CRANFIELD UNIVERSITY

EBIAKPO KAKANDAR

STUDY OF THE DEPENDENCIES BETWEEN IN-SERVICE DEGRADATION AND KEY DESIGN PARAMETERS WITH UNCERTAINTY FOR MECHANICAL COMPONENTS

SCHOOL OF AEROSPACE, TRANSPORT AND MANUFACTURING
Manufacturing Department

PhD Thesis
Academic Year: 2013 - 2017

Supervisor: Professor Rajkumar Roy and Professor Jorn Mehnen
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This thesis is submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy

ABSTRACT

The design features of machine components can impact significantly in its life while in-service, and only relatively few studies which are case specific have been undertaken with respect to this. Hence, the need for more understanding of the influence of geometric design features on the service life of a machine component. The aim of this research is to develop a methodology to assess the degradation life of a mechanical component due to geometric design influence in the presence of uncertainties and its application for the optimisation of the component in the presence of these uncertainties. This thesis has proposed a novel methodology for assessing the thermal fatigue life, a degradation mechanism based on the influence of design features in the presence of uncertainties. In this research a novel uncertainty analysis methodology that is able to handle simultaneously the presence of aleatory and epistemic uncertainties is proposed for a more realistic prediction and assessment of a components thermal fatigue degradation life estimated using finite element analysis. A design optimisation method for optimising the components design in the presence of mixed uncertainty, aleatory and epistemic uncertainties is also proposed and developed. The performance of the proposed methodology is analysed through the use of passenger vehicle brake discs. The novel uncertainty quantification methodology was initially applied on a solid brake disc, and validated for generalisability using a vented brake disc which has more complex design features. While the optimisation method as proposed was applied on the vented brake disc. With these this research proposes a validated set of uncertainty and optimisation methodology in the presence of mixed uncertainties for a design problem. The methodologies proposed in this research provide design engineers with a methodology to design components that are robust by giving the design with the least uncertainty in its output as result of design parameters inherent variability while simultaneously providing the design with the least uncertainty in estimation of its life as a result of the use of surrogate models.

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My greatest thanks to my parents Mr and Mrs Kakandar, particularly my father Felix Okosikeme Kakandar who I dedicate this thesis to, and also to Mr and Mrs Ekeremor. I also say thanks to my siblings and friends who have been a constant source of help to me. And particularly my wife and sweetheart Mimi.

Above all, I give thanks to the Almighty God for making this come true.

LIST OF PUBLICATIONS

Kakandar, E., Roy, R., Mehnen, J., and Addepalli, P. A Robust Design Optimisation Methodology in the Presence of Mixed Uncertainty. Engineering Optimisation, 2017 (Submitted for publication)

Kakandar, E., Roy, R., and Mehnen, J. An Uncertainty Based Thermal Fatigue Life Prediction Methodology Using Finite Element Analysis. Journal of Engineering Design, 2017 (Submitted for publication and under review)

Kakandar, E., Roy, R., and Mehnen, J. (2017) ‘A Simulation Based Approach to Model Design Influence on the Fatigue Life of a Vented Brake Disc’, *Procedia CIRP*, 59, pp. 41–46.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

ADT	Accelerated degradation test
ALTS	Accelerated life test
ANOVA	Analysis of Variance
Bel	Belief
BPA	Basic probability assignment
CAD	Computer aided design
CDF	Cumulative density function
COV	Coefficient of variation
DSSAvgwidth	Epistemic average width of nested intervals
EBDO	Epistemic based design optimisation
eFAST	Extended Fourier Amplitude Sensitivity test
FAST	Fourier Amplitude Sensitivity test
FEA	Finite element analysis
NDT	Non-destructive testing
OAT	One at a time
PBDO	Possibilistic based design optimisation
Pl	Plausibility
PRCC	Partial rank correlation coefficient
R^2	Coefficient of multiple determination of least squares
R^2 adj.	Adjusted coefficient of multiple determination of least squares
RBDO	Reliability based design optimisation
RDO	Robust design optimisation
SRC	Standardised regression coefficient
SRCC	Spearman rank correlation coefficient
Std.	Standard deviation
α - cut	Alpha cut
μ	Mean
ω	Angular velocity
σ_T	Total uncertainty

ϕ	Heat flux density

1 . INTRODUCTION

The health of machine components degrades over time while in service, eventually leading to a loss in functionality if not checked. This degradation of machines have received great attention over the last decades due to requirements for accurate assessment of current and future health states of machines by modern engineering asset management. This is seen in the constant demand for highly reliable machine systems. This demand arises from the desire to prevent failure with its attendant consequences that can at times be catastrophic (Kim et al. 2012). Most failures occur as a result of degradation mechanisms for which there are performance characteristics that change over time. Degradation has been defined in terms of performance or the change in component material or dimensions. Batchelor et al. (2002) defines degradation as a loss in performance of an engineering system. This loss in performance can be on the system level or on the component level. Considering degradation as a loss of performance at the component level, Son et al. (2007) described degradation as that loss in performance as result of material and dimensional changes in a component. These changes are often attributable to events such as time, wear, corrosion, fracture and temperature. Degradation of engineering components due to the effects of operating and environmental conditions produces time changing characteristics in the component structure which brings about changes in the system's performance over time.

Material degradation carries a cost, an undesirable financial burden. Various studies have attempted to analyse the costs of material degradation. These studies generally agree that degradation has a major impact on service cost

(Batchelor, Lam and Chandrasekaran, 2002; Zhao *et al.*, 2011). Due to degradation maintenance activities have to be carried out to ensure that an asset continues to perform its intended function. These maintenance activities carry a financial cost. An understanding of degradation mechanisms can lead to considerable cost savings in the utilisation of machine components. Hence, the need to design and develop engineering components that show better resilience to degradation. To develop mitigation measures against degradation, it is vital to identify what constitutes degradation in engineering systems and how it is influenced by events that drive it. These events that drive degradation are referred to as degradation mechanism. Degradation mechanisms are the phenomenon or processes that lead to a degradation or failure of the system. Such processes can be physical, chemical, thermal, biological or mechanical or in a combination of these, and they include processes such as fatigue, corrosion, creep, wear (Batchelor, Lam and Chandrasekaran, 2002; Suhir, 2007). In degradation studies of machine components, an investigation of degradation mechanisms alone is not enough. There should be an attempt not only to understand the progression of degradation mechanisms but also to understand the root cause and development of appropriate actions to minimize its reoccurrence (Morin, Shipley and Wilkinson, 1994).

The objective of this chapter is to present a review of the concept of degradation and the nature of the existing research on degradation. Based on the reviewed literature the research aims and objectives are presented. The chapter consists of several sections of which Section 1.1 gives a brief review of degradation mechanisms, and Section 1.2 focusing on fatigue degradation being the most common degradation mechanism as well as the approaches used for assessing the fatigue life of a component. Sections 1.3 and 1.4 gives the motivations for the study and the findings based on the initial review of literature respectively. In Section 1.5 the aim of the research thesis is presented. The research was funded and carried out in collaboration with supporting organisations, of which a synopsis of these organisations are presented in Section 1.6. The chapter concludes with Section 1.7 which provides a summary of the entire thesis document structure.

1.1 Degradation Mechanisms

Degradation mechanisms have been classified by several authors into different number of mechanisms. Batchelor, Lam, and Chandrasekaran (2002) gives the classification of materials degradation into three basic phenomena; physical, chemical and biological. In their classification, physical phenomena are degradation that arises as a result of the effect of heat, force and radiation. While the chemical and biological phenomena are a result of chemical and biological interactions of engineering materials with chemicals and life forms respectively. Tumer and Stone (2003) classified these mechanisms into thirty-four sub mechanisms under twenty-three main categories. Figure 1.1 shows a list of typical degradation mechanisms.

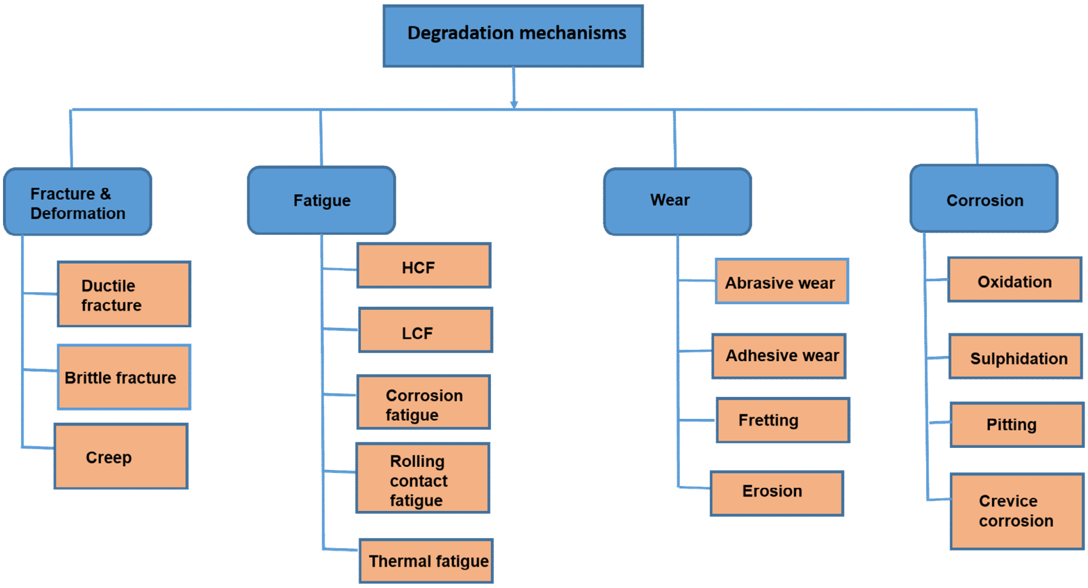


Figure 1.1 Types of degradation mechanisms

1.1.1 Mechanical Degradation Mechanisms: Fracture, Fatigue and Creep

Primary modes of the structural failure of engineering materials can usually be attributed to fracture, fatigue and creep. Fracture can be classified as brittle, ductile, static, dynamic, creep rupture, torsional; it is actually a broad term. In static fracture stress plays an important role. Static fracture occurs in uniaxial stressed tensile coupons as a result of excessive loading. In static stressed materials cyclic loads are not present. Ductile or brittle fracture normally occurs after a heavy load is applied to a structure (Batchelor, Lam and Chandrasekaran, 2002).

Fatigue failure refers to the sudden and often catastrophic failure of a material part as a result of the application of oscillating loads applied over a period of time that the material breaks into two or more pieces (Collins and Daniewicz, 2006). If a material is subjected to true constant stress, fatigue failure will not occur. Fatigue failure is characterised by two distinct mechanisms of cyclic stressing which are significantly different from each other, low cycle fatigue and high cycle fatigue. The difference between these two is in their loading and the number of cycles before failure. Low cycle fatigue is characterised by high loads and low number of cycles, and is normally associated with cycles of less than 10^5 cycles. While the high cycle fatigue the strain cycles are mainly confined to the elastic range, and is associated with lower loads and longer cycles of life to produce fatigue failure (Collins and Daniewicz, 2006). Failure by fatigue occurs by the initiation and progressive propagation of a crack until a stress level that is sufficient to cause rapid fracture is reached and the material fails by fracturing. Fatigue of engineering materials is influenced by several factors such as size, surface finish, loading effects, surface treatments, environment and temperature. In designing parts for high cycle fatigue, fatigue data can be plotted on a log-log plot of stress versus life. These plots provide design engineers basic information for machine parts that are subjected to cyclic loading and are called S-N curves. (Batchelor, Lam and Chandrasekaran, 2002; Collins and Daniewicz, 2006).

Creep as a mechanism of failure is mostly associated with failure of components operating in high temperature environments; the components can be static or rotating. Rotating turbine blades for instance are susceptible to creep damage due to their high temperature environment. Failure due to creep occurs when the accumulated creep strain results in the deformation of the machine component exceeding the design limits (Collins and Daniewicz, 2006). Susceptibility of materials to creep is not a function of temperature in the real sense but the ratio of service temperature to the melting point of the material.

Failures of component as a result of deformation caused by creep usually occurring at less than 50% the melting temperature expressed in units of Kelvin are common. Material degradation due to creep is initiated in the grain boundaries and progresses by sliding and separation. Deformation as a result of creep can be attributed to several mechanisms that all contribute to the overall creep rate additively. The dominance of a particular mechanism in a metallic alloy is dependent on the applied stress and temperature. Engineering material resistance to creep is usually evaluated in terms of the temperature threshold of accelerating creep. Based on these criteria refractory ceramics which suffer from negligible creep up to temperatures of 1000^{0}C are the most creep resistant materials. But these ceramics are limited by their brittle nature, necessitating the use of metals with high creep resistance. Resistance to oxidation as well as other corrosive influences is also an important attribute of good creep resistant metals. Manufacturing metallic alloys with large grain sizes or from a single crystal improves the creep resistant nature of alloys as it reduces grain length boundary which is a factor in creep deformation. (Batchelor et al., 2002; Collins and Daniewicz, 2006; Jata and Parthasarathy, 2011).

1.1.2 Chemical Degradation: Corrosion

Some of the common causes of material degradation are chemically mediated as they can proceed without the drive force of mechanical work. Chemical mediated materials degradation largely involves a reversion of metals to chemical equilibrium. There are different types of chemically facilitated materials degradation. Chemical degradation of materials can be grouped into three basic classifications which are aqueous, high temperature and biochemical. These forms of degradation are usually referred to collectively as corrosion. Corrosion is the undesired degradation of a material through chemical or electrochemical interactions with the environment or the destruction of materials by means other than purely mechanical action (Collins and Daniewicz, 2006). Corrosion is a complex process that involves several variables like the environment and certain electrochemical and mechanical variables. Though corrosion is a term usually associated with metals, polymers and ceramics are also prone to corrosion damage as a result of interactions with water and other solvents. For metals the term aqueous corrosion is equivalent to electrochemical corrosion, but for materials like ceramics and polymers this equivalence does not apply. What this implies is that for metals these terms can be used interchangeably. There are different types of metallic corrosion, some of which are as explained below:

Direct chemical attack: In this type of corrosion the material's surface is exposed to the corrosive material, and is attacked usually over the entire surface of the material resulting in the progressive damage and dimensional reduction of the load cross section (Collins and Daniewicz, 2006). The damage done by it can be reduced or prevented by several different methods singly or a combination of the methods. Some of these methods include the use of electroplating, cladding, flame spraying, hot dipping, and painting or vapour deposition to protect the material. It can also be controlled through proper selection of materials to suit the environment or by adopting other suitable design modification (Batchelor et al., 2002).

Electrochemical corrosion: It occurs when metallic atoms as a result of losing electrons become ions, going into solution. This corrosion occurs as a result of electrochemical reactions that occur at the metal- environment interface. It occurs most frequently in an aqueous medium in which ions are present in water or moist air. The metal surface can be in an elemental form or as an alloy. Rusting and galvanic corrosion are good representation of electrochemical corrosion (Collins and Daniewicz, 2006).

Pitting and Crevice corrosion: Pitting and crevice corrosion are initiated from the creation of a localised aggressive environment that breaks down the normally corrosion passivated surface of the metal. This localised environment normally contains halide ions such as chlorides and is generally created because of differential aeration, which creates potential drops between most of the surface and occluded regions that concentrate the halide at discrete locations. Control of pitting corrosion involves the proper selection of material to resist pitting. While in crevice corrosion it is important to eliminate the process through processes like seal welding existing lap joints, inspection and removal of corrosion deposits (Collins and Daniewicz, 2006).

Amongst the degradation mechanisms, fatigue degradation seems to be the most occurring degradation mechanism that causes failure. Fatigue a common degradation mechanism is said to account for 53% of actual failure modes (Sachs, 2014). A survey by Darlington and Booker (2006) suggests that fatigue failure is perceived as the highest occurring failure mode by engineers. According to Ralph et al. (2001) between 70% and 90% of mechanical damage of structures are as a result of fatigue during the course of their operations. A major reason for fatigue failures has been attributed to improper design decisions, bringing to fore the need for effective design methods against fatigue damage in industry (Darlington and Booker, 2004). Machine life assessment is performed to prevent failures and to maintain a safe functioning of machines. In order to carry out a machine life assessment it is important to define what constitutes a failure or degradation. Failure can be described as either structural or functional failure. Failures for machine components are often defined in terms of functional failure, which is the inability of the machine to perform its intended function reliably, safely and economically (Ramachandran, 2005). Using the structural definition failure can be as a result of the progressive damage of components. Coffin (1989) thus defines damage as a “progressive and cumulative change acting to degrade the structural performance of the load bearing component or components which make up the plant”. Fatigue fits into this definition of damage. In failure due to fatigue, failure occurs by the initiation and progressive propagation of a crack until a stress level that is sufficient to cause rapid fracture is reached and the material fails by fracturing.

1.2 Fatigue Life Assessment Approaches

To determine fatigue life as a result of fatigue degradation researchers have proposed several life prediction models using different approaches. These prediction models have found use in both computer modelling using numerical methods and the fatigue tests of specimens under complex loading conditions. Researchers have generally grouped the life prediction approaches into two categories, the total life approach and the damage tolerant approach (Suresh, 2004). The total life approach is concerned with crack initiation based on the assumption that the material is defect free. While the damage tolerant approach is concerned with crack propagation. This approach also referred to as the fracture mechanics approach is based on the premise that all engineering materials contain a pre-existing defect (crack) from which the fatigue life can then be determined as the time it takes for this defect to propagate from its initial size to a critical size. These two approaches can be used independently or in combination (Rice et al., 1988). These approaches can also either be deterministic or probabilistic (Karolczuk, 2008). Standard methods used in fatigue analysis are usually deterministic based on the assumption that the properties of materials including the size of defects are averaged predetermined values, and are based on standard stress-strain life plots or on the explicit determination of the number of cycles required for a crack to grow from an initial size to a predetermined final size (Wormsen, 2007) . A limitation of the deterministic methods of fatigue life assessment is that in real life fatigue is a highly stochastic process. Machine component elements are usually subjected to variable loading, experience non-uniform stress fields and are affected by several other factors such as uncertainties in material properties, operating environmental conditions, geometrical dimensions due to manufacturing tolerances, presence of defects etc. which all make the fatigue process indeterministic (Chang et al., 1998).

1.3 Research Motivation

A number of studies show the relationship between degradation and system performance (Kurz et al., 2009; Yokokawa et al., 2012). Though studies on degradation are not new, the influence of design features on degradation of mechanical components has not received much attention as most studies in component degradation are mostly limited to degradation of material used against the operating or environmental conditions thus leading to the need for increased understanding of design influence on the service life of machine components (Roy *et al.*, 2016). Though some reports show evidence linking the failure of components and design features these studies are not in themselves exhaustive, for they are limited to a few case studies. Witek (2006) through the use of FEA studied the occurrence of fatigue in a turbine disc of an aero engine. Based on a parametric study of the geometrical features of the aero engine turbine disc using the FEA analysis and visual inspection the most highly stressed or critical areas of the turbine disc were determined. The findings showed that altering the dimensions of certain features such as the radius of the lower slot and the dimensional tolerance of the dovetail rim area of the turbine would improve the life of the turbine. Carrying out a study of the effect of geometry on the fatigue properties of Nitinol wire, Norwich and Fasching (2009) showed that contrary to the generally held belief that strain controlled strain fatigue life is independent of wire diameter, that for Nitinol wires diameter has an influence on its fatigue life and properties. In this study by Norwich and Fasching (2009) only the effect of a single design feature was studied. Deng et al. (2013) presents the use of a 3D FE model of a 6208 deep grove ball bearing to evaluate the influence of design of the geometric design features on the bearings fatigue life. They applied the one factor at a time analysis, which involved changing the dimension of one parameter while keeping the other design feature parameters constant. Using this method they were able to establish that the geometric design features influences the fatigue life of the inner and outer race of the bearing. Their approach was a deterministic approach which is considered to have several limitations (Saltelli *et al.*, 2004). Kim (2012) to show the relationship between design features and fatigue, studied the influence of geometry on the fatigue life of bellows using FEA. Due to the complex geometry of bellows only the effect of specific features, the radius of convolution, quantity of pitch and the inner diameter on a bellows were analysed in this study. The results obtained in the study showed similar trends in a number of results though there were some differences between experimental results and the analytical FEA results. The findings showed that the fatigue life, for instance, decreases linearly with the increase in bellows radius. Amorim et al. (2006) in a study of the effect of design geometry on the thermal fatigue strength of a brake drum showed that the thickness of the drum influenced the location of the most highly stressed region of the brake drum. Using FEA their findings showed that in the original drum geometry, the highest level of stress was observed in the radial direction near the bolted flange. However, when the thickness of the disc was reduced the highest stress level was then observed to occur in the axial direction near the surface of the brake drum in contact with the brake drum shoes. Jianxiong et al. (2013) also demonstrated the effect of design feature on the fatigue life of a mechanical component. They studied the effect of design features and the dimensions on Aluminium alloy door frames with two different designs on fatigue life of a door frame. They used both physical fatigue testing and FEA to show that the door design features has an impact on the fatigue life of the door frames. (Liu *et al.*,

2010) also demonstrated the effect of detail design on the fatigue life of fastener hole. They studied the effect of the mode of the hole, countersunk rivet and countersunk bolt on the fatigue life of the fastener hole. They showed that the fatigue life of fastener holes using rivets is longer than that using bolts.

The above studies though cannot be said to be exhaustive but are a representation of the type of studies showing the influence of design on a type of degradation. These studies were deterministic, they did not take the stochastic nature of the influencing factors or design features into consideration in their analysis. These studies did not show the uncertainty contribution of the design features to the fatigue life estimation of the studied components. A stochastic based analysis would be able to take the uncertainties in the design features into consideration and thus help identify which features have the most significant influence and where design effort should be concentrated on. Using a probabilistic method would give more realistic results in design for fatigue life.

1.4 Findings on Initial Review of Literature

Based on the review of the pertinent literature in the previous sections of this chapter the following findings were made:

1. Though there exists several degradation mechanisms fatigue is considered the most prominent and most occurring degradation mechanism.
2. That the studies on design influence on degradation of a component aside from being limited compared to studies on material and environmental influence, are mostly deterministic and so may not give realistic results. A probabilistic approach is recommended due to the

uncertainties in real life situations and so should be used for a more realistic results.

3. There is the need for more studies on the influence of design feature on component degradation as previous studies have been case studies that are component specific.

Based on the findings from the initial review of literature this study is concerned with the study of the influence of design features on a degradation mechanism, fatigue. Fatigue degradation is selected as it has been shown to be the most prominent degradation mechanism in terms of occurrence. A probabilistic approach is adopted in this research contrary to most existing work on design influence on fatigue degradation so as to obtain more realistic results. A novel uncertainty quantification methodology is proposed to account for the different type of uncertainties that are present in the system, and existing stochastic approaches modified to study and explain the influence of design features on the fatigue life of a mechanical component and the uncertainty contribution of these features to the fatigue life. The proposed uncertainty quantification method is then used for the design optimisation of a component in the presence of uncertainty. Uncertainty as used in this research is defined as the variability in a system's outcome due to inherent variability in the systems input and also the lack of a complete knowledge of the factors that may influence the system.

1.5 Research Aim

The aim of this research is as stated below:

To develop a methodology to assess the thermal fatigue degradation life of a mechanical component due to geometric design influence in the presence of uncertainties and its application for the optimisation of the component in the presence of these uncertainties.

1.5.1 Scope of Research

The scope of this research is restricted to the study of the impact geometric design features have on a machine component degradation life. Due to the existence of several fatigue degradation mechanisms the study would be limited to thermal fatigue degradation. Based on this only machine components that are susceptible to thermal fatigue degradation and which have distinguishable design features shall be considered as candidate components for selection in this study. The success of this study shall be measured by the applicability and the generic nature of the proposed methodology for studying the influence of geometric influence on the thermal fatigue degradation life of the selected candidate component in the presence of uncertainties.

1.5.2 Research Questions

This research in investigating the influence of geometric design features on the degradation life of a mechanical component in the presence of uncertainties, would endeavour to answer the following research questions:

1. Based on the research requirements what is the current practice in industry for determining the influence of design features on in-service degradation of mechanical components?
2. What methodologies exists for handling the presence of aleatory and epistemic uncertainty in the design of mechanical components, and what are the shortcomings of these methods that can be improved upon?
3. What are the gaps in current methodologies used for the optimisation of the life of a mechanical component in the presence of uncertainties from different sources, and can the shortcomings of these methods be improved on?

1.6 The Collaborating Organisations

This section of the thesis presents the background and the business motivation of the collaborating organisations. The background and business motivation of the collaborating organisations informed the motivations that led to the initiation of the doctorate (PhD) project. The collaborating organisations refer to the sponsors of the research and the research centre in which this research was carried out. These organisations are the Niger Delta Development Commission (NDDC) and Through Life Engineering Services (TES) Centre in Cranfield University.

The research was sponsored by the Niger Delta Development Commission an intervention agency for the development of human and physical resources in the Niger Delta region of Nigeria. The NDDC was established in 2000 with the mission of facilitating the rapid, even and sustainable development of the Niger Delta into an area that is economically prosperous, socially stable and ecologically regenerative (Niger Delta Development Commission, 2017). To achieve this mission one of the aims of the NDDC is to provide the best educational opportunities and facilities for the indigenes for the development of their potentials. To fulfil this aim one of the core mandates of the NNDC is the training of Niger Delta indigenes in areas of competency key to the manpower developmental need of the Niger Delta area. One such area that has been identified for the manpower development of the region is engineering education. Hence, the researcher was sponsored to undertake a PhD research in engineering to boost the engineering manpower requirement of the region.

Through life Engineering (TES) Centre is a national research centre hosted by Cranfield and Durham Universities. The TES Centre is supported by several core industrial partners which includes Rolls Royce, Bombardier Transportation, BAE Systems, Babcock International and the UK MOD. Aside from these core partners there are several other supporting industrial partners. The mission of the Centre is to develop knowledge, technology and process demonstrators, novel methodologies, techniques and the associated toolsets to allow the concept design of high value engineering systems based on design and manufacturing for through-life engineering services (Through-life Engineering Services Centre, 2017). Roy *et al.*, (2013) provides a discussion on the various aspect of through life engineering, of which the management of degradation in machine components is considered a key aspect. Given the variety and complexities of degradation mechanisms which can be categorised into component and system levels, there is the need to understand the drivers of degradation and find ways to mitigate them (Roy *et al.*, 2013). Aside from managing degradation, through life engineering is also concerned with the optimisation of component/system life as well as the uncertainty modelling and simulation at the system and component levels (see Figure 1.2). This research fits into the research scope of the TES Centre as its title and aim indicates. The effect of the design features of a machine component on the degradation of life, as well as a presentation on incorporating uncertainty into the design of the component for its optimal performance even in the presence of these uncertainties are key aspects of this research, making this research a Through-Life Engineering research.

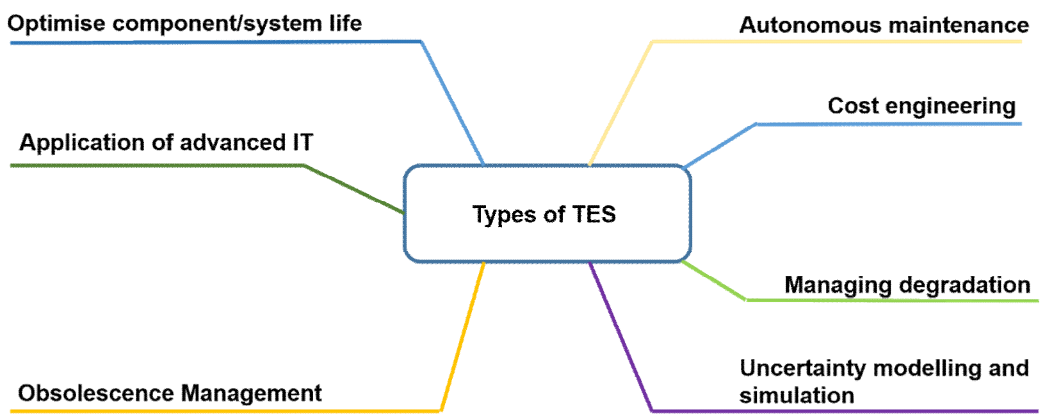


Figure 1.2 Types of Through-Life Engineering Services (Roy *et al.*, 2013)

1.7 Thesis Document Structure

This section provides a description of the thesis structure to give an overview of the entire thesis. The structure of the thesis is designed to show the progression of the research from the research motivation, aims and objectives to the findings of the thesis, the validations and conclusions. Chapter 1 provides an introduction to the research, its motivation and a review of literature that leads to the research aim and objectives. Chapter 2 provides a review of the literature in the domain area of the research topic. The review identifies gaps in the research study domain and the potential contributions to these areas. Chapter 3 provides a restatement of the research aim and objectives. The general research methodology adopted to achieve the research objectives are presented in this chapter. Chapter 4 discusses in detail an AS-IS study that is carried out to provide a basis for the selection of the case study type as used in this research. Chapter 5 presents the characterisation of the case study component material. It also presents the fatigue life modelling approach for the case study component material. The characterisation is done to establish the type and properties of the case study component material. Chapter 5 also includes the thermal analysis of two types of brake disc to identify the critical area of these brake discs for design purposes with respect to thermal fatigue life. Chapter 6 presents a novel approach for quantifying a mixed uncertainty, aleatory and epistemic uncertainty in the design of a component based on the use of surrogate models for design purposes. Chapter 7 presents a methodology for determining the sensitivity of the thermal fatigue life of the brake disc to the geometric design features. In Chapter 8 the uncertainty quantification methodology developed in chapter 6 is applied to the design optimisation of a component the brake disc in the presence of aleatory and epistemic uncertainties. Chapter 9 concludes the research by providing a discussion of the research contributions, its limitations and finally future research directions.

2 . LITERATURE REVIEW

The objective of this chapter is to review the literature on methods and techniques on uncertainty, sensitivity analysis and optimization under uncertainty. In this chapter a critical review is carried out to determine the merits and limitations in these methods and also identify the gap in research with respect to the above mentioned areas in research. This chapter is divided into several sections covering uncertainty in engineering design, sensitivity analysis and optimization under uncertainty. Section 2.1 presents uncertainty in engineering design, as well as the different methods for uncertainty quantification and the application of these methods. In section 2.2 a critical review of sensitivity analysis is presented, and section 2.3 discusses the concept of optimization under uncertainty. And finally section 2.4 provides a summary of the main points to conclude the chapter.

2.1 Uncertainty in Engineering Design

Engineering designs are generally deterministic and usually based on the assumption that the design variables are precisely known. However, in real life engineering design parameters are often non-deterministic in nature and possess a certain amount of natural variability. Lack of knowledge and data about the process can also introduce uncertainty in the analysis of engineering systems. The issue of variability in the design parameters and lack of knowledge both lead to non-negligible uncertainty in the parameters of a design model. With such uncertainty being propagated through the model also the output of the model becomes uncertain. Hence, for a more realistic assessment of engineering designs, these variabilities and uncertainties in the design input and output need to be accounted for to make the treatment of uncertainty manageable for further engineering decisions. In recent years there has been considerable effort to account for uncertainties in engineering design. This has become particularly necessary as it has been identified that not taking the effect of uncertainties in engineering applications sufficiently into account can lead to designs with unexpected or undesirable results. Hence, the use of uncertainty and sensitivity analysis techniques to assess and control these uncertainties (Marino *et al.*, 2008).

Helton et al. (2006) defined uncertainty analysis as “the determination of the uncertainty in analysis results that derives from uncertainty in analysis inputs”. Uncertainty is unavoidable in the modelling of engineering systems and is important for the understanding of the limit of operations of these systems (Zio and Pedroni, 2013). In classifying uncertainty, a distinction should be made between uncertainties in the physical models due to natural variability, and the state of the knowledge uncertainties about the parameters and assumptions of these these models (Aprostolakis, 1990). This distinction in uncertainty results in the classification of uncertainty as either aleatory or epistemic. Aleatory uncertainty is the inherent variation associated with the physical system or environment that is being studied, and is also referred to as irreducible uncertainty or stochastic uncertainty (Oberkampf *et al.*, 2002). Aleatory uncertainties are irreducible in the sense that the decision maker, designer, has no control over these type of uncertainties in the design of engineering systems. Aleatory uncertainties are usually modelled through random variables and mathematically represented as probability distributions (Oberkampf *et al.*, 2002). For instance the inherent variability of the geometric dimensions of machine parts due to the underlying manufacturing process is an example of an aleatory uncertainty. Common probability distributions used for modelling aleatory uncertainty include the exponential distribution, normal distribution, log-normal distribution, Weibul distribution etc. Amongs't these distributions the normal (Guassian) distribution is usually used for representing uncertainty where there is a lack of knowledge on what distribution that best describes the phenomena.

Epistemic uncertainties are those uncertainties in a system that are a result of lack of knowledge about the underlying phenomena of the system and are reducible through the collecting of more information (Zio and Pedroni, 2013). Epistemic uncertainties arise from imperfect modelling, simplification and limited availability of data (Zhang et al., 2010). Model uncertainty and parameter uncertainty are possible sources of epistemic uncertainty. Parameter uncertainty arises as a result of the use of the variables with fixed values but that are not precisely known, while model uncertainty is a result of uncertainty in the modelling hypothesis assumed (Zio and Pedroni, 2013).

2.1.1 Techniques Used for Uncertainty Analysis

Various methods have been proposed for representing and propagation of aleatory and epistemic uncertainty. Helton and Davis (2003) provide techniques for performing uncertainty analysis. The methods they describe are based on classical probability theory and are used for the treatment of aleatory uncertainty. These methods include Monte Carlo analysis, response surface methodology and differential analysis. Several researchers find the probabilistic framework for treating uncertainty to be inadequate with the argument that it is not possible to state precisely probability distributions associated with uncertain parameters particularly when limiting assumptions have been made that may reduce the relevance of the result. Helton, Johnson and Oberkampf (2004) have shown that epistemic uncertainty cannot be treated adequately using classical probability theory. This has given rise to methods for treating epistemic uncertainty such as interval analysis, evidence theory, possibility theory, and probability bound analysis (Oberkampf *et al.*, 2002; Zio and Pedroni, 2013). The following sections describe some of the common probabilistic and non-probabilistic methods used for uncertainty analysis.

2.1.1.1 Monte Carlo Method

Several methods have been described in literature for handling aleatory uncertainty such as Monte Carlo analysis, response surface methodology, and differential analysis. Monte Carlo method for uncertainty propagation has found wide usage amongst these methods. Monte Carlo simulation involves subjecting the input parameters to variation according to specific statistical distribution with their known variance and covariance taken into consideration. Farrance and Frenkel (2014) provides a description of the procedure for a Monte Carlo simulation for uncertainty analysis. For each input the Monte Carlo simulation generates a random number drawn from its respective PDF. This is done for all inputs a sufficiently large number of times to the known functional relationship of the output which is then used to produce a single numeric value as the output. An advantage of the Monte Carlo method for uncertainty analysis it's that the method avoids any errors associated with the linearization of the model, and it produces a distribution for the uncertain output as well as the mean and the standard deviation (Farrance and Frenkel, 2014). Monte Carlo simulation is usually done using random sampling, but there exists other efficient sampling methods like Latin hyper cube.

2.1.1.2 Differential Analysis

Differential analysis techniques are based mainly on the partial derivatives of the output function with respect to the input parameters (Helton and Davis, 2003). In the use of the differential analysis the model is approximated using Taylor series expansion. An advantage of using this method is that its uncertainty analysis based variance propagation is straightforward (Helton and Davis, 2003). The first order Taylor series approximation shall be used in this study to estimate the mean (expected value) and the variance of the uncertainty in the model output due to the input parameters variability. Although the second order Taylor series approximation gives more accurate results, its use is generally computationally expensive. Using the first order Taylor series, approximation of the mean and the variance can be determined as follows:

Given that $Y = f(X_1, X_2, \dots \dots \dots \dots, X_n)$

Expected value or mean of Y:

$$E(Y) \approx f(\mu_{X_1}, \mu_{X_2}, \dots \dots \dots, \mu_{X_n}) \qquad \textbf{(2-1)}$$

Variance of Y:

$$Var(Y) \approx \sum_{i=1}^{n} \left(\frac{\partial f}{\partial X_i}\right)^2 Var(X_i) + \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial f}{\partial X_i} \frac{\partial f}{\partial X_j} Cov(X_i, X_j) \qquad \textbf{(2-2)}$$

Assuming the variables are uncorrelated then the correlated covariance term can be neglected in the variance formulation. Then the estimate for the variance becomes:

$$Var(Y) \approx \sum_{i=1}^{n} \left(\frac{\partial f}{\partial X_i}\right)^2 Var(X_i) \qquad \textbf{(2-3)}$$

A determination of the expected value, the variance and the covariance gives the uncertainty analysis part of differential analysis. The resultant variance of the function output is dependent on the variance and covariance of the model's input. It should be noted though that the accuracy of the result obtained in a differential analysis is dependent on the order of the Taylor series.

2.1.1.3 Response Surface Methodology

Response surface methodology involves using an appropriate experimental design (Helton *et al.*, 2006; Khuri and Mukhopadhyay, 2010) to select the model inputs from which a response surface surrogate model can be developed for use in uncertainty and sensitivity analysis. Response surface methods are quite useful when it is computationally infeasible to perform Monte Carlo. In response surface methodology, scalar response variables are used to describe the output of a deterministic simulation for a given execution of the output (Langston *et al.*, 2009). Response surfaces are used to show the relationship between a response of interest and a number of associated input variables, for which the relationships are not generally known by approximating the response surface with a lower degree polynomial of the form (Khuri and Mukhopadhyay, 2010):

$$Y = F(x)\beta + \epsilon \qquad \textbf{(2-4)}$$

Where $x = (x_1, x_2, x_3, \ldots, x_n)$ are the input variable(s), *F(x)* is a function of *k* elements consisting of powers and cross products of powers of the input variables up to a given degree, β is a vector of k unknown constant coefficients called the parameters of the model, and ϵ is a random experimental error assumed to have a mean of zero. If Eqn.2-4 is assumed to adequately represent the response, the quantity *F(x)*β then gives the mean or expected value of Y and can be denoted as μ(x). A limitation of the response surface method is the difficulty in constructing an appropriate approximating meta-model to represent the modelled system, including thresholds, non-linearity and what is referred to as the curse of dimensionality, the possible need for a large number of design points as the number of input variables increases. Khuri and Mukhopadhyay (2010) provides a comprehensive review of the development, types and application of response surface methodology.

2.1.1.4 Evidence Theory

Evidence Theory also known as Dempster-Shafer Theory is a non-probabilistic method used for modelling epistemic uncertainty in systems where the available data provides information more than an interval but cannot give complete information on the specific probability distribution (Zio and Pedroni, 2013). It extends classic probability theory treatment of uncertainty as it is able to treat separately aleatory and epistemic uncertainty under a single framework. Evidence theory utilises two measures for the uncertainty description, i.e. Belief (Bl) and Plausibility (Pl). These measures are mutually dual and act as upper and lower bounds of classic probability to measure the likelihood of events without using probability distributions (Srivastava et al., 2013). These measures permit the treatment of uncertain information that is both random and imprecise at the same time. Shafer (1976) provides a comprehensive description of the mathematics of Evidence Theory.

