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Review

Uncertain design optimization of automobile structures: A survey

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Abstract: In real life, there are a lot of uncertainties in engineering structure design, and the potential uncertainties will have an important impact on the structural performance responses. Therefore, it is of great significance to consider the uncertainty in the initial stage of structural design to improve product performance. The consensus can be reached that the mechanical structure obtained by the reliability and robustness design optimization method considering uncertainty not only has low failure risk but also has highly stable performance. As a large mechanical system, the uncertainty design optimization of key vehicle structural performances is particularly important. This survey mainly discusses the current situation of the uncertain design optimization framework of automobile structures, and successively summarizes the uncertain design optimization of key automobile structures, uncertainty analysis methods, and multi-objective iterative optimization models. The uncertainty analysis method in the design optimization framework needs to consider the existing limited knowledge and limited test data. The importance of the interval model as a non-probabilistic model in the uncertainty analysis and optimization process is discussed. However, it should be noted that the interval model ignores the actual uncertainty distribution rule, which makes the design scheme still have some limitations. With the further improvement of design requirements, the efficiency, accuracy, and calculation cost of the entire design optimization framework of automobile structures need to be further improved iteratively. This survey will provide useful theoretical guidance for engineers and researchers in the automotive engineering field at the early stage of product development.

Keywords: automobile structures; design optimization; uncertain optimization; automobile design

1. Introduction

Optimization technology has always been an important means of decision-making for mechanical components and systems, playing an important role in improving performance objectives [1–4]. In the actual mechanical structure design, there are inevitable uncertainties in the structural design parameters and the external environment. The parameters related to each structure and the input of the external environment inevitably have uncertainties, including size parameters, material parameters, loads, highly nonlinear working conditions, etc. [5–7]. Ignoring the existence of uncertain factors, it is easy to cause huge differences in the performance response of the designed mechanical structures under very small input fluctuations. In serious cases, mechanical structures will directly cause failures, posing a threat to human life safety [8–16]. The failure source of automobile structures needs to consider the uncertainty factors inevitably. In our life, although the accident is sudden, the mechanical structure failure is not sudden. The failure of automobile structures largely comes from the accumulation of early uncertain factors, which is a typical evolution phenomenon of uncertainty from "quantity" to "quality" [17,18]. According to relevant reports, NASA has investigated the causes of structural failures in spacecraft and found that at least 21.4% of component failures in spacecraft structures are caused by uncertainties in the external environment, and 30.3% of component failures are caused by uncertainties in design and processing [19]. Therefore, to further reduce automobile structural accidents caused by uncertain factors and design more reliable or robust products, it is necessary to develop an uncertain design optimization method for automobile structures under the premise of comprehensive consideration of different structural performances. To more vividly describe the uncertainty propagation of automobile structures in the development stage, Figure 1 shows the uncertainty transmission of automobile structures from theoretical design to actual manufacturing.

Figure 1 shows the uncertainty propagation of the calculation input, the model uncertainty, the manufacturing process uncertainty, and the output performance response that may exist in the automobile structure design framework. Relevant research shows that the uncertainties in mechanical structure design mainly include three types [20–22]: (I) The uncertainty of design variables and other design parameters related to structural performance response, which mainly includes the uncertainty of design variables and related design parameters caused by structural design size [23], material characteristics [24], load changes [25], measurement and manufacturing installation errors [25], etc. (II) The inherent uncertainty in different theoretical models, which mainly includes the uncertainty caused by the conversion from the actual engineering model to the mathematical model and from the mathematical model to the computer simulation model [26]. It must be noted that in the process of converting the physical model of the actual project into a specific mathematical model, all nonlinear relationships in the physical model cannot be represented by an accurate mathematical model, and when using a variety of different simplified models for computer analysis and calculation, the final results of the computer output will also be inaccurate. (III) The uncertainty of relevant numerical models, mainly includes the influence of underestimation of errors in some numerical values and errors in the calculation of mathematical equations in the process of uncertainty modeling under actual conditions [21].

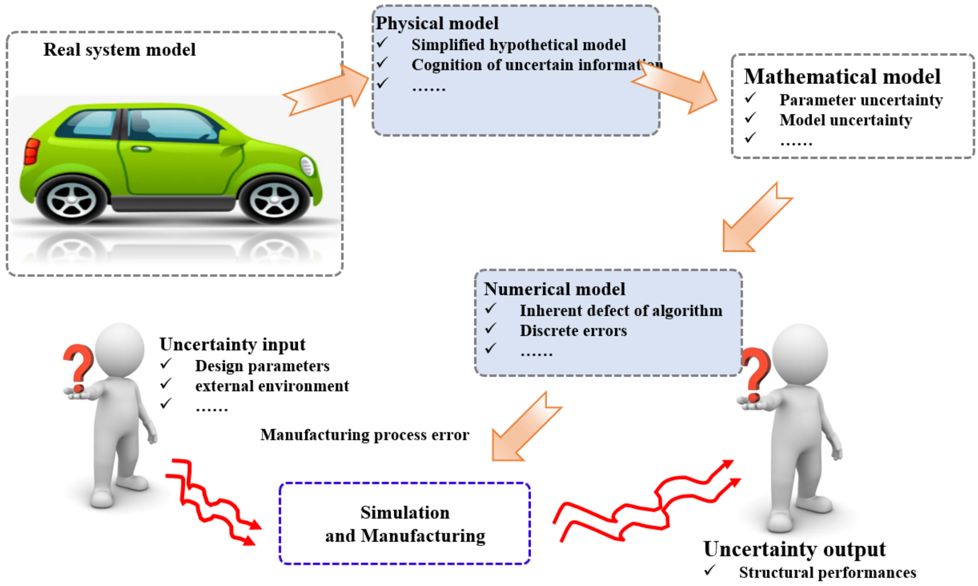

Figure 1. Schematic diagram of uncertainty propagation during automobile structure design.

According to different forms of uncertainty, the uncertainty representation methods can be roughly classified into random uncertainty and cognitive uncertainty [27–30]. Among them, the random uncertainty problem is generally used to describe the accidental factors of the physical system itself. This kind of uncertainty is usually caused by the randomness of the input data. At present, probability statistics is one of the best methods to describe this kind of uncertainty. In addition, cognitive uncertainty is caused by incomplete knowledge or incomplete information acquisition, and non-probabilistic analysis methods are commonly used to represent cognitive uncertainty. To a certain extent, based on different actual situations, cognitive uncertainty can be eliminated. As shown in Figure 2, the uncertainty forms are divided into random uncertainty and cognitive uncertainty, in which the cognitive uncertainty is mainly based on the existing knowledge system of humans to make design decisions on the uncertainty model.

