A CRITICAL DECONSTRUCTION OF THE OKISHIO THEOREM

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Abstract: As part of the literature denying Marx's Law of Tendency of Rate of Profit to Fall, the Okishio theorem exerts extensive influence. This theorem uses rigorous mathematical methods in an attempt to prove that the introduction of advanced technology in the basic goods sector of capitalism leads inevitably to an increase in the general rate of profit. In the literature criticizing the Okishio theorem, there are two main flaws identified. One is that the literature fails to investigate the validity of the production price, which forms the basis of the theorem. The other is that it does not examine the authenticity of the economic theory underlying the theorem. The analysis set forward here shows that the Okishio theorem's "production price" amounts to a complete departure from Marx's definition and is simply a product of the commodity circulation markup. The theorem follows the vulgar components of Smith's theory of value and Ricardo's theory of transformation. Okishio does not understand that the decline of the general rate of profit is precisely the compound result of individual capital pursuing excess profit, which represents the "prisoner's dilemma" of capitalists.

Key words: Marx; Law of Tendency of Rate of Profit to Fall; Okishio theorem

1. Introduction

The question of how to treat foreign Marxist research correctly is important for developing Marxism and further promoting its sinicization. As President Xi Jinping points out, "we should pay close attention to and study the new achievements of research on Marxism abroad, analyzing this research and taking a discriminating attitude towards it, neither rejecting it in blanket fashion nor copying

it wholesale" (Xi 2017). The Okishio theorem that is discussed in this article is among the key positions of the Japanese Marxist school of mathematical economics and has had a wide international impact. Involving a specious treatment of a series of major theoretical issues, it has triggered a long-standing debate in the field of Marxist economics at home and abroad.

The Law of Tendency of Rate of Profit to Fall (LTRPF) revealed in Marx's *Capital* is the core of Marx's theory of capital accumulation.[1] The law highlights the inherent contradiction between productivity and productive relations within the capitalist mode of production, and reflects in concentrated fashion a series of contradictions and conflicts that arise from the process of capital accumulation, affecting areas that include the production and realization of surplus value, production expansion and value proliferation, and questions of overpopulation and surplus of capital. These manifestations of the basic contradictions of the capitalist economy fully expose the historical limitations of the capitalist mode of production. Marx therefore attached great importance to the LTRPF: "In every respect, this is the most important law of modern political economy, and the most essential one for comprehending the most complex relationships. It is the most important law from the historical viewpoint" (Marx and Engels 2010b, 133).

On the surface, the LTRPF seems to conflict with Marx's theory of the production of relative surplus value, which explains how capital increases the rate of surplus value through scientific and technological progress. For this reason, this law has been the subject of continuous debate since its introduction, and has even become the most controversial theoretical issue in Marxist economics. In 1961 Okishio published "Technical Changes and the Rate of Profit," which has become part of the core literature attempting to refute this law. Okishio's article sets out a rigorous mathematical proof seeking to demonstrate his central proposition that the introduction of new technology in the basic goods sector of the capitalist economy will lead inevitably to an increase in the general rate of profit. This is the famous Okishio theorem that has aroused widespread controversy. The idea of questioning and denying the LTRPF from the perspective of technology choice had already existed in the relevant literature before Okishio propounded his theorem, but it was the Okishio theorem that came to occupy a central place in attempts to deny the LTRPF, as it was the first to offer an apparently sound mathematical proof. Since then, the theoretical debate on the tendency of the average rate of profit to fall has focused on the validity of the Okishio theorem.

Most of the foreign literature criticizing the Okishio theorem focuses on the conditions that Okishio ignored in his argumentation, on the non-dynamic nature of his theorem, and on its scope of application. Consequently, such criticisms do not fundamentally reject the Okishio theorem. We may take, for example, the criticism by A. Shaikh, who adheres to the LTRPF. Shaikh affirms that capitalist

competition can promote new technologies that reduce production costs, but argues that the Okishio theorem does not distinguish between production costs as flows and investment costs as stocks, and thus between the marginal rate of profit calculated on the basis of the former and the rate of profit calculated on the basis of the latter. It is assumed that capital must choose the technology that reduces the unit cost, and that this choice will lead to an increase in the marginal average rate of profit for a given real wage, but that reducing the price of unit goods requires an increase in expensive fixed capital, which in turn reduces the average rate of profit (Shaikh 1978). It is obvious that Shaikh restricts the Okishio theorem to marginal profits, and that the relation to the average rate of profit is not clear enough in the discussion for the Okishio theorem to be rejected. In contrast to this criticism, the "Temporal Single System" (TSS) presented by A. Kliman and A. Freeman in response to the challenge of Sraffa's price system (adopted by the Okishio theorem in terms of "value transformation") attempts to overturn the Okishio theorem at a fundamental level by criticizing Okishio's system of the general rate of profit determination (Ernst 1982; Kliman 1997). Nevertheless, the production price system advanced by the TSS school has also been severely criticized on both logical and conceptual grounds (Laibman 2011, 84–112; Mongiovi 2002). Where research in China on these questions is concerned, criticism of the Okishio theorem has been on a theoretically demanding level, focused mainly on the method of analysis and its confused nature, on questioning the basic assumptions of the Okishio theorem, and on addressing various factors that the Okishio theorem neglects (Gao 1991, 284; Xue 2010; Luo 2010; Wang 2011; Yu 2012; Li, Wang, and Pei 2016; Meng and Feng 2016). If the existing criticisms of the Okishio theorem are taken together, it may be seen that the literature has two major shortcomings. On the one hand, there is no understanding of the fact that production price forms the basis for the proof of the Okishio theorem, and as a result, there is no discussion of whether Okishio's treatment of production price is itself tenable. On the other hand, the literature does not take up the basic points of economic theory implicit in the Okishio theorem, and is therefore unable to refute the theorem in a fundamental way. These two major defects provide further space for the present research.

The first part of this article provides a brief review of the Okishio theorem. The second part, making up the core of the argument, subjects the theorem to a critical deconstruction. In the third part, the specific mechanism determining the tendency of the rate of profit to fall is explained from the perspective of methodology, i.e., game theory. The failure to employ game theory is a common feature of scholars such as Okishio, and one that hinders their understanding of the LTRPF. The final part presents the conclusions of the study and reflections on it.

2. A Brief Review of the Okishio Theorem

In "Technical Changes and the Rate of Profit," Okishio (1961) draws the following conclusions on the core logic of Marx's LTRPF. The competition between capitalists forces them to introduce new technologies to improve labor productivity, and these technologies can also increase the organic composition of capital. If the rate of surplus value remains unchanged, the increase in the organic composition of capital will cause a decline in the rate of profit. However, if the real wage rate remains unchanged, the increase in the labor productivity of the industry involved and of related industries will have a limited effect on hindering the decline of the rate of profit. As a result, the continuous introduction of new technologies that increase the organic composition of capital will result in a tendency for the rate of profit to fall.

Where the core logic of Marx's LTRPF is concerned, Okishio poses the following questions. Will the new technologies introduced by capitalists necessarily increase labor productivity, and will the technologies that improve labor productivity necessarily increase the organic composition of capital? Why should the rate of profit tend to decline if the new technologies not only increase the organic composition of capital but also increase the rate of surplus value? In response to these questions, Okishio proposes analytic prerequisites and assumptions different from those of Marx.

The first of his propositions has to do with the decision-making criterion employed by capitalists. According to Okishio, when capitalists introduce a new technology they do not follow a "productivity criterion" but a "cost criterion." Okishio applies the equation $t_i = \Sigma a_{ij} t_j + \tau_i \ (i = 1, 2, \cdots, n)$ to denote the amount of direct or indirect labor necessary to produce 1 unit of Commodity i. Where a_{ij} is the direct quantity of Commodity j necessary to produce 1 unit of Commodity i, τ_i is the amount of direct labor necessary to produce 1 unit of Commodity i, t_i is the total amount of labor necessary to produce 1 unit of Commodity i, and t_j is the amount of past labor consumption. Thus, the "productivity criterion" can be expressed as:

$$\Sigma a_{kj} t_j + \tau_k > \Sigma a'_{kj} t_j + \tau'_k, \tag{1}$$

where $(a'_{k1}, a'_{k2}, \cdots, a'_{kn}, \tau'_k)$ denotes the new technology in industry k, which is the condition under which a new technology can increase the labor productivity of Commodity k in industry k.

The "cost criterion" can be expressed as:

$$\Sigma a_{kj} q_j + \tau_k > \Sigma a'_{kj} q_j + \tau'_k, \tag{2}$$

where $q_j = p_j/w$, p_j and w denote the price and money wage rate of Commodity j, respectively, and q_i is the production price. In a capitalist economy there must be a positive profit in every industry, that is, the inequality $q_i > \Sigma a_{ij}q_j + \tau_i$ must be satisfied, so that there is $q_i > t_i$ for all i. For this reason, the "productivity criterion" is different from the "cost criterion." This is the reason why capitalists follow the "cost criterion." Okishio emphasizes that choosing new technologies on the basis of this criterion can reduce costs, but does not necessarily improve labor productivity or the organic composition of capital. At the same time, the impact on the general rate of profit of the new technologies introduced in accordance with this criterion should be examined now that capitalists follow the "cost criterion" in introducing new technologies.

Okishio's second contention involves the composition of capital, the rate of surplus value, and the division of industries. According to Marx, an improvement in labor productivity requires the circuitous degree of production to increase, and the amount of labor necessary for the production of goods must decrease in relation to the amount of labor required to create the necessary means of production. In Okishio's opinion, Marx fails to demonstrate his perspective fully when he measures the organic composition of capital in industry i using c_i/v_i (i.e., $\Sigma a_{ij}t_j/\tau_i\Sigma b_j t_j$, with b_j as the real wage rate). Therefore, Okishio proposes to replace Marx's organic composition of capital with "organic composition of production" $\Sigma a_{ij}t_j/\tau_i$ (i.e., $c_i/(v_i + m_i)$).