2.1.1.5 Probability Bound Analysis

The Probability Bounds Analysis (PBA) is also an epistemic uncertainty treatment methodology that can handle uncertainty due to random (variability) and imprecise information in engineering designs (Nikolaidis, 2005). Probability bounds analysis involves the combination of interval arithmetic and probability theory to produce a probability box (P-box) which is an imprecise cumulative density function (CDF) with left and right bounding distributions to represent the bounds within which all possible probability distributions may lie (Ferson and Tucker, 2008). The P-box simultaneously accounts for both aleatory and epistemic uncertainty; the horizontal span of the probability bounds are a function of the variability in the result and the vertical span of the bounds a function of the analysts lack of knowledge (Zio and Pedroni, 2013). An advantage of PBA is that it does not place strong requirements on the making of precise assumptions about the characteristics of the parameters and hence, can treat problems in which the distribution and dependency amongst the parameters are not precisely known (Ferson and Hajagos, 2004). PBA has a strong correlation with Dempster-Shafer theory in that one can be converted into the other and vice versa (Ferson *et al.*, 2003).

2.1.1.6 Interval Analysis for Epistemic Uncertainty

The use of interval analysis for uncertainty analysis is based on the work of Moore and Bierbaum (1979) and is predicated on the assumption that the statistical distribution of the input variables are unknown, and as such they are bounded by lower and upper limits to form an interval. This method provides rigorous solutions that can contain all the possible solutions. In non-probabilistic uncertainty analysis, interval modelling is usually done in its closed form (Möller and Beer, 2008) as shown in Eqn.2-5:

$$X = [x_l, x_u] = \{x \in R: x_l \ll x \ll x_u\} \quad \textbf{(2-5)}$$

where x_l is the lower bound and x_u the upper bound. Interval uncertainty analysis involves the mapping of interval input quantities to interval output results. Considering the interval input quantities to be represented by x_i and the output as Y_i, this mapping can be represented as shown in Eqn.2-6:

$$\{x_1, x_2, . . ., x_n\} \rightarrow \{Y_i, ..., Y_m\} \quad \textbf{(2-6)}$$

Its use however, poses some challenges as it is difficult to obtain tight bounds of system responses due to an overestimation effect of interval arithmetic (Liu, ZhuangZhuangTianShu and JunFeng, 2015). For a more comprehensive explanation of interval analysis see Moore, Kearfott and Cloud (2009).

2.1.1.7 Possibility Theory

One approach that has found wide use in uncertainty analysis for engineering designs is possibility theory. Zadeh (1978) proposed the possibility theory for uncertainty analysis with the argument that the majority of information on which decisions are based are best described as possibilities rather than as probabilities. This theory is formulated based on fuzzy sets. Possibility theory provides two measures for estimating the likelihood of an event, called the possibility, Π, and necessity, N. Using this theory, epistemic uncertainties are described using possibility distributions. According to Zio and Pedroni (2013), a rationale for this is that a possibility distribution can be used to define a family of probability distributions by bounding the probability distributions below and above by possibility and necessity functions.

Table 2.1 A comparison of uncertainty methods (Rao *et al.*, 2008; Zio and Pedroni, 2013)

Uncertainty method	Uncertainty representation	Uncertainty propagation	Advantages	Limitations
Monte Carlo simulation	Probability distribution	Simulation	Can use information about correlations among the variables. It can be easily implemented.	It requires a lot of empirical information. It confounds ignorance with variability It maybe computationally expensive.
Differential analysis	Moments of the parameters(mean and variance)	Analytical	An advantage of using this method is that its uncertainty analysis based variance propagation is straightforward.	As the order of its Taylor series approximation increases it becomes computationally expensive.
Probability bounds	P-boxes	The Cartesian product of intervals and probabilities	Its use does not require the need for precise assumptions about the distributions and dependencies of the variables. It is computationally faster than Monte Carlo method. It is guaranteed to bound answers.	Uncertainty must be represented by cumulative density functions. Obtaining optimal bounds may become challenging when there are repeated parameters.
Dempster-Shafer theory	Dempster-Shafer structures	Combination of analytical and simulation	Its use does not require the need for precise assumptions about the distributions	Not widely accepted as some consider it controversial.

Uncertainty method	Uncertainty representation	Uncertainty propagation	Advantages	Limitations
			and dependencies of the variables. The basic probability assignment does not depend on the type of data. It is simple to implement. It distinguishes between uncertainty and incertitude. It supports all standard mathematical operations.	
Possibility theory	Possibility distributions	Combination of analytical and simulation.	Possibility distributions can be applied subjectively. It can be applied to all kinds of uncertainty. Computations are simple and straightforward.	It may give overly conservative results. Repeated parameters may pose a challenge in its use.

2.1.2 Literature Review on Applications of Uncertainty analysis Techniques

The probabilistic methods have traditionally been used for uncertainty analysis. Motra et al. (2016) presented a probabilistic approach based on the Monte Carlo method to estimate measurement uncertainty when determining the properties of materials. The presented method gives a higher precision compared to the conventional method for quantifying measurement uncertainty, as well as a more stable result with smaller confidence interval. To estimate the uncertainty in fire design simulation, Upadhyay and Ezekoye (2008) presented an approach that was computationally efficient called the Quadrature Method of Moments (QMOM) for the propagation of uncertainty bounds in distributions.

Comparing the results obtained with fire models, CFAST and ASET, they showed that their method give accurate results at a significantly smaller computational cost. They concluded that the method has the potential to reduce the computational effort required in Monte Carlo simulations particularly for complex fire simulations. Fricker et al. (2011) applied probabilistic methodology for the uncertainty analysis of computationally expensive FE models making use of a Bayesian surrogate model as an emulator. They demonstrated the effectiveness of using a multivariate Gaussian process emulator in determining the uncertainty associated with multiple output FE models. The method was applied to the uncertainty analysis of a frequency response function of a structure obtained via a FE model. Huang and Du (2008) using a probabilistic approach proposed a method that utilizes the first order Taylor series expansion of a performance function at the mean values of the input variables which they called the mean-value first order Saddle point Approximation (MVFOSA). Their proposed method is considered an improvement to the traditional mean value first order Second Moment (MVFOSM) method, as it makes use of the more accurate saddle point approximation to estimate the cumulative density functions and the probability density functions of the linearised input variables. Two probabilistic techniques for the uncertainty analysis of simulation based multidisciplinary systems are presented by Du and Chen (2000). A multidisciplinary system is a system that consists of several subsystems that are coupled together, and in which the output of one subsystem becomes the input for another subsystem (Bloebaum et al., 1992). Du and Chen (2000) demonstrated the effectiveness of these two techniques through use of a mathematical example and an electronic packaging problem. They considered both parameter and model structure uncertainty in the proposed approach. Helton and Davis (2003) highlight the strengths and weaknesses of some common methods used in probabilistic uncertainty analysis

The non-probabilistic methods are also methods that have attracted attention in uncertainty analysis. Moller and Beer (2008) provide a comprehensive critical review of non-probabilistic methods for uncertainty analysis in engineering computation. Degrauwe et al. (2009) used fuzzy number theory to quantify uncertainty of the results of an updating procedure for damage assessment of bridge cables. The updating procedure is commonly carried out using a probabilistic framework. The fuzzy updating method was able to give a clear idea of the influence of the studied variables on the identified damage. Ferson and Hajagos (2004) proposed a non-probabilistic approach for uncertainty propagation and representation based on probability bounds analysis. It involved the combination of interval arithmetic and probability theory to produce probability bounds. A limitation of the probability bound method is that it is computationally demanding. Koroishi *et al.*, (2012) in evaluating the uncertainties affecting the dynamic behaviour of flexible rotors compared two methods of uncertainty analysis, Monte Carlo simulation and Fuzzy based analysis. Using the non-traditional method of fuzzy analysis the inherent variabilities of the system input parameters are modelled for the uncertainty analysis. They compared the results obtained using Monte Carlo simulation and the fuzzy method with experimental results. Both approaches based on comparison with the experimental results were found to give similar results. But they concluded that the fuzzy method seems to be more adequate as it could account for situations where the uncertain parameters of the bearings of the rotating machine is not well defined. Lara-Molina et al. (2015) carried out an uncertainty analysis of flexible rotors with the use of fuzzy parameters and fuzzy random variables based on a fuzzy finite element modelling and fuzzy stochastic finite element modelling. They compared the results obtained with a previous work (Koroishi *et al.*, 2012) in which stochastic approach was used. The results obtained were determined to be similar to that obtained in the previous study that utilised a stochastic approach. Several authors have successfully applied fuzzy approach to uncertainty modelling of different systems such as Pawar et al. (2012) for the uncertainty analysis of thin walled composite beams, Walz et al. (2015) for the uncertainty analysis of a controlled nonlinear system with unstable dynamics, and Chowdhury and Adhikari (2012) in a structural dynamics problem.

Dempster-Shafer theory also known as evidence theory is another non-probabilistic approach to uncertainty modelling that has been proposed to handle imprecise information. Ferdous et al. (2009) explored the fuzzy sets and evidence theory to address data uncertainties in comparison to Monte Carlo simulation for uncertainty estimation. These approaches were applied in an Event tree analysis using a case study of an LPG release facility. The results obtained demonstrated that only evidence theory of the three studied approaches is able to account for expert ignorance in defining the events probability. To account for incomplete knowledge and imprecise evaluation of evaluation of the related characteristic parameters of a distributed generation systems, Li and Zio (2012) utilised an uncertainty framework based on evidence theory. In this work aleatory and epistemic uncertainties are modelled using probability and possibility distributions respectively. They then used evidence theory to incorporate the aleatory and epistemic uncertainties under a single framework. They introduced a hybrid uncertainty propagation for the mixed uncertainty problem using the belief and plausibility functions of evidence theory. Comparing their results to the use of a purely probabilistic approach, they showed that their proposed hybrid method is able to express explicitly the imprecision in the knowledge of the distribution generation system. Several authors have also applied the use of Dempster-Shafer structures to determine epistemic uncertainty in the analysis of engineering systems (Limbourg et al., 2008; Rakowsky, 2007; Rao and Annamdas, 2008; Simon and Weber, 2009).

Jacquin (2010) present a method for the predictive uncertainty of a conceptual snowmelt runoff model. In this uncertainty determination method a possibilistic approach rather than probabilistic calculus was used. The obtained results of the study showed that the use of possibilistic method allowed the use of more information, which resulted in a reduction of the predictive uncertainty. The width of the uncertainty bounds were reduced without a having a corresponding significant increment in the number of unbounded observations. Possibilistic uncertainty modelling was also applied to a maintenance problem by Zio and Pedroni (2014). They applied their proposed method to a case study involving the degradation model of a check valve of a turbo pump lubricating system in a nuclear power plant for maintenance planning. The method involved elicitation of information from domain experts. The elucidated information is then coded as possibilistic distributions using fuzzy random variables which are then used in the uncertainty quantification. Their method was able to avoid the introduction of unjustified, biasing assumptions. Possibility method for uncertainty analysis has been applied to various areas such as engineering design and reliability (Sam and Chakraborty, 2010), event tree analysis (Baraldi and Zio, 2008), life cycle assessment of engineering systems (Liu *et al.*, 2013)

Other authors have used a combination of methods to carry out uncertainty analysis. Lee et al. (1987) compared two methods for analysing uncertainty in data due to combining information from multiple sources of remote sensing image data, a probabilistic method and evidential calculus based on Dempster-Shafer theory. The results they obtained indicated that both methods performed adequately for mixed multispectral data. The multispectral data has both numeric and non-numeric components. They noted that while the statistical scheme provided an advantage in the overall classification accuracy of the studied data, the evidential approach was able to handle the non-numerical data better. They concluded that a combination of both approaches might be a viable solution to the multisource data analysis problem. Zou et al. (2012) integrated interval analysis with response surface modelling for determining the uncertainty in reconstructing accidents. The modelled uncertain parameters which are given as intervals were divided into several sub intervals. Based on these intervals, simulated results of reconstructed accidents are calculated using these sub intervals. The results obtained indicated the high accuracy of the method even for the analysis of complex simulation models in accident reconstruction. Isukapalli et al. (2000) combined a the Stochastic Response

Surface Method (SRSM) with partial derivatives estimated by the derivative code generated by Automatic Differentiation of Fortran (ADIFOR) for carrying out an uncertainty analysis in environmental and biological systems. Their method was applied to case studies and the results obtained showed close agreement with results obtained using traditional Monte Carlo sampling methods, with the added advantage of a reduction in the required number of simulations. Lee et al. (1987). To reap the benefits of two different methods, Lee et al. (2003) integrated the use of Petri nets with Possibilistic reasoning to create a Possibilistic Petri nets model (PPN). The method was demonstrated by applying it to diagnosis in cracks in reinforced approach. André and Lopes (2012) applied possibility theory to the uncertainty analysis of life cycle inventory. They applied a hybrid method that involved Monte Carlo simulation for the random aleatory variables and fuzzy interval arithmetic for the Possibilistic variables so as to obtain upper and lower distribution functions for the output variable. They concluded that both the probabilistic and Possibilistic methods have advantages and limitations. So a hybrid approach that is able to incorporate both approaches for uncertainty analysis will provide the benefits of both, thus improving the reliability of the obtained results. To handle the uncertainties in a fault tree analysis, Wang et al.(2015) proposed a hybrid probabilistic-possibilistic framework. In this hybrid method they treated dependency in the model variables as epistemic uncertainty. Their approach allowed for the incorporation and combination of statistical knowledge involving the PDF and imprecise uncertainty statements made by experts. This method also results in the estimation of lower and upper cumulative distributions. The slopes of the lower and upper cumulative distributions gives the aleatory uncertainty due to the probabilistic inputs. While the interval between the two distributions account for the epistemic uncertainty due to the Possibilistic distributions. This provides a distinction between the aleatory and epistemic uncertainties under a unified framework.

The different approaches in treating uncertainty all have their limitations and advantages (Zio and Pedroni, 2013). And as such they are best suited for particular situations and less for others. Depending on the problem, where there is enough information the uncertainty can be represented using probability distributions, and where there is less fuzzy set can be used. Hence, selecting a method to perform uncertainty analysis would depend on certain criteria such as the type of uncertainty present, the source and the objective of the uncertainty analysis. Engineering design problems are often a combination of aleatory and epistemic uncertainties. Engineering design involves the making of decisions that often involves imprecise or incomplete information while still containing some random element. To account for the imprecision several methods have been proposed and used by researchers. The imprecision in data for uncertainty is usually handle by the use of methods such as evidence theory and possibility theories amongst others (Huang et al. 2013). These methods are generally classified by some authors as imprecise probability theories, and they include all mathematical models such as lower and upper probabilities, possibilities and necessities, belief and plausibility functions and other qualitative models (Kozine and Filimonov, 2000). To treat engineering design uncertainties would thus require approaches that can handle the inherent variation in the problem as well as the imprecision in knowledge of a system. Hence a methodology that can combine both aleatory and epistemic data for engineering design would be most appropriate. The review of the pertinent literature shows the advantages hybrid approaches that is approaches that are able to treat aleatory and epistemic uncertainty within a single framework offer. They combine the advantage of the traditional probabilistic methods with those of the non-probabilistic uncertainty treatment methods. A combination or hybrid method is able to account for the imprecision that traditional methods cannot account for while still providing for the random behaviour in the system.

2.1.3 Identified Gap in Uncertainty Quantification Methods

Based on the previous section the use of hybrid methods for handling engineering design problems is recommended. This is so as these methods are able to account for both the aleatory and epistemic uncertainties that may be present. Evidence theory compared to the other non-probabilistic uncertainty quantification methods can model both aleatory and epistemic uncertainty when there is incomplete information (Agarwal *et al.*, 2004). Evidence theory allows the treatment of aleatory and epistemic uncertainty independently within a unified framework using the belief and plausibility functions to assess data that is random and imprecise at the same time (Zio and Pedroni, 2013). The other uncertainty quantification methods, probability theory, possibility theory, interval analysis and probability bound analysis can all be said to be subsets of the evidence theory (Dai et al., 2004; Zio and Pedroni, 2013). Possibility theory is only used to handle epistemic uncertainty as opposed to probability theory which is used for aleatory uncertainty. As a result of this its advantage it has found application in the hybrid methods used for treating problems that contain both aleatory and epistemic uncertainty.

Tang et al. (2015) demonstrated the use of evidence theory and differential evolutionary method to quantify and propagate mixed aleatory-epistemic uncertainty for uncertainty analysis. They demonstrated that evidence theory can adequately handle problems with aleatory and epistemic uncertainty by applying the proposed methodology to two case studies involving semi-rigid jointed frames. Du (2006) investigated the feasibility of using a unified approach to handle problems with both aleatory and epistemic uncertainty. In this work probability theory was used to model the input parameters with aleatory uncertainty, while evidence theory using basic probability assignments was used for inputs with epistemic uncertainty. The uncertainty in the model output was represented using the belief and plausibility functions of evidence theory. In this work the ability of evidence theory to show model randomness and imprecision simultaneously was demonstrated. Baccou et al. (2008) presented a hybrid methodology based on evidence theory to model mixed uncertainty problems. Their work allowed the extension of the classical Monte Carlo simulation by relaxing assumptions related to the choice of probability distributions and the possible dependencies between the uncertain parameters. They illustrate their proposed method by applying it in the transfer of a radionuclide in the environment. The proposed method could handle and treat aleatory and epistemic uncertainties within a single framework. But a limitation of these previous methods based on evidence theory is that they have only considered the uncertainties associated with the input variables and their propagation into the output (Baccou et al., 2008; Baudrit, Dubois and Guyonnet, 2006; Chojnacki et al., 2010; Du, 2006; Tang, Su and Wang, 2015). Aside from the uncertainties in the input parameters, other uncertainties due to model prediction error and model form may also be present and have to be accounted for. Based on the ability of evidence theory to handle randomness and imprecision simultaneously a methodology based on it that can also handle other sources of uncertainties aside from those due to the input parameters would be desirable.

2.2 Sensitivity Analysis

Sensitivity analysis is the study of how the uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input (Saltelli *et al.*, 2008). Simulation models are made up of various information such as the parameters which contain some level of uncertainty due to inherent variability or precise information to describe their characteristic. The presence of these uncertainties in a simulation model reduces their reliability, hence the need for the simulation results to be tested for their sensitivity to changes in the model parameters. According to Saltelli et al. (2008), uncertainty and sensitivity analysis should be run alongside each other, with uncertainty analysis preceding the sensitivity analysis. Sensitivity analysis provides an extension to uncertainty analysis by identifying the influential contributing input variables to the variation or imprecision in the outcome variable; thus a sensitivity analysis quantifies how the output variables responds to changes in the individual input variables of the model (Iman and Helton, 1988). Carrying out a sensitivity analysis offers several benefits as it can be used to achieve the following (Christopher and Patil 2002):

1. Model validation and verification.
2. It assists in providing more information on the robustness of a model.
3. For the determination of a model’s performance when the model is extrapolated.

Sensitivity analysis has found wide application in different fields (Zi, 2011). In Chapter One it was established by a review of literature that the study of design influence on a component fatigue degradation have majorly been deterministic in approach. The approaches used are limited and would not provide realistic sensitivity results as they consider only single parameters at a time there by omitting the influence of other parameters that maybe influencing the system response at the same time. The succeeding sections under sensitivity analysis undertake a critical review of sensitivity analysis methods with a view to highlighting why the deterministic methods are not considered appropriate enough, and also to identify what other methods exist that can best satisfy the objective of this research.

2.2.1 Methods Used for Sensitivity Analysis

Hamby (1994) and Saltelli et al. (2008) provide some of the rationales for performing sensitivity analysis which include: (1) to corroborate the model i.e. to determine if the model is overly dependent on fragile assumptions; (2) determine which parameters are most deserving of further analysis; (3) model simplification by identifying which parameters are insignificant and can be eliminated from the final model, as well as the parameters that contribute most significantly to output variability; (4) identifying which parameters are most highly correlated to the output; (5) identifying critical regions in the space of the input parameters; (6) identifying parameters which interact and which may thus generate extreme values; (7) determine the consequence of changing a given input parameter on the model performance. Based on the rationales for carrying out sensitivity analysis, the different methods of sensitivity analysis can be classified as either local or global sensitivity (Saltelli *et al.*, 2004). Christopher Frey and Patil (2002) classifies sensitivity analysis based on methodology into mathematical, statistical and graphical method. According to Christopher Frey and Patil (2002) the other classifications of sensitivity analysis as local or global sensitivity analysis are based on the capabilities of the technique rather than on the methodology. Classifying sensitivity analysis schemes aids the modeller with an understanding of the applicability of a specific method to a particular model and analysis objectives (Christopher and Patil, 2002).

2.2.1.1 Local and Global Sensitivity Analysis

Sensitivity analysis is classified as either local (deterministic) or global sensitivity (probabilistic) analysis (Saltelli *et al.*, 2004). The local sensitivity analysis is a deterministic approach that involves the analysis of the impact of small input perturbations around nominal values of the input on the model output (Looss and Lemaitre, 2015) . Local sensitivity is often evaluated through partial derivatives of the outputs at specific points of an input while other inputs are kept constant (Zhou and Lin, 2008). They are relatively simple to implement and are intuitive. Global sensitivity analysis involves varying all their inputs of a model simultaneously and evaluating the sensitivity over the entire range of each input factor (Zhou et al. 2008). Carrying out a global sensitivity analysis can be computationally intensive. A limitation of local sensitivity analysis techniques is that where there is non-linearity in the model, local sensitivity analysis may not be able to account for the influence of interactions between the parameters, but of which the global sensitivity techniques are able to account for. Global sensitive analysis however, requires that the input variability space be well known, and where this is poorly known the conclusions made from it may be inaccurate (Pianosi *et al.*, 2016).Global methods for sensitivity analysis also require high number of model evaluations which increases with the number of input parameters. Table 2.2 presents a comparison of local and global sensitivity analysis.

Table 2.2 A comparison of local and global sensitivity analysis methods

	Local sensitivity analysis	Global sensitivity analysis
Characteristic	It considers the effect of only one parameter at a time while the other parameters are fixed at nominal values.	It considers the effect of a parameter while varying all the parameters simultaneously.
Type of distribution	All the input parameters are described with the same distribution based on assumed boundaries.	The use of different distributions for each of the input parameters is possible.
Model structure	Can treat only models that are monotonic and linear	Are usually independent of the model structure.
Correlation of inputs	It is based on assumptions that the input parameters are uncorrelated	It is based on assumptions that the input parameters are uncorrelated
Computational intensity	They are relatively less computationally intensive compared to the global methods.	They are more computationally intensive compared to the local sensitivity methods

2.2.1.2 Mathematical, Statistical and Graphical Sensitivity Analysis methods

The mathematical methods for sensitivity analysis refers to methods in which the sensitivity of a model's output is accessed to the range of variation of an input, and this involves calculating the output for a few values of an input that represents the possible range of the input parameters (Christopher and Patil, 2002). The mathematical methods do not take into consideration the variance in the output due to the input parameters variability (Christopher and Patil, 2002). The use of the mathematical methods has been helpful in exploratory initial analysis to identify non influential input parameters for screening purposes (Christopher and Patil, 2002). The mathematical methods are analogical to the local sensitivity methods and are quantitative in nature.

The statistical methods refer to the methods that involve simulations in which the input parameters are assigned distributions and assessing the effect of variance in inputs on the output distribution (Christopher and Patil, 2002). Based on Pianosi et al. (2016) classification, the Global sensitivity analysis are the statistical methods. The statistical methods can be grouped as what Pianosi et al., (2016) classifies as One-Factor–at-a-Time (OAT) or All-Factors-At-a-Time (AAT). These methods involve varying one factor at a time while keeping the other factors fixed at their baseline values, or varying all the factors at the same time respectively. The statistical methods allow for the inclusion of the effect of parameter interactions in the sensitivity analysis compared to the mathematical methods in which this cannot be done (Christopher and Patil, 2002; Saltelli *et al.*, 2008; Pianosi *et al.*, 2016). The statistical methods are also quantitative. The statistical methods include the elementary effects methods, sample based methods and variance based methods.

The graphical methods for sensitivity analysis refers to those methods that in which visual methods are used to provide an indication of the outputs response to the input parameter variability (Christopher and Patil, 2002; Pianosi *et al.*, 2016). These visual techniques include methods such as scatter plots, tornado plots, box plots, convergence plot, pattern plots, cobweb plots (Pianosi *et al.*, 2016). Pianosi et al. (2016) refers to these graphical methods that use visualisation tools to express sensitivity as qualitative methods for uncertainty analysis. The graphical methods can be used to complement any of the mathematical or statistical methods.

2.2.2 Techniques for Sensitivity Analysis

This section describes various sensitivity techniques based on a survey of literature. The methods to be presented here are applicable to various disciplines. For this study the methods would be classified based on the definition of global and local sensitivity analysis. A critical review of the methods shall be carried identifying their strengths and limitations. (Pianosi *et al.*, 2016) provides a classification of the sensitivity analysis methods based on their purposes and approaches as shown in Figure 2.1.

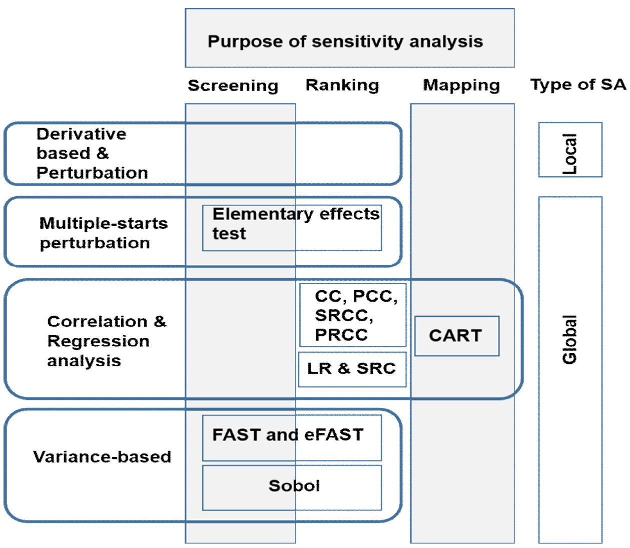


Figure 2.1 Classification of sensitivity analysis techniques (Pianosi *et al.*, 2016)

2.2.2.1 Differential Sensitivity Analysis

The differential sensitivity analysis technique forms the basis for other sensitivity analysis techniques. This technique also referred to as the direct method is based on the partial differentiation of the model in its aggregated form (Hamby, 1994). In the use of this technique a model's output is modelled as a function of the independent variables using the first order Taylor series expansion (Hamby, 1994; Norton, 2015), from which the variance is estimated using the error propagation formula as given in Eqn. 2-7:

$$V(Y) = \sum_{i=1}^{n} \left(\frac{\partial Y}{\partial X_i}\right)^2 V(X_i) \qquad \textbf{(2-7)}$$

The variance in the input variables, X_i weighted by the first order partial derivative of the response, Y with respect to the input variables gives the sensitivity of the model to the various input parameters (Hamby, 1994). To facilitate the comparison of sensitivities across input factors that have different units of measurements, the partial derivatives are usually rescaled (Norton, 2015). In a case where the partial derivatives have been rescaled for a situation where there is an algebraic equation describing the relationship between the output and the input parameters, a normalised sensitivity measure, $\emptyset_i$ can be obtained as shown in Eqn.2.8 (Hamby, 1994; Pianosi *et al.*, 2016):

$$\emptyset_i = \frac{\partial Y}{\partial X_i}\left(\frac{X_i}{Y}\right) \qquad \textbf{(2-8)}$$

Though computationally cheap, differential sensitivity analysis have the limitation of being only able to provide information about local sensitivity only (Norton, 2015), and not being able to handle non-linearity in the model adequately . This method is grouped under the techniques for performing local sensitivity analysis.

2.2.2.2 One-at-a-Time (OAT) Sensitivity Analysis

This provides a simple approach to sensitivity analysis. It involves varying (perturbing) one input variable at a time while keeping the other input variables at their nominal values and assessing the influence of the parameters on the simulation results using graphical or visual inspection of the simulated results under perturbed and nominal input parameters (Pianosi *et al.*, 2016). This method forms part of the local sensitivity analysis method as it only handles sensitivity relative to the point estimates selected and not the entire distribution of the variables (Hamby, 1994). This method is computationally cheap but determination of the right size of perturbation to capture the sensitivity of the output to the input maybe challenging. And a limitation of this method is that the sensitivity measures obtained are location dependent.

2.2.2.3 Elementary Effects Sensitivity Analysis

Elementary effects sensitivity analysis a global sensitivity method also called the Morris method belongs to the OAT class of sensitivity analysis methods. (Saltelli *et al.*, 2008). This method consists of individually randomised one-factor-at-a-time experimental designs (Morris, 1991). This method partly overcomes the limitations of the differential methods by making use of wider variation ranges for the inputs and averaging a number of local measures so as to eliminate the dependency on an individual sample point (Saltelli *et al.*, 2008). The elementary effects method consists of two measures, estimates of the mean, μ and the standard deviation, σ of the distribution of the elementary effects associated with each input parameter (Saltelli *et al.*, 2008). The mean measure, μ for the elementary effects is used to estimate the total influence of a variable on the output, while the standard deviation measure, σ gives estimates of the ensemble of the variable's effects whether non-linear or as a result of a variable's interaction with other model variables (Morris, 1991). Considering that the relationship between a model's output, Y and its input variables, X is given by Eqn. 2.9:

$$Y = F(X) \qquad \textbf{(2-9)}$$

Then the elementary effect of the input parameters for a given value *x* of X can be given as:

$$EE_i(X) = \frac{Y_j(x_1, \ldots\ldots, x_i+\Delta, x_{i+1}, \ldots\ldots\ldots, x_k) - Y_j(X)}{\Delta} \qquad \textbf{(2-10)}$$

Where Δ is a predetermined multiple of *1/ (p-1)*, p is the number of levels each model input parameter is assumed to vary in the space of the input parameters, and Y_j is the state parameter of interest. The finite distribution of the elementary effects, $EE_i(X)$ is obtained by random sampling of different X from its k-dimensional p-level parameter space, and is denoted as F_i .

The estimates of the sensitivity measures, the mean, and the standard deviation of the distribution, F_i are obtained by sampling *r* elementary effects from each F_i via an efficient design that constructs *r* trajectories of (*k+1*) points in the input space, each providing *k* elementary effects, one per input parameter, thus resulting in *r*(*k+1*) experiments (Campolongo et al., 2007). The results obtained using this method can easily be interpreted in that a large mean, μ , that is a mean substantially greater than zero indicates an input parameter that has a strong influence on the output, while a large value of the standard deviation, σ indicates an input with a non-linear effect on the output or that the input interacts with other input parameters (Saltelli *et al.*, 2004). A limitation of the elementary Effects (Morris) method is that if the distribution F_i has negative elements the results may not be reliable. To overcome this, Saltelli et al. (2004) suggest the use of the absolute value of the mean, denoted as μ^* as to avoid the occurrence of opposite signs. This method is computationally efficient and is independent of the nature of the model, though it has the limitation that the sensitivity measures it provides are qualitative and so cannot quantify the relative importance of one input parameter to another.

2.2.2.4 Sampling Based Methods

Regression methods and the correlation methods for sensitivity analysis are generally regarded as the sampling based methods for sensitivity analysis. They fall into the global sensitivity analysis methods. The use of Regressionand correlation methods for sensitivity analysis is based on the use of Monte Carlo simulations generated input and output datasets to obtain information about output sensitivity to the variability of the input parameters. Regression analysis for sensitivity studies measure the sensitivity of the output to input parameter variability as a by-product of regression analysis applied to the input/output sample. While correlation methods base the sensitivity analysis on the use of the derived correlation coefficients between the input parameter and the output as a measure for sensitivity. These two methods have been applied by several authors to sensitivity analysis; correlation (Iman and Helton, 1988; Helton and

Davis, 2002; Marino *et al.*, 2008; Fellin *et al.*, 2010; Gan *et al.*, 2014), and regression analysis (Levine and Renelt, 1992; Storlie and Helton, 2008; Sobie, 2009; Sobie and Sarkar, 2011). Pianosi *et al.* (2016) in their review for sensitivity analysis methods lists the different definitions of correlation techniques that are used for sensitivity studies to include Pearson correlation coefficient (CC), partial correlation coefficient (PCC), Spearman rank correlation coefficient (SRCC) or the partial rank correlation coefficient. For the regression analysis methods the simplest method is the linear regression analysis, and where the sensitivities to all individual inputs have to be obtained at once multiple linear regression is applied. Other regression techniques that have been used for sensitivity analysis exist like the Classification and Regression Trees (CART) which is a non-linear regression method (Pianosi *et al.*, 2016).

Regression analysis

The use of regression analysis for estimating sensitivity measures involves the use of multiple linear regression for the fitting of input parameter information to a theoretical model that can provide an estimate of the output with as little error as possible. According to Christopher et al. (2002) regression analysis serves three purposes which are; to describe the relationship between variables, the control of the input variables for a given value of the output, and the prediction of the output based on the input variables. The effect of the inputs on the output using regression analysis can be studied based on the relative magnitude of the regression coefficients as a means of applying sensitivity rankings to the model inputs (Hamby, 1994; Christopher and Patil, 2002). The sign of the regression coefficients gives an indication of whether the output increases or decreases as the corresponding input parameter increases. The multivariate generalised form of a simple linear regression model is shown in Eqn.2-11:

$$Y_i = \beta_0 + \sum_i \beta_i x_{ij} \qquad \textbf{(2-11)}$$

where Y_i is the predicted output variable, x_i the input variables of interest, β_0 the regression intercept, and β_j are the unknown regression coefficients to be determined. The regression coefficients are determined using the ordinary least square criterion. To estimate how well the regression model match the actual data, the model coefficient of determination, R^2 is used and is given as shown in Eqn.2-12:

$$R^2 = \frac{\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2} \qquad \textbf{(2-12)}$$

where $\hat{Y}_i$ is the estimated output that is obtained using the prediction model, with Y_i and $\bar{Y}$ being the actual output values and their means respectively. The value of R^2 gives an indication of how much of the variability of the in Y is due to the regression model, thus showing how well the regression model fits the actual data. If R^2 is close to 1, it implies that the regression model accounts for most of the variability in Y. But where R^2 is low, a non-linear behaviour could be present and so the linear approximation would not be adequate, thus requiring some alternative method to study the relationship between the inputs and output. A standardisation procedure for the regression analysis maybe required as result of difference in units and dimensions of the parameters to obtain the standardised regression coefficient (SRC) which is used for the sensitivity analysis. To obtain the standardised regression equation, the parameters of the regression model have to be normalised. Equation 2-13 gives the normalised expression for the regression model as:

$$\frac{\hat{Y}_i-\bar{Y}_i}{\hat{S}} = \sum_{j=1}^{k} \frac{\beta_j \hat{S}_j}{\hat{S}} \frac{X_{ij}-\bar{X}_j}{\hat{S}_j} \qquad \textbf{(2-13)}$$

where

$$\hat{S} = \left[\sum_{i=1}^{n} \frac{(Y_i-\bar{Y})^2}{N-1}\right]^{1/2}, \ \hat{S}_j = \left[\sum_{i=1}^{n} \frac{\left(X_{ij}-\bar{X}_j\right)^2}{N-1}\right]^{1/2} \qquad \textbf{(2-14)}$$

where $\hat{S}$ is the standard deviation of the output, $\hat{S}_j$ the standard deviation of the input parameters and $\beta_j \hat{S}_j / \hat{S}$ gives the standardised regression coefficient. To remove input parameters that are statistically insignificant stepwise regression can be employed rather than including all the input variables in the regression model (Draper and Smith, 2014). Regression based sensitivity analysis permit for the determination for the sensitivity of the individual model inputs while taking into account the simultaneous impact of the other model inputs (Christopher and Patil, 2002). A limitation of the regression method for sensitivity analysis is that its use is based on the assumption that the input variables are independent (Helton and Davis, 2002). Where this assumption of independence of the input variables is met the absolute value of the SRC can be used as a measure of the importance of the input variables (Helton and Davis, 2002). Regression analysis performs poorly for non-linear models and non-monotonic models (Saltelli *et al.*, 2004). The problem of non-linearity can be handled by carrying out rank transformations of the parameters.

Correlation methods

Correlation coefficients also called Pearson's moment correlation coefficients are used to determine the strength of the linear relationship between the input variables and output variable. The correlation coefficient between x_j and Y based on Eqn.2-11 is as shown in Eqn.2-15:

$$r_{x_jY} = \frac{\sum_{i=1}^{N}(x_{ij}-\bar{x})(Y_i-\bar{Y})}{\sqrt{\sum_{i=1}^{N}(x_{ij}-\bar{x})^2 \sum_{i=1}^{N}(Y_i-\bar{Y})^2}} \ , j = 1,2,3, \dots \dots \dots , k \qquad \textbf{(2-15)}$$

Where

$$\bar{Y} = \sum_{i=1}^{N} Y_i/N \ , \bar{x}_i = \sum_{i=1}^{N} x_{ij}/N \qquad \textbf{(2-16)}$$

The values of the correlation coefficient, r_{x_jY} ranges between -1 and +1. Correlation coefficient can measure only the effect of one input variable at a time on the output; it is not able to account for the possible influence on Y by the other uncertain input variables (Helton and Davis, 2002). Where the model is non-linear the Spearman or rank correlation is preferred, which is basically the same as correlation coefficients but with the use of the rank transformed data instead of the raw (Saltelli *et al.*, 2004).

To overcome the limitation of Correlation coefficients, Partial correlation coefficients (PCC) can be used to measure strength of the linear relationship between the input variables and output variable when all the linear effects of other variables have been removed from the analysis (Helton and Davis, 2002; Marino *et al.*, 2008). PCC are used when more than one input parameter is under consideration. It can be computed as the correlation between the residuals of the input variable and the output variable in sensitivity studies. The PCC between an input variable and the output is determined from the development of two regression models:

$$\hat{x}_j = c_0 + \sum_{p=1}^{N} c_p x_p \quad \textbf{(2-17)}$$

$$\hat{Y} = b_0 + \sum_{p=1}^{N} b_p x_p \quad \textbf{(2-18)}$$

$where\ p \neq j$

Using the results obtained from both regressions, two new variables are $(x_j - \hat{x}_j)$ and $(Y - \hat{Y})$ are obtained respectively. The PCC between $\hat{x}_j$ and $\hat{Y}$ is the Correlation coefficient between $(x_j - \hat{x}_j)$ and$(Y - \hat{Y})$. But where the relationship between the input variables and the output is non-linear but monotonic, Partial rank correlation (PRCC) can be used for the sensitivity analysis (Helton and Davis, 2002). Carrying out PRCC is similar to PCC, as it involves rank transformation of the raw data, which involves replacing the raw data with their corresponding ranks, after which the normal regression and correlation procedures are performed on the ranks (Helton and Davis, 2002; Marino *et al.*, 2008).Rank transformation removes the linear relationship between the two variables being evaluated. PRCC is quite a robust method for sensitivity analysis of non-linear but monotonic models given that the parameters are independent of each other (Marino *et al.*, 2008), and they provide a useful solution in the presence of long tailed data (Ekström and Broed, 2006).