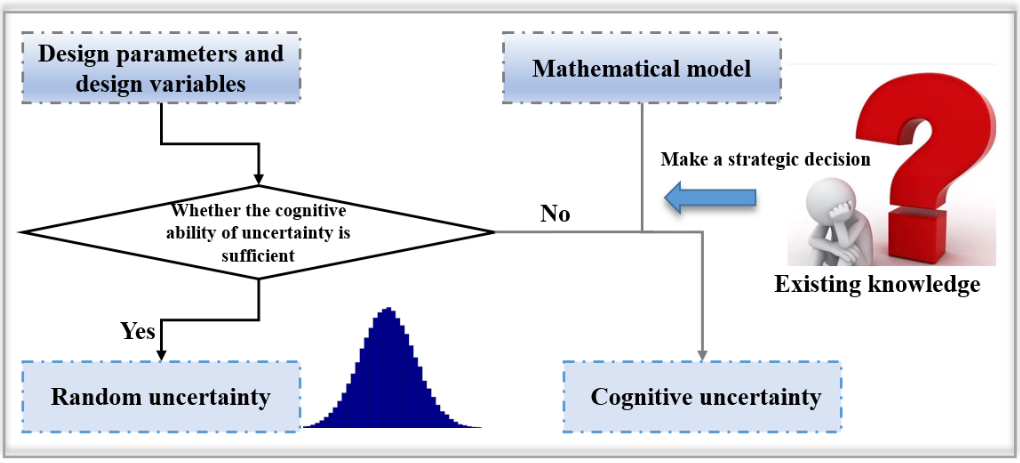

Figure 2. Classification of main types of uncertainties.

For complex vehicle structures, obtaining random uncertain information often requires a lot of test costs and simulation costs. Therefore, as an alternative uncertainty analysis model that does not require too much statistical data, the uncertainty analysis method with cognitive uncertainty is favored by more and more researchers and has a better engineering application prospect [31]. Cognitive uncertainty can be described mainly by fuzzy uncertainty [32], interval uncertainty [33], etc. Fuzzy uncertainty is an extension of fuzzy set theory, among which the fuzzy set theory was proposed by Professor Zadeh in his study [34]. When fuzzy sets are used to describe uncertain information, fuzzy sets can be regarded as sets with fuzzy boundaries. Each fuzzy set corresponds to a membership function distributed between 0 and 1 to indicate the possibility that elements belong to a set [35]. Inevitably, fuzzy uncertainty modeling needs to consider the selection of an appropriate membership function, and often how selecting an appropriate membership function requires certain experience or certain sufficient data information. Therefore, the nature of fuzzy uncertainty also limits its further engineering application development. For interval models, even if they are limited by test conditions or the data samples lack information about a probability distribution or membership function, they can still carry out uncertainty analysis according to the wrapping boundary of limited data. The interval model is developed based on the theory of interval numbers, which was proposed by Professor Moore in his literature [36]. The interval uncertainty analysis model is a method that uses the concept of "focusing on boundary information" to describe the uncertainty information. The model does not care about the specific distribution of uncertain data, but only about the upper and lower boundaries of its uncertainty value, which can realize the structural uncertainty analysis and design optimization of imprecise probability distribution under the condition of few samples [37]. As long as the uncertainty information in the vehicle structure design can be effectively described, no matter which uncertainty analysis method can be used. However, the design optimization of automobile structures needs to consider the actual data conditions and design costs. Therefore, how to carry out efficient design optimization considering uncertainties is also worth studying.

In this survey, some research related to uncertainty design optimization for automobile structures is discussed. Section 2 describes the uncertainty design optimization framework. In Section 3, relative uncertainty analysis methods are introduced. Section 4 describes some main multi-objective iterative optimization models for structural design. Finally, Section 5 presents the principal conclusions.

2. Uncertainty design optimization framework

The design optimization of automobile structures is one of the main means to improve lightweight and mechanical properties. In recent years, some research on single-objective, multi-objective, and multidisciplinary optimization of automobile structures has been carried out successively [38–50]. The conventional uncertainty optimization framework is mainly composed of nested optimization design parts, that is, the uncertainty analysis problem is regarded as an internal optimization problem, and the acquisition of the optimal scheme is regarded as an external optimization problem, as shown in Figure 3. The purpose of internal optimization is the uncertainty analysis module, which is mainly used to evaluate the propagation of uncertainty and feed it back to the external optimization route. Nested optimization requires multiple iterations and adding an additional optimization solver will lead to low computational efficiency. Therefore, some studies that can be used for reference have implemented the decoupling of uncertainty analysis to solve structural design problems with various uncertainties [51,52].

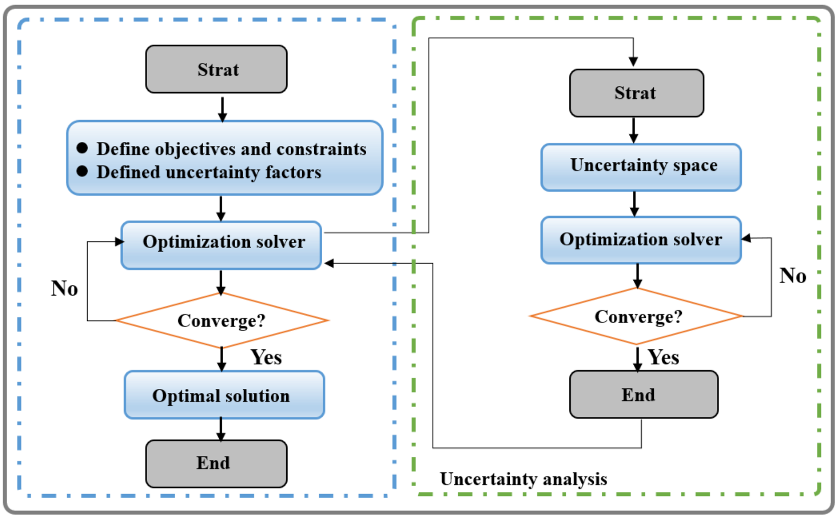

Figure 3. Nested uncertainty design optimization framework.

2.1. Design optimization considering single uncertainty factor

To improve the structural performance of automobile parts to resist the risk caused by uncertain factors, many researchers have gradually realized the importance of uncertain design optimization of automobile structures [53–59]. A lot of work in the field of design optimization considering a single uncertainty factor for automotive structures was carried out, as shown in Figure 4.

