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DOI:
10.1063/5.0140525

Citation for published version (APA):
Saberi, H., Xia, G., Islam, M. R., Liang, L., & Davut, C. (2023). Radiation reaction and its impact on plasma-based energy-frontier colliders. *Physics of Plasmas*, *30*(4), [043104]. https://doi.org/10.1063/5.0140525

Published in:
Physics of Plasmas

Radiation reaction and its impact on plasma-based energy-frontier colliders

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(Dated: 7 March 2023)

The energy-frontier TeV colliders based on plasma accelerators is attracting more attentions due to recent achievements in multi-stage laser acceleration, as well as the remarkable advances in electron- and proton-driven plasma accelerators. Such colliders may suffer a fundamental energy loss due to the radiation reaction (RR) effect, as the electrons lose energy through betatron radiation emission. Although the RR may not be critical for low energy accelerators, it will exert limitations on TeV-class plasma-based colliders that needs to be considered. In this paper, we have provided an extensive study of RR effect in all pathways towards such colliders, including multi-stage plasma acceleration driven by the state-of-the-art lasers and the relativistic electron beam, as well as the single-stage plasma acceleration with the energetic proton beams available at the CERN accelerator complex. A single-particle Landau-Lifschitz approach is used to consider the RR effect on an electron accelerating in the plasma blow-out regime. The model determines the boundaries where RR plays an energy limiting role on such colliders. The energy gain, the radiation loss and the validity of the model are explored numerically.

I. INTRODUCTION

Colliders are one of the most important scientific tools of discovery in particle physics, e.g. the Large Hadron Collider (LHC) at CERN led to the discovery of the new particle Higgs boson a decade ago. The future lepton colliders are going to be in the TeV energy range, however those will be huge and expensive if they are based on the conventional radio-frequency acceleration.[1–3] In other words, TeV-class colliders may rely on plasma accelerators which have shown a tremendous progress in the last decades.[4–6] In plasma accelerators, a much higher energy gradient than that in the conventional accelerators is generated in a plasma medium driven by an energetic driver, e.g., laser pulse,[7] electron-[8] or proton-beam[9]. Therefore, electrons potentially can gain high energy in a very short distance. On the other hand, plasma-based positron acceleration is very challenging due to the asymmetric response of the plasma, however, some ideas have been proposed and experimentally demonstrated the positron acceleration.[10–13] Although very promising for applications in high energy physics research, plasma-based colliders require considerable and long term research effort.

The laser wakefield accelerators (LWFA) have been proved to produce the shot-to-shot stable and high quality quasi-monoenergetic electron beams up to 1 GeV in a centimetre-long plasma.[14] Since then there have been lots of progress increasing the beam's energy gain in a single plasma stage with the energy gain record of 7.8 GeV in a 20 centimetre-long plasma.[15] With the successful coupling of two LWFA stages,[16] it is natural to think about the future compact and high energy colliders based on LWFA with several stages.[17–19]

The electron beam driven plasma wakefield accelerators (PWFA) have successfully demonstrated the energy doubling of a 42 GeV electron beam within an 85 centimetre long plasma at the Final Focus Test Beam facility at SLAC.[20] In addition, the recent experiments at Facility for Advanced Accelerator Experiment (FACET) have also shown the high efficiency electron beam acceleration with a considerable charge and low energy spread.[21] Therefore it is envisaged that a high energy collider can be designed based on PWFA scheme.[22,23] It will surely rely on the multi-stage acceleration to boost the beam energy up to TeV level due to the limited energy content of the electron beam driver.

On the other hand, the CERN accelerator complex has the high energy proton beams with an intrinsically significant energy content, usually two to three orders of magnitude greater than that of the electron beams nowadays, e.g. the 400 GeV Super Proton Synchrotron (SPS), and the 7 TeV LHC proton beams. The idea of proton-driven plasma wakefield acceleration (PDPWFA) has been proposed to accelerate an electron bunch up to the energy frontiers in a single stage plasma acceleration.[24] It opened a new avenue that a single stage PDPWFA-based collider can be realised using the existing CERN accelerators.[25] Soon after, the Advanced Wakefield Experiment (AWAKE) Run 1 demonstrated for the first time that a 400 GeV SPS proton bunch can excite plasma wakefield and accelerate a bunch of low energy electrons up to 2 GeV within a 10 metre long plasma.[26] Following the great success of the Run 1 experiment, the AWAKE Run 2 (2021-) aims at producing a stable and monoenergetic multi-GeV electron beam suitable for particle physics experiments.[27]

There are three common factors that limit the energy gain of plasma accelerators.[7] First is the energy depletion of the driver inside the plasma. Today's high power lasers and the electron drivers have almost the same amount of energy ($\leq$100 J), but much smaller than the huge amount of energy in the SPS ($\sim$19 kJ), and LHC ($\sim$129 kJ) proton beams. The second factor is the de-phasing of the driver and the accelerating electrons which put a limit on energy gain even if there is still energy in the driver. In fact, electrons overrun the driver and enter the deceleration stage. This effect, however, is more significant for LWFA. The third limiting factor is the laser pulse diffraction and the particle beams expansion. A combination of these factors puts limitation on the energy gain of the electrons specially on one stage LWFA and PWFA, however, it is still possible to increase the energy gain over multi-stage acceleration.

The blow-out (bubble) regime[28] is the most efficient plasma acceleration scheme. The energetic driver makes a bubble inside the plasma, ideally, free of electrons. This bubble exhibits strong longitudinally accelerating and transversely focusing electric fields. Therefore a well-suited electron beam inside the bubble can be accelerated to high energies while it is focused transversely. As electron acceleration accompanies transverse oscillations, the electron beam radiates electromagnetic fields in the forward direction known as betatron radiation.[29] This radiation spectrum lies usually in the X-ray range. During such interaction, non-linear effects like radiation reaction (RR) come into play which arises from the interaction between the betatron radiation and the accelerating/oscillating electrons. In this case, radiation emission dominates the electron motion and can lead to the electron beam energy loss.[30,31]

For the above-mentioned possible colliders based on plasma accelerators, the energy gain by the electron beam will be close to the TeV range, and so the accelerating beam is affected by the radiation loss.[32] There are already some studies on the RR and its impact on the particle dynamics in the plasma accelerators.[33–36] The effect of the RR due to the betatron radiation of a single electron in the plasma accelerators has been studied theoretically and numerically within a classical approach.[35] Recently, a three-dimensional model of the RR has been presented in which the long-term equations of motion have been derived without resolving the betatron frequency.[36] In this paper, an extensive study of RR effect in all pathways towards the plasma-based colliders is presented. A single-particle Landau-Lifschitz (LL) approach is used to consider the RR effect on an electron accelerating in the plasma blow-out regime. The model determines the boundaries where the RR plays an energy limiting role on such colliders. The energy gain, the radiation loss and the validity of the model are explored numerically. The paper is organised as follows: Section II showcases the theory of RR based on the LL approach. In Section III, the radiation loss by a test electron accelerating in a multi-stage LWFA and PWFA is presented. The radiation loss in a single-stage PDPWFA with both SPS and LHC proton drivers is investigated in Sec. IV. A comparative study between proton drivers with different energies, as well as various initial electron offsets is also presented. The conclusions are summarised in Sec. V.