Since Marx's formula m/v for the rate of surplus value does not reflect the role of the labor reproduction sector, Okishio uses his own notation and rewrites the formula for the rate of surplus value as:

$$\frac{m}{v} = \frac{\tau_i - \tau_i\Sigma b_j t_j}{\tau_i\Sigma b_j t_j} = \frac{1 - \Sigma b_j t_j}{\Sigma b_j t_j}. \tag{3}$$

From Equation (3) it can be seen that the rate of surplus value depends on the real wage rate and the labor productivity of wage goods. To emphasize the significance of the new expression, Okishio further identifies industry j as the wage-goods industry on the grounds of $b_j > 0$, and refers to all wage-goods industries and industries indivisible from wage-goods production as "basic industries" according to Inequality (1). For this reason, he concludes that if the real wage rate is given, the rate of surplus value depends solely on the production technology of the "basic goods industry." If the introduction of new production technology in a basic goods industry improves the productivity of labor, i.e., if t_i becomes smaller, then the rate of surplus value will inevitably increase provided that the real wage rate remains unchanged. However, changes in production technology in non-basic industries do not affect the rate of surplus value. Okishio further emphasizes that

the distinction between basic goods industries and non-basic industries is not only important for analyzing the factors affecting the rate of surplus value, but also crucial for analyzing the impact of technological change on the rate of profit. According to Okishio, Ricardo insisted on this point, while Marx did not adhere to the distinction between the two when analyzing the impact of technological change on the rate of profit. This is one of the reasons why, in Okishio's view, Marx made mistakes.

Okishio's third contention involves the method used for calculating the general rate of profit. According to Okishio, Marx's method of calculating the general rate of profit by dividing the total surplus value in the form of value by total capital, i.e., $m/(c + v)$, is not accurate. Okishio believes that the correct method for calculating the general rate of profit should be the one set out in the following equation:

$$q_i = (1+r)(\Sigma a_{ij} q_j + \tau_i) \qquad (i = 1, 2, ..., n)$$
$$1 = \Sigma b_i q_i \tag{4}$$

where $q_i = p_i/w$. On the basis of this analysis, two important propositions (the "Okishio theorem" is referred to in this thesis) are put forward. First, the general rate of profit is not affected if the industry where the new technology is introduced is a non-basic goods industry. Second, the general rate of profit is bound to rise if the industry where the new technology is introduced is a basic goods industry. The reason why Marx, in Okishio's view, fails to draw the right conclusion is that the general rate of profit is calculated based on the formula of total surplus value $r = m/(c + v)$ divided by total capital, including non-basic goods industries, without distinguishing the different roles played by non-basic goods industries and basic goods industries in the formation of the general rate of profit. The general rate of profit should not be calculated on the basis of such a simple quotient, but on Equation (4), because the change in the profit of the non-basic goods industry has only a negative effect on the general rate of profit.

Obviously, Okishio's proposition that the introduction of new technologies that satisfy the "cost criterion" in the basic goods industry will inevitably increase the general rate of profit is incompatible with Marx's LTRPF. Okishio attempts to show that the general rate of profit will invariably rise regardless of the increase in the "organic composition of production" as long as the introduction of new technology satisfies the cost criterion and the real wage remains unchanged. Meanwhile, every technology the capitalists introduce is cost-cutting in terms of the original prices and wages. In short, any technological innovation adopted by capitalists in the basic goods industry is bound to raise the general rate of profit unless the real wage rises sufficiently. In other words, Okishio attributes the decline in the

general rate of profit to increases in the real wages of workers, arguing that the trend of profit rate fluctuations is determined by the class struggle.

Okishio's fourth contention is proof of his theorem. In an effort to prove the above two propositions of the theorem, the following examples are used. It is assumed that industry I produces means of production, and that 1/2 of a unit of means of production and 10 units of direct labor are necessary to produce 1 unit of means of production. Meanwhile, industry II produces wage goods, and 1/4 of a unit of means of production and 15 units of direct labor are needed to produce 1 unit of wage goods. Industry III produces luxury goods, and the corresponding inputs are 1/5 of a unit of means of production and 16 units of direct labor. Let the real wage rate $b_j = 1/45$ of a unit of wage goods. The price of production is calculated accordingly, and the general rate of profit is determined by the following equations:

$$q_1 = (1+r)\left(\frac{1}{2}q_1 + 10\right),$$

$$q_2 = (1+r)\left(\frac{1}{4}q_1 + 15\right),$$

$$q_3 = (1+r)\left(\frac{1}{5}q_1 + 16\right),$$

$$1 = q_2 / 45.$$

Solving the system of equations yields the results that $r = 50\%$, $q_1 = 60$, $q_2 = 45$, and $q_3 = 42$, where $q_i = p_i / w$.

It is assumed that capitalists in the basic goods industry adopt new "Marxist" technologies that can improve labor productivity and the organic composition of capital, and reduce costs in terms of the original price and the real wage. One-third of a unit of means of production and 35/24 units of direct labor are now needed to produce 1 unit of wage goods. Using the original technology, the labor productivity is:

$$t_1 = \frac{1}{2}t_1 + 10,$$

$$t_2 = \frac{1}{4}t_1 + 15,$$

$$t_1 = t_2 = 20.$$

With the new technology, it is $t_2 = \frac{1}{3}t_1 + \frac{35}{24} = 8.125$ units. With the original technology, the "organic composition of production" is $\frac{1}{4}t_1 / 15 = \frac{1}{3}$, while using the new technology it is $\frac{1}{3}t_1 / \frac{35}{24} = \frac{32}{7}$, a huge increase. The cost with the original technology is $1/4\,q_1 + 15 = 30$, which using the new technology becomes $1/3\,q_1 + 35/24 = 21.5$. The equation $q_2 = (1+r)(1/3q_1 + 35/24)$ can be solved to obtain $q_1 = 80$, $q_2 = 45$, and $r = 60\%$. The general rate of profit rises.

Okishio's fifth contention takes the form of the criticism of Marx that the Japanese economist launches after proving his theorem. According to Okishio, there are two reasons why Marx could not reach the correct conclusion. One is that Marx's research on the so-called "transformation problem" was not sufficiently thorough; that is to say, Marx ignored the difference between basic goods and non-basic goods industries when analyzing the general rate of profit and using $m/(c+v)$ to calculate the general rate of profit. Okishio alleges that Marx did not reflect on his analysis despite realizing that his analysis of production prices was inadequate, and did not make the cost-price transformation. The second reason is said to be that Marx ignored important features of capitalist behavior in the adoption of new technologies; although in *Capital* Marx repeatedly elaborates on the limitations under capitalism of the choice of production methods, he is said to fail to consider this feature in his analysis of the general rate of profit.

A brief review of the content of the Okishio theorem indicates that Okishio explicitly rejects Marx's LTRPF and systematically modifies the conditions of this law, while confusing some key concepts. For this reason, it is necessary to make a comprehensive and critical deconstruction of the Okishio theorem.

3. Critical Deconstruction of the Okishio Theorem

3.1. The Production Price according to Okishio

The proof of Okishio's theorem depends on his formula for the production price $q_i = (1+r)(\Sigma a_{ij}q_j + \tau_i)$. The definition of Okishio's production price, as the decisive factor in the change of the general rate of profit, epitomizes the conditions that the change in the general rate of profit should follow, which is of decisive significance if the Okishio theorem is to be proved. Making a critique of the definition of Okishio's production price is thus the focal point for a deconstruction of the Okishio theorem. However, the literature on the Okishio theorem ignores this key issue, and affirms or actually admits that Okishio's definition is consistent with the

meaning of production price as defined by Marx. Instead, the criticism focuses on the Sraffian nature and non-dynamism of Okishio's production price system.

The price of production is an important category in Marxist economics.[2] Marx pointed out:

> The price of production of a commodity is equal to its cost price plus the profit, added to it in per cent, in accordance with the general rate of profit, or, in other words, to its cost price plus the average profit. (Marx and Engels 2010h, 156)

The cost price of a commodity is used to compensate for the sum of the price of the means of production consumed and the price of the labor used to produce the commodity, i.e., the value of the prepaid capital used to produce the commodity, which is necessary to maintain simple reproduction (Marx and Engels 2010h, 28, 158). The cost price of a commodity is determined completely by the expenditure of enterprises in various production sectors (158). The average profit is attributed to a certain amount of capital according to the general rate of profit (regardless of its organic composition), which is the product of the equalization of profit through competition based on the different rates of profit formed by the various sectors of production according to the market value of the goods sold by enterprises, and amounts to the average of all these different rates of profit (156, 157). The average profit reflects the total exploitation of all wage labor by the partnership of all capitalists (156, 169).

Let p denote the price of production, π the average profit, π' the average rate of profit, and $k = c + v$ the cost price. The price of production can then be expressed as $p = k + \pi$ or $p = k(1 + \pi')$ in line with Marx's definition of the price of production. According to the definition of the average rate of profit and Marx's arithmetic example, the average rate of profit is $\pi' = \Sigma m/(\Sigma(c + v))$, and c and v are calculated based on prepaid capital. When the value of a commodity has been converted into a price of production, the price of production of an industrial sector, together with the average profit contained therein, will be added to the cost price of the next industrial sector, and the cost price will no longer be calculated on the basis of the price of production instead of the market value (Marx and Engels 2010h, 159). The calculation of cost price on the basis of production price will cause the cost price to deviate from the market value, because the prices of production of cost-forming factor commodities and their average profits are higher or lower than the surplus value of these commodities. Marx analyzed such deviations of constant capital and variable capital (160–161). Although he did not express his ideas about the transformation of cost price in terms of models and arithmetic examples, his arguments indicated that he had solved the transformation of cost price theoretically.

To obtain the cost price using Marx's method, let P_c and P_v denote c and v as calculated from the price of production, and α and β denote the deviation coefficient of the price of production from the market value. The cost price expressed in the price of production is $k_p = \alpha c + \beta v = p_c + p_v$. Thus, the price of production can be expressed as:

$$p = (\alpha c + \beta v)(1 + \pi') = (p_c + p_v)(1 + \pi') = k_p(1 + \pi').$$

The assertion that the cost price of a commodity is always less than the value of the commodity remains true once the value of the commodity is transformed into the price of production. In the case of transformation, it is expressed in the fact that the cost price of a commodity is always less than its production price, i.e., $k_p < p$ (Marx and Engels 2010h, 164). The total profit of different sectors of production must be equal to the total surplus value, since the average profit is only a division of the total surplus value. At the same time, the sum of the production prices of the total products of society must be equal to the sum of their values (172).