A limitation of the use of the correlation methods is that they are predicated on the level of acceptability of linearity or monotonicity assumption between the input variables and the output, and a way to do this is visual inspection of the input/output relationship using scatterplots (Pianosi *et al.*, 2016).

2.2.2.5 Variance Based Methods

The variance based methods are classed under the global sensitivity analysis methods and they are able to determine to what amount the total variability of the output is due to the variability of the input parameters either in combination or singly. The variance based methods are used in quantifying the amount of variance each input parameter contributes to the uncertainty in the output (Ekström and Broed, 2006). The variability of the parameters is treated as being stochastic and is measured with their variance. According to Pianosi *et al.* (2016) the contribution to the variance of the output by a given input parameter provides a measure of the sensitivity.

Consider the function as defined in Eqn.2-9. Several sensitivity indices can be defined for this function either based singly, jointly or in combination with interactions, in terms of the input parameters. For instance the first-order indices also called main effects which is a measure of the expected reduction in output variance when a given input parameter is fixed is given:

$$S_{iF} = \frac{V(E[Y|X_i])}{V(Y)} \qquad \textbf{(2-19)}$$

where *E* is the expected value, and *V* is the variance. First order indices account for only the main effect contribution of the individual input parameters on the model’s output variance. They find good application when interactions between the input parameters are not considered significant contributors to the variance of the output. Higher order sensitivity indices can also be defined to determine joint effects of the input parameters on the output variance. To account for total contribution of an input parameter to the output's variance taking into consideration its direct effect and the effects due to its interaction with the other input parameters Homma and Saltelli (1996) presented total-order indices which is also called total effect sensitivity measure. The total order indices can be determined as shown in Eqn.2-20:

$$S_{iT} = \frac{E_x(V_x[Y|X_i])}{V(Y)} \quad \textbf{(2-20)}$$

The use of the first order and higher order indices is based on the assumption that the input parameters are independent, but when correlations are present this may lead to counter intuitive results (Pianosi *et al.*, 2016). To estimate the sensitivity indices using the variance based approach several methods have been suggested in literature and they include the Sobol method (Sobol, 1993), Fourier Amplitude Sensitivity Test (FAST) (Cukier *et al.*, 1973), and the Extended Fourier Amplitude Sensitivity Test (eFAST) (Saltelli et al., 1999).

Sobol's method

Sobol's method for sensitivity analysis is based on the decomposition of variance to determine sensitivity measures which are referred to as Sobol's sensitivity index (Sobol, 1993). Sobol's method involves decomposition of the model function $Y = F(X) = (X_1, X_2, X_3, \dots, X_k)$ into summands of variance with increasing number of input parameters. To determine the sensitivity of the model's output variation to that of the input parameters variation, the uncertain input parameters *Xi* are considered independent, and are assumed to be uniformly distributed over the range [0, 1]. Based on this the output function, $Y = F(X)$ is expanded into a series of orthogonal terms given by:

$$F(X) = F_0 + \sum_{i=1} F_i(X_i) + \sum_{i=1} \sum_{j>1} F_{ij}(X_i, X_j) + \cdots + F_{1,2,3,\ldots\ldots,k}(X_1, X_2, X_3, \ldots, X_k) \qquad \textbf{(2-21)}$$

where F_0 has to be a constant and the satisfaction of the condition that the integrals of the summands over their own variables is zero as shown in Eqn.2-22:

$$\int_0^1 F_{i1,i2,\ldots,is}\,(X_{i1}, X_{i2}, \ldots, X_{is})dx_{ik} = 0,\ for\ k = i_{i,}\, i_2, \ldots, i_s \qquad \textbf{(2-22)}$$

Squaring each term in Eqn.2-22 and integrating over the range [0, 1], the total non-conditional variance of the model's output is then given as:

$$V(Y) = \sum_{i=1} V_i + \sum_i \sum_{j>1} V_{ij} + \cdots + V_{1,2,\ldots,k} \qquad \textbf{(2-23)}$$

where

$$V_i = V_{xi}(E_{x\sim i}(Y|X_i)) \qquad \textbf{(2-24)}$$

$$V_{ij} = V_{xixj}\left(E_{x\sim ij}\left(Y\middle|X_i, X_j\right)\right) - V_{xi} - V_{xj} \qquad \textbf{(2-25)}$$

where *V* as previously mentioned is the variance and *E* the expected value. And V_i gives the partial variance of the individual input parameter when the parameter is fixed. With V_{ij} being the covariance when parameters X_i and X_j are fixed. Normalising Eqn. 2-22 by dividing with the total non-conditional variance, $V(Y)$ yields:

$$\sum_{i=1} S_i + \sum_i \sum_{i<j} S_{ij} + \sum_i \sum_{i<j} \sum_{j<k} S_{ijk} \ldots + S_{1,2,\ldots,k} = 1 \qquad \textbf{(2-26)}$$

The various Sobol's sensitivity indices can be obtained from Eqn.2.26 as:

$$1st\ order\ \ S_i = \frac{V_i}{V(Y)} \qquad \textbf{(2-27)}$$

$$2nd\ order\ \ S_{ij} = \frac{V_{ij}}{V(Y)} \qquad \textbf{(2-28)}$$

$$Total\ sensitivity\ index\ S_{Ti} = S_i + \sum_j S_{ij} + \sum_j \sum_k S_{ijk+\cdots} \qquad \textbf{(2-29)}$$

Sobol's method makes no assumption on the model's inputs and outputs, as well as having the ability to analyse the full range of each input parameter variation and the interactions between the parameters interactions. Sobol's method is computationally expensive compared to the other variance based methods (Saltelli *et al.*, 2008). This method also requires knowledge of the distribution (Ravalico *et al.*, 2005), and where this is not precisely known it can become a limitation to the method's reliability.

Fourier Amplitude Sensitivity Test (FAST) and Extended Fourier Amplitude Sensitivity Test (eFAST) Sensitivity Analysis Methods

The FAST method as developed by Cukier et al. (1973) is used to estimate the contribution of each input parameter in a model function to the expected value of the variance of the output. FAST method can be used for both monotonic and non-monotonic models as well as linear or non-linear models, as it does not make any assumption about a model's structure based on its monotonicity or linearity (Saltelli *et al.*, 2004). FAST can be used to estimate the sensitivity by considering the effect of only one input parameter, or that of simultaneously varying all the input parameters. FAST uses a pattern search procedure and can be used as an alternative to Monte Carlo method (McRae et al., 1982).

FAST method is limited in that it cannot efficiently handle higher order interactions, which has led to the development of eFAST(Saltelli et al., 1999). The eFast method can adequately handle both first order sensitivity and total order sensitivity.

FAST and the eFAST methods for sensitivity analysis are model independent and can be used for both monotonic and non-monotonic models (Saltelli *et al.*, 2004), as well as having the ability to allow arbitrarily large variations in the input parameters (Christopher and Patil, 2002). These methods cannot adequately treat models with discrete inputs (Saltelli *et al.*, 2004). FAST and eFAST methods for sensitivity analysis assumes a uniform parameter probability distribution which is an advantage when the distributions are not known, but where the distributions are known not to be uniform it can be become a limitation of the applicability of these methods (Ravalico *et al.*, 2005).

2.2.3 Criteria for Selecting an Appropriate Sensitivity Analysis Method

Looss and Lemaitre (2015) enumerated certain objectives that should be clearly defined before carrying out a sensitivity analysis, and as such an uncertainty analysis: identification of the most influential factors; identify the non-influential factors and fix them their nominal values; and map the output behaviour as a function of the inputs by focusing on a specific domain of inputs if necessary. The characteristics of the model to be analysed also plays a significant role in determining the sensitivity analysis technique chosen and also its scope. Hence, a comprehensive understanding of the model and its limitation is required to select a sensitivity analysis technique that is best suited for that particular model. Ascough et al. (2005) identified key features that need to be studied in a model for the application of sensitivity analysis to include the following:

I. The structure of the model needs to be identified so as to be able to determine the scope of the sensitivity analysis.
II. The input parameters and their feasible domain space to indicate their variability has to be determined
III. The required model response or output for the sensitivity analysis should be selected based on the assessment objective.
IV. Model modification should be done if it is not possible to apply sensitivity analysis techniques to the existing model.

After the development of the model, a sensitivity analysis technique best suited for the given model structure should be selected. This is necessary as different sensitivity analysis techniques will perform better on specific types of models than others. Models have different structures in terms of linearity, monotonicity, correlations and the number of uncertain input parameters. With the use of scatter plots as a starting point the model structure can be visually analysed for linearity, monotonicity, correlation so as to select a sensitivity analysis technique.

Sensitivity analysis techniques such as Pearson product moment correlation (CC), Standardised Regression Coefficients (SRC), and Partial Correlation Coefficients (PCC) are best suited for linear models that are monotonic. While sensitivity analysis techniques such as Spear-man Rank Correlation coefficient (RCC), Partial Rank Correlation Coefficient (PRCC) and Standardised Rank

Regression Coefficient (SRCC) would perform adequately for non-linear but monotonic models. For models that are linear but non-monotonic the Fourier Amplitude Sensitivity Test (FAST) is applicable. The other variance based methods such as Sobol's method, and the extended Fourier Amplitude Sensitivity Test (eFAST) are model independent. They can be used to analyse models that are linear, non-linear, monotonic, and non-monotonic in nature. Figure 2.2 shows the flowchart for selecting sensitivity analysis technique based on the model structure. Table 2.3 highlights the criteria on when to use given sensitivity analysis methods.

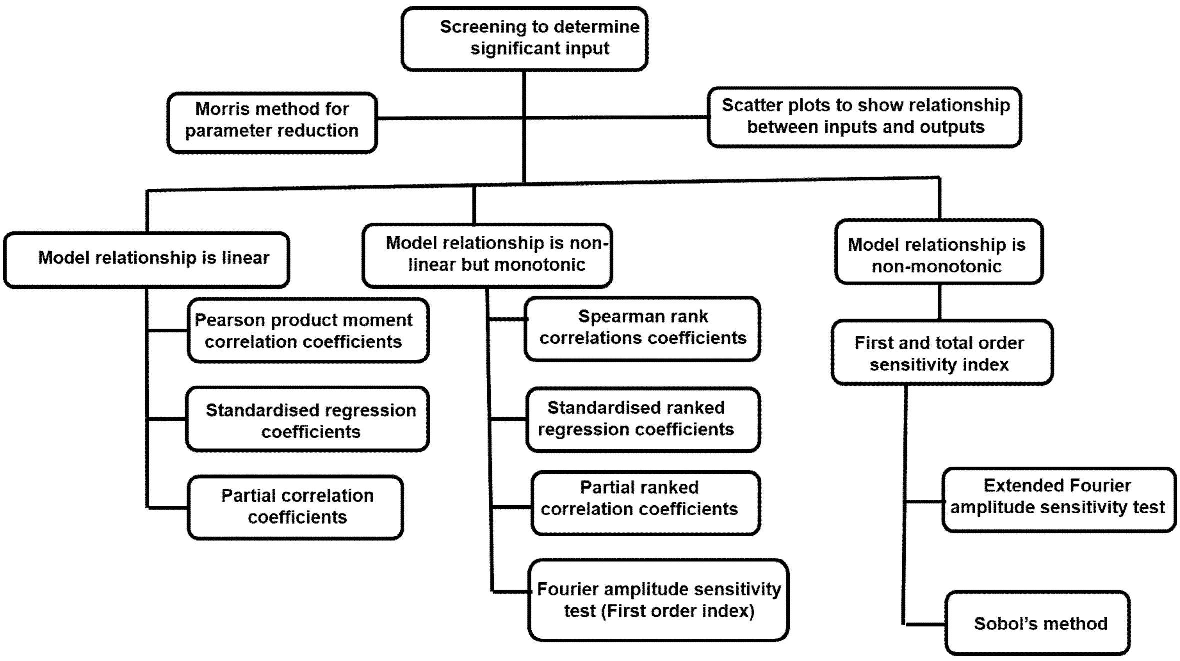


Figure 2.2 Flow chart showing sensitivity analysis techniques for different model structures (Ekström and Broed, 2006)

Table 2.3 Criteria on when to use given sensitivity analysis methods (Saltelli *et al.*, 2008)

	Sensitivity analysis methods				
Characteristics	Scatter plots	One-at-a-Time	Sample based	Elementary effects	Variance based
Coping with nonlinearity	Yes	No	No[a]	Yes	Yes
Coping with interactions	Yes	No	No	Yes	Yes
Samples taken from	Distributions	Levels	Distributions	Levels	Distributions
Number of input parameters	< 10		< 100	20-100	< 20
Cost of analysis	1000		500-1000	$r(k+1)$	$N(k+2)$

k: number of factors N: typically $N \approx 500/1000$.
[a] Recommended when $R^2 \geq 0.7$. In its ranked transformed version it can be quite effective for monotonic models.

For the sampling based methods PRCC and SRCC are considered the to be more robust and reliable (Saltelli and Marivoet, 1990; Hamby ,1994; Marino et al. 2008). For the variance based methods eFAST is considered more efficient and reliable compared to the other methods (Saltelli *et al.*, 2004). In this study the focus shall be on the PRCC and eFAST method as they have been shown to be reliable methods for sensitivity analysis. These methods as a result of their different approaches to sensitivity analysis measure two different model characteristics. PRCC measures model monotonicity due to an input when all the linear effects of other variables have been removed from the analysis, while eFAST provides measures for the fractional variance contribution of an input or combination of inputs variability to the variance of the model's output.

2.3 Design Optimisation in the Presence of Uncertainties

Real world optimisation problems of machine components are usually characterised with uncertainties and noise. These uncertainties arise due to several reasons such as manufacturing errors, inherent variability in design parameters, material properties variability, measurement errors, and fluctuations in operating conditions. These uncertainties such as variability in design parameters may be within acceptable tolerances but then they can still have significant impact on the intended behaviour of a component. Hence, unless uncertainties are taken into consideration, design solutions may become sensitive to input parameters variability that can lead to undesired outcomes in system performance, and even the violation of critical design constraints (Du and Chen, 2004). This highlights the need for the application of design procedures which consider uncertainties to ensure that the analysed systems perform within set boundaries. Two optimisation paradigms that are used to analyse optimisation problems in the presence of uncertainties are reliability based design optimisation (RBDO) and robust design optimisation (RDO) (Schuëller and Jensen, 2008).

2.3.1 Reliability Based Design Optimisation (RBDO) and Robust Design Optimisation (RDO)

Reliability based design optimisation (RBDO) is an optimisation method that seeks for the best compromise between cost and safety by considering system uncertainties (Aoues and Chateauneuf, 2010). RBDO involves a stochastic optimisation process in which it is the probabilistic functional of the objective function and constraints that is considered unlike deterministic optimisation. It involves the determination of the optimal design where the cost function is to be minimised by replacing the deterministic constraints in traditional deterministic optimisation with probability constraints (Enevoldsen and Sørensen, 1994; Youn and Choi, 2004; Schuëller and Jensen, 2008). It is usually the optimisation method of choice when the risk of a system or component failure is crucial. A major focus of RBDO is the maintaining of design feasibility at an expected reliability level (Zhuang et al., 2011).

Robust design as a design methodology was originally proposed by Taguchi to improve the quality of a product by minimising the product's performance variation without removing the source of the variability (Phadke, 1995). In recent years robust design methodology has been integrated to uncertainty based design optimisations to improve not only the products quality but also its reliability (Youn, 2005). A difference between robust design and RDO is that in the later the optimisation problem can be formulated to take the uncertainties into consideration. Robust design optimisation achieves its design objective by simultaneously optimising the mean performance while minimising the performance variance (Chen and Du, 2000). In robust design optimisation the uncertainty model unlike RBDO is not stochastic but rather deterministic and set based. Robust design optimisation aims to develop optimised solutions that are least sensitive to variations in the parameters of the nominal design. The performance and robustness of the optimum often tend to be in conflict with each other and so require trade-off decisions to be made, thus making the RDO a multi objective optimisation problem (Fang *et al.*, 2015).

2.3.2 A Comparison of RBDO and RDO

Robust design optimisation offers several advantages over RBDO in the optimisation of components in the presence of uncertainty. The use of RBDO requires the precise knowledge of the probability distributions of the random variables (Du *et al.*, 2006) as previously mentioned, making it strongly dependent on the assumptions made in obtaining these probability distributions. But in RBO there is no dependence on knowing precisely the probability distributions of these random variables. The use of RBDO requires the accurate determination of the limit state function to define the failure state of a component or structure, which in real life engineering applications this is usually not possible (Wiebenga, 2010). The RDO approach is also less sensitive to model errors and computationally more efficient compared to the RBDO approach (Kang, 2005; Wiebenga, 2010). In real life engineering designs precise or complete stochastic information on the distributions of the input variables may not be always be available, and so assumptions have to be made. Situations like this give rise to epistemic uncertainty and so it would not be appropriate to represent the random variables solely with probabilistic distributions. In design optimisation under uncertainty both types of uncertainty arising during the design of the product and its use should be taken into consideration. This requires the need then for design optimisation methods that can handle epistemic uncertainty. Non-probabilistic methods such as interval analysis, possibility theory and evidence theory offer alternative approaches to the probabilistic methods for design optimisation (Bae et al., 2004), as these have the ability to handle epistemic uncertainty. Thus the application of the aforementioned optimisation under uncertainty paradigms in several variants based on the type of uncertainty present, aleatory or epistemic as probabilistic and non-probabilistic methods of optimisation under uncertainty (Schuëller and Jensen, 2008).

In this study a non-probabilistic-based RDO shall be used as the optimisation paradigm of interest to achieve the aim of this chapter, which is the development of an optimisation framework to optimise the life of a component in the presence of aleatory and epistemic uncertainties. The RDO approach is selected due to its advantages over the RBDO approach as already highlighted.

2.3.3 Solution Methods for Multi-Objective Optimisation

Robust design optimisation as a previously mentioned involves the optimisation of a multi-objective problem, the maximisation of the mean of the objective function and the minimisation of its variance. The most common method that has been used for solving RDO problems is the weighted sum (WS) methods (Marler and Arora, 2004). This method involves the formulation of a single cost function by weighting the mean and the standard deviation values. Though this method can be easily implemented, there is the issue of selecting the most appropriate weights to capture the decision makers' preference. The conventional weighted sum approach has been shown to have serious limitations for the Pareto set generation as it fails to capture Pareto optimal points in a non-convex attainable region (Das and Dennis, 1997). Another limitation of the weighted sum method is that even with the consistent and continuous varying of the weights it may not be possible to obtain an even distribution of the Pareto optimal points and an accurate complete representation of the Pareto optimal set (Das and Dennis, 1997). Moreover the in the use of this method only a single solution rather than a set of solutions is returned at a time for a given combination of weights for the individual objectives. The second approach usually used in multi-objective optimisation is the determination of the entire Pareto optimal solution set or a subset of it. This is usually more desirable by decision makers as it enables trade-offs to be made. Pareto optimal solutions are those for which any improvements in one objective will result in the worsening of at least one other objective, thus requiring trade-offs to be made (Mattson et al., 2004). In real-life design problems, the optimisation often contain more than one objective which may be in conflict with each other. This often requires making a decision amongst several competing designs. Pareto optimal solutions thus provides a practical way for decision maker to make a compromise between the objectives.

Several methods for solving multi-objective optimisation problems have been proffered in literature, and they include other methods aside from the weighted sum approach, such as lexicographic method, the weighted min-max method, ε-constraints method, goal programming methods, physical programming and genetic algorithms (Marler and Arora, 2004). These methods all have their advantages and limitations. A criteria for selecting a multi-objective optimisation method is the ability of the method to generate a Pareto set that comprises an even distribution of Pareto solutions , able to generate all available Pareto points and generate only Pareto points (Mattson et al., 2004; Messac et al., 2003). According to Mattson et al., (2004) analytical methods that are able to meet this requirement are the physical programming method, normal constraints method, the normal boundary intersection methods. Table 2.4 gives a comparison of the effectives of the classical optimisation methods to generate Pareto solutions.

Aside from these classical optimisation methods the genetic algorithm has been applied successfully in multi-objective optimisation for generating a set of evenly distributed Pareto solutions optimisation (Deb *et al.*, 2002), and also meets the other criteria for selecting a solution method for multi-objective optimisation. An advantage of genetic algorithm over the classical optimisation methods is that it does not require gradient or hessian information (Roy et al., 2008).

Table 2.4 Effectiveness of classical methods to generate Pareto solutions (Messac et al., 2003)

Attributes			
	Generate even spread	Generates all available Pareto points	Generates only Pareto points
Physical programming	Y	Y	Y
Normal boundary intersection	Y	Y	N
Normal constraint	Y	Y	N
Weighted sum	N	N	Y
Compromise programming	N	Y	Y
Y: yes; N: no			

Their ability to simultaneously search regions of the design space gives them an added advantage as they are able to find a diverse set of solutions for difficult problems with multi-modal, discontinuous and non-convex solution spaces and do not require scaling of the objectives unlike the traditional optimisation methods (Konak et al., 2006). Studies show that these have made genetic algorithms more popular in usage for multi-objective optimisation than the classical optimisation algorithms (Roy et al., 2008).

2.3.4 A Review of Previous Studies on Robust Design Optimisation in the Presence of Mixed Uncertainty

In literature there exists an extensive volume on robust design optimisation methods and applications but majorly with respect to aleatory uncertainty represented by probability distributions (Zaman *et al.*, 2011). Relatively fewer studies have been reported in the literature on robust design optimisation that deals with epistemic or mixed uncertainty based optimisation. Some of the commonly used non-probabilistic optimisation methods under uncertainty include interval-based design optimisation (IBDO), possibility-based design optimisation (PBDO) and evidence-based design optimisation (EBDO) (Huang *et al.*, 2013). Interval based design optimisation has been applied to design optimisation in the presence of uncertain data (McWilliam, 2001; Solau *et al.*, 2011; Li *et al.*, 2013; Yoo and Lee, 2014; Cheng *et al.*, 2016). Though the use of IBDO is able to produce results that are considered rigorous, it cannot take into account distributions, dependencies and detailed empirical information even when these maybe available about a parameter (Zio and Pedroni, 2013), resulting in the discarding of valuable information. Possibility based design optimisation (PBDO) has also been applied in design optimisation majorly as an alternative to RBDO (Choi et al., 2004; Du et al., 2006; Mourelatos et al., 2005; Nguyen et al., 2015). The use of PBDO unlike IBDO can utilise additional information like the assumed probability distributions that may be given in the problem description (Zio and Pedroni, 2013), but then its use so far in literature is restricted to epistemic uncertainty (Huang *et al.*, 2013). The use of EBDO however offers some advantages over the IBDO and PBDO methodologies. Unlike interval based analysis and possibility theory that can handle only epistemic uncertainty, evidence theory allows the straight forward combining of aleatory and epistemic uncertainty (Liang, 2010). In this study a critical review of studies carried out using an evidence-based optimisation approach is undertaken. The aim of the review of literature on evidence based robust optimisation approaches is to determine limitations of this approach as demonstrated in literature. Though the review presented here is not exhaustive of all literature in this problem domain, but it is representative of the research within the problem domain.

In carrying out a multidisciplinary design optimisation in the presence of uncertain parameters, Agarwal et al. (2004) used evidence theory for the modelling. In this work the optimum design for a multidisciplinary system was determined using a thrust region sequential optimisation method. Surrogate models were used for the optimisation modelling to represent the uncertain measures as continuous functions. The imposed constraints in the optimisation problem were evaluated using the belief uncertainty measure from evidence theory. In their optimisation the minimum belief threshold was taken to be 0.99, and the effect of varying this value over a range was not carried out. The application of evidence theory to the test problem they studied worked fine. The study showed the efficacy of evidence theory in treating problems with mixed uncertainty.

Mourelatos and Zhou (2006) present the use of evidence theory for treating the uncertainty in design as a result of not having complete information. In this work a design optimisation is proposed based on evidence theory as it can handle a mixture of random and epistemic uncertainties. The proposed evidence-based design optimisation was tested using a cantilever beam and a pressure vessel example. The results obtained were compared with those obtained using RBDO and PBDO. The results obtained indicated that the EBDO design is less conservative compared to those obtained using RBDO and EBDO. The EBDO approach compared to RBDO and EBDO could be used to model expert opinion into the design opinion.

A robust multi-objective optimisation for a truss design using an EBDO approach is presented by (Su *et al.*, 2016). They demonstrated the application of an uncertainty quantification using evidence theory to the optimisation of shape and size of truss structures. Aside from optimising the objective function, an evidence based plausibility measure of failure of constraint satisfaction is minimised to formulate the robust design into a multi-objective problem. They investigated the problem by setting a plausibility measure of failure threshold at a predetermined level. A similar approach was used by Srivastava et al. (2013) for a pressure vessel design optimisation, the difference being in the algorithm procedures used in carrying out the design optimisation.

The previous paragraphs show the effectiveness of applying evidence theory to design optimisation in the presence of uncertainty. Despite its obvious advantages the approaches mentioned in literature for implementing it has some limitations. As presented by Shimoyama *et al.*, (2009) these approaches which involve creating an additional constraint in the problem, set the right hand side of the constraint as the limit of the robust constraint, and this poses some difficulty. Croisard et al. (2010) present methods for optimisation under uncertainty of the preliminary design of a space mission based on Evidence theory. Three different approaches for the optimisation under uncertainty were proposed and tested on a realistic preliminary space mission design. An objective of the optimisation is the maximisation of the belief function for an optimised performance measure. Three different approaches were used, the direct approach based on a multi-objective optimisation, a step method and a cluster approximation method. They solved optimisation under uncertainty problem by selecting a priori a threshold Evidence theory belief value for which the evidence of obtaining a performance function should not be greater than for the three approaches as a robustness constraint. The direct approach provided the best results. Different optimal designs were obtained for different levels of belief in all the approaches. This indicates the dependence of the results on the belief threshold level. For a belief threshold of 1, the same optimal points for a deterministic optimisation were obtained.

Salehghaffari et al. (2013) applied evidence theory for the optimisation of externally stiffened tubes under material parameter uncertainty. The EBDO formulation used by (Salehghaffari *et al.*, 2013) was to determine the optimum geometric dimensions for the maximisation of specific energy absorption, Pmax, while keeping the maximum crushing force less than a predetermined critical value. Pcric. Their formulation was a single objective optimisation problem subject to an added design constraint that imposed a limit on the plausibility that the *P*max does not exceed *P*cric. The value of the plausibility threshold is determined a priori, and it can take different values depending on the reliability. The obtained optimal design points varied with the value of the selected reliability factor or plausibility threshold.

Zhou et al., (2008) propose a method for carrying out optimisation in the presence of uncertainties using a combination of evidence theory and a Bayesian approach. A Bayesian approach was used in place of expert opinion to obtain the basic probability assignments. The proposed method was demonstrated using a pressure vessel example. In carrying out the optimisation, plausibility is used as the measure of uncertainty. In their algorithm the plausibility of failure had to be determined every time the optimiser evaluates the constraint. This required setting a plausibility of failure threshold value also as a limit of the robust constraints for considering the extent of robustness. Two threshold values were investigated for the demonstration of the proposed approach, plausibility thresholds of 0.45 and 0.35 respectively. The optimum values of the objective function obtained were 10718 and 9805. The optimum value of the objective function thus depends on the selected threshold.

2.3.5 Identified Gap in Optimisation under Uncertainty Studies

It is observed that the use of EBDO from the literature reviewed, the optimum design is determined according to some predefined criterion with respect to the feasibility robustness of the optimisation. In robust design optimisation under uncertainty two kinds of robustness, performance and feasibility robustness are usually studied in the literature (Chen and Du, 2000). In ensuring feasibility robustness, that is the satisfaction of the constraints under uncertainty (Chen and Du, 2000), the reviewed EBDO approaches required the a priori selection of an evidence based uncertainty limit for the belief or plausibility uncertainty measure. In these works an additional constraint is included in the optimisation formulation by setting the right hand side as the upper limit of the robust constraints for considering the degree of robustness. This creates some difficulty, as selecting this threshold or limit can be challenging and also introduce its own uncertainty into the optimisation, as there may not be precise information on the appropriate level to select for the problem in question. It is observed that optimal design points varied with the level set for the threshold belief or plausibility uncertainty measure. To remove the subjectivity in selecting these threshold or dependence on the threshold value requires design optimisation methods that are independent of the requirement to select a threshold as part of the objective function(s) formulation or optimisation problem formulation. Moreover the calculation of plausibility (or belief) for use as the robustness measure is computationally challenging in a practical implementation of evidence based design optimisation (Srivastava and Deb, 2011). There is the need then for the use of alternative robustness measures based on evidence theory that are less computationally challenging but practicable for design optimisation under uncertainty. In this thesis an evidence based optimisation methodology is proposed that does not depend on the use of a plausibility or belief threshold. The use of this method would remove the subjectivity in selecting this threshold, the dependence of the optimum design points on the thresholds, as well as reduce the computational complexity in obtaining the plausibility or belief uncertainty measures for the robust design optimisation.

2.4 Chapter Summary

The review presented in this chapter has presented features of design problems in relation to uncertainty quantification, sensitivity analysis and design optimisation in the presence of uncertainty. Existing techniques used in this disciplines were critically examined to establish their strengths and limitations. The literature analysis reveal that though effective methods and techniques exist for uncertainty, sensitivity and optimisation in the presence of uncertainty, there are research gaps. These gaps are identified as follows

- Existing methods of uncertainty quantification based on evidence theory that is able to handle mixed uncertainties have only been concerned with the propagation of input parameters uncertainty into the model output. These methods have not taken into consideration other sources of uncertainties such as model form and model prediction error uncertainty.
- In the use of evidence theory in design optimisation under uncertainty the use of a predetermined plausibility or belief threshold introduces its own uncertainty into the optimisation due to the subjectivity in choosing these values.
- The calculation of these measures (plausibility and belief) for use as the robustness measure is considered to be computationally challenging.

The filling of these gaps would satisfy the aim and objectives of this research as stated in chapter one. In filling these gaps the research shall attempt to develop an uncertainty based quantification method based on evidence theory that is able to handle aleatory and epistemic uncertainty both in the input and output of a model. An optimisation under uncertainty shall also be implemented using the proposed uncertainty quantification method with the use of a robustness measure that would be computationally less expensive to compute, and also remove the dependence of the optimal points on it. The next chapter presents the description of the research methodology that is used to achieve the aim and objectives of this research.

3 . RESEARCH METHODOLOGY

The review of pertinent literature presented in chapters one and two showed that most studies concerning the degradation of machines at the component level have mostly been limited to material degradation assessed against operating or environmental conditions. Aside from this, most studies involving fatigue life estimation of engineering components using finite element modelling have not involved the presence of mixed uncertainty, aleatory and epistemic and thus been of considerable limitation with respect to practical applications. In real life these uncertainties should be taken into consideration during design. The identified gaps based on the review of literature have led to the development of the aim, objectives, scope and adopted methodology of this research.

This chapter presents the research methodology and approach that are used to achieve the research aims and objectives. The research aim and objectives with the scope are restated in Section 3.2. In Section 3.3 methods and approaches used in research are discussed. Section 3.4 presents data collection methods in research, and in Section 3.5 the adopted research methodology for this study is presented.

3.1 A Restatement of Research Aim and Objectives

The aim of this research as previously stated in Chapter 1 is: To develop a methodology to assess the degradation life of a mechanical component due to geometric design influence in the presence of uncertainties and its application for the optimisation of the component in the presence of these uncertainties.

To achieve the aim of this research, the following objectives would be fulfilled:

1. To understand the impact of design variables and current practice in industry.
2. To determine the dependency of component life degradation on design features using selected candidate mechanical components.
3. To develop an uncertainty quantification methodology that can handle both aleatory and epistemic uncertainties that may be present in the design of the selected components.
4. To optimise the thermal fatigue life of a candidate mechanical component in the presence of design uncertainties based on the uncertainty quantification methodology developed in (3).
5. The validation of the proposed uncertainty quantification and optimisation under uncertainty methodologies for generalisability.

3.2 Research Design and Methodology

Research has been given different definitions by different persons based on their context of application. According to Shuttleworth (2008), research in its broadest sense is any gathering of data, information and facts so as to advance knowledge. This definition supposes research to be a data gathering and analysing activity that is done in a systematic manner involving a series of processes. Creswell (2008) provides these steps to make research a systematic process of collecting and analysing data so as to increase the understanding of an issue. The steps outlined in a sequential order are;

- Identify a research problem
- Review of literature
- Specify the research hypothesis
- Data collection
- Data analysis and interpretation
- Reporting and evaluation

A research study should involve the selection of a good research design and plan (Creswell, 2009). To develop a good research plan the purpose and the type of data to be analyzed should be known. Kumar (2005) based on this suggests that research design can be classified based on three major perspectives; application, objectives and inquiry mode (see Figure 3.1 for research classification based on perspective).

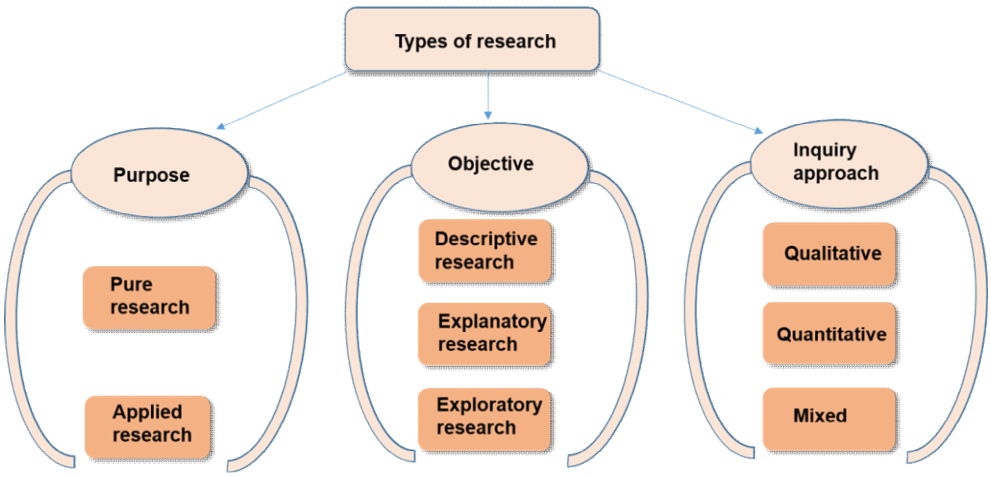


Figure 3.1 Classification of research types

3.2.1 Research Design Based on Application

A research if viewed from the perspective of its application can be classified into two broad categories, pure research and applied research. It is best to first identify the application of a research, then a strategy can be selected and appropriate data collection and analysis techniques are next employed. Pure research involves the developing and testing of theories or hypothesis that usually do not have practical application at the present time (Bailey, 1987). An applied research deals with solving practical problems by using existing methods and procedures that form the body of knowledge in that research domain to solve a specific issue (Kumar, 2005).

3.2.2 Research Design Based on Different Approaches

Authors use different terminology to refer to the research approach, they also use terms such as strategy or methodology (Wisker, 2001; Robson, 2002). There are basically two distinct approaches to research design, which are the qualitative and quantitative approaches (Gummesson, 2000). But then these two approaches are not as distinct as they appear. A research can tend to be more quantitative than qualitative and vice versa (Creswell, 2009). As a result of this a third approach arises, which Creswell (2009) refers to as the mixed method approach, a combination of qualitative and quantitative approaches.

Creswell (2009) describes qualitative research as a means for exploring and understanding the meaning ascribed to a human or social problem by individuals or a group. This approach is flexible as it does not require a rigid pattern; the research framework evolves as the research takes place (Robson, 2002).

Quantitative approach is a research design approach for testing objective theories by examining the relationships among variables using a numerical format (Creswell, 2009). These variables are measurable, so that numbered data can be analyzed statistically. A major feature of this research approach is the use of a controlled environment where the researcher is able to control the experimental environment and is expected to be detached to guard against having an effect on the research findings (Robson, 2002).

Mixed method approach is a research design approach that combines both qualitative and quantitative approaches to research. According to Creswell (2009) aside from combining the qualitative and quantitative approaches, it also involves some underlying philosophical assumptions. When appropriate a combination of qualitative and quantitative research is possible; as within certain limits all types of research are suitable for both approaches (Walliman and Bousmaha, 2005).

This research will make use of both qualitative information and quantitative data, hence the adoption of the mixed approach for this study. This approach offers some advantage in that it combines the advantages of qualitative and quantitative research. Using the mixed method approach allows the presentation of qualitative data using quantitative method of analysis enabling researchers to carry out well managed and well documented research (Wisker, 2001).

3.2.3 Research Inquiry Strategy

According to Yin (2003) research strategy can be classified based on research objectives as descriptive, explanatory, and exploratory. This classification is not exhaustive based on research strategy classification by other researchers (Wisker, 2001; Robson, 2002; Creswell, 2009). Explanatory research attempts to answer the question why and identify relationships between aspects of a phenomenon (Robson, 2002). A descriptive research seeks the description of a phenomenon in a systematic pattern. Its aim is to describe the what, why, where, when and how research questions. This strategy requires an extensive previous knowledge of the issue under study (Robson, 2002). Exploratory research is usually undertaken when the objective is to seek new insights particularly in areas where little is known. This strategy can also be employed to generate new ideas and hypothesis for future research (Robson, 2002). Though based on the objective perspective, research strategies are classified into these three classes, in practice several studies are a combination of them. Among these strategies, exploratory research familiarizes the researcher with the phenomenon being studied. (Yin, 2003) suggests the adaptability of exploratory research to many research strategies such as survey, experiment, and case study.

3.2.4 Research Strategies for Data Collection

Robson (2002) argues that a good research strategy should have good compatibility among the research purpose, theory, research questions and sampling strategy. He suggests that the research strategy should be concerned with how the research questions will be answered. The data collection method should provide answers to the research questions. Robson (2002) presented traditional research strategies for collection of research data in both quantitative and qualitative research. These research strategies includes experiments, case study, grounded theory, survey and ethnography study (Robson, 2002; Creswell, 2009)

3.3 Data Collection Methodology

Several methods exist for data collection depending on the type of research approach, qualitative, quantitative or mixed research design (Creswell, 2009) . Common methods of data collection include survey, interviews, literature review, observations and experiments (Robson, 2002; Creswell, 2009).