II. RADIATION DAMPING

Radiation reaction (also known as radiation damping) is the recoil force from the electromagnetic radiation emitted by an accelerating charged particle. This problem has been investigated over a century and the Lorentz–Abraham–Dirac (LAD) equation is believed to describe the RR effect fundamentally.[30,37] However LAD equation suffers from some self-inconsistent problems like run-away solution and the pre-acceleration without an accelerating force on some specific conditions. On the other hand, some papers are based on the LL equation, which is a variation of the LAD by assuming that the RR force is a small perturbation to the Lorentz force. The LL equation is believed to give more accurate solutions, but still there are debates about which equation is more appropriate in physics.[38] In LL approach, the equation of motion of an electron with charge e and mass m_e in an electromagnetic field is given by[39]

$$\gamma\frac{du^{\mu}}{dt} = \frac{cr_e}{e}F^{\mu\nu}u_{\nu} + \frac{2r_e^2}{3m_ec}\Big[F^{\mu}_{RR1} + F^{\mu}_{RR2} + F^{\mu}_{RR3}\Big], \quad (1)$$

where the RR force is defined as

$$F^{\mu}_{RR1} = \frac{e}{r_e}\frac{\partial F^{\mu\nu}}{\partial x^{\lambda}}u_{\nu}u^{\lambda}, \quad (2)$$

$$F^{\mu}_{RR2} = -F^{\mu\lambda}F_{\nu\lambda}u^{\nu}, \quad (3)$$

$$F^{\mu}_{RR3} = (F_{\nu\lambda}u^{\lambda})(F^{\nu m}u_m)u^{\mu}. \quad (4)$$

Here, $F^{\mu\nu}$ is the electromagnetic field tensor, u^{μ} is the four-velocity of the electron, γ is the Lorentz factor of the electron, $r_e = e^2/4\pi\varepsilon_0 m_e c^2$ is the classical electron radius, c is the speed of light and ε_0 is the vacuum permittivity. This RR force damps the particle energy and momentum, so it changes the electron trajectory in the individual Lorentz force.

Equation (1) is applicable if three conditions are satisfied:[39] i) the damping force is small compared with the Lorentz force, ii) the wavelength of the incident radiation is large compared with the electron radius, and iii) the incident radiation should be weaker than the Schwinger limit, i.e., $E_s \approx 1.3\times10^{18}$ V/m. In other words, the radiation can be treated in classical approach if the quantum electrodynamics (QED) parameter $\mathscr{X} \ll 1$, where[31]

$$\mathscr{X} = \gamma\frac{\sqrt{(\mathbf{E}+\mathbf{v}\times\mathbf{B})^2 - (\mathbf{E}\cdot\mathbf{v}/c)^2}}{E_s} \simeq \frac{\gamma|\mathbf{F}_{\perp}|}{eE_s}. \quad (5)$$

Here, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields of the incident radiation, respectively, $\mathbf{v}$ is the electron velocity, and $\mathbf{F}_{\perp}$ is the transverse force responsible for the electron oscillation.

One can simply model the bubble in the blow-out regime as a sphere comprising uniform ions with a radius of r_b that moves with the relativistic velocity v_p along the longitudinal $x-$axis. A phenomenological model for the electric and magnetic fields inside the bubble, is then given by[40]

$$\mathbf{E} = \frac{1}{4}\Big[(1+v_p)\xi\hat{x} + \mathbf{r}\Big], \quad (6)$$

$$\mathbf{B} = \frac{1}{4}[z\hat{y} - y\hat{z}], \quad (7)$$

where $\mathbf{r} = y\hat{y} + z\hat{z}$ is the electron off-axis position, $\xi = x - v_p t$ is the longitudinal co-moving coordinate which moves with the bubble velocity, x and t are the electron longitudinal position and the time, respectively, $\hat{x}$, $\hat{y}$, and $\hat{z}$ are the unit vectors. Hereafter, the variables are normalized as follow: $x \to k_p x$, $t \to \omega_p t$, $\mathbf{p} \to \mathbf{p}/m_e c$, and $\phi \to \phi/m_e c^2$, where $\omega_p = (n_0 e^2/\varepsilon_0 m_e)^{1/2}$ is the plasma frequency, n_0 is the plasma density, and $k_p = \omega_p/c$ is the plasma wave number. It is worth mentioning that a more accurate model of the bubble fields is given in[41], however, using simplified Eqs. (6) and (7) is justifiable as the difference with the accurate fields is negligible on betatron radiation in this work.[36]

The aforementioned electric and magnetic fields are then substituted into LL equation to obtain the equaitons of motion. In order to calculate RR force in Eqs. (2)-(4), it is assumed that $v_x \approx 1$ and $v_p \approx 1$, as both the electron and the bubble velocities are close to the speed of light. Moreover, the electron transverse velocity is far less than unity, i.e., $v_{y,z} \ll 1$, while its transverse position is typically on the order of 1. With a straightforward calculation, it can be shown that the RR force terms depend on different orders of the Lorentz factor, i.e., $F^{\mu}_{\mathrm{RR1}} \sim \mathcal{O}(\gamma^1)$, $F^{\mu}_{\mathrm{RR2}} \sim \mathcal{O}(\gamma^0)$ and $F^{\mu}_{\mathrm{RR3}} \sim \mathcal{O}(\gamma^2)$. Since the RR is significant in the highly relativistic regime, where F^{μ}_{RR3} is much larger than the other two terms, it is reasonable to neglect the terms F^{μ}_{RR1} and F^{μ}_{RR2}. Then, the simplified equations of motion in the normalized units are obtained as

$$\frac{dp_x}{dt} = -\frac{(1+v_p)\xi}{4} + \frac{\mathbf{r}\cdot\mathbf{p}}{4\gamma} - \frac{\gamma r_e r^2}{6} p_x, \quad (8)$$

$$\frac{d\mathbf{p}_\perp}{dt} = -\frac{\gamma + p_x}{\gamma}\frac{\mathbf{r}}{4} - \frac{\gamma r_e r^2}{6}\mathbf{p}_\perp, \quad (9)$$

$$\frac{d\xi}{dt} = \frac{p_x}{\gamma} - v_p, \quad (10)$$

$$\frac{d\mathbf{r}}{dt} = \frac{\mathbf{p}_\perp}{\gamma}. \quad (11)$$

Here, the electron motion is divided into parallel and perpendicular to the acceleration direction, so p_x and $\mathbf{p}_\perp = p_y\hat{y} + p_z\hat{z}$ represent the longitudinal and transverse momenta, respectively. The RR depends on the electron energy and its transverse amplitude of oscillations.

Equations (8)-(11) are solved numerically with SciPy in Python, that uses lsoda from the FORTRAN library odepack for solving a system of ordinary differential equations, to find the electron evolution with time. The electron is assumed to be initially located off-axis at the rear side of the bubble, where the accelerating field is maximum. It then accelerates till de-phasing happens, i.e. $\xi = 0$. The numerical calculation is considered over an acceleration stage to find the electron energy gain and radiation loss.