The production price as given by Okishio is set out in Equation (4) above. In formal terms, Okishio's production price is obviously different from that of Marx. According to Okishio, $\Sigma a_{ij} q_j + \tau_i$ in Equation (4) should be the production cost price of the commodity. If that is the case, the definition of Okishio's production price will be in line with Marx's definition of the production price. However, τ_i in Equation (4) is the direct labor cost of producing Commodity i, and according to Okishio's equivalence expression $\tau_i \Sigma b_j t_j = v_i$ and $\tau_i - \tau_i \Sigma b_j t_j = m_i$, so that $\tau_i = \tau_i \Sigma b_j t_j + m_i = v_i + m_i$. If the result is substituted into $\Sigma a_{ij} q_j + \tau_i$, the following is obtained:

$$\Sigma a_{ij} q_j + \tau_i = \Sigma a_{ij} q_j + \tau_i \Sigma b_j t_j + (\tau_i - \tau_i \Sigma b_j t_j) = \Sigma a_{ij} q_j + v_i + m_i.$$

In the above equation, $\Sigma a_{ij} q_j$ uses $q_j = p_j / w$ to calculate the constant capital c_i consumed in production. It is obvious that $\Sigma a_{ij} q_j + \tau_i$ is not the cost price of Commodity i, but the total value of Commodity i calculated by the wage rate after adjusting c_i. According to Okishio's equivalence relation and Marx's definition of cost, what is attributable to the cost or capital expenditure in τ_i is $\tau_i \Sigma b_j t_j$, i.e., the wage. Meanwhile, the other part $\tau_i - \tau_i \Sigma b_j t_j$ in τ_i is the surplus value or profit, which does not belong to the category of cost. Therefore, the cost of production or capital expenditure of Commodity i should be $\Sigma a_{ij} q_j + \tau_i \Sigma b_j t_j$, and accordingly, $\Sigma a_{ij} q_j + \tau_i \Sigma b_j t_j < \Sigma a_{ij} q_j + \tau_i$; that is to say, the cost of production of a commodity is less than the value of the commodity. However, if $\Sigma a_{ij} q_j + \tau_i$ is regarded as the cost of the commodity, this means that the producer of Commodity i incorporates his surplus value or profit into his cost, which is in conflict with the concept of cost

(Marx and Engels 2010h, 28, 159). Okishio mistakes the value of a commodity for its cost price.

Since Okishio's $\Sigma a_{ij}q_j + \tau_i$ is not the cost of a commodity but its full value, his q_i is not Marx's price of production. According to Okishio's equivalence relation, the production price can be equivalently expressed as:

$$q_i = (1+r)(\Sigma a_{ij}q_j + v_i + m_i) = \Sigma a_{ij}q_j + v_i + m_i + r(\Sigma a_{ij}q_j + v_i + m_i).$$

This means that the average profit is calculated on the basis of the total value of the commodity, and the average profit calculated in this way is added to the value of the commodity rather than to the cost price. The average rate of profit r can be obtained by expanding Okishio's production price:

$$r = \frac{q_i - (\Sigma a_{ij}q_j + \tau_i)}{\Sigma a_{ij}q_j + \tau_i} = \frac{q_i - (\Sigma a_{ij}q_j + v_i + m_i)}{\Sigma a_{ij}q_j + v_i + m_i}. \tag{5}$$

This is a clear indication that Okishio's average profit consists entirely of an increase over the value of the commodity. Such a profit is not in line with the reality of capitalism, and can only be understood as an artificial markup from circulation. The correct formula for calculating the rate of profit is:

$$\pi = \frac{p_i - (\Sigma a_{ij}p_j + w\tau_i)}{\Sigma a_{ij}p_j + w\tau_i}. \tag{6}$$

Obviously, $r \neq \pi$. According to $\tau_i = \tau_i \Sigma b_j t_j + (\tau_i - \tau_i \Sigma b_j t_j)$, only $\tau_i \Sigma b_j t_j$ belongs to the value of prepaid capital, so even if the price is calculated on the basis of Okishio's wage rate, the rate of profit should be calculated as:

$$r_q = \frac{q_i - (\Sigma a_{ij}q_j + \tau_i \Sigma b_j t_j)}{\Sigma a_{ij}q_j + \tau_i \Sigma b_j t_j}. \tag{7}$$

If Equations (5) to (7) are compared, $r \neq \pi \neq r_q$ is readily obtained.

It follows that Okishio's concept of profit and the rate of profit is fundamentally wrong, as are the average profit and the average rate of profit derived from it. Regardless of Okishio's subjective motives, his approach actually seeks to explain profit in terms of circulation, concealing the real source of profit and the exploitation of all wage labor by all capitalists. The price of production given by Okishio is not composed of cost price plus average profit, but of the value of the commodity plus the profit calculated at commodity value. Such a price of production has no theoretical basis and is not in accordance with Marx's rules on the price of production. The "proof" of the Okishio theorem that is based on this price is meaningless and cannot be used to negate Marx's LTRPF.

Okishio criticizes Marx for not addressing the issue of transformation with sufficient thoroughness to allow the cost-price transformation to be carried out. Marx's related thesis reveals that this criticism by Okishio is untenable. Nevertheless, Okishio unjustifiably takes the commodity price calculated according to the wage rate as the production price, while rejecting Marx's average rate of profit and price of production. According to the explanation provided by M. Morishima (1973, 62, 73), the price of a commodity calculated according to the wage rate represents the amount of labor that is obtained in order to provide a unit of a commodity, or the amount of labor that is administered in exchange. Only in this way can $\Sigma q_j a_{ji}$ and τ_i in Equation (4) have a uniform dimension and thus be additive. Nonetheless, Okishio not only returns to the false view of Adam Smith (which Marx rejected) that the value of a commodity is determined by the labor purchased in exchange, i.e., the wage, concealing the real source of surplus value and profit, but also clearly confuses the labor time (quantity of labor) consumed in producing the commodity with the labor time (quantity of labor) calculated according to the wage rate. This shows that what Okishio follows is not Marx's labor theory of value, but Smith's erroneous value theory.

Since $\Sigma a_{ij} q_j + \tau_i$ is not the cost, but the total value of the commodity after adjustment for the constant capital consumed at the prevailing wage rate, Okishio's production price must always be greater than the value of the commodity calculated according to labor consumption, i.e., $q_i > t_i$. This violates the proposition that the price of production of an individual commodity can be equal to, greater than, or less than its own value, as determined by the division of total surplus value (Marx and Engels 2010h, 162). If the calculation is made based on Okishio's production price q_i, it will inevitably result in the total production price being greater than the total commodity value and the total profit being greater than the total surplus value, which cannot satisfy Marx's constraint of the equality of the two aggregates.[3]

If q_i is expanded, the price of production or the transformation form of value $p_i = (1+r)(\Sigma a_{ij} p_j + w\tau_i)$ in monetary terms can be obtained. In effect, q_i is used to reverse the price of production p_i into the price of labor time or value in this sense (Smith's value) at the given wage rate. The value of the commodity obtained by this reversal is then used to calculate the average profit and the average rate of profit. This is a complete departure from Marx's theory of average profit and production price.

It can be seen that Okishio confuses the cost price of commodities with their commodity value. Based on this confusion, his production price has nothing to do with the essential provision of the production price resulting from the division of total surplus value, and is not the price of production as defined by Marx. For Okishio, the average profit and the average rate of profit do not arise out of the

total exploitation of all wage labor by all capitalists, but are derived from the arti-
ficial markup applied to commodities in circulation. Consequently, they are not
the average capitalist profit and average rate of profit that Marx elaborated.

3.2. The Positive Profit Condition and Capitalist Decision Criterion

The positive profit condition given by Okishio is $q_i > \Sigma a_{ij} q_j + \tau_i$. It should be
noted that this positive profit condition is widely used in the Western literature on
Marxist economics, without stirring notable criticism. However, there is some-
thing fundamentally wrong with this formulation, because it is neither consistent
with Marx's profit theory nor with empirical observations.

   According to Marx, profit is the transformation form of surplus value. In out-
ward appearance, profit is the difference between commodity price and cost price,
and cost price is the transformation form of prepaid capital value. To use Okishio's
notations, the condition of positive profit at the commodity price is that the com-
modity price (p_i) is greater than the cost price ($\Sigma a_{ij} p_j + w \tau_i$). Thus:

$$p_i > \Sigma a_{ij} p_j + w \tau_i. \tag{8}$$

   It is only when this inequality applies that there are positive profits under capi-
talism. In value terms, the amount of positive profit is the amount of surplus value,
that is, the amount of living labor consumption contained in the commodity, that
exceeds the sum of the past labor consumption and the labor consumption that
goes to compensate for labor reproduction. The positive profit condition or posi-
tive surplus value condition, in terms of the value of the commodity, is therefore
that the value of the commodity (t_i) is greater than the sum of the old value trans-
ferred in the commodity ($\Sigma a_{ij} t_j$) and of the value of the labor that must be compen-
sated ($\tau_i \Sigma b_j t_j$). The sum of these two is also the cost of the commodity
$\Sigma a_{ij} t_j + \tau_i \Sigma b_j t_j$. Hence:

$$t_i > \Sigma a_{ij} t_j + \tau_i \Sigma b_j t_j. \tag{9}$$

   If the commodity value is converted into production price, the positive profit
condition is that the production price must be greater than the transformed cost
price. The conditional expression of Positive Profit (9) is the essence of the posi-
tive profit conditional expression (8). It is only when Inequality (9) has been estab-
lished that Inequality (8) can be established. If both sides of Inequality (8) are
divided by the money wage rate w, that is, if a mathematical equivalent transfor-
mation is made, and the wage rate is used to calculate the commodity price, then
according to Okishio Inequality (8) will be converted into a positive profit condi-
tion. However, Okishio fails to explain or demonstrate why the wage rate should
be used to calculate the commodity price. This is a problem that students of the

Okishio theorem are compelled to try to solve.[4] According to Morishima, who uses the same method of transformation, the reason why the wage rate is used to calculate the commodity price is that Marx has a dual calculation system of price and value. There is no unified dimension between the two systems, so they cannot be compared. Morishima believes that prices can be standardized by labor. In other words, the wage rate is used to measure the commodity price $p_{i,w}$ (i.e., q_i of Okishio), so that the standardized price and value are measured by labor, the dimensional consistency between them is achieved, and they can thus be compared (Morishima 1973, 73). However, this can lead to erroneous results.

In purely mathematical terms, Okishio's positive profit condition is transformed by Inequality (8), so the two are mathematically equivalent. Nevertheless, they have completely different meanings in economics. The question is how the nature of wages and labor time τ_i should be understood.

