PARAMETER ESTIMATION FOR THE MARXIAN OPTIMAL GROWTH MODEL

Atsushi Tazoe

Atsushi Tazoe, graduate student, faculty of economics, Kyoto University. Co-author with Hiroshi Onishi of *Organic Composition of Capital, Falling Rate of Profit, and "Preferential Growth of the First Sector" in the Marxian Optimal Growth Model* (2011). Email: azusavantage177@gmail.com

Abstract: In this article, we conduct an empirical analysis of a model called the Marxian Optimal Growth Model proposed by Yamashita and Onishi (2002). Mathematically, the Yamashita-Onishi model is the same as the neoclassical optimal growth model. Therefore, if analysis is limited to the steady state in the Yamashita-Onishi model, it is possible to use the techniques of macro-econometrics based on the dynamic stochastic general equilibrium model developed in the field of mainstream macro-econometrics. Examination of the analysis results showed that most of the discrepancies between the model and data are explained by error terms. To a certain extent, this could have been foreseen from the unrealistic assumptions of the basic Yamashita-Onishi model, and the fact that it was confirmed through actual analysis is significant. In this article, we must also note the fact that, although a Bayesian estimation approach was used in the analysis, the posterior distribution of the parameters derived as a result of the analysis is strongly affected by the prior distribution. This trend is particularly striking due to the small number of data items in the macroeconomic field.

Key words: Marxian optimal growth model; parameter estimation; Japanese economy

Introduction

In this article, we conduct an empirical analysis of the Yamashita-Onishi model. This model was proposed by Yamashita and Onishi (2002), and they call it the Marxian Optimal Growth Model. One purpose of the Yamashita-Onishi model is to depict the historical growth path of capitalism. The conclusion of the paper by Yamashita and Onishi is that capitalist society arrives at a steady state in the sense that economic growth stops after reaching a certain point inherent to such societies.

This means that, although the mathematical methods of the neoclassical growth model and the Yamashita-Onishi model are the same, there are radical differences in the major point of interest. The steady state in the neoclassical growth model does not necessarily imply an end point to growth. It simply means that the ratios of various variables become constant. For example, even if the consumption/capital ratio is assumed to be fixed, that does not necessarily mean that the amounts of consumption and capital are fixed. This is because the consumption/capital ratio is constant if the rates of change in consumption and capital are the same.

The Neoclassical macro models began with the Solow-Swan model, evolved through the Ramsey model, and reached full maturity in the Real Business Cycle (RBC) model. The Dynamic Stochastic General Equilibrium (DSGE) model, which is the dominant tool in modern macroeconomics, is based on this RBC model. The Yamashita-Onishi model, in contrast, is mathematically based on the Ramsey model. This is one reason why the Yamashita-Onishi model has a different focus from the mainstream DSGE model. While the main focus of the Yamashita-Onishi model is the dynamics up to a steady state in the sense that growth stops, the focus of the DSGE model is analysis after arriving at a steady state in the sense that some defined variables become constant. In other words, the focus of the DSGE model is the state where the steady state in the sense of the Yamashita-Onishi model changes due to variables such as the technology term—i.e. the process of variation in the steady state. Thus there are qualitative differences between the Yamashita-Onishi model and the DSGE model.

In this article, we perform estimation for the Yamashita-Onishi model by applying a technique for estimation based directly on the DSGE model, which has developed rapidly in the field of mainstream macro-econometrics. However, there are a number of problems in carrying this out.

The technique used in this article requires that all variables comply with a weakly stationary process. As explained earlier, the DSGE model captures a process where the so-called steady state changes, and thus it is easy to convert so that it complies with a weakly stationary process. The Yamashita-Onishi model, in contrast, captures a process which arrives at a steady state, and thus it is difficult to convert to variables which comply with a weakly stationary process, and the author has yet to find a method to do that. Therefore, the techniques of mainstream macro-econometrics can be applied to the steady state of the Yamashita-Onishi model only in the steady state for the Yamashita-Onishi model. Naturally data is necessary to perform quantitative analysis, and this means that the actual world must be in a steady state in the sense of Yamashita-Onishi—i.e., it must be in a state where growth has stopped. This becomes a strong constraint.

The article is structured as follows. The first section examines whether or not it is valid to assume that the actually observed economy is in a steady state in the sense of the Yamashita-Onishi model. The next section gives a simple explanation of the Yamashita-Onishi model, discusses analysis techniques, and indicates the results of estimation. The final section discusses the validity of the obtained conclusions, and describes issues for the future.

Is Japan in a Steady State in the Sense of the Yamashita-Onishi Model?