In the following, different possible plasma colliders based on the laser pulse, electron- and proton beam drivers are investigated numerically. In all cases, the electron is initially located off-axis in the y direction, i.e., $y_0 k_p = 0.1$, and at the rear side of the bubble, i.e., $\xi_0 = -r_b$. Moreover, a small transverse momentum of $p_{y_0} = 10^{-3} p_{x_0}$ is considered where p_{x_0} is the electron's initial longitudinal momentum.

III. MULTI-STAGE PLASMA COLLIDERS

For multi-stage plasma-based colliders, the plasma cells are interconnected by vacuum regions. Each plasma cell is excited by an individual driver (either laser pulse or electron beam) at a constant time gap. At the onset of the process, the first driver excites the plasma wave and generates an energetic electron beam. Once the electron beam is at the de-phasing point $\xi = 0$, the second plasma cell is excited by a similar driver and let the electron beam to place at the correct phase. In this way one can multiply the energy gain. In the present model, the vacuum gap between the plasma cells are neglected.

The electron evolution is calculated numerically for each stage, till it reaches the de-phasing point $\xi = 0$, at which the electron enters the next stage. In the new stage the electron is located at the rear side of the bubble and with the final momentum and transverse position in the previous stage. The simulation runs over all acceleration stages to explore RR-dominated regime and the relevant radiation loss.

A. Multi-stage LWFA collider

In an LWFA, the energy gain scaling usually depends on the plasma density n_0 and the laser strength a_0. In other words, the maximum electric field amplitude of the wake is $E_{\max} \simeq \sqrt{a_0}E_0$ where $E_0[\mathrm{V/m}] \simeq 96 \times \sqrt{n_0[\mathrm{cm}^{-3}]}$ is the wavebreaking limit of the plasma. The associated de-phasing length is given by $L_d \simeq \lambda_p(\omega_0/\omega_p)^2$ assuming that $\omega_0/\omega_p \ll 1$, where λ_p is the plasma wavelength and ω_0 is the laser frequency.[7] For a plasma density of $n_0 = 10^{17}\mathrm{cm}^{-3}$, the corresponding field strength is nearly $E_{\max} \simeq 33$ GV/m, and the de-phasing length is about $L_d \simeq 1.2\,$m for a laser wavelength of $1\,\mu$m. Therefore, for an LWFA-based collider, one needs about 30 acceleration stages to achieve 1 TeV energy.

The corresponding laser and plasma parameters for numerical calculation are as follow: laser wavelength $\lambda_0 = 1\,\mu$m, laser strength parameter $a_0 = 1.2$, and the plasma density $n_0 = 10^{17}\,\mathrm{cm}^{-3}$. The electron initial velocity is the same as the laser group velocity. The bubble phase velocity is $v_p = (1 - \omega_p^2/\omega_0^2)^{1/2}$, and the bubble radius of $r_b k_p = 3.1$ is considered which is comparable to the analytical formula $r_b k_p = 2\sqrt{a_0}$.[42]

Figure 1 shows the electron trajectory indicating the oscillatory nature of the electron motion (a), energy gain (b), radiation loss (c), and the QED parameter (d) of a single electron

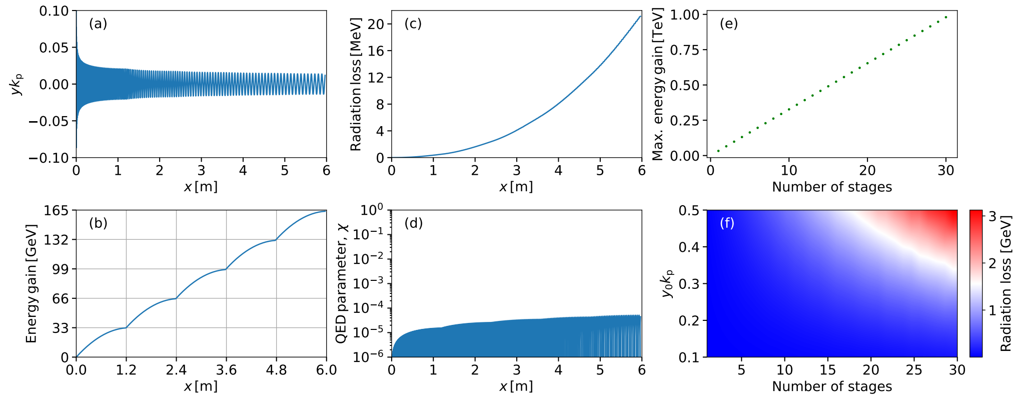


FIG. 1. A test electron accelerating in a staged LWFA. For plots (a)-(d) the number of stages are 5. The figure includes trajectory (a), energy gain (b), radiation loss (c), and the QED parameter (d). However, plots (e) and (f) correspond to 30 plasma stages to achieve 1 TeV energy. The maximum energy gain and a contour plot of energy loss are shown in plots (e) and (f), respectively. The laser and plasma parameters are $\lambda_0 = 1\,\mu$m, $a_0 = 1.2$, and $n_0 = 10^{17}\,\mathrm{cm}^{-3}$.

accelerated over 5 subsequent LWFA stages. The energy gain for each stage is about 33 GeV over the de-phasing length of 1.2 m, that leads to about 165 GeV energy gain in 6 m. The final radiation loss is about 20 MeV which is negligible. The energy loss is increasing with the number of staging as the energy gain increases. In other words, one can see from Eqs. (8) and (9) that the RR force depends on the electron energy mainly via γp_x term. The plot (d) confirms the validity of the classical approach as the QED parameter is much smaller than unity. For a TeV collider we may need about 30 LWFA stages which is investigated in plots (e) and (f). Plot (e) indicates the energy gain with respect to the number of stages. Plot (f) depicts the contour plot of the radiation loss as a function of the number of stages and the initial transverse position of the electron. The radiation loss is clearly significant when 30 stages are considered specially for larger initial transverse positions.

B. Multi-stage PWFA collider

High energy electron beams can be produced in conventional accelerators like SLC. Therefore, a PWFA-based collider has been conceptualized[23] by considering several stages of acceleration. Such colliders may suffer the limiting factor of RR effect. In this regard, a similar numerical approach to staged LWFA is followed and the electron evolution over several stages of PWFA is investigated.

Here, 10 PWFA stages are considered, with the plasma density of $n_0 = 2\times10^{16}\,\mathrm{cm}^{-3}$ which is consistent with the PWFA-based collider design.[23] Both the driver and the single electron energy are considered to be 25 GeV. The driver interacts with the plasma and creates a bubble, so the single electron at the rear side of the bubble accelerates. The bubble phase velocity is considered to be the same as the driver's velocity, and the bubble radius of $r_b k_p = 3.6$ is assumed.[42]

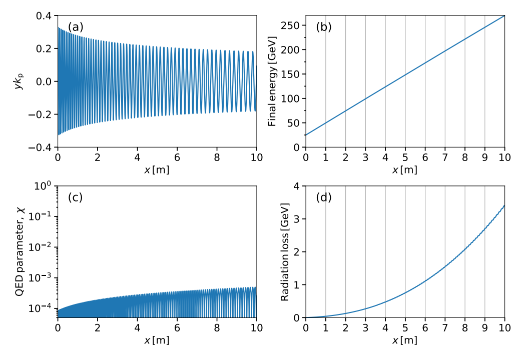


FIG. 2. A single electron accelerating in a multi-stage PWFA collider with 10 stages. The plots are, electron trajectory (a), energy gain (b), radiation loss (c), and the QED parameter (d). Here, $n_0 = 2\times10^{16}\,\mathrm{cm}^{-3}$, and both the driver and the single electron energies are 10 GeV.