In Inequality (8), $\Sigma a_{ij} p_j + w\tau_i$ is the cost paid by capitalists for the production of goods. Here, $w\tau_i$ is the cost of money wages at the hourly wage rate w for the labor time τ_i. There is no difference between $w\tau_i$ and $\Sigma a_{ij} p_j$ as phenomena, both belonging to the category of cost. However, the two are completely different in essence. According to Marx, wage is a transformation form of the value or price of labor power which conceals the essence of the division of workers' labor into necessary labor and surplus labor. The employed workers must provide surplus labor, which is a prerequisite for their combination with the means of production (employment). For this reason, the variable capital that capitalists invest in wages differs significantly from constant capital. On the one hand, the amount of wages represents a certain amount of materialized labor. On the other hand, its value is only an index of the amount of living labor it propels, and the amount of living labor it actually propels is always greater than the amount of labor expressed as wages; that is, it expresses itself as being greater than the value of variable capital (Marx and Engels 2010f, 225; 2010h, 143, 144). In their labor time τ_i, wage workers produce not only their wage equivalent but also the surplus value or profit; thus τ_i, which makes up the total labor time of a worker, contains both necessary labor and surplus labor. It can be seen that in Inequality (8), $w\tau_i$ is only the amount paid in money form for necessary labor, but it has the appearance of being payment for all the labor expended. If attention is not paid to this essential feature of wages, and if all that is done is to make an equivalent transformation for Inequality (8), a fundamental error must inevitably follow. Okishio ignores this essential feature of wages, and uses the wage rate to make a mathematical equivalence transformation of Inequality (8) regardless of the economic meaning of this inequality. The cost calculated by the price in the inequality thus becomes the past labor cost $\Sigma a_{ij} q_j$ and the total living labor cost of the worker τ_i calculated by the wage rate. As a result, the original cost of the commodity $\Sigma a_{ij} p_j + w\tau_i$ becomes the commodity

value $\Sigma a_{ij} q_j + \tau_i$, and the nature of the inequality has changed fundamentally. For all i, the prices calculated according to the wage rate must therefore be greater than the commodity value, and the positive profit can only be the increase over the commodity value, which is a phenomenon that occurs in the circulation field and has nothing to do with the production process. In addition, as Morishima (1973, chap. 7) proved, so long as such a positive profit condition is adopted, the total production price of all commodities will be greater than the total value of these commodities, the cost price of the commodities will be greater than their value, and the total profit will be greater than the total surplus value. It is thus clear that Okishio's positive profit condition is contrary to the law of value and the law of surplus value, and is therefore unacceptable. In other words, the mathematical transformation of Inequality (8), though mathematically equivalent, completely changes the nature of the problem in economic terms, and becomes a source of error.[5] It may be noted in passing that it is on the basis of this erroneous treatment and its results that Samuelson rejects Marx's labor theory of value and, in particular, his theory of surplus value (Samuelson 1957, 888). This is an important reason why Samuelson, when he participated in the debate on Marxist economics as a representative of the neoclassical synthesis school, concluded that Smith was correct and Marx incorrect.

The correct positive profit condition can only be $p_i > \Sigma a_{ij} p_j + w\tau_i$ and $t_i > \Sigma a_{ij} t_i + \tau_i \Sigma b_j t_j$. However, Okishio completely avoids the latter, in which the positive profit condition is calculated on the basis of value, and so avoids recognizing the essence of positive profit as surplus value. The uniform measurement of p_i and t_i makes the two comparable. It is an indisputable, objective fact that a uniform measurement of the wage rate can only lead to confusion and errors, leaving aside the question of whether or not it is practical.[6]

The false positive profit conditions $q_i > \Sigma a_{ij} q_j + \tau_i$ and $q_i > t_i$ are applied by Okishio to prove that the "cost criterion" is different from the "productivity criterion," but the specific meaning of the "cost criterion" is not clarified. According to Marx's argument, the "cost criterion" can be understood as the criterion of technological choice that can reduce the cost per unit of product without changing the labor productivity. According to this criterion, it follows from Inequalities (1) and (2) that

$$\Sigma a'_{kj} q_j + \tau'_k > \Sigma a'_{kj} t_j + \tau'_k.$$

Thus, it can be seen that

$$\Sigma a'_{kj} q_j > \Sigma a'_{kj} t_j.$$

Calculating the cost in terms of labor time (value) thus leads to the conclusion that on the basis of the "cost criterion," capitalists choose higher-cost technologies.

This paradox negates the "cost criterion" itself. It should be noted that Okishio, on the one hand, completely excludes $\Sigma a_{ij} t_j$ from the cost concept and, on the other, paradoxically defines $\Sigma a_{ij} t_j$ as Marx's constant capital c_i. Constant capital is obviously a cost element, in which t_j is the amount of past labor contained in a unit of product, and it is also a cost category. Higher productivity means decreasing costs according to labor time or value. It is thus clear that the "productivity criterion" is also the "cost criterion," and the technique of increasing productivity is also that of reducing costs.[7] The "productivity criterion" and the "cost criterion" are not necessarily satisfied at the same time, because Okishio applies this erroneous positive profit condition and uses its production price system to prove his theorem. It is for this reason that he concludes that capitalists follow only the "cost criterion" when introducing technology. In Okishio's production price system, however, an abnormal phenomenon arises in which the costs of industries are increased, assuming that the technology and productivity remain unchanged, while using new technology will mean that the costs of the industry are reduced. In the numerical example that he provides, the costs of industry I with constant technology and productivity increase from 40 to 50, and the costs of industry III increase from 28 to 32, if the technology for the basic goods industry satisfies both the "cost criterion" and the "productivity criterion." This abnormal phenomenon violates Okishio's "cost criterion," which shows that his average profit is obtained on the basis of the markup in circulation. It also indicates that Okishio's reproduction system is characterized by misconvergence and instability.

If capitalists follow only the "cost criterion" when introducing technologies, it should be observed empirically that capitalism will only develop technologies that are capable of reducing production costs in accordance with the capitalists' technology selection preferences. However, empirical observation shows that in a capitalist economy, technologies that increase productivity and those that reduce production costs exist at the same time and are integrated as one. A prominent feature of capitalism is its continuous striving to improve productivity, and cost reductions are usually the result of such increases in productivity. Improving productivity by reducing costs is the main means through which capitalists compete. In the actual capitalist world, a technology that can reduce costs must be one that improves productivity; otherwise, it will be eliminated by capitalist competition. This technology selection mechanism ensures that technologies aimed at improving labor productivity are of a cost-reducing nature. This is the spontaneous result of technological evolution under the condition of capitalism, and it is also the basic mechanism that underlies the production of relative surplus value. Marx says that the purpose of increasing labor productivity is "to shorten the labor time socially necessary for the production of a commodity, and to endow a given quantity of labor with the power of producing a greater quantity of use value." It is

through technological change alone that "the value of labor power [can] be made to sink," and hence, the production of relative surplus value increased (Marx and Engels 2010f, 319, 320). If capitalists choose technology only on the basis of the "cost criterion," it will be impossible to explain the empirical fact of the increase in capitalist productivity and of the organic composition of capital.

In fact, what capitalists follow when choosing technology is neither the so-called "productivity criterion" nor the "cost criterion," but the criterion of profit maximization or excess profit.

> The special productivity of labor in any particular sphere, or in any individual enterprise of this sphere, is of interest only to those capitalists who are directly engaged in it, since it enables that particular sphere, vis-à-vis the total capital, or that individual capitalist, vis-à-vis his sphere, to make an extra profit. (Marx and Engels 2010h, 196–197)

Here, revealed by Marx, is the decision-making criterion of capitalist economic operation in relation to capital investment. According to this criterion, whether a technology is worthy of adoption by capitalists depends entirely on whether the adoption of this technology can bring them higher profits. Similarly, the choice between different technologies still depends on which can bring higher profits. For capitalists, it is not sufficient simply to record positive profits. It is on the basis of the general decision-making criterion of capitalism, i.e., profit maximization, that Marx analyzes the law that capitalists apply when they choose technologies. If the "productivity criterion" rather than the "excess profit criterion" is regarded as Marx's explanation for capitalist decision-making behavior, the class interests behind capitalist production "rationality" remain concealed. Okishio does not understand the real process of capital accumulation, or the basis on which capitalist competition is conducted.

3.3. The Organic Composition of Capital and "Organic Composition of Production"

The organic composition of capital is an important category and basic concept of Marx's economics, one that is of great significance for understanding the LTRPF. Okishio, however, uses his formulation of the "organic composition of production" $\Sigma a_{ij} t_j / \tau_i = c_i / (v_i + m_i)$ to replace Marx's organic composition of capital $c_i / v_i = \Sigma a_{ij} t_j / \tau_i \Sigma b_j t_j$, and thus excludes the important concept of the organic composition of capital from his analysis. From the proof that Okishio advances for his proposition, we find that he regards the applicability of the "organic composition of production" as self-evident, and as an important constraint that must be followed. The question is whether the "organic composition of production" is a

clearer demonstration of Marx's view, as Okishio claims. It is necessary to review Marx's relevant arguments.

Marx puts forward a clear definition of the organic composition of capital: "I call the value composition of capital, in so far as it is determined by its technical composition and mirrors the changes of the latter, the organic composition of capital" (Marx and Engels 2010f, 608). Marx's category extends from the technological composition of capital through its value composition to the condensation of the organic composition of capital. His thinking proceeds dialectically, improving in echelon through a compound process from abstract to concrete, reflecting in this way the interaction between productive forces and production relations within the capitalist mode of production.

Premised on the enormous development of science, capitalist labor productivity increases with the rising scale of the means of production (Marx and Engels 2010b, 29, 30; 2010e, 190). Innovations in the performance and increases in the quantity of the means of production become the most general conditions for the material development of socialized production, and improvements in the technical composition of capital serve as the main symbol for the development of productivity. The numerator, which constitutes its ratio relation, is the amount of means of production, and the denominator is the amount of living labor. This is the technical condition of the labor process, if we temporarily abandon the social condition of the labor process.[8] Marx wrote:

> The growing extent of the means of production, as compared with the labor power incorporated with them, is an expression of the growing productiveness of labor. The increase of the latter appears, therefore, in the diminution of the mass of labor in proportion to the mass of means of production moved by it, or in the diminution of the subjective factor of the labor process as compared with the objective factor. (Marx and Engels 2010f, 617)

> On the side of material, as it functions in the process of production, all capital is divided into means of production and living labor power. This latter composition is determined by the relation between the mass of the means of production employed, on the one hand, and the mass of labor necessary for their employment on the other. (Marx and Engels 2010f, 608)

When analyzing the technical conditions of the labor process, it is enough to assume that the subject of labor occupies and dominates the means of production. According to this premise, the purpose of developing labor productivity is to save labor and shorten the working day by increasing the means of production.

