to some degree enhance achievement.

Relationship of Reading to Problem Solving

Reading is obviously important, since if the child cannot read the problem he will have difficulty in doing more than guessing how to solve it. It is suggested that reading and other interpretative skills specifically related to problem solving be developed in the problem solving program (Suydam and Weaver, 1970). Hater (1974) concurs with this statement in that he believes the best place to learn the reading of mathematics is in the mathematics class. The importance of reading in mathematics is further delineated by Treacy (1944) and Alexander (1960) in their finding that good and poor achievers in problem solving differed on many aspects of reading. Treacy concluded that reading should be regarded as a composite of specific skills rather than as a generalized ability. Within the same context, Barlow (1964) studied 468 sixth graders who had been classified by reading and computational levels. He reported that higher levels of problem solving abilities were positively correlated with higher levels of reading and computational abilities, but that perhaps much of this relationship apparently was the result of the high correlation of these abilities with IQ.

One of the problems in engendering understanding of problem solving, in the reading context, is the nonmathematical nature of the problem (Manheim, 1961). The nonmathematical nature of these problems is that they are stated in the English language, which requires reading and understanding what has been read, rather than being stated in mathematical symbolism (Bassler, et al., 1972). Kulm (1973) reported from a study that was conducted on the readability of mathematical literature that the variables that make mathematical material difficult to read are different from those affecting the reading difficulty of ordinary English. Other pertinent findings from his study are: (1) The percentage of difficult words, which is the best predictor of the readability of ordinary English, is not even among the best five predictors of the readability of elementary algebra material. (2) Vocabulary is important but is far outweighed by the difficulty of the symbolism of mathematics. Kulm pointed out that this result may be due, in some part, to the grade level of the textbooks from which passages were sampled. Concurring with Kulm is Freeman (1973) who reports that if students are to succeed in doing story problems, they must be able to perform in three areas--symbol perception and meaning, vocabulary comprehension, and basic operations. Jencks and Peck (1975) state that symbols ought to be tools for thinking, but that this can only be if students have meaningful referents.

The findings from the review of the literature seem to suggest that vocabulary ranks below symbolism as a source of difficulty (Kulm, 1973) in reading mathematical material. Among those who have experimented with the teaching of vocabulary was VanderLinde (1964), whose report states that specific instruction on quantitative vocabulary was effective in increasing problem solving scores when the tests typify the vocabulary that was used.

Another factor that impedes success of the problem solver is his speed of reading (Hater, Kane and Byrne, 1974). Barney (1972) noted this problem when he stated that some children read mathematics narrative problems so slowly that by the time they reach the words at the end of the sentence they have forgotten those at the beginning. When a child does this he is faced with an additional task of structuring, and rearranging the mathematical concepts such that he can produce a meaningful answer.

Several sources list factors that hinder the problem solver in his attempts to transform from ordinary English phrases to mathematics, and methods have been cited to enhance and whet skills associated with problem solving. One, of the many, aspects of the reading that is indigenous to verbal problems has not been covered. That aspect has to do with prescription. Call and Wiggin (1974), among others (Henry, 1971; Troutman and Lichtenberg, 1974; Hater, et al. 1974: Freeman, 1973), suggest that the mathematics teacher incorporate reading skills in his (her) instructional program. Call and Wiggin (1974) also write that the competent mathematics teacher would perhaps get considerably better results in his (her) teaching if he were trained to teach reading of the kind encountered in mathematics problems.

Catterson (1972) offers a promising approach to deal with overcoming difficulties associated with problem solving. She lists the following methods:

1. Directed reading lesson. Here questions are printed rather than asked orally.
2. Matching sentences to paragraphs. In this format, "main ideas of the paragraphs" sentences are supplied (some correct and some incorrect). The student is asked to match sentences to paragraphs.
3. Provide some form of "advance cognitive organizer" (Ausubel, 1963). That is, introducing, prior to the presentation of new ideas, some material that will provide "ideational anchorage" for the new ideas.

The Affective Domain (Attitude) and Related Measurements as They Relate to Mathematics

Aiken (1970) stated that attitude is commonly considered to be partially cognitive and partially affective or emotional. Within the last two decades research studies (Aiken, 1963; Aiken and Dreger, 1961) have been directed at studying the relationship of non-intellective variables to learning. Non-intellective factors as personality traits, socioeconomic status, interest and attitudes (Minder, Jones and Strowig, 1970) have been investigated. The studies on attitude have generally shown that attitudes toward mathematics and the learning of mathematics (of mathematics principles, operations, etc.) are positively correlated. That is, the more positive one's attitude toward mathematics, the greater is his ease of learning the fundamentals of mathematics. The more negative one's attitude toward mathematics, the greater is his difficulty in learning the fundamentals of mathematics.

Moreover, when attitude scores are used as predictors of achievement in mathematics a low but significant positive correlation is usually found (Neale, 1969). This is true at all levels of schooling, the elementary (Evans, 1971; Mastantuono, 1970; More, 1972), secondary (Burbank, 1968; Callahan, 1971; Crosswhite, 1972; Spickerman, 1965) college undergraduate (Edwards, 1972; Fenneman, 1973; Whipkey, 1969; Wilson, 1973) and postgraduate (Webb, 1971). Aiken (1975) reported that it is also true of students in other countries as recorded by do Carmo de Avila and Gillet (1970), Kulkarni and Naidu (1970), and minority groups in the United States (Jackson, 1974). An occasional study such as Webb (1971), finds attitude to be the most important predictor.

Spickerman (1965), found that attitude toward mathematics is directly related to both actual and aspired marks in mathematics courses. Other studies (Callahan, 1971; Evans, 1971; Jacobs, 1974) found attitude toward mathematics to be somewhat inversely related to grade level. The late elementary and early junior high grades are recognized as being particularly important to the development of attitude toward mathematics (Collahan, 1971; Taylor, 1969).

It is of interest to note that findings from at least one study (Straight, 1960) indicated a decline in the percentage of students, from the third through sixth grades, expressing a negative attitude toward mathematics. Perhaps part of this increase in positive attitudes expressed by students has resulted from their being told that mathematics is good for them and that a good attitude toward mathematics pleases the teacher (Aiken, 1970).

A trend that is evident in the studies is that mathematics starts to lose popularity in the junior high school and continues to become progressively more unpopular at the high school and collegiate levels. There is a possibility that this has occurred because of the students being introduced to abstract aspects of mathematics curriculum at too early a stage in their cognitive development. Reys and Delon (1968) studied the attitudes of prospective teachers toward mathematics and reported the junior high school years as being the period when attitudes reached a peak of development. Many researchers (Dutton, 1968) recognize the junior high school age period as being a critical point in the determination of attitudes toward mathematics. Dutton stated that there was a decline from 1958 to 1968 in the percentage of junior high school students expressing negative attitudes toward mathematics, but he also thought that many students lacked self-confidence in dealing with this subject.

Nevertheless, the goal of satisfaction with the state of affairs in mathematics and attitudes toward mathematics is yet to be obtained.

Personal and social factors affecting attitudes

At the junior-high levels, attitudes toward mathematics and achievement in mathematics are significantly related to a number of personality variables (Naylor and Gaudry, 1973; Neufield, 1967; Swafford, 1969). A few of the personality characteristics related to mathematics attitude and achievement are: a high sense of personal worth, a greater sense of responsibility, high social standards, high academic achievement, motivation, and greater freedom from withdrawing tendencies.

It is evidenced that children with positive attitudes toward mathematics tend to like detailed work, to view themselves as more persevering and self-confident (Aiken, 1972), and to be more intuitive than sensing in their personality type (May, 1971).

Studies also indicate that children who do well in mathematics are more conforming and obedient in school (Neale, 1969), and their parents are more possessive (Weston, 1968). The influence of the parents is demonstrated by the fact that pupils' attitudes and achievement in mathematics are positively related to the attitudes of their parents (Aiken, 1972; Burbank, 1968; Levine, 1972). However, the interactions of the attitudes of mothers and fathers with those of their daughters and sons are not completely clear from the research.

It has been shown that elementary school children who are good at mathematics are more intuitive in their thinking (May, 1971) and that high school students talented in mathematics are less sociable than those talented in English (Silverblank, 1972). Moreover, Silverblank found that mathematics majors tend toward extremes on the anxiety variable: they are either usually secure or severely anxious.

Trown (1970) found level of anxiety to interact with introversion-extroversion in its effects on performance in mathematics. He found that on original learning, retention, and transfer of a mathematics lesson, introverts were superior when rules were presented before examples. Extroverts, on the other hand, were superior when rules were presented after examples. Extroverts were also superior on retention tests when their anxiety levels are low. The introversion-extroversion variable also interacts with sex in its effects on performance; extroverted boys and introverted girls perform higher in their own sex groups (Lewis and Peng-Sim, 1973).

The effects of sex differences on attitudes

Although researchers conflict in their findings of significant sex differences (Jacobs, 1974; McClure, 1970; Merkel, 1974; Roberts, 1970), differences in attitudes and achievement in mathematics are frequently found to favor boys over girls at the junior high level and beyond (Hilton and Berglund, 1974; Keeves, 1973; Nevin, 1973; Simpson, 1974). Reports have also shown that mathematics test anxiety in appreciably higher for eighth-grade girls than for eighth-grade boys (Szetels, 1973). Greater interest and more positive attitudes toward mathematics and science on the part of males have been found in other countries as well as the United States (Keeves, 1973; Nevin, 1973). An example can be cited from Nevin's (1973) which reports that Irish girls have a deeper interest in human relationships than boys, a fact that he interprets as interfering with an interest in mathematics.

The correlation between attitude and achievement, as was previously cited, varies not only with grade level but also with the sex of the student, generally being somewhat higher for girls (Behr, 1973). That is, girls' mathematics marks are more predictable from their attitudes than are boys's marks.

The effects of instructional devices on attitude

There are several studies that have dealt with the effects of calculators on attitudes toward mathematics (Advani, 1972; Cech, 1972; Gaslin, 1972). The results of these studies show similar improvements in attitude with the "calculator" groups and the "no calculator" groups when simple machines are used. When more complex digital computers are used as instructional aids, the findings are more conflicting. For instance, Crawford (1970) reported more favorable attitudes toward mathematics as a result of computed-assisted instruction with seventh graders. After having worked with the same group, Johnson (1971) found no differential increase in positive attitude.

On several occasions machines and physical materials have been employed in attempts to improve attitudes toward mathematics. Only upon occasion do such machines, techniques and/or materials produce highly significant effects on attitudes (Burgess, 1969), but more often than not, the results are disappointing (Brown, 1972; Higgins, 1969; Simpson, 1973; Smith, 1973; Wilkinson, 1971).

The effects of teaching approaches on attitude

A great deal of research effort in mathematics education has resulted in a proliferation of answers, many conflicting, to the questions of whether an "experimental," student-centered, discovery, innovative approach to instruction results in greater achievement and more positive attitudes than a "control," teacher-centered, expository, traditional approach. Some of the tentative findings (Aikins, 1975) are:

1. Modern mathematics programs do not improve attitudes more than traditional programs (Demars, 1971; Joyner, 1973).
2. Compared to regular classes, "continuous progress" classes do not have a different effect on attitudes toward mathematics (Williams, 1973).

3. Discovery methods are not superior to expository methods in their effects on attitudes toward mathematics (Richards and Bolton, 1971; Struder, 1971).
4. Neither follow-up instructions (Avenoso, 1971) nor flexible scheduling (Faist, 1972) improves attitudes more than traditional instruction.
5. An individualized approach to instruction in elementary and junior high school mathematics sometimes has a more positive effective on attitudes than a traditional approach (Maguire, 1971; Malcolm, 1972; Scharf, 1971), but sometimes no difference in the effects of the two types of programs is found (Corbin, 1974; Ronshausen, 1971).
6. Certain units or topics in mathematics have a more positive (Duncan, 1970; McBride, 1974; Silverman, 1973) or a more negative (McCord, 1970) effect on attitudes than other units or topics.

Measurement of attitude

Quality in the measuring devises for attitude toward mathematics can be realized by a continuous effort on the part of researchers to become increasingly adept in the use of multivariate attitude scales, multivariate analysis of variance, multiple discriminant analysis and canonical correlation methodology. Presently, there are far too many "home-grown," unstandardized attitude scales (Aiken, 1975).

Perhaps the soundest conclusion that can be drawn from the results of the sheer volume of studies cited in literature such as this review is that changes in attitude toward mathematics involve a complex interaction among student and teacher characteristics, course content, method of instruction, instructional materials, parental and peer support, and methods of measuring these changes (Aiken, 1975; Leake, 1969).

State of the Art

The review of the literature indicates that in recent years there has been an increasing interest in affecting strategies to enhance a better attitude toward mathematics and to raise the level of literacy in mathematics. The task before the mathematics teacher dictates that he reassess old strategies and become cognizant of, and use, the current research in the field. The National Assessment of Educational Progress (NAEP) results seem to indicate very strongly that the practical aspects of mathematics are not being adequately taught.

CHAPTER III

METHOD

The nomenclature decided upon for the project was the Diagrammatic Method. The title seemed to typify the efforts, aims and strageties set forth in the dissemination process of skills and concepts. Diagramming each aspect of all problem situations was stressed from the outset of the project; therefore diagramming might be considered analogous to translation.

Organization for the Project

A program of instruction was devised to teach students how to solve verbal problems diagrammatically. It was intended that such an endeavor should aid in the facilitation of mathematical literacy and symbol recognition. On the first day of instruction, the students were instructed in the selection of variables which could represent unknown values in a problem. This instruction was common to both the control and experimental groups. Following this common instruction, students were introduced to a noncomputational-type problem in which the solution was best seen via pictorial or diagrammatic representation. Thus, the stage was set for a need in the use of symbols to represent abstract or physical situations. It was at this point that students were given practice in translating from the symbolism of the English language to mathematical symbolism or language; they were also given convenient mathematical symbolism or language which they could employ for problems involving addition and subtraction. It was at this time in the instructional program that the two groups diverged. The control groups considered whatever methods were given in the textbook or by the teacher, while the experimental groups used only the methods, materials, and procedures included in the prescribed program.

Teacher variations were minimized by having each teacher teach the same set of topics, with the teaching and learning objectives being the same for all classes for the duration of the four week unit.

Population and Description of Subjects

The target population of interests in this investigation of problem solving was composed of secondary school students. However, the accessible population was students in approximately 17 classes of general mathematics that were taught during the 1975-1976 school year in the schools of the Cache County School District, Cache County, Utah; Logan City School District, Logan, Utah; and Box Elder County School District, Brigham City, Utah. Cache County and Box Elder County are adjacent counties in the extreme northern part of the state of Utah. Cache County has Logan as its county seat, and is situated north of Box Elder County. According to the Utah Department of Employment Security (1973), a large portion of the working population of the districts concerned were employed in the areas of agriculture, manufacturing, trade, services, government, or were self employed. The largest portion were government employees. A small percentage of the working population were employed in the areas of construction, transportation, finance, insurance, and real estate.

The sample of students in the accessible population was chosen from the two junior high schools in the Cache County School District, the one junior high school in the Logan City School District, and one senior high school in the Box Elder School District. Of the teachers who were willing to participate in the investigation, a cluster sampling technique was employed to select the experimental and control groups for eight intact classes. Thus 210 students from the combined intact classes constituted the sample. There were 131 males and 79 females; 117 students were in the experimental groups and 93 students were in the control groups. It is also worth noting that the students enrolled in the general mathematics classes were either counseled or elected to take the subject in preference to Algebra I. Those who were counseled were encouraged to take general mathematics because of their past academic records and/or low achievement test scores.

Ordinarily, the general mathematics curriculum for the ninth graders consists of exercises with the four basic operations in whole numbers, fractions, decimals, and percents, and exercises in simple metric and non-metric geometry.

Description of Measures Employed

Two instruments were used in the collection of the data for this investigation. One of the instruments, the Mathematics Attitude Scale (Aiken, 1972) was employed to provide a general description of "enjoyment of mathematics"--which encompasses not only a liking for mathematics problems, but for mathematics terms, symbols, and routine computations. The test consists of 20 questions of which the correlation coefficient of reliability is .95 and the predictive validity is listed as .40. This instrument was used before and after treatment to assess attitudes. The time required to administer the Mathematics Attitude Scale is approximately 10 minutes.

The other instrument, Algebra Readiness Test, is a battery consisting of five subtests and six scores (fundamental operations, fractions, decimals, problem solving, general numbers and total). The general numbers section of the Algebra Readiness Test Consists of the kinds of problems that were designated as "problem solving" situations. Hence, a summation of the raw scores obtained for the two sections was made for the investigative attempts of "problem solving." The reliability coefficient of this test is given as .96 while the validity coefficient is given as .70. The time required for the administration of the Algebra Readiness Test is 30 minutes (Lueck, 1947).

Research Procedures and Design

Procedures

In order to test the set of hypotheses, The Solomon Four-Group Design was used. The Solomon (1949) Four-Group Design has high prestige and represents an explicit consideration of external validity factors (Campbell and Stanley, 1963).

The following procedures were used to facilitate the use of this design.

1. Requests for permission to do research in the general mathematics classes were sent to the Logan, Box Elder, and Cache County School Superintendents.
2. To insure familiarity and consistency with regard to procedures, the researcher visited with the selected teachers in two one hour sessions to teach and explain the project, and its purpose.
3. Each subject was assigned an identification number. The schools were numbered I, II, III, IV, and were assigned two consecutive numbers (1, 2), (3, 4), (5, 6), and (7, 8) respectively. The odd digits were used to identify the experimental groups and the even digits were used to identify the control groups. Thus, the numerals 1 through 8 were designated as digits for the thousands position in the identification numbers, which consisted of four digits. The

digits occupying the hundreds and tens position denoted the student's position in the respective teacher's roll book. The last digit, in the ones position, denoted the sex of the individual--1 for male and 0 for female. For example, a student with the identification number 1200 is a student from the experimental group of school number I, twentieth on the roll, and is female.

4. The experimenter collected the pretests and posttests two days after each had been administered. Students who missed a test were given an extra day to take it. Teachers were requested to read items to students if that was requested. No other assistance was to be given during the testing.
5. The tests were hand-scored by the researcher.
6. Test scores were analyzed by the Utah State University computer located in the University Computer Center.
7. The results of the study were sent to the participating schools in the form of an abstract.

Design

The data were analyzed as specified by the Solomon Four-Group Design, through the use of the University computer. In this design schools I and II received both the pretests and posttests. Schools III and IV did not receive the pretests, but were given the posttests.

The asymmetries of the design rule out the analysis of variance of gain scores. Disregarding the pretests, except as another "treatment," the posttest scores were analyzed with a two-way analysis of variance (Campbell and Stanley, 1963). What follows is a schematic representation of the design to facilitate an understanding of the analysis employed.

Variables: X refers to the treatment.

0 refers to the measurement. If 0 is located to the left it will be identified as a pretest, if 0 is on the right it will be identified as a posttest.

R refers to random assignment.

Schools	(I II) R R	(III IV) R R
Experimental	0_1 x 0_2	x 0_5
Control	0_3 0_4	0_6

Such a design leads to an increase in generalizability (Campbell and Stanley, 1963). In addition, the effect of X is replicated in four different fashions: $0_2 > 0_1$, $0_2 > 0_4$, $0_5 > 0_6$, and $0_5 > 0_3$. From the previous statements the following table was set up for two-way analysis of variance:

	No X	X
Pretested	0_4	0_2
Unpretested	0_6	0_5

From the columns means, an estimate of main effect of X is yield; from the row means, the main effect is pretesting, and from cell means, the interaction of testing with X.

The following delineation depicts specific steps that were entailed to insure full use of the design:

1. An ANOVA technique was used on the posttest scores to compare schools I and II with schools III and IV in order to find the effect of the pretests and the treatments and sex.
2. An ANOVA technique was used to compare pretest and posttest means of the pretested experimental groups to find the actual change (time) brought about by the experimental treatment and to check for sex differences.
3. An ANOVA technique was used to compare posttest scores of the two pretested schools in order to compare the treatment of the experimental groups and the treatment of the control groups, and to check for sex differences.
4. An ANOVA technique was used to compare posttest scores of the two unpretested schools in order to compare the

treatment of the experimental groups to the treatment of the control groups, and to check for sex differences.

5. An ANOVA technique was used to compare pretest scores of Schools I and II with posttest scores of the experimental groups in schools III and IV to check for treatment effects and to look for sex differences.

Summary

Through the facilitation of the Solomon Four-Group Design, the methods delineated in this chapter were realized, and the five hypotheses were tested in five different fashions--resulting in 25 tests.

CHAPTER IV

FINDINGS

Analytic Technique

A least squares analysis of variance (ANOVA) technique with unequal numbers of subjects per treatment was used to analyze the findings of five basic hypotheses that were tested in five different fashions using the Solomon Four-Group Design. The conclusions of the 25 hypotheses tests were then compared.

Analysis of variance is a statistical technique which allows the investigator to test hypotheses about the equality of several means. In applying the ANOVA technique, the investigator compares the variability between group means with the variability within groups. If these two sources of variability are about the same, then there is no statistical difference between the means. The investigator therefore "analyzes" the variation and he observes whether the variation is due to differences between or within groups. The ANOVA was used because this study involves the consideration of several means.

Description of Findings Relative to Each Hypothesis

Each of the five hypotheses were tested in five different fashions, as was previously stated. The organizational pattern which follows presents the purpose for each replication. Subsequent to the purpose are found the five tests and inferences made as a result of those tests.

In each factorial design that was set up for testing the hypotheses, different sources of variables were included. Not all variables as seen in the design are discussed. Some variables are included for use of others who might care to make use of the data, as the appropriate statistics for all data are located in specified appendices.

Test I: Pretested Vs Unpretested (Using All Subjects)

The purpose of Test I was to compare the posttest scores of the pretested schools to the posttest scores of the unpretested schools. Test I included all subjects. Such a test was needed to find the effect of the pretests; other variables included were treatment and sex.

The results of Test I are presented in Table 1C(A, B, C, D, E). The adjusted means for the table can be found in Appendix E.

Keeping in mind that the major hypothesis of this study is that there is no difference in the attitude and performance of students who received the diagrammatic method and students who received the traditional method (using the Mathematics Attitude Scale and Algebra Readiness Test), the following specific hypotheses were tested for all subjects in the study:

1.1 There is no difference in the treatment groups and control groups.

Table 1C (A, B, C, D, E). Attitude and algebra readiness for Test I

N = 210		A ATTITUDE		B FUNDAMENTAL OPERATIONS		C FRACTIONS		D DECIMALS AND PERCENTS		E PROBLEM SOLVING	
Source	df	Mean squares	F ratio	Mean squares	F ratio	Mean squares	F ratio	Mean squares	F ratio	Mean squares	F ratio
Sex	1	166.25	.31	98.80	2.96	35.76	3.57	6.64	.76	.996	.17
Treatment	1	2648.60	4.97*	230.57	6.90**	380.65	38.05**	5.57	.63	1913.73	32.01**
Test #	1	39991.56	74.99**	47.81	1.43	26.62	2.66	36.00	4.10*	2659.88	44.50**
Sex X treatment	1	2232.64	4.19*	39.52	1.18	3.19	.32	24.45	2.79	182.74	3.06
Sex X Test#	1	1087.82	2.04	.58	.17	1.72	.17	1.70	.19	54.74	.92
Treatment X test #	1	455.20	.85	461.43	13.82**	31.23	3.12	5.76	.66	44.29	.74
Treatment X sex X test #	1	3185.38	5.97*	40.48	1.21	9.24	.92	1.07	.12	7.81	.13
Error	202	533.32		33.38		75.90		8.78		59.77	

* Significant at the .05 level

** Significant at the .01 level

In table 1C(A, B, C, D, E) are the results of the analyses of the posttest scores of schools I, II, III and IV, where schools I and II (pretested) were compared with schools III and IV (posttested only). This table was computed using the ANOVA technique on the Mathematics Attitude Scale and the sub-tests of the Algebra Readiness Test (fundamental operations, fractions, decimals and percents, and problem solving). Hypotheses 1, 2, 3, 4, and 5 were tested through the use of Table 1C(A, B, C, D, E).

Under source, Test # refers to the number of tests taken by students (students have taken only a posttest or students have taken a pre and posttest).

1.2 There is no difference in the posttest scores of males and females.

1.3 There is no difference in the posttest scores of subjects who took pretests and posttests, and subjects who took posttest only.

These hypotheses were tested for each of the following:

1. Attitude
2. Fundamental Operations
3. Fractions
4. Decimals and Percents
5. Problem Solving

Attitude Toward Mathematics

1. The results of the analysis of variance (Table 1C-A) showed a significant F ratio of 4.94 for treatment. The posttest mean of the no-treatment group was 44.62 and the mean for the treatment group was 52.17 on the treatment variable. Therefore, the null hypothesis was rejected in support of the treatment. These significant levels imply that the diagrammatic method was more effective than the traditional method in engendering a better attitude toward work in mathematics. Also, males and females differed markedly in the affective domain, with the attitude of males improving to a much greater extent than females. Figure 1 illustrates the occurance of interaction with respect to sex and treatment.

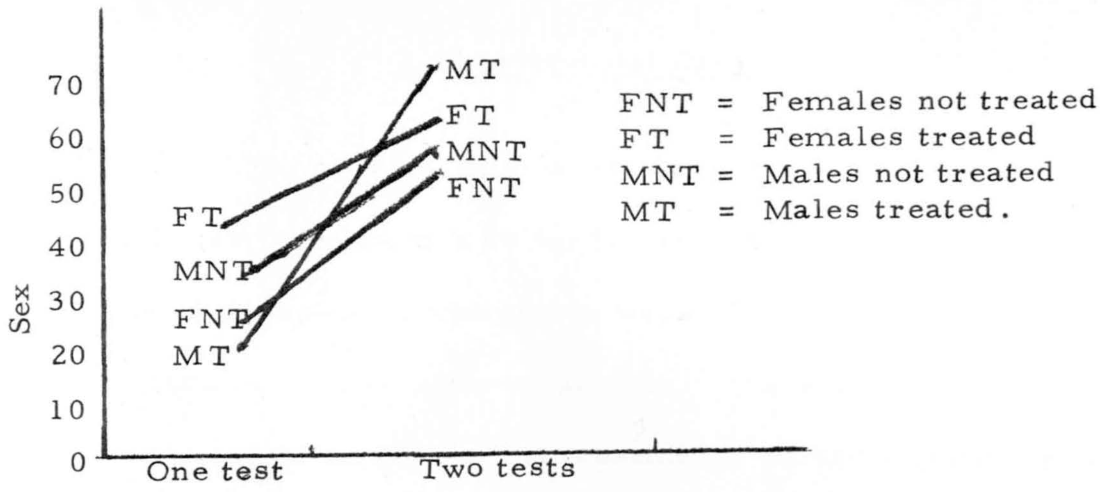

Figure 1. Interaction effects of attitude.

