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Mathematical models of erosive flash floods, huaycos and lahars

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ABSTRACT

1. Introduction

Huaycos - flash floods in the Peruvian Andes - and lahars – volcanic debris flows - are potent natural hazards that threaten regularly lives and livelihoods. They comprise debris-laden fluid that flows rapidly down steep slopes, bulking up considerably through erosion of the underlying bed as they progress. Owing to their rapid onset and the significant threat that they pose to communities and infrastructures, it is important to predict their motion to assess quantitatively some of the impacts that they may cause. This paper presents a new mathematical model of the physical processes that govern the motion and a new, free-to-use implementation of a numerical algorithm to integrate them, which may be used to predict the flows quantitatively.

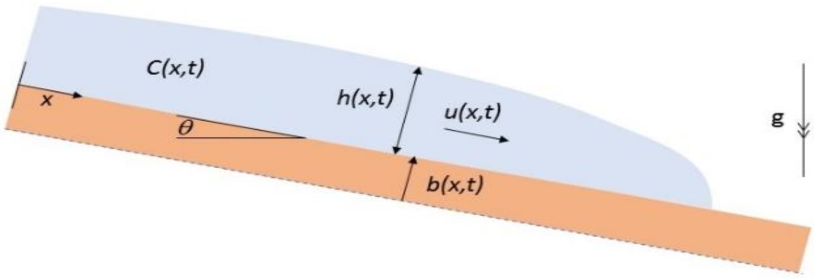

Fig. 1. Flow configuration for a model of a debris-laden flow down an inclined boundary, here shown on a constant incline and as a function of one spatial variable. Dependent variables are flow depth, h(x,t), flow velocity, u(x,t), concentration of particles, C(x,t) and bed elevation, b(x,t). Motion is driven by gravitational acceleration, g.

2. Methods

The mathematical model utilizes the relative shallowness of the flow (the flow depth is much less than the streamwise extent), which implies that the motion is predominantly parallel to the underlying slope and the pressure (or 'normal' stress) is hydrostatic. Figure 1 depicts a flow down a constant incline and defines the depth-averaged dependent variables; more generally the inclination spatially varies, and the dependent variable are functions of time, t and the two spatial coordinates parallel to the boundary, but for brevity here, we suppose that there is variation with only one spatial dimension. Conservation of fluid mass is given by

$$\frac{\partial}{\partial t}\big(h(1-C)\big)+\frac{\partial}{\partial x}\big(hu(1-C)\big)=-(1-C_b)\frac{\partial b}{\partial t}, \tag{1}$$

where C_b denotes the volumetric concentration of solids in the bed. This equation relates the rate of change of fluid volume with the divergence of the fluid flux and the addition of fluid from the saturated bed. Likewise, conservation of mass in the solid phase

$$\frac{\partial}{\partial t}(hC)+\frac{\partial}{\partial x}(huC)=-C_b\frac{\partial b}{\partial t}. \tag{2}$$

Here we have assumed that two phases move with the same depth-averaged velocity, $u(x,t)$. Bed erosion, E, and deposition, D, balance the rate of change of the bed elevation and are given by

$$C_b\frac{\partial b}{\partial t}=D-E. \tag{3}$$

Finally, relating the density, ρ, to the concentration, C, and densities of the solid and fluid phases, ρ_s and ρ_f, respectively, $\rho=\rho_f(1-C)+\rho_s C$, we may form the depth-averaged expression for the streamwise balance of the momentum of the entire mixture. This eliminates the need to model explicitly the inter-phase forces. The driving force is downslope gravitational acceleration, which is primarily balanced by the resistance at the base of the flow. Additionally, the expression for the rate of change of the momentum features the divergence of the momentum flux, the streamwise pressure gradient and the potential drag arising from the acceleration of entrained bed material.