Figure 2 shows the trajectory (a), energy gain (b), radiation loss (c), and the QED parameter (d) of a single electron accelerated over 10 PWFA stages. The energy gain for each stage is about 25 GeV over the distance of 1 m. It leads to about 250 GeV energy gain in 10 m. The plot (c) confirms the validity of the classical approach as the QED parameter is much smaller than unity. As one can see in plot (d), the energy loss is negligible in the early stages, but it increases sharply to above 3 GeV as the number of stages increases. The reason is that the RR force depends on the electron energy which increases with respect to the staging. That indicates the significant role of the RR at high energies which limits the energy gain from what is expected.

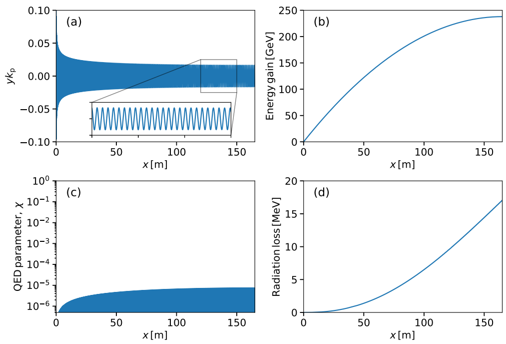


FIG. 3. A test particle in a PDPWFA with SPS driver. The plots are the electron trajectory (a), energy gain (b), QED parameter (c), and the corresponding radiation loss (d). Here, $n_0 = 7\times10^{14}\,\mathrm{cm}^{-3}$, and the proton driver energy is 400 GeV.

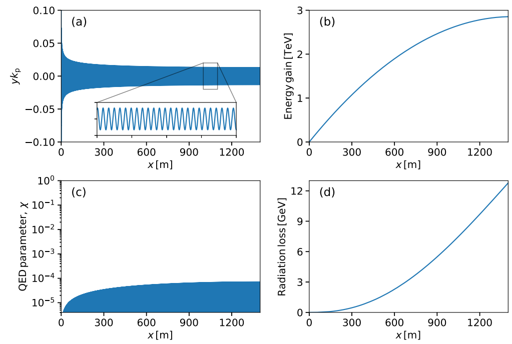


FIG. 4. A test particle in a PDPWFA with LHC proton driver. The plots are the electron trajectory (a), energy gain (b), QED parameter (c), and the corresponding radiation loss (d). Here, $n_0 = 7\times10^{14}\,\mathrm{cm}^{-3}$, and the proton driver energy is 1 TeV.

IV. SINGLE-STAGE PDPWFA COLLIDER

The CERN accelerators complex can deliver very energetic proton beams of 400 GeV and 7 TeV energies at the SPS and LHC, respectively. Such TeV-range proton beams can drive plasma wakefields for longer distances. The electron beam then can potentially gain energies high enough for a TeV collider over a single stage.[24,43]

After various theoretical investigations, the AWAKE experiment was proposed to exploit existing SPS proton beam to study the PDPWFA.[44] After the successful proof of principle of PDPWFA in AWAKE Run 1,[26] the AWAKE Run 2 experiment (2021-) aims at producing a monoenergetic and stable electron acceleration suitable for particle physics experiments.[27,45]

In the linear regime, the phase velocity of the bubble is smaller than the proton beam velocity but becomes comparable when going to the nonlinear regime.[46] Considering the nonlinear bubble, we can solve the Eqs.(8)-(11) to estimate the energy gain and the corresponding radiation loss for a test electron.

A. SPS driver: AWAKE experiment

In the AWAKE Run 2, the electron beam is accelerated over 10 metres of plasma.[27] However, the electrons potentially can ride the wakefield over hundreds of metres before de-phasing. In that case, the electrons achieve collider energies in a single-stage acceleration. It is shown that de-phasing is a limiting factor for sub-TeV proton beams.[47] Specially, within simulation it is shown that by the SPS driver, the maximum energy gain of the electron beam is 55 GeV.[48] Although this can be increased to about 250 GeV within a proper plasma density modulation.[49]

The parameters are chosen as follow: The plasma density of $n_0 = 7\times10^{14}\,\mathrm{cm}^{-3}$ is considered. The SPS proton driver has the energy of 400 GeV, and creates the bubble wakefield when interacting with the plasma. The single electron initially is located off-axis at the rear side of the bubble with the initial energy of 150 MeV, as in the AWAKE Run 2 case.[27] The bubble phase velocity is considered to be the same as the driver's, and the bubble radius of $r_\mathrm{b}k_\mathrm{p} = 2.26$ is assumed.[42]. The electron evolution is investigated in one acceleration stage till it reaches the de-phasing point.

Figure 3 shows the trajectory of the test electron with a partially magnified inset to show its oscillatory nature (a), the corresponding energy gain (b), the QED parameter (c), and its energy loss due to the radiation damping (d). The electron gains significant energy of ~ 250 GeV in one stage, which is suitable for high energy physics experiments. The negligible amount of the QED parameter in plot (c) indicates that the classical approach is still working here. The energy loss due to the radiation is also negligible as one can see in plot (d).

B. LHC driven PDPWFA

A multi-TeV electron/positron linear collider similar to the AWAKE, but with much higher energy has been proposed by employing the LHC proton beam as the driver.[25] Here, we investigate the RR effect on a single electron in the wakefield of a 1 TeV proton driver. The electron evolution is investigated in one acceleration stage till it reaches the de-phasing point.

The plasma density of $n_0 = 7\times10^{14}\,\mathrm{cm}^{-3}$ is considered. The initial electron velocity is assumed to be similar to the proton driver. The bubble phase velocity is considered to be the same as the driver's, and the bubble radius of $r_\mathrm{b}k_\mathrm{p} = \pi$ is assumed.[42].

Figure 4 presents the trajectory of the single electron with a partially magnified inset to elucidate its oscillatory behaviour in the bubble (a), the corresponding energy gain reaching $\sim$3 TeV at the de-phasing point (b), the negligible QED parameter confirming the validity of the classical approach (c), and its energy loss due to the radiation (d). Although the energy loss seems quite high, it is really small compared with the energy gain.

Initially different transverse offsets of the test electron can influence the RR impact. In this regard, a comparison study is done with different transverse initial positions of the test electron, i.e. $y_0 k_\mathrm{p} = 0.2, 0.4, 0.6$. Moreover, various proton beams with different energies in the range of 400 GeV SPS beam to 7 TeV LHC beam are explored. The parameter are the same as the LHC-driven acceleration. Figure 5 compares maximum energy gain (a), radiation loss (b) and the percentage of the radiation loss to the energy gain (c) for a single electron with the initial velocity equal to that of the relevant proton driver. Plot (a) elucidates the desired TeV energy gain especially with more energetic drivers. A non-trivial trend in this plot is that the particle with lower offset gains more energy than that with higher offset. This can be explained within plot (b). In fact the particle at higher offset loses more energy due to the radiation. This is because of the higher oscillation amplitude during acceleration, so it emits more radiation. In other words, the radiation loss depends on the particle offset from the propagation axis as well as its energy. Therefore, an electron with higher energy and higher offset loses more energy due to the radiation effect. Plot (c) indicates the importance of RR effect specially for those drivers with multi-TeV energies as the radiation loss percentage becomes considerable.