The significant F ratio of 4.19 for interaction can be better understood when the means are observed in Figure 1. As can be observed, males and females who were in the treatment groups had higher mean scores on the second testing than did males and females who were not in the treatment groups. The significant factor in this interaction is that the males who were in the treatment groups and took only one test (posttest), scored lowest of all the groups, while the males who were in the treatment groups and took two tests (pre and posttest) scored the highest of all the groups. This suggests that while improvement in attitude could have resulted from the treatment, there is a possibility of practice effects and this probably accounts for the highly significant F ratio of 74.99 for Test II. Thus the hypothesis was rejected in support of pretesting having an effect on posttest outcome (Test ≠).

Fundamental Operations

2. The results of the analysis of variance (Table 1C-B) showed a significant F ratio of 6.90 for the treatment source. The posttest means of the no-treatment group were 13.12, and 15.35 for the treatment group on the treatment variable. Therefore, the null hypothesis was rejected in support of the treatment. These significant levels indicate that the diagrammatic method was more effective than the traditional method in affecting improved performance on fundamental operations; additionally, they depict a difference in the treatment by test number. It appears that in the no-treatment groups pre and post testing had an adverse effect on posttest outcome, while the opposite is true of the treated groups. Perhaps this accounts for the significant F ratio of 13.82 for interaction effects.

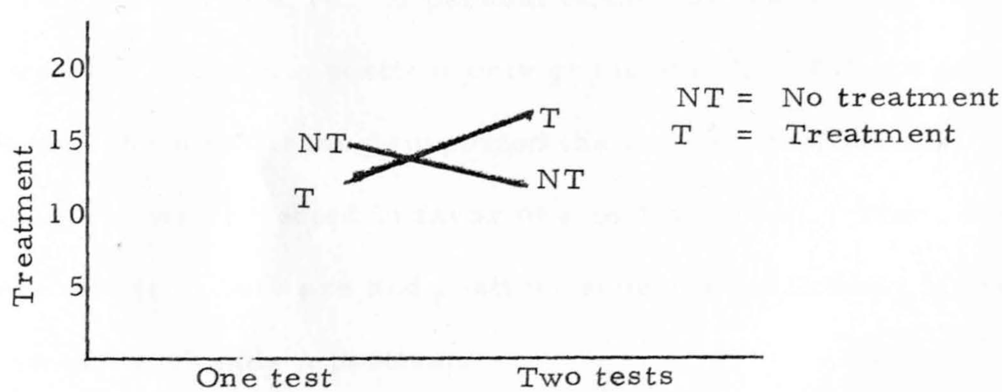

Figure 2. Interaction effects of fundamental operations.

Fractions

3. The results of the analysis of variance (Table 1C-C) showed a highly significant F ratio of 38.05 on the treatment variable. An examination of the posttest means revealed a score of 3.24 for the no-treatment group. Therefore, the null hypothesis was rejected in favor of the treatment variable, since the required F ratio for significance at the .05 level is 3.84. This significant level infers that the diagrammatic approach was more effective than the traditional approach in affecting improved performance on fractions. The number of tests taken was not significant.

Decimals and Percents

4. The results of the analysis of variance (Table 1C-D) did not show significance on the treatment, but the test variable was significant with an F ratio of 4.10. A perusal of the posttest means showed a score of 7.39 for the posttest only group and 8.27 for the pre-posttest groups. As significance supported the pre-posttest groups, the null hypothesis was rejected in favor of a testing effect. Thus, those students who took both pre and posttest scored significantly higher than those who took only a posttest.

Problem Solving

5. The results of the analysis of variance showed highly significant F ratios of 32.01 and 44.50 on the treatment and test variables, respectively. An examination of the posttest means revealed a score of 10.32 for the no-treatment group and 15.77 for the treatment group on the treatment variable for those students who had taken both pre- and posttests, means for those students who did not take pretests were 16.91 and 24.31, respectively. The null hypothesis was rejected in favor of the treatment variable and rejected in favor of test differences. This indicates that there is a significant difference in the posttest means of students who took the pretest and those who did not take the pretest; the difference is in the direction of those students who did not take the pretest. Thus, this factor implies that while the treatment was effective, pretesting did not increase performance means on problem solving.

Test II: Pretests Vs Posttests (Experimental Groups)

The purpose of Test II was to compare pretest scores to posttest scores of the experimental groups and to observe whether the difference brought about by the treatment was significant. The test served as a preliminary step in so far as the effectiveness of the treatment was concerned. If the test had revealed insignificance for the "time" (effect of treatment brought about from pretest to posttest) variable on most of the five replications, this would have been indicative of the fact that the treatment was indeed ineffective; moreover, any gain made could have been attributed to the number of tests taken by the subjects involved. Furthermore, if no significant gains had been realized from Test II, it would have been purely academic to continue the replications.

The results of Test II are presented in Tables 2A, 2B, 2C, 2D, and 2E. The adjusted means for the table can be found in Appendix F.

Keeping in mind that the major hypothesis of this study is that there is no difference in the performance of students who received the diagrammatic method and students who received the traditional method (using the Mathematics Attitude Scale and the Algebra Readiness Test), the following specific hypotheses were tested for all subjects in the experimental groups.

2.1. There is no difference in the measured scores from pretest to posttest (Time).

2.2. There is no difference in the measure of knowledge from pretest to posttest of males and females.

These hypotheses were tested for each of the following:

1. Attitude
2. Fundamental Operations
3. Fractions
4. Decimals and percents
5. Problem solving

Attitude Toward Mathematics

6. The results of the analysis of variance (Table 2A) showed a highly significant F ratio of 18.74 on the Time variable.

The null hypothesis was rejected on the time variable with significant gains between pretesting (62.56) and posttesting (69.05) for the experimental groups. This seems to indicate a more favorable attitude toward mathematics in the direction of the diagramatic method, when considering the posttest scores of the pretested experimental groups. The null hypothesis was retained on the sex source of variance.

Table 2A. Attitude for Test II

Source	df	Mean squares	F ratio
Sex	1	314.34	65
(Error A) Subjects (Sex)	57	316.21	---
Time	1	852.72	18.74**
Time X Sex	1	24.58	.54
(Error B) Time X Subjects (Sex)	57	45.51	---

N = 59
* Significant at the .05 level
** Significant at the .01 level

Table 2B. Fundamental operations for Test II

Source	df	Mean Squares	F ratio
Sex	1	40.19	.85
(Error A) Subjects (Sex)	57	47.26	---
Time	1	729.54	51.38**
Time x Sex	1	74.58	5.26*
(Error B) Time X Subjects	57	14.18	

N = 59
* Significant at the .05 level
** Significant at the .01 level

Fundamental Operations

7. This analysis of variance (Table 2B) represents another way of looking at the loss or gain in performance between the pretest administration and posttest administration. The results of the analysis showed a highly significant F ratio of 51.38 on the time variable and a significant F ratio of 5.26 on the time by sex interaction. The null hypothesis was rejected on the time, and time by sex interaction variables. It would thus appear that significant gains were made on fundamental operations during the time between pretesting (19.94) and posttesting (16.94) in favor of the diagrammatic approach. The interaction as viewed in Figure 3 illustrates the role of the sexes and time.

The null hypothesis was retained on the sex source of variance. While the pretest score of the males was lower than the female, the gain made by the males was greater than the gain of the female.

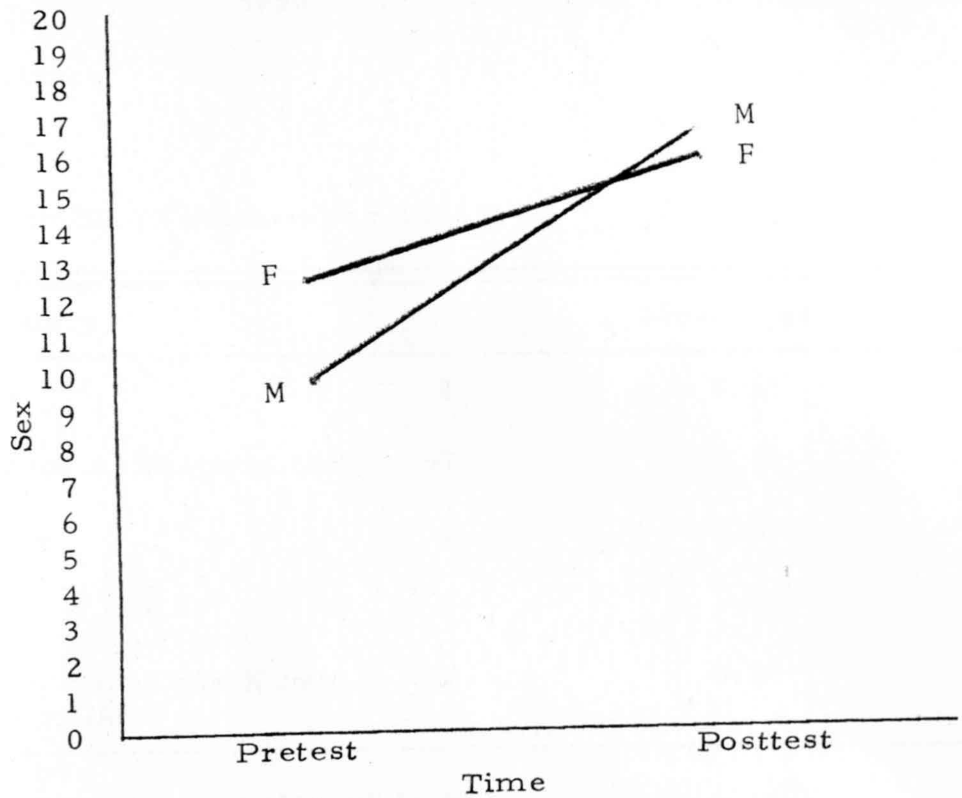

Figure 3. Interaction effects of fundamental operations.

Fractions

8. The results of the analysis of variance (Table 2C) showed a highly significant F ratio of 17.33 on the time variable, showing the pretest mean was 3.31 and the posttest mean was 5.07, indicating a positive gain in performance on fractions. Thus the null hypothesis was rejected in support of the diagrammatic method having positive effects on the posttest outcome. Figure 4 illustrates the role of sex differences on pre and posttesting.

The null hypothesis was retained on the sex source of variance.

Table 2C. Fractions for Test II

Source	df	Mean Squares	F ratio
Sex	1	8.53	.55
(Error A) Subjects (Sex)	57	15.43	---
Time	1	62.85	17.33**
Time X Sex	1	1.16	.32
(Error B) Time X Subjects (Sex)	57	3.63	---

N = 59
* Significant at the .05 level
** Significant at the .01 level

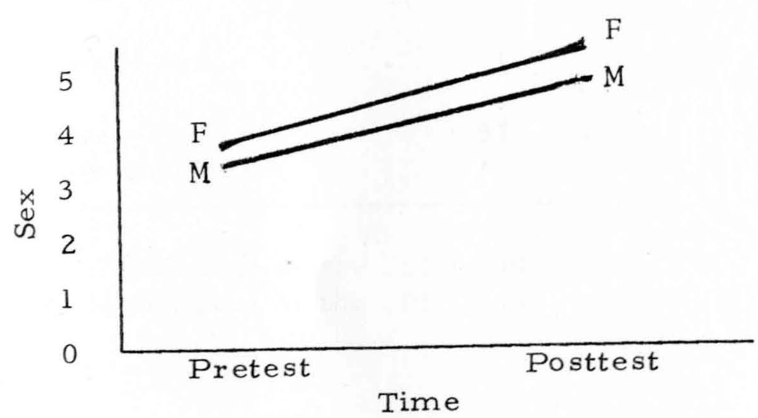

Figure 4. Interaction effects on fractions.

Decimals and Percents

9. The results of the analysis of variance (Table 2D) showed a highly significant F ratio of 20.54 on the time variable and 4.53 on the sex variable. The pretest mean was 5.77 and the posttest mean was 7.37. The posttest mean for females was 7.94 while the posttest mean for males was 6.79. Thus, both null hypotheses were rejected, suggesting a positive gain between pretest posttest means, and females performed significantly higher than males.

Table 2D. Decimals and percents for Test II

Source	df	Mean Squares	F ratio
Sex	1	51.27	4.53*
(Error A) Subjects (Sex)	57	11.32	---
Time	1	51.80	20.54**
Time X Sex	1	4.01	1.59
(Error B) Time X Subjects (Sex)	57	2.52	---

N = 59
* Significant at the .05 level
** Significant at the .01 level

Table 2E. Problem solving for Test II

Source	df	Mean Squares	F ratio
Sex	1	57.20	.78
(Error A) Subject (Sex)	57	73.23	
Time	1	1543.13	80.33**
Time X Sex	1	55.84	2.91
(Error B) Time X Subjects (Sex)	57	19.21	

N = 59
* Significant at the .05 level
** Significant at the .01 level

Problem Solving

10. The results of the analysis of variance (Table 2E) showed a highly significant F ratio of 80.33 on the time variable. The pretest mean was 6.37 while the posttest mean was 15.09. Thus, the null hypothesis was rejected. The large increase in the posttest mean probably indicates the effectiveness of the diagrammatic method. Figure 5 is indicative of the role of sex differences on testing (time).

Test III: Posttest Vs Posttests (Pretested Groups)

The purpose of Test III was to separate test-retest effects from treatment effects in the pretested groups. This was accomplished by analyzing the difference brought about from pretest to posttest of the control groups and experimental groups. Thus, the traditional treatment which the control groups received was compared to the treatment which the experimental groups received.

Tables 3A, 3B, 3C, 3D, and 3E present the data extracted from the least squares analysis of the posttest scores of all pretested groups. The adjusted means for the forementioned tables can be found in Appendix G.

Keeping in mind that the major hypothesis of this study is that there is no difference in the performance by students who received the diagrammatic method and students who received the traditional method (Using the Mathematics Attitude Scale and the Algebra Readiness Scale), the following specific hypotheses were tested for all subjects in the study who were pretested:

3.1. There is no difference in the posttest scores of the treatment groups and posttest scores of the control groups.

3.2. There is no difference in the measured scores from pretesting to posttesting (Time).

3.3. There is no difference in the posttest scores of males and females.

These hypotheses were tested for each of the following:

1. Attitude

2. Fundamental Operations

3. Fractions

4. Decimals and percents

5. Problem solving

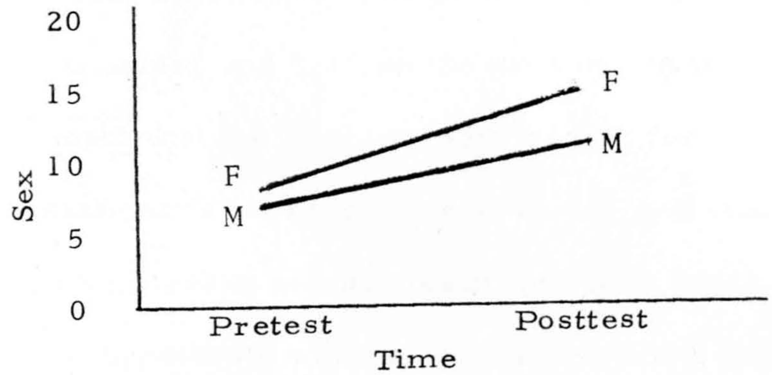

Figure 5. Interaction effects of problem solving.

Table 3A. Attitude for Test III

Source	df	Mean Squares	F ratio
Sex	1	521.77	1.52
Treatment	1	2764.86	8.04**
Sex X Treatment	1	4.85	.01
(Error A) Subjects (Groups)	99	343.86	
Time	1	651.72	13.46**
Time X Sex	1	14.53	.30
Time X Treatment	1	264.48	5.46*
Time X Treatment X Sex	1	118.96	2.46
Error B	99	48.41	

N = 103
* Significant at the .05 level
** Significant at the .01 level

Attitude Toward Mathematics

11. The results of the analysis of variance (Table 3A) showed significant F ratios of 8.04 on the treatment variable, 13.46 on the time variable, and 5.46 on the time by treatment interaction. An examination of the posttest mean scores for those students who received the treatment showed a score of 66.17, and students who received the traditional treatment (no treatment) had a mean score of 58.01. Thus the null hypothesis was rejected with results favoring the diagrammetic method on the treatment variable.

The control groups had a pretest mean score of 58.01 and a posttest mean score of 66.17 while the experimental groups had a pre-test mean score of 62.92 and a posttest mean score of 69.41. The gain made by the control groups were slightly higher than that made by the experimental group. Although the gain is slight, it is nevertheless significant. Therefore the null hypothesis was retained with respect to the time variable. The significant interaction effects are obviously due to shifts in means scores for the time variable. Figure 6 illustrates the level of interaction. The null hypothesis was retained on the sex source of variance.

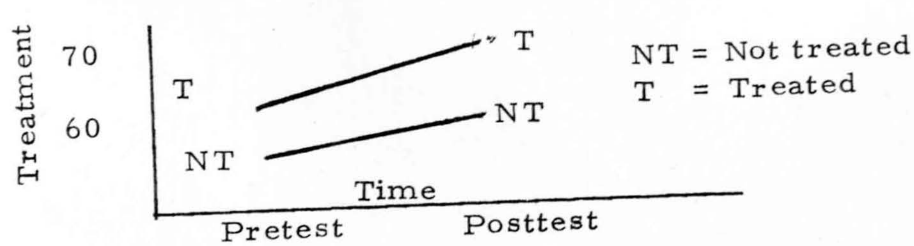

Figure 6. Interaction effects of attitude.

Fundamental Operations

12. The results of the analysis of variance (Table 3B) showed highly significant F ratios on the treatment and time variables and significant differences on the interactions of time, treatment and sex. The posttest mean for the treated groups was 14.36 and the posttest means for the groups that were not treated was 9.69. Thus, the increase was in the direction of positive learnings toward the treatment and led to rejection of the null hypothesis with respect to treatment. The time variable had a significant F ratio also. It was due to greater gains from pretesting to posttesting by the experimental groups. The experimental groups mean scores were incremented from 11.36 to 17.36, while the scores of the control groups were incremented from 7.40 to 11.98. Thus the null hypothesis was rejected with results favoring the diagrammatic method with respect to the time varizble.

The initial difference in the scores and the imbalance in "gain" performance led possibly to the significant interaction effects of time, treatment and sex. Figure 7 presents an illustration of the interaction effects. The null hypothesis was retained on the sex source of variance.

Table 3B. Fundamental operations for Test III

Source	df	Mean Squares	F ratio
Sex	1	141.67	3.34
Treatment	1	906.30	21.35**
Sex X Treatment	1	8.01	.19
(Error A) Subjects (Groups)	99	42.45	
Time	1	1160.12	79.84**
Time X Sex	1	10.28	.71
Time X Treatment	1	20.88	1.44
Time X Treatment X Sex	1	83.79	5.77*
Error B	99	14.53	

N = 103
* Significant at the .05 level
** Significant at the .01 level

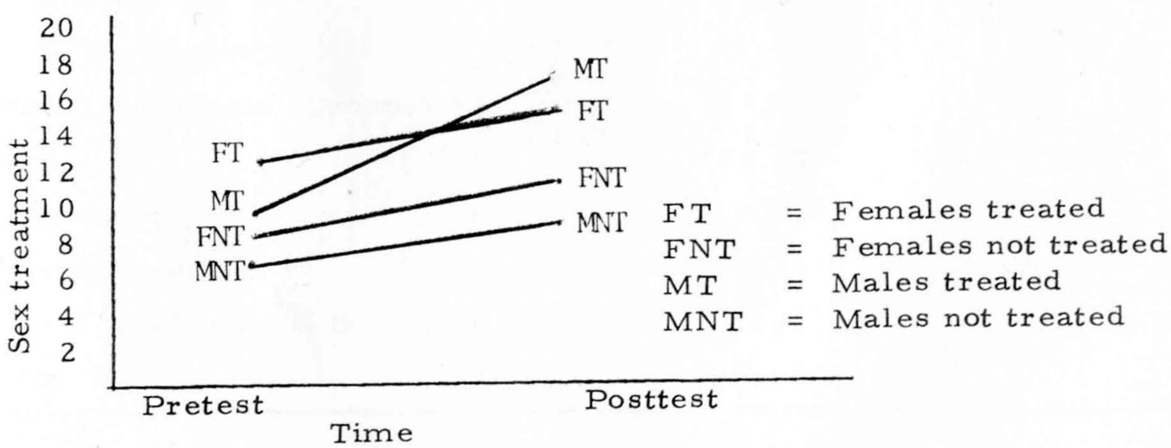

Figure 7. Interaction effects of fundamental operations.

Fractions

13. The results of the analysis of variance (Table 3C) showed significant F ratios of 8.85 and 13.13 on the variables of time, and time by treatment interaction respectively. In regard to the time variable the pretest mean score of the experimental groups was 3.42, while the mean score of the control groups was 3.31. The posttest mean score for experimental group was 5.18, while the mean score for the control groups was 3.14. Therefore the null hypothesis was rejected with results favoring the experimental treatment.

Table 3C. Fractions for Test III

Source	df	Mean Squares	F ratio
Sex	1	31.14	2.35
Treatment	1	48.09	3.63
Sex X Treatment	1	1.96	.15
(Error A) Subjects (Groups)	99	13.22	
Time	1	26.17	8.85**
Time X Sex	1	1.76	.60
Time X Treatment	1	38.82	13.13**
Time X Treatment X Sex	1	.05	.02
Error B	99	2.96	

N = 103
* Significant at the .05 level
** Significant at the .01 level

The time by treatment interaction was significant, and effects can be observed in Figure 8. As the experimental groups rose significantly, the control groups fail to show positive gains from pretest to posttest. The null hypothesis was rejected in support of the experimental treatment.

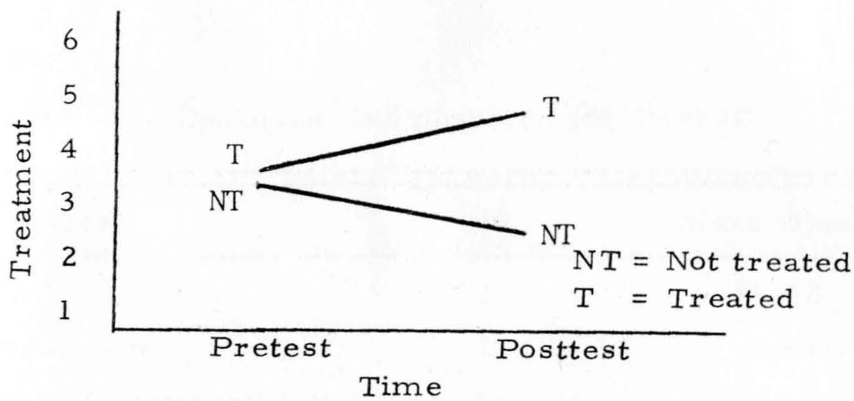

Figure 8. Interaction effects of attitude.

A perusal of Figure 8 shows that the treated groups realized an increase in mean scores while the groups not treated decreased in mean scores. This indicates the diagrammatic method seemed to be more effective in teaching fractional concepts than the traditional approach.

Decimals and Percents

14. The results of the analysis of variance (Table 3D) showed a highly significant F ratio of 25.18 on the time variable.

The mean score on the pretest for the experimental group was 5.98, while the mean score on the pretest for the control group was 5.92. The mean score on the posttest for the experimental group was 7.58, while the mean score for the control group was 6.88. Therefore, the null hypothesis was rejected on the time variable in the direction of the experimental treatment.

Table 3D. Decimals and percents for Test III

Source	df	Mean Square	F ratio
Sex	1	34.42	2.95
Treatment	1	5.93	.51
Sex X Treatment	1	19.19	1.64
(Error A) Subjects (Groups)	99	11.67	---
Time	1	67.81	25.18**
Time X Sex	1	4.80	1.78
Time X Treatment	1	4.27	1.59
Time X Treatment X Sex	1	.46	.17
Error B	99	2.70	---

N = 103
* Significant at the .05 level
** Significant at the .01 level

Problem Solving

15. The results of the analysis of variance (Table 3E) showed highly significant F ratios on the time and time by treatment interaction variable of 81.48 and 31.58, respectively.

The pretest mean score for the experimental groups was 6.73, while the pretest mean score for the control groups was 7.99. The posttest mean score for the experimental groups was 15.46 while the posttest mean score for the control groups was 10.02. Therefore, the null hypothesis was rejected with results supporting the experimental treatment.

Table 3E. Problem solving for Test III

Source	df	Mean Squares	F ratio
Sex	1	17.76	.27
Treatment	1	182.41	2.73
Sex X Treatment	1	43.69	.65
(Error A) Subject (Groups)	99	66.88	
Time	1	1200.61	81.48**
Time X Sex	1	3.42	.23
Time X Treatment	1	465.26	31.58**
Time X Treatment X Sex	1	78.22	5.30
Error B	99	14.73	

N = 103

* Significant at the .05 level

** Significant at the .01 level

The significant interaction of time by treatment variables was rejected also in support of the experiment treatment, as the experimental groups far exceeded the control groups in performance from pretesting to posttesting. Figure 9 gives a pictorial account of the distribution of scores for interaction purposes.

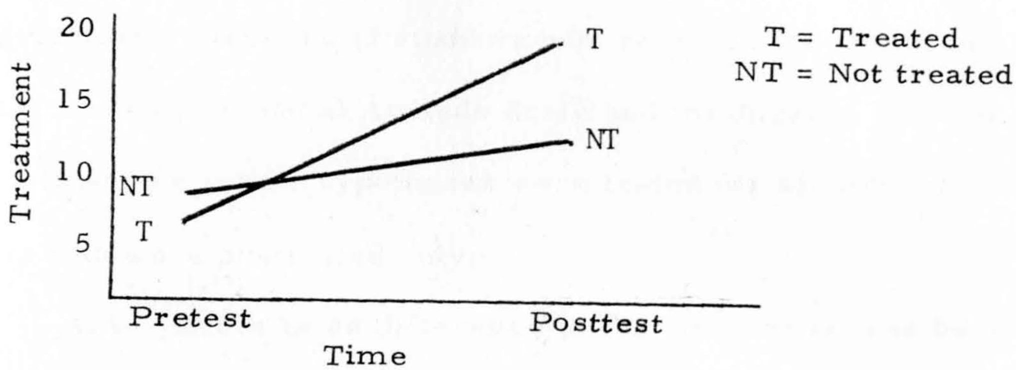

Figure 9. Interaction effects of problem solving.

This interaction seems to indicate that perhaps not-treated (control) groups failed to make the progress to the same degree as the treated (experimental) groups.

Test IV: Posttests Vs Posttests
(Posttested Only Groups

The purpose of Test IV was to assess the treatment effects, independent of testing effects. The use of Test IV entailed the inclusion of all unpretested or posttest only groups. Typically, such a test allows for yet another way to compare the treatment of the experimental groups to the treatment of the control groups. The comparisons in Test IV are made via mean scores of the posttest only groups.

