RADIOACTIVITY TRANSPORT IN WATER--
MATHEMATICAL SIMULATION

Technical Report--19

to the

U. S. Atomic Energy Commission
Contract AT-(11-1)-490

by

Allen White, Research Engineer

Earnest F. Gloyna, Project Director

CENTER FOR RESEARCH IN WATER RESOURCES
Environmental Health Engineering
Civil Engineering Department
The University of Texas at Austin

EHE-70-04
CRWR-52
ORO-19

ACKNOWLEDGMENTS

Financial support of the U. S. Atomic Energy Commission contributed to this study. The interest and direction of Mr. Walter Belter and Mr. Craig Roberts, Division of Reactor Development, U. S. Atomic Energy Commission are gratefully appreciated.

Acknowledgments are extended to Drs. F. D. Masch and Y. A. Yousef, from The University of Texas at Austin, for their participation.

SUMMARY

A mathematical model has been developed for routing a radionuclide through a model river system. The capabilities of this simulation model include the following characteristics:

1. It distributes radionuclide activity by advective and dispersional mechanisms along the longitudinal axis of the system.
2. It may be discretized into any number of segments (vertical planes normal to the longitudinal axis) as may be appropriate to describe spatial variations in radionuclide activity.
3. It is capable of treating instantaneous, continuous, or time-varying releases of radionuclide activity.
4. It provides for a temporal description of radionuclide activity throughout the system.
5. It provides for sorption and desorption by both plants and bottom sediments.

TABLE OF CONTENTS

LIST OF FIGURES

INTRODUCTION

There is increasing concern about the movement of radioactive materials in rivers, lakes and oceans since these materials can be transported and may be concentrated by sediments and biota. With the continued growth of the nuclear power industry, it is essential to estimate the movement of waste products. These wastes or radionuclides may enter a stream either as soluble or relatively insoluble materials and can be transported downstream during which time either dilution or possible concentration occurs.

Once in the stream, complex physical and chemical reactions occur. The radionuclides may undergo changes in state, remain in solution, or interact with various components of the aquatic system. The more important factors that influence the concentration of radionuclides are:

(1) the sorption of each radionuclide by bed sediments;

(2) the sorption of each radionuclide by aquatic plants;

(3) the desorption of radionuclides into flowing water;

(4) the migration of radionuclides through the bed sediments;

(5) the movement of sediment and attached radionuclides through the normal movement of suspended sediment or bed sediments;

(6) the period of contact and retention in the system, i.e., transport time associated with advection and dispersion.

A more comprehensive description of the factors that influence the transport of radionuclides is given by Gloyna and Ledbetter (1969).

Transport time is a function of many factors. For a given amount of radionuclides, the mode of release (constant, instantaneous, or varying), the receiving water (lake, river, etc.), and the hydraulics of the receiving water ultimately determine the dilution or reconcentration and exposure time to the ecosystem. Thus, considerable attention must be given to hydraulic

dispersion and convection in any study dealing with the transport of radioactive materials through an aquatic system.

The physical and chemical changes that occur in typical stream environments have been measured, and while considerable attention has been directed to finding solutions to the phenomena of hydraulic dispersion and convection, bottom sediment transport, sorption and desorption by the aquatic ecosystem, etc., the development of a general transport equation including all of these factors has yet to be undertaken.

OBJECTIVE AND SCOPE

Recognizing the need for a model to include interaction between the numerous factors mentioned above, the primary objective of this study was the development of a general mathematical model, in a form suitable for simulation on a digital computer, for routing a radionuclide through a model river (flume). As a necessary part of the development of this model it was also kept in mind that the model would eventually be applied to an actual river system; however, this study was restricted to flume data. To develop this model, several general requirements must be fulfilled. These are essential to the mechanics employed in the model and are necessary for describing the overall structure of the model and for assuring the practicality and utility of the model. This model must have the following basic capabilities:

(1) to treat advective and dispersional transfers of radionuclides along the longitudinal axis of a one-dimensional channel;

(2) to allow the channel to be discretized into elements of uniform length and cross-section (vertical planes normal to the longitudinal axis;

(3) to be capable of receiving instantaneous, continuous, or varying releases of radionuclides;

(4) to provide for a temporal description of the radionuclide activity throughout the system; and

(5) to be capable of treating the following as sources and/or sinks -

   (a) sorption and desorption by bottom sediments, and

   (b) sorption and desorption by aquatic plants.

PROCEDURAL DETAILS

Verification of the mathematical model was conducted in three phases. The first phase involved verifying the dispersion characteristics of the flume using dye-release data taken by Armstrong and Gloyna (1968). The second phase incorporated sorption and desorption of radionuclides by bottom sediments into the model. The data used in this phase is described by Shih and Gloyna (1967). The final phase was concerned with the inclusion of sorption and desorption by aquatic plants into the model, using data given by Armstrong and Gloyna (1968). Both of the above used ^{85}Sr as the radionuclide. All of the above data are described in Appendix A.

FORMULATION OF THE PROBLEM

Transport

The development of transport theory is, at this time, fairly well understood and is adequately documented in the literature (Fischer, 1966; Thackston and Krenkel, 1966).

Radionuclides, in tracer amounts, are transported in a moving fluid stream by turbulent and molecular diffusion and by differential transport. The combined effects of turbulent diffusion and differential transport are called dispersion. Molecular diffusion is usually considered negligible in relation to dispersion.

The basic transport equation can be written for a three dimensional system as:

$$\underbrace{\frac{\partial C}{\partial t}}_{(I)} = \underbrace{\frac{\partial}{\partial x}\left(D_x \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y \frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_z \frac{\partial C}{\partial z}\right)}_{(II)} \underbrace{- u\frac{\partial C}{\partial x} - v\frac{\partial C}{\partial y} - w\frac{\partial C}{\partial z}}_{(III)} \qquad (1)$$

where

C = concentration at any point (x, y, z)

D_x, D_y, and D_z = coefficients of dispersion in the x, y, and z directions respectively

t = time

u, v, and w = fluid velocities in each of the coordinate directions.

In Eq. (1), term (I) represents the temporal change in concentration, term (II) represents the transport by dispersion, and term (III) represents the mass transport by convection in the longitudinal, lateral, and vertical directions, respectively. The above equation in its complete form is almost impossible to solve, even with today's high-speed computers; however, by simplifying Eq. (1) in situations where some of the terms can be neglected, solutions can be obtained.

Taylor (1954) found that for a one-dimensional system, concentration curves could be represented by a simplified form of Eq. (1):

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} - \bar{u} \frac{\partial C}{\partial x} \tag{2}$$

where

D_L = longitudinal dispersion coefficient,
$\bar{u}$ = Q/A,
Q = flow rate, and
A = cross-sectional area.

Thus, for a one-dimensional channel with a uniform cross-section and mean velocity, an exact solution to Eq. (2) for a point source and a fixed coordinate system is:

$$C(x,t) = \frac{M}{A\sqrt{4\pi D_L t}} e^{-\frac{(x-\bar{u}t)^2}{4 D_L t}} \tag{3}$$

where

M = mass of the constituent introduced, and
x = distance downstream from the point of release.

Shih and Gloyna (1967) developed a transport equation for a one-dimensional channel to predict dispersion as well as uptake and release from the sorption phases. In the general case, it has the form

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} - \bar{u} \frac{\partial C}{\partial x} + \sum_{j=1}^{m} S_j \tag{4}$$

where

j = the index number for each sorption phase, and
S_j = uptake or release from the jth of m sorption phases.

Solutions to Eq. (4) are obtainable; however, difficulties arise in describing the dispersion coefficient and the sorption terms for the radionuclides. Therefore, considerable thought must be given to selecting dispersion coefficients and defining the various sources and sinks that are involved.

Longitudinal Mixing

Taylor (1954) was able to derive a predictive equation for the dispersion coefficient, D_L, in long straight pipes, as

$$D_L = 10.1\, r_o\, u^* \tag{5}$$

where

r_o = pipe radius,
u^* = $\sqrt{\tau_o/\rho}$ = average shear velocity,
τ_o = boundary stress, and
ρ = fluid density.

Some investigators have attempted to apply Taylor's expression to stream flow. Such applications are approximate, since there is no relation between the geometry or velocity distributions in stream flow and those occurring in a pipe.

Elder (1959) assumed that only the vertical velocity gradient is important in stream flow, and developed an expression analogous to Taylor's but with a coefficient equal to 5.93,

$$D_L = 5.93\, H\, u^* \tag{6}$$

where

H = depth of water.