Lightweight has always been the eternal theme of automobile design, in which structural optimization is one of the main means of automobile lightweight [60–62]. For example, to obtain a more reliable car door design, Fang et al. [63] proposed a door design optimization framework based on multi-objective reliability. This optimization framework uses approximate model technology to replace expensive finite element simulation and combines descriptive sampling technology with Monte Carlo simulation technology. The results show that the proposed optimization framework can generate a well-distributed Pareto boundary of reliable solutions, and it is recommended to select the best from the relatively insensitive regions. Niu et al. [64] proposed a hybrid multi-objective uncertainty design optimization method to make an appropriate trade-off between the lightweight and fatigue durability of the truck cab. They found that uncertainty may lead to unstable or even useless optimization design, which may be more serious in uncertain optimization. By using the Taguchi technology, according to the validation simulation model for the fatigue test, the interval of design variables can be refined, and subsequent optimization can be carried out. Sun et al. [65] proposed a multi-objective discrete uncertainty optimization algorithm (MODRO) framework for vehicle crash design. The multi-criteria decision model in the MODRO program is mainly composed of grey relational analysis (GRA) and principal component analysis (PCA), which convert multiple conflicting objectives into a unified cost function. Moreover, the continuous Taguchi method is used for iterative optimization, which avoids the limitation that the traditional Taguchi method cannot handle a large number of design variables and design levels. The results show that the algorithm can achieve an optimal design in a quite effective way due to its integration with the multi-criteria decision model. In addition, it is found that the optimal value is close to the corresponding Pareto front generated by other methods (such as non-dominated sorting genetic algorithm II).

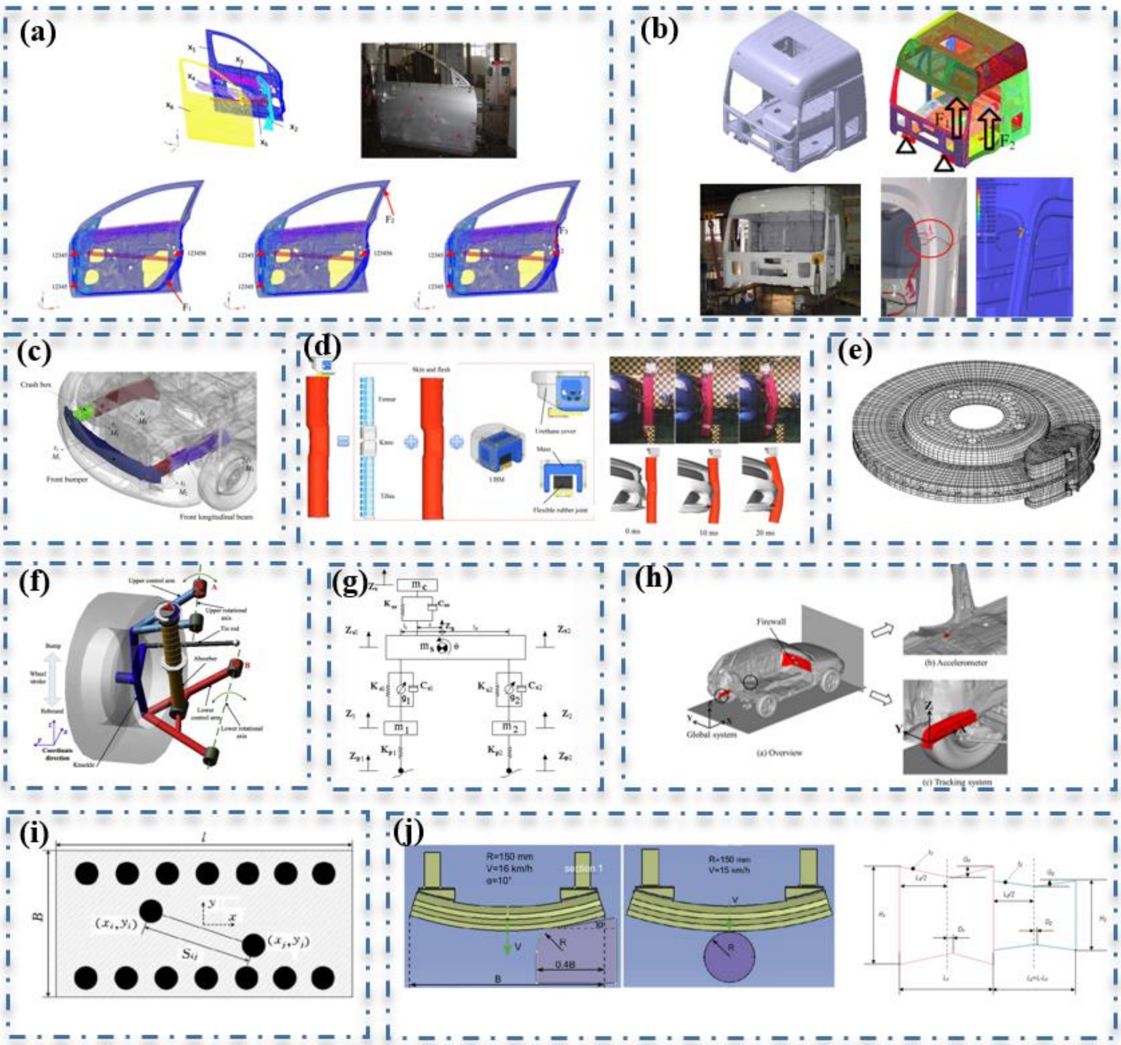

Figure 4. Design optimization considering single uncertainty factor. (a) [63], (b) [64], (c) [65], (d) [66], (e) [67], (f) [68], (g) [69], (h) [70], (i) [71], (j) [72].