The results of Test IV are presented in Table 4 (A, B, C, D, E). The adjusted means for the table can be found in Appendix H.

Keeping in mind that the major hypothesis of this study is that there is no difference in the performance of students who received the diagrammatic method and students who received the traditional method (using the Mathematical Attitude Scale and the Algebra Readiness Test), the following specific hypotheses were tested for all subjects in the study who were posttested only:

4.1. There is no difference in the posttest scores between treatment groups and control groups.

4.2. There is no difference in the posttest scores of males and females.

These hypotheses were tested for each of the following:

1. Attitude
2. Fundamental operations
3. Fractions
4. Decimals and percents
5. Problem solving

Attitude Toward Mathematics

16. The results of the analysis of variance (Table 4A) showed a significant F ratio of 7.19 on the interaction of sex by treatment. Figure 10 illustrates the role of the means involved in the interaction. The null hypotheses were retained on the sex and treatment variables.

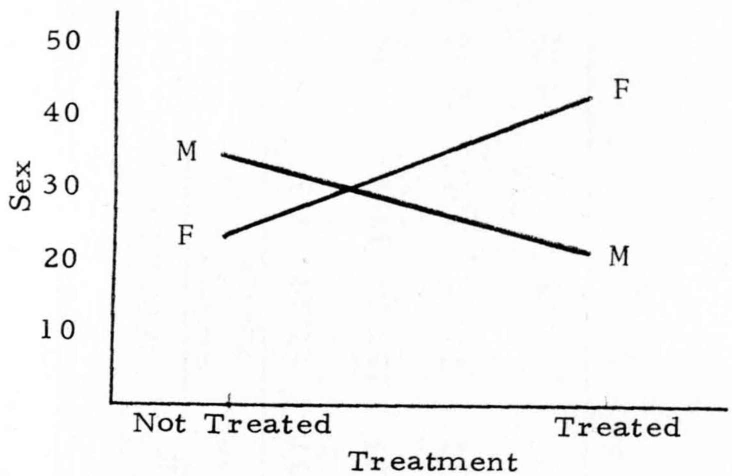

Figure 10. Interaction effects of attitude.

It appears that the treatment had a negative effect on the attitude of males and a highly positive effect on the attitude of females.

Fundamental Operations

17. The results of the analysis of variance (Table 4B) showed no significant F ratios. Therefore, the null hypotheses were retained with respect to sex differences and treatment effects.

Table 4(A, B, C, D, E). Attitude and Algebra Readiness for Test IV

		A ATTITUDE		B FUNDAMENTAL OPERATIONS		C FRACTIONS		D DECIMALS AND PERCENTS		E PROBLEM SOLVING	
Source	df	Mean Square	F ratio	Mean square	F ratio	Mean square	F ratio	Mean square	F ratio	Mean square	F ratio
Sex	1	1188.01	1.41	64.62	2.37	12.30	1.3	.91	.95	39.80	.71
Treatment	1	514.72	.61	22.48	.83	357.18	38.90**	.94	.97	1440.42	25.82**
Sex X treatment	1	6069.12	7.19**	.33	.12	13.14	1.43	20.18	2.10	64.91	1.16
Error	103	844.59		27.23		9.18		9.62		55.79	

N = 107

* Significant at the .05 level

** Significant at the .01 level

Fractions

18. The results of the analysis of variance (Table 4C) showed a highly significant F ratio of 38.90 on the treatment variable, in the direction of the diagrammatic method. Therefore, the null hypothesis was rejected on the treatment source and retained on sex source. An examination of the means revealed that the groups receiving the diagrammatic method had a score of 6.94 while those receiving the traditional approach had a score of 3.25. Thus, it appears that the diagrammatic method enhanced performance on fractions.

The null hypothesis on the sex source was retained.

Decimals and Percents

19. The results of the analysis of variance (Table 4D) showed no significant F ratios on sex and treatment. Therefore, the null hypotheses were retained.

Problem Solving

20. The results of the analysis of variance (Table 4E) showed a highly significant ratio of 25.82 on the treatment source. An examination of the means revealed a score of 17.05 for the untreated groups and 24.45 for the treated groups. Thus the null hypothesis was rejected with respect to the treatment source in support of the diagrammatic method. The null hypothesis was retained with respect to the sex source.

Test V: Pretested Groups Vs Post-testedOnlyExperimental Groups

The purpose of Test V was to provide another means of viewing and assessing the effects of the experimental treatment, independent of test-retest effects. Specifically, Test V compared the mean pretest scores of all students who were pretested (Schools I and II) to the mean posttest scores of the experimental groups in Schools III and IV. Schools III and IV were posttest only schools.

The results of Test V are presented in Table 5 (A, B, C, D, E). The adjusted means for the table can be found in Appendix I.

Keeping in mind that the major hypothesis of this study is that there is no difference in the performance of students who received the diagrammatic method and students who received the traditional method (using the Mathematics Attitude Scale and the Algebra Readiness Test), the following specific hypotheses were tested for the previously specified subjects in the study:

1.1. There is no difference in the measured scores between treatment groups and control groups.

1.2. There is no difference in the mean scores of males and females.

These hypotheses were tested for each of the following:

1. Attitude
2. Fundamental operations
3. Fractions

Table 5(A, B, C, D, E). Attitude and algebra readiness for Test V

		A ATTITUDE		B FUNDAMENTAL OPERATIONS		C FRACTIONS		D DECIMALS AND PERCENTS		E PROBLEM SOLVING	
Source	df	Mean squares	F ratio	Mean squares	F ratio	Mean squares	F ratio	Mean squares	F ratio	Mean squares	F ratio
Sex	1	1163.37	2.79	313.54	14.61**	82.05	9.33**	68.78	8.02**	344.91	4.29*
Treatment	1	24259.97	58.20**	397.53	18.52**	163.48	18.59**	.038	.004	84.86	1.06
Sex X treatment	1	615.79	1.48	17.09	.80	36.25	4.12*	59.16	6.90**	493.76	6.15*
Error	153	416.80		21.47		8.79		8.58		80.33	

N = 157

* Significant at the .05 level

** Significant at the .01 level

4. Decimals and percents

5. Problem solving

Attitude Toward Mathematics

21. The results of the analysis of variance (Table 5A) showed a highly significant F ratio of 58. 20 on the treatment source. An examination of means revealed a control group mean of 28. 96 for treatment and a mean of 64. 52 for the experimental groups. Therefore, the null hypothesis was rejected with respect to the treatment source in support of the diagrammatic method. The hypothesis was retained with respect to sex differences.

Fundamental Operations

22. To test the null hypothesis that there is no difference in the performance on fundamental operations among students who received the diagrammatic method and students who received the traditional method as measured by the Algebra Readiness Test, the ANOVA technique was used; the results of this technique are reported in Table 5B. The results of the analysis of variance (Table 5B) showed highly significant F ratios of 14. 61 and 18. 52 on the sex and treatment sources, respectively. An examination on the means showed males who were in the experiemental group received a posttest score of 10. 65 and females received a posttest score of 14. 24. The means of males and

females from the control groups were 8.44 and 10.29, respectively. Therefore, the null hypothesis was rejected with respect to sex differences, indicating that there was a difference in the performance on fundamental operations among males and females. The treatment scores were 12.44 for the experimental groups and 9.17 for the control groups. Thus, the null hypothesis was rejected in support of the diagrammatic method.

Fractions

23. The results of the analysis of variance (Table 5C) showed highly significant difference on sex (F ratio of 9.33) and treatment (F ratio of 18.59) sources, and significant difference on the interaction of the sex and treatment source (F ratio of 4.12). The scores associated with the significant levels are illustrated in Figure 11.

All the significant F ratios were in favorable directions of the treatment; the null hypotheses were rejected with respect to sex and treatment differences.

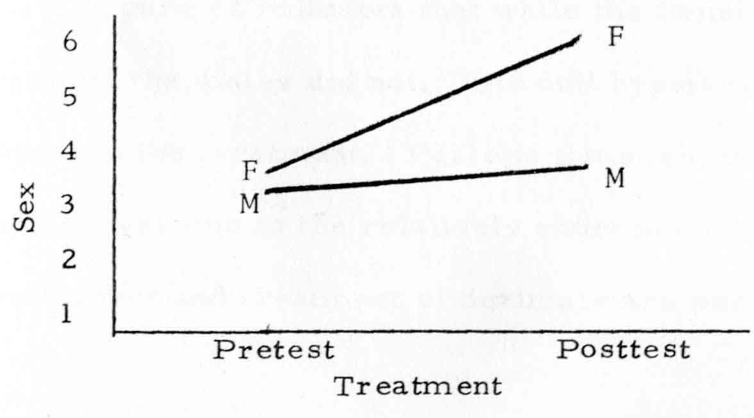

Figure 11. Interaction effects of fractions.

Decimals and Percents

24. The results of the analysis of variance (Table 5D) showed significant F ratios of 8.02 and 6.90 on sex and interaction of sex by treatment sources, respectively. To facilitate interpretation of the significant F ratios Figure 12 was drawn.

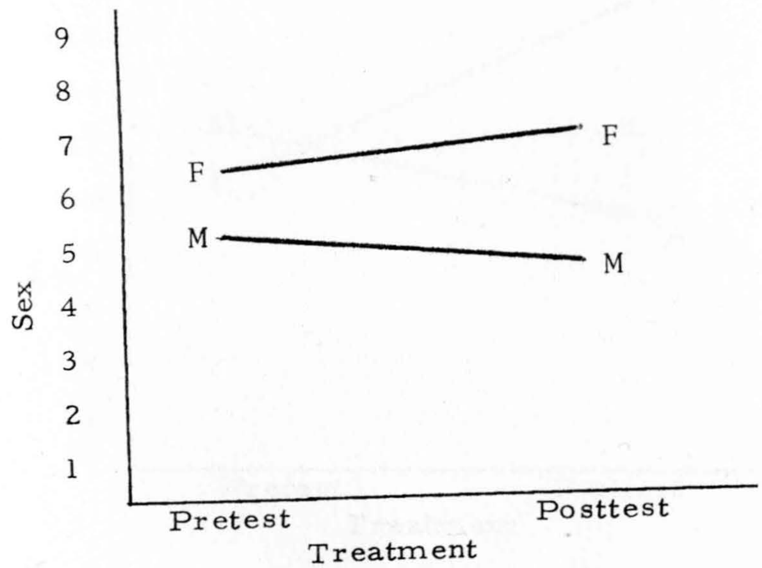

Figure 12. Interaction effects of decimals and percents.

Figure 12 indicates that while the females benefited from the treatment the males did not. The null hypothesis was retained with respect to the treatment. Perhaps the slight decline in performance of males was due to the relatively short period of time spent on the development and treatment of decimals and percents in the module.

Problem Solving

25. The results of the analysis of variance (Table 5E) showed significant F ratios of 4.29 and 6.15 on sex and interaction of sex by treatment sources, respectively. Figure 13 illustrates the role and effects of sex and interaction on the treatment.

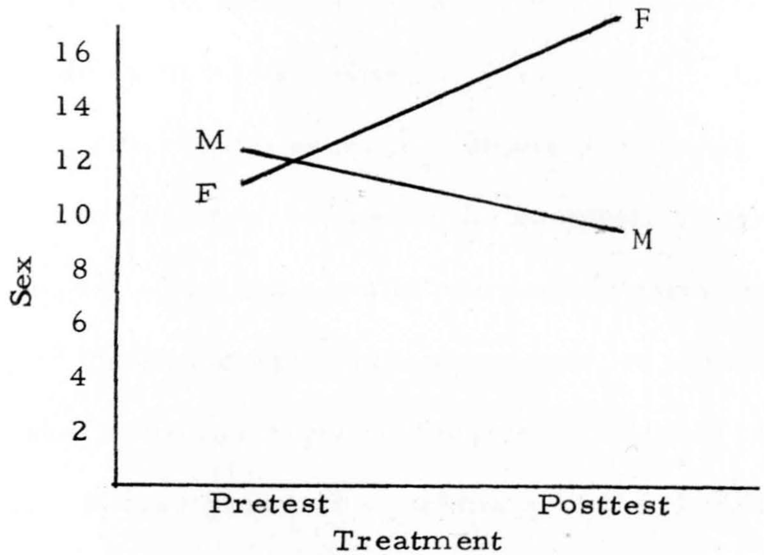

Figure 13. Interaction effects on problem solving.

Figure 13 illustrates that there are indeed differences between the sexes and interaction of the sexes and treatment. Therefore, the null hypothesis was rejected with respect to sex differences. The null hypothesis was retained with respect to the treatment source, as the F ratio of 1.06 was below the required significant level of 3.84. Moreover, the males were apparently not helped by the experimental treatment.

Summary

Having used the same instruments for pretesting and posttesting, the threat of the pretest to external validity was inherent (Campbell and Stanley, 1963). Therefore the Solomon Four-Group Design was chosen to test the five basic null hypotheses, which resulted in replication of the five in five different ways. The asymmetries of the design allow for overlap in testing effects.

The results seem to indicate that the pretested groups, overall, did indeed perform better on the posttests in regard to the Mathematics Attitude Scale. The results are further summarized in the next chapter.

Table 6 gives a capsule account of performance by students on subtests from the Algebra Readiness Test and the Mathematics Attitude Scale. It also presents a summary of the five different tests which were performed on each hypothesis.

Table 6 (A, B, C, D, E) Summation of replicated results

	TEST I				TEST II		TEST III				TEST IV		TEST V	
	Treatment		Test		Time		Treatment		Time		Treatment	Treatment	Treatment	Treatment
	Experi-mental	Con-trols	Post-test only	Pre-post	Experimental Groups Pretest	Experimental Groups Posttest	Experi-mental	Con-trol	Experi-mental	Con-trol	Experimental	Control	Experimental	Control
1. Attitude	*			*		*		*		*	--		*	
2. Fundamental operations	*		--			*	*		*		--		*	
3. Fractions	*		--			*	--		*		*		*	
4. Decimal and percents	--			*		*	--		*		--		--	
5. Problem solving	*		*			*	--		*		*		--	
	Purpose: Pretested Vs Posttested (All subjects)				Purpose: Pretests Vs Posttests (Experimental groups)		Purpose: Posttests Vs Posttests (Pretested groups)				Purpose: Posttests Vs Posttests (Posttest only groups)		Purpose: Pretested groups Vs posttested only experimental groups	

* Indicates that the mean for this group is significantly higher than the paired group.

-- Indicates that the mean for this group is not significantly higher than the paired group.

CHAPTER V

SUMMARY, CONCLSUIONS AND RECOMMENDATIONS

Summary of Hypotheses, Method and Findings

The purpose of this study was to evaluate an approved problem solving module as a model for use in the general mathematics curriculum, and assess its effectiveness in bringing about literacy in and a better attitude toward mathematics. The module focused on concerns of the National Assessment of Educational Progress (NAEP), that is, the inability of students and young adults to use numbers skillfully enough to meet the demands of a modern society. Topics covered in the module included a diagrammatic treatment of fundamental operations, fractions, decimals and percents, and application problems (problem solving).

The null hypotheses that were tested are as follows:

1. There is no difference in the attitudes among students who received the diagrammatic method and students who received the traditional method as measured by the Mathematics Attitude Scale.

2. There is no difference in the performance on fundamental operations among students who received the diagrammatic method and students who received the traditional method as measured by the Algebra Readiness Test.

3. There is no difference in the performance on fractions among students who received the diagrammatic method and students who received the traditional method as measured by the Algebra Readiness Test.

4. There is no difference in the performance on decimals and percents among students who received the diagrammatic method and students who received the traditional method as measured by the Algebra Readiness Test.

5. There is no difference in the performance on problem solving among students who received the diagrammatic method and students who received the traditional method as measured by the Algebra Readiness Test.

Conducting the study and testing the null hypotheses were facilitated through the cooperation of three school districts (Logan City, Box Elder County and Cache County), eight teachers, and eight intact classes of ninth grade general mathematics students. The total of the sample consisted of 210 students, 131 were males and 79 were females. There were 117 students in the experimental groups and 93 students in the control groups.

The cooperation of the districts was elicited during the third week of December, 1975. The end of the first week in January, 1976, marked the final positive response on the go-ahead with the project. In subsequent days contacts with school principals and teachers were made, and a random assignment of teachers was conducted to determine which of those chosen would have the experimental or control groups. Afterwards, the modules were distributed to the experimental group teachers which included lesson plans, possible questions, worksheets, and answers for a total of 20 lessons. When the teachers finished previewing the module, sessions were scheduled to accommodate each of his questions and concerns and to cement the philosophy of the module. Also, sessions were held with the control group teachers to delineate the kinds of experiences that would make for equitable comparisons. As the Solomon Four-Group Design, which was used, does not permit all groups to be pretested, of the four schools only two were pretested. Those which were pretested were given the Mathematics Attitude Scale and the Algebra Readiness Test during the fourth week of January, 1976 (The Mathematics Attitude Scale and the Algebra Readiness Test had reliability coefficients of .95 and .96 and validity of .40 and .70, respectively). During subsequent weeks, periodic visits were made to the schools to check on the progress of the project.

Pretests and posttests were collected two days after each had been administered. This gave students an extra day to take any test they had missed. Both tests were hand-scored by the researcher.

A least squares analysis of variance (ANOVA) technique with unequal numbers of subjects per treatment was used to analyze the findings relative to each hypothesis in five different fashions. The specifications for replicating the findings in four different fashions were given by Campbell and Stanley (1963) and Borg (1974). A fifth was chosen to check for sex differences.

Based on the findings of this study, the following conclusions were reached in regard to the treatment.

1. Three out of the five replications conducted on the null hypotheses concerning attitude toward mathematics were rejected. Therefore, it was concluded that the diagrammatic method was slightly more effective than the traditional approach in affecting a positive attitude toward mathematics.

2. Four out of the five replications conducted on the null hypotheses concerning performance on fundamental operations were rejected. Therefore, it was concluded that the diagrammatic method was more effective than the traditional approach in affecting improved performance on or literacy in the use of fundamental operations.

3. Four out of the five replications conducted on the null hypotheses concerning performance on fractions were rejected. Therefore, it was concluded that the diagrammatic method was more effective than the traditional approach in affecting improved performance on or literacy in the use of fractions.

4. Four out of the five replications conducted on the null hypotheses concerning performance on decimals and percents were retained. Therefore, it was concluded that the diagrammatic method was no more

effective than the traditional approach in affecting improved performance on or literacy in the use of decimals and percents. It was recognized that few practical experiences dealing with decimals and percents were provided in the module.

5. Three out of the five replications conducted on the null hypothesis concerning problem solving were rejected. Therefore, it was concluded that the diagrammatic method was slightly more effective than the traditional approach in affecting improved performance on or literacy in the use of problem solving strategies.

6. Sex differences were highly significant on one of the five replications, in favor of the females. In as much as the number of males far exceeded the number of females, a conclusion was not reached on the sex difference.

7. Students who took both pretests and posttests, using the attitude scale, scored significantly higher than those who took only the posttest. Therefore, it was concluded that pretesting served to raise scores on the attitude scale.

8. Students who took both pretests and posttests, using the readiness battery, did not perform significantly better than students who only took the posttest. Therefore, it was concluded that pretesting did not serve to enhance scores on the readiness battery.

Recommendations

On the basis of the information gathered from conducting this study and from the analyses of the data, the following recommendations are offered:

A. It is suggested that the study be replicated:

1. Using a sample from various school districts and localities with a near equal distribution of the sexes.
2. Utilizing more time in expressing anonymity of data reporting, so as to avoid threatened feelings of the teachers involved.
3. With the inclusion of more practical examples on decimals and percents.
4. Using a design other than the Solomon Four-Group design.

B. It is recommended that:

1. General mathematics teachers use this module or similar modules in their curriculum, and supplement the decimal and percent section to include more practical applications.
2. General mathematics teachers consider the use of more socially relevant materials in teaching or re-teaching basic operations.

3. General mathematics teachers develop or use methodologies from research that engender favorable attitudes.

4. General mathematics teachers look closely at their attitudes, and the possibility that students perceive and become as negative or as positive toward the subject as are teachers.

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APPENDIXES

Appendix A.

A Diagrammatic Problem Solving Unit

TRANSPARENCIES

Number	Description
1	Two river crossing problems
2	Symbol for a number
3	Sum of two numbers where one number is three more than the other
4	Vocabulary challenge
5	The "standard" river crossing problem
6	Sum of two numbers where one number is three times as large as the other
7	Two more river crossing problems
8	River crossing and the infamous love triangle
9	Basic diagram for developing fractional parts
10	Four-ninths of what number is 24?
11	Fractional increase and decrease
12	Quick review of per cent
13	Discounts during Community Bargain Days
14	More sales during Community Bargain Days
15	Illustration of the communative property
16	Quick checking squares - addition, subtraction
17	Quick checking squares - multiplication, division

TRANSPARENCIES (CONTINUED)

Number	Description
18	In a class of 27 students, five-ninths are boys. How many girls are there?
19	Compound relationships involving more than two numbers
20	The bottle and cork problem
21	The sum of two numbers is ___. If one-half of the smaller is the same as one-third of the larger, what are the numbers?
22	In a class of 28 students, there are five-ninths as many boys as girls. How many boys are there?
23	Bottles, Bottles Everywhere!

TRANSPARENCY EXPLANATION

The transparencies are keyed to various lessons, and are denoted with "T-" followed by a specific number. Typically, T-5 refers to transparency number five.

COLOR CODE EXPLANATION

Color coding has been used to separate the lesson plans from the student assignments. Each problem solving class will have a lesson plan. A sheet of buff paper will separate the lesson plan from the student classwork and homework for that class. Answers for the problems will be printed in red.

Problem Solving Class One

Objectives

1. To interest the students in problem solving by offering them an an intriguing non-computational problem.

2. To illustrate the advantages of using symbolism and diagrams in solving problems.

3. To provide the students with convenient mathematical symbolism which they can employ for problems involving addition and subtraction.

4. To give students practice in translating from the symbolism of the English language to mathematical symbolism or language.

Background

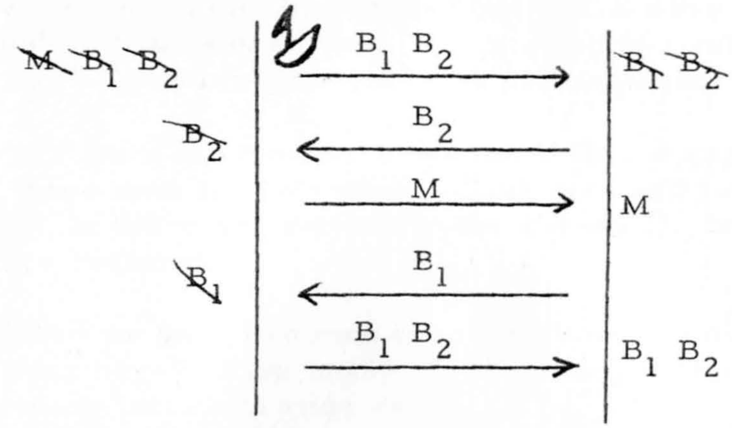
The river crossing problems are included in this lesson principally to engage students and to demonstrate the advantages of using diagrams and symbolism. The major content is the mathematical symbolism for addition and subtraction and its relationship to the English phrases.

With many classes this lesson could be expanded to cover two days of class time.

Procedure

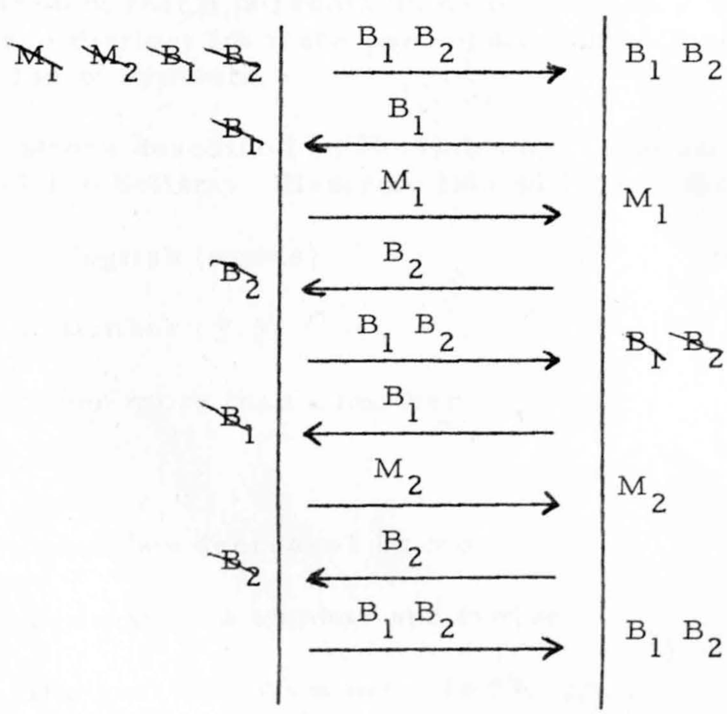
(T-1) Present the first problem on the transparency and begin discussion of its solution. Ask a student volunteer to read the problem. Perhaps another may wish to begin the explanation of the solution. The further one gets, the more complicated it is to follow what is being said. To be sure, after several students have offered their explanations, many of them will remain confused. When students have realized for themselves that it is very difficult to work through the problem using only language to describe the solution, move on to the next problem.

Set up the following diagram and have several students contribute to the problem's solution, which may look like this:



Ask if anyone can think of a different solution. For instance, B_1 could have returned instead of B_2 in the second crossing. What is the smallest possible number of crossings required to complete the task? (5) How many round trips is this? ($2\frac{1}{2}$)

Return now to the first problem and have the students speculate on the number of crossings it will require. Since one additional person is involved, they will probably reason that more than five crossings will be required. They may also see that an odd number of crossings will be necessary since the problem starts on one side of the river and progresses to the other side. A possible solution is shown below:



Now ask the same three questions which were asked after the previous problem had been solved: Are there other solutions? How many crossings? How many round trips are required?

At this point the teacher could ask: How many trips are required if there are three men and two boys? Four men and two boys? Five men and two boys? Is there any pattern to the answers? With twenty men how many trips are required?

Suppose we have two men but three boys. How many trips are required? Four boys? Five boys? Is there any pattern to this? With 50 boys how many trips are required?

Discuss with the class the fact that this type of problem is just a simplified mathematical model for a very complex situation in real life. What factors not included in the mathematical version would probably figure in an actual situation? (The ability of those selected to row the boat, the effect of the river current, weather, and the pressures of time are some of the answers you will probably get during this duscussion.)

(Include some questions designed to point up the advantages of using symbols.) When the advantages of using symbols and diagrams have been established; move on to explore their use in mathematics.

Comment that a person's name or nickname are symbols for the person and are distinct from the person himself. Then ask the class for other examples of symbols.