As noted in the previous section, in order to apply the techniques used in this article to the Yamashita-Onishi model, it is necessary to assume that the actual economy is in a steady state in the sense of the Yamashita-Onishi model. This section addresses whether it is valid to make such an assumption.

Assuming that the economy is in a steady state in the sense of the Yamashita-Onishi model amounts to assuming that the growth rate is zero. For the analysis, it was decided to use data from the Japanese economy. This is because the Japanese economy has been stagnant for a long time, even though it used to enjoy high growth and Japan is classified as a developed country, and thus it is likely valid to assume that its growth has stopped because it has reached a steady state in the sense of Yamashita-Onishi. In the following we make a subjective judgment on whether it is valid to assume that Japan is currently in a steady state in the sense of Yamashita-Onishi, by reviewing estimates of the potential growth rate in Japan, and examining the nature of those estimates.

Potential growth rate in Japan

In order to consider the nature of the growth rate in Japan, here we review the potential growth rate of the Japanese economy. There are a number of methods for estimating the potential growth rate. For example, well known techniques include the H-P filter and production function approach. In recent years, estimating the potential growth rate based on the DSGE model has become popular. Sekine et al. (2009) estimate the potential growth rate of Japan using a number of techniques. The techniques used include the H-P filter, production function approach and Phillips curve, and are based on the dynamic general equilibrium model. The obtained results were similar for all approaches, and the following gives an overview of the results.

Prior to 1992, the potential growth rate was approximately 5 percent, but this suddenly dropped to about 1 percent. However, in the first half of the 2000s, the potential growth rate exhibited a slow but steady recovery, improving just a little each year, and in 2007 it had recovered to about 2 percent. However, the rate dropped again in 2008.

Due to the above considerations, we claim it is not particularly inappropriate to assume that the Japanese economy is in a steady state in the sense of the Yamashita-Onishi model. Naturally if say 1 percent growth continues for ten years, it will result in about 10 percent growth. Therefore, in that case, we must be aware that the economy is not in a steady state in the sense that growth has stopped, as indicated by the Yamashita-Onishi model. However, since one of the main goals of the Yamashita-Onishi model is to describe the halt in the growth of capitalism as one historical stage, it is valid to assert that, if the annual growth rate is about 1 percent, the economy has almost reached a steady state from the historical perspective.

Estimation

This section provides an overview of the Yamashita-Onishi model, and simply describes the technique for estimation. The estimation process begins by solving the Yamashita-Onishi model and obtaining the Euler equation. Second, a log-linear approximation is developed (around the steady state) of equations such as the production function and Euler equation, together the equation for exogenous shock. Third, those equations are rewritten as state equations. Fourth, an observation equation is defined which links endogenous variables with observed variables. These two (i.e., the state equation and observation equation) comprise the state space representation for this model. A likelihood function is created using a Kalman filter from the model with this state space representation. In this article, Bayesian estimation is performed, and thus we obtain a posterior distribution by combining the prior distributions for each parameter with this likelihood function. For a more detailed, textbook style explanation, please see Canova (2007).

Here we would like to briefly note that the method used in this article is advantageous for other techniques such as the least squares method and the Generalized Method of Moments estimation. The model used here has a non-realistic framework in that capital is used only for the production of consumer goods. Therefore, it is impossible to obtain actual data corresponding to the model's variable for capital. Methods such as the least squares method cannot be used in cases like this where there is no data in the first place. With the method used in this article, in contrast, it is possible to carry out estimation even if there is no data directly corresponding to capital. This can be regarded as an advantage of the method used here.

Model

In this article we use the most basic framework of the Yamashita-Onishi model, and solve that model in a social planner framework. For details on the solution method, please refer to Kanae (2008). This model is comprised of a typical household budget and two production sectors. The first production sector produces consumer goods, and the second production sector produces producer goods. The typical household budget possesses all of the labor force and capital which exists in the model world, and supplies those to each sector. There are no externalities in the model treated in this article, and thus the solutions match for general equilibrium and social planner. Therefore, in this article, we solve the social planner problem because it can be solved more easily. The following describes the explicit form of the solution.