Based on the above methods, Lei et al. [66] developed a structure optimization framework for the front end of the vehicle which is affected by the flexible pedestrian leg impactor with upper body mass by modeling the flexible pedestrian leg impactor and the flexible pedestrian leg impactor with upper-body mass. The optimization framework is used to develop a fuzzy multi-attribute decision-making model by combining the ranking preference of ideal solution similarity with the fuzzy method. The optimization results show that compared with the structure impacted by the flexible pedestrian leg impactor, the vehicle front-end structure impacted by the flexible pedestrian leg impactor with upper body mass needs higher stiffness of the tibia contact area, but the stiffness of the knee and femur contact area is lower. This research provides automotive engineers with new insights into the front structure design based on injury biomechanics from the perspective of road safety. Lü et al. [67] proposed an optimization framework for brake noise suppression of vehicle disc brake systems considering the structural size uncertainty. The parameters such as friction coefficient, material properties, and thickness of worn parts are regarded as uncertain parameters. The stability analysis of the brake system in the squeal is studied, and the stability of the brake system is studied by using the complex eigenvalue analysis (CEA) method. The response surface method (RSM) is used to approximate the implicit relationship between the unstable modes and the system parameters. Finally, the disc brake system is optimized by using the genetic algorithm. The results show that the use of a harder back plate can reduce the tendency of brake squeal, and the optimization framework can effectively improve the stability of the disc brake system for vehicles with uncertain parameters. Wu et al. [68] proposed a suspension system design optimization framework considering the uncertainty of hard point position. This framework mainly explores the performance response of the structure, including camber, caster, kingpin inclination, and toe angle. By establishing a high-order response surface model with the zero point of the Chebyshev polynomial as the sampling point, the kinematics model is best approximated. The optimization results show that the suspension motion performance has been greatly improved compared with the initial conditions, and the optimization framework could have been applied to more complex mechanical systems. In addition, Jamali et al. [69] designed a Pareto optimal design framework for solving ten conflicting objective functions by combining a multi-objective uniform diversity genetic algorithm with Monte Carlo simulation technology to achieve the optimal design for the uncertainty of the vehicle vibration model in reality. In this study, the uncertainty is represented by a probability model and compared with the design obtained by using the deterministic method. The results show that the effect of uncertainty on the performance index can be obtained by using the optimal design of the vehicle vibration model considering uncertainty. Gu et al. [70] used the support vector regression (SVR) model to approximate the response between design variables and targets and introduced a hybrid kernel function (HKF) to overcome the shortcomings of the SVR single kernel function. At the same time, the particle swarm optimization (PSO) algorithm was used to optimize the parameters of the HKF-SVR model, and non-dominant sequencing genetic algorithm II (NSGA-II) and Monte Carlo simulation (MCS) were combined to carry out uncertain design for the safety of automobile structures. The results show that, compared with the initial design, this method not only improves the crashworthiness and light-weight of the vehicle but also improves the reliability and robustness of the design indexes. Liu et al. [71] used the modified Manson Coffin formula as the fatigue life calculation formula of the spot welding structure. Considering the uncertainty of the welding gun falling point in the process, they took the position coordinate of the welding point as the uncertainty variable. Through the optimization design of the welding point coordinate, they obtained the welding point coordinate with the maximum fatigue life of the structure, providing a calculation tool for the analysis and optimal design of the fatigue life limit of the spot-welding structure in engineering practice. The optimal design frame not only improves the fatigue life of the structure but also gives the fluctuation range of the fatigue life of the structure, which is conducive to improving the fatigue performance of the spot-welding structure. In the industry, automobile manufacturers and suppliers must find the best design solution for the safety bar subsystem to meet their conflicting requirements on functional performance and environmental impact. In view of the design problem of automobile bumper, Farkas et al. [72] considered the influence of parameter uncertainty and established an integrated method of mechanical structure multi-attribute design engineering to design and optimize the crashworthiness of vehicle bumper subsystem. Among them, the special platform for the automatic multidisciplinary design optimization process is realized by using the OPTIMUS software platform.

The above research mainly focuses on the optimization design of key automobile structures considering a single uncertainty factor, focusing on the influence of the manufacturing process and other uncertainties on design parameters. All research results can provide a more reasonable solution than deterministic optimization to resist potential uncertain disturbances.

2.2. Design optimization considering multiple uncertainty factors

The research on uncertainty design optimization of automobile structures in the above research mainly focuses on a single uncertain factor. In fact, there are still many uncertain factors to be considered in the initial design stage of automobile structures. Therefore, a lot of work in the field of design optimization considering multiple uncertainty factors for automotive structures was carried out, as shown in Figure 5.

In the past, researchers usually use the probability constraint of single failure mode to study the optimization of vehicle structural crashworthiness design based on uncertainty, which often has limitations. Therefore, Acar et al. [73] analyzed the influence of uncertainty under different failure modes and developed the uncertain crashworthiness design optimization for automobile structures. This method mainly considered the uncertainty of material properties and the error of finite element analysis. The proposed method can provide a reasonable tradeoff scheme under a variety of uncertain factors. Zhao et al. [74] developed the uncertain continuum structure topology optimization method based on the variable density method and carried out the uncertain topology optimization for the suspension control arm of a vehicle. In this method, the uncertainty decoupling strategy is developed, and the uncertainty fluctuations of material parameters and load parameters are considered. The results show that, compared with the deterministic topology optimization, the uncertain topology optimization method can meet the design requirements of economy and security to the maximum extent, and has better engineering applicability.

Since suspension system components are prone to failure under low cycle strain fatigue conditions, Grujicic et al. [75] developed an uncertainty optimization method for suspension system components of high-mobility multipurpose wheeled vehicles. In this method, the uncertainties of material properties, component shape and size caused by material processing and component manufacturing, as well as their effects on the main performance indicators of components, are considered. At the same time, the research results also demonstrate that it is necessary to consider a variety of potential uncertainties when realizing the performance indicators of key structural components for vehicle structures with complex systems. Rais-Rohani et al. [76] developed a framework for the optimization of vehicle structure shape and size under vehicle crash conditions, and examined the impact of different design constraints and related uncertainties on the performance and efficiency of optimization design indicators. By studying the material and geometric characteristics of the components, an alternative model is established for the intrusion distance and peak acceleration response at different vehicle locations. The research shows that the obtained solution provides insights into the influence of uncertainty in the optimization design of vehicle structures, and the optimization results are verified by the finite element simulation of vehicle crash scenes. Xu et al. [77] considered the manufacturing dimensions, materials and load input requirements of the planetary gear train as uncertain factors, and carried out the structural optimization design of the distributed electric drive motor reducer. In this optimization framework, the volume and transmission efficiency of the planetary gear train is considered optimization goals. The proposed optimization framework can meet the actual needs better than the deterministic optimization scheme under different uncertainties and has a higher ability to resist failure risk. The stamping process and geometric shape may transmit uncertainty to the assembly stage in the manufacturing stage, which may lead to uncertainty in vehicle structure performance response. The uncertainty of material properties, process parameters and final geometry can be propagated from the forming stage to the implementation stage of performance response in an uncertain environment. Therefore, Sun et al. [78] developed a multi-objective uncertain design optimization method for double-cap thin-walled structures to find the optimal design scheme. The optimization results show that the proposed method not only significantly improves the formability and crashworthiness but also improves the reliability of the Pareto solution. Gao et al. [79] developed a nondeterministic topology optimization framework for the layer direction of multi-fiber reinforced plastics (FRP) materials (such as carbon fiber reinforced plastics and glass fiber reinforced plastics composites) when the load size and direction are both uncertain factors. The optimization framework is developed based on discrete material optimization (DMO) technology. Four material design examples have been used to verify the effectiveness of this method, and the method has been applied to the design of battery suspension points of electric vehicles. The topology optimization results show that when the load fluctuates, composite structures with appropriate ply directions have more stable performance indicators. Mierlo et al. [80] developed a design optimization method for vehicles under uncertain boundary conditions in the crash simulation process. This method considers the uncertainty of the unknown mechanical response of adjacent structures inside the vehicle and verifies the applicability of the method through a crash optimization case of a rectangular energy-absorbing box.