The significant energy gain over a single acceleration stage without considerable energy loss, makes PDPWFA more interesting for particle physics applications of a plasma-based TeV collider.

V. CONCLUSIONS

The huge progress in laser wakefield accelerators and specially multi-stage LWFA, as well as electron-driven PWFA at SLAC, and proton-driven wakefield experiment AWAKE at CERN is promising to an energy frontier collider for particle physics applications. The concepts of such colliders have already been discussed. However, the future TeV plasma-based colliders may suffer from the RR effect limiting its continuing energy gain. In this paper an extensive study of RR effect in all pathways of plasma accelerators towards energy-frontier colliders including staged-LWFA and PWFA, as well as the single-stage PDPWFA has been done. A single-particle Landau-Lifschitz approach is used to consider the RR effect on an electron accelerating in the bubble regime. The model determines where RR plays a limiting role on the electron energy gain. It is shown that while the radiation damping is negligible in a single LWFA or PWFA, it becomes important within several stages. Moreover, in a single stage PDPWFA, significant energy gain over a single stage of acceleration without considerable energy loss is achieved.

ACKNOWLEDGEMENTS

We gratefully acknowledge the support of the Cockcroft Institute Core Grant and the STFC.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

[1] A. Abada, M. Abbrescia, S. S. AbdusSalam, I. Abdyukhanov, J. A. Fernandez, A. Abramov, M. Aburaia, A. O. Acar, P. R. Adzic, P. Agrawal, and et. al., "Fcc-ee: The lepton collider," The European Physical Journal Special Topics **228**, 261–623 (2019).
[2] T. Behnke, J. E. Brau, B. Foster, J. Fuster, M. Harrison, J. M. Paterson, M. Peskin, M. Stanitzki, N. Walker, and H. Yamamoto, *The International Linear Collider Technical Design Report*, Vol. 1: Executive Summary (arXiv, 2013).
[3] T. Charles, P. Giansiracusa, T. Lucas, R. Rassool, M. Volpi, C. Balazs, K. Afanaciev, V. Makarenko, A. Patapenka, I. Zhuk, and et al., "The compact linear collider (clic)—2018 summary report," Tech. Rep. (CERN Yellow Rep. Monogr., 2018).
[4] F. Zimmermann, "Accelerator technology and beam physics of future colliders," Frontiers in Physics **10** (2022).
[5] V. Shiltsev and F. Zimmermann, "Modern and future colliders," Reviews of Modern Physics **93** (2021), 10.1103/RevModPhys.93.015006.
[6] V. D. Shiltsev, "High-energy particle colliders: past 20 years, next20 years, and beyond," Physics-Uspekhi **55**, 965–976 (2012).
[7] E. Esarey, C. B. Schroeder, and W. P. Leemans, "Physics of laser-driven plasma-based electron accelerators," Rev. Mod. Phys. **81** (2009), 10.1103/RevModPhys.81.1229.
[8] M. J. Hogan, "Electron and positron beam–driven plasma acceleration," Reviews of Accelerator Science and Technology **9**, 63–83 (2016).
[9] E. Adli and P. Muggli, "Proton-beam-driven plasma acceleration," Reviews of Accelerator Science and Technology **9**, 85–104 (2016).
[10] S. Gessner, E. Adli, J. M. Allen, W. An, C. I. Clarke, C. E. Clayton, S. Corde, J. Delahaye, J. Frederico, S. Z. Green, and et. al., "Demonstration of a positron beam-driven hollow channel plasma wakefield accelerator," Nature Communications **7** (2016), 10.1038/ncomms11785.
[11] Y. Li, G. Xia, K. V. Lotov, A. P. Sosedkin, and Y. Zhao, "High-quality positrons from a multi-proton bunch driven hollow plasma wakefield accelerator," Plasma Phys. Control. Fusion **61** (2019), 10.1088/1361-6587/aaf121.
[12] A. Doche, C. Beekman, S. Corde, J. M. Allen, C. I. Clarke, J. Frederico, S. J. Gessner, S. Z. Green, M. J. Hogan, B. O'Shea, and et al., "Acceleration of a trailing positron bunch in a plasma wakefield accelerator," Scientific Reports **7** (2017), 10.1038/s41598-017-14524-4.
[13] S. Diederichs, C. Benedetti, E. Esarey, J. Osterhoff, and C. B. Schroeder, "High-quality positron acceleration in beam-driven plasma accelerators," Physical Review Accelerators and Beams **23** (2020), 10.1103/PhysRevAccelBeams.23.121301.
[14] W. P. Leemans, B. Nagler, A. J. Gonsalves, C. Tóth, K. Nakamura, C. G. R. Geddes, E. Esarey, C. B. Schroeder, and S. M. Hooker, "Gev electron beams from a centimetre-scale accelerator," Nature Physics **2**, 696–699 (2006).
[15] A. J. Gonsalves, K. Nakamura, J. Daniels, C. Benedetti, C. Pieronek, T. C. H. de Raadt, S. Steinke, J. H. Bin, S. S. Bulanov, J. van Tilborg, and et al., "Petawatt laser guiding and electron beam acceleration to 8 gev in a laser-heated capillary discharge waveguide," Phys. Rev. Lett. **122** (2019), 10.1103/PhysRevLett.122.084801.
[16] S. Steinke, J. van Tilborg, C. Benedetti, C. G. R. Geddes, J. Daniels, K. K. Swanson, A. J. Gonsalves, K. Nakamura, B. H. Shaw, C. B. Schroeder, and et al., "Staging of laser-plasma accelerators," Physics of Plasmas **23** (2016), 10.1063/1.4948280.
[17] W. Leemans and E. Esarey, "Laser-driven plasma-wave electron accelerators," Physics Today **62**, 44 (2009).

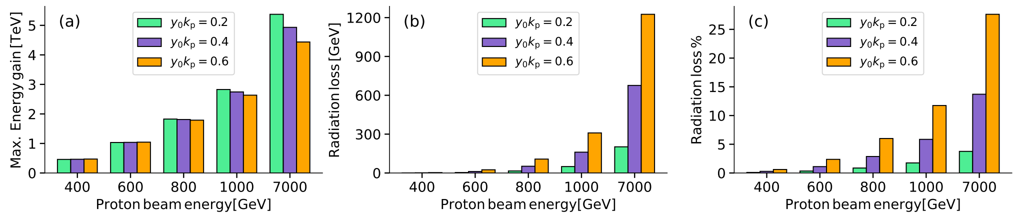


FIG. 5. A comparative study of a test particle in PDPWFA with different initial transverse position and various proton driver energies. The maximum electron energy gain (a), the radiation loss (b), and the corresponding radiation loss percentage (c) are shown.

[18] C. B. Schroeder, E. Esarey, C. G. R. Geddes, C. Benedetti, , and W. P. Leemans, "Physics considerations for laser-plasma linear colliders," Physical Review Accelerators and Beams **13** (2010), 10.1103/PhysRevSTAB.13.101301.