However, the technical combination of workers and the means of production is not the same as the social combination of the two. As the primary objects of study

in *Capital*, the workers and the means of production are "the productive mode of existence of . . . capital," and become "a function of capital, the capitalist process of production" (Marx and Engels 2010g, 43). Therefore, it is necessary to investigate the special way in which workers and the means of production are combined in history, that is, the capitalist mode of production. There is a special need to study the social characteristics of productive capital, because "human labour power is by nature no more capital than are the means of production" (Marx and Engels 2010g, 43). As Marx pointed out, it is the "change in the technical composition of capital," that is, the "change in the composition of capital's material constituents," that approximately reflects the growth of constant capital in relation to variable capital in the composition of capital value (Marx and Engels 2010f, 618). The social form of the technical composition of capital is production capital, which is expressed as the inner unity of the labor process with the process of value formation and value multiplication in direct production. The prepayment of money capital G for the purchase of labor goods A and means of production Pm in the market is the precursor stage of the production process. Wage laborers lose the conditions for the realization of their own labor power, i.e., the ownership of their means of production and means of livelihood. Only by selling their labor power, becoming a disposable variable capital component of production capital and providing unpaid surplus value for production capital, can they engage in production activities in combination with the means of production, and make monetary capital as the real production capital. As Marx emphasized, G–A "is the essential condition for the real transformation of value advanced in the form of money into capital, into a value-producing surplus value. M–MP is necessary only for the purpose of realizing the quantity of labor bought in the process M–L." In essence, it is in the interaction G–A that "the class relation between capitalist and wage labourer . . . exists" (Marx and Engels 2010g, 35, 37). Abstract productivity has become concrete capital productivity. The means of production belonging to the subject of labor has been transformed into the capitalist's constant capital, and the scale and performance of its use value are directly proportional to the development of productivity. Living labor is transformed into the use value of variable capital purchased and disposed of by capitalists, and the development of productivity means that the necessary labor expended by wage labor to compensate for the value of labor shrinks relative to surplus labor, in inverse proportion to the increase in constant capital. The increase in relative surplus value cannot abolish necessary labor and the rate of surplus value cannot be infinite, while the direct application of science and technology means that capital productivity takes on a tendency to expand indefinitely and to experience a continuous erosion, until it disintegrates the exchange value as the basis of the capitalist mode of production. This is the class content of Marx's organic composition of capital. Compared with the

technical combination of the technical composition ratio, its social combination undergoes qualitative changes in both numerator and denominator.

By replacing Marx's organic composition of capital with the "technical composition of production" and, in particular, by replacing the amount of necessary labor (v) required for the reproduction of labor commodities with the amount of living labor ($v + m$), Okishio removes the class opposition between constant capital (means of production) and variable capital (price of labor) in prepaid capital based on the ownership relation.[9] Meanwhile, he erases the actual subordination of the surplus value created by wage labor, which cannot be possessed by labor itself but only by production capital, without payment. He further conceals the conflict between the increase in the organic composition of capital on the one hand, and the growth of the reserve army of labor and the increased impoverishment of the proletariat on the other. For capitalists, an increase in the productivity of labor means only that "the same capital sets more surplus labor in motion, therefore less necessary labor" (Marx and Engels 2010a, 315). The increase in relative surplus value is premised on an increase in the organic composition of capital, and the price capital must pay for this is a decrease in the rate of profit. As Marx points out, "The rate of profit is inversely related to the growth of relative surplus value or relative surplus labor, to the development of the productive forces, and to the size of the capital employed in production as [constant] capital." As development of the productive power of capital proceeds, he explains, the rate of profit experiences a tendency to fall (Marx and Engels 2010b, 147). The fact that variable capital has decreased in comparison with constant capital is also reflected in the change in the value composition of unit commodity prices, where, as Marx states,

> Since the mass of the employed living labor is continually on the decline as compared to the mass of objectified labor set in motion by it, i.e., to the productively consumed means of production, it follows that the portion of living labor, unpaid and congealed in surplus value, must also be continually on the decrease compared to the amount of value represented by the invested total capital. (Marx and Engels 2010h, 211)

According to Okishio, changes in the real wage rate will also cause changes in the organic composition of capital even if the production technology remains unchanged. This is the direct reason for Okishio to abandon Marx's concept of the organic composition of capital. It follows from Marx's concept that the necessary labor in the denominator (v), excluding surplus labor, is smaller than all the living labor in the denominator ($v + m$) of Okishio's "organic composition of production." The rise in the actual organic composition of capital reflected in Marx's concept is artificially reduced by Okishio. According to Okishio, the "organic composition of

production" reflects the development of productivity and the increase of labor productivity directly in terms of the ratio of indirect labor to direct labor per unit of commodity, and he assumes that capitalist relations of production can automatically adapt to the development of productivity. This completely erases the class antagonism implicit in the contradictory movement between productivity and production relations under the conditions of the capitalist mode of production.

3.4. Non-basic Goods Industries and the Equalization of Profit

Non-basic goods industries, according to the view put forward by Ricardo and adhered to by Sraffa (Ricardo 1976, 100, 112; Sraffa 1963, 13–14), do not participate in the equalization of the rate of profit. This view is also the main basis on which Okishio queries Marx's LTRPF. Unlike Sraffa, Okishio sets out to provide direct proof of the Ricardian proposition that non-basic goods industries do not participate in the equalization of the rate of profit, proceeding directly from his rewritten surplus value rate in Equation (3).

On the surface, there is no difference between Okishio's formula for the rate of surplus value and Marx's concept, but in fact they are not the same. According to Marx, the rate of surplus value depends on the size of the prepaid variable capital, the amount of surplus value, and the length of the working day. Of these, only the size of prepaid variable capital is relevant to the basic goods sector, and this depends on the necessary labor time of the labor force employed in reproduction. Marx's first stipulation is that the rate of surplus value will depend on the length of the working day and be positively correlated with it if the production technology and labor productivity in the basic goods sector remain unchanged. His second stipulation is that the rate of surplus value depends on the labor time required for labor reproduction, and thus on the production technology in the basic goods sector and related industries, only if the length of the working day remains constant. The first of these stipulations belongs to the production of absolute surplus value, and the second to the production of relative surplus value. In practical production within the capitalist economy, they are usually combined. As a result, the correct formula for expressing the rate of surplus value can only be m/v, and the rate of surplus value $\left(\dfrac{1-\Sigma b_j t_j}{\Sigma b_j t_j} \right)$ in Okishio's Equation (3) is actually the formula for relative surplus value. Here b_j is the real wage rate expressed in terms of consumer goods, and t_j is the labor consumed in producing those wage goods. If the real wage rate is constant and productivity increases, i.e., if $t_j > t'_j$,[10] then $1 - \Sigma b_j t'_j > 1 - \Sigma b_j t_j$ must be true for all industrial sectors. In other words, there is a relative increase in surplus labor or a relative increase in surplus labor time, which corresponds to the relative production of surplus value. According to

Okishio, what he provides is a general formula for the rate of surplus value; there-after, the rate of surplus value depends on the real wage rate and the labor produc-tivity of wage goods. Under the condition that the actual wage rate remains unchanged, the rate of surplus value depends on the production technology employed in the "basic goods sector." This ignores the length of the working day, another important factor in determining the rate of surplus value. It also ignores the fact that as part of their competitive behavior, individual capitals may increase their rate of surplus value by extending the workday or improving their own labor productivity, without relying on the labor productivity of the basic goods sector.

More seriously, Okishio believes that the rate of surplus value must increase, provided that the wage rate remains unchanged, if the basic goods sector intro-duces new technologies to improve the labor productivity of various wage goods. Nevertheless, he contends that the change of production technology in non-basic goods industries will not affect the rate of surplus value, and will not affect the general rate of profit. For this reason, he maintains, non-basic industries do not figure in the determination of the general rate of profit, but only passively accept the general rate of profit determined by the basic industries. In other words, according to Okishio's logic, the general rate of profit will definitely not be affected since profit is an expression of surplus value, and technological changes in the non-basic goods industries do not affect the rate of surplus value. It may be seen that Ricardo's view, which was criticized by Marx and upheld by Sraffa, completely confuses the determination conditions and formation mechanism of relative surplus value and average profit.

The production of relative surplus value is certainly the direct result of the decline in the value of commodities that constitute the value element of labor, and its price depends directly on the labor productivity of the basic goods industry. Nevertheless, the basic goods industry is an element in the production of the means of subsistence and is part of the second category, except for luxury goods. Under the conditions of socialized production, the investment of fixed capital and work-ing capital in the basic goods industry is absolutely inseparable from the produc-tion of capital goods in the first category. Meanwhile, the raising of labor productivity and the pursuit of technological progress in the basic goods industry are absolutely inseparable from their counterpart processes in the production of fixed and floating capital inputs in the first category. Marx states:

> The general rate of surplus value is, therefore, ultimately affected by the whole process, only when the increase in the productiveness of labour, has seized upon those branches of production that are connected with, and has cheapened those commodities that form part of, the necessary means of subsistence, and are therefore elements of the value of labour power. (Marx and Engels 2010f, 324)

But the value of a commodity is determined, he insists, "not only by the quantity of labour which the labourer directly bestows upon that commodity, but also by the labour contained in the means of production" (Marx and Engels 2010f, 320). A fall in the value of labor power may also be brought about

> by an increase in the productiveness of labour, and by a corresponding cheapening of commodities in those industries which supply the instruments of labour and the raw material, that form the material elements of the constant capital required for producing the necessaries of life. (Marx and Engels 2010f, 320)

The expanded reproduction of the total social capital involves the compensation for and renewal of the material goods and value created by enterprises in two major sectors through the process of circulation. It is in the complex competitive process of cross-sector capital flows that the surplus value produced by enterprises in different sectors and industries is transformed into average profit, and their respective rates of surplus value are transformed into the general rate of profit. Equal amounts of capital of various industrial enterprises compete in a rat-race for profit maximization, resulting in the formation of different rates of profit on account of the different organic composition of capital and the speed of capital turnover, and in the transformation of the average rate of profit, which in the field of capital bears no direct relation to the particular features of basic goods industries. Marx pointed out clearly that capitalist production is not only "a matter of obtaining an equal mass of value in another form—be it that of money or some other commodity—for a mass of values thrown into circulation in the form of a commodity," but also "a matter of realising as much surplus value, or profit, on capital advanced for production, as any other capital of the same magnitude, or pro rata to its magnitude in whichever line it is applied." In the form of the price of production, "capital becomes conscious of itself as a social power in which every capitalist participates proportionally to his share in the total social capital" (Marx and Engels 2010h, 194). In accordance with the fact that capital is a social power, and in response to Ricardo's erroneous view that non-basic goods industries in the luxury category do not participate in the process of equalization of the rate of profit, Marx pointed out in particular that even for luxury goods, technological improvements can raise the general rate of profit, because the rate of profit in these fields of production, like the rate of profit in all other fields of production, participates in the process of averaging all special rates of profit into the average rate of profit (Marx and Engels 2010d, 55). It is obvious that both basic goods industries and non-basic goods industries should participate in profit equalization and determine the formation of the general rate of profit through the overall circulation process of social reproduction. The view according to which only basic goods

industries take part in the production of relative surplus value, determining the value of labor and the general rate of profit, is fundamentally untenable.