Situations described by English words can also be described in mathematical symbolism. Illustrate this with the following examples:

English (words)	Math (diagrams)
1. A number (T-2)	
2. Seven more than a number	
3.	9 + ∆
4. A number decreased by four	
5. The sum of a number and twelve	
6. The sum of two numbers is 57. (T-3) If one of the numbers is three more than the other, what are the two numbers?	

7. Provide other examples to help your class understand "+" and "−"

Now distribute the classwork and offer help to those students who need it.

Have the students select the words in part A which mean addition, and those which indicate subtraction.

Distribute the homework.

Class 1 In-Class Dyna-m-i-t-e! Name: ______________

A. In each example, try to give either the math or the English symbols.

Symbolism

	English (words)	Math (diagram)
1.	a number	
2.	6 more than a number	
3.	a number plus 12	
4.	add four to a number	
5.	a number decreased by 5	
6.	five less than a number	
7.	five taken away from a number	
8.	V subtracted from a number	
9.	a number increased by 5	
10.	the sum of a number and 3	
11.	the difference of a number and 3	
12.		$\triangle + 6$
13.		$\odot - 7$
14.		$5 + \square$
15.		$12 - \bigcirc$

B. Use diagrams to solve these problems

1. Five more than a certain number is 12.

2. The sum of two numbers is 63. If one number is 3 more than the other, what are the numbers?

(Lest I forget)

$\frac{1}{4} + \frac{2}{3} =$

Class 1 | Followup Assignment | Name: ________

A. Can you provide the missing symbols?

Symbolism

English (words) ↔ Math (diagram)

	English (words)		Math (diagram)
1.	a number plus eight	⟷	
2.		⟷	□ − 17
3.		⟷	12 − ○
4.	The sum of 5 and a number	⟷	
5.	The difference of a number and 7	⟷	
6.	Twelve subtracted from a number	⟷	
7.		⟷	□ + 13
8.	Add six to a number	⟷	
9.	Six less than a number	⟷	

B. Use diagrams to find the numbers in these problems:

1. The sum of two numbers is 57. If one number is 3 more than the other, what are the two numbers?

2. A certain number increased by 5 is 14.

C. Now here is a real tough one! Can you solve this? Use a diagram to make it easy.

You must carry a wolf, a goat, and a cabbage across the river in a boat so small that you can take only one thing at a time with you. You must always be present to keep the wolf from eating the goat, and the goat from eating the cabbage. How can you take them all safely across the river?

(Lest I forget: $\frac{2}{5} + \frac{3}{4}$)

Problem Solving Class Two

Objectives

1. Heighten students' awareness of mathematical symbolism.

2. Provide practice in translating English phrases for addition and subtraction into mathematical symbolism.

3. Introduce a convenient mathematical symbolism which can be used for problems involving multiplication and division.

Background

The relationship of English phrases for multiplication and division and their mathematical equivalents forms the heart of this lesson. For many classes this lesson could be expanded to take two days without overworking the material.

Procedure

(T-4) Review part A of last night's homework. Use the transparency to support the vocabulary effort. Have students select and circle English phrases which indicate addition. Have them illustrate their choices by giving an English word example. Have the class put mathematical phrases on the board for each of their illustrations. Repeat with subtraction phrases.

Review part B of last night's homework. The teacher might ask the following questions about problem number 1 of part B of the homework. Would the answer come out without fractions if the sum of the numbers was to be 56? 54? 53? 52? For which numbers will the answer come out so it doesn't involve fractions? All odd numbers you say? What about 3? What about 1?

Suppose the sum of the numbers was to be 57, but one number was 4 more than the other. Does the answer involve fractions? 5 more than the other? 6 more than the other? 7 more than the other? 8 more? 9 more? When does the answer involve fractions and when does it not? It doesn't for odd numbers? How about when one number is 57 more than the other? 59 more? 61 more?

(T-5) Have two or three students assist in the solution of the river crossing problem in part C of last night's homework. Are there any other solutions? How many?

(T-2) Begin discussion of mathematical symbols for English phrases which involve "X" and "÷".

Class 2 In-Class Practice Name:______________

A. Try to write in the missing type of symbolism:

	English (words)	Math (diagram)
1.	a number	
2.	two times a number	
3.		4⊙ or ⊙⊙⊙⊙
4.	five times a number decreased by the number	
5.	3 more than two times the number	
6.		□□□□□ +7
7.		⌓ − 4
8.		12 + ▱
9.		◁◁◁ −5
10.	ten plus two times a number	
11.	a number multiplied by five	
12.		◯ × 3

B. Can you solve these with diagrams?

1. The sum of two numbers is 64. One number is three times as large as the other. What are the numbers?

2. The sum of your age and the age of your teacher is 42. If your teacher is twice as old as you, what is your teachers age?

3. What number when multiplied by four is five less than sixty-nine?

(Lest I forget: 2 1/3 + 4 1/2)

Class 2 Class Preparation (HW) Name: ______________

A. Fill in the missing symbolism:

	Math (diagram)		English (words)
1.		________	6 times a number
2.		________	the product of a number and 4
3.	12 − △△△	________	
4.	◖◖ + 3	________	
5.		________	one-half of a number
6.		________	two-ninths of a number

B. Try your skill at solving these with diagrams:

1. Two numbers have a sum of 35. One of them is four times the size of the other. What are the numbers?

2. Three times a certain number is sixty-three. What is that number?

C. Can you figure this out? Try to make your answer picture perfect!

Two jealous husbands and their wives, Mr. and Mrs. Nurn and Mr. and Mrs. Bleeb, must cross a river in a boat that holds only two persons. How can this be done so that a wife is never left with the other woman's husband unless her own husband is present?

(Lest I forget: $3\frac{2}{3} + \frac{3}{4}$)

Problem Solving Class Three

Objectives

1. Heighten students awareness of mathematical symbolism.

2. Provide practice in translating English phrases for addition and subtraction into mathematical symbolism.

3. Introduce a convenient mathematical symbolism which can be used for problems involving multiplication and division.

Background

The relationship of English phrases for multiplication and division and their mathematical equivalents forms the heart of this lesson. For many classes this lesson could be expanded to take two days without overworking the material.

Procedure

(T-4) Review part A of last night's homework. Use the transparency to support the vocabulary effort. Have students select and circle English phrases which indicate addition. Have them illustrate their choices by giving an English word example. Have the class put mathematical phrases on the board for each of their illustrations. Repeat with subtraction phrases.

(T-5) Review part B of last night's homework. The teacher might ask the following questions about problem number 1 of part B of the homework. Would the answer come out without fractions if the sum of the numbers was to be 56? 54? 53? 52? For which numbers will the answer come out so it doesn't involve fractions? All odd numbers you say? What about 3? What about 1?

Suppose the sum of the numbers was to be 57, but one number was 4 more than the other. Does the answer involve fractions? 5 more than the other? 6 more than the other? 7 more than the other? 8 more? 9 more? When does the answer involve fractions and when does it not? It doesn't for odd numbers? How about when one number is 57 more than the other? 59 more? 61 more?

(T-5) Have two or three students assist in the solution of the river crossing problem in part C of last night's homework. Are there any solutions? How many?

(T-2) Begin discussion of mathematical symbols for English phrases which involve "X" and "÷".

English	Math
1. Three times a number	
2. Twice a number	
3. Six times a number increased by four	
4. Double a number	
5.	□□+5
6.	17-△△△
7. The sum of two numbers is 54. If one of the numbers is twice as large as the other, what are the numbers?	

(T-6) Pass out the classwork - Use the new slide with part B. With problem 1 in part B, several interesting questions can be asked. When the sum of the numbers was 64 the answer did not involve fractions. What if the sum were 65? 66? 67? 68? 69? 70? 71? 72? Can you state a general rule for when the answer won't involve fractions? What are the only fractions that ever seem to get into any of the answers? Why do you think these are the only ones?

Optional

(T-7)Another river crossing problem if time permits. A one problem pop quiz might be good here to check understanding of assignment number 1. For example:

Give the mathematical phrase for:

1. A number decreased by 7?

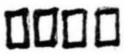
2. A number increased by 5?

3. Fourteen subtracted from a number?

Distribute homework

Class 3 "First-Class" Work Name:______________

A. Give the missing symbols:

	Math (diagram)	English (words)
1.		
2.		five times a number
3.	 -3	
4.		a number increased by five
5.	 +7	
6.		five more than number
7.		a number decreased by seven
8.		
9.	 +2	
10.		four decreased by a number
11.		a number times three
12.		

B. Draw the diagram and then find the answer to the following problems:

1. A number plus ninety-six is three hundred-five. What is the number?

2. Five times a number is twenty-six. What is the number?

(Lest I forget: . 2 x . 04 = ___)

Class 3 Homework B S T T Y B ! Name:__________

A. Yep! More symbolism:

	English (words)	Math (diagram)
1.	a number minus 5	
2.		6□
3.	six added to a number	
4.	five minus a number	
5.	5 times a number minus the same number	
6.		▽×▽
7.	3 times a number plus a number	
8.		6○ - 6
9.		5◹
10.		4+3+2+▭
11.		◆+9
12.	17 more than a number minus 3	

B. Diagram and answer:

1. A number added to five is equal to seven. Find the number.

2. What is a number?

3. Five times a number equals the same number. What is the number?

(Lest I forget: .004 x .03)

Class 3

A traveler wishes to stay at an inn for several days. He has no money, but has a valuable gold chain. The innkeeper agrees to let him stay on the condition that each day he pay one link of his chain. The traveler must pay every day; that is, at the end of one day, the innkeeper must have one like, at the end of two days, two links, and so forth. The traveler must cut his chain in order to make these payments. If the chain has seven links and the traveler stays for seven days, show how by cutting only one link in all he can pay for his lodging every day.

Problem Solving Class Four

Objectives

1. Develop a convenient and effective mathematical symbolism which can be used to translate English phrases involving fractions.

2. Give students a more intuitive feel for problems involving fractions.

Procedure

(T-8) Review last night's homework. In question 1, part B, of the homework in regular lesson 2 the teacher could ask the following: If one number were five times as large as the other would the answer involve fractions? If one number were six times as large? Seven times as large? There are two other numbers besides four and six for which the answer doesn't involve fractions. What are they? (34 times as large, 0 times as large.) What does 0 times as large mean? Involve students in this discussion. Then have them work the river crossing problem.

(T-4 Use this transparency to pick out, illustrate and translate the illustrations to mathematical phrases which represent multiplication. Repeat for phrases which involve division.

(T-9)Start discussion of phrases involving fractions. Discuss different ways of dividing a number up:

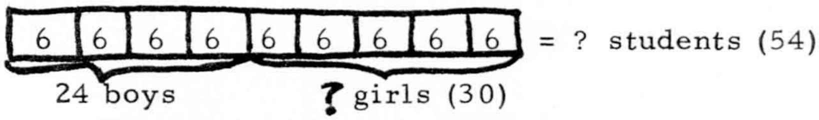
$$\frac{1}{2}, \frac{1}{6}, \frac{1}{4}, \frac{1}{8}, \frac{1}{10}, \frac{1}{9}, \frac{1}{12}$$

(T-10)The fractions above can all be illustrated easily on a diagram with a little planning. $\frac{1}{5}, \frac{1}{3}$ and $\frac{1}{7}$ are not quite as easy to represent with precision. Why?

Develop a meaning for fractions.

This represents a number divided into 1/3's. Since two parts are shaded, 2/3 of the number is shaded. Present more examples if needed.

(T-10) Use this symbolism for fractions in a problem. If 4/9 of the eighth grade students at your school are boys and there are 24 boys in the grade, how many eighth grade students are there? How many girls?



Distribute classwork. Work with individuals. Perhaps you would like to extend this material for certain students. Look at question 5 of part A in the classwork. How many different ways can you shade $\frac{7}{12}$? How many different ways could you shade

$$\frac{6}{12}? \ \frac{5}{12}? \ \frac{4}{12}? \ \frac{3}{12}?$$ Is there any regular pattern to this?

Examine question 5 of part C in the classwork? 90 is five-sixths of what number? How much smaller than 90 is five-sixths of 90? How much larger than 90 is a number such that 90 is five-sixths of that number?

Distribute the homework.

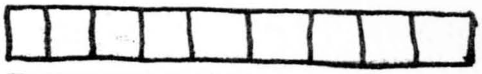
Class 4 Classroom Curios Name: ______________

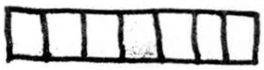
A. What part of the following sketches have been shaded in?

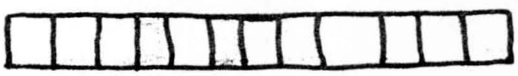
Answers

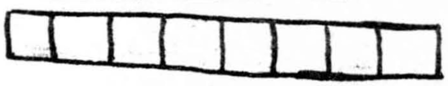
1.  ________

2.  ________

3.  ________

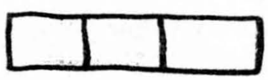
4.  ________

5.  ________

6.  ________

7. Can you think of another name for the shaded portion of the diagram in the previous problem? __________

B. Draw sketches for numbers and shade in the suggested parts. Do this neatly. The first one has been done for you.

1. $\frac{2}{3}$ 

2. $\frac{3}{5}$

3. $\frac{4}{9}$

4. $\frac{5}{8}$

5. $\frac{7}{12}$

6. $\frac{4}{7}$

C. Now try using your skill at dividing a number into parts in order to solve the following problems:

1. If 1/3 of a certain number is 24, what is that number?

2. One-eighth of a certain number is nine. What is that number?

3. Two-thirds of a certain number is thirty-six. What is that number?

4. Five-ninths of what number is forty-five?

5. Five sixths of 90 is what number? (Be careful!)
(Lest I forget $.25 \times \frac{3}{4}$)

Class 4 Brain Strain (HW) Name: ____________

Do you remember? (Use diagrams to picture the problems.)

1. Four times a certain number is 96. What is the number?

2. Fourteen more than a particular number is 72. What is the number?

3. The sum of two numbers is 81. If one number is two times as large as the other, what are the numbers?

4. When two numbers are added together, the result is 81. The larger is two more than the smaller. What are the numbers?

5. If two numbers have a sum of 79 and the smaller number is 3 less than the larger, what are the numbers?

6. What number is three-fourths of 360?

7. Five-eighths of what number gives 35?

8. Three-sevenths of what number is 9?

9. Four-ninths of 72 is what number?

10. Three-eighths of 48 is what number?

(Lest I forget: .02 + .002 + .2 + 2)

Problem Solving Class Five

Objectives

1. To translate English sentences into mathematical sentences using the mathematical symbolism we have developed.

2. To use mathematical symbolism to assist in solution of a type of problem that is not easy to solve by direct intuition.

Background

This lesson concentrates more on the mathematical sentence and its relationship to English sentences than earlier lessons have. Here we use the mathematical phrases of earlier lessons in sentences expressed in our mathematical symbolism.

Procedure

Review a few of the harder problems from last night's homework. It is a good idea to turn several of the problems around to point up contrasts. In problem number 10, for example, you might also ask, "48 is three eighths of what number?" Is this number larger or smaller than 48? How much? Is three eighths of 48 larger or smaller than 48? Again, how much?

On the overhead, present these patterns. Try to get your students to give descriptions of the patterns.

3, 7, 11, 15, _____, _____, _____
1, 22, 333, _____, _____, _____
AZ, BY, CX, _____, _____, _____

boy, three, girl, four, woman, _____

You might have the students make a few and present them.

Using the first prediction, discuss the idea of "increase". Have the students make up a pattern that shows the idea of "decrease."

(T-11) Discuss problems of the types:
A number decreased by two-fifths of itself is 45.
What is that number?
Would your answer involve fractions if we decreased a number by two-fifths of itself and got 46? 47? 48? 49? 50? 51?
Do you see a general rule?

What fractions seem to be the only ones that show up? Why is this?
A number increased by one-fifth of itself is 42. What is that number?

Give out the classwork. You may wish to collect these and examine them to find a few good patterns to present to the class.

Distribute homework.

Class 5 Classtime Labor Name: ________________

A. Diagram the answers to these problems:

1. A number increased by one-third of itself becomes 20. What is that number?

2. A number decreased by three-fifths of itself is 16. What is that number?

3. What number increased by one-half of itself gives a sum of 75?

4. If one-third of a certain number is added to the number, the sum is forty. What is that certain number?

5. What number increased by one-sixth of itself becomes 63?

6. A boy runs two-thirds of the way home in sixty-eight minutes. If he continues at the same rate, how much longer will it take him to get home?

7. Joe tried very hard to sell newspapers this week. He found that the number he sold was one-third more than he had sold the week before. If he sold 84 newspapers last week, how many did he sell this week?

B. Do you like predictions? Try these and remember the rules for them.

1. 1 4 8 13 19 ___ ___ ___

2. 1 5 3 7 5 9 ___ ___ ___

3. B21R C23P D25N _____ _____ _____

4. AZ BX CV _____ _____

5. 1 1 2 3 5 8 13 ___ ___ ___

You make up a couple...if they are really tough, we may use them to challenge other students in the class.

6.

7. (Lest I forget 3.04 + 31.2)

Class 5 Mental Gyrations for Tomorrow (HW) Name:________

A. Predictions anyone? Complete the following, write down the rule you discovered for each.

1.	ABC	ACB	BAC	BCA	CAB	____		
2.	23	28	51	56	79	____	____	____
3.	A1	C3	E5	G7	____	____	____	

B. Use diagrams to solve these problems:

1. The sum of two numbers is 140 but one of the numbers is 20 more than the other. What are the numbers?

2. A number, when increased by one-fourth of itself, becomes forty-five. What is that number?

3. Because the students have been more careful, the accident rate has been reduced by one-fourth. If there were 48 accidents last session, how many have happened this session?

4. A farmer had 29 sheep and all but 14 of them died. How many did he have left alive?

5. A man learns that he has lost three-fourths of his money and knows that he has $120 left. How much did he have originally?

C. Remember this? Diagram:

A man and his three sons wish to cross a river by boat. The boat will safely carry at most two hundred pounds. The man weighs 165 pounds and his sons weigh 85, 100, and 110 pounds, respectively. Is it possible for the four of them to cross the crocodile infested river? If you think it is, diagram your solution in the space at the right.

(Lest I forget: .42 - .05)

Problem Solving Class Six

Objectives

1. Review the standard mathematical meaning and symbolism for percentage.

2. Give students practice in interpreting problems involving percentage in terms of simple fractions.

Background

(T-7) For some classes this may be review material. For other classes this material may require more than the two days allotted to it.

Procedure

Review and perhaps collect homework. Have your students assist you in this, particularly with the river crossing problem.

Discuss where we find the percentage in our daily world:

> discount, sales, weather reports, interest, statistical reports, hog prices, carrying charges, etc.

Develop the idea that per cent is related to another word, century, which most people know. Also, they know there are 100 cents in a dollar. Per cent simply means the number of parts per one-hundred.

(T-12) Illustrate the basic fractions $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$ as percentages. Which fractions have convenient forms that don't involve a fraction when expressed as a decimal? Why do these have no remainder? This is a toughie, but may be of interest to some of the students as a topic to investigate.

Point out how difficult our method of diagramming problems would be if we used $\frac{25}{100}$ instead of $\frac{1}{4}$.

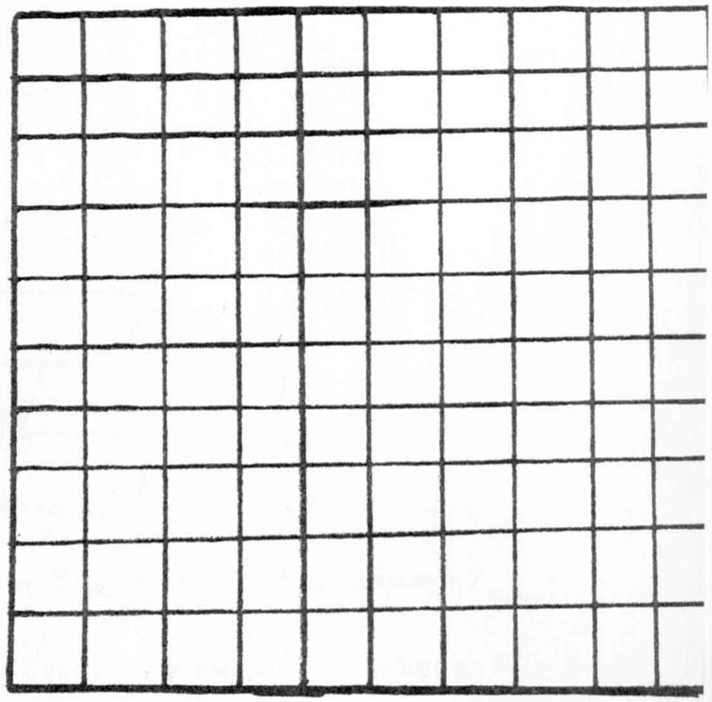
Distribute classwork. After students have had a chance to work on this, review a few of the earlier problem types and some of the basic percentages. In part B of the assignment several of your students will be able to solve the problems without mechanically converting the more commonly used percentages into fractions. See if they can explain how to arrive at these fractional values.

Distribute the homework.

Class 6 Classy Tasks Name: ________________

A. Fill in the missing information. Use the 100 squares if you need them.

%	fraction with 100 as denominator	simplest fraction
30%	a)	b)
c)	$\frac{10}{100}$	d)
e)	f)	1/4
g)	$\frac{100}{100}$	h)
i)	j)	1/3
$12\frac{1}{2}\%$	k)	l)
m)	n)	1/2
1%	o)	p)



B. Use diagrams in solving these problems:

1. Twenty percent of a number is 12. What is that number?

2. If twenty-five percent of a number is added to that number the sum will be thirty-five. What is that number?

3. Twelve and one-half percent of what number is 5?

4. How far can a dog run into the woods?

5. If 25% of a certain number is added to that number the sum is 105. What is that certain number?

6. A number decreased by 33 1/3% of itself becomes 48. What is the number?

7. The product of two numbers is 1. If one of the numbers is one-third, what is the other number?

(Lest I forget 2.3 - .085)

Class 6 Playing the Percentages (HW) Name:____________

A. Fill in the missing information:

%	fraction with 100 as denominator	simplest fraction
75%	a)	b)
c)	d)	$\frac{7}{10}$
e)	$\frac{66\ 2/3}{100}$	f)
g)	h)	$\frac{1}{20}$

B. Use diagrams to solve these problems.

1. Seventy percent of a number is 21. What is that number?___

2. A number increased by 3/4 of itself becomes 21. What is the number? ______.

3. If only one person out of fifty has been deep-sea fishing, what percentage is this? _____

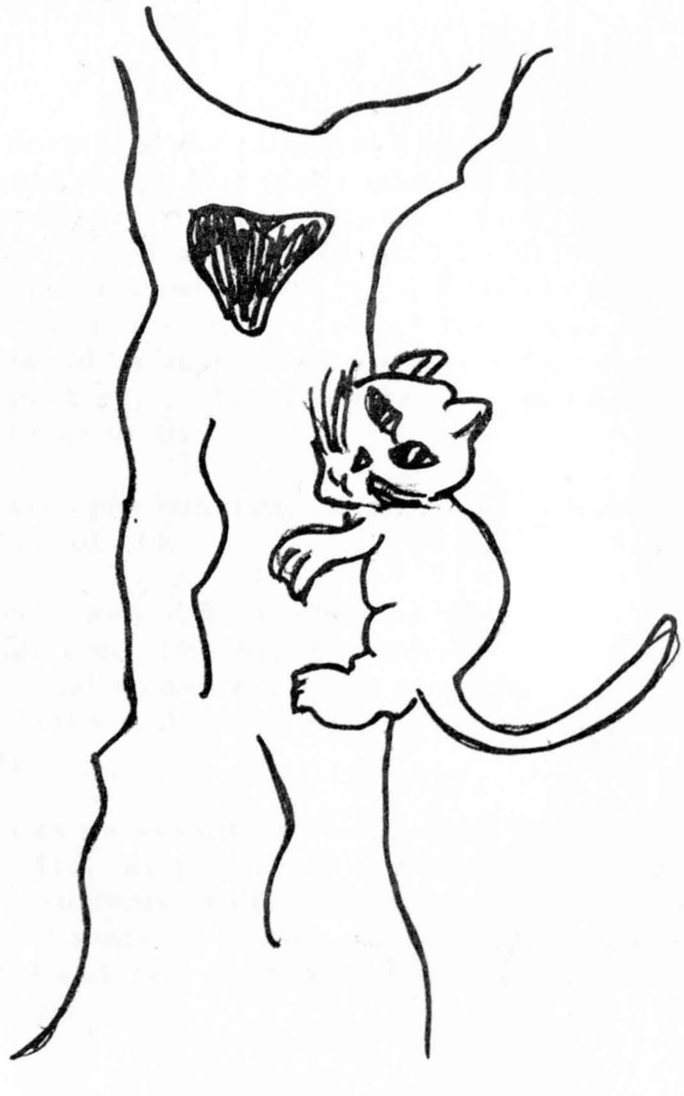
4. A boy went into a store with seventeen dollars. He spent five dollars there and 50% of what was left in a second store. How much money did he have when he left the second store?___

5. The product of two numbers is 1. One of the numbers is 1/4. What is the other number? _____

6. Seventy-five percent of <u>what</u> number is 39? _____

(Lest I forget: 3.04 - 1.876)

Johnson's cat went up a tree,
Which was sixty feet and three.
Every day she climbed eleven;
Every night she came down seven.
Tell me, if she did not drop,
When her paws would touch the top?!



Problem Solving Class Seven

Objectives

1. Emphasis again that percentages are simply a special form for writing fractions.

2. Give students practice in solving problems which involve percentages by applying our previous methods.

Background

For classes with above average background in percentages this lesson could follow directly after lesson four. The students should simply be encouraged to translate the percentages into fractions and proceed as before.

Procedure

Start with a careful review of the homework paying particular attention to problems three and five. For many students problem number 5 of part B of the homework will have been too hard. Ask these what is 1/4 of 40? 1/4 of 36? 1/4 of 32? 1/4 of 28? Is there a pattern to the questions? Is there a pattern to the answers? When will the answer be 1? So we have 1/4 of 4 is 1. 1/4 x 4 = 1 is our symbolism for this. These problems should be used to illustrate the fact that diagrams do not aid in solving problems. The diagram is a tool which should be used only when it helps do the job.

(T-12) Discuss the basic "parts per hundred" idea of percentage and the percentage form of 1/8, 1/5, and 3/4.

(T-13) You might discuss such uses of percentage as discount, sales, interest, weather forecasting, etc. Present the advertising slides, and get students to give the fractional equivalent of the percentage involved. Then have them set up a mathematical sentence for each of the problems and present solutions. (T-14)

Additional problems can be worked in class which might involve percent of increase in population, weight, death rates, accident rates, etc. A useful idea is to have students "pawn off" on each other some items like a last year's license plate, a broken bat, a punctured football, and a 1966 calendar at large discounts. The class can then figure the discount involved or the sale price.