[19] C. B. Schroeder, C. Benedetti, E. Esarey, and W. P. Leemans, "Laser-plasma-based linear collider using hollow plasma channels," Nuclear Instruments and Methods in Physics Research A **829**, 113–116 (2016).

[20] I. Blumenfeld, C. E. Clayton, F.-J. Decker, M. J. Hogan, C. Huang, R. Ischebeck, R. Iverson, C. Joshi, T. Katsouleas, N. Kirby, and et al., "Energy doubling of 42 gev electrons in a metre-scale plasma wakefield accelerator," Nature **445**, 741–744 (2007).

[21] M. Litos, E. Adli, W. An, C. Clarke, C. Clayton, S. Corde, J. Delahaye, R. England, A. Fisher, J. Frederico, and et al., "High-efficiency acceleration of an electron beam in a plasma wakefield accelerator," Nature **515** (2014), 10.1038/nature13882.

[22] E. Adli, "Plasma wakefield linear colliders-opportunities and challenges," Phil. Trans. R. Soc. A **377** (2019), 10.1098/rsta.2018.0419.

[23] E. Adli, "Towards a pwfa linear collider — opportunities and challenges," Journal of Instrumentation **17** (2022), 10.1088/1748-0221/17/05/T05006.

[24] A. Caldwell, K. Lotov, A. Pukhov, and F. Simon, "Proton-driven plasma-wakefield acceleration," Nat. Phys. **5**, 363–367 (2009).

[25] G. Xia, O. Mete, A. Aimidula, C. P. Welsch, S. Chattopadhyay, S. Mandry, and M.Wing, "Collider design issues based on proton-driven plasmawakefield acceleration," Nuclear Instruments and Methods in Physics Research A **740**, 173–179 (2014).

[26] E. Adli, A. Ahuja, O. Apsimon, R. Apsimon, A.-M. Bachmann, D. Barrientos, F. Batsch, J. Bauche, V. K. B. Olsen, M. Bernardini, and et.al., "Acceleration of electrons in the plasma wakefield of a proton bunch," Nature **561**, 363–367 (2018).

[27] E. Gschwendtner, K. Lotov, P. Muggli, M. Wing, R. Agnello, C. C. Ahdida, M. C. A. Goncalves, Y. Andrebe, O. Apsimon, R. Apsimon, and et. al., "The awake run 2 programme and beyond," Symmetry **14**, 1680 (2022).

[28] A. Pukhov and J. M. ter Vehn, "Laser wake field acceleration: the highly non-linear broken-wave regime," Applied Physics B **74**, 355–361 (2002).

[29] S. Corde, K. T. Phuoc, G. Lambert, R. Fitour, V. Malka, A. Rousse, A. Beck, and E. Lefebvre, "Femtosecond x rays from laser-plasma accelerators," Reviews of Modern Physics **85**, 1 (2013).

[30] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (New York:John Wiley and Sons, 1999).

[31] A. Gonoskov, T. G. Blackburn, M. Marklund, and S. S. Bulanov, "Charged particle motion and radiation in strong electromagnetic fields," Reviews of Modern Physics **94** (2022), 10.1103/RevModPhys.94.045001.

[32] F. Zimmermann, "Possible limits of plasma linear colliders," IOP Conf. Series: Journal of Physics: Conf. Series **874** (2017), 10.1088/1742-6596/874/1/012030.

[33] P. Michel, C. B. Schroeder, B. A. Shadwick, E. Esarey, and W. P. Leemans, "Radiative damping and electron beam dynamics in plasma-based accelerators," Physical Review E **74** (2006), 10.1103/PhysRevE.74.026501.

[34] I. Y. Kostyukov, E. N. Nerush, and A. G. Litvak, "Radiative damping in plasma-based accelerators," Physical Review Accelerators and Beams **15** (2012), 10.1103/PhysRevSTAB.15.111001.

[35] M. Zeng and K. Seto, "Radiation reaction of betatron oscillation in plasma wakefield accelerators," New Journal of Physics **23** (2021), 10.1088/1367-2630/ac12fa.

[36] Y. Liu and M. Zeng, "Modeling of three-dimensional betatron oscillation and radiation reaction in plasma accelerators," Physical Review Accelerators and Beams **26** (2023), 10.1103/PhysRevAccelBeams.26.031301.

[37] D. A. Burton and A. Noble, "Aspects of electromagnetic radiation reaction in strong fields," Contemporary Physics **55**, 110–121 (2014).

[38] A. D. Yaghjian, "Lorentz-abraham-dirac and landau-lifshitz equations of motion and the solution to a relativistic electron in a counterpropagating laser beam," Physical Review Accelerators and Beams **24** (2021), 10.1103/PhysRevAccelBeams.24.114002.

[39] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, Vol. 2 of Course of Theoretical Physics (Amsterdam:Pergamon, 1975).

[40] I. Kostyukov, A. Pukhov, and S. Kiselev, "Phenomenological theory of laser-plasma interaction in bubble regime," Phys. of Plasmas **11** (2004), 10.1063/1.1799371.

[41] W. Lu, C. Huang, M. Zhou, M. Tzoufras, F. S. Tsung, W. B. Mori, and T. Katsouleas, "A nonlinear theory for multidimensional relativistic plasma wave wakefields," Physics of Plasmas **13** (2006), 10.1063/1.2203364.

[42] W. Lu, C. Huang, M. Zhou, W. B. Mori, and T. Katsouleas, "Nonlinear theory for relativistic plasma wakefields in the blowout regime," Phys. Rev. Lett. **96** (2006), 10.1103/PhysRevLett.96.165002.

[43] A. Caldwell and K. V. Lotov, "Plasma wakefield acceleration with a modulated proton bunch," Physics of Plasmas **18** (2011).

[44] G. Xia, R. Assmann, R. A. Fonseca, C. Huang, W. Mori, L. O. Silva, J. Vieira, F. Zimmermann, and P. Muggli, "A proposed demonstration of an experiment of proton-driven plasma wakefield acceleration based on cern sps," Journal of Plasmas Physics **78** (2012), 10.1017/S0022377812000086.