If, as Okishio believes, non-basic industries cannot participate in the determination of the general rate of profit, and can only passively accept the so-called general rate of profit r determined by the basic industries, let the special rate of profit of the non-basic industries be r_f, supposing that the special rate of profit of the non-basic industries is lower than Okishio's general rate of profit, i.e., $-\Delta r = r_f - r$. It follows that the non-basic goods industries can only make up for the shortfall $(-\Delta r)$ by artificially increasing their prices to a level above that corresponding to the total profit, so as to achieve the average profit attributable to them under the condition that the average profit of other industries at this general rate is not affected. If this is the case, the so-called profits used to make up for the shortfall must have no value basis. If, on the contrary, the special rate of profit of the non-basic goods industries is higher than Okishio's general rate of profit, i.e., $\Delta r = r_f - r$, then according to Okishio the higher part (Δr) can neither be divided among other industries nor remain in the non-basic goods industry. For a capitalist economy to function in this way is inconceivable. It should be noted that the enterprises in the first category that produce the means of production tend to be capital-intensive, so their organic composition of capital is higher than that of the labor-intensive second category; consequently, the rate of profit of the first category enterprises will be lower than that of the second category enterprises before the equalization of profit, under the condition that labor can flow freely between the two categories and the rate of surplus value is the same. Denying the participation of non-basic goods industries in the equalization of profit implies that the first category enterprises that provide them with means of production do not participate in the equalization of profit. In that case, the rate of profit of the basic goods industry is bound to be overestimated, and the decline in the general rate of profit of the whole society to be underestimated. In addition, the decline in the general rate of profit must be compensated by an increase in the amount of profit, which will promote a continuous increase of the minimum amount of prepaid capital. As Marx points out, the tendency to such an infinite expansion of capital will have the following results: "In order to use an absolutely increased variable capital in a situation where higher composition or constant capital is relatively increased to a greater extent, the total capital should not only increase in proportion to the higher composition, but also increase more rapidly," since in the process of capital accumulation, "a large capital with a small rate of profit accumulates faster than a small capital with a large rate of profit." He continues: "At a certain high point this increasing concentration in its turn causes a new fall in the rate of profit" (Marx and Engels 2010h, 249). Under the influence of the law of surplus value, the infinite expansion of capital brings an absolute excess of capital, and at a certain point

the general rate of profit will experience "a steep and sudden fall" (250). Okishio confuses the conditions of determination and the mechanism of formation of relative surplus value and average profit, replaces the organic composition of capital with "the technical composition of production," accuses Marx falsely of maintaining that basic goods industries do not participate in the equalization of profit rate, and then denies the LTRPF. As a result, Okishio conceals the conflict, revealed by Marx, between the limited purpose of capital proliferation under the conditions of the capitalist mode of production and its unconditional development of social productivity, the conflict that creates the class antagonism between wage labor and capital in the production process. As a consequence, the fundamental cause of periodic economic crises is unknowable (248, 249).

Another important reason why Okishio draws this erroneous conclusion is the Sraffian mathematical model he uses to prove his proposition. In Okishio's model, as in that of Sraffa, non-basic goods industries do not participate in any way in determining the general rate of profit. Okishio regards this as an objective fact, fundamental to the nature of capitalism. Neither empirical observation nor capital's code of conduct, however, supports this view, only Ricardo's erroneous opinion—criticized by Marx, but adhered to by Sraffa—and Okishio's model. The model is an indeterminate equation with unknowns greater than the number of equations that can be solved by any given $1 = \Sigma b_i q_i$. Once q_2 or q_3 is given, r and other q can be determined, but different, ungiven q_2 or q_3 do not work for Okishio's general rate of profit r. For instance, let $q_3 = 60$; then $q_1 = 100$, $q_2 = 650/9$, and $r = 2/3$. At this point, $r = 2/3$ will not be changed no matter how the known parameters of the basic goods industry q_2 are altered, unless the numerical value of q_3 is changed. According to this logic of the indeterminate equation found in Ricardo's model, the general rate of profit r is determined by the non-basic goods industries q_3, independent of the basic goods industries q_2. In other words, the basic goods industries do not participate in the process of equalizing the profit rate. The result of solving the indeterminate equation is the exact opposite of the conclusion that Okishio presupposes. This conclusion is obviously absurd, but it flows necessarily from Okishio's mathematical model. At the same time, Okishio argues that the non-basic goods industries do not participate in the determination of the general rate of profit, and still less does he accept the law, revealed by Marx, according to which the profit rate is averaged into the general rate of profit.

3.5. The Okishio Paradox

It follows from the Okishio theorem that the introduction of new technology in the basic goods sector necessarily increases, rather than decreasing, the general rate of profit (r). For this reason, a decline in the general rate of profit must have different causes. In line with this logic, Okishio argues explicitly that the technological

innovations introduced by capitalists will not reduce the general rate of profit unless there is a sufficient increase in the real wage rate. The bourgeoisie is able to raise the rate of profit as long as the workers fail to increase their wages, because the movement of the rate of profit is determined by the success or failure of the workers' class struggle for higher wages (in practice, the economic struggle of the unions). The rise in the real wage rate b_{ij}, determined by the class conflict, is the only reason for the fall in the general rate of profit.

There is no doubt that income distribution is the main element in labor conflict, or that class conflict is an important factor determining the real wage rate in a capitalist economy. Classical political economy, the New Cambridge School and Marxist economics all prove that wages and profits (surplus value) are opposites, while Marx explicitly characterized this opposition as a determinant of change in the general rate of profit, making a theoretical investigation of this point (Marx and Engels 2010h, 168). If the Okishio theorem is tenable, however, the opposite conclusion can also be drawn: that a rise in the real wage rate does not hinder the rise of the general rate of profit. Class conflict, this would indicate, is not the reason for the decline of the general rate of profit, since this general rate is determined not only by the real wage rate but also, under the cost constraint that accompanies the general rate of profit condition given by Okishio, by the "organic composition of production" $\Sigma a_{ij} t_j / \tau_i$, which in turn depends on the amount of direct inputs a_{ij} and the amount of direct labor τ_i per unit of goods that embodies the production technology. As a general rule, wage workers can determine the real wage rate b_{ij} and the amount of direct labor τ_i under special conditions (such as assuming that there is no industrial reserve force), but they cannot determine the choice of technology a_{ij} or the "organic composition of production," which are determined and controlled by the capitalists. If forced by special conditions to make concessions on the real wage rate, capitalists, according to the Okishio theorem, can increase the general rate of profit by choosing new technologies at the new real wage rate. Therefore, every real wage rate in any sequence of rising real wage rates can be defined as a given, and there must be a corresponding rising sequence of the general profit rate, with significance for Okishio's arguments.

In the numerical examples given by Okishio, for example, it is assumed that the real wage rate rises from 1/45 to 1/20. If other conditions remain unchanged, the general rate of profit of the capitalist will be reduced from 50 to 0%. However, if the capitalist adopts a new technology, so that $a_{21} = 1/8$, $\tau_2 = 31/37$ and the labor required to produce a unit of wage goods is reduced from 20 units to 3.34 units, the "technical composition of production" rises from $\Sigma a_{ij} t_j / \tau_i = 0.333$ to 3, and the cost falls from $\Sigma a_{21} q_1 + \tau_2 = 30$ to 8.34. Okishio's general rate of profit r will then rise from 0 to 64%. It is assumed that the real wage rate rises further from 1/20 to 1/15. If the capitalist adopts a new technology so that $a_{21} = 1/25$ $\tau_2 = 1/5$ and if the

labor required to produce a unit of wage goods is reduced from 3.34 units to 1 unit, the "technical composition of production" rises from 3 to $\Sigma a_{ij} t_j / \tau_i = 4$, and the cost falls from 8.34 to $\Sigma a_{21} q_1 + \tau_2 = 2.6$. Okishio's general rate of profit r will then rise further, from 64 to 81.9%. This result is the exact opposite of what Okishio anticipates, since the increase in the real wage rate does not reduce the general rate of profit, but rather causes it to rise further. It is thus obvious that the capitalist class can still raise Okishio's general rate of profit significantly by adopting new technologies, even if the Okishio theorem ensures that the working class raises the real wage rate through economic struggle. The fact that the real wage rate and Okishio's general rate of profit r raise each other is the "Okishio paradox" that is derived necessarily from Okishio's theorem. This outcome completely negates Okishio's efforts to explain the decline in the general rate of profit on the basis of class struggle.

If class struggle, instead of reducing the general rate of profit, had become a factor contributing to its rise, there would be no need for capitalist countries to dread labor conflicts and to formulate various policies in a vain effort to avert them.

3.6. The Basic Assumption and Essence of the Okishio Theorem

The basic assumption of the Okishio theorem is that the real wage rate is constant; that is to say, the Okishio production price in basic goods industries is not affected by the technical choices that these industries make. From the example he gives, Okishio also appears to assume that the technological choices made by capitalists must satisfy both the "cost criterion" and the "productivity criterion." If this assumption is made, the Okishio theorem is always tenable. However, the assumption is erroneous, a product of Okishio's confusion of living labor with necessary labor, and of relative surplus value with average profit. Through this confusion, Okishio denies the fetishistic nature of the capitalist mode of production, in which the objective means of production dominate the subject of living labor, and he also denies the class antagonism in which all wage workers are actually and absolutely subordinate to the equal social power of all capitalists. Okishio's judgment on the use by capitalists of improved technology to raise the general rate of profit is not based on the laws of motion of the rate of profit as determined by the contradictions inherent in capitalism and the power of capital. Instead, the resulting general rate of profit is held to be determined by the marking up of commodities rather than by the division of the total surplus value, and the mutual markup of circulation is a zero-sum game. The Okishio theorem is nothing more than transcendentalism divorced from the reality of capitalism.