3.3.1 Review of Literature

Creswell (2009) states that the review of literature helps to determine if a research topic is worth studying, as well as provide insights into ways in which the researcher can limit the scope to a needed area of inquiry. He further enumerates the other purposes a literature review accomplishes in a research. A literature review extends the knowledge gained in previous studies by filling in the gaps. He states further that the review of pertinent literature provides a basis for establishing the relevance of a study and the comparison of the study results with previous findings. The literature review hence provides an exploration of what is already known, what methods have been used, and possible limitations of existing knowledge about a problem domain.

3.3.2 Interviews

Research interviews provide a flexible and adaptable way of finding things and it provides the potential for providing rich informational material (Robson, 2002). According to Cohen et al. (2013) interviews unlike the use of questionnaires where the respondents are expected to record in some way their responses, it involves verbal interactions. Cohen et al. (2013) further highlights that an advantage of interviews is that it permits for more elaborate data collection method compared to other methods of data collection. Despite the advantages offered by interview to other methods, Robson (2002) states that they lack standardization, and are prone to biases, thus raising concerns about their reliability.

The structure of an interview is a major consideration ; the questions interviewer asks and how they interpret the answers they are given influence the nature of the knowledge produced are all influenced by the interview structure (Mason, 2002). Mason (2002) argue that the interview structure should allow space for free association which it can achieve by not enforcing a particular set of questions. She goes on to say that the irony of this is that even free association narratives require some kind of structure. Based on this in the use of interview as a data collection method the researcher should determine the amount of structure that should be included within an interview. Robson (2002) list three types of interviewing techniques based on structure that a researcher can use: structured, semi-structured and unstructured.

Structured interviews: In this type of interview the interviewer uses a rigid procedure in which the interview questions are organized and presented in a way that allows little or no opportunity for modifications (Cohen et al., 2013; Robson, 2002). The questions are close ended and so same questions can be posed to different respondents from which a comparison of their responses can be made.

Semi-structured interviews: These are interviews that have predetermined questions but the order of the questions can be modified based upon the interviewer's perception of what is best or most appropriate (Robson, 2002). It provides more flexibility than the structured interview.

Unstructured interview: This type of interview is open-ended, non-standardized and in-depth and offers a greater flexibility compared to the other interview techniques. At times it takes an informal approach and in this method the interviewer has a general area of interest, but lets the interview develop within this area. In the use of this technique the interviewer may lose control of the interview and also have difficulty in the analysis and interpretation of the result (Robson, 2002).

3.3.3 Experiments

In research, the intent of an conducting an experiment is to test the impact of a treatment (or an intervention) on an outcome, controlling for all other factors that might influence the outcomes (Creswell, 2009). It helps researchers to study cause and effect relationships between the independent variables and the dependent variables. In carrying out experiments the variables have to be identified and specified, the dependent and independent variables. In carrying out experiments the type of experimental design to be used should be identified, what is being compared in the experiment should also be identified.

3.4 General Research Methodology Adopted

The adopted research design is based on the research aim, the objectives the research intend to achieve and the research questions. The research questions help to determine what is to be studied and the scope of the study. The present study being an applied research follows an applied research methodology by using existing information, methodology and techniques that already exist in the body of knowledge to solve an industry need. This research is exploratory as it seeks new insights into strategies for assessing the degradation life of a mechanical component.

The study will involve a mix of the qualitative and quantitative research design approach. The type of data to be collected and analyzed is a determinant of the research design approach employed. Both qualitative and quantitative data shall be sought in the course of the study hence the combined approach.

A focus of this research is to propose a methodology that can be used for initial exploratory characterization of the influence of geometric design features on the in-service degradation of a machine component rather than develop a new theory as in grounded theory or describe a community's experience. Hence, the selection of the case study data collection methodology which requires the development of detailed, intensive knowledge about a case, which in this study is a mechanical component. Literature review, experiments, computer simulations, physics based analysis, interviews, and document analysis will be employed as strategies for elucidating the required knowledge to achieve the aim of this study. The integration of experiments into case studies is becoming a current approach though case studies are traditionally more suited for qualitative research. Yin (2003) asserts that case study can be used as an experimental inquiry that investigates an issue within real life framework, and can include a combination of quantitative or qualitative evidence. The requirements of the present investigation are the motivation of the selected research design approach and data collection methodologies.

This research investigates the dependency of component degradation on geometric design features. In this research a methodology for fatigue life degradation assessment in the presence of uncertain parameters of an engineering component is developed using the brake disc of a passenger car as a case study. The rationale for selecting the brake disc is based on the findings of a case study selection process carried out in industry (presented in chapter 4), the review of literature, as well as the availability of brake disc material and design data in the open literature. The brake disc being a component that is subjected to cyclic high temperature operations suffers from thermal fatigue degradation damage, and also has distinguishable design features (Okamura and Yumoto, 2006). All these fits the requirements for this study. This study shall then validate the developed methodology using black body validation approach. This research is an applied research done in collaboration between academia and industry. It is borne out of the desire to minimise costs associated with component degradation, so that component designers can design components that are less prone to degradation as a result of better understanding of design influence on degradation, and how uncertainties in design parameters impact on degradation life estimation. Figure 3.2 presents a graphical illustration of the research methodology adopted in this thesis.

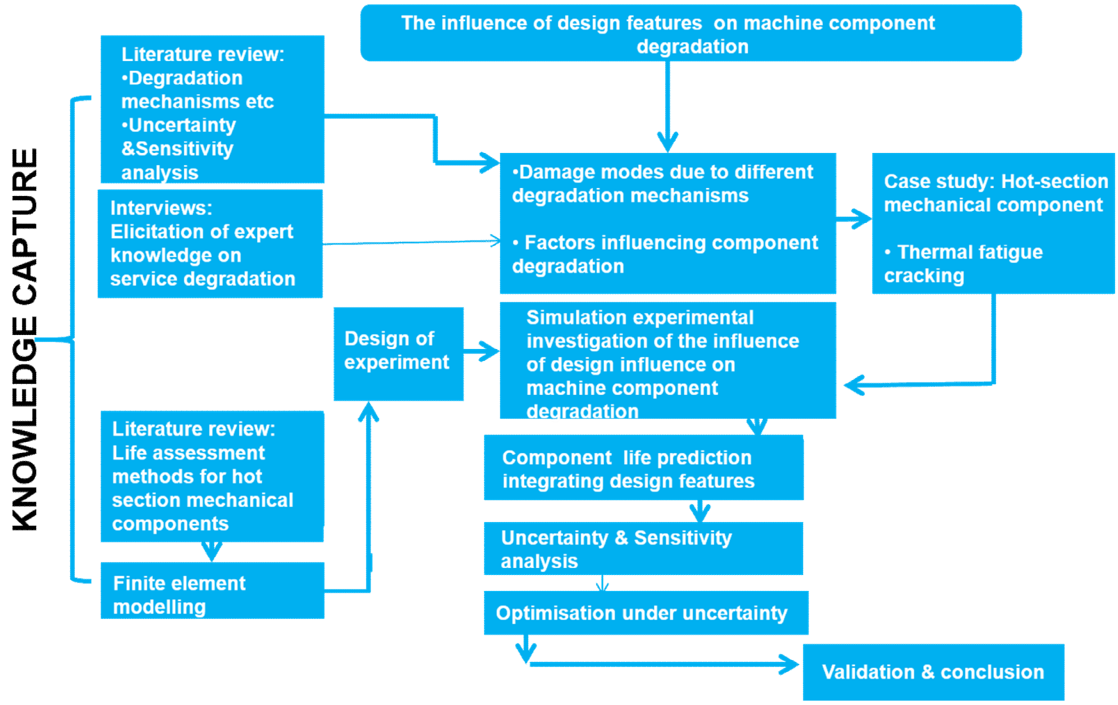


Figure 3.2 Graphical illustration of adopted research methodology

Data Collection

Data for this research was collected using three main sources (1) published works such as journal and conference papers, as well as technical reports, (2) Experiments, and (3) Expert interview and industrial observation.

A review of literature was carried out to identify existing methods for analysing the fatigue life of components. The literature survey provided a comprehensive understanding of existing research, methods for degradation analysis and uncertainty analysis, their strengths and limitations, and as well as the knowledge gap in the study of design impact on the degradation life, fatigue life of components in the presence of uncertainties.

The industry visit and interview of experts were carried out to understand how the research fits into an industrial context. The interviews and industry observation helped to identify a component of interest that fits the research aim and would also be of benefit to industry. The observation was done in a maintenance section of a company where vehicle components are maintained. Face to face interviews as well as observations were done. The procedure followed in conducting the interviews are presented in chapter 4.

3.4.1 Research Variables

This research studies the influence of design features on the fatigue life of a machine component in the presence of uncertainties. Due to the plethora of engineering components, a case study approach is selected. The degradation mechanism under study is the thermal fatigue life, and the component selected to characterize it is the vehicle brake disc. In carrying out this study certain variables and parameters have been identified to be of interest. These variables include the dependent variables which are the temperature, stress and thermal fatigue life. The independent variables refer to the dimensions of the geometric design variables and their shape. Aside from these variables there are also the design parameters. The design parameters as used in this study refer to those parameters whose values are fixed but are required for a proper investigation of the phenomena under study. The design parameters in this study include material properties of the component material, the applied heat flux and the modelling boundary conditions.

3.4.2 Research Instruments and Materials

The research utilises the use of experiments to achieve its aim and objectives. Two types of experimental methods were employed in this research. The type of experiments used were physical experiments and computer simulated experiments. Physical experiments to characterise the vented brake disc material were carried out. The material characterisation was carried out to determine and to confirm the brake disc material type as well as its surface roughness and inherent residual stress which are design parameters required for fatigue life estimation. This was done to provide data for further use in the computer experiments. The computer simulated experiments were used to study the relationship between design features and thermal fatigue life of the case study component, the vented brake disc. Computer simulation experiments was selected for this study based on its relatively lower time and cost requirement. Appropriate design of experiments are selected based on the literature review.

3.5 Research Validation

The validity of a research refers to the extent to which a theory, concept or model describes real life. Hence a validation is carried out to determine the accuracy and validity of the work. The developed surrogate model was validated by comparing the results obtained from actual simulation experiments to the statistical measures obtained by using the regression model obtained from a design of experiment. The proposed uncertainty quantification was also tested for generalisability by testing the method using randomly generated configurations of the solid and vented brake disc respectively. The newly proposed uncertainty methodology and optimisation under uncertainty for the brake disc were validated using a black box approach for modelling and method accuracy. Finally the second case study was used to demonstrate the effectiveness of the new proposed uncertainty methodology.

3.6 Chapter Summary

This chapter presents the aim, objective and scope of the research. It also presents the methodology used in attaining the aim and objectives of the research. A literature review of research methods and approaches is undertaken with an aim to selecting the most appropriate methodology. A mixed study research approach was adopted, integrating both qualitative and quantitative research methods. Due to the plethora of engineering components and degradation mechanisms, a case study approach is selected. Based on the nature of the study the variables and experimental methods to be used are highlighted, and finally a validation procedure is presented to validate the effectiveness of the proposed uncertainty and optimisation modelling in the presence of uncertainties. In the next chapter details of the methodology and procedures followed in eliciting expert knowledge and observation on different degradation mechanisms affecting the brake disc as observed in real life, and possible reasons for them.

4 . CASE STUDY SELECTION

The research aims and objectives were restated in chapter 3 and a literature survey of research methodology undertaken to determine a most appropriate methodology for this research. To fulfil the research aim and objectives a case study approach was chosen to be the most appropriate method. The aim of this chapter is to select a component and degradation mechanism of interest based on service life conditions in real life operations of machine components. To achieve the chapter aim a case study selection process was undertaken. In this chapter the case study selection to reflect current practice is presented and is carried out through use of questionnaires, semi-structured and unstructured interviews and observations. Conducting the interviews were considered necessary to determine real life assessment of degrading machine components, possible causes of the degradation and what can be done to minimise degradation of these machine components.

The structure of this chapter consists of various sections and their respective sub-sections. Section 4.2 provides a description of the case study information gathering methodology. Section 4.3 describes the data collection method. In section 4.4 a presentation of the analysis of the gathered information is done. Section 4.5 gives a summary of the entire chapter.

4.1 Information Gathering Methodology for Case Study

A case study selection procedure was used in this study to obtain fundamental though not necessarily a detailed insight about machine components degradation in real life applications. The gathering of information about the research domain with respect to machine component degradation was done through the use of interviews supported with questionnaires and observations. The primary purpose of the interview is to elicit real life machine degradation knowledge from experts conversant with the problem domain in industry. Through the use of carefully designed questionnaires with structured set of questions and informal interviews tacit knowledge was elicited from industry experts. The interviews were carried out in two phases. The initial phase was unstructured to elicit as much information as possible from the experts so as to be able identify machine components they considered critical in terms of degradation and that fits into the requirements of this research, and also to aid in the development of a structured set of questions for further interviews. The second phase consisted of a semi-structured interview using questionnaire which was developed based on the outcome of the first phase. The questionnaires for the semi-structured interview was were made available to the participants in advance to intimate them of the research area of interest so as to ensure the adequate elicitation of required information. The interviews, both the unstructured and structured phases were arranged to take place in the participant's location. For the interviews a presentation to introduce the aim and objectives of the case study selection was made. At the end of each interview session a review of the process was carried out. Job floor observations integrated with informal (unstructured) interview was also carried out to give the researcher a first-hand experience of the different types of degradation and their impact on the life of the components in real life settings. Both phases of interview and observations were carried out with an OEM company also involved in the maintenance of vehicle components. A detailed explanation of the procedures used in the case study selection is presented in the subsequent sections of this chapter. Interviews and observations were used for the effective elicitation of expert knowledge and also as a result of the nature of the research.

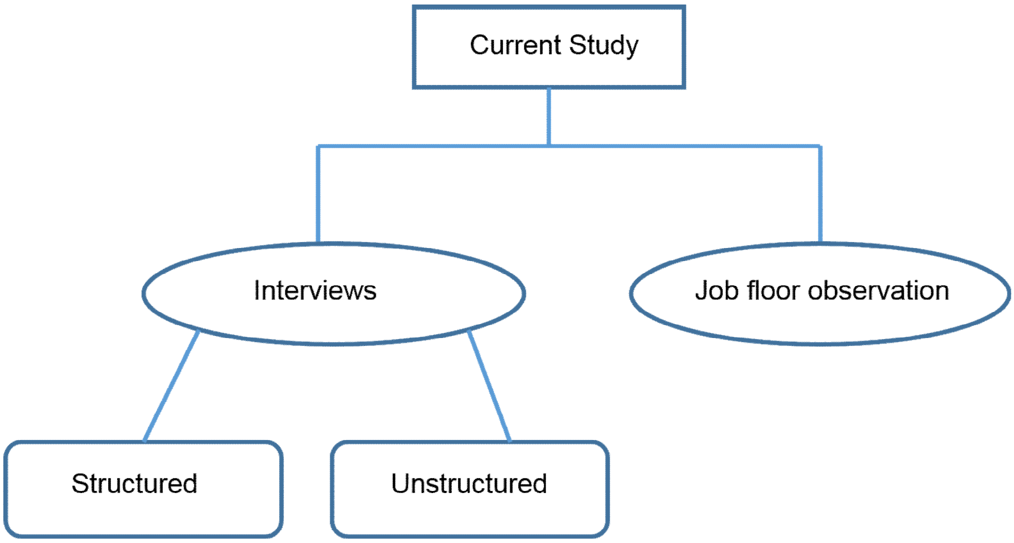


Figure 4.1 Phases for the current study practice

4.2 Data Collection

This section provides details of the data collection in the case study selection. Resource constraint requiring that a participating company have a base in the United Kingdom, and participant willingness were used as basis for selecting participating companies. The case study selection was conducted with the involvement of an OEM in one of their maintenance sites. This company was chosen based on its expertise and willingness to partake in the survey. In the initial phase of the case study selection, a presentation outlining the research aim and objectives with the study requirements were made to management representative of the participating company. Based on the presentation it was agreed that the maintenance section of the company would best fit the study requirement. The interview involved fivepersons with varying skills and level of experience suggested by the company to fit the case study selection requirement. A brake disc design expert outside the company also was a participant in the study. Table 4.1 lists the participants showing their area of expertise and years of experience.

Table 4.1 Participants expertise and experience level

Participant	Job title	Years of experience
A	Senior engineer- maintenance	15 years
B	Braking systems engineer	12 years
C	Maintenance technician	5 years
D	Maintenance technician	2 years
E	Brake design expert	20 years

4.2.1 Conducting the Interviews

The researcher opted for both unstructured and semi-structured interviews to get an in depth response to the different responses in the questionnaire and other questions asked in the course of the unstructured interview. The interview sessions were in phases, consisting of a preliminary phase described as phase 1 to elicit knowledge from participants A and B using an unstructured interview method, and phase 2 which involved semi-structured interviews with participants A and B, and unstructured interviews with participants C, D and E.

Phase 1

The phase one involved brainstorming sessions to identify a component of interest that will fit the research requirements and likely questions that should form the questionnaire. The research requirements required a component that is considered a safety critical component in its system, is affected by a degradation mechanism that is critical to its operations, and has distinguishable design features. This process was reiterated three times to select a component of interest to be used as the case study component, as well as the development of a set of questions that can capture the required information and knowledge in the subject domain for the selected component. Participants A and B are industry experts with the participating firm.

Phase 2

The semi-structured interviews for the phase 2 knowledge elicitation were conducted through the use of the developed questionnaire with participants A and B. The semi structured interviews were taped and transcribed to ensure that the reporting of the collected information is accurate and make use of it to the best detail. Aside from the interviews job floor observations were also undertaken by the researcher with a view to obtain real life knowledge of degradation of the selected component through observation and also informal (unstructured) interviews with the maintenance technicians who in this case were participants C and D. The observations and answers are described in subsequent section of this chapter. The interviews and observation visits were all conducted on site at the company maintenance location. While participant E is a brake disc design expert but not from the participating firm. Participant E was selected after the identification of the component based on the outcome of the initial interviews with the industry based experts. The purpose of the informal interview with participant E was to obtain independent expert opinion on the design and modelling of the selected case study component based on the findings from the phase 1 interviews.

4.2.1.1 Questionnaire development

This section describes the questionnaire development and the rationale for the questions. The questionnaire aims to elicit knowledge about the brake disc and its degradation. The interview as already described was semi-structured to allow in-depth elicitation and give the interviewees more opportunity to give robust answers. Table 4.2 lists the questions and the rationale for the questions. The questions were not necessarily asked based on their chronological order. Certain questions were rephrased based on the response of the interviewee being that it was a semi-structure interview. The transcripts of the interview are presented in Appendix A.

Table 4.2 Questionnaire themes and aim

Part A		
S/N	Question	Rationale
1	Job role of industry expert	To determine the relevance of the expert to the study
2	Years of relevant experience	To determine the level of expertise of the expert with respect to the study
Part B		
3	What are the types of damages that affect the brake discs	To determine the damage modes that are particular to the brake disc
4	Can you rank these damage modes in terms of criticality to brake discs life	To gauge the damages in terms of importance
5	How are the damages identified	To determine what methods are used to identify that there is a damage

6	How is the extent of damage measured	To determine what methods are used to assess the extent of damage
7	Who is responsible for determining the brake discs examination criteria	To determine if there is a standard for damage examination
8	On what basis where these criteria established	To determine why the criteria are used
9	The root cause(s) of these damages are they known	To determine what influences or cause the brake disc to get damaged
11	If the root causes are known, what are these root causes	To determine what are the likely causes
12	Has there been a formal study to determine the root cause(s) of the damages	To determine if the organization had undertaken its own investigation on the causes of the failures
13	What is the expected life of these brake discs	To determine the design life of the brake disc
14	What is the average service life of the brake discs	To determine the life of the brake disc while in service
15	What are the materials used for the manufacturing of these brake discs	To determine what type of material is used for the type of discs they use in their vehicles.
16	Are the brake disc of the same design or they are they different in their design configurations	To determine if the brake disc have same or different geometric design features
17	Are particular kind of damages more common to a particular brake discs design	To determine if the geometric design influences brake disc damage
18	In the design of brake discs what are the required design parameters	To determine if there are particular design features that are common to brake discs
19	Can these parameters be listed in terms of importance to brake discs design	To determine the hierarchy of importance of the design parameters to the life of a brake disc under actual working conditions
20	And if yes to the previous question, how would you list them in terms of importance to brake discs life	To determine if based on real life applications the influence of these features are known

21	Do these parameters affect brake discs life, and in what manner	To get the experts opinion based on experience on the impact of these features on disc life
22	Are the brake discs repaired	To determine if it is possible to repair damaged disc, and what extent of damage is the end life of the brake disc
23	Is there any standard for the repair	

4.3 The Analysis and Presentation of the Case Study Selection Results

The purpose of the knowledge elicitation process is to identify key degradation mechanisms that can affect a critical component while in service, factors that influence it in real life applications, methods for assessing or measuring the extent of given degradations, how they can be treated, and what can be done to minimise degradation. It is important to process and define the responses of the experts to the interview questions for the analysis and presentation of the interview feedback. The interview process for knowledge elicitation in the problem domain as conducted in this study was completely reliant on the responses given by the experts based on their own assessment in the subject domain being surveyed based on their experience. The responses from the experts in some respects are subjective, and hence, a qualitative sorting of their responses is required for the feedback analysis. The responses to the questions given by the experts have their similarities and differences, with some responses being unique. These differences and similarities no doubt can be attributed to the experts' experience. The answers should be treated in such a way that meaningful results can be extracted from the interviews while still being able to account for the differences and similarities or uniqueness of an answer. The responses were analysed and conclusions made by following the procedure in the sequence as shown below:

(i) List the questions and their respective responses as taken in the interview.

(ii) Carry out a categorisation of the responses based on the similarities and differences in the responses.

(iii) Summarise the response feedback

(iv) Make decisions based on the analysis of the responses.

The response feedback for the different interview phases is summarised and presented.

4.3.1 Knowledge Elicitation for Component That Fits the Research Study Requirements

The phase one interview was conducted using an unstructured interview process with the aim of identifying a component that fits the research aim and hence, can be used for a case study. This stage of the interview was recorded by the researcher through notes. The recommendations and basis for the suggestions proffered by the interviewees, Participant A and B were noted.

The informal interviews were initially conducted separately with both participants to enable the researcher obtain views that are independent and without bias or participants opinion influencing each other. The results obtained were compared for similarities and differences. In the course of the interview the various components that they handle in terms of maintenance were mentioned. In a question about what components are critical to their operations in terms of severity and occurrence both participants mentioned the axle and the brake disc. Another interview meeting was conducted with both participants present to harmonise their view for the purpose of the selection of a single component to be used for the case study. The researcher in the course of the interview re-stated that a key requirement is that the component should have distinguishable design features, and if these features are known to impact the life of the component. Based on the brainstorming a brake disc was finally selected as a component that fits the study requirement. The selection of the brake discs as meeting the study requirements were based on the considerations that a major degradation of the brake disc is fatigue in the form of thermal fatigue degradation. As earlier been discussed in Chapter 1 based on the review of literature fatigue is considered a major degradation that impacts critically on the life of components. The brake disc was also selected as it is a safety critical component, and also has design features that are distinguishable as is also shown in literature (Okamura and Yumoto, 2006).

4.3.2 Knowledge Elicitation on Brake Disc Degradation

This section presents the observations the researcher made based on the answers provided by the interviewees. Based on the analysis of the results the responses are summarised and presented as shown below:

(1) There are basically three degradation mechanisms or damage modes that affect the brake disc. They are

- Cracking
- Wear
- Disc distortion

(2) The brake disc is expected to wear while in service, so wear is not considered critical. Thermal cracking of the disc is considered the most critical as it could have serious consequences on the life and particularly the safety of the brake disc.

(3) Wear and distortion are identified through the taking of prescribed measurements of the disc rotor, while thermal cracks are basically identified by visual inspection.

(4) The disc manufacturer provides the criteria for assessing the extent of damage based on their own tests. There are also standards that provide the minimum extent of permissible damage for the different damage modes.

(5) The service life is usually shorter than the design life. But then the service life is dependent on several factors such as route topography, frequency of braking, the braking mode, type of brake pad material used etc.

(6) The brake disc come in different design configurations, and it has been observed that the extent of damage and damage location varies with the design of the brake disc.

(7) The brake disc are made of similar material which is grey cast iron, though the grade type of grey cast iron may be different. But the experts believe that it is the geometric design features that influence the occurrence of damage the most.

(8) Formal investigations have been carried out through use of metallographic tests. But these test could not determine why different designs of the brake disc degrade differently.

(9) The configurations they have include circular, semi-circular, brake discs with bolt holes on the rotor, brake discs holes on the rotor, and the type of mounting. A ranking of the influence of these design features on brake disc damage could not be ascertained as no formal study has been undertaken for this.

(10) For the type of disc being used repairs are undertaken based on the extent of damage and the location of the damage. There is no certainty if repairing of the brake disc impacts on the life.

Based on the results obtained the researcher was able to make the following conclusions:

- Thermal cracks are significant to the life of a brake disc
- Geometric design features impact significantly on the damage and life of the brake disc.
- Metallographic analysis would not be able to explain how design impacts on brake disc damage and life.
- Finite element analysis provides a better method for a study of the influence of design on a component's life.
- There is a need to study how and why these design configurations impact on brake disc damage and life

- In modelling and designing of the brake disc there are features such as the groove whose designs are considered critical to the life of a brake disc.

4.3.2.1 Analysis of the Unstructured Interviews

Observations carried out on the job floor as well as the informal interview of the maintenance technicians, participants C and D corroborated the findings of the responses obtained from the semi-structured interview using the questionnaire. The maintenance technicians considered the formation of cracks as more critical, and that a higher percentage of brake disc that are put out of service due to a damage was due to thermal cracks. The maintenance technicians both agreed that based on their observations and experience the formation and the location of the thermal cracks were different for different brake disc designs.

The unstructured interviews with participant E which were conducted after the identification and selection of the case study component was in agreement with the findings from the interviews with the industry based experts. The analysis of participant E responses indicated that design of the brake disc is a major factor that affects the thermal behaviour of the brake disc, aside from other factors such as the type of brake pads, the braking mode and the route. Participant E being a brake disc design and modelling expert also provided insights on the thermal modelling of brake disc. As a result of this the expert opinion of participant E was made use of during the course of the thermal modelling of the brake disc. The summary of the observations made from the informal interviews with participant E is as shown below:

(1) Brake disc thermal analysis can be done using physical experimentation or computer based simulations.

(2) That brake disc thermal behaviour is affected by several factors, and of which no single method can adequately capture all these factors simultaneously.

(3) The method of brake disc thermal analysis should be based on the study requirements.

(4) A fast and effective method for brake disc thermal analysis is Finite element modelling and computational fluid dynamics.

(5) That due to the fact that certain limiting assumptions have to be made in the use of finite elements for brake disc thermal modelling, the results can be used for exploratory design, and if necessary, further experimental and real life findings could be undertaken.

Literature findings are also in agreement with the responses obtained from the participants with respect to the brake disc thermal modelling. These literature findings are shown in Chapter 5 which discusses the thermal modelling of the brake disc.

4.4 Chapter Summary

This chapter presented the case study selection methodology used in this research. Data collection techniques comprising of semi-structured interviews, informal interviews and job floor observations were used to identify the mechanical component and a degradation mechanism which are used for the case study in this research. The case study selection study was performed to extract expert opinion based on their experience in the maintenance of degrading components to ascertain what factors influence degradation and should be of concern for real life applications. Findings from this study showed that geometric design features of machine component impact significantly on the life of components. From these findings a case study using the brake disc as the component and thermal fatigue degradation as the degradation mechanism based on the research requirement was identified and selected.

The next chapter presents the characterisation of the brake disc material used in this study and its modelling. The chapter gives a review of the properties of grey cast iron, and its finite element thermal modelling method. The characterisation was carried out to determine the brake disc material hardness for confirmation of the grey cast iron grade, as well as the surface roughness and residual stress of a sample brake disc used for this study.

5. GREY CAST IRON CHARACTERISATION AND BRAKE DISC THERMAL FATIGUE LIFE MODELLING

This chapter presents a review of the properties of grey cast iron. The material characterisation of a sample brake disc specimen to determine certain properties required for identification of the brake disc material and its fatigue life modelling are also included in this chapter. The chapter also includes the FE thermal modelling of two types of brake disc, solid and vented brake disc so as to identify the region of interest in modelling for fatigue life in brake discs. This chapter consists of four sections which are structured as follows. In Section 5.1 a review of grey cast iron material and its properties are presented. Section 5.2 describes the brake disc material characterisation procedures and the basis for the particular characterisations performed. Section 5.3 gives a brief description of the brake discs types, its geometry, design features and their dimensions as used in this study. In section 5.4 the thermal modelling of both brake discs types is carried out and the inferences made from the results are highlighted. In Section 5.5 the summary of the chapter is presented.

5.1 Grey Cast Iron

Grey cast iron which is also known as flake graphite cast iron has found significant use in the construction of engineering components that are exposed to thermal fatigue. One of such components for which the grey cast iron has been used extensively for its construction is the brake disc rotor. Several factors affect the performance of a brake disc and they include the brake disc geometry, dimension, the brake disc material response to thermal inputs and the durability. One of this factors the brake disc material response to thermal inputs has been considered quite critical in brake disc design and has influenced the general use of grey cast iron in brake disc rotors (Maluf et al., 2004). Brake disc are subjected to high stresses from repeated thermal cycling during use. As a result a brake disc material should be able to resist these high thermal stresses, by being able to absorb and dissipate the heat generated during braking quickly. Grey cast iron is able to do so due to its high thermal conductivity and diffusivity. Though there are other materials that have high thermal conductivity and diffusivity, cast iron has mostly been used in brake discs due to its better metallurgical stability behaviour, lower cost of production, comparatively cheaper cost of production, outstanding castability, high resistance to wear, excellent vibration damping capacity, and moderate resistance to thermal shock (Maluf et al., 2004; Nayar, 1997). Grey cast iron like other cast irons also have the advantage that its specific heat increases with temperature thereby improving the ability of the component to absorb more heat during normal operating conditions as well as also possessing a low coefficient of expansion value (Maluf et al., 2004). Figure 5.1 shows a classification of cast irons into its different types with their constituent composition and morphology. Figure 5.1 shows a chart for the classification of cast iron.

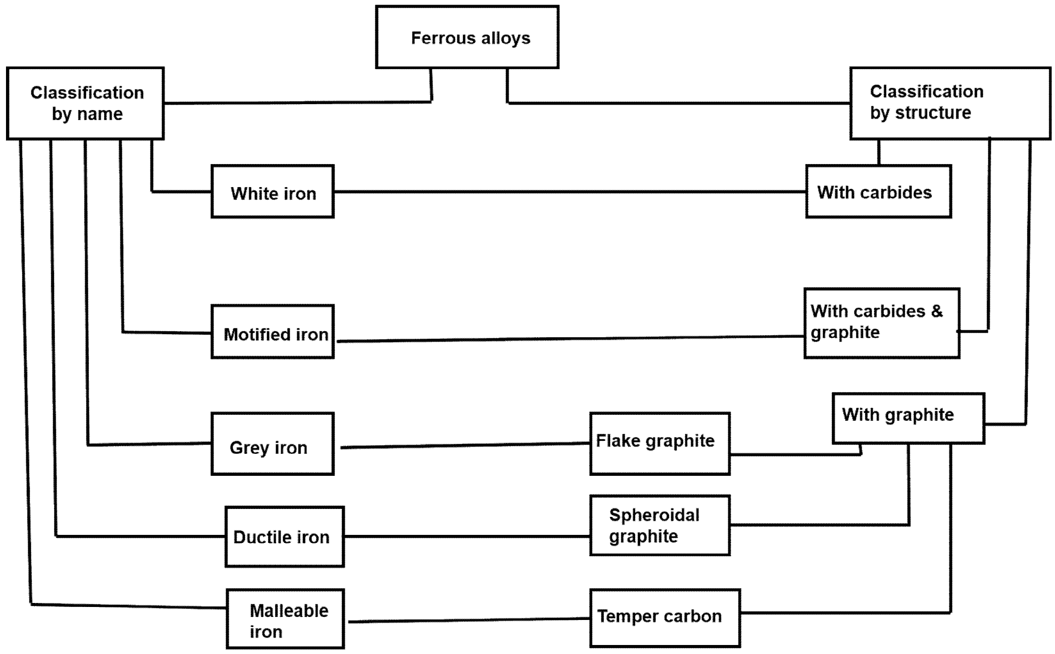


Figure 5.1 Cast Iron classification (Davis, 1996)

5.2 Brake Sample Material Characterisation

A sample brake disc was obtained and its material characterised using a hardness test to determine the grade of the brake disc to establish its properties required for the thermal modelling of the brake disc (Mackin *et al.*, 2002). Residual stress test and surface roughness test were also carried out on the obtained sample. Residual stress tests and the surface roughness tests were performed as these have been shown to impact on fatigue life, and are required for a more realistic estimation of fatigue life (James *et al.*, 2007; Rossini *et al.*, 2012; Deng *et al.*, 2013)

5.2.1 Hardness Test

The hardness test was performed using the Vickers hardness test method. The hardness test was carried out on grey cast iron samples three times and the results averaged. The obtained hardness value is then compared to published hardness values to confirm the class of grey cast iron. The hardness test was done to confirm the class of the cast iron brake sample. The hardness test was done using a Vickers hardness tester- Armstrong pedestal machine. An advantage of the Vickers hardness test method is that measurements with high accuracy can be obtained with it, and it can be used for testing the softest and hardest of materials. The use of Vickers hardness test requires good surface preparation. Hence to prepare the samples for the Vickers hardness test metallographic processes were carried out. The metallographic processes involved grinding and polishing of the samples cross sectional surface. Small cross sections of the brake disc was cut and mounted in. The sample was mounted in Bakelite resin to ease the mechanical grinding, polishing and the analysis with the Vickers hardness machine microscope. The sample cross sectional surface was grind with silicon carbide paper with gradually decreasing roughness, beginning with grinding paper of grade 240 progressively to grade 1200. A constant grinding speed of 3000rpm was maintained for all grades of the grinding paper. At the end of each grinding paper type the sample was rotated at 90 degrees. After the grinding process, polishing is done using a 3μm diamond step, and OP-AN Alumina suspension for the final polishing. Polishing is done until a mirror like surface is obtained. The Vickers hardness test is performed according to ASTME384-11 standard guidelines for performing Vickers hardness test. For the test a 30kgf was set for the indenter and applied for 15s. Figure 5.2 shows a diagram of the sample preparation for the hardness test process. The measurements were carried out on three specimens obtained from the same cross section of a sample brake disc. The average of the obtained results was computed as the Vickers hardness value. Table 5.1 shows the obtained results. The averaged value compared to published hardness values indicates the grey cast iron to be of the ASTM48 Class30 type of grey cast iron (MatWeb, 2016)

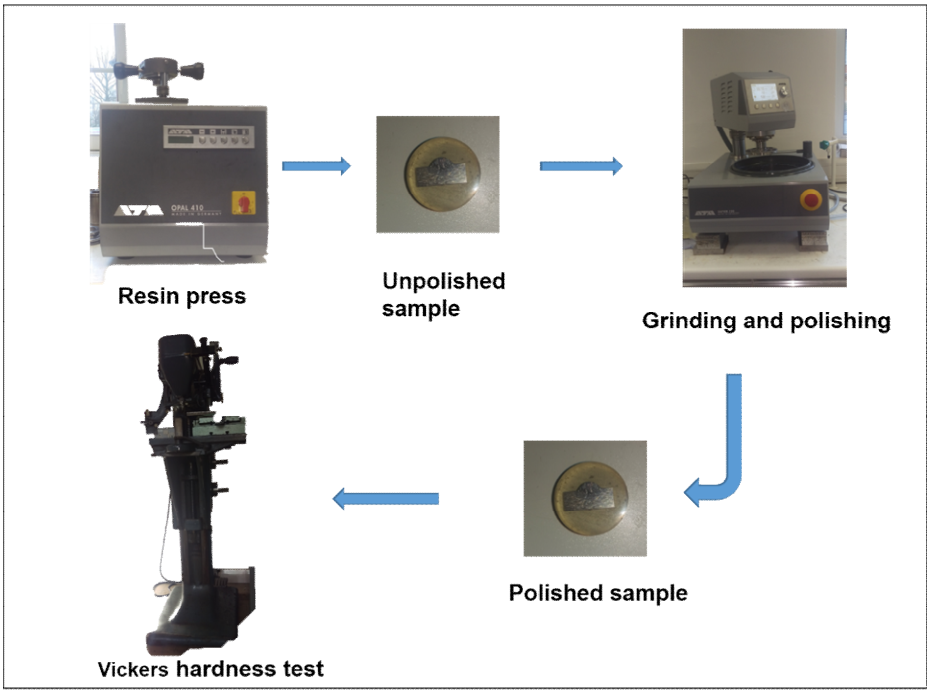


Figure 5.2 Sample preparation process for hardness test

Table 5.1 Hardness test result for sample brake disc material

Parameter	Specimen 1	Specimen 2	Specimen 3
Hardness (HV)	220	190	220
Average Hardness (HV)	210		

5.2.2 Residual Stress Test

For this study the measurement of the residual stresses in brake disc sample is carried using the incremental hole drilling method. The incremental hole drilling method which is a variant of the hole drilling method is used to improve the accuracy of the hole drilling method by making the strain measurements at a series of small depth increments as the hole is drilled from zero to the complete depth (Schajer, 2010). Ten degrees cross sectional specimens from a sample brake disc were used for the residual stress measurements. The residual stress measurements were carried out using the incremental hole-drilling technique using Stress Craft equipment. The centre of the friction face of the brake disc rotor was chosen for the strain gauge position. The sample surface was prepared by swab etching with glycerol and then acetone. The strain gauges were attached using glue to the specimen surface. The incremental drilling was done using an orbital drilling technique with a three axis PC controlled drilling stage. For the measurement a CEA-XX-062UL-120 strain gauge with an electrical resistance of 120Ω±0.4% was used. The strain gauge installation and hole drilling were carried out in accordance with the National Physical Laboratory good practice guide (Grant et al., 2002). The Young's modulus and Poisson ratio were not provided by the disc manufacturer, and so were obtained from literature as $E = 1.14 \times 10^5$ MPa and $\mu = 0.26$ respectively (Nayar, 1997). The residual stresses were obtained from a single location in the disc, and so cannot represent the stress distributions across the disc profile, but it does provide information about the relative amount of residual stress present in the disc sample.

The tensile residual stresses were selected for use in the study as they are the stresses that impact negatively on the brake disc, and as well as the stresses that are developed on cooling of the disc as at the end of braking. The residual stresses obtained for the transverse direction are chosen for use in the estimation of the thermal fatigue life of the brake disc as this has been shown to be more influential in brake discs (Shin *et al.*, 2013). The residual test results are as presented in Table 5.2. The average based on tensile transverse residual stress value obtained is 33 MPa.