Other investigators derived similar expressions for D_L and found that it is extremely sensitive to lateral velocity profiles. However, Elder's expression seems to be quite adequate in one-dimensional situations where the channel is not too wide. For very wide channels, Fischer (1964) has shown that half-width, rather than depth, is the dominant scale and therefore is

important in defining the longitudinal dispersion coefficient. Such relations for the dispersion coefficient usually can be written in terms of the Manning Equation and other variables characteristic of stream channels.

Shih and Gloyna (1967), based on the results of their dye studies, were able to develop a relationship between the dispersion coefficient, D_L, and channel velocity, $\bar{u}$. It was of the form:

$$D_L = 29\ (\bar{u})^{0.9} \tag{7}$$

where D_L had units of 10^{-2} ft^2/sec and $\bar{u}$ had units of 10^{-3} ft/sec. Armstrong and Gloyna (1968) found that using this equation for the dispersion coefficient in their model overpredicted the cloud dispersion and underestimated its velocity. This same observation was verified in the course of this study.

Armstrong and Gloyna (1968) also used observed concentration curves from their study to obtain dispersion coefficients for the model river. These data are shown in Fig. 1. All the dispersion coefficients, D_L, obtained from the flume studies under various flow conditions, were fitted to a generalized equation in the form:

$$D_L = 3.26\ (\bar{u})^{0.607} \tag{8}$$

where D_L had units of ft^2/min and $\bar{u}$ had units of ft/min. This equation gives dispersion coefficients which seem to work very well for the model river. However, one should not try to extrapolate this equation to other systems as it holds true for the flume only and even then only under a small range of conditions.

Sources and Sinks

In general, the sorption terms for plants and bottom sediments have the following form:

$$S_j = K_j [W_j - g_j (C)] \tag{9}$$

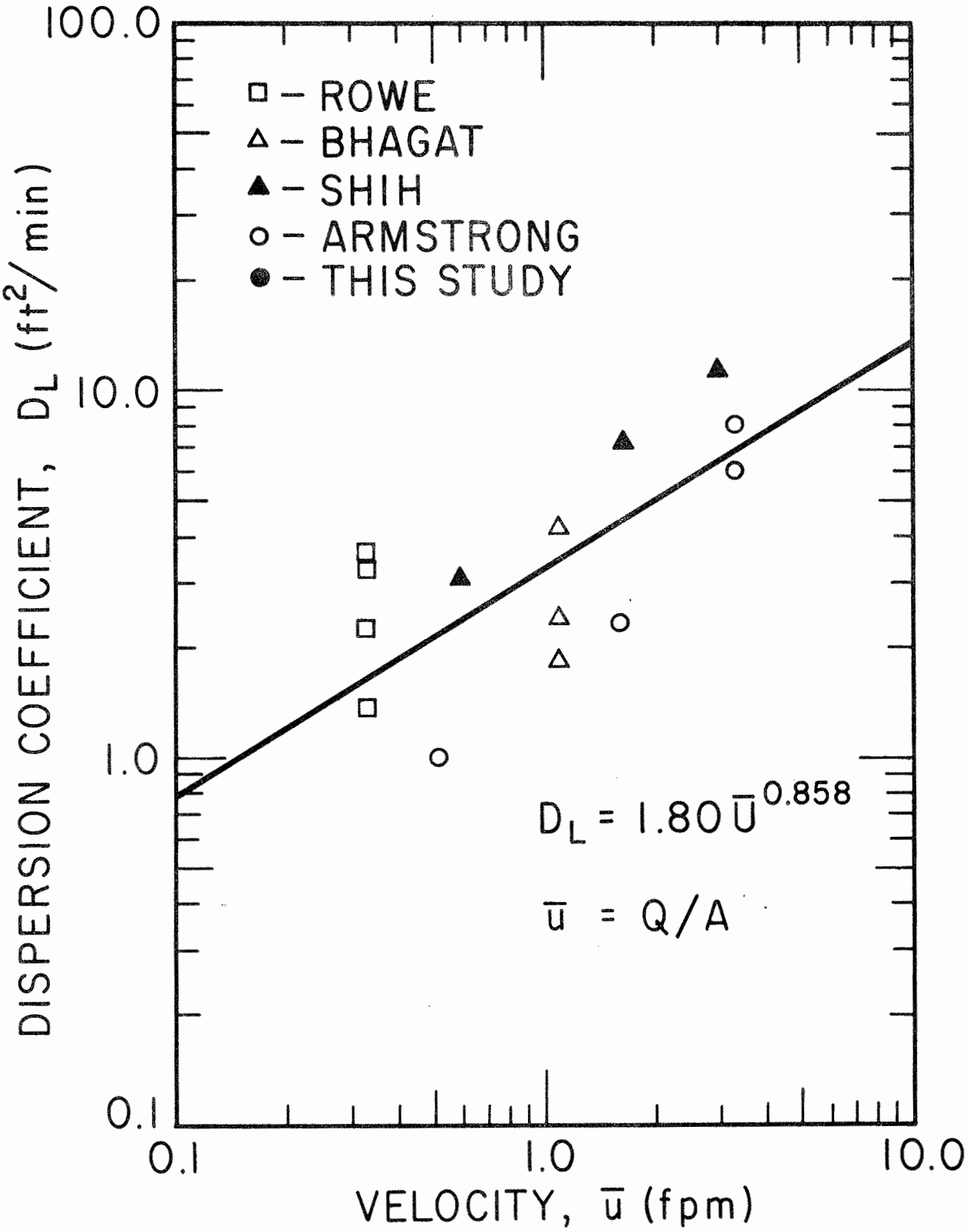

FIG. 1. LONGITUDINAL DISPERSION COEFFICIENT VS. Q/M VELOCITY

where

K_j = the mass transfer coefficient for phase j,

$g_j(C)$ = the transfer function relating the concentration of activity in water to the equilibrium level in phase j, and

W_j = the specific activity in the jth of m sorption phases.

The activity in W_j is determined by:

$$\frac{\partial W_j}{\partial t} = K_j [g_j(C) - W_j] \tag{10}$$

Usually the equilibrium concentration of radionuclide in a sorbent, g(C), has the Freundlich isotherm form,

$$g(C) = KC^n \tag{11}$$

where K and n are constants pertaining to a given radionuclide and sorption phase. For uptake of most radionuclides n = 1.0 and K becomes Kd, the equilibrium distribution coefficient. Thus, the non-equilibrium radionuclide sorption reaction within a given material is derived as,

$$\frac{\partial W_j}{\partial t} = K_j [Kd_j C - W_j] \tag{12}$$

and it follows that,

$$S_j = K_j [W_j - Kd_j C] \tag{13}$$

MODEL DEVELOPMENT

Equation Development

Since the dispersion equation is written as a function of time, T, and distance, L, let i and n be increments of the variables x and t where $x = L/I$, I being an integer, and $t = T/N$, N being an integer. The set of points in the x-t plane forms a net and is given by $x = i\Delta x$ and $t = n\Delta t$, where $i = 1, 2, \ldots, I$ and $n = 0, 1, 2, \ldots, N$. The concentration, C, at each mesh point is approximated by $C(i\Delta x, n\Delta t)$ which is denoted by C_i^n. Using the implicit method in which derivatives are approximated by expanding C_i^n in a Taylor series and averaging over the time step, Δt, the dispersion equation may be written as:

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} = \frac{D_L}{2}\left\{\frac{\left(C_{i+1}^{n+1} - 2C_i^{n+1} + C_{i-1}^{n+1}\right)}{\Delta x^2} + \frac{\left(C_{i+1}^{n} - 2C_i^{n} + C_{i-1}^{n}\right)}{\Delta x^2}\right\}$$

$$-\frac{\bar{u}}{2}\left\{\frac{\left(C_{i+1}^{n+1} - C_{i-1}^{n+1}\right)}{2\Delta x} + \frac{\left(C_{i+1}^{n} - C_{i-1}^{n}\right)}{2\Delta x}\right\} \qquad (14)$$

where

$i = 1,2, \ldots, I$, and

$n = 0, 1, 2, \ldots, N$.

The rigor for this method has been established by Smith (1965). The boundary conditions for instantaneous release are

$C_i^o = 0;\ i = 1, 2, \ldots, I$

$C_i^n = 0$ as $i \to \infty$

$C_i^n =$ finite value as $n \to \infty$

and the initial condition is:

$$C_o^{t+t_o} = \frac{M}{A\sqrt{4\pi D_L(t+t_o)}}\, e^{-\frac{[\bar{u}(t+t_o)]^2}{4\pi D_L(t+t_o)}} \tag{15}$$

where

t_o = a time slightly greater than zero for which a finite solution for Eq. (15) can be obtained.