In this theoretical world, there is a social planner, who attempts to maximize the utility function of the typical household budgets indicated by the following expression (1).

$$\sum_{t=0}^{\infty} \beta^t U(C_t) \tag{1}$$

In this article, the utility function takes the following form:

$$U(C_t) = \ln C_t$$

The social planner faces constraint conditions given by the following equations (2) and (3):

$$C_t = AK_t^{\alpha}\left(s_t L\right)^{1-\alpha} \tag{2}$$

$$K_{t+1} = (1-\delta)K_t + B(1-s_t L) \tag{3}$$

Equation (2) is the production function for the consumer goods production sector, and equation (3) is the capital accumulation equation. Equation (3) takes a characteristic form. In this model, capital goods themselves are not used for the production of capital goods. The above model is the most basic Yamashita-Onishi model, but this model may be inadequate to depict the real world. There are a number of extended Yamashita-Onishi models to better handle this situation. One of which is Kanae (2008). But we decided to use the most basic Yamashita-Onishi model in this article. The reason why is because there has still been very little estimation done for the most basic Yamashita-Onishi model. Liu (2010) has conducted estimation using the conventional panel analysis technique and the most basic form of the Yamashita-Onishi model, which is the same as that used in this article.[1]

The social planner solves the following sort of maximization problem:

$$\max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t \ln C_t \quad \text{s.t. equations (2) and (3)} \tag{4}$$

The following discusses the first order conditions, and the process of deriving the equations used in this article. From equations (1), (2), and (3), the dynamic Lagrangian is:

$$L=\sum\beta^{t}\left\{\ln A+\alpha\ln K_{t}+(1-\alpha)\ln s_{t}L+\beta\phi_{t+1}\left[(1-\delta)K_{t}+B(1-s_{t})L-K_{t+1}\right]\right\} \quad (5)$$

The first order conditions are:

$$\beta^{t}\phi_{t}(-1)+\beta^{t}\left[\frac{\alpha}{K_{t}}+\beta\phi_{t+1}(1-\delta)\right]=0 \quad (6)$$

$$(1-\alpha)\frac{1}{s_{t}}+\beta\phi_{t+1}(-BL)=0 \quad (7)$$

The Euler equation for this model is derived as follows from equations (6) and (7):

$$\frac{(1-\alpha)}{\beta Bs_{t-1}L}=\frac{\alpha}{K_{t}}+\frac{(1-\alpha)}{Bs_{t}L} \quad (8)$$

This model does not originally include any exogenous shocks. However, when performing estimation, the same number of exogenous shocks as the number of observed variables must be included in the model. In our estimation, the only observed variable is C which indicates consumption. Therefore, we add a single exogenous shock to this model. The equation for the exogenous shock is defined as follows:

$$z_{t}=\eta z_{t}+e_{t} \quad (9)$$

In order to make the above z_t conform to a weakly stationary process, the parameter η must be constrained so that $0<\eta<1$. Also, e_t is set to conform to independent and identically distributed $N(0,\sigma)$. Based on the above, equation (2) is replaced as follows.

$$C_{t}=e^{z_{t}}AK_{t}^{\alpha}(s_{t}L)^{1-\alpha} \quad (10)$$

This is the same as adding an exogenous shock to the Total Factor Productivity (TFP) of the consumer goods production function.

Next, the model must be solved, but the first order conditions of this model have non-linearity. Therefore it is difficult to obtain an explicit solution on the transition pathway to the steady state. This fact is one of the technical requirements which constrain the discussion in this article around the steady state when dealing with this model.

If estimation is attempted with the approach used to handle the DSGE model in this article, then all of the variables must obey the weakly stationary assumption. However, the focus of the Yamashita-Onishi model is growth theory, and thus all of the endogenous variables included in the model do not conform to a weakly stationary process until the steady state is reached. Also, the author has not been able to find a method for converting variables to a weakly stationary process. This is also a technical requirement which constrains the discussion of this article around the steady state. If some method were available to convert the variables of the Yamashita-Onishi model to variables conforming to a weakly stationary process, then it would be possible to remove the strong constraint that the economy must be in a steady state in the sense of the Yamashita-Onishi model which had to be imposed to conduct estimation in this article. However, at the present stage, the author has found no such method.

The model is solved by performing log-linear approximation around the steady state. For this article, we used MatLab® and Dynare[2] to do that.

Observed variables and data

It was decided that the only variable observed in this model would be consumption. Please note that the variable indicating capital is not observed. This corresponds to a special assumption of the Yamashita-Onishi model. In this model, a sector for producing capital goods is not used to produce capital. This fact shows that there is no direct correspondence between capital in the theoretical model, and capital observed in reality. We estimate this model by using quarterly data for Japan. The data is the seasonally adjusted household final consumption expenditures in the national economic accounting, from the first quarter of 1992 to the fourth quarter of 2009. The logarithm is taken for all the model variables.[3]

Determination of prior distribution

The technique of Bayesian estimation is used in this article. Therefore it is necessary to determine the prior distribution of each estimated parameter. There are four parameters in this model: α, A, B, and σ. Determining prior distributions is an important problem for Bayesian estimation, but unfortunately there is no theoretical guide for determining prior distributions. However, a number of clues can be obtained from the economic background for the DSGE model. For example, for parameters indicating individual preferences, the value from previous empirical research is used as the mode value of the prior distribution. In other words, incorporating existing information from prior research is possible if Bayesian techniques are used.