Now distribute the classwork, and use problem number one to emphasize once again that drawing the diagram is a tool to be used it makes solving the problem easier, but that it is not indispensible. Problem 7 of the classwork can be used to give some students a better idea of percent of decrease. Suppose High Tide's percent of decrease had been 5%, how many people would be left? 10%? 15%? 20%? 25%? 50? 60%? 75%? 90%? Now it can be made very interesting. What does 99% of 400 people mean? 99 1/4%? 99 1/2%? 99 3/4%? 99 7/8%? Does this last question mean anything? Now the question can be turned around. If there are 200 people left, what was the percent of decrease? 204 left means what percent of decrease? 208? 212? 216? 220? 221? 222? 223? 224?

Suppose the population decreased 25% one year. How many people were left? Now suppose the following year it decreased 25% again. Now how many are left? Could it decrease exactly 25% again the year after that? (Not without a saw.)

Distribute the homework.

Class 7 Tough Life (or luck) Assignment Name:__________

"Slim Pickins"

Slim Pickens was an easy mark in these days of socalled "easy money." He was married and had 3 kids. To hear him tell it. "I ain't go no education." To be sure, Slim didn't have much book learnin' and there was some folks who thought he was pretty thick between the ears. As a matter of fact, poor old Slim really wasn't too swift in his thinking.

But he was a good, honest guy who loved his family dearly. He worked hard and was a very reliable person at the furniture manufacturing plant which employed him.

Things were going along pretty well until old Mr. Stork paid them another visit. Some folks say "when it rains, it pours," meaning of course, that when your luck turns bad, it goes bad all the way.

Keep in mind now that before this visit by the stork, his family, including himself, consisted of ____ people. Well, the visit increased the size of his family by 40%. How many people were now in Slim's family? ____

What do you suppose had happened? __________________

Well, this meant more food, more clothes, and more doctor bills: His two doctors, Dr. Al K. Seltzer and his brother, Bromo, were pretty good to him. Slim had already owed them $300. They increased this amount by four fifths of itself and told him that it could be paid to them at the rate of $20 a month. How much did Slim now owe them? _____. How many months would it take to pay off this debt? _______

Now keep in mind that he didn't do too poorly in his pay. He earned $100 a week. This is about _____ a month. But he didn't receive all this money. His monthly income was decreased by 25% because of deducations for income tax and social security. This left him how much money each month? _________

Twenty percent of this amount went for a new car payment. This was $_____

One-third of his take-home pay was spent on food. This was $______.
5% was for a television payment. How much was this? $______.
1/10 of his money paid his gasoline bill. This is $_______.
Each month he spent $80 for rent and about $20 for clothing.

We can now see Slim's problem.

Take-home pay = ________

Monthly Expenses:

$20 Clothing

Car payment

TV payment

Doctor bill

Food

Rent

Gasoline

Total = ________

Do you see what his situation is?

It's pretty grim, isn't it? Actually, it's worse than the figures above indicate because all of the regular monthly expenses are not listed. Can you think of some which might have been included?

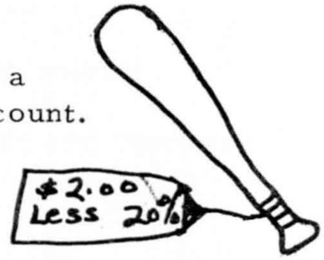
What types of alternatives are possibly open to Slim as solutions to this "tough" problem?

Chicken Hearted Solutions	Possible Solutions	Actual Solutions
1.	1.	1.
2.	2.	2.
3.	3.	3.

Class 7 What % will you get right (HW) Name:____________

Use diagrams to solve these problems:

1. A baseball bat originally cost two dollars. In a special sale it was marked down on a 20% discount. What was the sale price?



2. If 33 1/3% of a certain number is added to that number the sum is 52. What is that number?

3. A certain number is increased by 25% of itself and the result is 75. What is that certain number?

4. If 67 people out of 100 have black hair, what % has black hair?

5. What percent of the people in problem 4 <u>do not</u> have black hair?

6. The product of two numbers is 1 and one of the numbers is 7/8. What is the other number?

7. Thirty-seven and one-half percent of what number is thirty. ($37\frac{1}{2}\%$ is either 1/8, 3/8, or 5/8. Do you know which one?)

8. Barry Plump went on a "crash" diet. He lost one-twelfth of his weight the first week. During the second week he lost one-eleventh of what he weighed at the end of the first week. During the third week he lost one-tenth of what he weighed at the end of the second week. If he then weighed 135 pounds, how much did he weigh at the beginning of the first week?

(Lest I forget: 3/4 - 1/2)

Problem Solving Class Eight

Objectives

1. Provide a "break" in the routine of the problem solving.

2. Introduce the ideas of "commutative operations" and "inverse operations."

Background

The order in which two numbers are combined does not affect the outcome in either addition or multiplication. Thus $6 + 3 = 3 + 6$ and $6 \times 3 = 3 \times 6$. This is not true, however for subtraction or division. In fact $6 - 3 \neq 3 - 6$ and $6 \div 3 \neq 3 \div 6$.

We say that addition and multiplication are "commutative operations" while division and subtraction are not "commutative". "Commutative" then is a word which applies to operations and tells whether or not the order in which the numbers are combined is important.

Inverse operations are operations that undo each other. For example addition and subtraction undo one another. Addition and subtraction form one pair of inverse operations. Another example of such a pair are the operations of multiplication and division.

Procedure

Discuss with the students the reason for using diagrams. Hopefully they will see these diagrams as a tool to help them work through problems they encounter in life. Use the ideas of diagrams and review the homework of the day.

(T-15) Place these simple problems on the overhead.

$6 + 3 =$	$6 - 3 =$	$6 \times 3 =$	$6 \div 3 =$
$3 + 6 =$	$3 - 6 =$	$3 \times 6 =$	$3 \div 6 =$

Ask the students what they can conclude. You hope to get them to decide that order doesn't matter in addition and multiplication, but does matter in subtraction and division. Thus addition and multiplication are commutative. In certain special cases the order of subtraction can be turned around. Can you find one? $(6 - 6 = 6 - 6)$ Another? What is true of the two numbers if you can turn subtraction around and get the same answer?

Are there special cases when division can be turned around? Again what can be said about the two numbers if you can turn division around and get the same answer?

Discuss the fact that some actions and things have opposites while others do not. For example, in automobiles forward motion can be undone by going in reverse. Opening a door is the opposite of closing the door. Ask the class for examples. They might give picking up a pencil, raising a window, adding, buttoning a coat, cutting off a dog's tail, standing up, jumping out of a flying airplane, subtracting, building a brick wall, dividing, multiplying, etc.

(Note: Doing something and then undoing it, is not quite the same as doing something and not doing it. For example, picking up a pencil and putting it back down are actions that undo each other. Picking up a pencil and not picking up a pencil are not really actions which undo each other.)

In general all operations have theoretical opposites, but some like jumping out of a flying airplane do not have practical opposites.

The four operations of arithmetic do have opposites and these pairs of operations are called inverse operations. How can this idea be useful? For checking our work.

(T-16)Introduce quick checking squares by working two with the students to illustrate the technique. Have the class try the third addition example. Now try the subtraction example. Did it work? How do the diagonals work? (Add them and then subtract the results) Why is this? (Subtractions is not commutative)

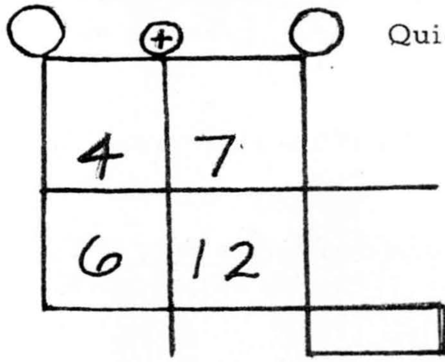
Distribute the class work. Circulate while the students work on this. Particularly encourage them to make up their own examples. Is there any trick to making up addition examples? What about subtraction examples? Will the squares work out if you allow negative numbers? What about fractions? Could you make up a square using decimals? Percentages? These squares are three by three squares, if you leave off the "ears." Can you design a four by four square that works for addition? A five by five? How about a three by five rectangle?

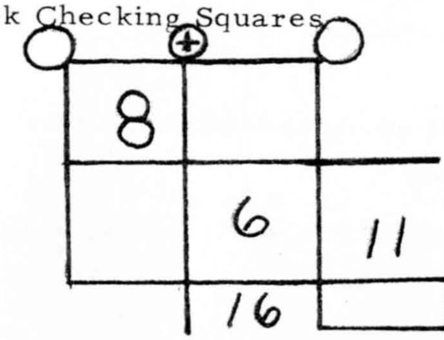
Distribute the homework.

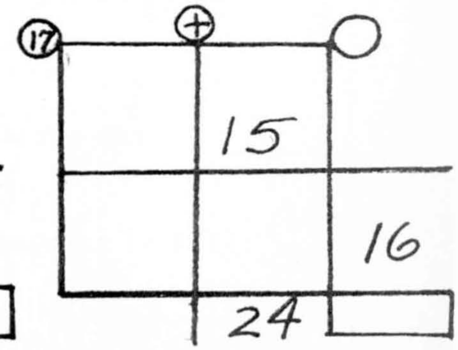
Class 8 Work ? ? & Q / / Name: ____________________

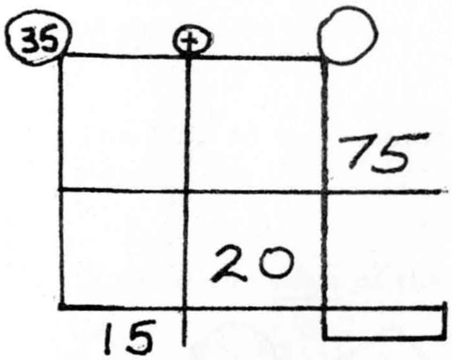
See what you can do with these. Look at the sign of the operation before you begin the work!

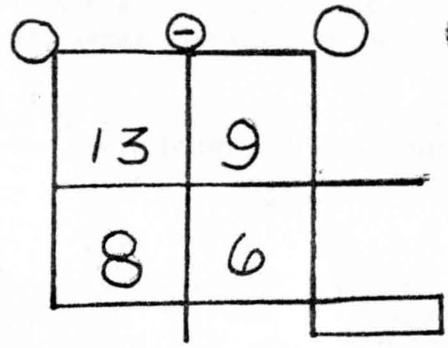
Quick Checking Squares

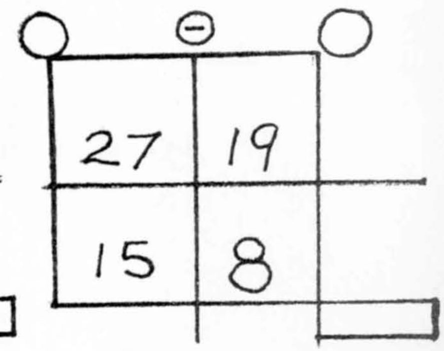


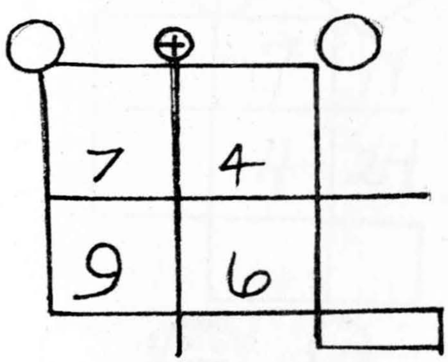


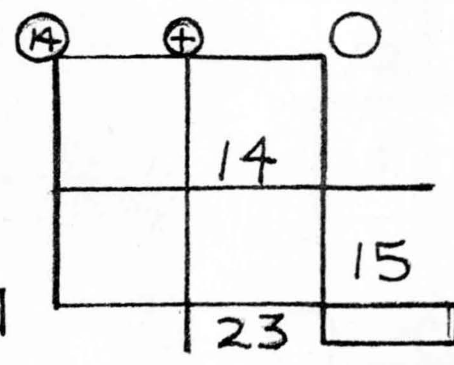


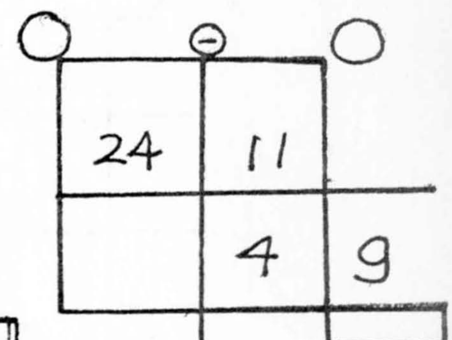


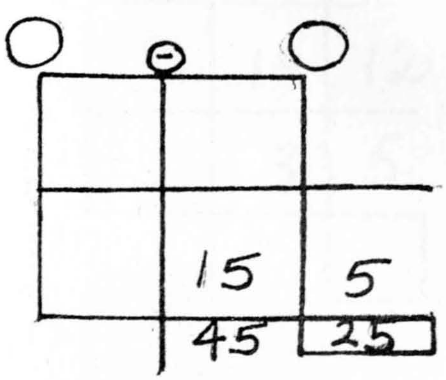












Now you try making up two, then complete them.

Lest I forget $.3\overline{)9}$ or $9 \div .3$)

Class 8 More Work? (HW) Name:____________

A. Remember these? Use the diagram to solve them.

1. If two numbers have a sum of 74, and the larger is 6 more than the smaller, what are the two numbers? ____, ____.

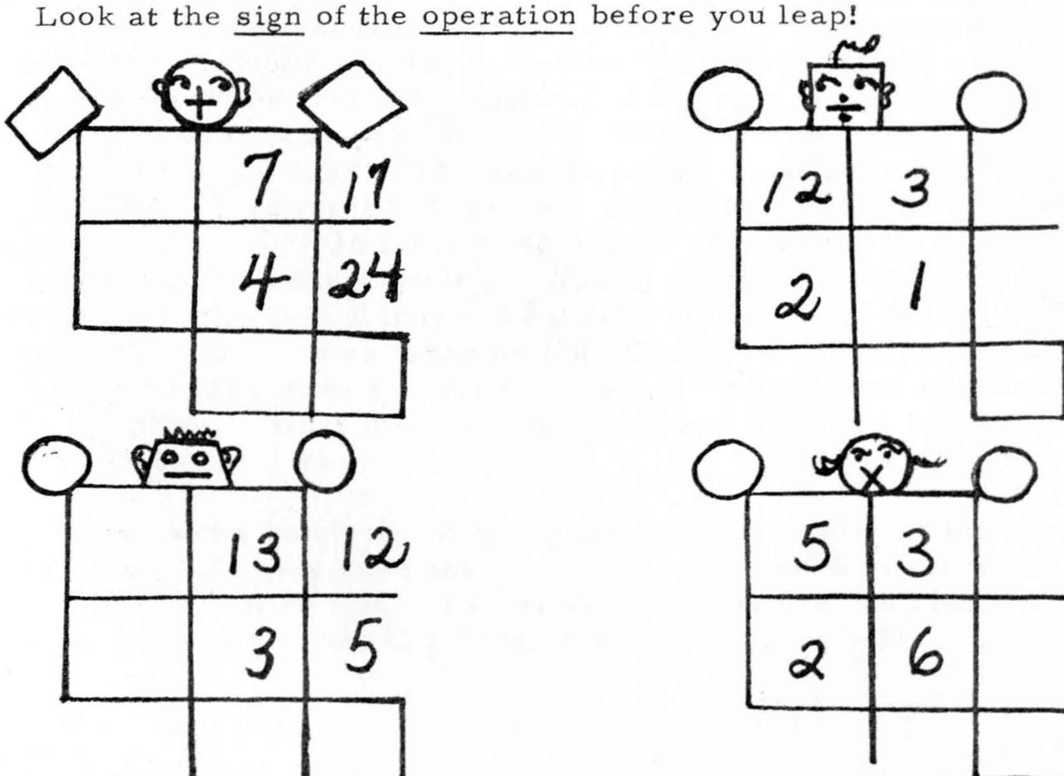
2. Ninety-eight subtracted from what number gives thirty-three? ____.

3. The product of two numbers is one. One of the numbers is 7/11. What is the other? ____.

4. One number is three times the size of a second number. If the sum of these two numbers is 36, what are the numbers? ____, ____.

5. The sum of two numbers is one. If one of the numbers is 3/4, what is the other? ____.

6. Look at the sign of the operation before you leap!



(Lest I forget $.3\overline{)\,.9}$ or $.9 \div .3$)

Problem Solving Class Nine

Objectives

1. To review the ideas of mathematical symbolism we have developed.

2. To provide a review of the problem solving course so far.

3. To use the inverse relationships of multiplication and division in a few quick checking squares.

Background

This lesson presents only a little new material. It is designed as a comprehensive review of the unit so far.

Procedure

(T-9) Start by reviewing the first five problems of the homework. The students should by now have had very little trouble with these. Take as much time as necessary to clear up any misconceptions the students may have about these problems. Discuss number five particularly since it can be diagrammed and illustrates the idea of inverse operations. You might also vary the conditions of the problems in some systematic way and have the students look for systematic variations in the answers. For example in problem four what happens if the sum of the numbers is to be forty? 64? 48? 52? 56? 60? 61? 62? 63? 64? $64\frac{1}{2}$? 65? $65\frac{1}{2}$? 66? $66\frac{1}{4}$? $66\frac{1}{2}$? 66 3/4? 67? What happens if one number is to be 4 more than three times as large? 8 more? 12 more? 16 more? 20 more? 28 more? 32 more? 36 more? 40 more? What does this last question involve? Does it mean anything? What if one number is to be 4 less than three times as large? 8 less? 12 less? 36 less? 40 less? 44 less? If one number is twice as large as the other, does your answer involve fractions? What if they are the same size? What if one is four times as big as the other? What other examples of this sort will your answer not involve fractions? Why?

Discuss the quick checking squares which involved addition and subtraction. Did they have any problems when asked to construct their own? Did they notice that if we are to avoid negative numbers then it is necessary that $A \geq B$ and $C \geq D$ and that $A \geq C$ and $B \geq D$?

(T-17)Discuss the multiplication and division quick checking squares from the homework. Did they see that the diagonals must be multiplied in the division example? Why? (Division, like subtraction, is not commutative.)

Distribute the classwork. Notice how the students handle the review problems. When a diagram is useful the students should be getting into the habit of using it. Problem 12 of the classwork lends itself very nicely to diagraming, since the icicle can actually be drawn to represent itself. Your better students can be asked "if after Thursday night the icicle begins to lose 1/4 of its length everyday on that day will the length of the icicle drop below 1 inch? Below 1/4 inch? When will the icicle finally be completely melted?

Distribute the homework. This is light, but a good review for tomorrow's classwork which will be regarded as a "graded classwork" or a "quiz."

Class 9 Bad News - More Work! Name:______________

Use diagrams to solve these "jewels."

1. A number decreased by seven. ______.

2. The sum of a number and five. _____.

3. Three times a number decreased by twelve. _____.

4. The sum of two numbers is 47. If one number is 5 more than the other, what are the numbers? ____, ____.

5. The sum of two numbers is 48. If one number is three times as large as the other, what are the numbers? ____, _____

6. Four-sevenths of what number is 12? ____

7. A boy went into a store with twenty dollars. If he spent five dollars in that store and half of what was left in a second store, how much did he have when he came out of the second store?____

8. The product of two numbers is 1. If one of them is 4/9, what is the other one? _____

9. A number decreased by three-fifths of itself is 14. What is the number? ____

10. Waldo weighed 160 pounds in 1965. If he has increased his weight by 1/5(20%), what does he weigh now? _____

11. What number divided by four gives twenty-seven as the quotient?__

12. When it was very cold for several days during the winter, Maxwell closely watched a large icicle that hung from the roof of his dog's house. On Monday it was very long. It began melting on Tuesday and lost 1/8 of its length. Wednesday it continued to melt and lost 3/7 of Tuesday's length. Thursday night it was only one-half as long as it had been on Wednesday. When he measured it on Thursday night it was 4 inches long. How long had it been on Monday?____

Class 9 Independent Explorations (HW) Name: ____________

A. Draw diagrams

1. If the sum of two numbers is 1 and one of them is 5/8, what is the other? ______

2. 60% of what number is 39? _____

3. The sum of two numbers is 120. If one of the numbers is three more than twice as large as the other, what are the numbers? _______, _______.

4. Forty-seven subtracted from what number gives eighty-four? ____.

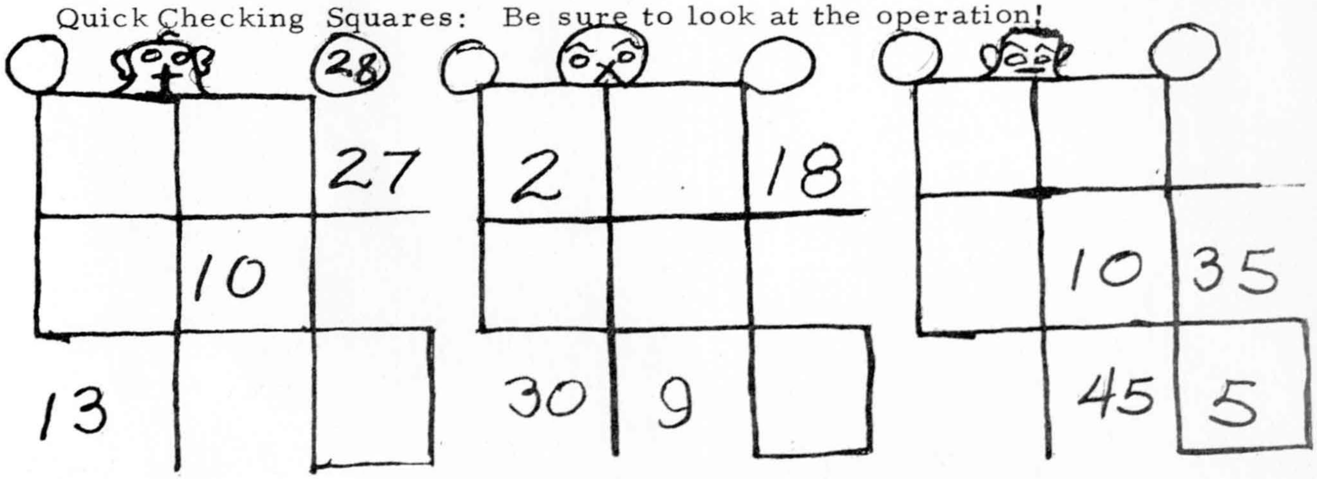
5. Two-fifths of what number is twenty-eight? _____.

6. A number increased by one-fourth of itself becomes 40. What is it? ____.

7. A number decreased by one-sixth of itself becomes 40. What is it? _____.

8. A silver-plated toothpick was priced to sell for two dollars. It didn't get bought so it was put on sale at 1/4 off. What was the sale price? ______.

B. Quick Checking Squares: Be sure to look at the operation!



(Lest I forget: $.25\overline{)6.25}$ or $6.25 \div .25$)

Problem Solving Class Ten

Objective

Evaluate how much the students have learned about the use of diagrams as a tool for solving word problems.

Procedure

Begin by reviewing the homework and answering any questions which might arise.

Discuss the following questions, perhaps leaving them open for the students to investigate:

> Is it possible to make a quick checking square with just three numbers given? If you are successful will everyone who works your square get the same answers everywhere? How many free choices will he have? (1)
>
> Can a quick checking square be made with four numbers given in such a way that it cannot be worked?

Give the classwork as a quiz. This is to be graded.

Distribute the homework. This assignment is a "Just for fun" assignment and shouldn't be taken seriously although rather provocative discussions will probably result from it.

Class 10 Pass in Review! Name:________________

Use diagrams to solve these problems:

1. Two numbers have a sum of 63. If one number is 5 more than the other what are the two numbers? ____, ____

2. Two-fifths of a certain number is 36. What is the number? _____

3. The product of two numbers is 1 and one of the numbers is 3/4. What is the other number? ___

4. If one-fifth of a certain number is added to that number the sum is 72. What is that certain number? ____

5. Jim did three-fourths of his math homework in 12 minutes. If he works at the same rate, how much longer will it take him to finish the assignment? ____

6. The sum of two numbers is 1. If one of the numbers is 3/13, what is the other number? ____

7. What number is three-eights of 56? ____

8. Two numbers have a sum of 52. If one number is three times as large as the other, what are the two numbers? _____, ____.

9. Before going on his diet, "Blimp" Ferguson weighed 180 pounds. He was able to decrease his weight by 20%. What does he weigh now?___

10. A cook can buy rice for 16 cents for half a pound. How much will she have to pay for 5 pounds of rice? _____

11. At a certain sale the clothes were marked down 10%. If my coat was $27.00 at the sale, what was its price before the sale? ___

12. Mike and Butch collected bottles. They have a total of 51. Mike has 3 more than twice as many as Butch. How many does each have? Mike ______ Butch ______

13. Check over your work carefully before turning in your paper.

Class 10 I.Q. Test? (HW) Name: ________________

1. If you went to bed at 8 o'clock p.m. and set the alarm to get up at 9:00 in the morning, how many hours of sleep would you get? _____

2. Is there a Fourth of July in England? _____

3. How many birthdays does the average man have? _____

4. Why can't a man living in Logan, Utah, be buried east of the Mississippi River? _____

5. If you had a match and entered a room in which there were a kerosene land, an oil heater, and a wood burning stove, which would you light first? ______

6. Some months have 30 days, some have 31 days, how many have 28 days? _____

7. If a doctor gave you three pills and told you to take one every half-hour, how long would the pills last you? _____

8. A man builds a house with four sides to it, and it is rectangular in shape. Each side has southern exposure. A big bear comes wandering by. What color is the bear? _____ Why?

9. What four words appear on every denomination of U. S. coin?

10. What is the minimum of active baseball players on the field during any part of an inning? _____ How many outs in an inning? ______

11. I have in my hand two U. S. coins which total 55 cents in value. One is not a nickel. Can you explain this?

12. A farmer had 17 sheep. All but nine died. How many were left? __

13. Divide 30 by one-half and add 10. What is the answer? _____

14. Two men were playing tennis. They played five games and each won the same number of games. How do you explain this?

15. Take two apples from 3 and what do you have? ___

16. An archeologist claimed he found some gold coins dated 46 B.C. Do you think he did? _____ Explain:

17. A woman gives a beggar 50 cents. The woman is the beggar's sister, but the beggar is not the woman's brother. How come?

18. How many animals of each species did Moses take aboard the ark with him? ______

19. Is it legal in Utah for a man to marry his widow's sister?

20. What word in the test is mispalled?

(Lest I forget: $.21\overline{)4.2}$)

Problem Solving Class Eleven

Objectives

1. Introduce ratio as another meaning of a fraction.

2. Present problems which involve ratios and illustrate the use of diagrams in solving these problems.

Background

This lesson involves the new idea of ratio, but also concentrates heavily on review material. The lesson can be used effectively to illustrate problems of the same type as those that your class had the most difficulty with on the quiz.