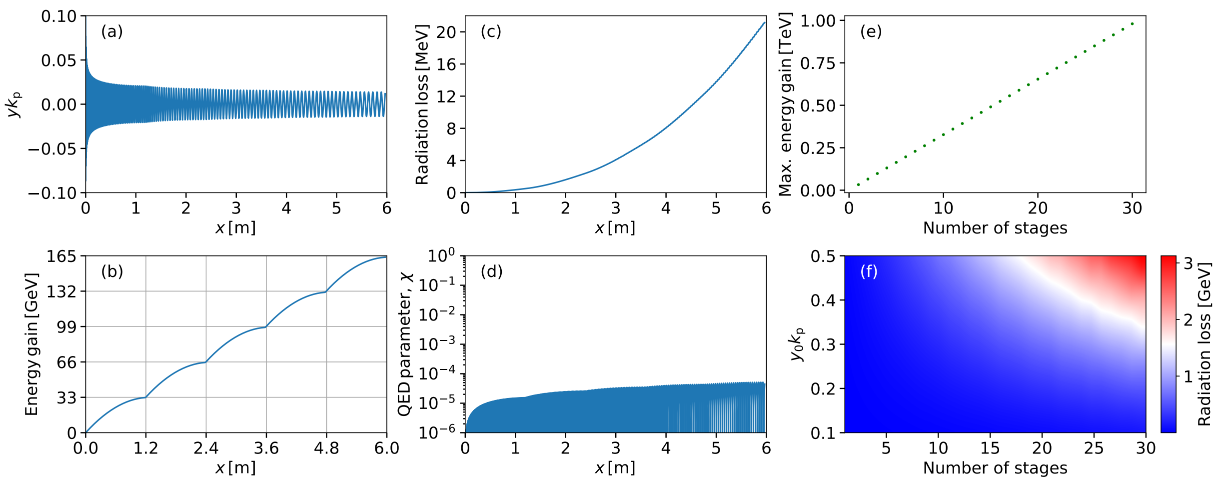
[45] P. Muggli, "White paper: Awake, plasma wakefield acceleration ofelectron bunches for near and long term particle physics applications," Archive (2022).

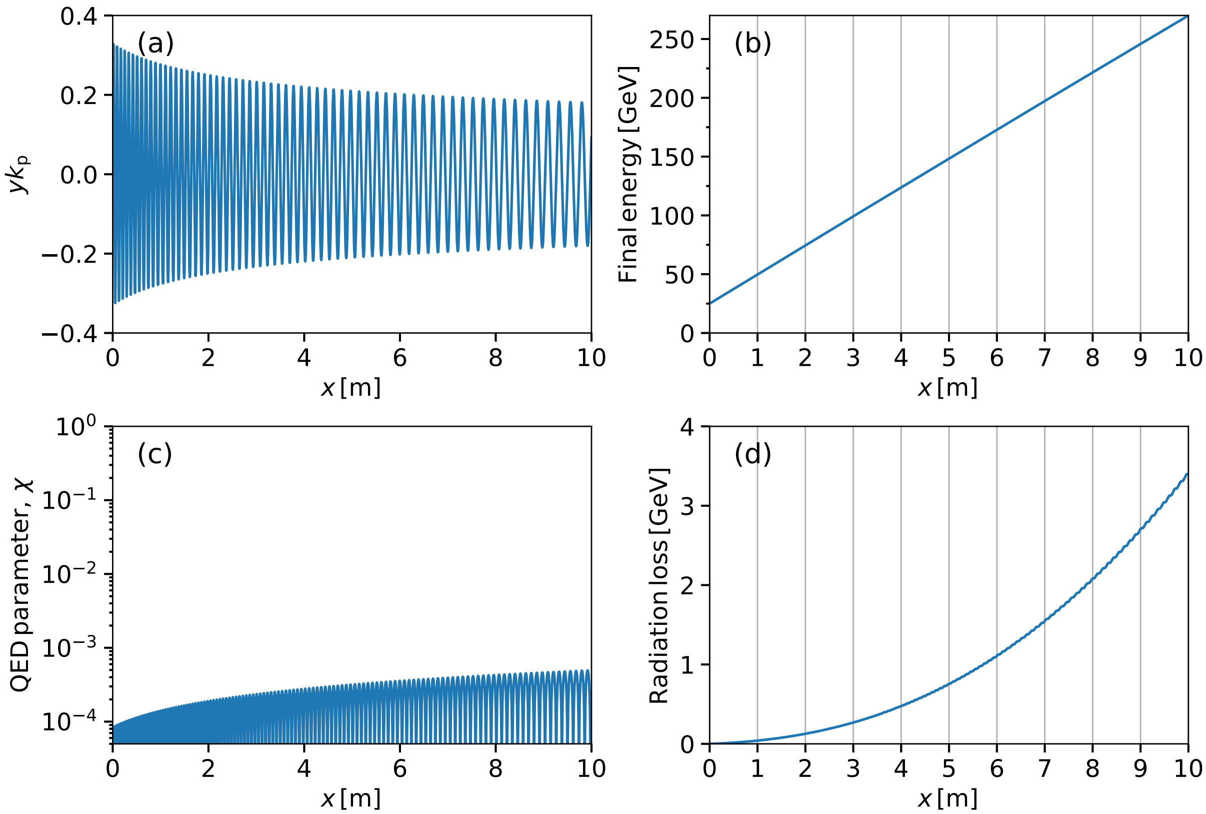
[46] A. Pukhov, N. Kumar, T. Tückmantel, A. Upadhyay, K. Lotov, P. Muggli, V. Khudik, C. Siemon, and G. Shvets, "Phase velocity and particle injection in a self-modulated proton-driven plasma wakefield accelerator," Phys. Rev. Lett. **107** (2011).

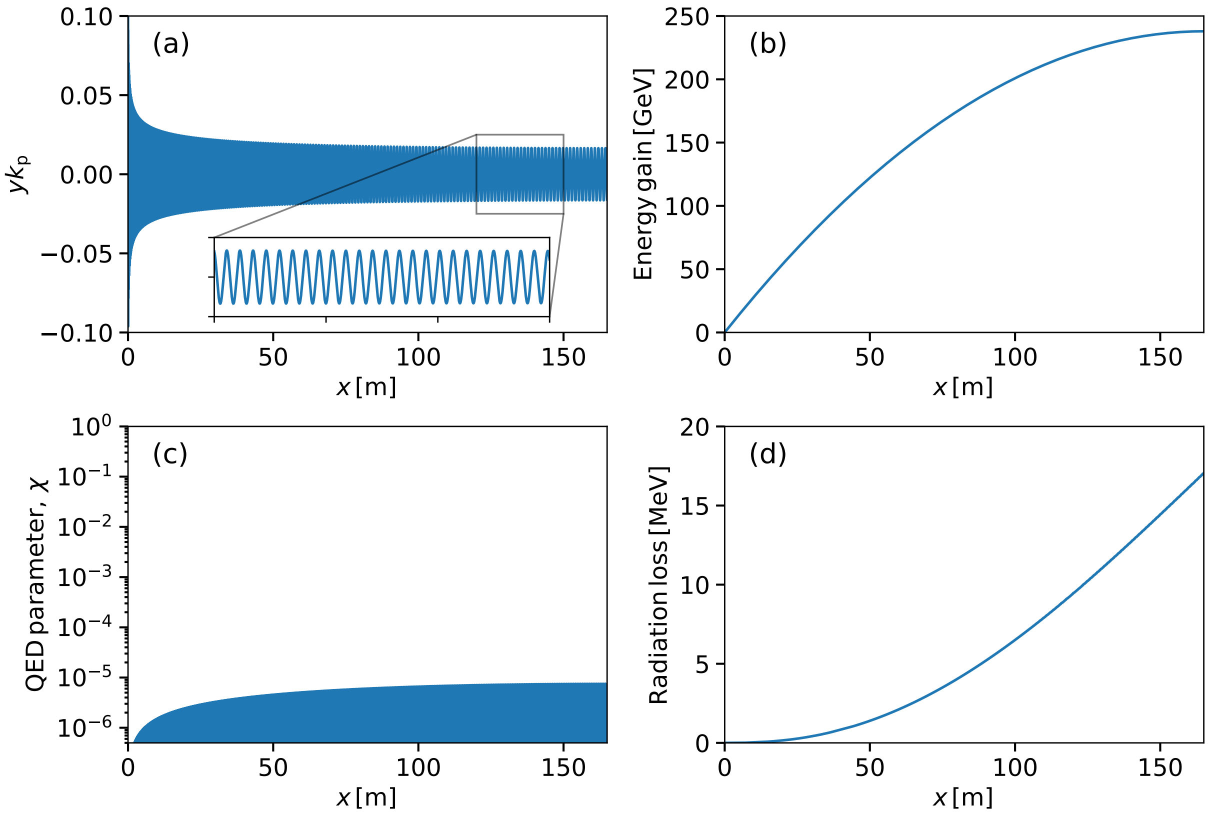
[47] K. V. Lotov, "Simulation of proton driven plasma wakefield acceleration," Phys. Rev. ST Accel. Beams **13** (2010), 10.1103/PhysRevSTAB.13.041301.