Roxianguli (2010) conducted parameter estimation with the least squares method using Kanae's extended Yamashita-Onishi model. In our model, the model structure is different because we use the most basic Yamashita-Onishi model. As noted earlier, Liu (2010) is an example of estimation using the same model as that used in this article. However, we are aware of the difficulties in interpreting the estimation results in Liu (2010), and thus we refrained from using that paper here. It's certainly true that the model structure used in this article is different, but we adopt the values estimated by Roxianguli as the modes of the prior distributions. The results of Roxianguli are as follows:

$$\ln A = 4.2, \alpha = 0.35, \ln B = 5.48$$

We decided on the following values for other non-estimated parameters.

$$\delta = 0.05, \eta = 0.8, \beta = 0.98$$

Next we determined the form of the distribution for each parameter. α must be constrained due to the theoretical background of the economic model so that $0<\alpha<1$. Therefore, it was decided to using the beta distribution as the prior distribution for α. The other parameters A, B, and σ have the constraint that they must be at least 0. Therefore, the inverse gamma distribution is used as the prior distribution for σ, and the gamma distribution is used as the prior distribution for A and B.

Here we would like to say a word about Bayesian estimation. As is clear from previous remarks, the prior distributions were determined to fit with the economic model. Since the amount of data is small for a macro model, the effect of the prior distribution on the posterior distribution becomes greater. Therefore, in a certain sense it is natural to assume that the form of the posterior distribution falls into an economically valid range. Analysis using the Bayesian estimation technique employed in this article assumed that model is valid rather than conducting pure econometric analysis and it may be more valid to regard it as inferring what the parameters would be in that case.

Estimation results and interpretation

This section gives the results of conducting estimation, and their interpretation. Reiterating, the estimated parameters are: A which is interpreted as the TFP of the consumer goods producing sector, α which is the elasticity relating to production of capital in the consumer goods producing sector, B which is interpreted as the TFP of the producer goods producing sector, and an error term for exogenous shock. The following also shows the estimated and smoothed curves for the endogenous variables C, K, s, and z. Among these, C is an observed variable, and the value given exogenously as data is further smoothed. The results are shown in Table 1.

Table 1

	Prior mean	*Post. mean*	*Standard deviation*
α	0.35	0.3589	0.02
ln*A*	4.2	4.2017	0.2
ln*B*	5.4	5.4816	0.2
σ	0.01	1.4962	∞

For visual understanding, the forms of the prior distribution and posterior distribution for each parameter are shown in Figure 1. The estimated smoothed graphs for the endogenous variables *C*, *K*, *s*, and *z* are shown in Figure 2. How can the above results be interpreted?

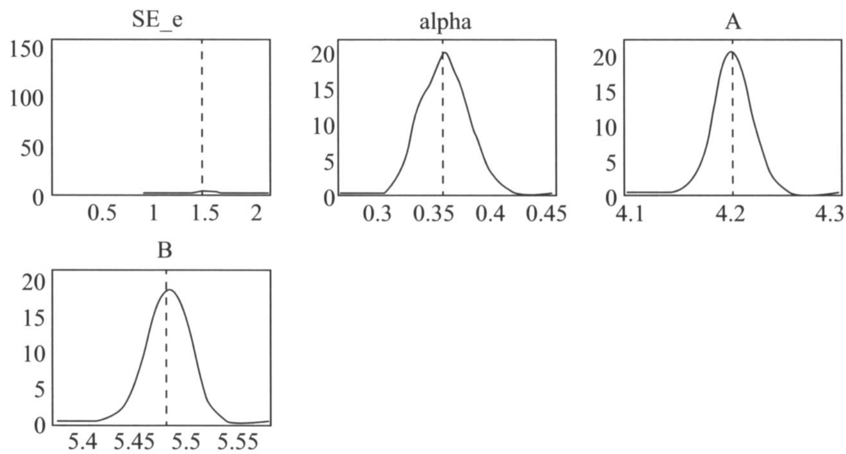

Figure 1 Form of prior and posterior distributions

Looking first at the parameters, almost no difference is evident between the prior and posterior distributions for α, A, and B. At a glance, it appears that the selection of the prior distribution was appropriate, but that is not the case. The σ which indicates the variance of the error term shows that there has been a change to a larger value. In addition, it is evident from Figure 2 that the value for *z* remains stopped at around 7. This means that e_t imposes large positive shock on z_t. This is the reason why the variance of the error term increased. A large *z* value means that the difference between the theoretically derived value of *C* in the steady state and the value of *C* as actual data is absorbed by the exogenous shock term. This signifies that the model is not consistent with the data.