4. The Mechanism of the Law of Tendency for General Rate of Profit to Fall: A Game Perspective

From Okishio's analysis, it can be seen that Okishio actually believes that the first and second volumes of *Capital* are contradictory. Since capitalism only chooses new technologies that can increase profits, and since a rise in the organic composition of capital implies an improvement of labor productivity and leads to an increase in the production of relative surplus value that raises the rate of surplus value, then it cannot be concluded that the general rate of profit decreases with an increase in the organic composition of capital. Here, Okishio's analytical logic is identical to that of Robinson and Sweezy, who reject the LTRPF (Robinson 1962; Sweezy 1997). The technical choices of all capital and their consequences are viewed and understood in exactly the same way, in terms of maximizing the decisions of individual capital, assuming that the decisions that benefit individual capital will also make all capital equally profitable. It can be found that this analysis of the behavior of "decision theory" capital reflects a failure to understand the universality of the general law of proportional distribution of total labor, which is reflected in the special antagonism of the capitalist private economy that results from the post factum distribution of labor shares by exchange value. The result will be a failure to understand the contradiction in Marx's analysis of capitalism. Analysis of the technical choices of capitalism reveals that the technical choices made by all individual capitals on the basis of excess profit will lead to completely opposite results, with the individual subjective motivations and purposes of all capitals influenced by the value law of capital competition (Marx and Engels 2010f, 338–339; 2010h, 228–230, 263). This is an expression of the conflict between the "individual rationality" and "collective rationality" of the capitalist mode of production, and represents a "prisoner's dilemma" arising out of the drive for profit maximization. The "prisoner's dilemma," as a specific mechanism of the LTRPF, is the contradiction between the social price of production and the individual price of production, and is determined by the internally antagonistic contradiction of total labor distribution under capitalism. It is necessary to analyze the mutual strategic behavior of the production parties as determined by the internally antagonistic contradiction of total labor distribution under capitalism, and to analyze the competition of capital for maximum profit along with its consequences. This is the fundamental difference between Marx's analysis of the behavior of capital and that of Okishio, Sweezy, Robinson, and others, and it is also an important reason why the latter analysis makes it difficult to understand the LTRPF.

To illustrate the mechanism of action of the LTRPF as revealed by Marx, the general mathematical model of Marx's LTRPF is given first. From Marx's formula for the rate of profit $\pi' = \pi/C = m/(c+v)$, the following can be obtained:

$$\pi' = \frac{m'}{k+1}, \tag{10}$$

where $k = \dfrac{c}{v}$ is the organic composition of capital, unlike the previous k.

Full differentiation of Equation (10) can be obtained:

$$d\pi' = \frac{1}{k+1}dm' - \frac{m'}{(k+1)^2}dk. \tag{11}$$

If nothing else is considered, then according to Equation (11) there is a positive correlation between the rate of profit and the rate of change of surplus value, and a negative correlation between the rate of profit and the change in the organic composition of capital. If the rate of surplus value is constant ($dm'/dt = 0$), the rate of profit will decrease as the organic composition of capital increases. We then find:

$$d\pi' = -\frac{m'}{(k+1)^2}dk.$$

The above correlations are still tenable when the capital of different sectors is considered as a whole. Assuming that a capitalist economy has n production sectors, let $m_S = \sum\limits_{i=1}^{n} m_i$, $c_s = \sum\limits_{i=1}^{n} c_i$ and $v_s = \sum\limits_{i=1}^{n} v_i$, where the subscript s denotes the whole society. The production sector $i = 1, 2, \cdots, n$, and the average rate of profit p'_s is:

$$p'_s = \frac{m_s}{c_s + v_s} \rightarrow p'_s = \frac{m'_s}{k_s + 1}.$$

The mathematical model of the LTRPF can be obtained from the full differentiation of the above equation:

$$dp'_s = \frac{1}{k_s + 1}dm'_s - \frac{m'_s}{(k_s + 1)^2}dk_s. \tag{12}$$

Obviously, if the rate of surplus value is constant ($dm'_s/dt = 0$), the general rate of profit decreases with an increase in the organic composition of capital. The following is then obtained:

$$dp'_s = -\frac{m'_s}{(k_s + 1)^2}dk_s.$$

Assuming that the socially necessary labor time for the production of a certain type of commodity in a capitalist economy is t_s, the value of the commodity determined by the socially necessary labor time is transformed into the social price of production P_s after the rate of profit is averaged out through competition.[12] At a given social price of production P_s, individual capital can raise labor productivity by improving its technology so as to lower the individual price of production, and by then selling commodities at the social price of production for an excess profit. This will give rise to competition between individual capitals for excess profits. The result of this competition is on the one hand an increase in the organic composition of capital and labor productivity, and, on the other, the disappearance of excess profits and a decrease in the average rate of profit (Marx and Engels 2010f, 270). For convenience of analysis, the intricate competition between individual capitals within and across sectors is simplified below. Assuming that the above process begins with Sector j, the social price of production of the sector on a per unit commodity basis can be expressed as:

$$p_{s,j} = \frac{K_{g,j}T^{-1}}{Q_j} + K_{l,j} + \frac{N_j w_j + \pi_{P,j}}{Q_j}. \tag{13}$$

Here, K_g and K_l are fixed capital and current constant capital at production prices, respectively, and T is the useful life of fixed capital. Q_j is the total output of the sector at a given time, and the consumption of working capital is in a linear correlation $K_l Q_j$ with output, so the consumption of working capital per unit of output is K_l. N_j is the number of workers employed in Sector j, w_j represents the money wages per capita at the production prices of wage goods, and $N_j w_j$ is the total wages, i.e., the total prepaid variable capital. $\pi_{p,j}$ is the average profit of Sector j calculated on the basis of the technology and social production conditions. It is assumed that Sector j adopts new technologies that improve labor productivity, and has lower individual production prices for excess profits. According to Marx, an increase in the productiveness of labor through an alteration in the labor process endows a given quantity of labor with the power of producing a greater quantity of use value (Marx and Engels 2010f, 319, 320). This effect appears in the diminution of the mass of labor in proportion to the mass of means of production applied by it. This incorporation of labor power with increasing quantities of the means of production is thus an expression of the growing productiveness of labor, and represents an increase in the technical composition of capital (Marx and Engels 2010f, 617). It is assumed that when Sector j adopts new technologies that improve labor productivity, this is in order to raise the technical composition of capital, and assuming that the production prices of wage goods and investment goods in Sector j remain unchanged, then the increase in the technical composition of capital in Sector j is manifested as an increase in the value composition of

capital and thus in the organic composition of capital. Let us suppose that the growth rate of fixed capital usage is b, and that the number of workers employed is reduced in proportion α. The rate of output growth following the increase in labor productivity is g, and $g > b$, $\alpha < 1$. Since the technology of the production price and its social production conditions remain unchanged, the Sector j still obtains the same social average profit $\pi_{p,j}$. The individual production price of Sector j is then:

$$p_{j,j} = \frac{K_{g,j}(1+b)T^{-1}}{Q_j(1+g)} + K_{l,j} + \frac{N_j(1-\alpha)w_j + \pi_{p,j}}{Q_j(1+g)}. \tag{14}$$

It follows from $(1+b)/(1+g) < 1$ and $(1-\alpha)/(1+g) < 1$ that $p_{j,j} < p_{s,j}$. At the same time, the capital composition of Sector j in terms of production prices has increased significantly. The result is that

$$\frac{K_{g,j}(1+b)+K_{l,j}}{N_j(1-\alpha)w_j} > \frac{K_{g,j}+K_{l,j}}{N_j w_j}.$$

Depending on the supply and demand situation, Sector j can obtain a certain amount of excess profit $\Delta\pi$ so long as the sector can set the selling price of the product p_j at $p_{j,j} < p_j < p_{s,j}$ at the profit level $\pi_{p,j} + f(p_j)$, $p_{j,j} < p_j < p_{s,j}$. This will lead to competition among industry sectors for excess profits. The capital composition and technical conditions of different sectors are different, but the competitive strategies adopted by these sectors in order to obtain excess profits are the same. In other words, they reduce individual production prices by increasing labor productivity, and make use of the difference between individual production prices and social production prices to obtain excess profits. This strategy, which is effective for individual sectors, is nonetheless ineffective when applied to all sectors. The reason is that the social necessary labor time for the production of various commodities and the technical and social production conditions that determine the production price will change once all sectors adopt the same competitive strategy to improve labor productivity generally, and once the individual production prices of enterprises or sectors undergo a general decrease. With the general improvement of labor productivity in all sectors, social production prices will converge around the individual production prices with the largest weight of commodities in all sectors, or around the individual production prices organically composed of the median capital of commodities in all sectors. The difference between individual production prices and social production prices will disappear, and excess profits will disappear, so that $\Delta\pi = 0$. As a result, the organic composition of capital undergoes a general increase, and the general rate of profit decreases. It is assumed that until labor productivity is generally improved on account of

competition (period t), the technical and social production conditions expressed in terms of the organic composition of capital are:

$$k_t = \frac{K_{g,s} + K_{l,s}Q_s}{N_s w_s}. \tag{15}$$

Accordingly, the social production price and the average profit rate are respectively:

$$P_{s,t} = (K_{g,s} + K_{l,s}Q_s + N_s w_s)(1 + p'_{s,t}),$$

$$p'_{s,t} = \frac{\pi_t}{K_{g,s} + K_{l,s}Q_s + N_s w_s}. \tag{16}$$

The social price of production per unit of commodity is:

$$p_{s,t} = \frac{K_{g,s}T^{-1}}{Q_s} + K_{l,s} + \frac{N_s w_s + \pi_{p,t}}{Q_s},$$

where $\pi_{p,t}$ is the average profit of the sector calculated at the average social rate of profit for the period t. Assuming that after the general increase in labor productivity through competition (period $t + 1$), the individual production conditions of Sector j become technical and social production conditions, and consequently, the individual production prices of Sector j become social production prices and the production prices of their individual units of goods become social production prices of units of goods, we then have the following:

$$k_{t+1} = \frac{K_{g,j}(1+b) + K_{l,j}Q_j(1+g)}{N_j(1-\alpha)w_j}, \tag{17}$$

$$P_{s,t+1} = [K_{g,j}(1+b) + K_{l,j}Q(1+g)_j + N_j(1-\alpha)w_j](1 + p'_{s,t+1}),$$

$$p_{s,t+1} = \frac{K_{g,j}(1+b)T^{-1}}{Q_j(1+g)} + K_{l,j} + \frac{N_j(1-\alpha)w_j + \pi_{P,t+1}}{Q_j(1+g)}.$$