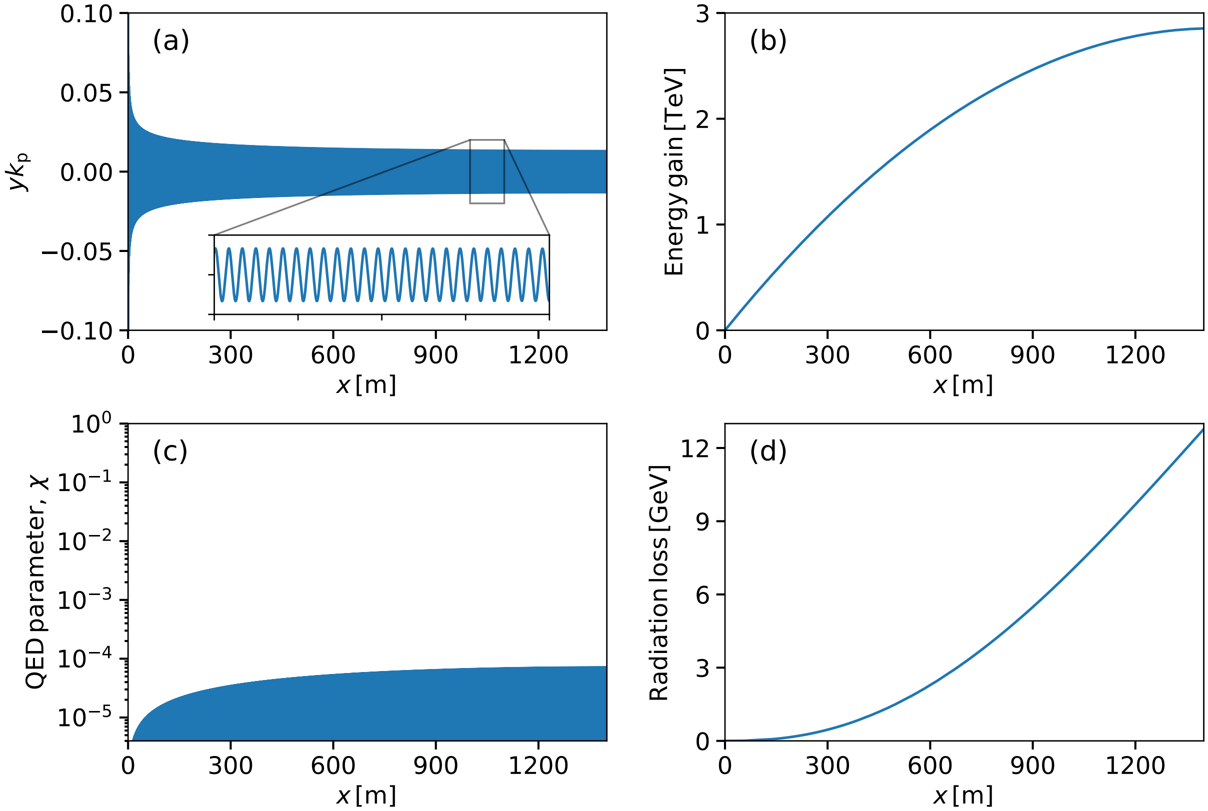
[48] W. Bartmann, A. Caldwell, M. Calviani, J. Chappell, P. Crivelli, H. Damerau, E. Depero, S. Doebert, J. Gall, S. Gninenko, and et al., *AWAKE++: the AWAKE Acceleration Scheme for New Particle PhysicsExperiments at CERN* (CERN-PBC-REPORT-2018-005, 2018).

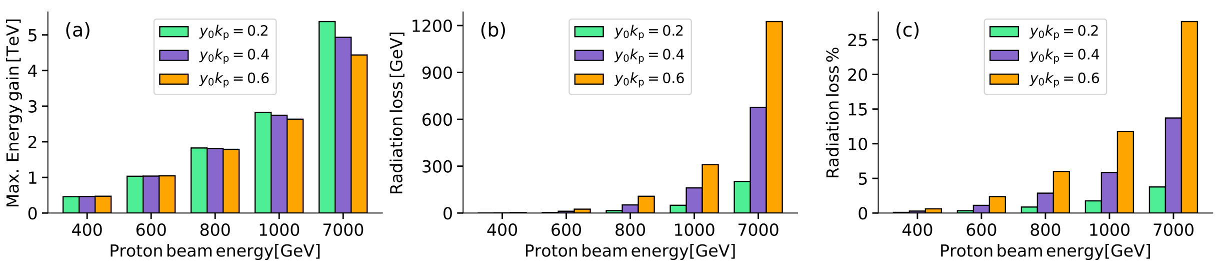
[49] K. V. Lotov and P. V. Tuev, "Plasma wakefield acceleration beyond the dephasing limit with 400 gev proton driver," Plasma Physics and Controlled Fusion **63** (2021).


(a)
yk_p
x [m]
(b)
Energy gain [GeV]
x [m]
(c)
Radiation loss [MeV]
x [m]
(d)
QED parameter, χ
x [m]
(e)
Max. energy gain [TeV]
Number of stages
(f)
y_0k_p
Number of stages
Radiation loss [GeV]


(a)
(b)
(c)
(d)
yk_p
Final energy [GeV]
QED parameter, χ
Radiation loss [GeV]
x [m]


(a)
yk_p
0.10
0.05
0.00
−0.05
−0.10
0
50
100
150
x [m]
(b)
Energy gain [GeV]
250
200
150
100
50
0
0
50
100
150
x [m]
(c)
QED parameter, χ
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
50
100
150
x [m]
(d)
Radiation loss [MeV]
20
15
10
5
0
0
50
100
150
x [m]


(a)
yk_p
0.10
0.05
0.00
−0.05
−0.10
0
300
600
900
1200
x [m]
(b)
Energy gain [TeV]
3
2
1
0
x [m]
(c)
QED parameter, χ
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
x [m]
(d)
Radiation loss [GeV]
12
9
6
3
0
x [m]


(a)
Max. Energy gain [TeV]
$y_0k_p = 0.2$
$y_0k_p = 0.4$
$y_0k_p = 0.6$
Proton beam energy[GeV]
400
600
800
1000
7000
(b)
Radiation loss [GeV]
(c)
Radiation loss %