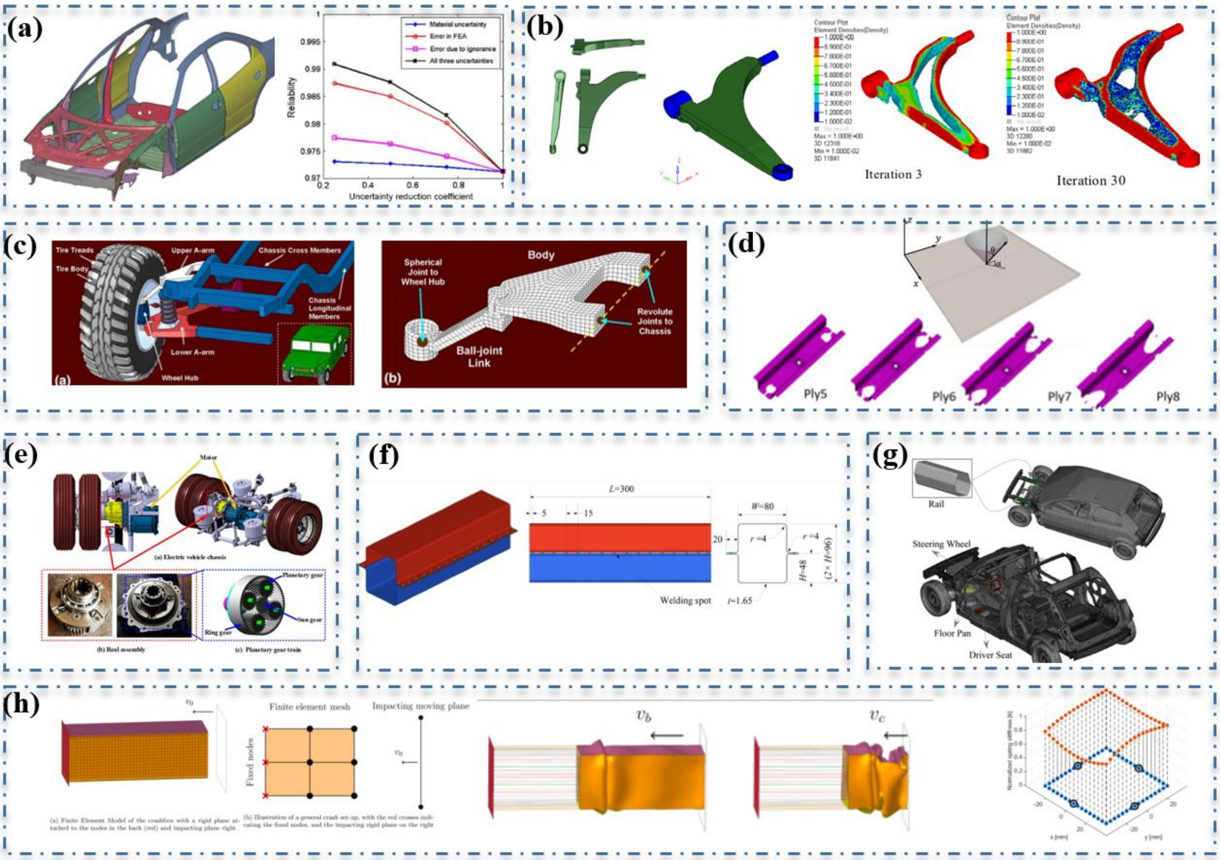

Figure 5. Design optimization considering single uncertainty. (a) [73], (b) [74], (c) [75], (d) [76], (e)[77]; (f) [78], (g) [79], (h) [80].

The main uncertain factors are usually ignored in the conventional deterministic optimization of automobile structure design optimization, and the structural parameters, external environment and performance response are all regarded as deterministic values. It can be predicted that such methods can obtain nominally good performance indicators at the initial stage of design but cannot overcome the objective impact of actual uncertainty [81–84]. A complete structural design optimization process includes the specific physical system model to the mathematical model, and then to the optimization algorithm. The whole structural design optimization process has more or fewer uncertainty problems. As a system assembly integrating a huge mechanical structure, it is particularly critical to consider the relevant uncertainties in the structural design of each component.

3. Uncertainty analysis methods

Due to the limitations of the manufacturing process, there are inevitable differences between the ideal design and the actual engineering parts. These differences come from uncertainties in the production process, and even small uncertainties may lead to large fluctuations in the output response [85–90]. It should be noted that the uncertainty response analysis is particularly important in the whole uncertainty design optimization framework of structures. Taking structural reliability design as an example, as shown in Figure 6, the optimization results without considering uncertainty are usually attached to the boundary of the constraint boundary. Once the design variables fluctuate slightly, their constraint function values are easily beyond the constraint boundary. Therefore, to further improve the reliability of the structure at the design constraint boundary, it is necessary to establish a structural optimization model considering uncertainty factors to reduce the risk of constraint failure.

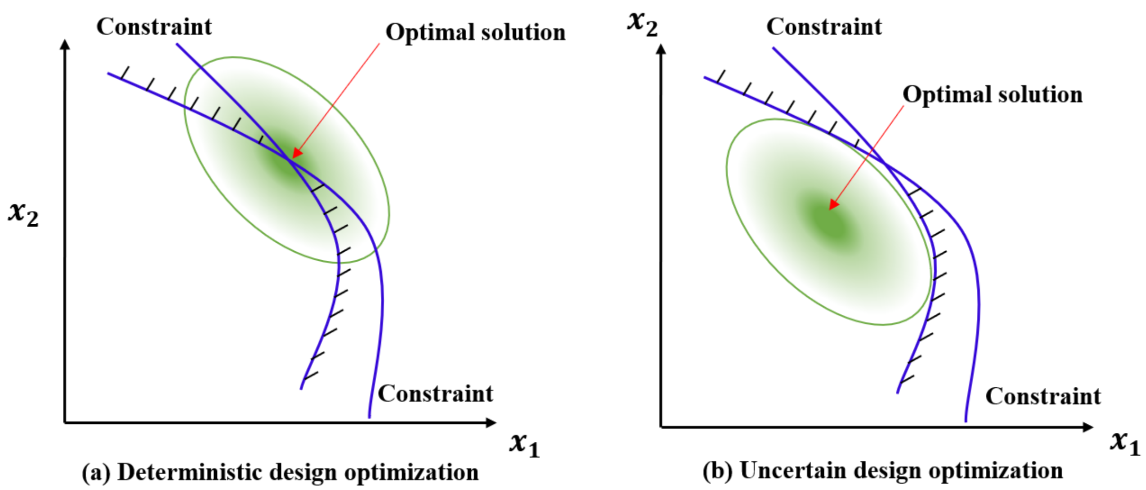

Figure 6. Deterministic and reliability design optimization results.

In addition, taking the structural robustness design as an example, as shown in Figure 7, if the structural performance is optimized, it is easy to determine the optimal solution x_2 through the conventional deterministic optimization algorithm. It is worth noting that once the design variables fluctuate slightly, the value of the performance response function will change significantly. Therefore, to reduce the high sensitivity dependence of the design response on the design variables, it is necessary to design a robust optimization model that considers uncertainties to improve the stability of the performance response value, as shown the point x_3 in Figure 7. Structural uncertainty analysis is the basis of structural uncertainty optimization. A complete uncertainty optimization process includes iterative updating in the optimization direction and uncertainty analysis of output response in each iteration step.