For a continuous release, the boundary condition is:

$$C_o^n = \frac{M}{QT} \tag{16}$$

for all n where T is the time span over which the continuous release occurs.

Eq. (14) may also be expressed as:

$$a C_{i-1}^{n+1} + b C_i^{n+1} + c C_{i+1}^{n+1} = \frac{D_L \Delta t}{2\Delta x^2}\left(C_{i+1}^n - 2C_i^n + C_{i-1}^n\right) - \frac{\bar{u}\Delta t}{4\Delta x}\left(C_{i+1}^n - C_{i-1}^n\right) + C_i^n \tag{17}$$

where

$$a = -\frac{D_L \Delta t}{2\Delta x^2} - \frac{\bar{u}\Delta t}{4\Delta x}, \tag{18}$$

$$b = 1 + \frac{D_L \Delta t}{\Delta x^2}, \text{ and} \tag{19}$$

$$c = -\frac{D_L \Delta t}{2\Delta x^2} + \frac{\bar{u}\Delta t}{4\Delta x}. \tag{20}$$

For the transport equation with sorption terms, Eq. (17) becomes:

$$a C_{i-1}^{n+1} + b C_i^{n+1} + c C_{i+1}^{n+1} = \frac{D_L \Delta t}{2 \Delta x^2} \left(C_{i+1}^n - 2 C_i^n + C_{i-1}^n \right)$$

$$- \frac{\bar{u} \Delta t}{4 \Delta x} \left(C_{i+1}^n - C_{i-1}^n \right) + \Delta t \sum_{j=1}^{m} K_j \left[W_{j,i}^n - Kd_j C_i^n \right] \tag{21}$$

Bottom Sediments - For bottom sediments Shih and Gloyna (1967) used Eq. (12) in the following form:

$$\frac{\partial W_{1,i}}{\partial t} = K_1 \left[Kd_1 C_i - W_{1,i} \right] \tag{22}$$

and Eq.(13) as:

$$S_{1,i} = \frac{K_1}{H A_s} \left[W_{1,i} - Kd_1 C_i \right] \tag{23}$$

where

A_s = cross-sectional area of a sediment sample, and
H = effective depth of sediments.

In finite-difference form Eq. (22) and Eq. (23) become:

$$W_{1,i}^n = W_{1,i}^{n-1} + \Delta t K_1 \left[Kd_1 C_i^{n-1} - W_{1,i}^{n-1} \right] \tag{24}$$

and

$$S_{1,i} = \frac{K_1}{H A_s} \left[W_{1,i}^n - Kd_1 C_i^n \right] . \tag{25}$$

Plants - For plants Armstrong and Gloyna (1968) in a similar fashion used Eq. (12) and Eq. (13) in finite difference form as:

$$W_{2,i}^n = W_{2,i}^{n-1} + \Delta t K_2 \left[Kd_2 C_i^{n-1} - W_{2,i}^{n-1} \right] \tag{26}$$

and

$$S^{n}_{2,i} = Mb\, K_2\left[W^{n}_{2,i} - Kd_2\, C^{n}_{i}\right] \tag{27}$$

where

Mb = biomass of plants.

With the sorption terms for bottom sediments and plants included, Eq. (21) becomes:

$$a\, C^{n+1}_{i-1} + b\, C^{n+1}_{i} + c\, C^{n+1}_{i+1} = \frac{D_L\, \Delta t}{2\, \Delta x^2}\left(C^{n}_{i+1} - 2\, C^{n}_{i} + C^{n}_{i-1}\right)$$

$$- \frac{\bar{u}\, \Delta t}{4\, \Delta x}\left(C^{n}_{i+1} - C^{n}_{i-1}\right) + \frac{K_1\, \Delta t}{H A_s}\left(W^{n}_{1,i} - Kd_1\, C^{n}_{i}\right)$$

$$+ Mb\, \Delta t\, K_2\left(W^{n}_{2,i} - Kd_2\, C^{n}_{i}\right) \tag{28}$$

where

i = 1,2, ..., I and

n = 0, 1, ..., N.

Solution Technique

If Eq. (28) is rewritten at some point in time, n, for all values of i in matrix notation it becomes $A\,C = d$ or

$$\begin{bmatrix} b_1 & c_1 & & & & \\ a_2 & b_2 & c_2 & & & \\ & a_3 & b_3 & c_3 & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & C_{I-1} & \\ & & & a_I & b_I & \end{bmatrix} \begin{bmatrix} C^{n+1}_1 \\ C^{n+1}_2 \\ C^{n+1}_3 \\ \cdot \\ \cdot \\ \cdot \\ C^{n+1}_{I-1} \\ C^{n+1}_I \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \cdot \\ \cdot \\ \cdot \\ d_{I-1} \\ d_I \end{bmatrix} \tag{29}$$

where:

A = represents the coefficient matrix,

C = represents the column matrix of unknowns, and

d = represents the column matrix of known quantities in Eq. (28).

The matrix [A] is tridiagonal since any row excluding the top and bottom rows contains only three terms, the diagonal and two immediately adjoining off-diagonal terms. The bottom and top rows which express the boundary conditions contain the diagonal and one offdiagonal term.

Eq. (29) may be solved by a number of different methods. The solution methods differ considerably in computational effort and hence in computer cost. For the problem at hand a unique and highly efficient solution method described by Ralston and Wilf (1967) was used.

Briefly, this algorithm takes advantage of the fact that the matrix [A] is tridiagonal, hence the algorithm is very simple and fast. The algorithm is recursive, allowing the following sequence of steps:

(1) Set $f_1 = c_1/b_1$ and $g_1 = d_1/b_1$

(2) Then operate from row 2 to the last row, I, using the recursive relations below for i = 2, I. Compute

$$f_i = \frac{c_i}{b_i - a_i f_{i-1}}$$

$$g_i = \frac{d_i - a_i g_{i-1}}{b_i - a_i f_{i-1}}$$

Store these quantities since

$$C_i^{n+1} = g_i - f_i \, C_{i+1}^{n+1} \tag{30}$$

The last row, I, is such that

$$C_I^{n+1} = g_i$$

(3) With C_I^{n+1} computed, operate with Eq. (30) from row I-1 to row 1, i.e., back substitution.

The back substitution yields the set of concentrations, C_i^{n+1}, to advance the solution over the time step Δt.

DISCUSSION

In compliance with the objective of this study, a mathematical model has been developed in a form suitable for simulation on a digital computer for routing a radionuclide through a model river system (flume).

Model Capability

The model was designed so that it has the following capabilities and characteristics:

(1) It distributes radionuclide activity by advective and dispersional mechanisms along the longitudinal axis of the system.

(2) It may be discretized into any number of segments (vertical planes normal to the longitudinal axis) as may be appropriate to describe spatial variations in radionuclide activity.

(3) It is capable of treating instantaneous, continuous, or time-varying releases of radionuclide activity.

(4) It provides for a temporal description of radionuclide activity throughout the system.

(5) It provides for sorption and desorption by both plants and bottom sediments.