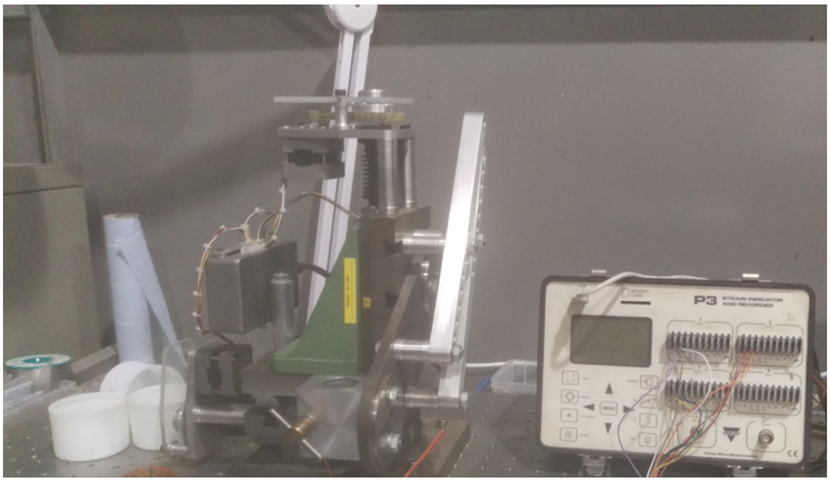


Figure 5.3 Residual test equipment set up

Table 5.2 Residual stress test results

Residual stress Test	Residual stress (MPa)
Test 1	34
Test 2	34
Test 3	32
Average measurement	33

5.2.3 Surface Roughness Test

Surface roughness which is the measure of the texture of a surface can be determined using different roughness geometric parameters of which the most common is the mean roughness (Roughness Average, Ra). The mean roughness, Ra is the arithmetic average of the roughness profile coordinates. The units of Ra are micrometres or micro inches. Mathematically the mean roughness measurement is given as:

$$R_a = \frac{1}{L}\int_0^L |Z(x)|dx \qquad \textbf{(5-1)}$$

Another surface roughness measurement metric similar to Ra is the Root mean squared roughness, R_q. It is expressed mathematically as:

$$R_q = \sqrt{\frac{1}{L}\int_0^L Z^2(x)dx} \qquad \textbf{(5-2)}$$

where, Z(x) = profile ordinates of roughness profile, L is the length, and x the distance along the measurement. Ra is used as the surface roughness measuring metric in this thesis.

There are several techniques for measuring surface roughness but for engineering applications the mechanical or optical methods have found more general use (Bhushan, 2001). For this research a coherence correlation interferometry optical method was used to measure the surface roughness of a sample vented brake disc. The equipment used was a Taylor Hobson Talysurf CC16000 white light interferometer. White light interferometry is a non-contact optical method 3D method for measuring the surface profiles. The equipment has a measurement resolution of 0.1 Armstrong for vertical resolution and 0.4-

0.6μm optical resolution with a fast measurement time of typically 5-10s. The obtained result of the surface roughness is as shown in Table 5.3.

Table 5.3 Surface roughness values

Roughness parameter Arithmetic mean (Ra)	Gaussian filter	Roughness value
Test 1	25 μm	0.214 μm
Test 2	25 μm	0.208 μm
Test 3	25 μm	0.214 μm
Average Test measurement	0.212 μm	

5.3 Thermal Fatigue Cracking

Thermal fatigue occurs a result of a metal being subjected to alternating heating and cooling that can eventually lead to cracking. Metals when subjected to these alternating thermal solicitations expand and contract by the same amount which results in the metal being stained continually in alternate directions. As result of being subjected to constant stain the metal eventually yields after a number of cycles leading to cracks. In thermal fatigue this cyclic deformation is imposed as the result of the constrained differential thermal expansion within a solid caused by the temperature gradients induced during alternating heating and cooling leading to high stresses and strains (Halford, 1986). Crack initiation, to engineers is related to the perception of significant crack which is used as a measure for the integrity of components (Maillot *et al.*, 2005). (Maillot *et al.* (2005) presented measurement criteria for crack initiation determination. The criteria are that (1) Crack initiation occurs when at least of 50 – 150 μm crack length is observed at the surface using optical microscopy, and (2) crack initiation is said to occur when at least one of 2 mm cracked length can be observed at the surface visually. Example of components subjected to thermal fatigue are turbine blades (Yang *et al.*, 2014), diesel engine pistons (Szmytka *et al.*, 2015), brake discs (Li *et al.*, 2015), nuclear plants components (Rudolph *et al.*, 2011) etc.

5.4 Disc Brakes

The brake disc system is used to bring a vehicle to a stop or slow it down. The disc brake system primarily consists of the disc brake rotor, the brake pads and a calliper (BOSCH, 2007). The disc brake rotor is circular in shape and is mounted to the wheel hub, and so rotates with the wheel. The brake calliper which is mounted to the chassis of the vehicle carries the piston that presses the brake pads against the rotating disc rotor surface. And the brake pads are used to grip the disc rotor to either slow it down or bring its rotation to a stop when the brakes are applied.

There are basically two types of disc brakes, the solid brake disc and the ventilated brake disc. The solid disc brake consists of the circular friction surface and a top hat section which are connected through a section of the disc called the neck (Bae and Wickert, 2000; Okamura and Yumoto, 2006). While the ventilated brake disc consist of an inboard and outboard friction surfaces with cooling vanes in-between the friction surfaces. The cooling vanes permit the air cooling of the brake disc friction surfaces by creating a passageway for air to flow through the brake disc structure. There are basically two types of vented brake disc with their respective variants; front vented and back vented brake disc (Okamura and Yumoto, 2006). Figure 5.2 shows a typical disc brake system.

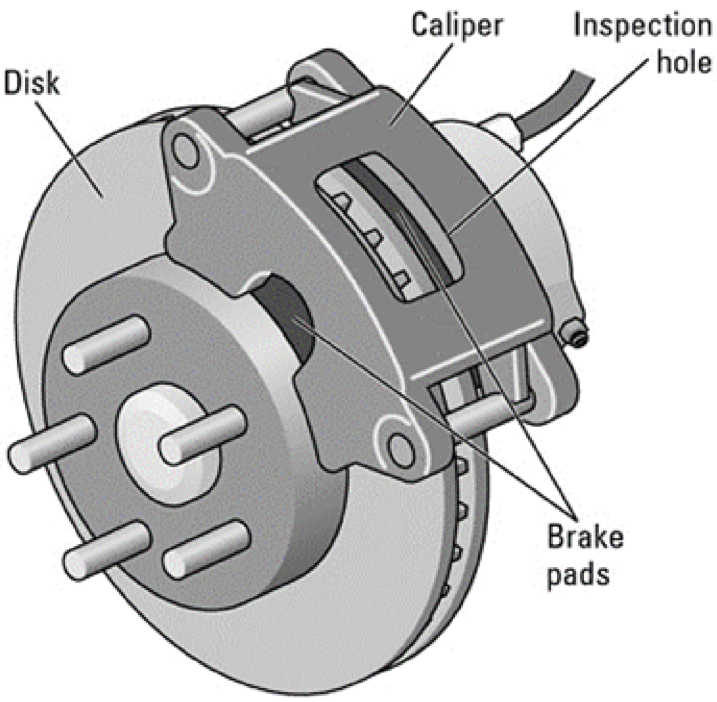


Figure 5.4 Brake disc system showing the brake parts (Sclar, 2011)

5.4.1 Finite Element Application in Brake Disc Thermal Analysis

The major aim of brake thermal analysis can be related to brake sizing, design optimisation, investigation of material suitability and component fatigue (Tirovic, 2004). This brings to fore the need to model and measure relevant parameters for a comprehensive understanding of thermal effects on the brake disc. Researchers have studied the thermal effects on brake disc performance and life using methods such as experiments (Gunther and Klingelhoeffer, 2000;

Gigan *et al.*, 2014), empirical analysis (Limpert, 2011) and with numerical methods (Dufrenoy and Weichert, 2003; Gao *et al.*, 2007; Belhocine and Bouchetara, 2012). In the use of these methods, finite element modelling, a numerical method has found consistent usage in the study of brake discs.

Belhocine and Bouchetara (2013) made use of a numerical method, finite element analysis to model the temperature distribution in a vented disc brake in order to identify the factors and the entering parameters associated with the time of braking such as the braking mode, geometric design and the brake material. They used a sequentially coupled thermal-structural FE analysis. Their study with FEA involved the study of braking mode on the thermal behaviour of a vented brake disc using three different types of cast iron. The results they obtained using FEA were considered satisfactory when compared with other studies in brake disc thermal behaviour in literature.

Using both experiments and finite element modelling, Sasada et al. (2000) studied the effect of hole layout on crack ignition in brake disc. They used a ¼ model of the brake disc for the FE analysis. They observed from the FE analysis that high compressive stress in the circumferential direction occurs around the holes that outside of the disc. Based on the obtained FE and experimental results they concluded that fatigue cracks do not occur on the disc surface only due to cyclic torque at steady states even at high temperatures, but that the compressive stress developed around the holes as a result of heating at braking affects crack initiation.

Huang and Chen (2006) in their work studied the effects of design and boundary conditions on temperature distribution of brake disc by using a three-dimensional finite element model. Their FE model involved the use of the application of a uniform heat flux on the friction surfaces of a three dimensional segment of a vented brake disc. A periodic ten stop repeated braking was used in the thermal analysis. Their analysis showed that the size of the fillet radius of the disc has significant effect on the temperature profile of the brake disc. And that changing the thickness of the disc support has an insignificant influence on the temperature.

Bagnoli et al., (2009) identified with finite element analysis the areas of crack development in fire fighting vehicles grey-iron brake discs. They carried out their analysis by applying a heat flux based on vehicle and braking data on the friction surface of a vented brake disc to obtain the temperature distribution. Once the temperature profile was obtained they carried out a linear elastic mechanical analysis taking into consideration pad constraints and the braking pressure. They indicated the FEA findings were in agreement with morphological evidences obtained from experiments, which showed that the cracks on the disc surface are due to local residual stresses developed at braking.

Okamura and Yumoto (2006) carried out a series of CAE/FEM experiments using Taguchi method to demonstrate the effect of basic configurations of brake discs on their thermal behaviour. In their work they presented experimental results to confirm the influence of brake disc design configuration on its thermal behaviour, as well as the correlation between FEA simulation and physical experimental results. They supported their findings with simplified models based on classical theory on the strength of materials to show and discuss the effect of geometric dimensional parameters on the thermal distortion of a brake disc.

The results obtained by Valvano and Lee (2000) in using finite element modelling to investigate the influence of thermal parameters on brake disc stress and rotor deformation. Utilising a PC-based programme to calculate the required thermal parameters as inputs for a finite element based thermal stress analysis. The method they used provides a method to determine the heat input and cooling behaviour of a brake disc and also the resultant thermal stress and thermal distortion in a brake disc. The results obtained showed good agreement with experimentally obtained temperature and distortion data.

Kim et al. (2008) with the use of a three dimensional FEM performed thermal stress analyses on a railway disc on whether the pressure distribution on the friction surface of the brake disc is uniform or non-uniform. Their findings showed that the application of variable pressure on the disc contact surface gave a more reliable result than that of applying uniform pressure. They also showed in comparison to an actual brake with thermal cracks that the region of the maximum Von-Mises stress in the FEM thermal analysis coincided with where actual cracks were located on the actual brake. They validated the results obtained by comparing their findings to experimental results available in literature.

Gao et al., (2007) using a fully coupled method performed thermal-mechanical analysis on a solid brake disc with the use of a three dimensional FE model.

The method they used allowed for the analysis of the effects of a moving heat source, i.e. the brake pad on the friction surface of the disc. The obtained results showed that during an emergency braking the maximum surface Von-Mises stress may get greater than the material yield strength and lead to a plastic damage accumulation in the brake disc.

To determine the temperature distribution, thermal stress and disc conning in a solid brake disc, Hwang et al., (2009) used a thermal-mechanical coupled FE simulation. They simulated repeated braking by applying a moving heat source defined by time and space on the contact surface of the disc. The method they used involved the use of only the disc rotor which resulted in simulation time reduction. Their results suggest that the thermal load on the rotor friction surface is dependent on the velocity and braking pressure applied on the brake pad.

These researches show the benefit of using finite element modelling in the analysis as well as development of the brake disc. Finite element based on previous studies have been shown to give comparable results with physical experiments. This makes the use of FEA in brake disc analysis an appealing alternative to physical experiments. The use of FEM can lead to considerable savings in time and money as it does not involve physical experiments that can be costly and time consuming (Okamura and Yumoto, 2006), while giving approximately equivalent results.

5.4.2 Approaches for FE the Thermal-Mechanical Modelling of Brake Discs

The thermal behaviour and hence, fatigue life of brake discs are modelled usually based on three types of loading: isothermal in which the brake disc is subjected to constant temperature with variable mechanical load; thermal in which the brake disc is subjected to abrupt temperature changes with no imposed mechanical load; and thermal-mechanical in which the temperature and mechanical load vary (Maluf et al., 2004). Hence, the modelling of a brake disc constitutes a thermomechanical problem. In finite element modelling two approaches are basically used to model the thermomechanical problem of a brake disc, sequentially coupled thermal-stress analysis and the fully coupled thermal-stress analysis (Abaqus Analysis User's Guide, 2013). Sequentially coupled thermal stress analysis is used when the thermal field is the driving force for the stress analysis, i.e. the stress depends on the temperature. In this method nodal temperatures obtained from an initial thermal analysis are read into a subsequent stress analysis to estimate the stresses. The fully coupled method is applicable in situations where the stress analysis depends on the temperature field and the temperature solution depends on the stress solutions. The fully coupled thermal-stress analysis involves only a single analysis.

Thuresson (2000) using a fully coupled thermal-mechanical finite element model studied the temperature and pressure distribution between two sliding bodies in contact, the brake disc and pad. Thuresson (2000) used a continuum thermal-mechanical wear model to determine the interface temperature and pressure distribution between the brake disc and pad. The model was discretised by finite elements and solved using a Newton type solver. Thuresson (2000) concluded that the model results are promising as it had the capability to investigate the temperature and contact pressure distribution in a brake disc-pad system by taking into account the effect of wear. Gao et al., (2007) demonstrated the use of a fully coupled thermal-mechanical finite element model to identify the source of fatigue in brake disc-pad interaction during braking. Their model allowed the analysis of the effect of a moving heat source, i.e. the brake pad with variable speed on the friction surface of a solid brake disc. Gao et al., (2007) validated the results obtained with experimental results in literature, and based on this suggested ways for avoiding fatigue fracture propagation in brake discs. Hwang and Wu (2010) investigated the temperature and thermal stress in a vented disc brake using a fully coupled thermal-mechanical analysis. Their study was done with a full three-dimensional finite element model of a brake disc and pads for a single stop braking mode. They compared their analytical results with experimental results which they suggest were in close agreement.

In modelling of a brake discs using the sequential approach, the modelling is simplified with the assumption that the pads are smeared on the disc friction surface (Abaqus Analysis User's Guide, 2013). Koetniyom et al., (2002) modelled a vented brake disc using a three-dimensional segment by taking the rotational symmetry of the disc into consideration with the sequentially coupled method. Dufrénoy and Weichert (2003) also employed a FE disc segment to study damage mechanisms on a brake disc due to thermal-mechanical solicitations. Both studies involved the smearing of the pad on the friction surface through the application of a heat flux. Using the sequentially coupled method Nguyen-Tajan et al. (2005) developed a transient heat transfer Eulerian analysis followed by a steady state mechanical analysis for simulation of the thermal-mechanical analysis of a solid brake disc using a full three-dimensional FE model. Strömberg (2011) using the Eulerian method also modelled the frictional heating in discs-pad systems. Strömberg (2011) demonstrated the approach by solving a two-dimensional model as well as a three-dimensional model of a brake disc by sequentially coupling the heat transfer and mechanical analysis.

The best method to model a brake disc is the thermal-mechanical method, that is the fully coupled approach (Maluf et al., 2004). The fully coupled method is best used in brake disc modelling as temperature changes in the brake disc lead to axial and radial distortion, of which this change in shape, in turn affects the contact between the pad and the disc and thus account for variation in temperature and contact pressure, of which the sequentially coupled method is limited in this respect (Abaqus Analysis User's Guide, 2013). In FE software the fully coupled method involves the Langrangrian approach in which the FE mesh used to discretise the brake disc rotates relative to a brake pad, and so thermal and mechanical analysis are performed simultaneously (Abaqus Analysis User's Guide, 2013). But this method is computationally expensive and for the aim of this research it is considered to be computationally too expensive to be used in this study. The sequentially coupled method is significantly computationally less expensive compared to the fully coupled method. Its use involves the application of a calculated heat flux based on vehicle and braking characteristics on the disc friction surfaces at the heat transfer analysis, and the application of the resulting temperature distribution to a stress analysis. Tirovic (2004) outlines possible classification of thermal-mechanical FE brake discs analyses based on three criteria which are as shown in Table 5.4. The criteria suggests that for the FE analysis of a disc segment a sequentially coupled linear elastic FE analysis with the application of a uniform heat flux is permissible based on the desired objectives of the analysis.

Table 5.4 FE Thermal-mechanical classification criteria for brake discs (adapted from Tirovic, 2004)

<table>
<tr><td>Friction surfaces modelled</td><td>▪ 3D
▪ Pad and Disc
▪ Pad
▪ Disc
▪ Disc segment</td><td>▪ 2D Axisymmetric
▪ Pad and Disc

Disc</td><td>• 2D
▪ Pad</td></tr>
<tr><td>FE analysis type</td><td colspan="3">• Elasto-plastic
• Elastic
▪ Fully coupled
▪ Sequentially coupled
▪ Non-linear
▪ Linear</td></tr>
<tr><td>Heat flux modelling</td><td colspan="3">• Thermo-elastic instability
▪ Including TCR
▪ Excluding TCR
• Uniform
• Non-uniform</td></tr>
</table>

5.4.3 Brake Disc Geometric Design Features and Dimensions

In this study a solid and front vented brake disc of a passenger car are used as the case study components. The brake discs of passenger cars was selected due to the availability of data about it in the open literature The geometric design features of the solid and vented brake disc as used in this study are based on a previous work by Okamura and Yumoto (2006). The pictures of a typical solid brake and vented brake disc are as shown in Figure 5.4.

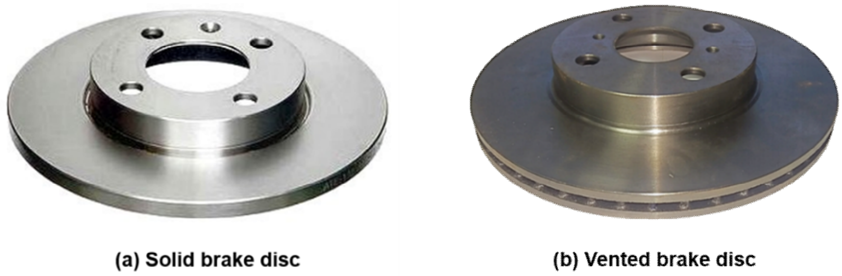


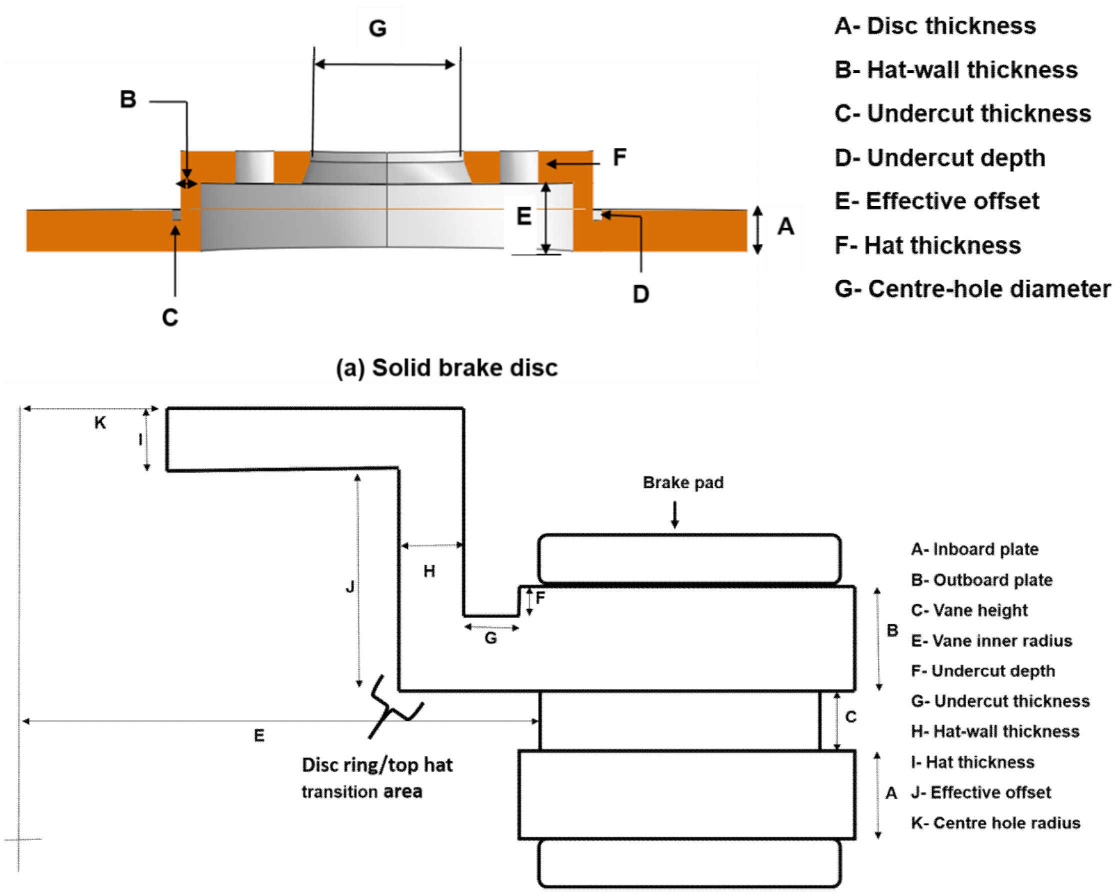
Figure 5.5 Typical solid and vented brake discs

A sample solid brake disc and a front vented brake disc were used for this study. The dimensions of the brake disc were measured using a calliper, and are used for the development of a symmetric 3-D model that would be used for the finite element thermal analysis of the brake disc. The design features of the solid and vented brake disc as used in this study are as shown in Figure 5.5. The geometric dimensions of the solid brake and the vented brake discs are as presented in Table 5.5.

Table 5.5 Brake disc dimension for solid and vented brake

Description	Dimensions	
	Solid brake disc	Vented brake disc
Disc outer diameter, mm	259	255
Disc inner diameter, mm	155	155
Inboard friction plate thickness, mm	-	7
Outboard friction plate thickness, mm	-	7
Rotor thickness, mm	9	20

Number of cooling vanes	-	42
Brake disc height, mm	30.5	46



(a) Solid brake disc

(a) Vented brake disc

Figure 5.6 Schematic diagrams of the brake discs indicating their design features

5.5 FEA Thermal Fatigue Life Modelling

Mechanical damage of structural materials of machine components are generally attributable to factors such as load, temperature, corrosion, time and their interactions which in connection with component design features, manufacturing process and mechanical properties can intensify the damage (Bedowski, 2014) . One such component prone to fatigue damage is the brake disc. Fatigue in the form of thermal fatigue is a problem of the brake disc as a result of its being subjected to alternating thermal loads (heating and cooling), and constrained in a manner that restricts its free contraction and expansion (Gunther and Klingelhoeffer, 2000).

Conducting fatigue tests can be quite expensive, hence the need to develop models and computer software that can simulate fatigue damage. In the application of Finite element analysis to fatigue life determination, the machine component is simulated under static loads and from the results indications of the fatigue behaviour are extracted. In the application of finite element analysis, local cyclic stress-strain states of the component subjected to fatigue loading are obtained. The stress results obtained in the finite element stress analysis are used in estimating the fatigue life. Generally in the use of finite element method in fatigue analysis the material properties and behaviour of the component are assumed to be linearly elastic, hence the estimates of the component stress are calculated elastically (Conle, 2013). These elastic stress estimates at the critical points are then corrected into elastic-plastic stress and strains using certain correction methods such as Neuber's rule (Conle et al., 1988). The corrected plastic stresses and strains are placed in an overall local hysteresis loop from which fatigue damage is calculated given the maximum and minimum stress-strain of the tips of each hysteresis loop set (Conle, 2013).

Zoroufi (2004) based on experimental work and FEA analysis concluded that the linear FEA plus Neuber's correction method is a feasible approach to life prediction of components especially considering the fact that it does not require more complicated data requirement as the time consuming nonlinear FEA. As the material of interest is grey cast iron it was modelled with the Downing method based on the use of the Smith Watson Topper (SWT) biaxial parameter (Weinacht and Socie, 1987). The fatigue life determination was done using FE-Safe® version 6.5. The stress and strain results obtained from the finite element analysis are used as inputs for the fatigue life estimation using the FE-Safe® software. This is a third party Abaqus® software which has a routine for implementing the described method for fatigue life estimation. Figure 5.6 presents the FE thermal-mechanical modelling approach used for the modelling of the brake disc thermal fatigue life.

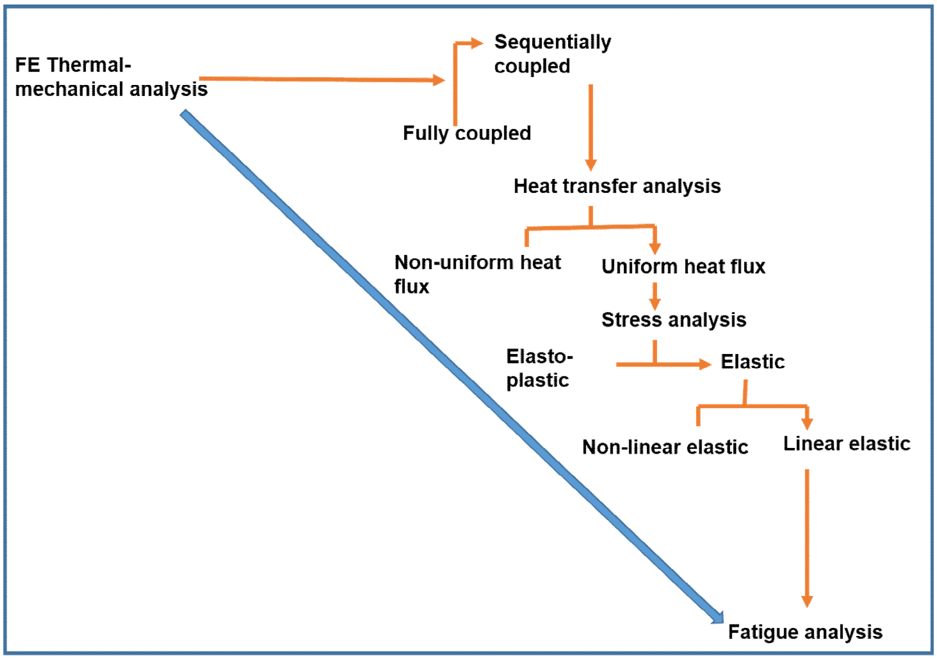


Figure 5.7 FE Thermal-mechanical adopted modelling procedure for fatigue life estimation

5.5.1 Finite Element Thermal Modelling Methodology for a Brake Disc

This section presents a linear elastic Finite element thermal analysis of a vented brake disc. The purpose of this section is to identify the part of a nominal vented brake disc with the worst thermal fatigue life. The thermal fatigue life as used here refers to the time to crack initiation. The sequentially coupled thermal-stress analysis method is used in this study. Using this method a transient heat transfer analysis is done to determine the transient temperature response on the application of a heat flux on the braking surfaces of the disc. The obtained temperature profile is then used as a thermal load input for a subsequent stress analysis. The stress and strain results obtained from the FE stress analysis are next used for estimating the thermal fatigue life of the brake disc with respect to time to crack initiation. The single stop braking mode is selected for the braking cycle. For a single stop braking mode elastic stresses and strains are completely reversible and non-cumulative (Koetniyon, 2000), so the brake is investigated for only one cycle of braking. This involves bringing the vehicle to a complete stop from an initial vehicle speed.

To carry out the FE simulation the applied heat flux on the brake friction surface has to be determined. This is done to simulate the heat generated on the friction surfaces due to the friction between the brake pads and the rotor on braking. To do this typical braking data for a complete stop braking mode were used in this study (Sarip, 2013). The vehicle is brought to a stop from an initial velocity of 28m/s with a wheel rotational speed of 93rads/s. The initial disc temperature was set at 20 ^{0}C. A braking force was applied to bring the vehicle to a stop at a braking time of 4 seconds, which in an actual car is equivalent to a deceleration of 7m/s^2. During braking the heat flux is inputted through the disc friction surface during the 4s of braking. During this period the heat transfer by conduction as result of the friction is greater than the convective heat effects (Limpert, 2011), so convective heat transfer can be neglected at this stage. For a single stop braking mode, the cooling period has no significant influence on the thermal behaviour of the brake disc, hence a cooling period is not included in the analysis (Limpert, 2011).

The applied heat flux can be estimated using basic energy considerations which are as presented in Koetniyon (2000) and Limpert (2011). The heat flux estimation begins with the consideration of a vehicle of mass, *M* decelerating on a level surface from a higher initial velocity, V_i to a final velocity, V_f. On the assumption that all the kinetic energy of the vehicle is transferred to heat, the vehicle braking energy would then be given as:

$$E_{brake} = (\frac{M}{2})(V_i^2 - V_f^2)(\frac{I}{2})(\omega_i^2 - \omega_f^2) \quad \textbf{(5-3)}$$

Where *I* is the mass moment of inertia of the rotating parts, kg/m^2

ω_i is the initial angular velocity of the rotating parts, rad/s

ω_f is the final angular velocity of rotating parts, rad/s

A consideration of where the vehicle comes to a complete stop, $V_f = \omega_f = 0$, the rotating parts given in terms of $V = R\omega$, Eqn.5-3 becomes:

$$E_{brake} = \frac{M}{2}(1 + \frac{I}{R^2M})V_i^2 \quad \textbf{(5-4)}$$

Taking $(1 + \frac{I}{R^2M})$ to be approximately *K*

$$E_{brake} = \frac{KMV_i^2}{2} \quad \textbf{(5-5)}$$

where *K* is a correction factor for the rotating mass

R is the rolling tire radius, m

If the deceleration, *a,* is constant the instantaneous velocity, *V* would be given as:

$$V = V_i - at \tag{5-6}$$

the braking power $P_{braking}$ is then given as:

$$P_{braking} = KMa(V_i - at) \tag{5-7}$$

Based on Eqn.5-7 the braking power is not constant, It is maximum at the beginning of braking (t=0) and zero when the vehicle stops, giving rise to an average braking power.

Given that the time, t_s to stop the vehicle is given as:

$$t_s = V_i/a \tag{5-8}$$

The average braking power can then be expressed as:

$$P_{avg} = KMaV_i/2 \tag{5-9}$$

Limpert (2011) provide typical range for K to lie between 1.05 and 1.15 for passenger vehicles. In this study a value of 1.1 is assumed for K as a correction factor for the total mass for the rotating parts. During braking the force of braking on the front axle is usually higher than that on the rear axle. For this study it is assumed that 60% of the braking force acts on the front axle for the solid brake disc and 60% for the vented brake disc. Based on this a correction factor, X_f of 0.4 and 0.6 for each type of brake disc is introduced into the Eqn.5-9 to account for this distribution of the braking force. During braking the generated thermal energy is partitioned between the brake pads and the rotor. For this research a thermal partitioning factor Kp of 0.95 is assumed (Limpert, 2011). And since the vehicle has two brake pads per wheel, the average braking power applied on each friction surface is then given as:

$$\phi = KK_pX_fMaV_i/2n\,A_F \quad \textbf{(5-10)}$$

Where ϕ is the heat flux density and for this research based on the vehicle braking characteristics it was calculated to be 9036.2 W/m^2 and 1440140.63 W/m^2 for the solid and vented brake disc respectively. The basic vehicle braking data used for calculating the heat flux are as presented in Table 5.7.

Table 5.6 Braking Characteristics

Parameter	Value
Deceleration, a (m/s^2)	7
Correction factor for rotating mass, K	1.1
Heat proportion transferred to disc, K_p	0.95
Braking force fraction of front wheel, X_f	0.4 and 0.6
Mass of vehicle, M (kg)	1500
Number of braking pads per wheel, n	2

The brake disc under consideration is made of grey cast iron with a density of 7100 kg/m^3, Poisson's ratio is 0.26, Young's modulus of 114000 MPa, yield strength of 214 MPa, a conductivity of 53.3 W/m K, a specific heat capacity of 430 J/kg K, and a thermal expansivity of 0.0000011 m/K (Nayar, 1997). Aside from the material properties of the brake disc, other parameters such as the residual stress and surface roughness due to manufacturing were obtained through tests of a sample brake disc used in this study as 34 MPa and 0.24 µm respectively. These parameters were included in the analysis to improve the prediction ability of the FEA simulation as literature has shown that these influence fatigue life (Ralph *et al.*, 2000). Table 5.8 shows the values for the parameters for estimating fatigue life in this study.

Table 5.7 Fatigue life estimation parameters

Yield strength , σ_f'	Fatigue ductility coefficient, ε_f'	Fatigue strength exponent, b	Fatigue ductility coefficient, c
214 MPa	0.007	0.1176	0.3011

For the work presented here a single stop braking mode of a vehicle is analysed using both a solid brake disc and a front vented brake disc. The sequentially coupled thermal mechanical analysis method using Abaqus® version 6.14 software was used. For the FE model an eight degree symmetric 3D CAD model of a front vented brake disc was used for purpose of symmetry and a ten degree symmetric 3D CAD model for the solid brake disc. The FE model for the thermal analysis for both discs were meshed using linear solid hexahedral elements of type DC3D8. While for the FE stress analysis linear solid hexahedral elements with reduced integration of type C3D8R with hour glass control were used.

To perform FE stress analysis in Abaqus® coupled displacement-temperature 8 nodes solid elements, either with full (C3D8T) or with reduced integration and hourglass control (C3D8RT) as found in the Abaqus® element library can be used. The C3D8T element is known to have both shear and volumetric locking issues which are undesirable (Haddag *et al.*, 2010). The use of reduced integration elements can remove these undesirable effects, as well as bring about a reduction in the FE analysis time of 3-D components. But use of reduced integration elements can lead to severe mesh distortion, and to minimise this hour glass control is recommended (Abaqus User's Guide, 2013). Hence, the selection of a reduced integration element with hour glass control.

To carry out FE simulation a time varying uniform heat flux was applied to the friction surfaces of the brake disc.

5.5.2 Boundary Conditions for Thermal Modelling of Brake Disc

In carrying out the FE thermal modelling of the brake discs certain assumptions are made. These assumptions are as shown below.

1) The material properties required for the heat transfer analysis are assumed constant. Though material properties are known to change with temperature variations, Evans and Newcomb (1961) suggest that at temperatures below 400 ^{0}C the effect of material property change with temperature during braking on the disc thermal behaviour is sufficiently small , so it can be assumed not to be a significant source of error in brake disc thermal modelling. Due to the short braking time of a single stop braking mode adopted in this study, this assumption can be made as the temperature rise is relatively lower.
2) For this study the single stop braking mode is to be modelled, hence the assumption that convective cooling can be ignored. According to Limpert (2011) in a single stop the short braking time is usually less than the time taken for the generated heat to be conducted through the brake rotor, and as such no convective cooling occurs as all braking energy can be assumed to be absorbed by the brake rotor and the pad. Also Newcomb

(1960) in the investigation of temperature behaviour of brake disc at braking concluded that convective losses have little effect on the disc surface temperature during a single stop braking mode.

3) This study assumes a constant temperature for the non-heated surfaces of the brake disc such as the hub. This simplifying assumption is based on the assumption that the contact between the brake hub and its mounting to the vehicle axle has perfect thermal conductive properties. As a result of this the vehicle axle act as a heat sink thereby making the temperature not to deviate significantly from the ambient temperature. This assumption can be considered valid for a single stop braking mode as the heat loss due to convection is very small at the hub compared to the heat conducted into the brake. As a result the hub and other non-heated surfaces can be modelled to be at ambient temperature isolated from the heat transfer through conduction (Noyes and Vickers, 1969).
4) It is assumed that the applied heat flux on the brake friction surfaces is uniform. Due to the high revolution of the brake disc the pad can be assumed to make a ring contact with the brake rotor and as such the heat input around the surface of the disc is assumed uniform (Huang and Chen, 2006).
5) In this study it is assumed that the thermal effects are mainly responsible for the induced stress on the disc as the centrifugal forces due to its spinning and the load due to the pads pressure on the disc are negligible (Medonos, 1983).

6) The brake disc in real life the hub of the disc is rigidly fixed to its mounting to form a connection of high stiffness. To simulate this in the FE stress analysis, all nodes on the hub section of the disc in contact with its mounting faces are constrained fully by means of fixation in all degrees of freedom. The representation of this constraint is as shown in Figure 5.8
7) Circumferential constraints are applied on the brake disc segment due to geometrical symmetry of the vented brake disc to simulate its response as a complete disc. By constraining the brake discs circumferentially as shown in Figure 5.8. Only free movement in the axial and radial direction would be permitted thus making the segment to behave as would a segment of the complete vented brake disc.

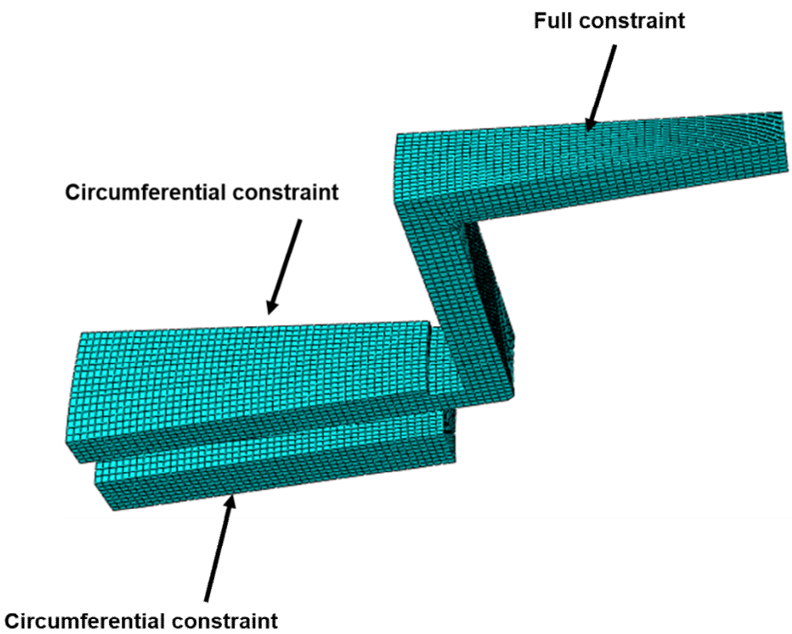


Figure 5.8 Brake model showing modelling constraints for the stress analysis

5.5.3 Mesh Convergence Test

A mesh density sensitivity analysis was carried out to investigate the optimum number of global element size to use for the thermal and stress FE analysis. Four different levels of global mesh sizes were used ranging from 0.5 to 2.5 in increments of 0.5. The highest temperature and stress at a selected point of the FE model at the end of braking were compared for the different mesh sizes to determine the optimal mesh size. Table 5.9 shows the mesh sensitivity analysis results and Figure 5.8 the graphical representation of it.