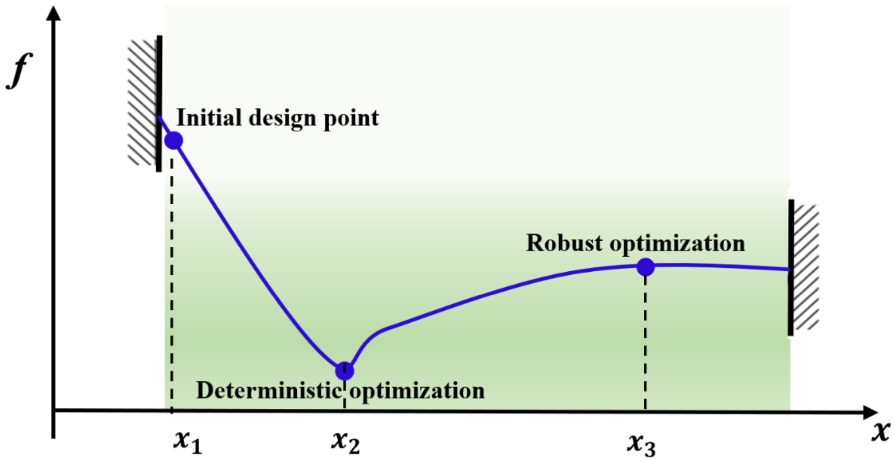

Figure 7. Deterministic and robust design optimization.

3.1. Analysis methods related to probabilistic models

The influence analysis of uncertainty is an important subject in structural engineering design. When the uncertainty information is sufficient, researchers usually regard the uncertainty of model parameters and performance response as a probability model that can be described by random distribution parameters [91,92]. For example, Qiu et al. [93] analyzed the uncertainty of the proxy model and finite element numerical simulation model in structural optimization by using the sample statistical characteristics of data test design and established the uncertainty analysis model of multi-cell thin-walled structure under load. Song et al. [94] proposed a theoretical method to analyze the sum of lognormal random variables using the effective numerical integration method for uncertainty analysis in probabilistic safety assessment. The uncertainty analysis method changes the variables from the relevant random variables with complex integration regions to independent random variables with unit hypercube integration regions to obtain effective numerical integration. The research shows that the uncertainty analysis method proposed by him provides an effective way to calculate the probabilistic safety assessment. In addition, to explore the problem of nuclear reactor safety analysis, Walton et al. [95] developed and demonstrated a random uncertainty analysis framework based on the random sampling of the benchmark universal fluoride-cooled high-temperature reactor core. The uncertainties mainly include manufacturing, nuclear data, operational process, etc. Du et al. [96] designed a sequential optimization and reliability assessment (SORA) method to improve the efficiency of uncertainty analysis of probabilistic models in structural optimization. Farland et al. [78] used the method of random probability to predict the changes in structural system performance and assess the risks caused by the randomness of model inputs (such as material properties, loads and boundary conditions). Lei et al. [97] proposed two probabilistic structural damage detection methods respectively to explain the potential uncertainty of structural parameters and external excitation. One is the structural damage detection algorithm based on statistical moment combined with the sensitivity analysis of damage vector to uncertain parameters, and the other is the probabilistic structural damage detection method. It is based on the combination of structural damage detection using instantaneous moments and sensitivity analysis of damage vectors to uncertain parameters in each period of measuring response time history. Inevitably, probabilistic uncertainty models require a large number of test data samples, which is a great challenge for researchers who have limitations in their own test environment.

3.2. Analysis methods related to non-probabilistic models

To avoid the shortcomings of traditional probabilistic methods, some alternative uncertainty analysis methods, such as the fuzzy set model [98–100], the D-S evidence theory model[101–103], and the interval number model [104–106], are commonly used in the description of cognitive uncertainty and have been widely used to quantify uncertainty. Since the non-probabilistic model does not need to consider statistical information, it is more practical than the probability model in uncertainty analysis [107–110]. The interval model mainly considers the upper and lower bounds of uncertainty information, and the boundary of structural response is solved by the so-called interval analysis method. If we pay more attention to the boundary information of uncertainty, the unknown but bounded interval uncertainty model can be used as a better choice [111,112].

Recently, a series of interval uncertainty analysis models have been proposed gradually [113,114]. For example, Zhao et al. [115] proposed an effective analysis method by using an interval process model to find the response boundary of the vibration system under time-varying uncertainty, considering the external excitation and the inherent uncertainty in the system parameters. Their research results were verified by the corresponding MCS technology and were further applied to lunar soil coring. In addition, Wang et al. [116] developed an interval non-probabilistic reliability method to quantify the safety of active vibration systems based on the performance of PID controllers. Moreover, the sub-interval model is used to obtain the response with large uncertainty, and the research results are successfully applied to the analysis of a discrete mass spring damper system. In some studies, the MCS method applicable to random probability uncertainty analysis can be used to calculate the upper and lower bounds of uncertain response, that is, the upper and lower bounds of uncertain response can be predicted by random sampling technology [117,118]. It can be predicted that MCS can gradually converge to an accurate interval by increasing the calculation samples in the uncertainty space. However, since the relationship between the upper and lower bound numerical accuracy of interval response and the number of samples calculated by MCS is random, it usually requires a large number of MCS calls to converge. There is no doubt that MCS inherits the characteristics of the probabilistic model, which requires a large amount of calculation. Therefore, MCS is not suitable for solving the interval uncertainty analysis problems in practical projects, and more effective alternative methods need to be studied. Therefore, some scholars introduced the auxiliary optimization algorithm to solve the interval model. For example, Li et al. [119] developed an interval uncertainty analysis by using the adaptive Kriging method, and applied Sequential Quadratic Programming as an internal optimization solver to search the upper and lower bounds of structural performance response. In addition, since the biological heuristic algorithm provides superior global optimization capability based on natural biological behavior, the interval limit of structural response can be calculated by optimization [120]. For example, considering the intergenerational mapping genetic algorithm (IP-GA) combined with micro genetic algorithm (IP-GA) and alternate mapping (IP) operators have good global convergence performance [121]. Jiang et al. [122] used IP-GA as the optimization operator in each iteration step to seek the optimal value of the response surface in the uncertainty interval, to obtain the upper and lower bounds of the interval uncertainty of the structural response. Cheng et al. [123] proposed a direct interval optimization algorithm for a reliability model combining GA and Kriging, which avoids complex transformation of multiple indirect models and can effectively calculate the solution of interval response. It is undeniable that the intelligent evolutionary algorithm has significant advantages in searching for the global optimal solution. However, because its accuracy is closely related to the number of iterations, it often costs a lot of computing costs to obtain an accurate solution set. Therefore, the intelligent evolutionary algorithm has obvious limitations for the uncertainty analysis of some complex engineering problems. In addition, another improved response uncertainty analysis method is the interval vertex method (IVM). It is found that IVM has high applicability in solving interval linear problems, but it is still challenging for highly nonlinear complex engineering problems [124–126]. Compared with the optimization strategy of an intelligent evolutionary algorithm, IVM can quickly find the value of the uncertainty boundary under the premise of meeting the accuracy requirements. For example, Qiu et al. [127,128] gave a mathematical proof of the vertex solution theorem and applied it to interval response analysis for calculating unknown but bounded parameters. By expressing the static response analysis problem of the structure in the form of linear interval equations, where the coefficient matrix and the right term are interval matrices and interval vectors respectively, and then using Kramer's rule to solve the linear interval equations, the upper and lower bounds of the interval solution set can be quickly found. Unfortunately, IVM is mainly applicable to monotone mathematical models, which may cause large errors in complex nonlinear problems. The Taylor expansion approximation method has also aroused the interest of researchers for uncertainty analysis. In many existing studies, on the premise of meeting the accuracy requirements, the Taylor series usually only needs to retain the first or second-order Taylor series term to quickly quantify the uncertainty. For example, Qiu et al. [129,130] proposed an interval perturbation method to characterize uncertainty, and the Taylor series was used to expand interval matrices and interval vectors. In addition, Qiu et al. [131] studied the performance response of uncertain nonlinear vibration systems, and estimated the interval range of nonlinear dynamic response using the mathematical interval method based on the second-order Taylor series expansion. In nonlinear systems without expressions, it is difficult to obtain derivative information directly by traditional auxiliary methods, which hinders the expandability of Taylor approximate expressions.