Verification, Application, and Results

The results obtained from the dye-release data, Appendix A, are illustrated in Figs. 2a and 2b. The shapes of the dye curves at the four stations on the east side of the flume, Fig. 2a, were well predicted except for the peaks. From the shape and maximum values of the observed data, it is apparent, as Fischer (1966b) has pointed out, that initial dispersion of a point source does not follow Taylor's Eq. (2), but rather takes the shape of a skewed normal distribution because of the dominance of convection over diffusion. However, at some point downstream, the curves approach a form where the Taylor expression is valid. The point is made, however, that dispersion was well predicted and velocity was only slightly overestimated. The

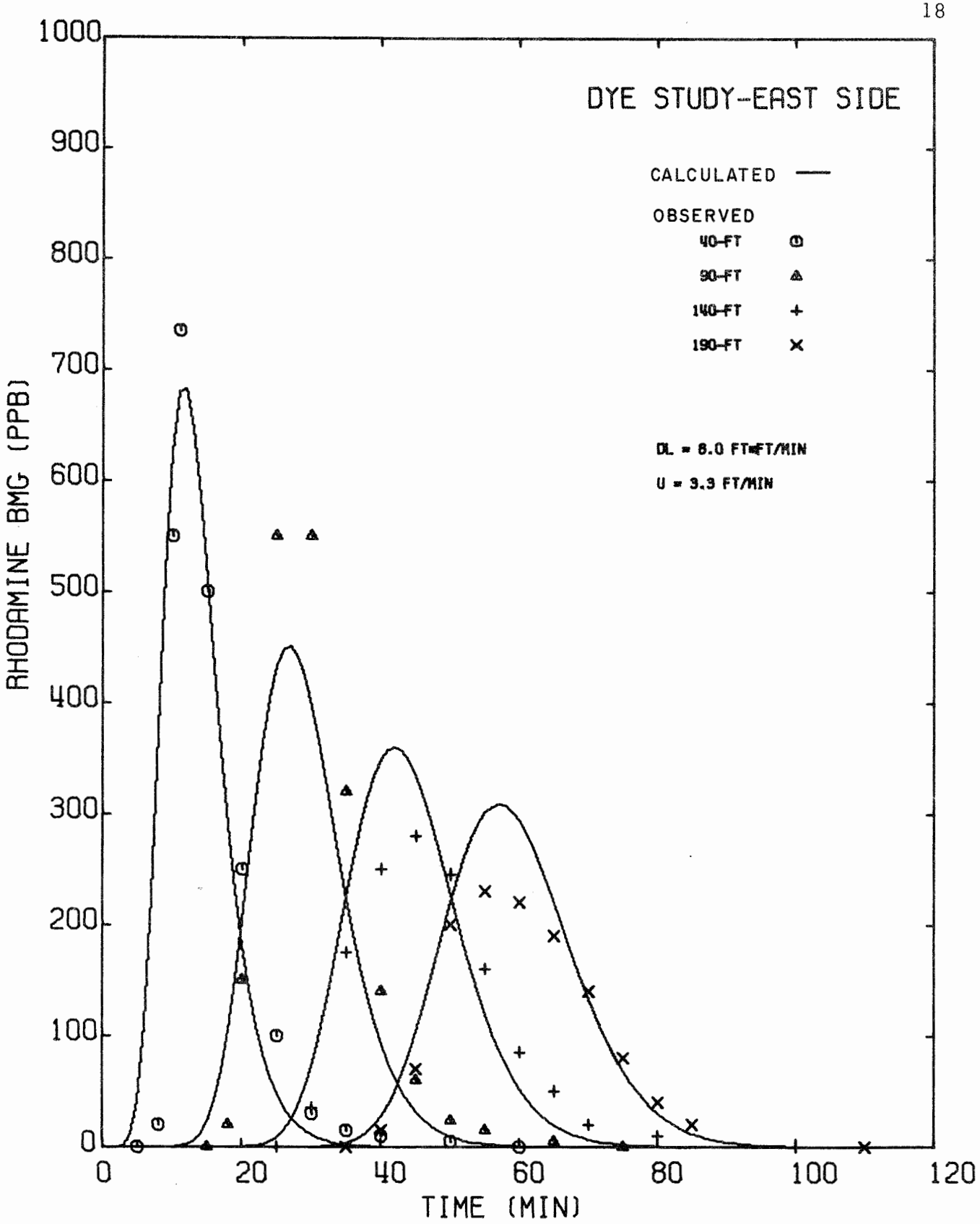

FIG. 2a. OBSERVED AND PREDICTED DYE STUDY CURVES

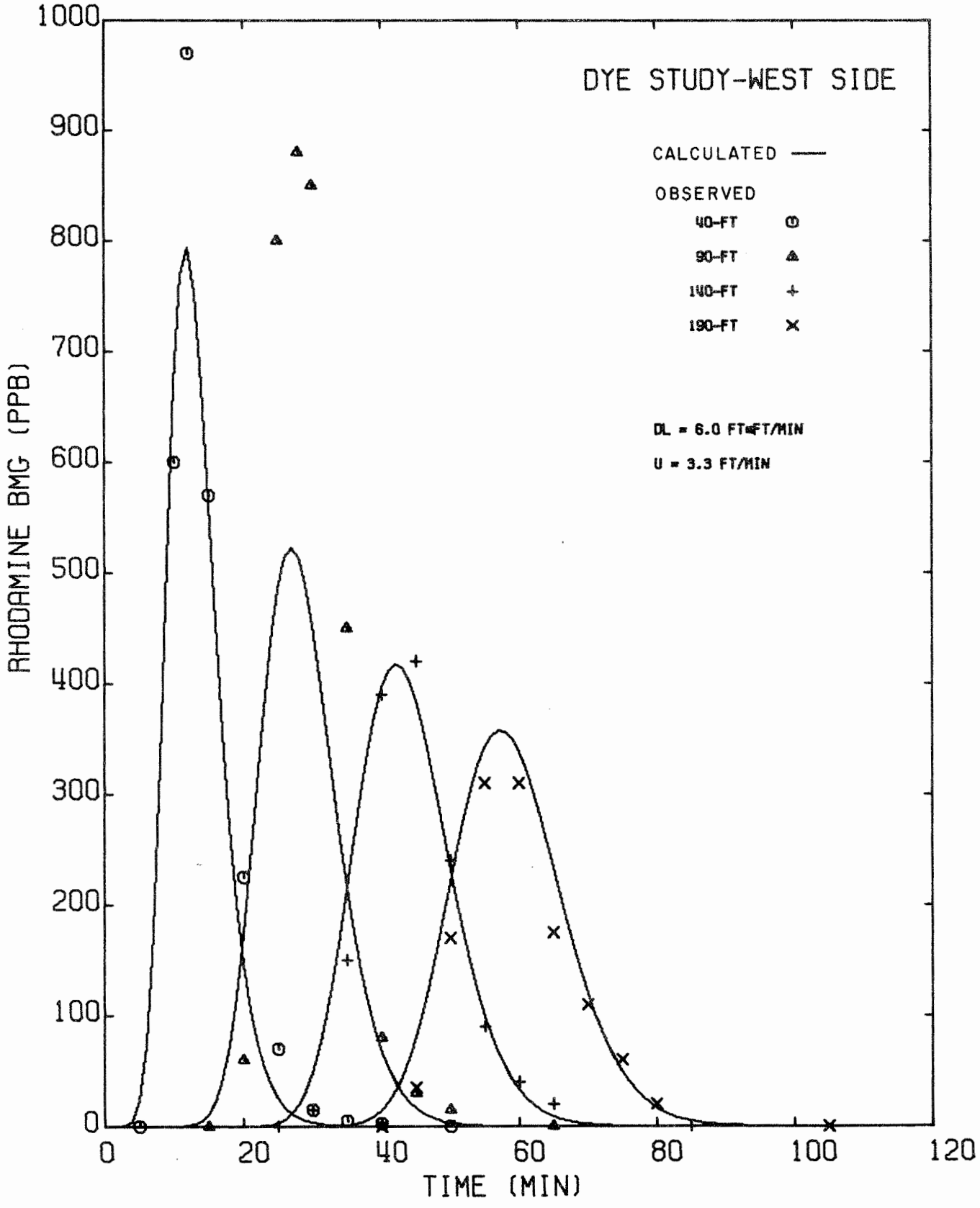

FIG. 2b. OBSERVED AND PREDICTED DYE STUDY CURVES

results of the west side, Fig. 2b, again show that dispersion and velocity were fairly well predicted except for the 90-ft. station.

The results of this part of the study verified the observation of Armstrong and Gloyna (1968). When they applied Eq. (7) used by Shih and Gloyna (1967) their predicted curve overestimated the cloud dispersion and underestimated the velocity; however, when they used dispersion coefficients determined from their own studies, very good agreement between observed and predicted curves was obtained.

The results obtained with a sorption term for bottom sediments added to the dispersion equation were predicted using the factors in Appendix A and are shown in Fig. 3. The fit to the observed data was good, although, as in the dye studies, velocity was slightly overestimated. Again, when Eq. (7) was used to determine the dispersion coefficients, the predicted curves overestimated the cloud dispersion; however, when Eq. (8), developed in this study, was used, good agreement was obtained under test conditions.

The results obtained with a sorption term for plants added to the dispersion equation are shown in Fig. 4. The agreement between the predicted and observed curves was extremely good in this instance.

Continuous releases of radionuclides may also be modeled with this numerical solution. Using data taken by Armstrong and Gloyna (1968), an equilibrium level of 53 dpm/ml was inserted into the model and the curve illustrated in Fig. 5 was obtained. The transport equation with sorption terms for both bottom sediments and plants was used in the continuous release.