Procedure

(T-18) Start with a few congratulations on the class performance on the quiz. Then go carefully over the entire quiz concentrating on the problems that gave the most trouble. Problems 2, 4, 9, 11, and 12 should be carefully worked. This is probably not the last place to expand on or vary the problems. The students here are interested in where they went wrong on the problem that was given.

Start discussion of this problem:

In a class of twenty-seven students, five-ninths are boys.
Are there more boys or girls? (Boys)
How do you know? ($5/9 > \frac{1}{2}$)
For every 5 boys there are ____girls (4)
The comparison of boys to girls is ____ to ____ (5 to 4). This is called a ratio.
The ratio of boys to girls is ____? (5 to 4) This is written 5:4.
Another name for ratio is __________? (fraction) (5/4)
The ratio of girls to boys is ________? (4:5)
How many boys are there? _____ (15) How many girls? (12)
What is the ratio of students to teacher? _____ (27:1)
What is the ratio of teacher to students? _____ (1:27)
What is the ratio of boys to students? ______ (5:27)
What is the ratio of girls to students? ______(4:27)
What is the ratio of students to girls? _____ (27:4)

Distribute the classwork and work part of it with the class as a whole. Concentrate on the ratio problems in particular. Also cover review problems of the types often missed by your class on the quiz.

In problem number one of the classwork the teacher might well ask: What is the rate of the car in miles per minute? Will it be more or less than the rate in miles per hour? Has the speed of the car changed any? Has the rate? What has changed? (The way in which the rate is expressed, ie, the time interval used for comparison with the distance traveled.) What is the rate in miles per half hour? In miles per 1/4 hour? In miles per 3/4 hour? In miles per 1/6 hour? In miles per ten minutes?

What is the rate of the car in half miles per hour? In half miles miles per half hour? In quarter miles per hour? In quarter miles per quarter hour? In quarter miles per half hour?

In problem number four of the classwork, how many games are lost when the team plays 11 games? 22 games? 33 games? 44 games? 55 games? 56 games? This will create some lively discussion. Can you win or lose a fraction of a game? What would that mean anyway? Can you win one quarter in football, but lose the game? An inning in baseball? How many are won out of 77 games? 88 games? 99? 110? 121?

Distribute the homework.

Class 11 The Daily Grind Name: ______________

1. If a car travels 28 miles in 30 minutes, what is the rate of the car in miles per hour? _____

2. In a group of fifty-six students, the ratio of boys to girls is three to five? How many boys are in the class? ____

3. A baseball team lost three times as many games as it won. If the team played 136 games, how many did they win? _____ Lose? _____

4. A pretty "sorry" baseball team played 121 games. The number of games won was in a ratio of 3 to 8 to the number of games lost. How many games were won? ____

5. Jimbo takes his allowance to town, spends one-half of it in a store and one-half of what is left at the movie. If he than has 90 cents, how much is his allowance? _______

6. A car travels 72 miles in 2 hours. During the first hour it travels 6 miles farther than it does during the second hour. How far does the car travel each hour? _____, ____

7. The product of two numbers is 1. If one of the numbers is $\frac{23}{14}$, what is the other number? ____

8. A boy spent three-sevenths of his money and had $2.52 left. How much money did he have originally? _____

9. Find a number which when multiplied by 3 is 7 less than 46. _____

10. If an airplane can travel 900 miles in $4\frac{1}{2}$ hours, what is the rate in miles per hour? _____

11. A number increased by three-fourths of itself is 42. What is that number? _____

12. If twenty-four is divided by 3 and 6 is added to the result, what is the sum? _____

(Lest I forget: $1.3\overline{)39}$ or $39 \div 1.3$)

Class 11 Betcha Ya Can't DO'em (HW) Name:______________

1. For every six fords seen on USU's campus, there are seven Chevies. If there are 65 cars in the West Parking Lot, how many Fords are there? ________ Chevies _______

2. For every dollar that Dave saves, his father will give him four. If at the end of this week, the money that he has saved plus his father's gifts, total \$15, how much did he save and how much did his father give him? Save ________ Father gave________

3. Paula went into Grand Central and spent $\frac{2}{5}$ of her money on Elton John's records and had \$1.62 left. How much money did she have originally? _________

4. A number increased by one-third of itself becomes 40. What is the number?

5. In the last school election, for every three girls, voting there were two boys voting. A total of 850 ballots were cast. How many girls voted? ______ boys _______

6. If you can peddle your bike 10 miles in 30 minutes, what is your rate of speed in miles per hour? _______

7. (Tough problem) If you can peddle your bike six miles in 15 minutes, what is your rate of speed in miles per hour? ________

8. If your mom usually drives to Salt Lake City, which is 90 miles away, in $1\frac{1}{2}$ hours, what is her rate of speed? ______

(Lest I forget: $4\frac{1}{2} - 2\frac{3}{4}$)

Problem Solving Class Twelve

Objectives

1. Distinguish between the mathematical phrases.

$$6 \div 2 \text{ and } 6 \div \tfrac{1}{2}$$

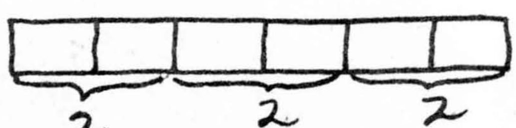
2. Introduce problems that involve relationships between three numbers.

Background

(T-9) Our problems so far have contained conditions on only one or two numbers. Now we present problems that involve conditions that must be satisfied by three numbers.

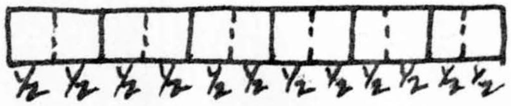
Procedure

Start with the usual review of the night before's homework. Discuss the meaning of the phrase $6 \div 2$. Most of your students will have some idea that this asks for the groups of "2's" contained in six. This can be diagramed by



three groups of two parts each

Then discuss $6 \div \frac{1}{2}$ as asking "how many groups of "$\frac{1}{2}$'s" are contained in six. This diagrams as



12 parts of $\frac{1}{2}$ unit each

Some student will be sure to bring up the idea of "invert and multiply." Write it down for them to see. Ask for it to be explained. See if they are interested or become curious, but do not explain the meaning behind the "rule" at this point.

(T-19)Have students read this problem and attempt to solve it. Some of the better students will be able to diagram it. Work the problem and others like it quite carefully as examples of problems with conditions involving three or more numbers. This problem lends itself to the posing of additional questions. For example, what other numbers besides 108 could have been used to yield only whole numbers as answers. How many of these other numbers are there? Is there a smallest one? What is it? Is there a largest "other number?" What is it? Do you think we could use negative numbers? Give an example.

(T-20)Present the transparency and let the students try solving the problem. This problem is more than just for fun. It lends itself very nicely to diagraming.

Distribute the classwork and circulate about the class paying particular attention to the problems which involve conditions on three or more numbers. Encourage the better students to play with these problems by varying the conditions on the numbers in systematic ways to observe how the resulting answers vary.

Distribute the homework.

Class 12 "Cheaper by the Dozen" Name: ______________

1. Jim bought a pen and a book. The pen was sixty-five cents more than the book. If the total cost was $5.05, how much did each cost?
Pen ______, Book ______

2. If twenty-four is divided by one-third and eight is added, what is the sum? ______.

3. A board which is 46 inches long is cut into two pieces so that one piece is four inches longer than twice the length of the other. What is the length of each piece? _____, ______.

4. A student finishes 2/3 of his assignment in twenty-four minutes. If he continues to work at the same rate how long will it take him to finish the assignment? _____.

5. If twenty-four is divided by three and eight is added, what is the sum? _____.

6. I have three numbers. The middle one is twice the smallest. The largest number is three times the smallest. If the sum of the three numbers is 48, what are they? _____, _____, ______.

7. 

If a plane travels 840 miles in 2 and 1/2 hours, what is the rate of travel in miles per hour? _____

8. Three-eights of a certain number is 72. What is that number? _____

9. What number is three-eights of 72? _____.

10. If three-eighths of a certain number is added to that number, the sum is 77. What is the certain number? _____.

11. The sum of two numbers is 145. If one of the numbers is 15 greater than the other, what are the two numbers? ______, _____.

12. A man is able to paint five-twelfths of a fence in ten hours. How much longer would it take the man to finish painting the fence? ____
(What other question might we ask about this problem?)

Class 12 Homework Name: ____________

Two trains left Ogden for St. George at the same time traveling on parallel tracks. Both arrived at St. George at the same time. One train made the trip in 80 minutes, while it took the other train 1 hour, 20 minutes. Explain.

(Lest I forget: $24\frac{2}{8} - 13\frac{5}{14}$)

Problem Solving Class Thirteen

Objectives

1. Employ diagrams to illustrate and aid in the solution of a type of problem that would be difficult without the use of the diagram.

2. Re-enforce the diagraming idea for problems that involve conditions on several numbers.

Background

This lesson emphasizes what is perhaps the most difficult type of problem in this unit. Diagrams make this problem type easy to handle and should emphasize to the student the value of their use.

Procedure

Review, as usual, the homework of the day before. Get the students to do most of this. Entertain any questions and spend time on the THINK before going into the day's lesson.

(T-21)Present the problem and ask for ideas of what a good diagram might look like. After a few tries if none is successful give the diagram, but not the solution. Have a student complete the problem.

Work several examples with the class emphasizing the diagrams. After several examples have been worked, let someone from the class make up an example. See if anyone can decide how to avoid getting fractions as the answer. (The sum of the diagramed parts must be a factor of the sum of the numbers.)

Work part of the classwork with the students, then let them finish it on their own. In problem 3 of the classwork we can have: What if the sum of the numbers is 91? 98? 105? 112? What if the sum is 77? 70? 63? What if the sum is 35? 28? 14? 7? 0? Now the hard ones: If the sum is 85? 86? 87? 88? 89? 90?

Suppose that one tenth of the smaller number is equal to $\frac{1}{4}$ of the larger. Does the answer involve fractions if the sum of the number is 84? 91? 98? 105? What if the sum is 77? 70? 63? 35? 28? 14? 7? Is there any pattern which determines whether or not the numbers involve fractions? Is that pattern related in any way to the pattern observed in problem three where 1/3 of the smaller was equal to 1/4 of the larger?

Pass out the homework.

Class 13 A Little Challenge! Name: ______________

Draw diagrams as an aid to solving these problems.

1. If one-third of a certain number is subtracted from that number, the result is 42. What is that certain number? _____

2. If I pay 36 cents for one-third of a pound of butter, how much will I pay for six pounds? ______

3. The sum of two numbers is 84. If one-third of the smaller is the same as one-fourth of the larger, what are the numbers? ___, ___.

4. Wilhelm spent one-fifth of his money in one store and one-half of the remainder in a second store. If he has 50 cents left, how much did he begin with? ____

5. Ted hiked 95 miles in three days. The second day he walked 5 miles more than the first. The third day he walked 9 miles more than he did the first. How far did he walk each day? 1st ____, 2nd ____, 3rd _____

6. Two numbers have a sum of 120. If one-third the larger is the same as one-half the smaller, what are the numbers? ____, ____.

7. A certain number increased by one-third of itself becomes 48. What is it? _____.

8. Al is three years older than Joe. If the sum of their ages is 35, how old is each boy? ____, ____.

9. Three-eighths of what number is 27? ____.

10. Twenty percent of a certain number is 30. What is that number? ____

11. Here's a toughie: Two numbers have a product of 39 and a sum of 40. What are they? _____, ____.

(Lest I forget: $\frac{3}{4} \div \frac{2}{5} =$)

Class 13 Pictures, please! (HW) Name: ______________

(Use diagrams to help solve these problems)

1. The sum of two numbers is 124. One of the numbers is three times as large as the other. What are the two numbers? ______, _____.

2. A number decreased by two-fifths of itself becomes 60. What is it? ______.

3. The sum of two numbers is 63. If one-third the smaller is the same as one-fourth the larger, what are the numbers? ____, ____.

4. Al spent five-sixths of his money and had $1.45 left. How much did he have originally? ______

5. The sum of two numbers is sixty. If one-half the smaller is the same as one-fourth the larger, what are the numbers? ____, _____.

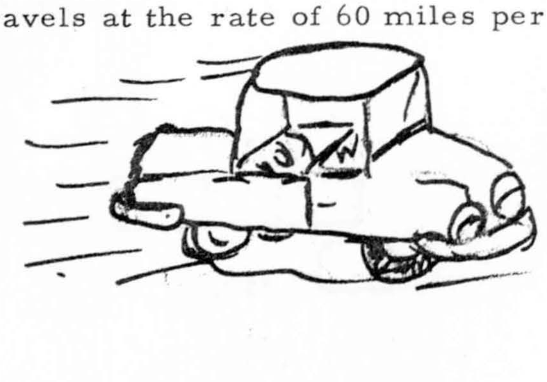
6. Three-sevenths of what number is twenty-four? _____.

7. What number increased by one-half of itself becomes ninety? _____.

8. One-half of a certain number is 54. What is that number? _____.

9. Don't get run over: A car travels at the rate of 60 miles per hour. What is the rate in miles per second? __________

What is the rate in feet per second? __________



(Lest I forget: $7\frac{1}{4} - 3\frac{2}{3}$)

Problem Solving Class Fourteen

Objectives

1. Introduce the idea of "as many."

2. Provide practice and additional examples of problems involving relatively complex relationships.

Background

This lesson and the following lesson are principally review lessons designed to re-enforce the idea of the second part of this course. Diagrams should be stressed.

Procedure

Start with discussion of the homework from the night before. Problems 3 and 5 are particularly interesting.

(T-22)Present the transparency and discuss the meaning of the phrase "as many." Are there more boys or girls? (Girls) What is the ratio of boys to girls? (5:9) What is the ratio of girls to boys? (9:5) What fraction of the students are boys? (5/14) What fraction of the students are girls? (9/14) How many boys are there? (10) How many girls? (18)

Vary the conditions of this problem and have the students set up diagrams for each of your variations.

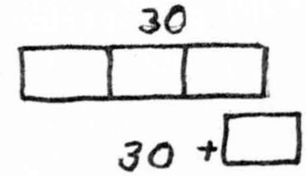
(T-23)Present the problem "Bottles, Bottles - Everywhere." Encourage all of your students to diagram this one for themselves and arrive at a solution. This is a challenging problem.

Here is what each group has:

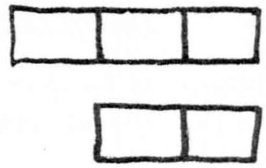
group 1

group 2

group 3



Since group 2 has as many bottles as groups 1 and 3 combined, we can set up the following relationship:



= 30 + (30 + □)

= 60

= 30

We can now conclude that:

Group 1 has 30 bottles

Group 2 has 90 bottles

Group 3 has 60 bottles

Distribute the classwork and circulate helping your weaker students with their diagrams. Questions similar to those we have asked for earlier problems can be used effectively to focus a student's attention on any problem type which is giving him trouble. For example, in problem 7 of the classwork: What is the price per 1/4 dozen eggs? What is the price per 1/8 dozen eggs? What is the price per 1/16 dozen eggs? What is 1/16 dozen eggs? What is the price per egg? What is the price per 2 eggs? 3 eggs? 1/4 dozen eggs? What is the price per 1/2 egg? 1/4 egg? Do all of these questions provide useful and meaningful answers?

Pass out the homework.

Class 14 Confucius say: "Man who dilly dally daily get behind in his work." Name:__________

Don't just sit there: try <u>diagraming</u>!

1. A certain number increased by one-fourth of itself gives 60 as a sum. What is that certain number? ______.

2. Two numbers have a sum of 84. If one number is twice as large as the other, what are the two numbers? ______, ______.

3. The sum of two numbers is 85. If the larger number is one more than five times the smaller, what are the numbers? _____, _____.

4. A herd made up of sheep and goats totals 112 animals. If there are three-fourths as many goats as sheep, how many of each are in the herd? _____goats, ___sheep.

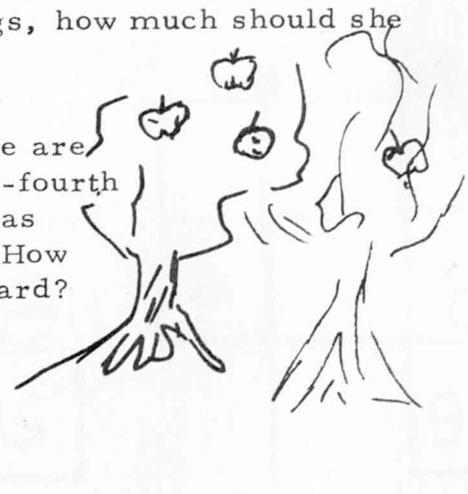
5. 

Joe has written the first 3 chapters of a book for his communications class. The first chapter is 20 pages long. The third chapter is as long as the first plus 1/5 of the second. The second is as long as the first and third chapters combined. How many pages has Joe written?

6. What number multiplied by 15/47 gives a product of one? ________.

7. If your cook pays $1.28 for two dozen eggs, how much should she pay for half a dozen eggs? _______.

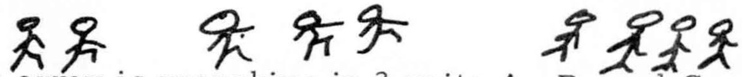
8. A man has 147 trees in his <u>orchard</u>. Some are apple, and others are peach trees. One-fourth of the number of apple trees is the same as one-third of the number of peach trees. How many of each type of tree are in the orchard? Peach, ______, apple _______.



(Lest I forget: $3\frac{2}{3} \div 2\frac{1}{2} =$)

Class 14 Another Apple from Dr. Math (HW) Name: ______________

1. What number is three-fourths of 360? ______



2. An army is marching in 3 units A, B, and C. Unit A is 35 yards long. Unit C is as long as the sum of A and one-eighth of B. Unit B is as long as the sum of A and C. How long is each unit? ______, ______, ______.

3. A man has fifty-five coins. If they are all nickels and dimes, and he has two-thirds as many nickels as dimes, how many of each type of coin does he have? _____, nickels ______ dimes.

4. Seven-twelfths of what number is eighty-four? ______

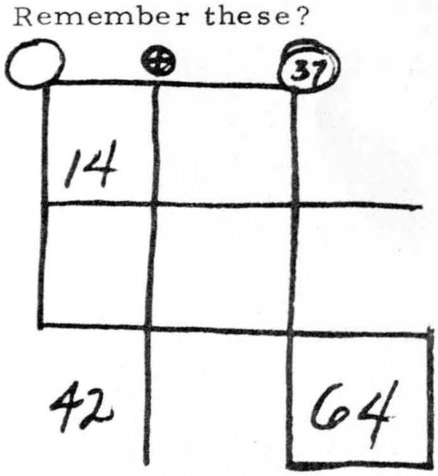
5. A number decreased by one-third of itself becomes 54. What is that number? ______.

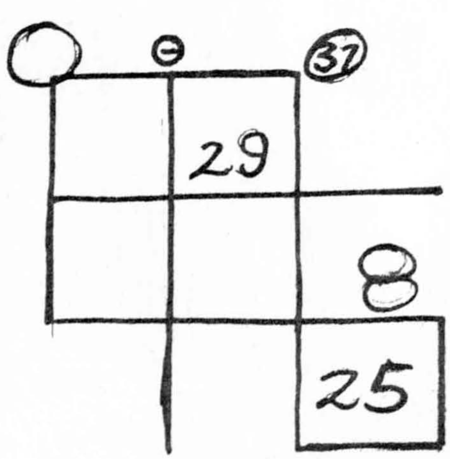
6. The sum of two numbers is 105. If one-third of the smaller number is the same as one-fourth of the larger, what are the numbers? ______, ______.

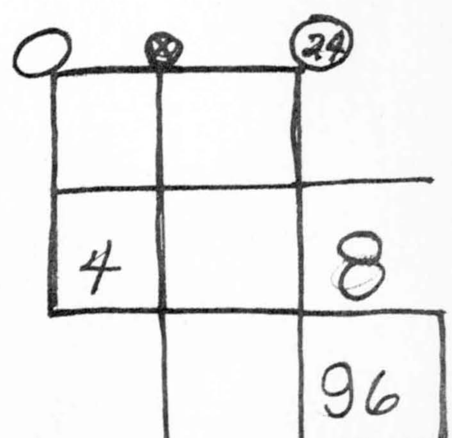
7. 

If a car travels 72 miles in 90 minutes, what is the rate in miles per hour? ______.

Remember these?







(Lest I forget: $12\frac{2}{9} \div \frac{2}{3} =$)

Problem Solving Class Fifteen

Objectives

1. Review and prepare for the quiz in the next lesson.

2. Give drill and practice in the use of the diagram technique.

Procedure

Carefully go over all of last night's homework in class. Place particular emphasis on problems 2, 3, and 6.

(T-17)Use the quick checking squares as a change of pace. Let the students try to make up examples of a division quick checking square. Can they do it so that none of the numbers will involve fractions? Can they do it using only odd numbers? Only even numbers? Only square numbers? Can they do it so the number in the upper left hand corner is the same as the number in the lower right hand corner?

Pass out the classwork and help those who need assistance. Have the students who are doing well assist others. The teacher can ask students happens to their answers if the conditions of a problem are changed in some particular way. Can they see a pattern in what happens and explain it?

Distribute the homework.

Class 15 A Rest from the Rigors of Conversation Name: ______

Use diagrams

1. Two numbers have a sum of 56. If one number is 8 more than the other, what are the numbers? ____, _____.

2. A certain number increased by two-fifths of itself gives 126 as a sum, what is that certain number? _____.

3. Two numbers have a sum of 80. If one-third of the larger number is the same as one-half the smaller, what are the numbers? _______, _______.

4. A man marks up the price of a gun to make 33 1/3% profit. If the bun then sold for $56, what was the original price? ________.

5. The product of two numbers is 1. If one of the numbers is 23/14, what is the other number? _______.

6. If 30 is divided by 3 and the quotient added to 10, what is the result? _____.

7. If 30 is divided by 1/3 and the quotient added to 10, what is the result? _____.

8. Ed spent one-fourth of his money in a drug store and one-third of what was left was spent at a movie. If he then had $1.50 left, how much money did he begin with? ____.

9. Two numbers have a sum of 130. If the larger number is 10 more than twice the smaller, what are the numbers? ____, _____.

(Lest I forget: $\frac{3}{9} \times \frac{3}{12}$)

Class 15 Another Rockpile ?!&? (HW) Name: ______________

D-Day! (Diagram Day for problems No. 1, 3, 5.)

1. A number is multiplied by four and then seven is added to the product. If the sum is then 63, what was the number? ______.

2.  If eggs sell at the rate of 3 for 14 cents, how much would you pay for two dozen? ______.

3. Seventy-five percent of what number is 36? _______.

4. If 38 is divided by one-half and 10 is added to the quotient, what is the final sum? _______.

5. A student finished three-fourths of his assignment in 27 minutes. If he works at the same rate, how much longer must he work to finish? ______.

6. A gambler began the day with 34 dollars and lost all but 14 dollars of it. How much did he have left? _______.

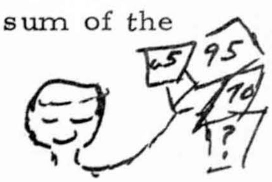
7. If 25% of a certain number is added to that number, the sum is 35. What is that certain number? _______.

8. What number added to 14/31 gives a sum of 1? ______.

9. Two numbers have a sum of 144. If one-fifths of the smaller is the same as one-seventh of the larger, what are the numbers? ____, _____.

Brainbusters

10. The difference between two numbers is four. If the sum of the numbers is 22, what are the numbers? _____, _____.



11. A boy has grades of 65, 95 and 70 on three tests. What grade must he have on the fourth test if he is to average 80? _______.

(Lest I forget: $3\frac{2}{5} \times 4\frac{1}{5}$)

Problem Solving Class Sixteen

Objective

To evaluate the students' ability to use the tool we have developed for solving problems.

Procedure

Review carefully the homework emphasizing problems 1, 3 and 5.

Give the quiz.

Distribute the assignment for the next day. The teacher might give the students a few puzzles to solve or the fun exercise that is included by the researcher.

Class 16 | A Periodical Review of Mathematical Literature | Name: ____________

Be sure to use diagrams as an aid - especially on No. 3, 6, 10, 11, 12, 13.

1. Two numbers have a sum of 67. If one number is 9 more than the other, what are the numbers? ____, ____.

2. Two numbers have a sum of 92. If one number is three times as large as the other, what are the numbers? ____, ____.

3. What number is three-sevenths of 91? _____.

4. The product of two numbers is 1. If one of the numbers is 3/14, what is the other number? ____.

5. If 36 is divided by 3 and 10 is added to the quotient, what is the sum? ______.

6. Six-sevenths of a certain number is 54. What is that number? _____

7. The sum of two numbers is 1. If one of the two numbers is 2/17, what is the other number? _____.

8. Twenty-five percent of what number is 51? _____.

9. If 36 is divided by 1/3 and 10 is added to the quotient, what is the sum? _____.

10. If one-sixth of a certain number is added to that number the sum is 84. What is that number? _____.

11. Doug and Ken are saving wishbones from the chickens they eat and have a total of 77. If one-third of Doug's wishbones is the same as one-fourth of Ken's amount, how many does each have? ____, ____.

12. An airplane travels 1400 miles in $3\frac{1}{2}$ hours. What is the rate of the plane in miles per hour? ________.

13. In counting cars on the highway we found there were twice as many Fords as Plymouths, and 3 times as many Chevrolets as Fords. If there were 108 cars of the three types, how many were Chevrolets? ______ Fords ______ Plymouths? ________.

Problem Solving Class Seventeen

Objectives

1. Begin making the transition from the mathematical symbolism we have developed to the standard mathematical symbolism of algebra.

2. Introduce the idea of a "variable" or an "unknown."

3. Reinforce the student's intuitive understanding of equality.

Background

This lesson is designed to bridge the gap between our rather pictorial symbolism for mathematical language, and the more usual and more efficient symbolism of algebra.

The teacher with a slower class may wish to omit the remaining three lessons of the unit. On the other hand, these lessons can be nicely expanded into a neat introduction to algebra. As they now stand, the lessons give a nice introduction to algebraic symbolism and some examples of its easier applications.

Procedure

Review the quiz of the preceding lesson very carefully. Have students assist in this.

Discuss with the students what we have done so far in the unit. Emphasize the use of mathematical notations, in the form of our diagram, to solve various problems. Explain that in place of a "△", or a "□" we can use a letter for our "unknown or "variable." (These words should not be emphasized or explained too much at this point.) For example:

Words	Diagram	Variable
a number	□	n (or any other letter)
5 more than a number	□ + 5	n + 5
3 times a number	3 · □ or □□□	3 · n

The teacher and the class can extend this list to many other examples. The teacher or one of the students can give any one of the three forms - words, diagram, or variable - and ask for the other two forms.