Table 5.8 Mesh sensitivity analysis

Mesh size	Number of elements	Temperature (^0C)	Stress (MPa)	Percentage change in stress (%)	CPU time for stress analysis (s)
0.5	92782	152.90	109.18	-	1039.60
1.0	21408	153	109.13	0.05	229.20
1.5	10576	153.10	107.41	1.56	113.90
2.0	5896	153.3	105.90	1.43	69.60
2.5	3776	153.6	102.97	2.86	39.50

It can be observed from Table 5.9 that increasing the mesh density from mesh size 1 to 0.5 produces the smallest increment in the stress. This is also observable in Figure 5.8 where it is observed that the convergence occurs between mesh size 0.5 and 1. Based on the mesh sensitivity analysis a mesh size of 1 was selected for the analysis. It gives approximately about the same result with a mesh size of 0.5 while requiring a much relatively lesser CPU time than the 0.5 mesh size.

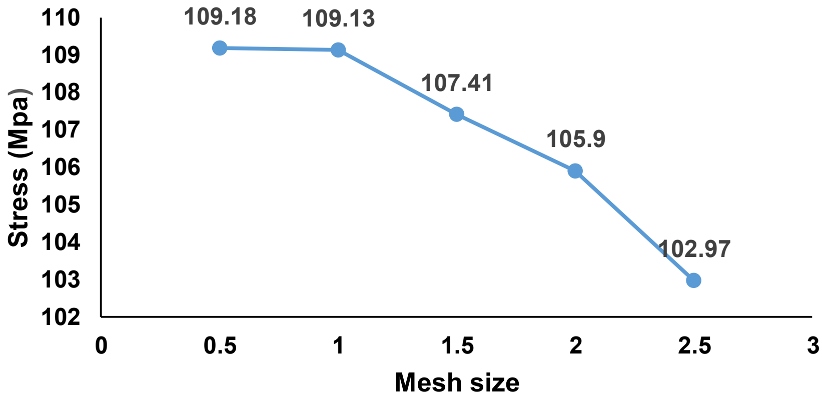
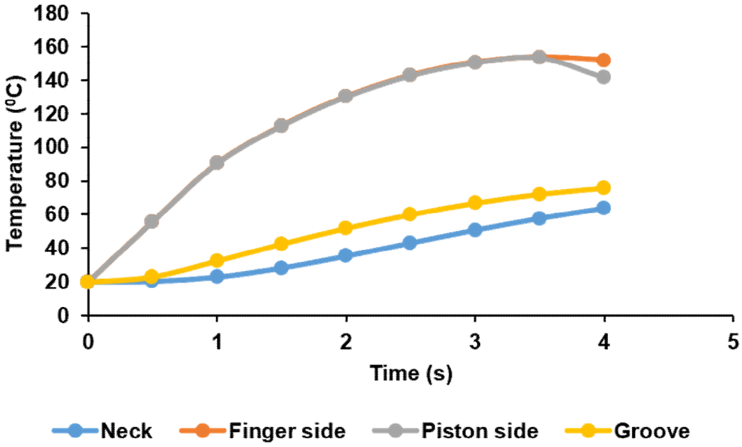


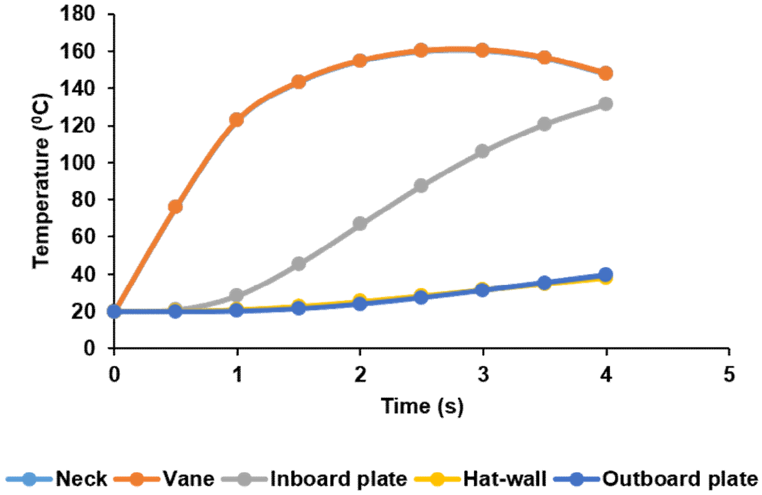
Figure 5.9 Mesh convergence plot

5.5.4 Analysis of Thermal Modelling Results

At the braking process thermal stresses develop as a result of the temperature differential across the disc profile. Figure 5.9 shows the temperature evolution over time at selected points of the solid brake and the vented brake disc. It is observed that the temperature evolution over time shows a similar pattern for the finger and piston side of the solid brake disc. The maximum temperatures at the end of braking occur at these areas at about 150 ^{0}C. At the neck region the temperature is 63.56 ^{0}C. A similar trend is shown also observed with the front vented brake disc. It is observed that the temperature evolution over time shows a similar pattern for the inboard and outboard plate. The maximum temperatures at the end of braking occur at these areas, about 147 ^{0}C. At the neck region the temperature is 39.35 ^{0}C. The high temperature gradient at the neck region for both disc types is due to heat conduction to the hub based on the modelling assumption that the hub area remains at ambient temperature.



(a) Solid brake disc

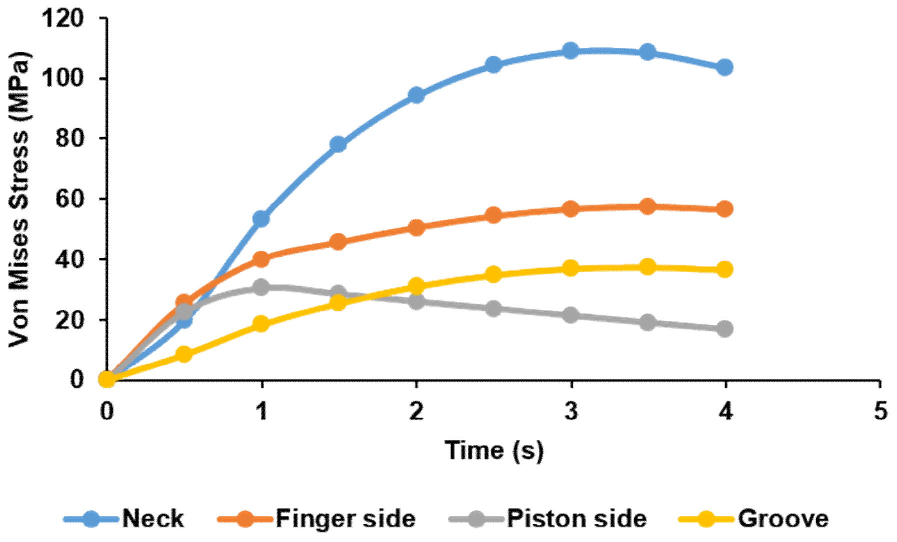


(b) Vented brake disc

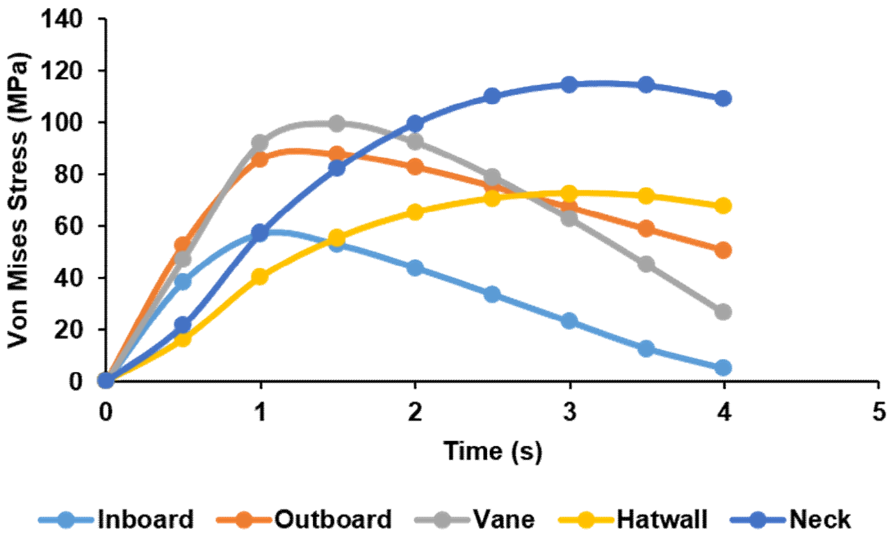
Figure 5.10 Temperature profile at different sections during braking

The highest Von Mises stress at the end of braking occurs at the neck region area at the end of braking for both brake types Figure 5.10. This is because the brake rotor is heated uniformly and expands in relation to the relatively small change or no change in temperature of the top hat section of the brake disc. Coning in the brake disc also contributes to the presence of the high stresses at the neck region. The highest Von Mises stress, 103.49 MPa for the solid brake and 109.14 MPa for the vented brake disc are both less than the yield point of the grey cast iron, 214 MPa. For the heat load applied in this study and the corresponding induced stresses this justifies the use of a linear elastic FE model to simulate the thermal fatigue life of the brake disc.

The contour plot of the thermal fatigue life of the brake disc are as shown in Figures 5.11 and 5.12 for the solid brake disc and vented brake disc respectively. The worst thermal fatigue life is observed to be at the neck area of both brake types, 4.5675 and 4.4492 log life repeats respectively. According to Le Gigan et al. (2016) the use of the Smith Watson Topper criterion for modelling the fatigue life of the brake as which was used in this study would indicate the neck area as being the critical area instead of the disc friction surface. Though a more realistic modelling in terms of heat flux loading would show the friction surfaces to have the worst thermal fatigue life indicating the hot spots. According to Tirovic (2004) the disc/ring top hat transition (neck) area is the crucial area in brake disc design as disc designers can do little about friction surface stresses. Hence, in brake disc modelling and design efforts should be made to reduce stress at the neck area.



(a) Solid brake disc



(b) Vented brake disc

Figure 5.11 Stress profile at different sections during braking

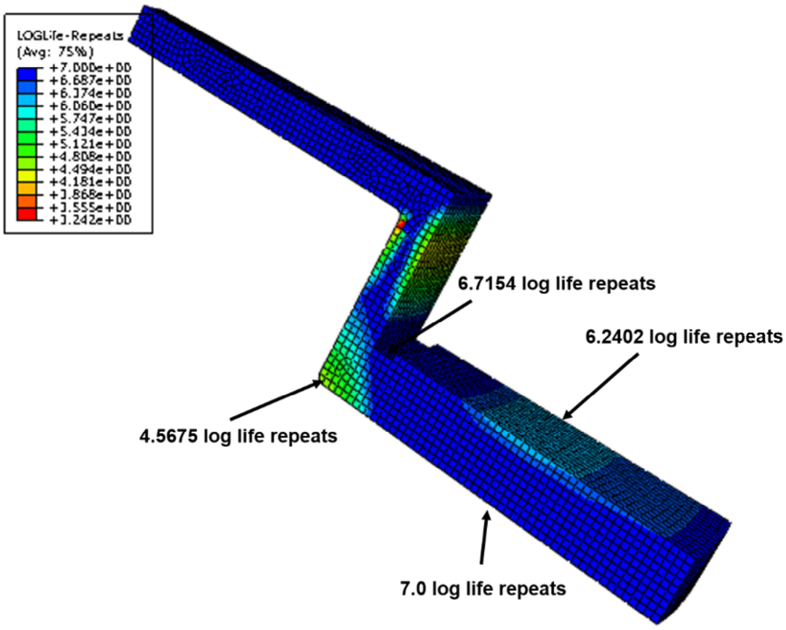


Figure 5.12 Contour plot for fatigue life for solid brake disc

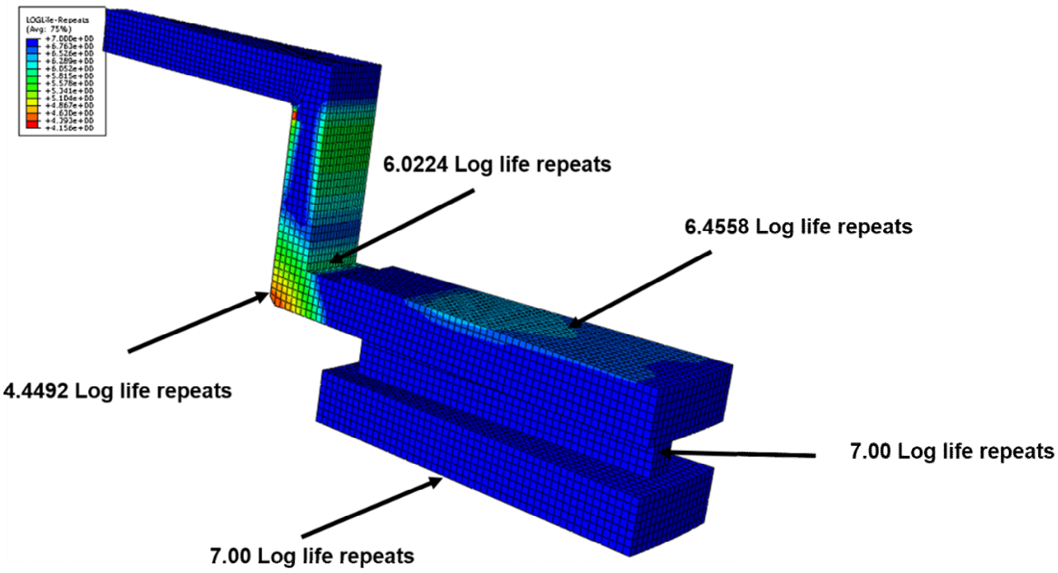


Figure 5.13 Contour plot for fatigue life for vented brake disc

5.6 Chapter Summary

This chapter presented a literature overview of the cast iron material and its properties. In this chapter the reason grey cast iron has found wide use in the making of brake discs is discussed. Material characterisation tests are carried out to determine the hardness of the vented brake disc sample so as to confirm the grade of grey cast iron of the brake disc sample used in this study. The surface roughness and residual stress present in the brake disc sample are also tested for. The surface roughness and residual stress test were carried as they are required in the estimation of the fatigue life of a component. This chapter also included the FE modelling of the thermal fatigue life of the solid and vented brake disc. The sequential coupled method is applied in this study to model the thermal induced stress in the brake disc. The results obtained showed that the use of a linear FE analysis for the thermal modelling was appropriate based on the highest Von Mises stress present in the brake discs at the end of the braking which was less than the yield strength of the grey cast iron. Based on the FE analysis the disc/ring top hat transition (neck) area was determined to be the critical area of the brake disc, and this is also in agreement with findings in brake disc thermal modelling in literature. The next chapter presents a novel uncertainty quantification methodology to determine the uncertainty in predicting the thermal fatigue life at the disc/ring top hat transition area of the brake disc based on the influence of the design features. This area is selected for the uncertainty analysis as it has been indicated to be the critical area of the brake disc design effort should be concentrated on. This novel uncertainty quantification method is demonstrated with the solid brake disc, and validated for general applicability using the vented brake disc.

6. A METHODOLOGY FOR A MIXED UNCERTAINTY ANALYSIS

Computer simulations and the accompanying surrogate models that can be built from computer simulations have found widespread use in analysing and predicting the behaviour of complex systems. Different sources can introduce uncertainties into the representations of complex systems. These sources can be generally categorised into a) uncertainty due to variability in model inputs, b) uncertainty due to modelling assumptions (model form uncertainty), c) uncertainty due to numerical approximations, and d) uncertainty due to limited knowledge of the precise characteristics of the model parameters (Azene *et al.*, 2010; Roy and Oberkampf, 2011).

The objective of this chapter is to develop an uncertainty quantification method that can handle the presence of aleatory and epistemic uncertainty in terms of parameter uncertainty in the use of a surrogate prediction model for estimating thermal fatigue life of a vented brake disc. The proposed method is tested on a case study involving the use of a surrogate prediction model showing the relationship between identified significant geometric design parameters of the brake discs (solid and vented brake discs) and the thermal fatigue life at the disc ring/top hat transition area. The disc ring/top hat transition area is considered the crucial area in brake disc design (Tirovic, 2004). In this thesis the geometric design parameters are treated as aleatory, while uncertainty in the surrogate model output due to the random prediction error is considered to be epistemic as there is no precise information on its characteristics and probability distribution. Once the uncertainties and their sources have been identified, the next step was to develop a method that can quantify the total uncertainty in the model output by treating the design inputs as aleatory while considering the output as epistemic.

This chapter is made of several sections, of which Section 6.1 presents the development of the proposed novel uncertainty quantification method. Section 6.2 demonstrates the application of the proposed uncertainty quantification method to the solid brake disc. In Section 6.3 the results with its discussions on the application of this method to the solid brake disc are presented. Section 6.4 describes the validation of the method for generalisability through the use of a second case study using the vented brake disc. And finally a summary of the chapter is presented in Section 6.6.

6.1 A New Uncertainty Quantification Methodology

The type of uncertainty present should determine the method to be used to characterise the uncertainty. The use of a surrogate model can introduce prediction errors. The estimation of the model surrogate prediction error cannot be done with precision due to lack of complete information of its characteristics. This leads to the introduction of epistemic uncertainty and as such the model error term should be modelled as an interval (Roy and Oberkampf, 2011). There is also parameter uncertainty due to the assumption of a probability distribution of the surrogate model error and hence, the model's output leading to epistemic uncertainty (Ferson and Hajagos, 2004). This assumption is a result of not having enough data to describe the distribution of the surrogate model error and output (in the case study this will relate to data on fatigue life). Aside from these uncertainties, there is the natural variability of the design variables which is aleatory. All these uncertainties combined together lead to the prediction uncertainty in using the developed surrogate model. According to Chojnacki et al., (2010) there is a need to differentiate between these uncertainties and a way to do this is to model aleatory uncertainty with classical probability distributions and epistemic uncertainty by non-probabilistic distributions, and then propagate them by their respective calculus. Chojnacki et al., (2010) inferred that the result of such a propagation is a fuzzy random variable.

The aim of this chapter is to present a new methodology that can incorporate and combine imprecise probability analysis with a traditional probability distribution under a unified framework to evaluate the uncertainty involved in using a surrogate model for predicting the thermal fatigue life of an engineering component as a result of variability of the design variables. Previous methods based on evidence theory in literature have only considered the uncertainties associated with the input variables and their propagation into the output. These methods did not take into consideration other probable sources of uncertainties such as uncertainty due to modelling error, uncertainty due to sparse data etc. (see Chapter 2). The proposed method in this research however considers both the uncertainty in the input variables and their propagation into the output, as well the uncertainty in predicting the model output as result of using a surrogate model to replace actual FE simulations. In this work a unified methodology that integrates the notion of possibilistic mean and variance of a fuzzy number, and probabilistic uncertainty quantification using Taylor series expansion method with Dempster-Shafer theory, to generate a Probability box to represent the lack of knowledge in using the surrogate model for prediction is applied for the first time. This section describes a novel method for handling these uncertainties using this fuzzy approach. Figure 6.1 shows a flow chart of the proposed methodology.

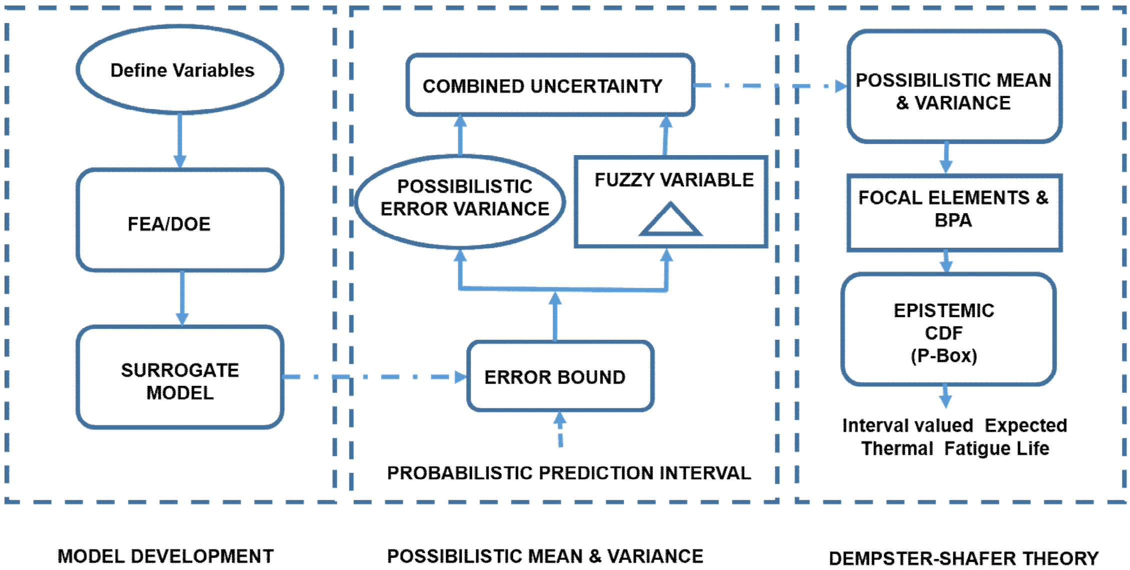


Figure 6.1 Flow chart of the uncertainty based thermal fatigue life prediction methodology.

In developing the uncertainty analysis methodology, in this thesis it is assumed that the error and predicted output value follow a Gaussian distribution. An assumed probability distribution can be used when a P-box is used to estimate the uncertainty. The use of the P-box does not place stringent requirements on the selection of a probability distribution, as the resulting lower and upper CDF of the P-box represents the bounds in which all possible probability distributions will lie. A Gaussian distribution is assumed based on the following two premises: A) the distribution is stable under addition, i.e. the sum of the uncertainties of two independent Gaussian random variables A_1 (σ_1) and A_2 (σ_2) is also a Gaussian variable A_C ($\sqrt{\sigma_1^2 + \sigma_2^2}$), and also that regardless of the distribution of a population as the sample size increases it approaches a

Gaussian distribution. This implies that in the event of more data the distribution has a high likelihood of being a normal distribution. B) The error term is assumed to follow a Gaussian distribution with mean and variance given as $(0,\sigma_{\varepsilon}^{2})$, respectively.

Fuzzy number and its possibilistic mean and variance

Carlsson and Fullér (2001) showed that it is possible to obtain the interval-valued possibilistic mean, crisp possibilistic mean value and variance of a continuous possibility distribution that are consistent with the extension principle (Zadeh, 1978) to deal with fuzzy sets and with the established definitions of expected value and variance in probability theory. They defined these concepts using the notion of lower possibilistic mean and upper possibilistic mean. Carlsson and Fullér (2001) proposed that if A is a fuzzy number is the characteristic function of the crisp interval [a, b], that the possibilistic mean of A is given as:

$$Mean\ (\mathrm{A}) = \frac{\underline{M}\ (\mathrm{A}) + \overline{M}(\mathrm{A})}{2} \tag{6-1}$$

and its variance given as:

$$Variance(A) = \frac{1}{2}\int_0^1 \alpha\left(\overline{A} - \underline{A}\right)^2 d\alpha \tag{6-2}$$

With

$$\underline{M}\ (A) = 2\int_0^1 \alpha\underline{A}\, d\alpha \ \text{ and } \ \overline{M}\ (A) = 2\int_0^1 \alpha\overline{A}\, d\alpha. \tag{6-3}$$

And where the fuzzy number is a triangular fuzzy number described by a triplet of single numbers [a, b, c] with “b” being the mode and a > 0 and c> 0 the left and right width of the fuzzy number respectively, the possibilistic mean is then given by Hao, Liu, and Wang (2008) as:

$$M(A) = \frac{1}{4}(a + 2b + c) \qquad \textbf{(6-4)}$$

And the variance determined as:

$$Variance(A) = \frac{1}{24}(c - a)^2. \qquad \textbf{(6-5)}$$

where $\underline{A}$ and $\overline{A}$ are referred to as the lower and upper bounds of the alpha cut (α-cut) of the fuzzy number respectively. The alpha cut of a fuzzy set is defined as the crisp set of elements that belong to the fuzzy set A at least to the degree alpha (α). The statement is expressed mathematically as shown in Eqn.6-6 (Zimmermann, 2011).

$$A_\alpha = \{x: \mu_A(x) \geq \alpha\}\ where\ \alpha \epsilon [0,1] \qquad \textbf{(6-6)}$$

Dempster-Shafer basic probability assignment, belief and plausibility functions

The basic probability assignment, belief and plausibility are primitive fundamental functions of the Dempster-Shafer theory. The Basic Probability Assignment (BPA) usually represented as “m(A)” is a measure of the degree of evidence that the element in question belongs exactly to a set, say “A” but not to any subset of “A” (Klir and Wierman, 1999). The BPA defines a mapping of the power set to the interval [0, 1]. The formalism for the Dempster-Shafer theory is as follows:

Consider X to be the universal set given as X={x_1, x_2, x_3, … , x_n}.

For this universal set X the frame of discernment is the power set P(X) which is the set of all possible sub sets of X including the null set and can be represented as

$P(X) = \{ \emptyset, \{x_1\}, \{x_2\}, \{x_3\}, \ldots,\{x_1, x_2\},\ldots,\{x_1, x_n\},\ldots,\{x_1, x_2, x_3, \ldots, x_n\}\}$

In the Dempster-Shafer theory a belief mass, i.e. the basic probability assignment (PBA) is assigned to each subset of the power set such that the two axioms of the theory of evidence are satisfied which are:

$$\mathrm{M}(\emptyset) = 0 \qquad \textbf{(6-7)}$$

$$\sum_{A \epsilon P(x)} m(A) = 1 \qquad \textbf{(6-8)}$$

Whereby the focal elements in P(x) with m ≠ 0 give the belief or Dempster-Shafer structure of X. Applying the BPA the other evidential functions of belief and plausibility can be determined, and the Dempster-Shafer structures converted to probability bounds (Ferson *et al.*, 2003). Belief and Plausibility are two non-additive continuous measures. In the Dempster-Shafer theory the total degree of belief in A is described using the belief and plausibility functions as shown below:

$$Bel(A) = \sum_{B \backslash B \subseteq A} m(B) \qquad \textbf{(6-9)}$$

$$Pl(A) = \sum_{B \backslash B \cap A \neq \emptyset} m(B) \qquad \textbf{(6-10)}$$

with the belief and plausibility related to each other by the expression:

$$Pl(A) = 1 - Bel(\overline{A}) \qquad \textbf{(6-11)}$$

From the BPA the other evidential functions of belief and plausibility are determined, and the Dempster-Shafer structures converted to probability bounds (Ferson *et al.*, 2003).The interval between belief and plausibility bounds give the epistemic uncertainty, with the probability of proposition, "A" being bounded as *Bel (A) ≤ P (A) ≤ Pl (A)*. To obtain the BPA in this paper, the method proposed by (Ali and Dutta, 2012) is adopted. In this method the fuzzy variable is discretised by considering a finite number of equally spaced alpha cuts, to give a family of nested intervals. The focal elements are obtained using the alpha cuts as:

$$\alpha A_i = \{x: \mu_A(x) \geq \alpha_i\} = [\alpha A_{iLower}, \alpha A_{iUpper}], \ \ (i = 1,2,3, \dots \dots \dots . n)$$

Where $\alpha \in [0\ 1]$

and the basic probability assignment (BPA) is then given as:

$$m(\alpha A_i) = \frac{1-\alpha_i}{\sum_{i=1}^{n}(1-\alpha_i)} \quad \textbf{(6-12)}$$

Based on the derivations of the above formalisms, the epistemic uncertainty can be determined through the following steps as proposed as part of this new methodology:

(i) Determine the estimate of the surrogate model error bound in interval form from the prediction interval which is then expressed as [0, a]. From the obtained fuzzy interval determine the possibilistic variance of the error term as σ_a^2 based on the earlier assumption that the error term follows a Gaussian distribution with mean and variance given as (0,σ_ε^2), respectively.

(ii) Express the model output as a triangular fuzzy number with the predicted value "A" as the mode and the left and right width obtained using the upper bound "a" obtained from the error bound. The triangular fuzzy number can then be expressed as [A-a, A, A+a]. The possibilistic mean and variance of the obtained triangular fuzzy number is M_r and $\sigma^2{}_r$, respectively.

(iii) From the quantified uncertainty components of each identified uncertainty source, calculate the combined uncertainty using the method of quadrature. The following expression gives the combined uncertainty:

$$\sigma_E = \sqrt{\sigma_\varepsilon^2 + \sigma_r^2} \quad \textbf{(6-13)}$$

The mean is given by the estimated model possibilistic mean obtained in (ii).

(iv) The uncertainty in the model output, σ_F due to the aleatory uncertainty present in the model's input, is estimated by taking the square root of the variance obtained from using the Taylor series approximation. With the inclusion of the uncertainty due to input parameter variability, the total uncertainty for the model prediction is then given as:

$$\sigma_T = \sqrt{\sigma_\varepsilon^2 + \sigma_r^2 + \sigma_F^2} \quad \textbf{(6-14)}$$

Note that the result of the propagation of the aleatory uncertainty due to the input parameter variability and the possibilistic uncertainty is a fuzzy variable (Chojnacki et al., 2010).

(v) Determine the focal elements with their corresponding basic probability assignments of the obtained Gaussian fuzzy number without and with the inclusion of the input parameter aleatory uncertainty. Based on the focal elements and basic probability assignments plot, the Probability box for the belief and plausibility functions to give the epistemic cumulative density functions (ECDF) for both scenarios.

6.2 Application of Proposed Method to the Solid brake Disc

The following sections demonstrates the application of the proposed uncertainty analysis method using the solid brake disc as the case study component

6.2.1 Development of the Prediction Model for the Solid Brake disc

This section deals with the development of a parametric mathematical model to relate the influence of geometric design variables on the thermal fatigue life expressed as log-life repeats at the hat-friction plate corner of a vented brake disc. The thermal fatigue life as used in this thesis refers to the time to crack initiation. Computer simulations using a design of experiment was used for the development of a surrogate model to represent this relationship. A design of experiment approach using Computer Assisted Engineering (CAE) was utilized.

To achieve the purpose of the work which is a study of the influence of geometric design features on brake disc thermal life, an integrated CAE/DOE approach incorporating finite element analysis was used. The research method involved a simulation of the thermal stresses on the brake discs as result of brake application from which the fatigue life is then determined. The thermal stress and fatigue life of the brake discs are determined through use of finite element analysis a numerical method.

The procedure for obtaining the fatigue life is as described in chapter five. A combination of Taguchi method of experimental design and Latin hyper cube design were used to determine the relative significance of geometric design parameters on the fatigue life of the brake disc, as well as carry out an uncertainty and sensitivity analysis of the thermal fatigue life of the brake discs to these parameters. The area of interest for the vented brake disc under study is the disc ring/top hat transition area. Based on the FE modelling and thermal fatigue life estimation as shown in chapter five, as well as based on literature (Tirovic, 2004) this region of the vented brake disc is considered as the design crucial region.

In this chapter a design of experiment methodology to make an otherwise deterministic FEM experiments become probabilistic is used. A series of computer experiments are carried out using design of experiments methodology so that the data can be obtained and analysed in a manner that valid and objective statistical conclusions can be made. The simulation experiments used in this study are deterministic and so cannot be efficiently analysed statistically by classical design of experiment methods, as they there is no randomness in them (Santner et al., 2013). Space filling designs fulfil this condition of introducing randomness into computer experiments by covering the design space as evenly as possible, so they can be evaluated statistically (Wang and Shan, 2007). Two design of experiment methods are used sequentially to achieve the objective of the case study, Taguchi method and Latin hyper cube design of experiments. Seven design features of the solid brake disc were selected for analysis based on previous work (Okamura and Yumoto, 2006; Sarip, 2012). The Taguchi method was used initially to screen the design features so as to identify the significant design features that should be analysed further. When the input parameters are relatively many, screening experiments are used to select only those parameters that are influential to the model output (Kleijnen, 2005).

Taguchi method identifies those factors that minimise product variability by determining the significance of the factors to product performance (Green and Johrendt, 2010; Simpson and Chen, 2010). After screening with the Taguchi method, Latin hyper cube design of experiments which is a space filling design method was used to analyse and develop a surrogate model with those design features that were identified to be significantly influential to the thermal fatigue life of the brake disc. Latin hypercube designs introduces randomness into deterministic experiments and so provide a means for increasing the statistical efficiency of computer experiments (Santner et al. 2013; Hora and Helton, 2003). The use of Latin hyper cube sampling as the sampling method in a design of experiment rather than simple random sampling is that its use results in estimates of the output that have a higher precision or smaller variance and thus require significantly fewer number of sample runs for same level of precision of using random sampling (Hora and Helton, 2003).

6.2.2 Taguchi Method

Taguchi method which has found wide application is a design of experiment method that is used for minimizing product performance variation, and for getting the performance characteristic as close as possible to the targeted mean. Taguchi method is based on orthogonal array (OA) experimental matrix. The use of OA causes a reduction in the variance for the experimental runs resulting in optimum setting of the product/process parameters. Coupled with the use of OA, Taguchi proposed the analysis of product variation using an appropriately selected measure called Signal – to - Noise ratio (SNR) which is derived from the quality loss function (Phadke, 1995) and can be used as the objective function for optimisation purposes. An advantage of the SNR is that it can reflect the variability in the response, and does not induce unnecessary complications such as control factor interactions (Phadke, 1995).The use of Taguchi analysis of the SNR involves three kinds of quality characteristics; smaller the better, larger the better, and nominal the better.

Larger the better,

$$S/N = -10log10\left[\frac{1}{n}\sum_{i=1}^{n}(1/Y_i^2)\right] \quad \textbf{(6-15)}$$

Smaller the better,

$$S/N = -10log10\left[\frac{1}{n}\Sigma_1^{\mathrm{n}}(\mathrm{Y}_{\mathrm{i}}^2)\right] \quad \textbf{(6-16)}$$

Nominal the best,

$$S/N = -10log10(S_i^2/Y_i^2) \quad \textbf{(6-17)}$$

where *n* is the number of experiments and Y_i the measured *i*th quality, which is response indicator.

A schematic diagram of the vented brake disc showing the design features is as shown in Figure. 6.2. The geometric parameters to be evaluated in this study using the Taguchi method are shown in Table 6.1. To evaluate these parameters initially using Taguchi, three levels are chosen for each, and so an experimental layout of L_{27} orthogonal array was selected. For this study the larger the better Taguchi characteristics was selected as a most obvious aim would be to obtain the design that gives the best life. Table 6.2 shows the L_{27} orthogonal array in which 27 runs are carried to determine the influential geometric parameters.

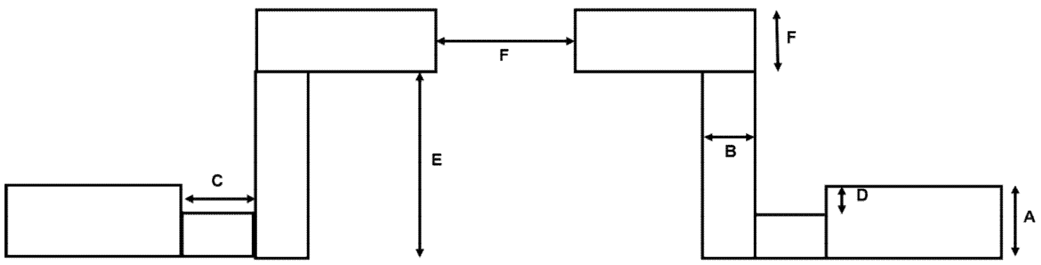


Figure 6.2 Solid brake disc sketch showing design features

Table 6.1 Geometric parameters and their levels

Designation	Factors	Levels		
		1	2	3
A	Disc thickness, mm	8	10	12
B	Hat-wall thickness, mm	5	6	7
C	Undercut thickness, mm	2	4	6
D	Undercut depth	1	2	3
E	Effective offset, mm	20	35	50
F	Hat , mm	5	7.5	10
G	Centre hole diameter, mm	55	60	65

Table 6.2 L27 – orthogonal array of the simulation runs and response values

A (mm)	B (mm)	C (mm)	D (mm)	E (mm)	F (mm)	G (mm)	Fatigue life (Log life repeats)
8	5	2	1	20	5	55	4.3654
8	5	2	1	35	7.5	60	4.0848
8	5	2	1	50	10	65	4.4451
8	6	4	2	20	5	55	4.0511
8	6	4	2	35	7.5	60	3.8736
8	6	4	2	50	10	65	3.8706
8	7	6	3	20	5	55	4.1937
8	7	6	3	35	7.5	60	3.8591
8	7	6	3	50	10	65	3.8599
10	5	4	3	20	7.5	65	4.4547
10	5	4	3	35	10	55	4.4531
10	5	4	3	50	5	60	4.4670
10	6	6	1	20	7.5	65	4.5811
10	6	6	1	35	10	55	4.4680
10	6	6	1	50	5	60	4.4837
10	7	2	2	20	7.5	65	4.4465
10	7	2	2	35	10	55	4.5385
10	7	2	2	50	5	60	4.6755

A (mm)	B (mm)	C (mm)	D (mm)	E (mm)	F (mm)	G (mm)	Fatigue life (Log life repeats)
12	5	6	2	20	10	60	4.8691
12	5	6	2	35	5	65	4.9246
12	5	6	2	50	7.5	55	4.9508
12	6	2	3	20	10	60	5.2726
12	6	2	3	35	5	65	5.1807
12	6	2	3	50	7.5	55	5.2076
12	7	4	1	20	10	60	5.2761
12	7	4	1	35	5	65	5.0328
12	7	4	1	50	7.5	55	5.0663

6.2.3 Taguchi Results and Discussions

Table 6.3 shows the analysis of variance (ANOVA) results for the SN ratios for the fatigue life of the brake disc for the eleven design parameters based on the larger the better Taguchi quality characteristic measure. The analysis was carried out at a confidence level of 95%. Parameters with a P-value less than 0.05 are considered to have significant influence on the fatigue life. Phadke (1995) recommends that for further analysis, the significant parameters should be selected from the analysis of variance of the SN ratios. The factors that have significant effect on the SN ratios are thus selected as the significant factors.