Application to a Real River

In light of the results obtained in this study, the utility of this model has been sufficiently demonstrated; however, it should be recognized that the model does have its limitations. Thus it seems appropriate at this point to consider how the model might be adapted to a real river system.

Model Resolution and Sensitivity - The mode of conceptually representing a natural conveyance channel will depend to a considerable degree on the detail required in the space and time description of the radionuclide activity. If flows are steady and the geometric properties rather

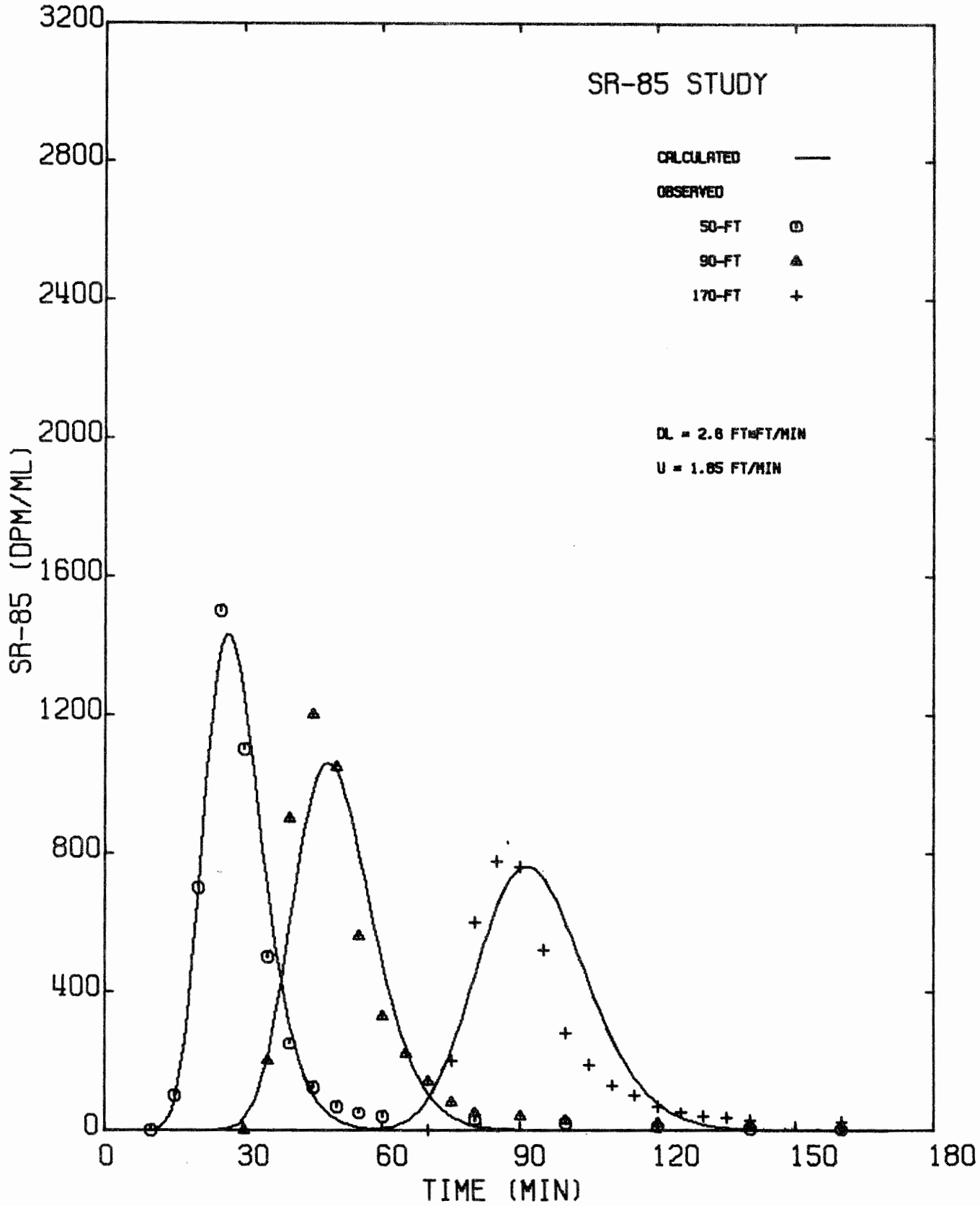

FIG.3. INSTANTANEOUS RELEASE CURVES FOR WATER AND SEDIMENTS

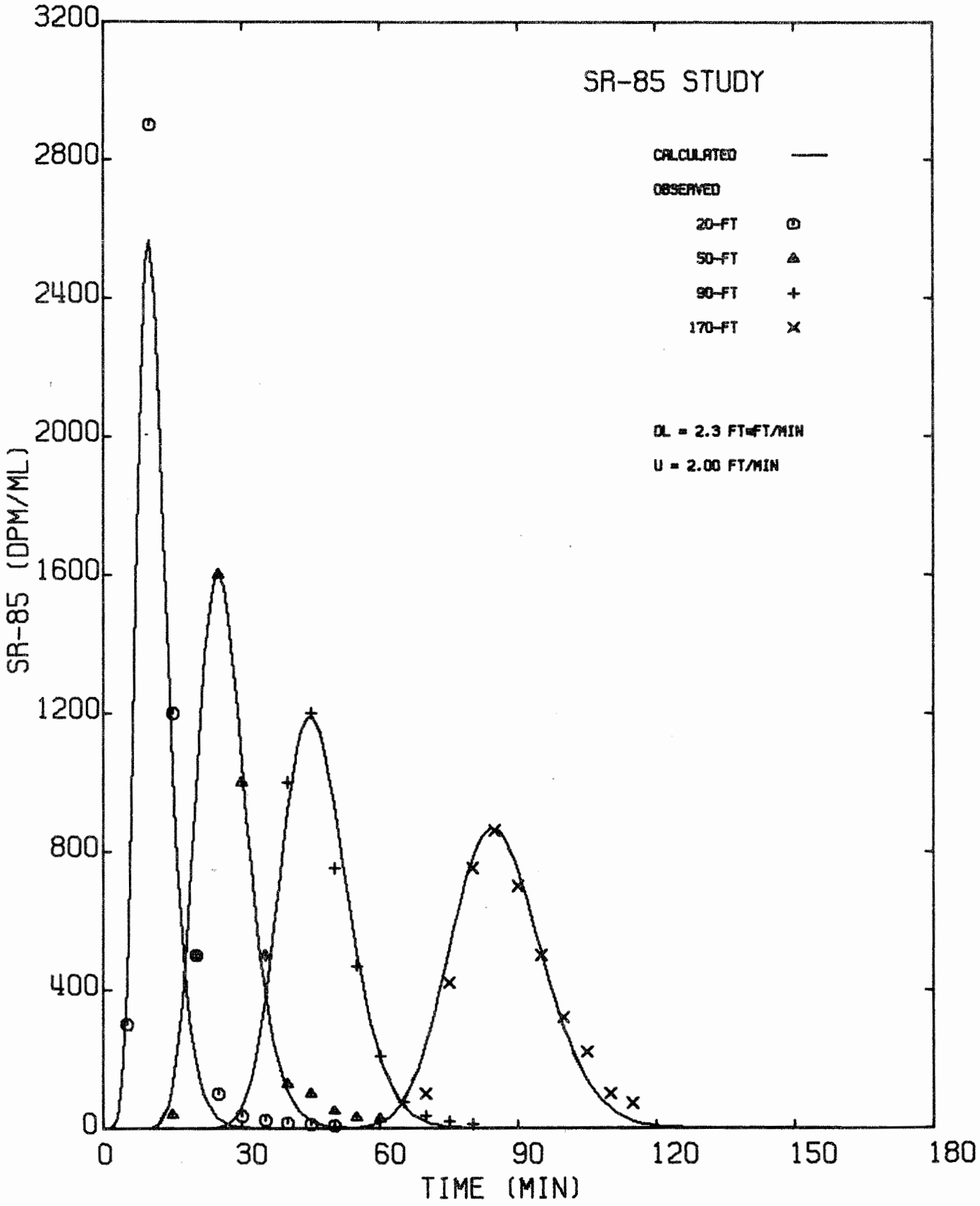

FIG. 4. INSTANTANEOUS RELEASE CURVES FOR WATER AND PLANTS

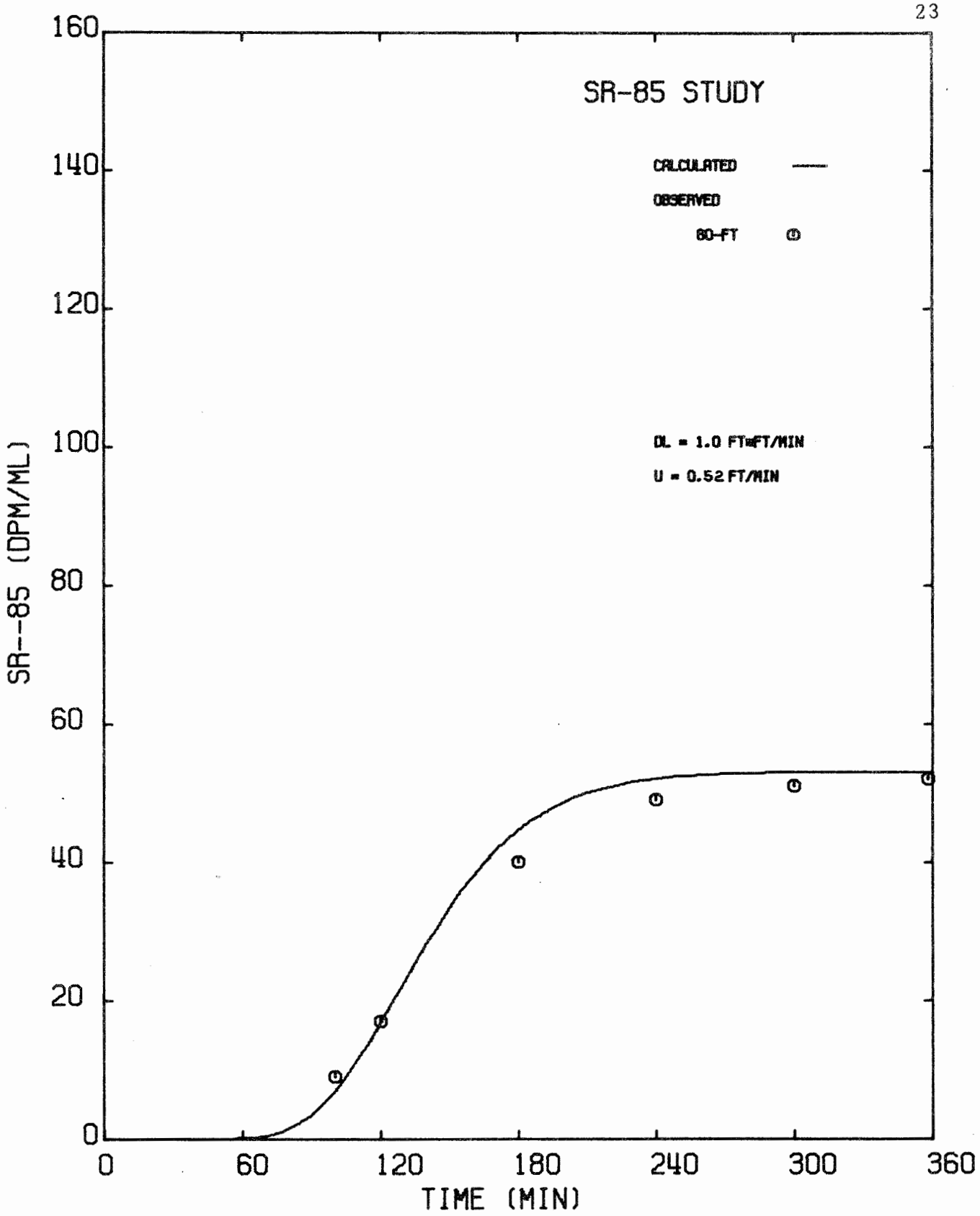

FIG. 5. CONTINUOUS RELEASE CURVES FOR WATER AND PLANTS

uniform from place to place, as for example in the flume, it is sufficient to represent the system with a few prismatic segments of fixed geometry. The length of channel segment depends, in such cases, more on the variations in input data than on flow or cross-section configuration, the optimum length being chosen to provide the necessary detail required to describe the radionuclide activity along the axis of flow.

However, if the flows are nonsteady, with large changes from time to time in discharge, water depth, water surface area, and velocity, it may be necessary to discretize the system into a greater number of elements. The choice of number and size of individual elements may be dictated, in part, by the technique by which nonsteady flows are determined.

Transport - In natural channels wide variation in geometry may be expected along the axis of flow and the effects of these on dispersion and convective transport may be further magnified by the inherent unsteadiness of natural flows. Consequently, in such cases, a satisfactory description of the radionuclide transport will probably require maximum detail. This might entail a need to look at two-dimensional modeling, i.e., lateral dispersion and convection as well as longitudinal.

Sources and Sinks - Although hydrodynamic mixing is believed to be the dominant factor involved, it is recognized that the interaction of radionuclides associated with the liquid phase and various solid phases, i.e., the biota, the bottom sediments, and the suspended debris, etc., of a river may produce a significant effect on the net transport of radionuclides. These components or sorption phases manifest themselves in time-concentration relationships as previously described. The mass-transport and distribution coefficients used in these relationships vary with contact time, temperature, velocity, type of sediment, etc. Thus, to be able to adequately describe the effects of these various components a detailed knowledge of the biota, types of sediments, and sediment transport characteristics of a natural river will be required.