The average rate of profit is:

$$p'_{s,t+1} = \frac{\pi_{t+1}}{K_{g,j}(1+b) + K_{l,j}Q(1+g)_j + N_j(1-\alpha)w_j}. \tag{18}$$

Since the social price of production converges around the price of production determined by the new technology and the new social conditions of production, the excess profit disappears, that is, $\Delta\pi = 0$. Comparing the situations before and after the increase in labor productivity, it is obvious that $k_{t+1} > k_t$ and $p'_{t+1} < p'$. It

is thus clear that the result of competition is on the one hand to increase the organic composition of capital and labor productivity, and on the other, to decrease the average rate of profit. This is a standard "prisoner's dilemma." All sectors are considered as players A in the game, and the players are divided into i and $-i$, while there is the same strategy set in which technology and labor productivity (e) can be improved and technology and productivity (b) remain constant, i.e., $S_{i,-i} = \{e, b\}$. The payment matrix is as follows, where $\pi + \Delta\pi > \pi > \pi_j > \pi - \Delta\pi$.

		A_{-i}	
		b	e
A_i	b	π, π	$\pi - \Delta\pi, \pi + \Delta\pi$
	e	$\pi + \Delta\pi, \pi - \Delta\pi$	π_j, π_j

Obviously, whether for A_i or A_i the strategy e is strictly dominant, and the optimal strategy combination is $s_{i,-i} = (e, e)$. From Equation (17) and Equation (18), the result must be an increase in the organic composition of capital and a decrease in the general rate of profit, i.e., $k_{t+1} > k_t$ and $p'_{t+1} < p'_t$, "which is, therefore, wholly independent of the will of the capitalist" (Marx and Engels 2010h, 264). The action mechanism of the law of the tendency of the general rate of profit to fall has been analyzed and illustrated above by means of a static game model using complete information. A dynamic model will not alter this conclusion, but will be more in line with Marx's analysis.

5. Conclusion

Analysis in this article shows that the Okishio theorem is erroneous, and cannot be used to refute the general law of the tendency of rate of profit to fall revealed by Marx.

First, the production price in the Okishio theorem is not the production price defined by Marx. Marx's production price is based on the theory of labor value and surplus value, which reveals the total exploitation relationship that applies between all capitalists and all employed workers. The production price in the Okishio theorem confuses the labor consumed in production with the labor calculated at the wage rate, and the cost of the commodity with the value of the commodity. The average profit calculated with this production price is the product of an artificial markup occurring in the course of circulation and lacking a value basis. This distorts the essence of the transformation of capitalist surplus value into profit. A capitalist production system constructed using this production price will inevitably result in the error that the total production price is greater than the

total commodity value, and the total average profit is greater than the total surplus value. This cannot meet Marx's constraint that the two aggregates must be equal. In essence, it reflects not the division of the total surplus value by all capitalists, but the behavior of capitalists when they mark up and offset each other in the process of circulation. The general rate of profit obtained from such a production system has nothing to do with the capitalist mode of production, and deviates from Marx's average profit production price theory.

Second, Okishio obtains his so-called production price by reversing Marx's original production price at the monetary wage rate. This shows that Okishio does not understand the essence and payment characteristics of capitalist wages, but replaces the dialectical logic of Marx's economics with mathematical formal logic. Okishio's calculation of commodity prices in terms of wage rates does not follow Marx's scientific insight that the abstract labor consumed in production determines the commodity value and surplus value, but Smith's erroneous view that the value of commodities is determined by the labor purchased in exchange for them, i.e., the wages. Okishio confuses the role of relative surplus-value production with the average profit formation mechanism, and denies that the non-basic goods sector also participates in the process of equalization of profit. As this indicates, Okishio disguises the inherent antagonistic contradictions of the capitalist mode of production and ignores the equal social power of capital to exploit wage labor, following the erroneous ideas of Ricardo. The theoretical basis of the Okishio theorem is not Marxist economics, but the vulgar elements of classical political economy.

Third, and based on the above mistakes in the Okishio theorem, the unreasonable assumption that the real wage rate is constant comes to replace Marx's scientific assumption, in his "rise from the abstract to the concrete" argument for the price of production, that the rate of surplus value is constant. This further shows that Okishio does not understand the scientific method that should be followed for a consistent examination of the general rate of profit, and that he changes the nature of the problem under discussion. This is one of the reasons why the process of demonstrating the Okishio theorem falls short methodologically.

Fourth, the capitalist production system as constructed in the Okishio theorem takes its lead from Sraffa's system, in which the price of production and the average rate of profit are determined simultaneously (Sraffa 1963, 12). This is considered to be a prominent advantage of such a system. In reality, this "simultaneous determination" theory is the system's major defect. The "simultaneous determination" system is relatively static, and completely excludes the possibility that individual enterprises can gain excess profits by improving their technology and selling at the social production price while lowering their individual production prices. At the same time, it also excludes the resulting competition between

individual capitals in different industries. It is this competition that eventually leads to the disappearance of excess profits, as well as to changes in production price, technology and social production conditions, and that therefore results in the tendency of the general rate of profit to fall. As a result, the "simultaneous determination" system fails to take account of the strategic behavior of capital as it engages in competition as determined by the internal contradictions of capitalism. Instead, it proposes that the operations of individual capital are based on "decision theory," rather than being dynamic and of a "game theory" nature. Consequently, it cannot provide a basis for understanding the LTRPF. The "simultaneous determination" system in the Okishio theorem is in fact a set of mathematical simultaneous equations which in essence have either no solution or infinite solutions. It does not amount to a concrete reproduction of the dialectical theory of capitalist social reproduction.

The study of the classical propositions of Marxist economics using mathematical methods is in line with the heuristics of Marxist economics. In this respect, Nobuo Okishio may be regarded as the first prominent scholar to study the LTRPF using rigorous modern mathematical methods. This is part of a trend in the development of Marxist economics, and in this respect Okishio provides a useful reference. The long-term debate triggered by the Okishio theorem has objectively advanced the study of the LTRPF in Marxist economics, and has deepened understanding of the LTRPF among scholars. Accordingly, study of the LTRPF using mathematical methods has become a prominent feature of the field, and this may be rated as one of the contributions Okishio made. However, the prerequisite for employing mathematical methods is to follow the logic and principles of Marxist economics; otherwise, the way is open to the "mathematical abuse" criticized by Paul Romer (2015). Unfortunately, this is precisely the problem with the Okishio theorem, and the contribution to Marxist economics is thus negated. The debates surrounding the Okishio theorem over the years have ignored its fundamental errors in the areas of mathematical application and theoretical foundation.

Notes

1. In this article, the general rate of profit and the average rate of profit are synonymous. Therefore, they are used alternately based on the writing style.
2. For details, see Marx and Engels (2010h, chapters 9 and 10). Cost price was used by Marx in the "Economic Manuscripts of 1861–63" (Marx and Engels 2010d, 401–404).
3. See Marx and Engels (2010h, 172). Morishima demonstrates that these two conditions of Marx cannot be satisfied by calculating commodity prices using the wage rate (Morishima 1973, 79–80).
4. In the contemporary economics literature, Keynes was the first to use the wage rate to calculate price, using this method to eliminate the effects of price changes. See Keynes (1983).
5. Marx uses the term "production cost" in relation to different scenarios. The first is the real production cost based on the labor theory of value, and is the sum of the materialized labor and direct

labor spent on the production of goods, that is, the value of the goods. The second is the prepaid capital required for the purchase of the means of production and labor commodities. The third is the production price determined by the price of capital advanced plus average profit (Marx and Engels 2010d, 264). Vulgar economics denies or obscures Marx's theory of surplus value. "Thus, from the viewpoint of capital, the production cost is not the actual cost of production, this is because surplus labor does not cost capital a penny." "This has caused much confusion: people either say that profit is not realized in exchange, but is generated from exchange . . . or say that capital has a magic to make something out of thin air" (Marx and Engels 2010b, 142).

6. According to this thesis, uniform measurement has no meaning.

7. What capitalists are compelled to observe is neither Okishio's untenable cost concept, nor the cost concept calculated in terms of value or labor time, but the cost concept calculated in monetary price, that is, the cost concept in the form of prepaid capital conversion. It is impossible for capitalists to follow Okishio's "cost criterion."

8. The unity of the opposition between the technical and social conditions of the labor process constitutes the movement of the mode of production, see Marx and Engels (2010f, 319, 320). These are "not two different actual processes, but the same process examined on the basis of its content and its form respectively" (Marx and Engels 2010c, 140).

9. The means of production are still capital even if outside the production process, while labor cannot survive without the production process. See Marx and Engels (2010g, 43, 44).

10. The labor consumption under the new technology is $t_j^{'}$. The increase in productivity means a decrease in labor consumption, so $t_j > t_j^{'}$.

11. This calculation is merely the average rate of profit for industry, not the full average rate of profit, including business capital. According to Marx and with reference to the study by Luo (1990), the full average rate of profit can be expressed by the equation: $\bar{P}' = \dfrac{\bar{P} - (K+b)}{(c+v) + (B+K+b)}$, where $c + v$

is the total industrial capital prepaid, and $B + K + b$ is the capital invested by commercial capital in the purchase of goods, the commercial constant capital (material operating expenses), and the commercial variable capital, respectively. The quantitative limits of the average profit that commercial capital can divide are defined on the basis that industrial capital not only creates surplus value, but also converts value into the price of production. Since Marx's analysis of the LTRPF is an analysis of the creation of surplus value by industrial capital, this thesis still employs the industrial rate of profit and the resulting price of production. It can be shown that the result of this analysis will not be changed if the full average rate of profit is used.

12. The social price of production is the weighted average price of production based on the market value formation mechanism, which is formed from individual production price through the market competition in various sectors.

Acknowledgements

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