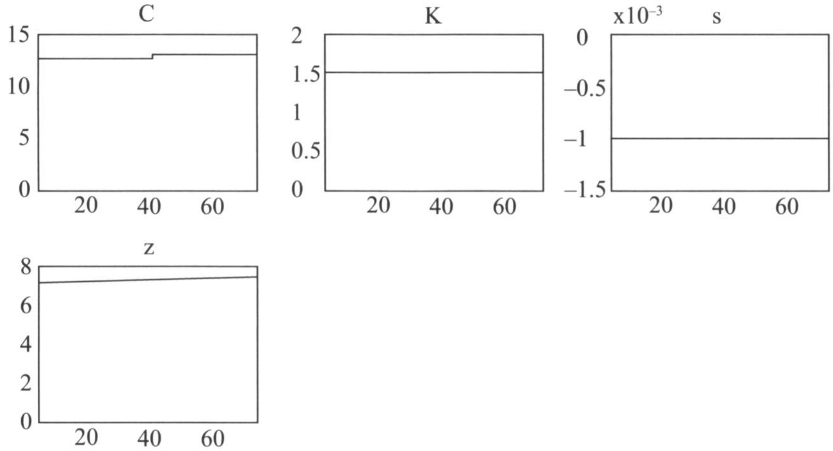

Figure 2 Smoothed endogenous variables

Conclusion and Issues for the Future

In this article, we conducted estimation for the Yamashita-Onishi model, by applying a technique for estimation based directly on a theoretical model established in the field of mainstream macro-econometrics. As explained in the previous section, the results showed that most of the difference between the data and theoretical model was absorbed in the error term. Therefore, we cannot really say that this analysis was effective. This is something which could have been foreseen to a certain extent due to the structure of the basic Yamashita-Onishi model, and it is important to confirm that with actual estimation.

In order to apply the method used in this article to the Yamashita-Onishi model, it is necessary to assume that the economy has reached a steady state in the sense of the Yamashita-Onishi model. This is the same as assuming that growth has completely stopped. However, even the Japanese economy is continuing a low-level of growth which amounts to some growth, and if we consider that fact, there may be no economy which has reached a steady state in the sense of the Yamashita-Onishi model. Naturally, if we consider that the Yamashita-Onishi model was originally an historical theory, then it may be valid to describe low-level growth as a steady-state in an approximate sense, but when conducting empirical analysis, we must note that this may cause major problems.

The technique of Bayesian estimation is used in this article. In interpreting the results, we can regard it as valid to take the interpretation that we are estimating what values the parameters will take when the correctness of the model is assumed as a condition. This point is different from analysis using an ordinary likelihood function.

In terms of future issues, it will be necessary to conduct estimation based on a more realistic model such as the two-sector model presented by Kanae (2008), and investigate how the theoretical variables contained in the Yamashita-Onishi model correspond with actual data. If we take into account the fact that the focus of the Yamashita-Onishi model is analysis of the transition pathway to the steady state, then it is important to not limit estimation to just the steady state as in this article, but to extend further to estimation along the transition pathway. To do this, it will be necessary to convert the variables contained in the condition equations for the Yamashita-Onishi model to variables which conform to a weakly stationary process, and finding a method to do that is the most important point. If this becomes possible, it will enable us to remove the strong constraint that the economy must be almost stopped, as was assumed in this article, and it would be significant in that sense too.

Notes

1. The author is aware that there are a number of difficulties in the interpretation of these estimation results. Therefore, the results of Liu (2010) are not used in this article.
2. MatLab® is a popular calculation software package for economics. Dynare is a toolbox for solving DSGE models.
3. The Yamashita-Onishi model itself is indicated with levels. However, it turns out this way because log-linear approximation is done around the steady state. However, the reader may feel a sense of incongruity even so. Ordinarily, after making a log-linear approximation, the variables are indicated by percent deviation from the steady state, and thus the reader may feel it is inappropriate to simply take the logarithm of observed values. Here solution is done using Dynare, and thus we use the results of simply taking the logarithm of observed values.

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