Data Requirements - The following brief discussion presents the data requirements for what may be regarded as a normal river simulation problem.

(1) Hydraulic Data

(a) For steady flow: water surface area of each channel segment.
For unsteady flow: depth-surface area curve for each channel segment.

(b) For steady flow: length and cross-sectional area of each channel segment.
For unsteady flow: length and depth-cross-sectional area curve for each channel segment.

(c) Longitudinal dispersion: velocity profiles, roughness, and sinuosity of stream.

(d) Waste discharge: amount, quality, and location.

(e) Tributary inflows: amount, quality, and location.

(2) Radionuclide Data

(a) Detailed description of release conditions and amount released.

(b) Spatial and temporal description of radionuclide activity after release. This will require a comprehensive sampling program throughout the river system. This data can be used to compute dispersion coefficients which in turn can be used to check against dispersion coefficients computed empirically from the hydraulics and geometry.

(3) Environmental Data

(a) Detailed description of the biota throughout the river system.

(b) Detailed description of the bottom sediments throughout the river system.

(c) Detailed description of the sediment transport characteristics throughout the river system.

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APPENDICES

APPENDIX A - DATA - RADIONUCLIDE TRANSPORT MODEL

1. Dye Release Data (after Armstrong - 1968)

   a. East Side

   $M = 1000$ mg

   $Q = 5.15$ cfm

   $A = 1.56\ ft^2$

   $D_L = 8.0\ ft^2/min$

   b. West Side

   $M = 1000$ mg

   $Q = 5.15$ cfm

   $A = 1.56\ ft^2$

   $D_L = 6.0\ ft^2/min$

2. Instantaneous Release Data for Sediments (after Shih - 1967)

   $M = 0.55\ \mu c$

   $Q = 1.93$ cfm

   $A = 1.04\ ft^2/min$

   $D_L = 2.6\ ft^2/min$

   $a = 4.16 \times 10^{-3}\ ft^2$

   $k_1 = 1.83 \times 10^{-4}\ min^{-1}$

   $Kd_1 = 140$ dpm/core/dpm/ml

3. Instantaneous Release Data for Plants (after Armstrong - 1968)

   $M = 0.504\ \mu c$

   $Q = 1.67$ cfm

   $A = 1.04\ ft^2$

   $D_L = 2.30\ ft^2/min$

   $Mb = 9.04 \times 10^{-5}$ gm/ml

   $k_2 = 5.0 \times 10^{-4}\ min^{-1}$

   $Kd_2 = 260$ dpm/gm/dpm/ml

4. Continuous Release Data for Plants (after Armstrong - 1968)

$M = 2.08\ \mu c$

$Q = 0.54$ cfm

$A = 1.04\ ft^2$

$D_L = 1.0\ ft^2/min$

$Mb = 9.04 \times 10^{-5}$ gm/ml

$k_2 = 5.0 \times 10^{-4} min^{-1}$

$Kd_2 = 260$ dpm/gm/dpm/ml

APPENDIX B - COMPUTER PROGRAM - RADIONUCLIDE TRANSPORT MODEL

Data Input

A relatively small amount of data is required for use of the program. The data may be grouped as follows:

1. Title of study
2. Control
3. Time and space dimensions
4. Initial conditions and hydraulics
5. Parameters and coefficients for bottom sediments
6. Parameters and coefficients for plants
7. Printout control
8. Printout titles