$$\frac{\partial}{\partial t}(\rho u h) + \frac{\partial}{\partial x}\left(\rho u^2 h + \tfrac{1}{2} g \cos\theta \rho h^2\right) = \rho h g \sin\theta - \tau_b - \rho_b u_b \frac{\partial b}{\partial t}, \tag{4}$$

where u_b and ρ_b are the basal streamwise velocity and density, respectively.
There are two key changes that must be made to the governing system of equations before they can be used for predictions. First as posed in (1)-(4), the system is mathematically ill-posed. This technical result has a crucial consequence: it is not possible to compute grid-resolved solutions. Instead, we reinstate a small, neglected term, the streamwise variation of stress, which is sufficient to regularise the system (see Langham *et al.* 2022). Additionally, we compute solutions on measured topography. For example, we have conducted drone surveys of catchments in Chosica, Peru and used photogrammetry to construct a high-resolution digital elevation model (DEM). Therefore, it is convenient to compute in a rotated and projected coordinate system - Earth-centred coordinates - which enable the measured DEM to be used directly without further manipulation.

3. Results

Our model is integrated using efficient modern numerical algorithms, driven by source conditions such as a prescribed hydrograph upstream in the catchment, or an instantaneous release of material The model is available to use free of charge, at www.laharflow.bristol.ac.uk, with computations made using servers in Bristol. Figure 2 shows a computed result that simulates a huyaco event at La Libertad de Chosica, Peru. Note that the DEM has street-scale features and that the flow is directed by this topography. The flow reached quite high flow depths (~3m) and flow speeds (~10 ms^{-1}). It was also highly erosive, picking up deposited material and tearing up the unpaved street. These predictions are consistent with observations of the 2016 event and evidence the plausibility of our modelling framework and computations. Other examples in different international settings and at much larger scales will be reported, along with their use in hazard assessment.

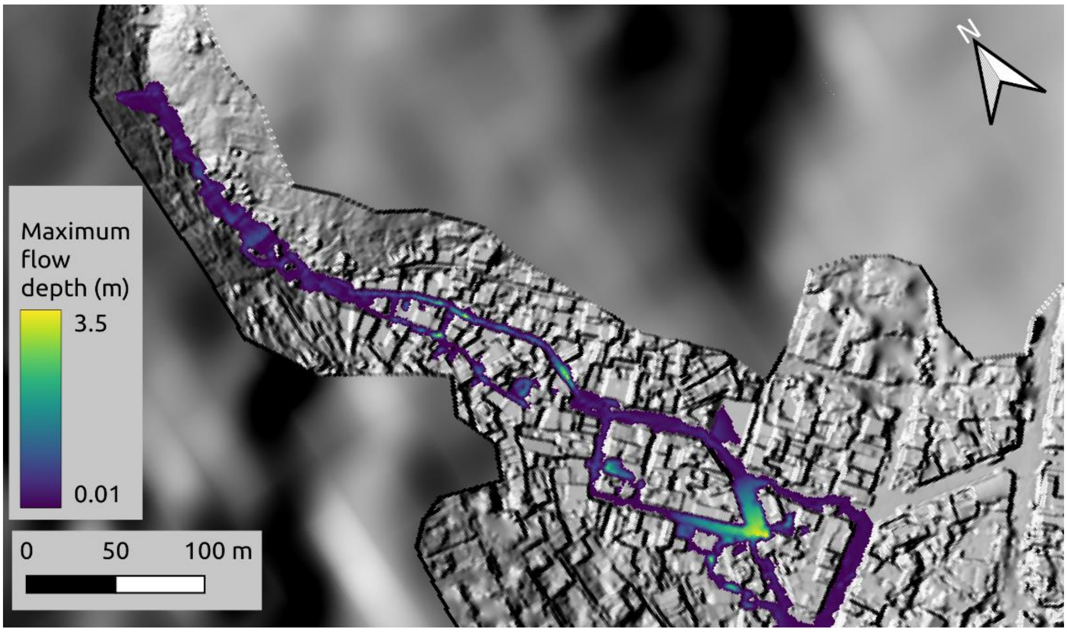

Fig. 2. Plan view of maximum flow depth during a huayco event simulated at La Libertad de Chosica, Peru. The source hydrograph is specified at the northerly end of the quebarda, and the flow downstream is predicted by the model. The DEM is depicted in grey, showing the resolution of street-scale features that guide the flow.

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