Give out the classwork and have them do part A.

Use the variable notation to set up, but not to solve, several problems using the transparencies. At this point we wish to emphasize that if we do the same thing to both sides of an equation, we maintain a state of equality. This will be the key point in the solving of these problems.

A balance beam can be used to demonstrate this idea very neatly. Select several pairs of objects in such a way that each member of any given pair weighs the same amount as the other object of the pair. Have students select items and attach them to the beam. When the same thing is done on both sides of the beam the balance is preserved. When different things are done to each side, the balance is destroyed. This is a very vivid, concrete demonstration of what happens when the same thing is done to both sides of an equation.

Now use this idea to solve the problems which were set up on the transparencies earlier in the class. By doing the same thing to both sides of each equation it can be shown that the unknown can be eventually isolated on one side with its actual value on the other side.

Have the students do part B of the classwork. The teacher might ask for problem 1 of part B: How does the equation change if the two numbers have a sum of 57? 58? 59? How does the equation vary if the numbers differe by 9? 10? 11? How do the answers vary in each case.

Distribute the homework.

Class 17 Name: ____________

Confucius say:
"Man is smart who listens
to man who is smarter."

A. Symbolism: Supply the missing information.

Words	Diagram	Algebra
1. A number plus 5		
2. Four times a number		
3.		n + 7
4.	□ □ □	
5. Three more than twice a number		
6.		3b + 5
7.	9 – □	
8.		2n – 3
9.	□□ + 8	
10. Double a number		

B. Try writing the number sentences for these problems and then use your knowledge of algebra to solve them.

1. Two numbers have a sum of 56. If one number is 8 more than the other, what are the numbers? ____, ____.

2. Two numbers have a sum of forty. If one of the numbers is 4 times as large as the other, what are the numbers? ____, ____.

3. Seventh-six subtracted from what number leaves forty-four? ____.

4. The sum of two numbers is 1. If one of the numbers is 3/7, what is the other number? ____.

(Lest I forget: $4\frac{2}{5} - 2\frac{3}{4} =$)

Class 17 Name: ________________

Confucius say:
"Man who is careless with lens - grinding machine, may make spectacle of himself." (HW)

A. Can you supply what is missing below?

	Algebra	Diagram	Words
1.	3n + 2		
2.		□□ -5	
3.			Three more than twice a number
4.	2n − 4		
5.			A number increased by five
6.		17− □□□	

B. Try writing the number sentence for these problems and then see if you can solve them. (The odds are in your favor!) Use your algebra!

1. What number subtracted from 108 is twice thirty-eight? ______.

2. The sum of two numbers is 160 but one of the numbers is twenty greater than the other. What are the numbers? _____, _____.

3. The sum of two numbers is 120. If one of the numbers is three times as large as the other, what are the numbers? _____, _____.

4. The sum of two numbers is 63. One of the numbers is three more than twice the other. What are the two numbers? _____, _____.

5. Bill has twice as much money as Al, and Jim has three times as much as Al. Together they have $3.60. How much does Al have? ______: Bill: _____: Jim? _____.

(Lest I forget: $7\frac{2}{5} + \frac{1}{4} - 2\frac{4}{5} =$)

Problem Solving Class Eighteen

Objective

1. Reinforce the ideas of lesson sixteen on algebraic symbolism.

2. Discuss the use of reciprocols and equivalent fractions for the solution of equations.

Background

Discuss and clarify the homework of the night before. Discuss the work "variable" at this point a little more formally than before. " □ was a president of the U. S." The variable here is □ , and as the name suggests its "value can vary" over all the names you may wish to use. Only certain of these values, however, will make the sentence true. Similarly only certain values for x will make the sentence " x is a member of the varsity football team" true.

The same analysis applies to "$N + 5 = 8$." Here "n" is a variable and varies in fact over numbers. No matter how hard they try, your students will only find one number which makes the statement true (although they will be able to name the number in a variety of ways).

Stress again that the same operation involving the same numbers done on both sides of an equation preserves equality.

Example:

Solve $3/4 \times n = 1$

(a) $4/3 \times (3/4 \times n) = 4/3 \times 1$

(b) $(4/3 \times 3/4) \times n = 4/3 \times 1$

(c) $\frac{12}{12} \times n = 4/3 \times 1$

(d) $1 \times n = 4/3 \times 1$

(e) $n = 4/3 \times 1$

(f) $n = 4/3$

Have the students give the reason for each step here very carefully. (Be sure to do this slowly and put in all the steps. What seems obvious to the teacher may seem obscure to students who are unfamiliar with the algebraic processes.) This problem gives a good opportunity to review the commutative property of multiplication, multiplication of fractions, simplification of fractions, reciprocals and the fact that $1 \cdot n = n$.

The teacher could ask? Why did we select 4/3 to multiply both sides of the equation by? Why would have happened if we had not been so far sighted and had multiplied instead by 8/3? By 8/6? By 12/6? By 13/7?

Similarly, we have used the fact that 1 x n = n where n is any rational number. The students know this for whole numbers, but is it true when it is a fraction? What if we substitute 12/12 for the 1. Then is 12/12 x n = n? Is 13/13 x n = n? How about a/a x n = n? What could "a" be here? A whole number? a fraction? a decimal?

A second equation is given below which gives an opportunity to review the properties discussed in solving the first example.

Example:

$$3/4 \times n = 6$$

$$4/3 \times (3/4 \times n) = 4/3 \times 6$$

$$(4/3 \times 3/4) \times n = 4/3 \times 6$$

$$\frac{12}{12} \times n = 4/3 \times 6$$

$$1 \times n = 4/3 \times 6$$

$$n = 4/3 \times 6$$

$$n = 4/3 \times 6/1$$

$$n = \frac{24}{3}$$

$$n = 8$$

Distribute the classwork and discuss it with the students, then pass out the homework.

Class 18 Name: ____________________

What's purple and rides a great white horse?

Suggestions for problem solving:

I. Read the problem carefully.

II. Let the variable be what you are trying to find, the unknown.

III. Write the number sentence.

IV. Try to find the solution.

V. Does it answer the question which was asked?

Here they are! Do your best and see how many you can solve.

1. Two numbers have a sum of 86. One number is 8 more than the other. What are the numbers? ____, ____.
Let n =

2. One-half of what number is 38? ____
Let n =

3. The product of two numbers is 1. If one is 3/7, what is the other number? ____

4. If a certain number is increased by one-seventh of itself, it becomes 56. What is that certain number? ____
Let x =

5. When your age is added to mine the sum is 52. My age is 4 less than three times your age. How old am I? ____
Let n = your age
____ = my age

6. A number decreased by two-fifths of itself becomes 60. What is the number? ____
Let n = the number

Class 18 Name: ____________________

Confucius say:
"Man who can't count his blessings,
is poor at mathematics." (HW)

Use diagram or algebra to solve these gems:

1. Twenty percent of a certain number is 16. What is that number? ___

2. The sum of two numbers is 63. One number is twice as large as the other. What are the two numbers? _____, _____.

3. A baseball team won three more than twice as many games as they lost. If they played 27 games, how many games were lost? _____

4. A boy cuts three-eighths of his lawn in twenty-seven minutes. At this rate, how long will it take him to finish the lawn? _______

5. A certain number is multiplied by 3 and 8 is added to the product to give 41. What is that certain number? _____

6. The product of two numbers is zero. If one of the numbers is 27/29, what is the other number? ______

7. In a group of 54 students twice as many have watches as those who have rings. Three times as many have charge cards as those who have watches. How many have rings? _____, watches? _____, charge cards? ______.

A Real Brainbuster!

8. Bill and Ed together can paint a boat in four hours. If Bill can do it alone in eight hours, how long would it take Ed if he worked alone? _________

Problem Solving Class Nineteen

Objective

1. To present tough challenges to the students which, if they use algebra they will be able to solve.

2. To reinforce the ideas of lessons sixteen and seventeen.

Background

This lesson should present students with practice in using and understanding algebraic symbolism and its relationship both to English and to the intermediate pictorial symbolism we developed earlier.

Procedure

Review the last assignment carefully, while encouraging the students to handle most of the explanztions. (Optional) Problem 8 may give the students some difficulty. It can be handled as a rate problem if the teacher so desires. What is Bill's rate of boat painting in boats per hour? In boats per minute? In boats per 8-hour day? In boats per 40-hour week? What is the combined rate of Bill and Ed in boats per hour? In boats per minute? In boats per $\frac{1}{2}$ hour? In boats per 8-hour day? In boats per 40-hour week? From this the problem is easily solved. Let x = Ed's rate of boat painting. Then 1/8 boat/hour + x = 1/4 boat/hour. Thus x = 1/8 boat/hour, or it takes Ed eight hours to paint one boat by himself.

Review the ideas of "variable" and "preserving equality by doing the same thing to both sides of an equation."

Distribute the classwork. Work with small groups and individuals, and encourage student group effort. Have selected students write their better solutions on clear transparencies. Have other students explain the solution. Stress importance of moving on when a problem proves to be too tough - but be sure to at least <u>try</u> all the problems.

Pass out the homework early to get them off to a good start. An interesting observation of the Casino Problem is that it is easier to solve by reasoning backwards than forwards. The algebraic solution requires only the knowledge of the distributive property and the meaning of such grouping symbols as parentheses, brackets, and braces.

We trust that this short visit through the topic of problem solving has, like Gamby's visit to the casinos, been engaging and exciting. It is here that we should drop the analogy as we would like to think that the unit has proven to be more profitable to you and your students than Gamby's misadventure was to him.

Class 19 Name: ______________

What is green and swims in the ocean?

Use either a diagram or algebra as an aid in finding the solutions.

1. If an airplane travels 450 miles in forty minutes, what is the rate of the plane in miles per hour? ______.

2. Twenty percent of a certain number is thirty. What is that number? ____

3. A plane travels 700 miles in two hours and twenty minutes. How far does it travel per hour? ____

4. Chicken-In-Honey Drive-In has reduced the price of sandwiches by $12\frac{1}{2}\%$. If the sale price of a sandwich is 84 cents, what is the regular price of the sandwich? ______

5. Three groups of boys are collecting Mud Comic Books. Group I has 30 books. Group III has as many as Group I plus one-third as many as group II. Group II has as many as group III and group I together. How many books does each group have? I ____, II _____, III ____.

6. If you have twenty-four dollars and spend one-third of it for clothing and one-fourth of it for records, how much will you have left? _____.

7. Try this one using a diagram and algebra. The sum of two numbers is sixty. If one-half of the smaller number is the same as one-third of the larger number, what are the two numbers? _____, _____.

Let x =

(Lest I forget: $1.8\overline{)96}$)

Class 19 Name: ____________________

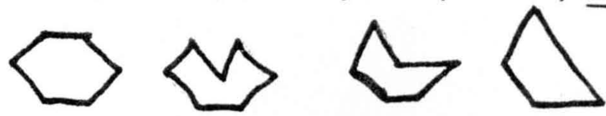
Confucius' brother say:
"Confucius talk too much!" (HW)

A. Predictions

1. 2 4 2 8 2 16 ___ ___ ___ ___

2. 1 2 6 4 5 9 ___ ___

3. man, three, warm, four, chair, ___

4. 

5. 3600 1800 600 150 ___ ___

6. 3 12 18 9 36 42 ___ ___

B. Have you ever been faced with a problem that seems easier to solve backwards rather than forwards? Here's one that's called "The Casino Problem."

Gamby went to Lost Vegas to "make a million." He visited three gambling casinos and the following shows what happened:

Casino I				Casino II				Casino III	
$10 in	Doubled his money	$10 out	———	$10 in	Doubled his money	$10 out	———	$10 in	Doubled his money	$10 out

As you can see, Gamby at each casino paid $10 to get in, then doubled his money, and paid $10 to leave! After he left the third casino he was broke!

1. What is a reasonable question to ask about Gamby in this problem? ______________________________
____

2. Can you solve this problem by reasoning backwards? If so, what is your answer? ________

3. Try setting the problem up using algebra. Be careful, it's rather tricky!!

(Lest I forget: (.302 x .05)

Class Twenty

Class twenty is given as an exercise for motivation and fun. If the instructor sees a need to incorporate a different set of problems for the given class period, it is permissable.

Class 20 Using Symbols Name: ______________

HOW WELL DO YOU FOLLOW DIRECTIONS ? ? ? ?

If you do the following exercise correctly, you will change the name of an American president into the name of a European state. Check your method, speed and accuracy in following directions.

Write the words GEORGE WASHINGTON. Take out all the e's counting only the remaining letters, add an l after each seventh letter. Move the second g to the beginning and put the last letter in its place. Whenever three consonants appear together, change them in order so that the first consonant in the group becomes the last, the one in the second place takes the first position, and the one in third position becomes the middle consonant of the group. Take out the last two vowels. Where a double consonant appears, take out both letters. Beginning with the third letter from the left, interchange each two letters. Take out the last two letters. Move the last letter so it will be the first letter. Add a d after each fourth letter and at the beginning. Replace every s with an n. Take out the middle three letters. Take out the final letter and put the first letter in its place.

If you did the exercise correctly with no errors at all consider how you achieved this perfection in following directions and how you can transfer this skill to your mathematical thinking and daily living.

If you gave up altogether, consider . . . .

Class 20 Using Symbols Name: __________

Can you find your way?

DIRECTIONS: Here is a tough one! You will have to follow each step very carefully. Remember to rewrite the word in the blank as it is changed each time and so use the newly changed word with the next direction. Each time the word should be different, and you should work from the new change.

The name is Thomas Jefferson

1. Place the two words side by side as one word in the blank. __________
2. Drop the fourth letter of the word, and in it's place add the letter which follows it in the alphabet. __________
3. Drop the second e in the word, and in its place add the letter which precedes it by two in the alphabet. __________
4. Drop the double consonant in the word and in its place add double t. __________
5. Drop the a and the e in the word. Add an i to take the place of each. __________
6. Take the first i in the word and place it after the first s, and take the second i in the word and place it after the second s. __________
7. Take the tenth letter of the word and place it before the first t. __________
8. Drop the third, the eighth, and the eleventh letters of the word. __________
9. Take the second letter of the word and place it after the first s. __________
10. Drop the second s in the word and in its place add the letter which follows it by two in the alphabet. __________
11. Change the position of the eighth and ninth letters of the word, so that the eighth letter becomes the ninth one and the ninth letter becomes the eithth one. __________

Did you know that many of Thomas Jefferson's ideas went into this document?

Appendix B

Mathematic Attitude Scale

NAME: ______________________________

MATHEMATICS ATTITUDE SCALE

irections: Please write your name in the upper right-hand corner. Each of the :atements on this opinionnaire expresses a feeling or attitude toward mathematics.)u are to indicate, on a five-point scale, the extent of agreement between the ttitude expressed in each statement and your own personal attitude. The five)ints are: Strongly Disagree (SD), Disagree (D), Undecided (U), Agree (A), Strongly gree (SA). Draw a circle around the letter or letters giving the best indication f how closely you agree or disagree with the attitude expressed in each statement.

1.	I am always under a terrible strain in a mathematics class.	SD	D	U	A	SA
2.	I do not like mathematics, and it scares me to have to take it.	SD	D	U	A	SA
3.	Mathematics is very interesting to me, and I enjoy arithmetic and mathematics courses.	SD	D	U	A	SA
4.	Mathematics is fascinating and fun.	SD	D	U	A	SA
5.	Mathematics makes me feel secure, and at the same time it is stimulating.	SD	D	U	A	SA
6.	My mind goes blank and I am unable to think clearly when working mathematics.	SD	D	U	A	SA
7.	I feel a sense of insecurity when attempting mathematics.	SD	D	U	A	SA
8.	Mathematics makes me feel uncomfortable, restless, irritable, and impatient.	SD	D	U	A	SA
9.	The feeling that I have toward mathematics is a good feeling.	SD	D	U	A	SA
10.	Mathematics makes me feel as though I'm lost in a jungle of numbers and can't find my way out.	SD	D	U	A	SA
11.	Mathematics is something that I enjoy a great deal.	SD	D	U	A	SA
12.	When I hear the word mathematics, I have a feeling of dislike.	SD	D	U	A	SA
13.	I approach mathematics with a feeling of hesitation, resulting from a fear of not being able to do mathematics.	SD	D	U	A	SA

(continued on back side)

14. I really like mathematics.	SD	D	U	A	SA
15. Mathematics is a course in school that I have always enjoyed studying.	SD	D	U	A	SA
16. It makes me nervous to even think about having to do a mathematics problem.	SD	D	U	A	SA
17. I have never liked mathematics, and it is my most dreaded subject.	SD	D	U	A	SA
18. I am happier in a mathematics class than in any other class.	SD	D	U	A	SA
19. I feel at ease in mathematics, and I like it very much.	SD	D	U	A	SA
20. I feel a definite positive reaction to mathematics; it's enjoyable.	SD	D	U	A	SA

Appendix C

Algebra Readiness Test

Test 1 — FUNDAMENTAL OPERATIONS (2 minutes)

DIRECTIONS. Find the answers to the exercises below. Write the answers under the exercises. Work rapidly.

	a.	b.	c.	d.	e.	f.	g.	h.
1.	18 +5	17 —8	34 ×7	$8\overline{)64}$	6 +23	14 —9	47 ×6	$9\overline{)54}$
2.	29 +5	37 —8	58 ×8	$8\overline{)72}$	37 +6	46 —7	76 ×4	$8\overline{)56}$
3.	17 +5	15 —9	94 ×6	$9\overline{)81}$	23 +8	34 —7	63 ×6	$9\overline{)63}$
4.	7 +19	26 +5	19 —5	26 —9	$7\overline{)42}$	67 ×9	9 +14	18 +9
5.	21 —7	28 —9	$6\overline{)54}$	89 ×9	39 +6	18 +7	33 —8	27 —9
6.	$7\overline{)63}$	76 ×8	8 +26	27 +4	18 —6	25 —6	$9\overline{)72}$	78 ×7

Score (No. right ÷ 2)..........................

Have you studied algebra before?..........................

Teacher.......................... Date..........................

School.......................... City..........................

Name.......................... (Last) (First) Age.......... Sex..........

TEST	SCORE	PERCENTILE
1		
2		
3		
4		
5		
TOTAL		

ALGEBRA READINESS TEST

William R. Lueck

Prod. No. 77922

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Page 2

Test 2 — FRACTIONS (4 minutes)

DIRECTIONS. Work these exercises. Write the answers in lowest terms on the dotted lines at the right of each exercise. Work down the page.

1. $2\frac{1}{6} + \frac{5}{6} =$ Ans. ....
2. $12 - 8\frac{1}{4} =$ ....
3. $\frac{3}{5} \times \frac{7}{6} =$ ....
4. $\frac{3}{6} \div \frac{3}{8} =$ ....
5. Change $\frac{1}{2}$ percent to a fraction. ....
6. $\frac{3}{8} + \frac{2}{3} =$ ....
7. $7\frac{2}{5} - 3\frac{1}{4} =$ ....
8. $2\frac{3}{4} \times 8 =$ ....
9. $4\frac{1}{2} \div 6 =$ ....
10. Change $\frac{7}{16}$ to per cent. ....
11. $4\frac{2}{3} + 7\frac{3}{4} =$ ....
12. $17\frac{2}{7} - 5\frac{3}{4} =$ ....
13. $5\frac{3}{5} \times 1\frac{2}{3} =$ ....
14. $3\frac{3}{4} \div 2\frac{1}{2} =$ ....
15. $5\frac{2}{3} + 2\frac{3}{8} - 4\frac{1}{4} =$ ....

Score (No. right)..................

Test 3 — DECIMALS (4 minutes)

DIRECTIONS. In the answers written to the right of the exercises below, all the decimal points and some zeros have been omitted. Point off the answers by putting decimal points in the correct places and inserting zeros if you need them to make the answers correct.

1. Add 1 to .14.	1 1 4
2. 3.46 × 1.2 =	4 1 5 2
3. 1 — .03 =	9 7
4. Write 7 cents as a part of a dollar.	7
5. Divide .0288 by .48.	6
6. Change 385 per cent to a decimal.	3 8 5
7. Multiply .375 by .005.	1 8 7 5
8. .088 ÷ .02 =	4 4
9. Change $\frac{7}{32}$ to a decimal.	2 1 8
10. 8.3 + 71 + .239 =	7 9 5 3 9
11. From the sum of 6.47 and 57.6 subtract .968.	6 3 1 0 2
12. 6.8 divided by .034 =	2
13. 3.52 ÷ 32 × .022 =	2 4 2
14. 6.08 + 14.3 — 3.415 =	1 6 9 6 5
15. Change $\frac{1}{2}$ per cent to a decimal.	5
16. 16 ÷ .4 =	4
17. Change $\frac{1}{16}$ to a decimal.	6 2 5
18. .24 ÷ 48 =	5

Score (No. right)..................

Test 4 — PROBLEM SOLVING (8 minutes)

DIRECTIONS. Read the problem carefully and solve it. Then look at the four answers to the right of the problem. Circle the answer you have found to be correct.

1. Pencils are selling at 4 for 5 cents. How many can be bought for 35 cents?	7	16	28	120
2. In four subjects Bob got final grades of 90, 86, 80 and 72. Find his average grade.	82	78	74	88
3. Jane spent half of her money for clothes and $5 more for a gift. If she had $11 left, how much money had she at first?	6	32	15	55
4. If the circumference of a circle is divided by its diameter, what is the quotient?	.78	1.25	3.14	6.28
5. Mr. Hanson purchased some city lots at $120 each. He sold them at $150 each. His total profits were $210. How many lots did he buy?	3	7	15	30
6. The sum of three numbers is 99. One of the numbers is 21. The other two numbers are equal. Find one of the equal numbers.	26	33	78	39
7. Henry drove his car at an average rate of 36 miles per hour. How long did it take him to make a trip of 126 miles?	2.8	3.1	4.2	3.5
8. The length of Dick's step while walking will average $\frac{3}{4}$ yd. How many steps must he take to walk 48 yd?	72	32	36	64
9. Nell is 17 years of age. Her aunt Sara is 15 years older than Nell. What is the sum of their ages?	49	32	45	22
10. Find the value of D in the formula D = RT if R = 20 and T = 7	27	87	140	207
11. How many cubic ft. are there in 400 bushels? (1 cubic ft. = 0.8 bu.)	50	200	320	500
12. It required 45 square yd. of carpet to cover the floor of a large room. The room is 15 ft. wide. What is its length in feet?	3	9	27	30
13. After paying 25 per cent of his car, Henry found that $180 would pay the remainder. What did he pay for the car?	45	215	240	540
14. Using the formula H = RS + 5, find H if R = 2 and S = 10.	17	25	30	100
15. After picking 120 quarts of berries, Don found that he had finished $\frac{4}{5}$ of his work. How many quarts remained to be picked?	24	30	60	96
16. Tom paid a bill of $2.40 with 20 coins of two denominations. Seven of the coins were quarters. What was the value of one of the other coins?	1	5	10	25
17. The perimeter of a rectangle is 92 ft. The width is 17 ft. Find the length.	21	29	36	75
18. The base of a triangle is 8 ft. The area is 16 square ft. Find the altitude.	4	2	8	16
19. How many minutes will it take to cut a piece of cloth 15 yd. long into one-yard towels if one towel is cut off each half minute?	7	7.5	15	30
20. A horse and bridle cost $124. The horse cost $110 more than the bridle. What was the cost of the bridle?	4	14	9	7

Score (No. right)..................

Page 4

Test 5 — GENERAL NUMBERS (8 minutes)

DIRECTIONS. This is a lesson in some easy algebra. First study the lesson and the samples that follow. The letters in this lesson stand for numbers. You will add, subtract, multiply, or divide these numbers depending on what you are asked to do.

Subtraction. The expression A — B means that the number B is subtracted from another number A. Seven less than the number R is R — 7. However, 7D — 4D = 3D.

Multiplication. Five times the number N is written 5N. Notice that the N is written after the 5. A times B = AB.

Addition. The sum of the two numbers C and D is C + D. However, 3N + 6N = 9N.

Division. The number R divided by 10 is written $\frac{R}{10}$ or $\frac{1}{10}$ R.

Now write the answers to the exercises below on the dotted lines at the right side of the page. Study the lesson again if you need to. Be sure your answers are in exactly the same form as given in the lesson even though you have learned to write them in other ways. Notice the samples.

Sample A. How many crayons are there in 7 boxes if each box contains N crayons? 7N

Sample B. Sam had D dollars in each of three pockets. He spent $5. How many dollars had he left? 3D — 5

1. If the number N is added to the number 5N what is the result? ....
2. A man's present age is M years. Write his age 3 years ago. ....
3. What is the simplest answer to B — B? ....
4. If the number of feet in Y yards is 3Y, write the number of inches in F feet. ....
5. How many yards are there in F feet? ....
6. If in 7 dimes there are 10 times 7 cents, how many cents are there in D dimes? ....
7. In the statement 4X + 3 = 15, what definite number does X stand for? ....
8. John had D dimes. He gave 6 cents to Tom. How many cents had he left? ....
9. If a train goes a distance of D miles in 3 hours, what is the rate in miles per hour? ....
10. The length of a rectangle is B feet and the width is A feet. What is the perimeter? ....
11. What is the result if you subtract the number N from twice the number P? ....
12. How many gallons are there in T quarts? ....
13. Each of B bags of peanuts weighs H pounds. The total weight of all the bags is P pounds. Write a formula for the total weight using B, P, and H. ....
14. The sum of two numbers is 17. One of them is N. What is the other number? ....
15. How many square yards are there in S square ft.? ....
16. Write a number that is three more than five times the number X. ....
17. Sam is X years old. His sister is 5 years older. What is the sum of their ages? ....
18. James paid C cents for a bat and 15 cents less for a ball. What did he pay for both articles? ....

Score (No. right × 2)..........