The quantity identification, corresponsing units, and related format specifications are given as:

First Card	Units	Format
1. RADNUC - designates title of study		A6
Second Card		
1. MODE - specifies release conditions		I5
2. NPR - number of printout locations		I5
Third Card		
1. DELT - specifies time interval over which solution is advanced	min.	F10.0

		Units	Format
2.	TMAX - total time period over which a solution is obtained	min.	F10.0
3.	DELX - distance increment for which solution is obtained	ft.	F10.0
4.	LENGTH - total distance over which solution is obtained	ft.	F10.0
Fourth Card			
1.	CBKGRN - background concentration	dpm/ml	F10.0
2.	COMASS - total mass of radionuclide introduced	dpm	F10.0
3.	QFLOW - flow rate of water through system	cfm	F10.0
4.	AREA - cross-section area of channel	ft^2	F10.0
5.	DEPTH - depth of water	ft	F10.0
6.	DL - longitudinal dispersion coefficient	ft^2/min	F10.0
Fifth Card			
1.	K1 - mass transfer coefficient for bottom sediments	1/min	F10.0
2.	KS - equilibrium distribution coefficient for bottom sediments	dpm/core/ dpm/ml	F10.0
3.	AS - Area of sediment core sample	ft^2	F10.0
Sixth Card			
1.	K2 - mass transfer coefficient for plants	1/min	F10.0
2.	KP - equilibrium distribution coefficient for plants	dpm/gm/ dpm/ml	F10.0
3.	BIOMSS - Biomass of plants per unit volume	gms/ml	F10.0
Seventh Card			
1.	IPR(I) - indexes of downstream locations to be printed out		16I5
Eighth Card			
1.	Title (I) - distances from source for locations being printed out		12A6

RESTRICTIONS

1. The maximum number of elements or reaches is fixed at 500.
2. No more than 500 printout times are allowed.
3. No more than 6 locations can be printed out.
4. The only restriction on the total time for which the program is allowed to run is that TMAX/DELT $\leq$ 500.
5. The program is economical to employ. As an example 180 min of real time can be simulated on the UNIVAC 1108 system in approximately 15 sec (less than 5 sec on CDC 6600 System).