In the actual project, due to the lack of cognition of complete information of uncertainty, there may be multiple mixed uncertainties in structural response analysis. The factors of cognitive uncertainty can be represented by a simplified interval model, and the information of random uncertainties that have been mastered can continue to be regarded as random probability models with specific distribution types. As for the response analysis method of mixed uncertain structures with interval probability. Different uncertain response analysis methods can be used for different specific structural objects according to design requirements [132–135], as shown in Figure 8. When the input of design variables are interval variables and probability variables respectively, the uncertainty of structural performance response presents a probability distribution with a boundary effect. The random response value contains interval information and the mean value of the random response changes within an interval. Uncertainty response analysis is a crucial step in the process of uncertain design optimization. A conventional single type of uncertainty optimization process needs to analyze the uncertainty in each iterative calculation process, while for multiple mixed uncertainty optimization problems, multiple nested uncertainty analysis is required, which significantly increases the calculation cost in the optimization process. Therefore, it is necessary to further develop a mixed uncertainty representation model for design optimization problems.

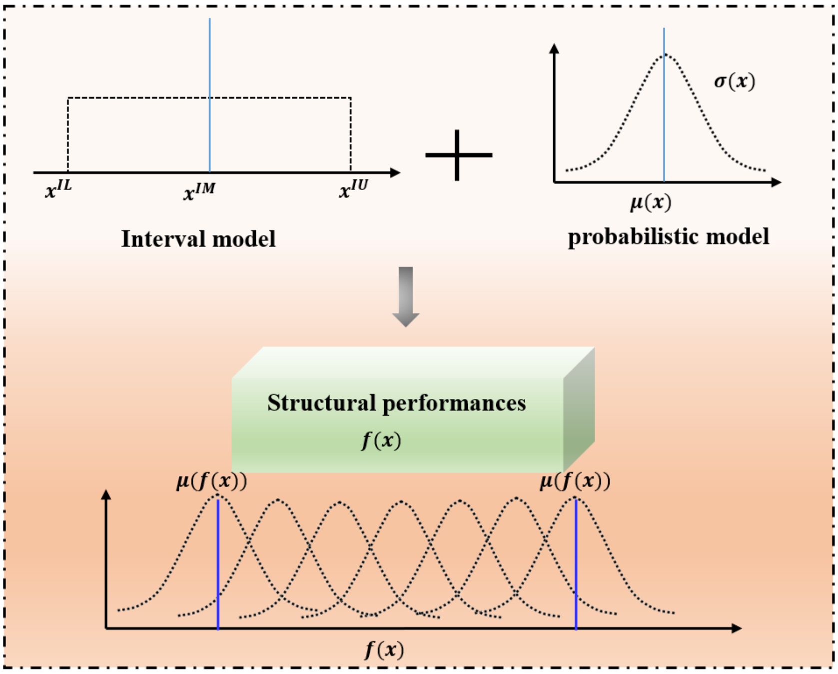

Figure 8. Schematic diagram of mixed uncertainty.

4. Multi-objective iterative optimization models

4.1. Probabilistic uncertainty optimization models

Inevitably, performance optimization in specific vehicle structure design often involves multiple objectives optimization, and sometimes some performance objectives are conflicting [136–141]. Therefore, it is necessary to carry out multi-objective uncertain design optimization of structures. For the multi-objective optimization of the probabilistic uncertainty model, Su et al. [142] proposed an uncertain multi-objective optimization method for truss optimization design to improve the robustness of truss structures when uncertainties in materials and loads are considered. Liu et al. [143] proposed an effective decoupling strategy to transform the initial three-layer nested uncertain multi-objective optimization model into a two-layer nested optimization problem, and then used IP-GA and micro multi-objective genetic algorithm as the inner and outer optimization operators to solve the multi-objective uncertain optimization problem. Vo-Duy et al. [144] developed an effective coupling method to solve the multi-objective uncertain optimization problem of truss structures, in which probabilistic constraints are considered as approximate deterministic constraints to minimize the weight and displacement of the truss. Lobato et al. [145] considered reliability and robustness in structural design at the same time obtained reliability indexes by using probability models and established a reliability-based robust design optimization (RBRDO) model that meets reliability. Wang et al. [146] carried out multidisciplinary and multi-objective uncertainty design optimization for dynamic and static performance and structural lightweight of vehicle structures. Considering the uncertainty reliability optimization results of product design and production process, product stability can be guaranteed. Yin et al. [147] established a multi-objective uncertainty optimization model by combining Kriging, multi-objective particle swarm optimization (MOPSO) algorithm, "k-sigma" robust design theory and MCS method, designed the absorption characteristics of foam-filled thin-wall and compared with the Pareto front obtained by the traditional multi-objective deterministic optimization algorithm, the uncertainty optimization results are more reliable than the deterministic optimization results. Khakhali et al. [148] used a neural network model and MCS technology to establish a multi-objective robust design optimization model considering the probability uncertainty of the material and geometric parameters of the front S-beam of the car body. The optimization results can obtain a more robust Pareto design scheme.