Table 6.3 Analysis of variance of SN Ratios for solid brake disc

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Disc thickness	2	17.1677	17.1677	8.58384	224.19	0
Hat-wall	2	0.0141	0.0141	0.00706	0.18	0.834
undercut thickness	2	0.982	0.982	0.49098	12.82	0.001
Undercut depth	2	0.5611	0.5611	0.28054	7.33	0.008
Effective offset	2	0.2684	0.2684	0.13419	3.5	0.063
Hat thickness	2	0.1833	0.1833	0.09167	2.39	0.133
Centre hole diameter	2	0.0835	0.0835	0.04173	1.09	0.367
Residual Error	12	0.4595	0.4595	0.03829		
Total	26	19.7195				

Model Summary: S = 0.1957 R-sq. = 97.7% R-sq. (adj) = 95%

For factors that have no significant effect on the SN ratios, their optimal values based on the response of means analysis are selected to become the nominal values. From the analysis of variance (ANOVA) of the SN ratios using the P-values, the significant design parameters are the brake disc thickness, undercut thickness, and undercut depth. The Taguchi method in this study was used to screen out the insignificant design parameters. The parameters that are significant are then used as input values at their given levels, and the insignificant parameters set at their nominal values for the uncertainty and sensitivity analysis that would be carried out subsequently using regression analysis based on a Latin hypercube design of experiments. Figure 6.3 shows a chart for the surrogate modelling procedure for the uncertainty and sensitivity analysis used in this study. The process of using a design of experiment involving a lower order polynomial such as the Taguchi matrix, and ANOVA as a basis of screening is carried out based on (Kleijnen et al. 1995).

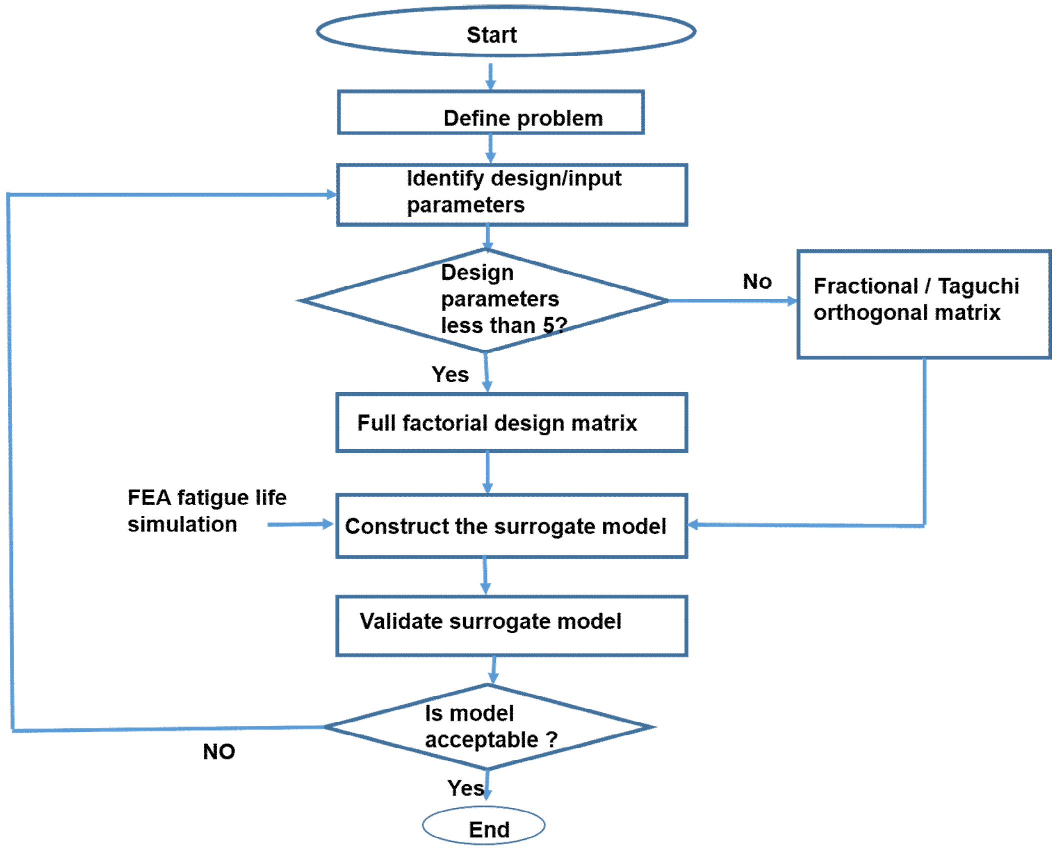


Figure 6.3 Surrogate modelling flow chart for thermal fatigue life determination of the brake disc.

6.2.4 Latin Hypercube Design of Experiment for Surrogate model development

A Nearly Orthogonal Latin Hypercube (NOLH) experimental design (MacCalman, Vieira and Lucas, 2013) was used for the development of a second order polynomial model to represent the underlying relationships of the issues under study. The NOLH design is an orthogonal variant of Latin hypercube design of experiments. For polynomial models orthogonal or nearly orthogonal Latin hypercube designs are considered directly useful as they ensure that the estimates between the first and second order effects are uncorrelated (Gu and Yang, 2013; Evangelaras and Koutras, 2017) The NOLH design is used to develop the surrogate model based on the identified geometric design factors. The selected NOLH design allows for the fitting of a full second order polynomial with nearly uncorrelated coefficient estimates between all the terms, while adequately exploring the interior of the design region (MacCalman et al., 2013). The use of a low order polynomial such as the second order polynomial used in this study makes it easier to comprehend main effects, two-factor interactions and quadratic effects as it makes it easier to display these effects graphically (Kleijnen and Sargent, 2000)

It also permits for the treatment of the input parameters as being random. The use of a the surrogate polynomial model provides the advantage of describing complex simulations with parametric coefficients that are easier to interpret while showing the functional relationship between output responses and the input design variables. Three design variables were identified as being statistically influential to the fatigue life at the hat-friction plate corner of the vented brake disc. The geometric design variables were assumed to be normally distributed with means and standard deviation. The means of the design variables were nominal measurements obtained from a sample vented cast iron brake disc, while the standard deviations are obtained from on an assumed coefficient of variation (tolerance) based on expert opinion. Three brake disc manufacturing firms were contacted on appropriate tolerances expressed in terms of coefficient of variation (COV) for brake disc design and manufacture. Baseline value for this were given irrespective of the geometric design feature of the brake discs. Based on their responses a coefficient of variation (COV) value of 0.02 was assumed in this study for the dimensions of the various features of the brake disc. The tolerance values as obtained from the companies are as shown in Table 6.4. Appropriate lower and upper boundaries for the nominal dimensions were selected to provide a range for the design space so that modelling issues caused by parameter variability can be accounted for. Table 6.5 lists the design variables and their corresponding design space. Table 6.6 gives a sample of the design of experiment matrix from which the surrogate model is developed. The obtained surrogate prediction model is then used for the uncertainty and sensitivity analysis.

Table 6.4 Design tolerance COV for brake disc design and manufacture

Company	A	B	C
COV	0.04	0.02	0.02

Table 6.5 Design variables and their corresponding design space used for model development and uncertainty analysis for the solid brake disc.

Designation	Design variable	Mean(mm)	COV	Design space
x_1	Disc thickness	8	0.02	$8 \leq x_1 \leq 12$
x_2	Undercut thickness	4	0.02	$2 \leq x_2 \leq 6$
x_3	Undercut depth	1	0.02	$1 \leq x_3 \leq 3$

Table 6.6 Sample of NOLH design of experiment matrix with the corresponding thermal fatigue life

Disc thickness (mm) (x_1)	Undercut thickness (mm) (x_2)	Undercut depth (mm) (x_3)	Fatigue life (log life repeats
11.5	5.43	1.23	4.8621
9.02	3.86	2.3	4.4335
10.71	3.57	2.07	4.8910
8	3.19	1.36	4.2582
11.57	5.18	2.57	4.8913

Surrogate model

The surrogate model is approximated using a quadratic response surface. The model for the fatigue life response surface of the brake disc is generated by fitting a second order polynomial that accounts for the model's main, quadratic and interaction effects to the results from the FE simulations. The surrogate model is based on fifteen FE simulation runs. The design points are sampled using a nearly orthogonal Latin hypercube (NOLH) sampling. A stepwise regression was performed to obtain the fit using Matlab® software. This resulted in the following model:

$$Fatigue\ life = 2.585 + 0.24871x_1 - 0.30802x_2 + 0.43004x_3 + 0.022637x_2^2 - 0.10073{x_3}^2 \quad \textbf{(6-18)}$$

Where x_1-x_3 are the input design factors as defined in Table 6.5. To determine the accuracy of the surrogate model, statistical measures of how well the model fits the data have to be evaluated. These measures are the R-squared (R^2) and the R-squared adjusted (R^2 adj.). R^2 and R^2 adj. provide a measure of the percentage of the variation in the response explained by the input variables in the model. The higher the R^2 value the lower variation between the actual and predicted values and hence the better the model. The surrogate model presented in this work gave a R^2 and R^2 adj. values of 0.99 and 0.98 respectively a p-value of 6.734E-08. The obtained surrogate prediction model is then used for the uncertainty and sensitivity analysis.

Model validation for predictive accuracy

A surrogate model has to be validated for predictive accuracy in that the model within its domain of applicability possesses a satisfactory range of accuracy that is consistent with its intended application (Kleijnen and Sargent, 2000). This requires specifying validity measures and their required values such as absolute error and absolute percentage error for the surrogate model in comparison to the actual experimental result (Kleijnen and Sargent, 2000). The developed surrogate model is validated for predictive accuracy by taking eight randomly generated design points within the design space. The FE modelling procedure and thermal loading conditions as previously used in the main simulations are applied for the validation simulations. The validation simulation input data and the predicted thermal fatigue life are as listed in Table 6.7.

Table 6.7 Validation design points and predicted thermal fatigue life of solid brake disc

Run	X_1 (mm)	X_2 (mm)	X_3 (mm)	Surrogate Prediction (Log life repeats)
1	10.6	2.2	1.9	5.1068
2	9.8	5.4	1.1	4.3704
3	8.2	5.8	2.7	4.0263
4	11.8	4.2	2.9	5.0255
5	10.2	4.6	2.3	4.6403
6	8.6	2.6	2.5	4.5217

The prediction results obtained from the randomly generated design points for the six validation runs as shown in Table 6.6 using the prediction surrogate model are compared with actual FE simulation results using those same design points. The comparison is done to determine the percentage error deviation of the predicted results from the actual FE simulation results. The aim of doing this is to determine if the prediction surrogate model gives a good representation of the actual FE simulation generically. The comparison results are as listed in Table 6.8. The results show percentage errors that are below 5% in the comparison of the predicted results to the actual results. A percentage error of 5% is accepted conventionally to be within a statistically acceptable error margin. This indicates that the second order polynomial prediction surrogate model can be used to represent the studied response surface adequately.

Table 6.8 Comparison of results to validate prediction surrogate model

Run	Surrogat Predicted (Log life repeats)	Actual FE (Log life repeats)	Absolute error (%)
1	5.1068	5.0331	1.46
2	4.3704	4.4698	2.22
3	4.0263	4.0136	0.32
4	5.0255	5.0402	0.29
5	4.6403	4.6486	0.18
6	4.5217	4.4905	0.69

6.3 Results and Discussions for the Solid Brake Disc

Using the method outlined in Section 6.3, the required uncertainties were determined for the sample solid disc with nominal dimensions as given in Table 6.9. The uncertainties were determined from the probabilistic prediction interval of [4.2777 4.5871] Log life repeats and the surrogate model predicted life of 4.4324 Log life repeats. Based on their evaluation the possibilistic mean, variance and hence, the uncertainties for the prediction surrogate model with and without input parameter variability were obtained. The obtained uncertainties expressed as a Gaussian distribution were then used in obtaining the intervals at selected alpha cuts for use with their respective BPA's for plotting of the belief and plausibility functions to generate lower and upper intervals to characterise the epistemic uncertainty of the prediction model. The obtained estimates of the various uncertainties are as shown in Table 6.9.

Table 6.9 Nominal dimensions of the sample solid brake disc

Disc thickness (mm)	Hat-wall thickness (mm)	Undercut thickness (mm)	Undercut depth (mm)	Effective offset (mm)	Hat thickness (mm)	Centre-hole diameter (mm)
9	5.1	3	1	25	5.5	55

Table 6.10 Error bound and the various possibilistic uncertainties

Measurement	Value
Error bound (Log life repeats)	[0, 0.1547]
Uncertainty without input parameter variability, $\boldsymbol{\sigma_c}$ (Log life repeats,)	0.10
Uncertainty due to input parameter variability, $\boldsymbol{\sigma_F}$ (Log life repeats)	0.046
Overall uncertainty with input parameter variability, $\boldsymbol{\sigma_T}$ (Log life repeats)	0.11

Using Monte Carlo sampling the joint BPA of the focal elements were propagated to generate the bounds of the belief and plausibility cumulative density functions to form the required P-box. The BPA's are calculated using the method described in Section 3 with the corresponding alpha cuts chosen as α_0=1, α_1=0.8, α2=0.6, α_3=0.4, α_4=0.2, and α_5=0. The BPA of the null set, and the summation of the BPA's of all the subsets of the power set are confirmed to be zero and unity respectively. The focal elements and the corresponding BPA obtained from the fuzzy numbers based on the alpha cuts is presented in Table 6.11.

Table 6.11 Focal elements and their respective BPA

Alpha-cuts	Focal elements		BPA
	Without input parameter variability	**With input parameter variability**	
0.8	[4.4071 4.4577]	[4.4045 4.4603]	0.067
0.6	[4.3800 4.4848]	[4.3747 4.4901]	0.133
0.4	[4.3483 4.5166]	[4.3398 4.5250]	0.200
0.2	[4.3042 4.5606]	[4.2914 4.5734]	0.267
0	[4.0059 4.8589]	[3.9633 4.9015]	0.333

Based on the P-box plots for both determinations, that is with and without including the input parameter variability the epistemic (expected) intervals were obtained. For the epistemic uncertainty without including input parameter variability the expected interval is determined to be [4.2890 4.5760] Log life repeats. The actual FE thermal fatigue life of 4.5675 Log life repeats falls within the epistemic expected interval determined by the use of the proposed method. The obtained expected interval for the uncertainty without input variability is observed to be tighter than that of the probabilistic prediction interval of [4.2777 4.5871] Log life repeats. This is desirable as it reflects a reduction in the uncertainty in the use of this method compared to the probabilistic prediction interval. This is quite observable when the results are expressed in cycles to crack initiation as shown in Figure 6.4.

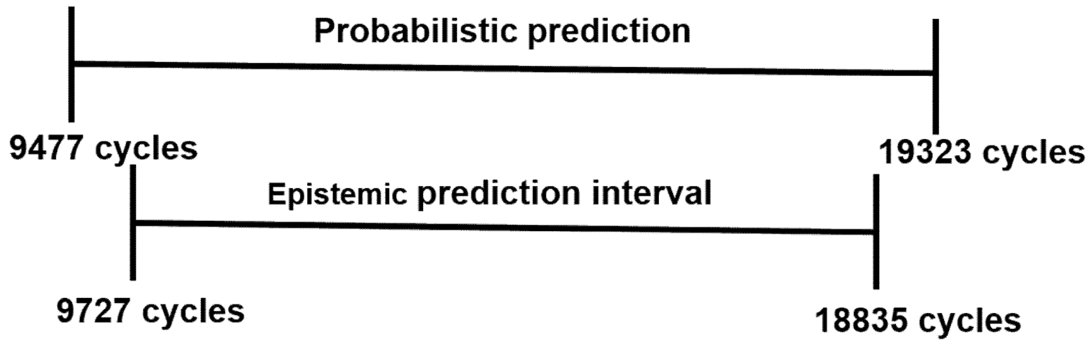


Figure 6.4 Comparison of the probabilistic and epistemic intervals in cycles to crack initiation for solid brake disc

While that for the epistemic uncertainty with input material variability is estimated to be [3.5015, 4.9541] Log life repeats. Figure 6.5 shows the P-box plot for the epistemic uncertainty of the surrogate model without considering input parameter variability. The epistemic CDF for the uncertainty with input parameter variability is seen to be wider than that of the epistemic without input parameter variability as shown in Figure 6.6. This is due to the added uncertainty due to input parameter variability. The inclusion of input parameter variability makes the uncertainty prediction more realistic. The plots are generated using the Imprecise Probability Toolbox for Matlab® (Imprecise probability toolbox for Matlab, 2006). The predicted epistemic life interval of the solid brake disc taking input parameter variability into consideration when expressed in number of cycles to crack initiation gives an estimated time to crack estimation of [9727 - 18835] cycles which has a wider margin compared to that without including input parameter variability [9418 - 19452] cycles. The wide margin shows that there is a wide uncertainty in predicting the life of a brake disc taking input parameter variability into consideration within the limits of the used FE modelling method. This highlights the importance of taking uncertainties into consideration during design. It has been demonstrated basically that for the solid brake disc, the proposed uncertainty quantification method reduces the uncertainty bound of the of the traditional probabilistic prediction interval thereby improving the confidence in the obtained results.

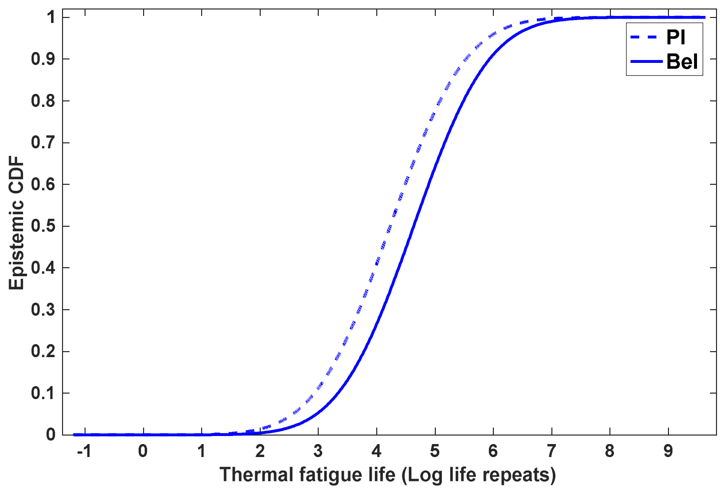


Figure 6.5 Epistemic cumulative density plot for uncertainty without parameter variability

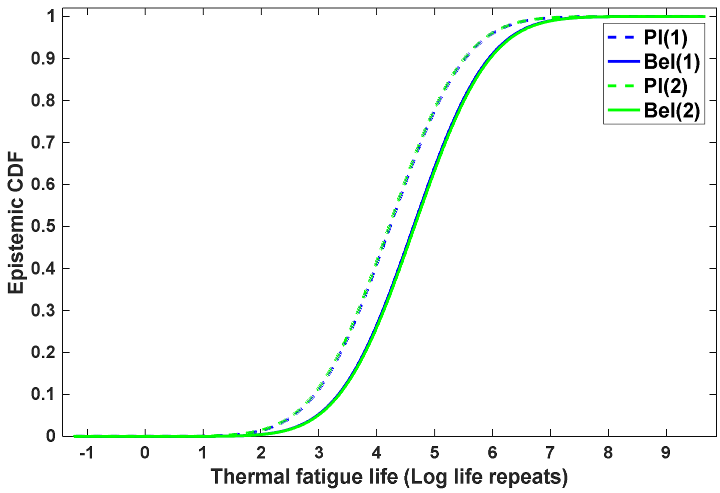


Figure 6.6 Epistemic cumulative density plot for uncertainty without including (1) and with (2) parameter variability

6.4 Case Study: Vented Brake Disc

The presented uncertainty quantification method is tested using the vented brake disc to validate its general applicability. The application of the method to determine the uncertainty in predicting the thermal fatigue life of the vented brake disc at the disc ring/top hat transition area is a follow up of the same procedures used for the solid disc brake. The vented brake disc has eleven design features, which are initially screened to determine the significant design features. A schematic diagram of the vented brake disc showing the design features is as shown in Fig. 6.7. The geometric parameters to be evaluated in this study using the Taguchi method are shown in Table 6.12. For the Taguchi analysis an experimental layout of L_{27} orthogonal array was selected. Three levels are chosen for each design parameter, and for this case study the larger the better Taguchi characteristics was selected. Table 6.13 shows the L_{27} orthogonal array in which 27 runs are carried to determine the influential geometric parameters.

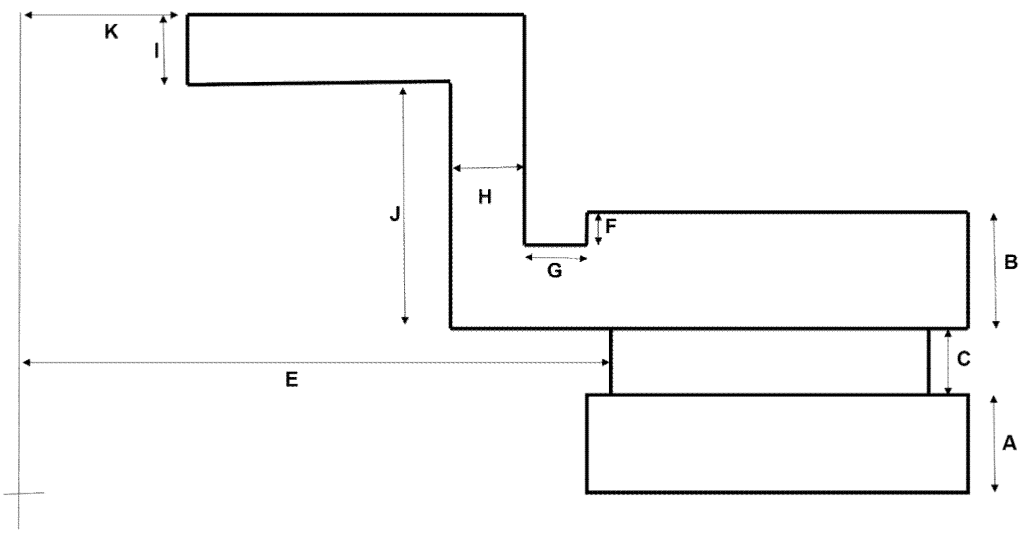


Figure 6.7 Vented brake disc diagram showing design features

Table 6.12 Geometric parameters and their levels for vented brake disc

Designation	Parameter	Level1	Level 2	Level 3
A	Inboard plate	5	7	9
B	Outboard plate	5	7	9
C	Vane height	4	6	8
D	Vane thickness	3	4	5
E	Vane inner radius	78.5	80.5	82.5
F	Undercut depth	1	2	3
G	Undercut thickness	2	4	6
H	Hat-wall thickness	4	6	8
I	Hat thickness	4	6	8
J	Effective Offset	15	25	40
K	Center hole radius	25	27.5	30

Table 6.13 Sample of L27 – orthogonal array results

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**J**	**K**	**L**	**Fatigue life (Log life repeats)**
1	1	1	1	1	1	1	1	1	1	1	2.4955
1	1	1	1	2	2	2	2	2	2	2	4.3902
1	1	1	1	3	3	3	3	3	3	3	3.9594
1	2	2	2	1	1	1	2	2	2	3	4.8796
1	2	2	2	2	2	2	3	3	3	1	5.0632

The analysis of variance (ANOVA) of the SN ratios identified five design variables to be statistically influential based on their P-values being less than 0.05. The analysis was done at a 95% confidence level. Table 6.14 provides the results for the ANOVA analysis.

Table 6.14 Analysis of Variance for SN Ratios for vented brake disc

Source	DF	Seq SS	Adj SS	Adj MS	F-Value	P-Value
A	2	13.4267	13.4267	6.71336	63.59	0.001
B	2	6.0252	6.0252	3.01261	28.53	0.004
C	2	1.4315	1.4315	0.71573	6.78	0.052
D	2	1.0731	1.0731	0.53654	5.08	0.080
E	2	0.1688	0.1688	0.08438	0.80	0.510
F	2	3.1404	3.1404	1.57018	14.87	0.014
G	2	2.6171	2.6171	1.30856	12.39	0.019
H	2	0.0349	0.0349	0.01744	0.17	0.853
I	2	0.0953	0.0953	0.04766	0.45	0.666
J	2	5.0164	5.0164	2.5082	23.76	0.006
K	2	0.102	0.102	0.05098	0.48	0.649
Residual Error	4	0.4223	0.4223	0.10558		
Total	26	33.5536				

Model Summary: S = 6.84027, R-sq = 98.7%, R-sq (adj) = 91.8%

Using the same procedure as previously described the non-significant design features were fixed at the nominal value of the obtained sample brake disc. The significant design features of the vented brake disc are the brake inboard thickness, outboard thickness, undercut depth, undercut thickness and the effective offset. Figure 6.8 shows the diagram of the vented brake disc with the significant design features. The significant design features with their corresponding design space and coefficient of variation (COV) is as listed in Table 6.15. A sample of the NOLH design matrix of 39 simulation runs with their respective response used for developing the surrogate model is shown in Table 6.16 where X_1-X_5 are the input design factors as defined in Figure 6.8.

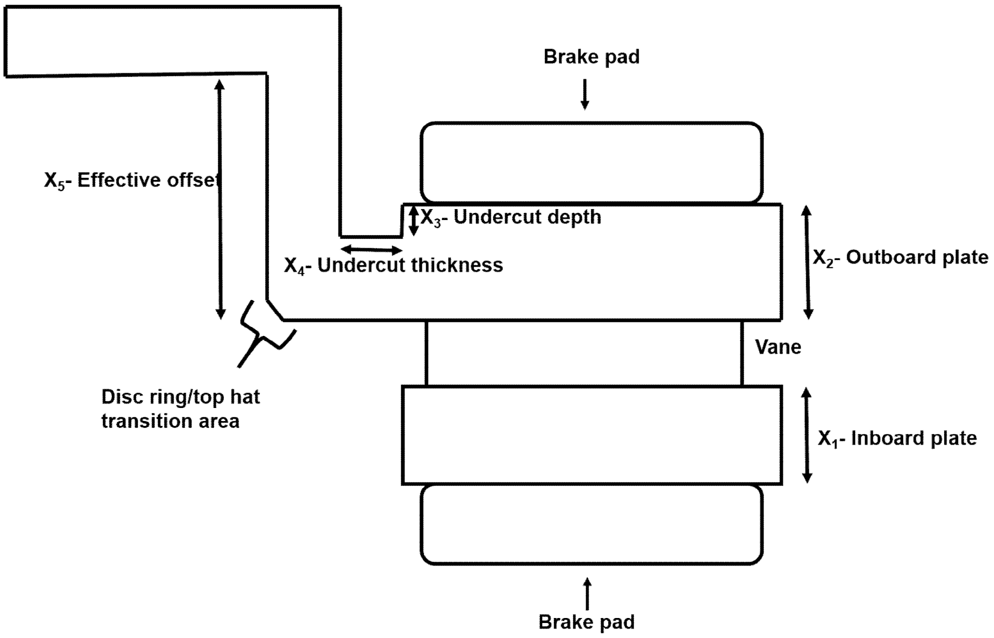


Figure 6.8 Diagram of a vented brake disc showing the significant design features

Table 6.15 Design variables and the corresponding design space used for model development and uncertainty analysis for the sample vented brake disc.

Designation	Design variable	Nominal (mm)	COV	Design space
x_1	Inboard plate	7	0.02	$5 \leq x_1 \leq 9$
x_2	Outboard plate	7	0.02	$5 \leq x_2 \leq 9$
x_3	Undercut depth	1	0.02	$1 \leq x_3 \leq 3$
x_4	Undercut thickness	6	0.02	$2 \leq x_4 \leq 6$
x_5	Effective offset	27	0.02	$15 \leq x_5 \leq 40$

Table 6.16 Sample of NOLH design of experiment matrix showing design inputs and the respective thermal fatigue life

Run	X_1	X_2	X_3	X_4	X_5	Thermal fatigue life (Log life repeats)
1	7.05	7.26	2.13	4.68	25.81	4.4844
2	6.84	8.74	1.66	2.00	29.01	4.6300
3	8.04	8.09	1.18	5.74	35.18	4.2448
4	6.79	5.26	2.76	4.26	31.24	3.5678
5	5.89	5.05	1.23	2.58	29.14	4.2374
6	5.58	7.37	1.39	5.11	26.18	4.2530
7	5.51	7.97	1.12	3.21	38.7	4.4877
8	6.00	6.27	1.76	3.32	34.69	4.3055

Surrogate model: The surrogate model is approximated using a quadratic response surface. The model for the fatigue life response surface of the brake disc is generated by fitting a second order polynomial that accounts for the model's main, quadratic and interaction effects to the results from the FE simulations. The surrogate model is based on thirty nine FE simulation runs. The design points are sampled using a nearly orthogonal Latin hypercube (NOLH) sampling. A stepwise regression was performed to obtain the fit using Matlab® software. This resulted in the following model:

$$Fatigue\ life = 5.5983 - 0.14095x_1 + 1.1816x_2 - 1.7708x_3 - 0.47759x_4 - 0.16741x_5 - 0.098692x_2^2 + 0.0020047{x_5}^2 + 0.024294x_1 \cdot x_2 + 0.072232x_1 \cdot x_3 - +0.045315x_2 \cdot x_3 + 0.023179x_2 \cdot x_4 + 0.075855x_3 \cdot x_4 + 0.016073x_3 \cdot x_5 \quad \textbf{(6-19)}$$

6.4.1 Vented brake disc surrogate model and uncertainty quantification method validation

This section involves the statistical validation of the surrogate model for the vented brake disc and the validation of the proposed uncertainty quantification method. The regression model obtained for the thermal fatigue life at the disc ring/top hat transition area of the vented brake disc is verified to make sure the model is able to give reliable predictions. The uncertainty quantification method is validated for trend and generalisability. The uncertainty quantification method is validated using the vented brake disc to determine if it shows the same trend as the results obtained with the solid brake disc.

6.4.2 Statistical validation

The surrogate model obtained from the regression analysis is analysed to check that the model meets statistical acceptance criteria as reviewed in literature. The criteria used are based on three statistical measures for regression models. These measures of R-squared (R^2) and the R-squared adjusted (R^2 adj.) and the model's P-value. A value of R^2 and R^2 adj. close to 1 indicates a high fit. The higher the value of R^2 and R^2 the lower the variation between the predicted results and the actual, and so a more reliable model. The P-value provides a measure of the statistical significance of the model. A model with a P-value less than 0.05 is considered acceptable. The surrogate model presented in this work gave a R^2 and R^2 adj. values of 0.96 and 0.94 respectively a P-value of 2.14E-14. These values indicate that the surrogate model is statistically acceptable for use in place of the actual FE simulation. The surrogate model is built using the given data, hence the model has to be validated for predictive accuracy. This is to determine that the model within its domain of applicability possesses a satisfactory range of accuracy that is consistent with its intended application. The validation of the model for general ability and consistency shows that the model can be used in place of the actual simulation model as the comparison percentage errors are less than 5%. Table 6.17 shows the validation results matrix.

Table 6.17 Comparison of results to validate prediction surrogate model for vented brake disc

Run	X_1	X_2	X_3	X_4	X_5	Surrogate Predicted (Log life repeats)	Actual FE (Log life repeats)	Absolute error (%)
1	7	5.79	1.68	3.68	18.71	4.7245	4.6694	1.18
2	6.69	8.8	1.39	4.44	16.51	5.0101	5.0634	1.05
3	8.33	6.54	2.8	2.42	37.05	4.7522	4.6716	1.73
4	6.06	6.15	2.48	2.21	19.48	4.522	4.6118	1.95
5	8.95	5.21	1.92	3.81	28.47	4.1741	4.3597	4.26
6	7.08	8.3	2.81	3.75	34.46	4.452	4.6032	3.29
7	8.03	7.89	2.49	3.77	37.11	4.7443	4.7367	0.16
8	8.92	5.86	2.37	5.06	24.1	4.3337	4.3578	0.55

6.4.3 Uncertainty Quantification Method Validation using vented brake disc

The uncertainty quantification methodology was applied on the vented brake disc to determine if the obtained results follow the same trend as that obtained in the result analysis of the solid brake disc. Analysing the results it is seen that the results obtained for the vented brake disc follow a similar trend. The nominal dimensions of the obtained sample vented brake disc are used to demonstrate this (see Table 6.5). Applying the described methodology in section 6.4 the fuzzy Gaussian variable with expected life 4.2278 was obtained with uncertainty without including input parameters variability standard deviation of 0.18, and the uncertainty when input parameter variability is included to be 0.19. The obtained uncertainties expressed as a Gaussian distribution were then used in obtaining the intervals at selected alpha cuts for use with their respective BPA's for plotting of the belief and plausibility functions to generate lower and upper intervals to characterise the epistemic uncertainty of the prediction model.

A million joint BPA of the focal elements were propagated to generate the bounds of the belief and plausibility cumulative density functions to form the required P-box using Monte Carlo simulation. The BPA's are calculated using the method described in Section 7.3 with the corresponding alpha cuts ranging between [0, 1] in incremental steps of 0.2. The BPA of the null set, and the summation of the BPA's of all the subsets of the power set are confirmed to be zero and unity respectively. The focal elements and the corresponding BPA obtained from the fuzzy numbers based on the alpha cuts is presented in Table 6.18.

Table 6.18 Focal elements and their respective BPA of the vented brake disc

Alpha-cuts	Focal elements		BPA
	Without input parameter variability	With input parameter variability	
0.8	[4.1822, 4.2734]	[4.1797, 4.2759]	0.067
0.6	[4.1334, 4.3222]	[4.1282, 4.3274]	0.133
0.4	[4.0763, 4.3793]	[4.0679, 4.3877]	0.200
0.2	[3.9971, 4.4585]	[3.9843, 4.4713]	0.267
0	[3.4601, 4.9955]	[3.4175, 5.0381]	0.333

The epistemic prediction interval obtained for the vented brake disc exhibited the same trend as that of applying the method on the solid brake disc. The actual FE simulated thermal fatigue life of 4.4242 Log life repeats falls within the expected epistemic prediction interval which was obtained to be [3.9700, 4.486] Log life repeats. The epistemic prediction interval is also observed to be tighter than that of the probabilistic prediction interval of [3.9459, 4.5096] Log life repeats. For the purpose of making this difference in both prediction bounds obvious both intervals are expressed in cycles to crack initiation, with their comparison shown in Figure 6.9. This same trend was observed with the solid brake disc. This validates the generic application of the proposed uncertainty quantification method. The combined epistemic CDF plots for uncertainty without including input parameters uncertainty and that of the combined uncertainty (i.e. addition of the input parameter uncertainty and epistemic prediction error uncertainty) also show a similar trend which intuitively should be the expected trend. The combined uncertainty have a wider epistemic prediction interval than that of the uncertainty without including the input parameter variability. The epistemic CDF for both plots is shown in Figure 6.10.

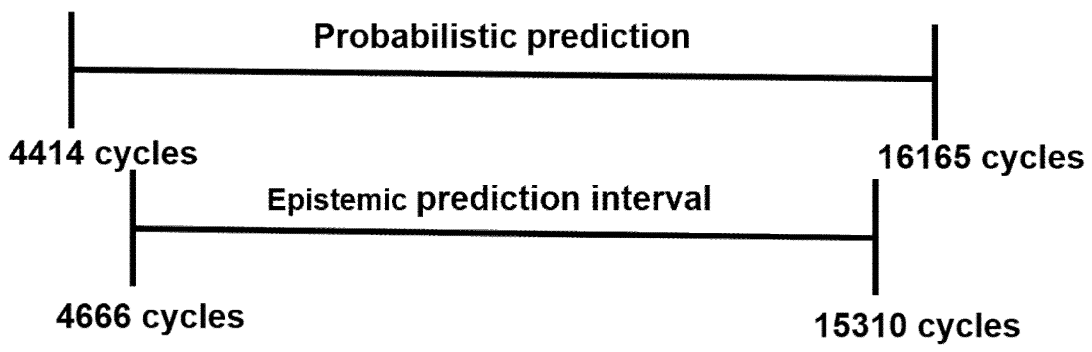


Figure 6.9 Comparison of the probabilistic and epistemic intervals in cycles to crack initiation for vented brake disc

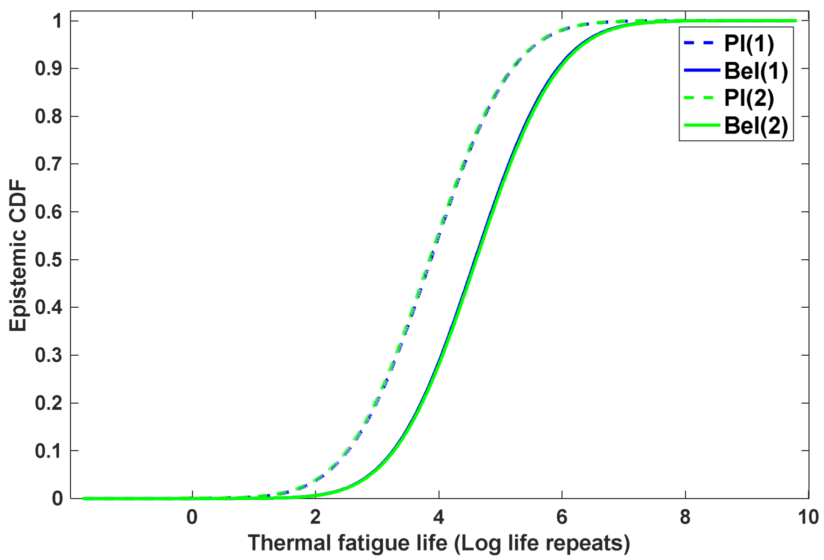


Figure 6.10 Epistemic cumulative density plot for uncertainty without (1) and with (2) parameter variability for vented brake disc

The FE modelling method used here based on the application of a uniform heat flux and the use of the Smith Watson Topper criterion for fatigue life determination gives the neck area of which the disc ring/top hat transition area as the area with the worst thermal fatigue life. This agrees with the work of previous authors on the most highly stressed area in a brake disc based on the adopted method (Koetniyon, 2000; Le Gigan *et al.*, 2016). According to Le Gigan *et al.*, (2016) a more realistic heat flux loading would indicate the friction surfaces as the areas with the worst life. In this study taking the input variability into consideration makes the prediction more realistic. Expressing the prediction in terms of an interval gives the designer what can be described as the worst possible and best possible life the component can attain. The use of a P-box with Dempster-Shafer belief and plausibility functions allows the designer to integrate randomness and imprecision in one single framework. With this method the designer can access the lower and upper bounds of the thermal bounds by specifying predetermined belief and plausibility function values based on experience or designer preference. Based on the meanings of belief and plausibility in Dempster-Shafer theorem, the upper bound gives the maximum value of the expected life.