PROGRAM LISTING

A listing of the program is presented on the following pages.

```
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C             THIS PROGRAM IS AN IMPLICIT-FINITE-DIFFERENCE MODEL       C
C                  WHICH WILL ROUTE A RADIONUCLIDE THROUGH A MODEL      C
C                  RIVER SYSTEM (FLUME). IT PROVIDES FOR --            C
C                       1. TRANSPORT BY DISPERSION AND ADVECTION       C
C                       2. SORPTION AND DESORPTION BY BED SEDIMENTS     C
C                       3. SORPTION AND DESORPTION BY PLANTS           C
C                       4. INSTANTANEOUS RELEASES                      C
C                       5. CONTINUOUS RELEASES                         C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      DIMENSION CONC(500),CONCN(500),IFLAG(500),A(500),B(500),C(500),
     *          D(500),CBS(500),CBSN(500),CP(500),CPN(500),IPR(6),
     *         CPR1(500,6),TITLE(6),Z(500),TIM(500)
      REAL LENGTH,KS,K1,KP,K2,MCTDPM
      PI=3.1416

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C                MCTDPM CONVERTS MICROCURIES TO DPM                     C
C                FTTOCM CONVERTS FEET TO CENTIMETERS                    C
C                CFTTML CONVERTS 1/FT**3 TO 1/ML                        C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      MCTDPM=2.22E+09
      FTTOCM=30.45
      CFTTML=3.54E-05

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C STEP  1 -- READ NAME OF RADIONUCLIDE                                   C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      READ 1,RADNUC
    1 FORMAT (12A6)

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C STEP  2 -- READ TYPE OF RELEASE AND PRINT CONTROL                      C
C                MODE = 1 FOR INSTANTANEOUS RELEASE                     C
C                MODE = 2 FOR CONTINUOUS RELEASE                        C
C                NPR = NO. OF LOCATIONS TO BE PRINTED OUT               C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      READ 2,MODE,NPR
    2 FORMAT (2I5)

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C STEP  3 -- READ TIME AND SPACE DIMENSIONS                              C
C                DELT = TIME INTERVAL (MIN)                             C
C                TMAX = LENGTH OF TIME AXIS (MIN)                       C
C                DELX = SPACE INTERVAL (FT)                             C
C                LENGTH = LENGTH OF SPACE AXIS (FT)                     C
```

```
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      READ 3,DELT,TMAX,DELX,LENGTH
    3 FORMAT (8F10.0)

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C STEP  4 -- READ INITIAL ACTIVITY AND HYDRAULIC CONDITIONS              C
C                  CBKGRN = BACKGROUND ACTIVITY (DPM/ML)                C
C                  COMASS = TOTAL ACTIVITY RELEASED (MICROCURIES)       C
C                  QFLOW = INFLOW RATE OF WATER (CFM)                   C
C                  AREA = X-SECTIONAL AREA (FT**2)                      C
C                  DEPTH = WATER DEPTH (FT)                             C
C                  DL = LONGITUDINAL DISPERSION COEFFICIENT (FT**2/MIN) C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      READ 3,CBKGRN,COMASS,QFLOW,AREA,DEPTH,DL

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C STEP  5 -- READ COEFFICIENTS FOR BED SEDIMENTS                         C
C                  K1 = MASS TRANSFER COEF. (1/MIN)                     C
C                  KS = EQUILIBRIUM DISTRIBUTION COEF. (ML/CORE)        C
C                  AB = SAMPLING AREA OF SEDIMENTS IN CORE (FT**2)      C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      READ 3,K1,KS,AB

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C STEP  6 -- READ COEFFICIENTS FOR PLANTS                                C
C                  K2 = MASS TRANSFER COEF. (1/MIN)                     C
C                  KP = EQUILIBRIUM DISTRIBUTION COEF. (ML/GM)          C
C                  BIOMSS = BIOMASS OF PLANTS (GMS/ML**3)               C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      READ 3,K2,KP,BIOMSS

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C STEP  7 -- READ INDEXES OF LOCATIONS TO BE PRINTED OUT
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      READ 4,(IPR(I),I=1,NPR)
    4 FORMAT (6I5)

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C STEP  8 -- READ DISTANCES FROM SOURCE FOR POINTS TO BE PRINTED OUT     C
C                  TITLE = ( XX-FT)                                     C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      READ 1,(TITLE(I),I=1,NPR)
      PRINT 2055
 2055 FORMAT (1H1)

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C STEP  9 -- ECHO PRINT ALL INPUT DATA                                   C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
```

```
      PRINT 11,RADNUC
   11 FORMAT (10X,15HINPUT DATA FOR ,A6,6H STUDY,//)
      PRINT 12,PTIME,NPR
   12 FORMAT (15X,F10.2,I5,/)
      PRINT 13,DELT,TMAX,DELX,LENGTH
   13 FORMAT (15X,4F10.2,/)
      PRINT 14,CBKGRN,COMASS,QFLOW,AREA,DEPTH,DL
   14 FORMAT (15X,8F10.3,/)
      PRINT 15,K1,KS,AR
   15 FORMAT (15X,F10.6,F10.2,F10.5,/)
      PRINT 16,K2,KP,BIOMSS
   16 FORMAT (15X,F10.6,F10.2,F10.7,/)
      PRINT 17,(IPR(I),I=1,NPR)
   17 FORMAT (15X,16I5,/)
      PRINT 18,(TITLE(I),I=1,NPR)
   18 FORMAT (15X,12A6,/)
      VEL=QFLOW/AREA
      VO4DX=VEL/(4.0*DELX)
      DODX2=DL/(2.0*DELX*DELX)
      COMASS=COMASS*MCTDPM*CFTTML
      PTIME=DELT
      DEPTH=DEPTH*FTTOCM
      AR=AR*(FTTOCM)**2
      NMAX=LENGTH/DELX+1
      NMAXL1=NMAX-1
      IF (MODE.EQ.1) GO TO 88
      TIME=0.0
      CSTART=COMASS/TMAX/QFLOW
      GO TO 87
   88 DELT=DELT/8192.0
      TIME=DELT
      CSTART=0.5*COMASS/(AREA*SQRT(PI*DL*DELT))*EXP(-(VEL*DELT)**2/
     *       (4.0*DL*DELT))
   87 CONTINUE

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C STEP 10 -- SET FLAG FIELD TO FIX BOUNDARY CONDITIONS FOR EACH REACH  C
C                 IFLAG = 1 FOR EXT. BOUNDARY CELL (INST. RELEASE PT)  C
C                 IFLAG = 2 FOR INT. BOUNDARY CELL ADJ. TO RELEASE     C
C                 IFLAG = 3 FOR INT. CELL NOT ADJ. TO A BOUNDARY       C
C                 IFLAG = 4 FOR INT. BOUNDARY CELL FOR   C/  X = 0     C
C                 IFLAG = 5 FOR EXT. BOUNDARY CELL FOR   C/  X = 0     C
C                 IFLAG = 6 FOR EXT. BOUNDARY CELL (CONT. RELEASE PT)  C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      DO 5 I=1,NMAX
      CONCN(I)=CBKGRN
      IF (I.NE.1) GO TO 6
      IFLAG(I)=1
      IF (MODE.NE.1) IFLAG(I)=5
      CONC(I)=CSTART
      GO TO 5
    6 IF (I.NE.2) GO TO 7
      IFLAG(I)=2
      CONC(I)=CBKGRN
```

```
      CBS(I)=CBKGRN
      CP(I)=CBKGRN
      GO TO 5
    7 IF (I.NE.NMAXL1) GO TO 8
      IFLAG(I)=4
      CONC(I)=CBKGRN
      CBS(I)=CBKGRN
      CP(I)=CBKGRN
      GO TO 5
    8 IF (I.NE.NMAX) GO TO 9
      IFLAG(I)=5
      CONC(I)=CBKGRN
      GO TO 5
    9 IFLAG(I)=3
      CONC(I)=CBKGRN
      CBS(I)=CBKGRN
      CP(I)=CBKGRN
    5 CONTINUE
      NTT=-1
      M=1
      TIM(M)=TIME
      TMAX=TMAX-0.1
      PTIME=PTIME-0.1
      DO 20 I=1,NPR
      K=IPR(I)
      CPR1(M,I) = CONC(K)
   20 CONTINUE
      PRINT 2055
      PRINT 2054,RADNUC
 2054 FORMAT (39X,9HCOMPUTED ,A6,15H CONC. (DPM/ML),//,53X,2HVS,//,
     *        44X,20HDISTANCE FROM SOURCE,//)
      PRINT 2056,(TITLE(I),I=1,NPR)
 2056 FORMAT (19X,4HTIME,6X,6(4X,A6))
      PRINT 165,TIME,(CPR1(M,I),I=1,NPR)

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C STEP 11 -- COMPUTE COEFFICIENT AND VECTOR MATRICES                     C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

   99 TIME=TIME+DELT
      VTO2DX=DELT*VO4DX
      DO2DX2=DELT*DODX2
      DO 100 I=1,NMAX
      IFL=IFLAG(I)
      GO TO (101,102,103,104,105,100), IFL
  101 CONCN(I)=0.5*COMASS/(AREA*SQRT(PI*DL*TIME))*EXP(-(VEL*TIME)**2/
     *          (4.0*DL*TIME))
      GO TO 100
  105 CONCN(I)=CSTART
      GO TO 100
  102 ITRID=I
      A(I)=-(VTO2DX+DO2DX2)
      B(I)=1.0+2.0*DO2DX2
      C(I)=VTO2DX-DO2DX2
      SOBS=K1/(DEPTH*AR)*(CBS(I)-KS*CONC(I))
```

```
      CBSN(I)=CBS(I)+DELT*K1*(KS*CONC(I)-CBS(I))
      SOP=BIOMSS*K2*(CP(I)-KP*CONC(I))
      CPN(I)=CP(I)+DELT*K2*(KP*CONC(I)-CP(I))
      D(I)=DO2DX2*(CONC(I-1)-2.0*CONC(I)+CONC(I+1))-VTO2DX*
     *      (CONC(I+1)-CONC(I-1))-A(I)*CONCN(I-1)+CONC(I)+
     *      (SOBS+SOP)*DELT
      GO TO 100
  103 A(I)=-(VTO2DX+DO2DX2)
      B(I)=1.0+2.0*DO2DX2
      C(I)=VTO2DX-DO2DX2
      SOBS=K1/(DEPTH*AB)*(CBS(I)-KS*CONC(I))
      CBSN(I)=CBS(I)+DELT*K1*(KS*CONC(I)-CBS(I))
      SOP=BIOMSS*K2*(CP(I)-KP*CONC(I))
      CPN(I)=CP(I)+DELT*K2*(KP*CONC(I)-CP(I))
      D(I)=DO2DX2*(CONC(I-1)-2.0*CONC(I)+CONC(I+1))-VTO2DX*
     *      (CONC(I+1)-CONC(I-1))+CONC(I)+(SOBS+SOP)*DELT
      GO TO 100
  104 NTRID=I
      A(I)=-(VTO2DX+DO2DX2)
      B(I)=1.0+DO2DX2+VTO2DX
      C(I)=VTO2DX-DO2DX2
      SOBS=K1/(DEPTH*AB)*(CBS(I)-KS*CONC(I))
      CBSN(I)=CBS(I)+DELT*K1*(KS*CONC(I)-CBS(I))
      SOP=BIOMSS*K2*(CP(I)-KP*CONC(I))
      CPN(I)=CP(I)+DELT*K2*(KP*CONC(I)-CP(I))
      D(I)=DO2DX2*(CONC(I-1)-2.0*CONC(I)+CONC(I+1))-VTO2DX*
     *      (CONC(I+1)-CONC(I-1))+CONC(I)+(SOBS+SOP)*DELT
  100 CONTINUE

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C STEP 12 -- SOLVE TRIDIAGONAL MATRIX                                   C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      CALL TRIDAG(Z,A,B,C,D,ITRID,NTRID)
      DO 110 I=ITRID,NTRID
      CONCN(I)=Z(I)
  110 CONTINUE
      CONCN(NMAX)=CONCN(NMAXL1)

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C STEP 13 -- REPLACE OLD SOLUTIONS WITH NEW SOLUTIONS AT TIME + DELT     C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      DO 130 I=1,NMAX
      CONC(I)=CONCN(I)
      CBS(I)=CBSN(I)
      CP(I)=CPN(I)
  130 CONTINUE
      IF (TIME.GT.PTIME) GO TO 95
      DELT=2.0*DELT
      GO TO 125
   95 DELT=PTIME+0.1
      M=M+1
      TIM(M)=TIME
```

```
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C STEP 14 -- PRINT OUT SELECTED LOCATIONS IN SYSTEM                        C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      DO 150 I=1,NPR
      K=IPR(I)
      CPR1(M,I) = CONC(K)
  150 CONTINUE
      NTT=NTT+1
      IF (NTT.NE.30) GO TO 164
      NTT=0
      PRINT 2055
      PRINT 2054,RADNUC
      PRINT 2056,(TITLE(I),I=1,NPR)
  164 CONTINUE
      PRINT 165,TIME,(CPR1(M,I),I=1,NPR)
  165 FORMAT (18X,F5.1,6X,6F10.2)
  125 CONTINUE
      IF (TIME.LT.TMAX) GO TO 99
      END
```

```
      SUBROUTINE TRIDAG (Z,A,B,C,D,ITRID,NTRID)

C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C
C              THIS SUBROUTINE WILL SOLVE A SYSTEM OF SIMULTANEOUS       C
C                   EQUATIONS OF THE MATRIX FORM (A * C = D), WHERE      C
C                        A IS THE TRI-DIAGONAL COEFFICIENT MATRIX,       C
C                        C IS THE VECTOR OF UNKNOWN VALUES, AND          C
C                        D IS THE VECTOR OF KNOWN VALUES.                C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * C

      DIMENSION Z(1000),A(1000),B(1000),C(1000),D(1000),W(1000),G(1000)
      W(ITRID)=C(ITRID)/B(ITRID)
      G(ITRID)=D(ITRID)/B(ITRID)
      I=ITRID+1
      DO 10 K=I,NTRID
      DENOM=B(K)-A(K)*W(K-1)
      W(K)=C(K)/DENOM
      G(K)=(D(K)-A(K)*G(K-1))/DENOM
   10 CONTINUE
      Z(NTRID)=G(NTRID)
      IK=NTRID-1
   20 Z(IK)=G(IK)-W(IK)*Z(IK+1)
      IK=IK-1
      IF (IK.GE.ITRID) GO TO 20
      RETURN
      END
```