4.2. Non-probabilistic optimization models

Most of the above works use the parameters related to probability to establish an uncertain design optimization model. Considering the complexity of uncertain factors, Ebenuwa et al. [149] calculated the expected value of the membership function based on the fuzzy output variable, and proposed a fuzzy-based multi-objective design optimization method for the optimization analysis of buried pipelines. Fuzzy set theory and multi-objective optimization algorithms are used to consider the variability related to uncertain parameters to ensure that the impact of pipeline structure on uncertainty has acceptable performance. It is difficult to obtain the accurate probability distribution and membership function of fuzzy sets in practical projects [150]. Therefore, multi-objective optimization based on the non-probabilistic interval uncertainty model is gradually applied to engineering structure design. For example, Liu et al. [151] used the nonlinear interval number programming method to convert each uncertain objective function in the multi-objective optimization problem into a deterministic single objective optimization problem, and used the constraint penalty function to re-establish the deterministic multi-objective unconstrained optimization problem. Xie et al. [152] proposed an efficient sequence multi-objective optimization (MORO) method based on the support vector machine (SVM) to consider interval uncertainty. Chagraoui et al. [153] proposed a new method of robust multidisciplinary design optimization (MDO) problem for multi-objective optimization of structures with frequency, mass, and displacement indicators. The multi-objective optimization framework decomposes the optimization problem into two structural layers to solve the robust optimization problem of complex Y-shaped stiffened plate structures with interval uncertainty. Li et al. [154] regarded the lightweight and safety of thin-walled beams as a multidisciplinary multi-objective optimization problem, taking into account the interval uncertainty of design parameters. In the optimization process, the uncertain optimization problem is transformed into a conventional deterministic optimization problem using the interval number programming method. Zhang et al. [155] developed a new effective design optimization method with high computational efficiency because of the uncertainty problems in structural design optimization. In the optimization process, the manufacturing tolerance of each dimension of the structure is defined as the regional limit of the design variables, and the physical planning is used in the multi-objective optimization problem, thus avoiding the problem of providing weights without physical significance for the traditional evaluation methods. Li et al. [82] proposed a multi-objective optimization framework based on the interval uncertainty model to design the passive safety problem of vehicle structures, in which the uncertainty of the main structural parameters of vehicles is described by interval models. Considering lightweight and safety performance, structural weight and peak acceleration are selected as targets, and occupant distance is considered as a constraint. Finally, the uncertain optimization problem is transformed into the classical deterministic multi-objective optimization problem by the interval number transformation method, which effectively solves the uncertain optimization problem of vehicle crashworthiness.

Generally, the uncertainty optimization model is transformed into a conventional deterministic multi-objective optimization problem by calculating the interval possibility degree. Cheng et al. [156] proposed a multi-objective uncertainty optimization framework combining interval model analysis and radial basis function, and verified the feasibility of this method by designing a press slider considering uncertain material properties. Feng et al. [157] used Chebyshev polynomials to model a complex nonlinear suspension system with interval uncertainty, and regarded the characteristics of the suspension bushing as design variables and uncertain parameters to optimize the K&C characteristics of the suspension. The optimization process adopts a double cycle nested optimization process, in which the inner cycle is the boundary of the calculation interval design function, and the outer cycle is actually to optimize the K&C characteristic target of the suspension. Li et al. [158] proposed an interval multi-objective optimization model based on the nonlinear interval analysis method, and applied this method to the optimization of ten truss structures and commercial vehicle frames. In this optimization process, the nonlinear interval optimization problem is transformed into the conventional deterministic optimization problem by using the interval possibility theory. Finally, the transformed deterministic multi-objective problem is calculated by using the genetic algorithm. Xie et al. [159] developed a multi-objective uncertainty optimization model by designing a tolerance index to describe the overall interval uncertainty for all design variables. Meanwhile, the probability degree of the interval is used to express the reliability of the constraint function under uncertainty.

It should be noted that most of the current multi-objective optimization models are designed with non-dominated strategies. The common practice is to convert the initial uncertain optimization framework to the conventional deterministic optimization framework by using the mathematical programming designed by researchers and applying the conventional iterative intelligent evolutionary algorithm as the solver to obtain the final Pareto solution set. In the process of the non-preference multi-objective optimization, the non-dominated updating strategy in its iterative optimization algorithm is still regarded as a deterministic problem in essence, and this kind of method is easy to lose the uncertain information in the iterative process of non-preference multi-objective optimization.

5. Conclusions

This survey discusses the current situation of the uncertain design optimization framework of automobile structures and summarizes the uncertain design optimization of key automobile structures, uncertainty analysis methods and multi-objective iterative optimization models. It can be found that although current uncertainty analysis and design optimization methods have been well developed and applied in the initial design of different automobile structures, there are still some points that need to be improved, mainly in the following aspects:

(1) For these interval models represented by non-probabilistic models, the main parameters in the current interval model are mostly derived from the given conditions, lacking the design optimization of the interval model parameters themselves, which reduces the diverse demand of structural design schemes. In addition, for the problem of structural performance response analysis with interval

uncertainty, most models with explicit functions or simple computational frames can be analyzed using conventional perturbation methods, but the application of perturbation methods is limited due to the lack of derivative information for some complex or non-explicit engineering structures. Due to the completeness and inconsistency of uncertainty information, mixed uncertainty is inevitably prone to occur in the process of structural analysis or design optimization, and there are still deficiencies in the analysis and design optimization of mixed uncertainty.

(2) There are many highly nonlinear problems in automobile structure problems, such as collision, so it is a common method to improve the structural design framework by using the auxiliary agent model. However, in the uncertainty analysis and design optimization problems based on the auxiliary approximate agent model, it is usually assumed that the approximate agent model and its numerical simulation model are correct. However, the error between the approximate substitute model and the numerical simulation noise is an inherent problem that is difficult to eliminate, which will lead to the final solution obtained by auxiliary approximate substitute model optimization not being easy to meet the actual design requirements.

(3) Most of the current multi-objective optimization models are designed using non-dominated strategies. Therefore, multi-objective uncertain optimization usually uses the mathematical programming designed by researchers and the traditional iterative intelligent evolutionary algorithm as the solver to convert the initial uncertain optimization framework to the traditional deterministic optimization framework to obtain the final Pareto solution set. Taking interval uncertain multi-objective optimization as an example, the nominal deterministic value or interval midpoint value is taken as the objective function response in the optimization model, and the multi-objective design optimization is carried out by using the conventional optimization algorithm as the solver. It should be noted that for the non-biased multi-objective optimization problem of heuristic algorithm design if the uncertainty of target performance is not considered, the feasible solution is easy to lose the uncertainty information in the non-dominated operation.

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Conflict of interest

The authors declare there is no conflict of interest.

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