Appendix D

Performance and Attitude Scores for Students of Schools I, II, III and IV

School I (Experimental) performance raw scores

	School	Pre	Post	Attitude	Algebra Readiness A_1	A_2	A_3	A_4
1101	1	0		47	02	00	04	00
1101	1		1	67	03	00	01	03
1241	1	0		69	07	07	05	06
1241	1		1	81	11	08	09	24
1121	1	0		74	16	06	10	12
1121	1		1	89	16	08	12	38
1181	1	0		39	08	02	03	07
1181	1		1	56	08	07	06	17
1091	1	0		64	02	02	06	11
1091	1		1	69	05	03	08	17
1051	1	0		61	08	01	04	04
1051	1		1	74	08	03	06	09
1041	1	0		77	05	00	03	05
1041	1		1	98	08	00	04	08
1211	1	0		53	14	09	07	08
1211	1		1	51	15	07	07	09
1071	1	0		62	09	03	09	09
1071	1		1	78	16	06	08	20
1010	1	0		70	14	07	11	14
1010	1		1	72	16	07	09	34
1081	1	0		72	09	07	05	15
1081	1		1	82	14	08	05	26
1230	1	0		55	05	00	06	01
1230	1		1	70	09	03	05	07
1031	1	0		74	09	03	05	05
1031	1		1	58	13	02	07	12
1140	1	0		67	09	01	04	06
1140	1		1	77	11	05	05	09
1161	1	0		72	08	04	09	11
1161	1		2	84	08	03	09	17
1021	1	0		91	21	02	04	07
1021	1		1	91	20	04	08	21
1060	1	0		62	08	03	05	05
1060	1		1	74	17	03	07	18
1131	1	0		47	06	00	05	07
1131	1		1	55	07	02	05	16
1220	1	0		50	16	07	11	10
1220	1		1	55	13	07	12	33

School I (Experimental) performance raw scores (continued)

	School	Pre	Post	Attitude	Algebra Readiness A_1	A_2	A_3	A_4
1110	1	0		76	13	05	12	16
1110	1		1	64	16	07	15	42
1151	1	0		68	07	04	03	08
1151	1		1	77	08	04	11	24
1170	1	0		63	10	03	05	07
1170	1		1	65	11	07	09	24

School I (Control) performance raw scores

	School	Pre	Post	Attitude	Algebra Readiness A_1	A_2	A_3	A_4
2200	1	0		40	08	06	11	12
2200	1		1	33	07	03	08	12
2231	1	0		72	07	05	05	06
2231	1		1	68	22	05	07	07
2051	1	0		76	12	05	11	07
2051	1		1	71	16	03	14	13
2031	1	0		65	11	03	06	10
2031	1		1	80	12	01	08	08
2061	1	0		70	06	03	07	12
2061	1		1	77	06	02	08	31
2021	1	0		47	06	01	02	13
2021	1		1	49	08	01	05	16
2181	1	0		66	08	05	09	12
2181	1		1	80	09	03	08	18
2120	1	0		67	06	05	08	12
2120	1		1	66	08	03	09	11
2140	1	0		63	09	05	04	08
2140	1		1	67	12	04	06	10
2111	1	0		54	04	02	08	07
2111	1		1	49	04	02	05	09
2101	1	0		45	04	01	05	11
2101	1		1	40	07	01	05	16
2130	1	0		49	08	05	04	16
2130	1		1	55	09	02	04	23
2171	1	0		68	05	02	07	09
2171	1		1	69	07	02	08	15
2010	1	0		70	04	01	00	01
2010	1		1	70	03	01	01	00
2221	1	0		56	04	05	07	19
2221	1		1	52	07	04	08	29
2161	1	0		88	09	07	07	11
2161	1		1	74	12	04	09	09
2091	1	0		55	03	01	05	03
2091	1		1	56	05	00	05	01

	School	Pre	Post	Attitude	Algebra Readiness A_1	A_2	A_3	A_4
2241	1	0		65	06	06	07	07
2241	1		1	62	08	03	08	04
2210	1	0		24	12	05	07	08
2210	1		1	51	16	04	07	12

A_1 = Fundamental operations

A_2 = Fractions

A_3 = Decimals and percents

A_4 = Problem solving

School I (Experimental) performance raw scores (continued)

	School	Pre	Post	Attitude	Algebra Readiness A_1	A_2	A_3	A_4
2091	1	0		55	03	01	05	03
2091	1		1	56	05	00	05	01
2241	1	0		65	06	06	07	07
2241	1		1	62	08	03	08	04
2210	1	0		24	12	05	07	08
2210	1		1	51	16	04	07	12

A_1 = Fundamental operations

A_2 = Fractions

A_3 = Decimals and percents

A_4 = Problem solving

School II (Experimental) performance raw scores

	School	Pre	Post	Atti-tude	Algebra Readiness			
					A_1	A_2	A_3	A_4
3151	2	0		63	11	06	03	00
3151	2		1	75	24	05	05	02
3291	2	0		53	12	01	04	04
3291	2		1	59	20	04	07	11
3301	2	0		48	10	03	07	13
3301	2		1	52	23	09	10	18
3311	2	0		66	04	01	05	09
3311	2		1	73	16	05	04	16
3401	2	0		74	03	03	03	07
3401	2		1	74	20	02	05	16
3181	2	0		70	10	01	07	07
3181	2		1	76	16	03	11	11
3061	2	0		53	03	02	04	05
3061	2		1	66	03	02	06	11
3351	2	0		69	12	05	05	10
3351	2		1	72	23	13	06	25
3141	2	0		36	15	02	01	06
3141	2		1	47	23	01	04	10
3421	2	0		59	07	00	06	09
3421	2		1	67	16	02	08	07
3491	2	0		60	06	00	03	00
3491	2		1	65	22	00	05	06
3431	2	0		56	18	04	06	09
3431	2		1	78	23	09	08	21
3451	2	0		73	21	06	07	04
3451	2		1	78	23	10	07	13
3441	2	0		55	12	03	05	11
3441	2		1	68	23	01	05	13
3411	2	0		56	12	01	07	06
3411	2		1	53	24	03	10	05
3191	2	0		82	06	07	05	06
3191	2		1	84	19	08	09	11
3030	2	0		36	16	03	07	03
3030	2		1	43	23	01	08	04
3370	2	0		49	16	01	05	03
3370	2		1	58	16	05	02	08
3460	2	0		45	13	04	05	04
3460	2		1	45	21	05	07	13

School II (Experimental) performance raw scores (continued)

	School	Pre	Post	Attitude	Algebra Readiness			
					A_1	A_2	A_3	A_4
3111	2	0		50	14	06	05	11
3111	2		1	60	23	08	09	17
3100	2	0		90	15	03	11	02
3100	2		1	92	23	07	15	12
3090	2	0		64	18	07	05	09
3090	2		1	69	23	11	08	08
3480	2	0		61	16	04	07	09
3480	2		1	74	23	06	07	12
3121	2	0		54	11	04	05	03
3121	2		1	60	23	00	12	11
3071	2	0		95	13	04	08	03
3071	2		1	93	24	09	09	09
3211	2	0		49	03	00	08	06
3211	2		1	40	20	01	07	06
3391	2	0		54	09	07	08	13
3391	2		1	68	23	08	08	18
3471	2	0		50	01	01	00	01
3471	2		1	36	22	02	00	03
3501	2	0		69	06	02	08	10
3501	2		1	67	19	01	07	12
3161	2	0		71	07	03	04	00
3161	2		1	73	15	00	09	06
3381	2	0		70	14	08	07	14
3381	2		1	73	23	12	08	28
3361	2	0		64	14	05	03	02
3361	2		1	54	24	04	09	07
3101	2	0		73	12	06	03	02
3101	2		1	82	23	14	09	08
3011	2	0		60	18	03	07	05
3011	2		1	93	24	05	09	14
3081	2	0		58	13	04	04	11
3081	2		1	70	23	03	07	23
3131	2	0		65	07	03	08	06
3131	2		1	99	24	12	09	12
3341	2	0		60	10	00	01	06
3341	2		1	69	21	00	05	08

School II (Control) performance raw scores

	School	Pre	Post	Attitude	Algebra Readiness A_1	A_2	A_3	A_4
4050	2	0		48	11	03	05	03
4050	2		1	40	13	03	06	01
4060	2	0		50	03	00	02	05
4060	2		1	46	11	00	07	08
4241	2	0		33	05	01	02	04
4241	2		1	35	05	01	05	04
4231	2	0		71	12	08	08	12
4231	2		1	69	23	13	13	16
4250	2	0		24	04	05	08	10
4250	2		1	34	05	11	09	13
4221	2	0		46	06	01	05	07
4221	2		1	45	05	02	07	06
4140	2	0		41	64	01	07	08
4140	2		1	33	15	01	06	09
4131	2	0		64	06	01	04	04
4131	2		1	64	12	01	05	07
4121	2	0		55	05	02	05	02
4121	2		1	46	02	01	08	11
4211	2	0		50	01	00	00	00
4211	2		1	35	04	00	02	01
4201	2	0		54	02	01	05	05
4201	2		1	58	19	02	12	08
4110	2	0		64	10	06	12	15
4110	2		1	73	21	08	12	18
4101	2	0		44	08	03	06	11
4101	2		1	45	14	03	08	16
4190	2	0		65	13	07	03	12
4190	2		1	68	23	06	10	13
4181	2	0		59	08	04	08	01
4181	2		1	67	09	03	07	07
4090	2	0		54	05	03	07	08
4090	2		1	73	24	01	05	10
4040	2	0		72	13	04	05	05
4040	2		1	46	08	06	08	06
4170	2	0		72	08	03	08	06
4170	2		1	72	10	03	06	02
4160	2	0		36	10	05	09	11
4160	2		1	47	23	04	08	13

School II (Control) performance raw scores

	School	Pre	Post	Attitude	Algebra Readiness			
					A_1	A_2	A_3	A_4
4031	2	0		82	08	05	09	12
4031	2		1	70	09	08	10	13
4151	2	0		37	06	01	06	06
4151	2		1	37	10	01	02	09
4080	2	0		38	07	03	06	01
4080	2		1	67	23	02	08	03
4021	2	0		57	16	03	07	06
4021	2		1	67	24	01	04	03
4011	2	0		58	06	00	03	04
4011	2		1	50	10	00	04	05
4070	2	0		76	10	02	09	11
4070	2		1	79	13	07	08	05

School III (Experimental) performance raw scores

	School	Post test	Atti- tude	Algebra Readiness			
				A_1	A_2	A_3	A_4
5050	3	1	65	16	06	04	14
5131	3	1	62	10	06	06	20
5251	3	1	72	16	09	05	29
5081	3	1	68	24	10	07	23
5301	3	1	73	20	08	08	16
5041	3	1	50	15	08	08	36
5190	3	1	41	13	08	09	16
5140	3	1	75	19	11	10	28
5240	3	1	57	09	10	10	14
5070	3	1	48	19	08	04	17
5020	3	1	32	15	11	11	30
5030	3	1	83	12	08	12	30
5180	3	1	81	14	10	12	25
5290	3	1	55	22	14	09	31
5280	3	1	56	14	05	08	21
5100	3	1	72	13	13	04	20
5271	3	1	71	05	06	06	19
5121	3	1	70	19	12	12	27
5220	3	1	60	12	05	07	29
5150	3	1	53	17	07	09	26
5200	3	1	75	11	09	12	35
5160	3	1	55	15	05	09	14
5210	3	1	75	23	13	11	31
5260	3	1	85	15	12	15	43
5111	3	1	78	18	09	10	33
5311	3	1	65	13	10	08	30
5011	3	1	67	20	10	12	32
5060	3	1	68	18	06	09	15

School III (Control) performance raw scores

	School	Post test	Atti-tude	Algebra Readiness			
				A_1	A_2	A_3	A_4
6031	3	1	83	15	01	07	11
6210	3	1	87	17	02	09	12
6020	3	1	61	09	01	04	10
6320	3	1	57	12	01	08	15
6080	3	1	71	15	01	14	17
6230	3	1	65	15	06	11	32
6060	3	1	64	20	02	08	19
6120	3	1	40	16	05	09	21
6281	3	1	52	11	05	14	30
6311	3	1	74	12	08	16	36
6200	3	1	76	17	06	09	16
6300	3	1	60	12	03	08	18
6130	3	1	67	13	11	10	25
6220	3	1	74	13	06	07	22
6240	3	1	43	12	03	02	11
6171	3	1	66	16	02	06	24
6291	3	1	55	09	06	10	32
6161	3	1	30	11	05	10	11
6090	3	1	76	09	01	06	10
6181	3	1	50	07	05	11	30
6101	3	1	69	09	04	04	21
6141	3	1	48	13	03	09	16
6041	3	1	54	09	01	07	16
6190	3	1	68	09	00	08	10
6121	3	1	63	08	01	07	24
6151	3	1	35	18	03	12	22

School IV (Experimental) performance raw scores

	School	Post test	Atti-tude	Algebra Readiness			
				A_1	A_2	A_3	A_4
7280	4	1	71	07	02	10	30
7071	4	1	56	06	05	11	22
7061	4	1	46	07	02	08	27
7100	4	1	63	09	01	06	11
7351	4	1	67	09	04	06	21
7320	4	1	88	17	10	13	41
7051	4	1	46	08	05	06	24
7241	4	1	53	05	01	03	06
7130	4	1	62	14	06	12	31
7251	4	1	53	08	04	06	20
7191	4	1	75	23	07	12	20
7161	4	1	73	10	05	06	23
7271	4	1	55	06	03	07	23
7201	4	1	80	10	05	08	22
7210	4	1	45	10	01	06	22
7021	4	1	63	10	05	10	23
7121	4	1	88	16	09	15	27
7011	4	1	59	10	02	05	10
7171	4	1	48	11	06	06	20
7221	4	1	85	23	06	07	24
7031	4	1	82	08	07	06	26
7260	4	1	91	12	04	06	20
7041	4	1	64	14	04	06	27
7231	4	1	60	12	01	06	25
7151	4	1	60	14	04	06	21
7347	4	1	76	07	07	10	30
7140	4	1	57	06	03	03	10
7331	4	1	35	04	06	05	20
7301	4	1	78	17	09	11	37
7311	4	1	56	08	08	11	30

School IV (Control) performance raw scores

	School	Post test	Atti-tude	Algebra Readiness			
				A_1	A_2	A_3	A_4
8070	4	1	67	17	01	05	12
8161	4	1	44	02	01	05	06
8190	4	1	46	09	01	12	16
8010	4	1	89	14	07	09	09
8021	4	1	79	23	11	12	14
8121	4	1	72	21	01	08	12
8221	4	1	60	22	01	14	06
8060	4	1	44	18	02	11	11
8051	4	1	61	18	01	05	16
8251	4	1	53	13	02	09	10
8150	4	1	71	15	01	04	14
8140	4	1	61	22	03	06	09
8270	4	1	53	21	07	11	21
8041	4	1	60	20	00	05	17
8181	4	1	71	08	06	12	22
8201	4	1	52				
8260	4	1	59	15	00	05	11
8131	4	1	52	23	01	04	14
8101	4	1	75	18	05	09	19
8111	4	1	54	10	05	10	14
8081	4	1	52	18	02	11	26
8241	4	1	75	12	02	09	22
8090	4	1	46	22	02	08	21

Appendix E

Means for Table 1C(A, B, C, D, E)

Variables	Sex	Treatment	Test	Means
4	0	0	1	33.72
5	0	0	1	13.73
6	0	0	1	5.03
7	0	0	1	8.27
8	0	0	1	20.61
4	0	0	2	63.07
5	0	0	2	14.75
6	0	0	2	4.27
7	0	0	2	7.39
8	0	0	2	13.04
4	0	1	0	44.62
5	0	1	0	13.12
6	0	1	0	3.22
7	0	1	0	7.66
8	0	1	0	7.66
4	0	1	1	31.51
5	0	1	1	14.19
6	0	1	1	3.19
7	0	1	1	8.28
8	0	1	1	16.91
4	0	1	2	57.73
5	0	1	2	12.05
6	0	1	2	3.25
7	0	1	2	7.04
8	0	1	2	10.32
4	0	2	0	52.17
5	0	2	0	15.35
6	0	2	0	6.08
7	0	2	0	8.01
8	0	2	0	20.04

Variables	Sex	Treatment	Test	Means
4	0	2	1	35.93
5	0	2	1	13.27
6	0	2	1	6.87
7	0	2	1	8.27
8	0	2	1	24.31
4	0	2	2	68.41
5	0	2	2	17.44
6	0	2	2	5.29
7	0	2	2	7.74
8	0	2	2	15.77
4	1	0	0	49.34
5	1	0	0	14.97
6	1	0	0	5.09
7	1	0	0	8.02
8	1	0	0	16.76
4	1	0	1	37.09
5	1	0	1	14.52
6	1	0	1	5.37
7	1	0	1	8.37
8	1	0	1	19.100
4	1	0	2	61.59
5	1	0	2	15.42
6	1	0	2	4.81
7	1	0	2	7.68
8	1	0	2	13.51
4	1	1	0	42.09
5	1	1	0	14.32
6	1	1	0	3.52
7	1	1	0	7.49
8	1	1	0	12.55
4	1	1	1	27.26
5	1	1	1	14.97
6	1	1	1	3.17
7	1	1	1	7.93
8	1	1	1	1.60

Variables	Sex	Treatment	Test	Means
4	1	1	2	56.93
5	1	1	2	13.66
6	1	1	2	3.88
7	1	1	2	7.04
8	1	1	2	9.59
4	1	2	0	56.59
5	1	2	0	15.62
6	1	2	0	6.65
7	1	2	0	8.56
8	1	2	0	20.96
4	1	2	1	46.93
5	1	2	1	14.06
6	1	2	1	7.57
7	1	2	1	8.80
8	1	2	1	24.48
4	1	2	2	66.26
5	1	2	2	17.18
6	1	2	2	5.73
7	1	2	2	8.32
8	1	2	2	17.44
4	1	2	2	48.67
5	2	0	0	13.51
6	2	0	0	4.21
7	2	0	0	7.64
8	2	0	0	16.90
4	2	0	1	30.35
5	2	0	1	12.94
6	2	0	1	4.68
7	2	0	1	8.18
8	2	0	1	21.23
4	2	0	2	64.55
5	2	0	2	14.07
6	2	0	2	3.73
7	2	0	2	7.11
8	2	0	2	12.57

Variables	Sex	Treatment	Test	Means
4	2	1	0	47.14
5	2	1	0	11.93
6	2	1	0	2.91
7	2	1	0	7.83
8	2	1	0	14.69
4	2	1	1	35.76
5	2	1	1	13.41
6	2	1	1	3.20
7	2	1	1	8.62
8	2	1	1	18.32
4	2	1	2	58.53
5	2	1	2	10.45
6	2	1	2	2.62
7	2	1	2	7.05
8	2	1	2	11.05
4	2	2	0	47.76
5	2	2	0	15.08
6	2	2	0	5.51
7	2	2	0	7.45
8	2	2	0	19.12
4	2	2	1	24.94
5	2	2	1	12.48
6	2	2	1	6.17
7	2	2	1	7.74
8	2	2	1	24.14
4	2	2	2	70.56
5	2	2	2	17.69
6	2	2	2	4.85
7	2	2	2	7.17
8	2	2	2	14.10

Averaged over = 0

Sex

Female = 1

Males = 2

Treatment

Not treated = 1

Treated = 2

Test

Pretest = 1

Posttest = 2

Attitude = 4

Fund. Opr = 5

Fractions = 6

Dec & Per = 7

Prob Solv = 8

Appendix F

Means for Tables 2A, 2B, 2C, 2D, and 2E

Variables	Sex	Test	Means
3	0	1	62.56
4	0	1	10.94
5	0	1	3.30
6	0	1	5.77
7	0	1	6.37
3	0	2	69.05
4	0	2	16.94
5	0	2	5.07
6	0	2	7.37
7	0	2	15.09
3	1	0	64.20
4	1	0	14.64
5	1	0	4.51
6	1	0	7.36
7	1	0	11.57
3	1	1	61.51
4	1	1	12.61
5	1	1	3.51
6	1	1	6.79
7	1	1	6.38
3	1	2	66.90
4	1	2	16.68
5	1	2	5.51
6	1	2	7.94
7	1	2	16.76
3	2	0	67.41
4	2	0	13.24
5	2	0	3.86
6	2	0	5.77
7	2	0	9.89

Variables	Sex	Test	Means
3	2	1	63.61
4	2	1	9.28
5	2	1	3.10
6	2	1	4.75
7	2	1	6.36
3	2	2	71.20
4	2	2	17.19
5	2	2	4.62
6	2	2	6.79
7	2	2	13.42

Variables:
Average over = 0
Females = 1
Males = 2
Attitude = 3
Fundamental Operations = 4
Fractions = 5
Decimals and percents = 6
Problem Solving = 7

Appendix G

Means for Tables 3A, 3B, 3C, 3D, and 3E

Variables	Sex	Treatment	Test	Means
4	0	0	1	60.11
5	0	0	1	9.38
6	0	0	1	3.37
7	0	0	1	5.95
8	0	0	1	7.36
4	0	0	2	64.07
5	0	0	2	14.67
6	0	0	2	4.16
7	0	0	2	7.22
8	0	0	2	12.74
4	0	1	0	58.01
5	0	1	0	9.69
6	0	1	0	3.22
7	0	1	0	6.40
8	0	1	0	9.00
4	0	1	1	57.29
5	0	1	1	7.40
6	0	1	1	3.31
7	0	1	1	5.92
8	0	1	1	7.99
4	0	1	2	58.73
5	0	1	2	11.98
6	0	1	2	3.14
7	0	1	2	6.88
8	0	1	2	10.02
4	0	2	0	66.17
5	0	2	0	14.36
6	0	2	0	4.30
7	0	2	0	6.78
8	0	2	0	11.10

Variables	Sex	Treatment	Test	Means
4	0	2	1	62.92
5	0	2	1	11.36
6	0	2	1	3.42
7	0	2	1	5.98
8	0	2	1	6.73
4	0	2	2	69.41
5	0	2	2	17.36
6	0	2	2	5.18
7	0	2	2	7.58
8	0	2	2	15.46
4	1	0	0	60.31
5	1	0	0	12.95
6	1	0	0	4.20
7	1	0	0	7.04
8	1	0	0	10.38
4	1	0	1	58.04
5	1	0	1	10.55
6	1	0	1	3.70
7	1	0	1	6.57
8	1	0	1	7.54
4	1	0	2	62.59
5	1	0	2	15.34
6	1	0	2	4.60
7	1	0	2	7.51
8	1	0	2	13.21
4	1	1	0	56.06
5	1	1	0	10.83
6	1	1	0	3.76
7	1	1	0	6.51
8	1	1	0	8.81
4	1	1	1	54.20
5	1	1	1	8.08
6	1	1	1	3.77
7	1	1	1	6.15
8	1	1	1	8.34

Variable	Sex	Treatment	Time	Means
4	1	1	2	57.92
5	1	1	2	13.58
6	1	1	2	3.77
7	1	1	2	6.87
8	1	1	2	9.29
4	1	2	0	64.57
5	1	2	0	15.06
6	1	2	0	4.62
7	1	2	0	7.57
8	1	2	0	11.94
4	1	2	1	61.87
5	1	2	1	13.03
6	1	2	1	3.62
7	1	2	1	6.99
8	1	2	1	6.74
4	1	2	2	67.26
5	1	2	2	17.10
6	1	2	2	5.62
7	1	2	2	8.15
8	1	2	2	17.13
4	2	0	0	63.86
5	2	0	0	11.10
6	2	0	0	3.33
7	2	0	0	6.13
8	2	0	0	9.72
4	2	0	1	62.17
5	2	0	1	8.21
6	2	0	1	3.04
7	2	0	1	5.32
8	2	0	1	7.18
4	2	0	2	65.55
5	2	0	2	13.100
6	2	0	2	3.62
7	2	0	2	6.94
8	2	0	2	12.27

Variables	Sex	Treatment	Test	Means
4	2	1	0	59.95
5	2	1	0	8.55
6	2	1	0	2.68
7	2	1	0	6.28
8	2	1	0	9.19
4	2	1	1	60.37
5	2	1	1	6.72
6	2	1	1	2.86
7	2	1	1	5.69
8	2	1	1	7.63
4	2	1	2	59.53
5	2	1	2	10.37
6	2	1	2	2.51
7	2	1	2	6.88
8	2	1	2	10.74
4	2	2	0	67.77
5	2	2	0	13.66
6	2	2	0	3.98
7	2	2	0	5.98
8	2	2	0	10.26
4	2	2	1	63.98
5	2	2	1	9.70
6	2	2	1	3.21
7	2	2	1	4.96
8	2	2	1	6.72
4	2	2	2	71.56
5	2	2	2	17.61
6	2	2	2	4.74
7	2	2	2	7.00
8	2	2	2	13.79

Variables:
Averaged over = 0
Females = 1
Males = 2
Treated = 2
Not treated = 1
Pretested = 1
Posttest = 2
Attitude = 4
Fundamental operations = 5
Fractions = 6
Decimals and percents = 7
Problem solving = 8

Appendix H

Means for Table 4 (A, B, C, D, E)

Variable	Sex	Treatment	Mean
3	0	1	30.84
4	0	1	13.98
5	0	1	3.25
6	0	1	8.32
7	0	1	17.05
3	0	2	35.26
4	0	2	13.05
5	0	2	6.94
6	0	2	8.31
7	0	2	24.45
3	1	0	36.42
4	1	0	14.30
5	1	0	5.44
6	1	0	8.41
7	1	0	20.13
3	1	1	26.59
4	1	1	14.76
5	1	1	3.24
6	1	1	7.97
7	1	1	15.64
3	1	2	46.25
4	1	2	13.84
5	1	2	7.63
6	1	2	8.85
7	1	2	24.62
3	2	0	29.67
4	2	0	12.73
5	2	0	4.75
6	2	0	8.22
7	2	0	21.36
3	2	1	35.09
4	2	1	13.20
5	2	1	3.26
6	2	1	8.67
7	2	1	18.45

Variable	Sex	Treatment	Mean
3	2	2	24.26
4	2	2	12.26
5	2	2	6.24
6	2	2	7.78
7	2	2	24.28

Variables

zero = averaged over
Treated = 2 not treated = 1
Female = 1 Male = 2
Attitude = 3
Fundamental operations = 4
Fractions = 5
Decimals and percents = 6
Problem Solving = 7

Appendix I

Means for Table 5 (A, B, C, D, E)

Variable	Sex	Treatment	Mean
3	0	1	38.96
4	0	1	9.17
5	0	1	3.31
6	0	1	6.86
7	0	1	11.73
3	0	2	64.52
4	0	2	12.44
5	0	2	5.41
6	0	2	6.83
7	0	2	13.25
3	1	0	48.94
4	1	0	12.26
5	1	0	5.11
6	1	0	7.53
7	1	0	14.01
3	1	1	34.13
4	1	1	10.29
5	1	1	3.56
6	1	1	6.91
7	1	1	11.44
3	1	2	63.75
4	1	2	14.24
5	1	2	6.65
6	1	2	8.14
7	1	2	16.59
3	2	0	54.54
4	2	0	9.36
5	2	0	3.62
6	2	0	6.17
7	2	0	10.97
3	2	1	43.79
4	2	1	8.06
5	2	1	3.06
6	2	1	6.81
7	2	1	12.03

Variable	Sex	Treatment	Mean
3	2	2	65.28
4	2	2	10.65
5	2	2	4.17
6	2	2	5.52
7	2	2	9.90

Averaged over = 0
Female = 1
Male = 2
Not treated = 1
Treated = 2
Attitude = 3
Fundamental Operations = 4
Fractions = 5
Decimals and percents = 6
Problem Solving = 7

VITA

Leo Edwards, Jr.

Candidate for the Degree of

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