6.5 Uncertainty Method Validation

The proposed uncertainty quantification method is validated for generalisability by randomly selecting design inputs for different configurations of the solid and vented brake disc. The same procedure used in the uncertainty analysis of the nominal brake discs configurations is applied to these randomly generated designs. The validation is done without including the uncertainty due to input parameter variability. The validation simulation input data and the predicted and actual FE thermal fatigue life for the solid and vented brake discs are as listed in Table 6.18 and 6.19 respectively. The obtained results with respect to the epistemic expected interval for these random vented brake discs are compared with their respective traditional probability prediction intervals. Tables 6.19 and 6.19 show the obtained results with their respective prediction interval comparisons. It can be seen that the epistemic expected intervals are tighter than their respective traditional probabilistic prediction intervals, and the actual FE thermal fatigue life fall within the epistemic prediction (expected) interval. This validates the effectiveness of the method. The proposed uncertainty estimation method can be applied only when it is possible to determine a probabilistic prediction interval.

Table 6.19 Uncertainty methodology validation for solid brake disc

Run	X1 (mm)	X2 (mm)	X3 (mm)	Predicted (Log life repeats)	Actual FE (Log life repeats)	Probabilistic prediction interval (Log life repeats)	Epistemic prediction interval (Log life repeats)
1	10.6	2.2	1.9	5.1068	5.0331	4.9564-5.2571	4.9680-5.2460
2	9.8	5.4	1.1	4.3704	4.4698	4.2222-4.5185	4.2330-4.5070
3	8.2	5.8	2.7	4.0263	4.0136	3.8694-4.1832	3.8820-4.1710
4	11.8	4.2	2.9	5.0255	5.0402	4.8715-5.1794	4.8830-5.1670

Table 6.20 Uncertainty methodology validation for vented brake disc

Run	X1 (mm)	X2 (mm)	X3 (mm)	X4 (mm)	X5 (mm)	Actual FE prediction (Log life repeats)	Probabilistic prediction interval (Log life repeats)	Epistemic prediction interval (Log life repeats)
1	7.00	5.79	1.68	3.68	18.71	4.6694	4.4772-4.9717	4.495-4.954
2	6.69	8.80	1.39	4.44	16.51	5.0634	4.7290-5.2911	4.751-5.270

Run	X1 (mm)	X2 (mm)	X3 (mm)	X4 (mm)	X5 (mm)	Actual FE prediction (Log life repeats)	Probabilistic prediction interval (Log life repeats)	Epistemic prediction interval (Log life repeats)
1	7.00	5.79	1.68	3.68	18.71	4.6694	4.4772-4.9717	4.495-4.954
3	8.33	6.54	2.80	2.42	37.05	4.6716	4.4701-5.0342	4.910-5.013
4	6.06	6.15	2.48	2.21	19.48	4.6118	4.2544-4.7856	4.275-4.765

6.6 Chapter Summary

In this chapter a framework for handling the various uncertainties present in the use of a surrogate prediction model is presented. The uncertainties treated by the method are those due to imprecise knowledge of the model output distribution, and that due to aleatory uncertainty as a result of the variability of the input parameters. A fuzzy based approach integrating possibilistic mean and variance with Dempster-Shafer method is used. This method offers the advantage of giving the uncertainty analyst the flexibility to model in the presence of incomplete information. As in this case not having precise information on what probability distribution to describe a parameter with as a result of surrogate model prediction error and sparse data. The mean and variance of the fuzzy numbers are used as the nominal value from which the epistemic uncertainty is determined using a P- box generated through the application of Dempster-Shafer theory. The method also permits the simultaneous treatment of aleatory and epistemic uncertainty using the additive property of Gaussian distribution. The effectiveness of this method is demonstrated through a case study involving the thermal fatigue life estimation of vehicle brake discs. The obtained epistemic intervals give tighter intervals compared to the intervals obtained with classical probability method. With the use of this method it was demonstrated that the uncertainty due to input material variability contributes significantly to the uncertainty in estimating the thermal fatigue life of the brake disc at the region of interest. Thus highlighting the need for uncertainty estimation in fatigue life design. Understanding the uncertainties in engineering design is necessary as it would enable the design of components that are robust and hence less sensitive to uncertainties. The proposed method offers the possibility of its application to the design optimisation of machine components such as in this case the vented brake disc in the presence of uncertainties. The finite element modelling was done based on simplifying assumptions to the obtained thermal fatigue life. This method is best suited to black box models where it is possible to obtain prediction interval for future predictions. The next chapter presents a sensitivity analysis in which the vented brake disc is used as the case study. Probabilistic sensitivity methods are used to study the influence of each design feature on the thermal fatigue life while simultaneously considering the effect of the other features. As has been shown in chapter one previous sensitivity studies have majorly been deterministic thereby limiting the quality of the sensitivity results obtained.

7 . SENSITIVITY ANALYSIS

Sensitivity analysis is usually preceded by an uncertainty analysis. Uncertainty in the parameters of a model reduces the reliability of the models, hence the need for the simulation results to be tested for their sensitivity to changes in the model. The review of literature in chapter 1 show that majorly sensitivity studies of design influence on a degradation, fatigue of a component has generally been deterministic. The use of deterministic methods have been shown not to present a realistic evaluation of the sensitivity of a response to the input parameters of a model (Saltelli *et al.*, 2008). In this thesis the global sensitivity analysis methods, a stochastic approach for sensitivity analysis are used. The sensitivity analysis is performed using the vented brake disc as a case study. Two different global sensitivity methods that treat sensitivity using different perspectives are used for complimentary purposes. This is done to provide a better insight into the sensitivity of the thermal fatigue life at the disc/ring hat friction area of the vented brake disc to the brake discs design features. The chapter is divided into several sections. Section 7.1 describes the methodology adopted for the sensitivity analysis as carried out in this chapter. Section 7.2 presents the model input and their distributions as well as the pre-sensitivity analysis results to check if correlation and non-monotonicity exists in the design parameters and the model. Section 7.3 presents the sensitivity analysis methods used in this study PRCC and eFAST as well as the results and their discussions. In section 7.4 a general discussion of result for the sensitivity study is presented. And in section 7.5 a summary of the chapter is made.

7.1 Methodology Used for Sensitivity Analysis

The sensitivity of the fatigue life at the hat-friction ring area of the vented brake disc to the five previously identified design features in chapter six are to be determined in this section. A detailed sensitivity analysis shall be carried out to determine the features that contributes the most to the uncertainty in predicting the thermal fatigue life at the region of interest. This will assist in indicating where design efforts should be concentrated on, as in this study for the brake design optimisation. In this study both local and global sensitivity analysis methods are employed to corroborate the results that would be obtained. Scatter plots showing graphical relationship between the input parameters and the output to visually determine their relationship in terms of linearity shall be used as starting point. Next monotonicity plots for each input parameter against the output are obtained by keeping the other parameters fixed at their nominal values and varying the input across the design space (One-Factor-at-a-Time) to check for monotonicity. And finally a detailed sensitivity analysis is carried out. According to Saltelli and Marivoet (1990), Hamby (1994), and Marino et al. (2008) PRCC and SRCC in general are considered the most robust and reliable sensitivity methods in the sampling based methods. For the variance based methods eFAST is considered more efficient and reliable compared to the other methods (Saltelli *et al.*, 2004). In this study the focus shall be on the PRCC and eFAST method as they have been shown to be the most reliable methods for sensitivity analysis. These methods as a result of their different approaches to sensitivity analysis measure two different model characteristics. PRCC measures model monotonicity due to an input when all the linear effects of other variables have been removed from the analysis, while eFAST provides measures for the fractional variance contribution of an input or combination of inputs variability to the variance of the model's output. PRCC is selected for the previously obtained regression model in chapter 7 is non-linear, and PRCC can be used to handle non-linear but monotonic models. Hence, for a robust and complete uncertainty analysis which sensitivity analysis provides support for, Marino et al. (2008) proposes that both sensitivity index be calculated. Chan, Saltelli and Tarantola (1997) provides a general methodology for conducting a sensitivity analysis which shall be adopted in this study. This procedure consist of the following:

i. Define the model and its input and output variables.
ii. Assign appropriate probability distributions to each input parameter.
iii. Generate an input matrix through an appropriate sampling method, evaluating the output.
iv. Carry out a detail sensitivity analysis by assessing the influences of each input parameter on the model's output.

7.2 Definition of Model Input and Output Variables and Their Distributions

The design features are input parameters, while the thermal fatigue life is the output. The design features selected for the sensitivity analysis are the five features which were found to be influential to the disc thermal fatigue life after the screening carried out using Taguchi's method in chapter seven. These features are restated in this section, and they include the Inboard thickness, Outboard thickness, Undercut depth, Undercut thickness and the Effective offset. A uniform probability distribution is assumed due to the sampling requirements for the eFAST sensitivity analysis. This sampling distribution is provided as a range, and can also be used for the PRCC analysis. The variation of the random design variables and their corresponding design space for the sensitivity analysis is shown in Table 7.1

Table 7.1 Design variables and the corresponding design space used for model development and sensitivity analysis.

Designation	Design variable	Mean (mm)	Design space
x_1	Inboard plate	7	$5 \leq x_1 \leq 9$
x_2	Outboard plate	7	$5 \leq x_2 \leq 9$
x_3	Undercut depth	2	$1 \leq x_3 \leq 3$
x_4	Undercut thickness	4	$2 \leq x_4 \leq 6$
x_5	Effective offset	27.5	$15 \leq x_5 \leq 40$

7.2.1 Surrogate Model

The surrogate model obtained in Chapter 6 through use of the Nearly Orthogonal Latin Hypercube (NOLH) experimental design for the vented brake disc used for all subsequent sensitivity analysis in place of the actual simulation runs (see Eqn.6-19).

7.2.2 Test of Monotonicity in Model

The test for monotonicity is required to determine the suitability of given methods for the sensitivity analysis which are best suited for particular models based on the model's attribute. Partial rank correlation provides a measure of the level of monotonicity between a given input parameter and the output of a model. In the use of Partial Rank Correlation Coefficient (PRCC) only monotonic models can be adequately treated by the use of PRCC. The extended Fast Fourier Amplitude Sensitivity Test (eFAST) sensitivity method being model independent can handle both monotonic and non-monotonic models. Thus, for the use of PRCC a monotonic investigation should be carried out initially to determine the suitability of the method for the uncertainty analysis. Depending on the results obtained, modifications can be carried out on the design space where the model does not show monotonicity between the input and output variables to make the model relationship monotonic. To carry out the monotonicity investigation for the model under study, nominal values of all other input parameters are determined and fixed as baseline values, while the input parameter under investigation is varied across its design space. The shape of the plot for the respective outputs made against the varying input parameters while the other parameters are fixed would provide an indication of monotonicity. The monotonic test is carried using the developed surrogate model as this has been shown in the previous chapter to adequately represent the studied response surface. Both methods however assume no correlation between the model input parameters. This requires the use of scatter plots to assess if correlation exists, and if it does appropriate measures are to be taken to remove them.

7.2.3 Analysis of the Scatter and Monotonicity Plots

For this study as has been stated only one outcome measure, the thermal fatigue life of the vented brake disc at the hat-friction plate corner is evaluated against the required design features. Scatter plots of each design feature against the thermal fatigue life are made to check visually the nature of relationship between the inputs and the outputs and also the dependencies between the input variables. The scatter plots are obtained using the raw data in Figure 7.1. The correlation coefficients between the input variables are very small that it can be safely assumed that they are independent of each other. The correlation coefficients are shown in the top left hand corner of each box in the correlation matrix. Though scatter plots provide an indication of the relationship between the output and the individual input variables, they may not be able to adequately capture curvilinear relationships such as non-linear or non-monotonic relationships (Jacoby, 2000). The monotonicity plots as previously described shall be used to confirm if there are non-monotonic relationships between the input variables and the output

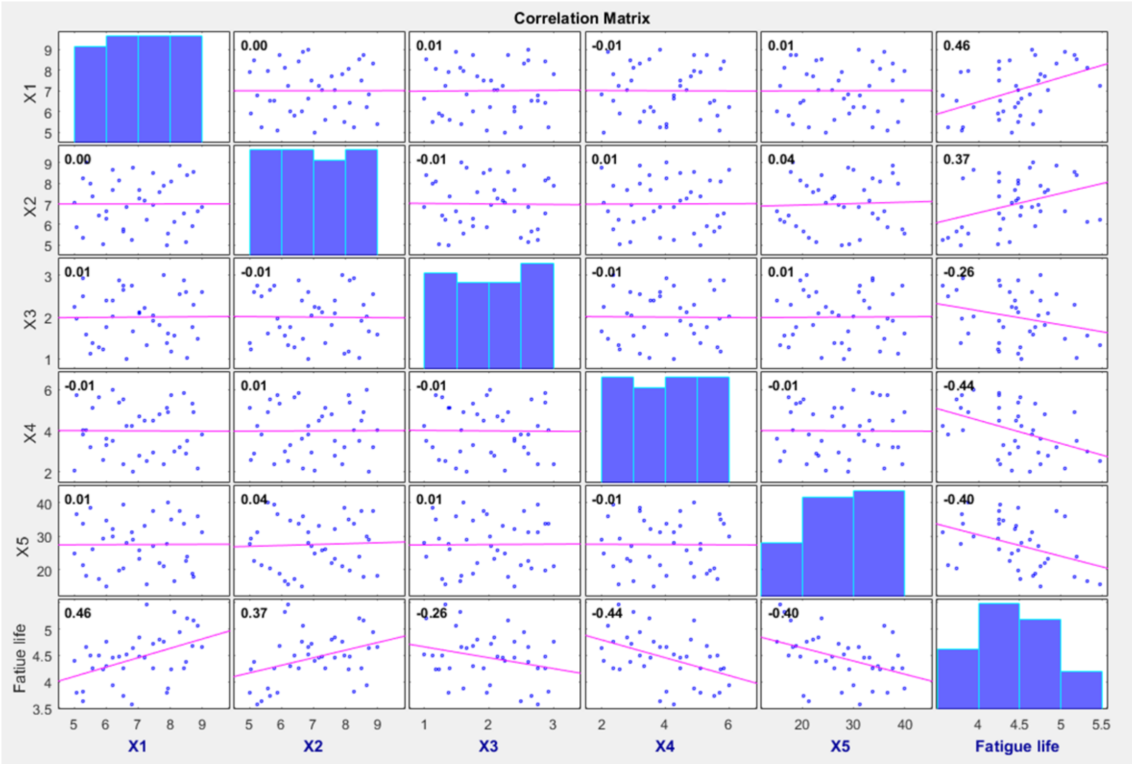


Figure 7.1 Scatter plots for input parameters and output showing correlation matrix

The monotonicity plots are as shown in Figure 7.2 indicate that two of the input parameters X_2 and X_5 show a non-monotonic relationship. As a result of this an adjustment of the selected ranges for X_2 and X_5 has to be made for it to be possible to implement PRCC. To adjust for the range in situations where non-monotonic relationship exists, a consideration is to truncate the range into two possible halves and analyse the truncated halves separately in the PRCC analysis. From the plots of Figure 7.2 the point of inflection for X_2 and X_5 occur at 7 mm and 35 mm respectively. The design space for both X_2 and X_5 are truncated at these points to make the plots monotonic. Two PRCC analysis shall be carried out to take into consideration the truncation of the design spaces of x_2 and x_5 into two halves. Truncating the design space to remove non-linearity or non-monotonicity is necessary as Marino et al. (2008) have shown that misleading conclusions can be made even for simple models when PRCC is used for the sensitivity analysis of non-monotonic relationship between input and output. Since eFAST is model independent as previously highlighted the design space does not need to be adjusted for this analysis. Table 7.2 lists the new sampling intervals on truncation of the design spaces for X_2 and X_5.

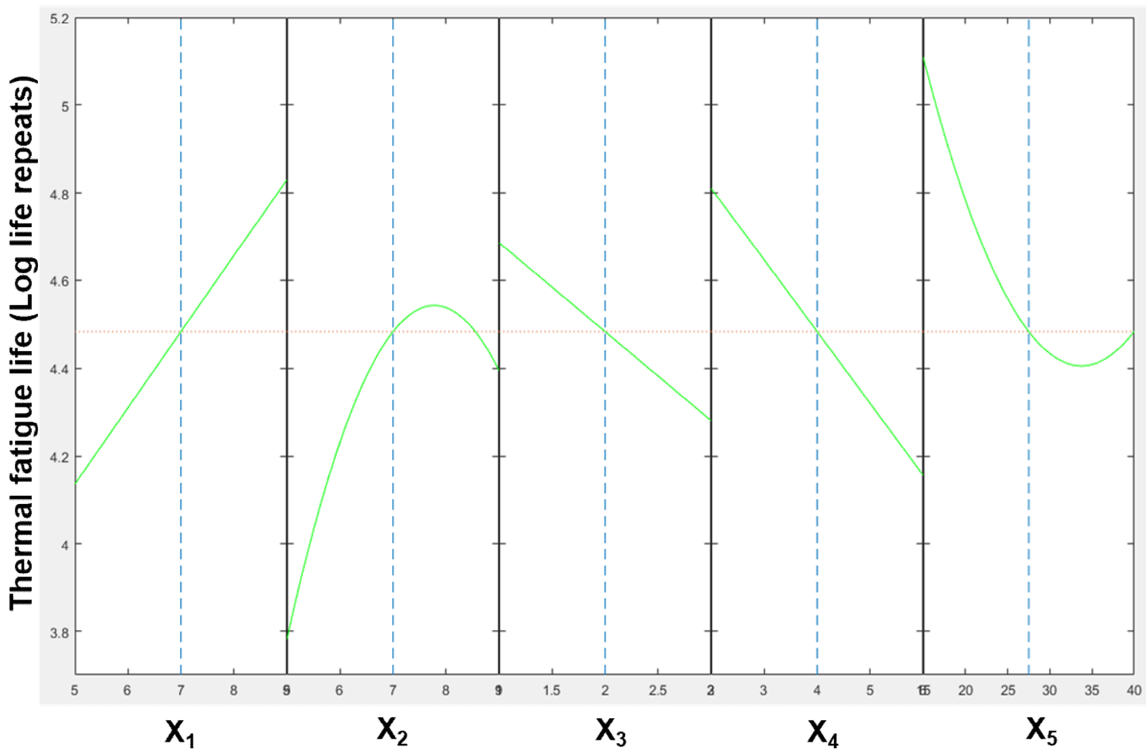


Figure 7.2 Monotonicity plots for design variables

Table 7.2 Adjusted design space of random variables used for PRCC sensitivity analysis

Designation	Design variable	Design space (mm)	
		A	B
X_1	Inboard plate	$5 \leq X_1 \leq 9$	$5 \leq X_1 \leq 9$
X_2	Outboard plate	$5 \leq X_2 \leq 7$	$7 \leq X_2 \leq 9$
X_3	Undercut depth	$1 \leq X_3 \leq 3$	$1 \leq X_3 \leq 3$
X_4	Undercut thickness	$2 \leq X_4 \leq 6$	$2 \leq X_4 \leq 6$
X_5	Effective offset	$15 \leq X_5 \leq 35$	$35 \leq X_5 \leq 40$

7.3 Sampling Procedure for PRCC and eFAST Sensitivity Analysis

Though there exists several sampling methods such as simple random, sampling, importance sampling, or Latin hypercube sampling (LHS) (Marino *et al.*, 2008), LHS is selected as the sampling method for the PRCC analysis as it permits the un-biased estimation of the average model output, and also requires a smaller sample size than simple random sampling to achieve the same level of accuracy (McKay et al., 1979; Helton and Davis, 2003). A sample size of 1000 simulations using Latin hypercube sampling was used for the PRCC analysis in this study. For the LHS/PRCC sensitivity analysis each of the uncertain five input parameters are obtained by the sampling of a uniform probability distribution across their respective design space. Figure 7.3 shows a pictorial description of the PRCC method steps. The eFAST sensitivity analysis was carried out using the SimLab (version 2.2) software for sensitivity analysis. A sample size of N= 1280 was generated using the extended Fast (eFAST) method to achieve an adequate estimation of the sensitivity indices (Saltelli *et al.*, 2008). Using the relationship $N(k+2)$ based on Saltelli's method where k is the number of input parameters, a total run of 8960 model executions were carried out.

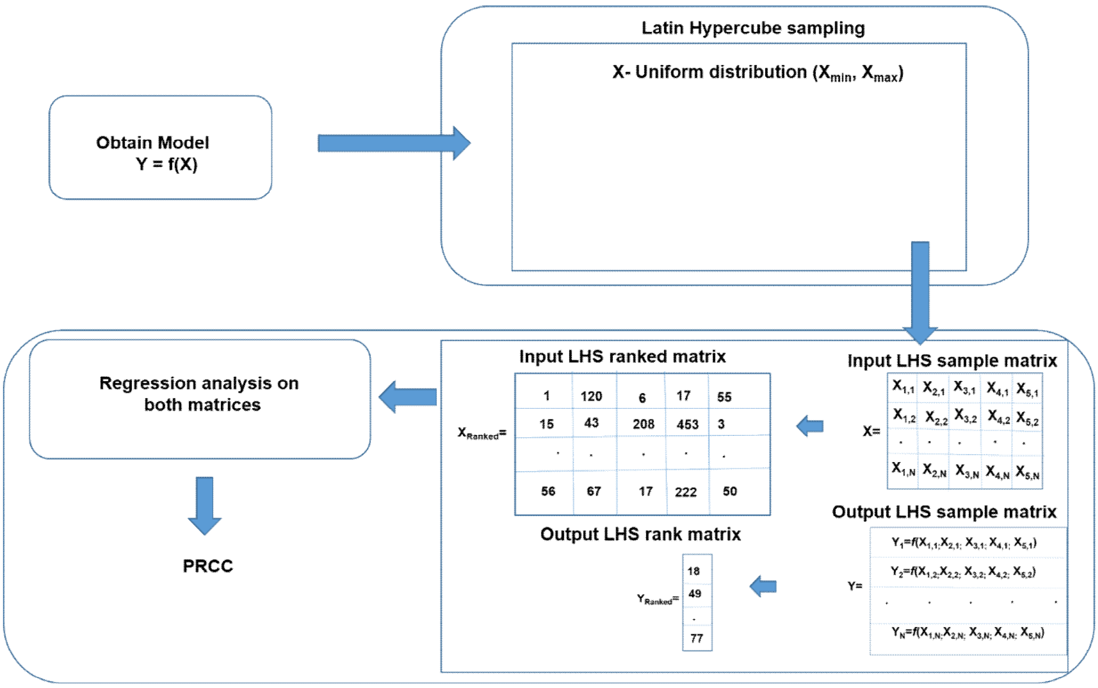


Figure 7.3 Pictorial representation of PRCC method steps

7.3.1 Analysis of PRCC Results

To carry out the PRCC analysis, the raw data for both the input parameters and the output are ranked and then the PRCC analysis is carried out. The purpose of ranking is to standardise the data so as to remove any effect difference in dimensions or units may have on the sensitivity analysis. The corresponding p-values of the obtained PRCC values are used to determine the statistical significance of the results obtained. Input parameters with PRCC p-values less than 0.05 are considered statistically significant (Biau et al., 2010). It should be noted though that a statistically significant correlation coefficient does not necessarily imply that the parameter is influential (Taylor, 1990). For this study two criteria are used to select influential parameters: a p-value less than 0.05 as well as with a ($|PRCC|>0.5$) (Sanchez and Blower, 1997).

The PRCC scatter plots for both truncated design spaces are shown as Figure 7.4 and 7.5 respectively. The PRCC plots show a correlation between the design parameters and the output. For the truncated design space A, the design parameters X_1 and X_2 show positive linear relationship with the output that is the thermal fatigue life increases as X_1 and X_2 increases. While for X_3, X_4 and X_5 there is a negative linear relationship with the output that is the thermal fatigue life reduces as these design parameters increases. The truncated design space B show also show a strong correlation between the design parameters and the output for several of the design parameters. Within the range of values used for truncated design space B, it is observed that X_1 and X_5 show a positive correlation, while X_2, X_3 and X_4 show a negative correlation with the output. For the design parameters X_1 and X_5 the thermal fatigue life increases as they increase. While for X_2, X_3 and X_4 the thermal fatigue life reduces as these design parameters increases. Truncating the design space for X_2 and X_5 at their turning points make the PRCC sensitivity analysis accurate as it cannot provide reliable analysis for non-monotonic function.

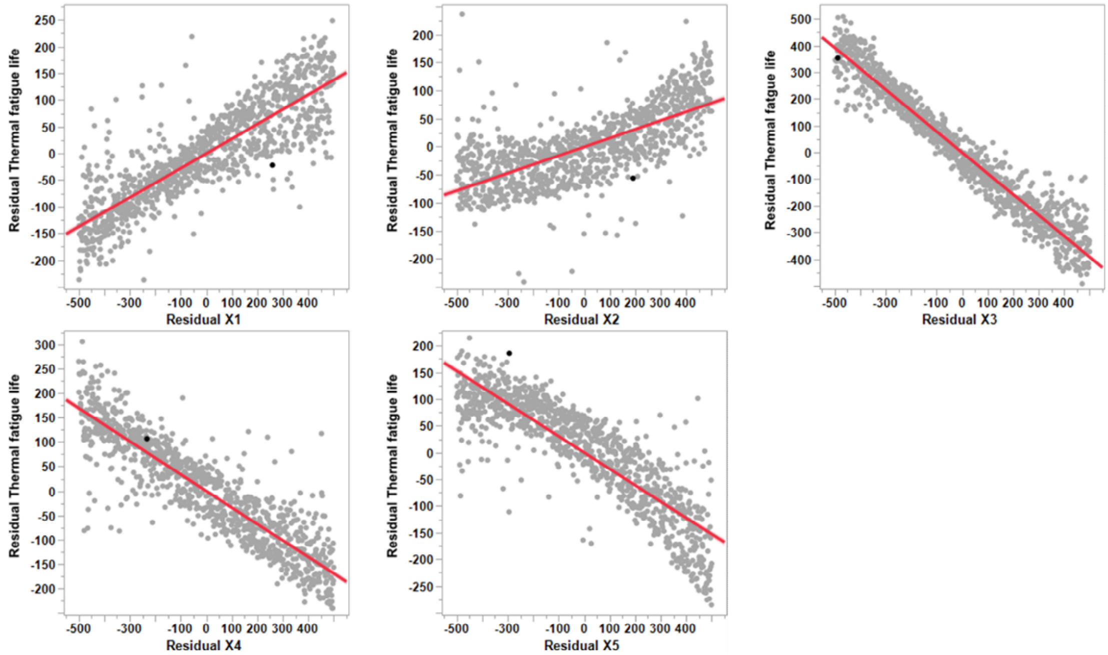


Figure 7.4 PRCC plots for truncated design space A

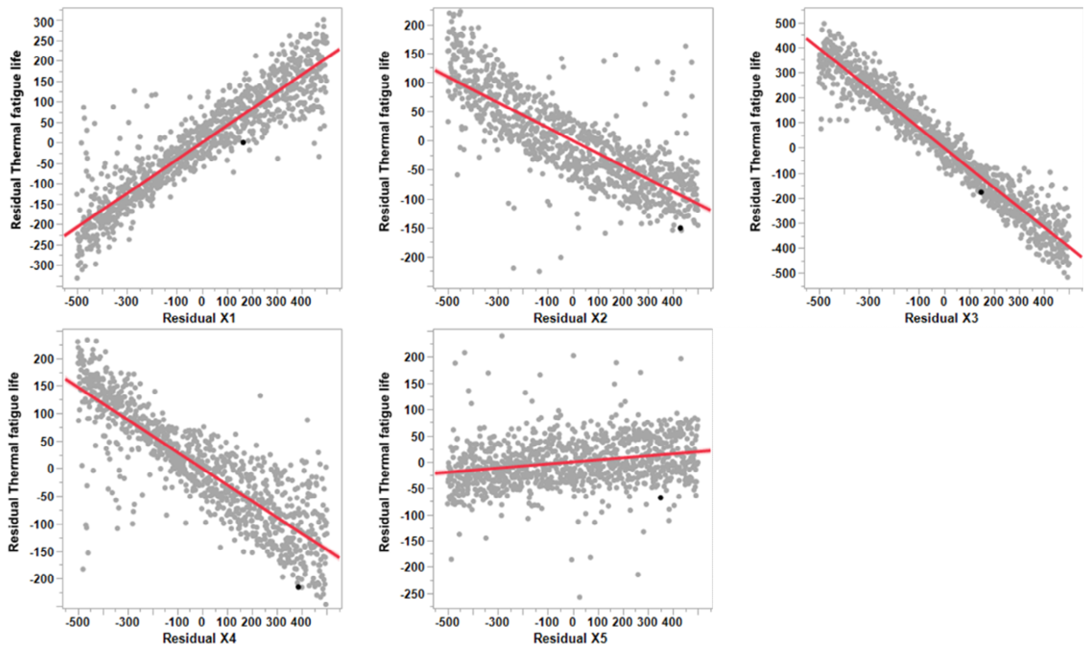


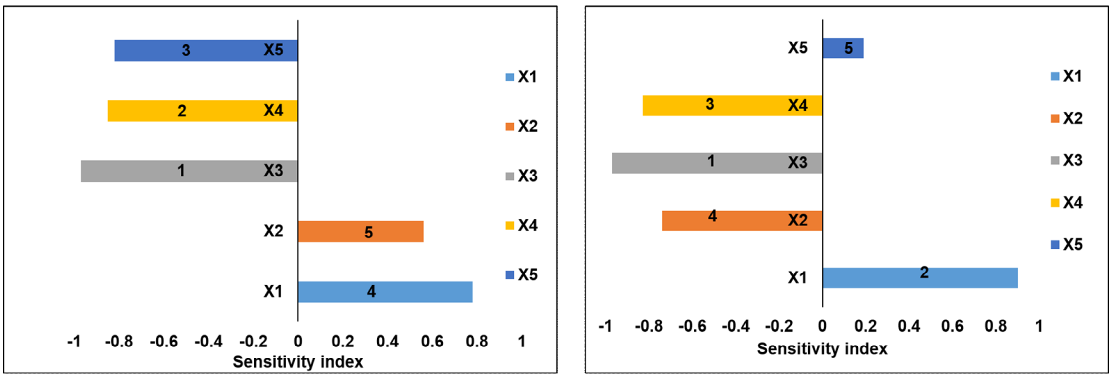
Figure 7.5 PRCC plots for truncated design space B

The p-values Of the PRCC for each design parameters for both analysis as indicated in Table 7.7 show all the design parameters to be statistically significant, as they are all less than 0.05. With this the second criteria

(│PRCC│>0.5) for selecting influential parameters is used to identify the influential design parameters. Table 7.3 show that at the selected ranges all the design parameters within design space A are influential to the thermal fatigue life of the vented brake disc, while for design space B the design parameters are influential except for design parameter X_5 (it has a PRCC value less than 0.5). This shows agreement with the monotonic plot and the PRCC residual plot for X_5. The PRCC values are ranked based on their correlation strengths ranging from very high correlation to little if any correlation (Asuero et al., 2006). Within the range of design space A, the parameter X_3 is very highly correlated with the output (PRCC values range between the absolute values of 0.9 - 1.0), parameters X_1, X_4 and X_5 as highly correlated to the output (PRCC values range between the absolute values of 0.7- 0.89), and parameter X_2 as moderately correlated (PRCC values range between the absolute values of 0.5 - 0.69) with the output. While within the range of design space B, the parameter X_1 and X_3 has a very highly correlated with the output (PRCC values range between the absolute values of 0.9 - 1), parameters X_2 and X_4 as highly correlated to the output (PRCC values range between the absolute values of 0.7-0.89), and parameter X_5 as weakly correlated (PRCC values range between the absolute values of 0.00 - 0.29) with the output. Figure 7.6 present a tornado plot for the PRCC analysis with the ranking of the parameters indicated in the plot. The most influential parameter for both truncated design spaces is the X_3 parameter which is the undercut depth.

Table 7.3 PRCC Results for the truncated Design Spaces

Parameter	Designation	Design space					
		A			B		
		PRCC value	P-value	Rank	PRCC value	P-value	Rank
X_1	Inboard plate	0.78	4.21E-207	4	0.90	0.00	2
X_2	Outboard plate	0.56	1.63E-92	5	-0.74	8.98E-173	4
X_3	Undercut depth	-0.97	0.00	1	-0.97	0.00	1
X_4	Undercut thickness	-0.85	4.52E-272	2	-0.83	1.27E-255	3
X_5	Effective offset	-0.82	1.97E-239	3	0.19	7.62E-10	5



(a) (b)

Figure 7.6 PRCC sensitivity plots showing ranking: (a) Design space A; (b) Design space B

7.3.2 Analysis of eFAST Results

For the eFAST sensitivity analysis first order (S_i) and total order (S_{Ti}) indices were obtained using the SimLab software version 2.2 for sensitivity analysis. The sensitivity measures estimated using the eFAST method are listed in Table 7.4 and graphically represented as shown in shown in Figure 7.7 for the five input parameters. X_1 showed first and total order sensitivity indices of 0.1062 and 0.1178, X_2 (0.0386 and 0.04482), X_3 (0.6146 and 0.6470), X_4 (0.0961 and 0.1075), and X_5 (0.1107 and 0.1246) respectively. The first order sensitivity indices (S_i) show that the interactions between the input factors have only a negligible impact on thermal fatigue life of the brake disc at the hat-ring friction plate corner, since the output variance explained by the single effects of the inputs is 96.62%. This implies that input parameter interactions and non-linearities account for only 3.38% of the output variance. This value is small and hence can be neglected. The most influential parameters in order of influence for the eFAST sensitivity indices are Undercut depth (X_3), Effective offset (X_5), Inboard plate (X_1), Undercut thickness (X_4) and Outboard plate (X_2). The eFAST analysis shows that Undercut depth (X_3) individual effect (S_i) accounts for about 62% of the output variability due to single effects of the input parameters. The other parameters account for the remaining 38% with the lowest being X_2 which individual effect accounts for only 3.9% of the output variability.

Table 7.4 Sensitivity indices for the thermal fatigue life

Parameter	Designation	First order (S_i)	Total order (S_{Ti})	$S_{Ti} - S_i$
X_1	Inboard plate	0.1062	0.1178	0.0116
X_2	Outboard plate	0.0386	0.0482	0.0096
X_3	Undercut depth	0.6146	0.6470	0.0324
X_4	Undercut thickness	0.0961	0.1075	0.0114
X_5	Effective offset	0.1107	0.1246	0.0139
Sum		0.9662	1.0451	0.0789

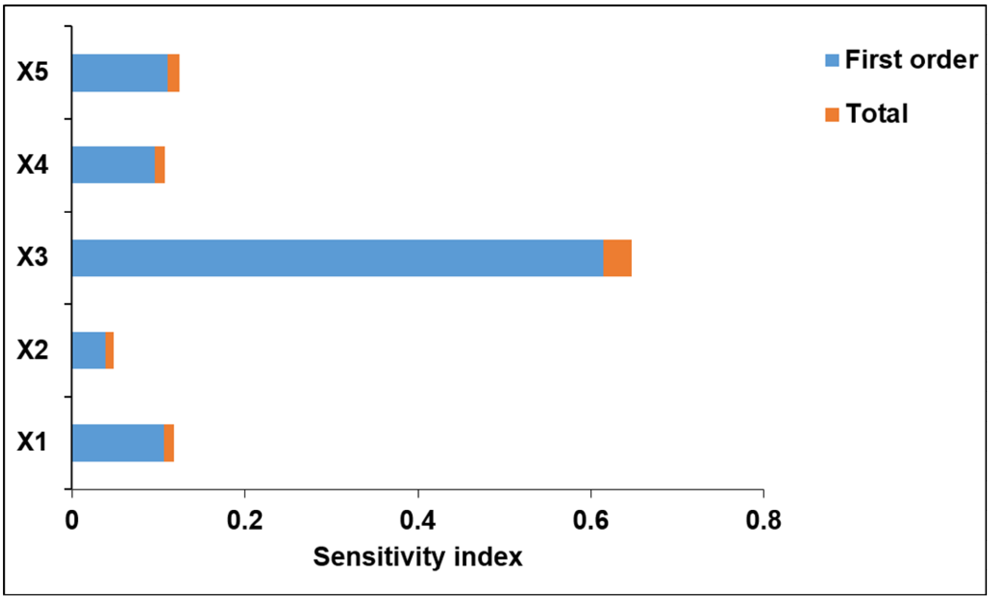


Figure 7.7 Global sensitivity indices calculated with eFAST method

7.4 Key observations from the sensitivity results

The results presented in this chapter are discussed in a broader context so as to render the findings of this study generic and also relevant for similar studies. The results obtained and hence conclusions to be arrived at are limited to the design parameters selected and their respective design spaces as used in this study. Hence, care should be taken with the extrapolation of the methodological conclusions of this study to other similar studies or models. A comparison of the order of ranking of the sensitivity indices of PRCC and eFAST show that the individual positions of the input parameters could differ between both sensitivity measures. This difference in the order of the ranks between PRCC and eFAST has been reported in works by other authors (Marino et al., 2008; Nguyen and Reiter, 2015; Waikhom et al., 2016). This is attributable to the reason that they both measure different characteristics. Both methods though identified the undercut depth, X_3, to be the most influential parameter and outboard plate, X_2, as the least influential. Though the PRCC result for the truncated design space B show X_5 to be the least influential. This is attributable to the relatively smaller interval for assessing parameter X_5, but it is observed then that X_2 for the truncated design space B takes 4^{th} position in the ranking. PRCC sensitivity indices provides a measure of the influence of the input parameters based on their linear relationship to the output. PRCC is able to provide information on the range over which this relationship exists, as well as the direction of this relationship. PRCC helps to determine what input parameters to target in order to achieve specific goals (Marino *et al.*, 2008) as it determines how the output is impacted positively or negatively over the input parameters design space. While eFAST provides a measure of each inputs contribution to the variability in the model's output. The eFAST indices tell what amount of uncertainty in the output is due to a particular input. The eFAST measures help identify the parameters to concentrate design effort on so as to reduce uncertainty to get more reliable estimates on selected parameters of the model.

The methodology used in the PRCC analysis makes the obtained results using the PRCC method more reliable, by removing the non-monotonicity through the truncation of the design space into two parts. It reduces the uncertainty in assessing the sensitivity of the response to the input variables. Other authors in carrying out sensitivity analysis using PRCC did not carry out a model monotonicity check (Gilbert et al., 2014; Hickson et al., 2011; Li et al., 2014; Marino et al., 2008; Park et al., 2011; Waikhom et al., 2016), thereby limiting the reliability of the obtained measures. The non-removal of the non-monotonicity has been reported to give misleading sensitivity measures when using PRCC for sensitivity analysis (Marino *et al.*, 2008), and this is further confirmed by Nguyen and Reiter (2015) in a performance comparison of different sensitivity methods. By truncating the design space for the parameters that show a non-monotonic behaviour with respect to the output, the study is able to capture the actual non-linear relationship of the sensitivity of the output to these input parameters. If these design spaces had not been truncated the PRCC sensitivity analysis would have indicated a linear relationship. For instance, the results show that from 5 mm to 7 mm the thermal fatigue life increases moderately with the outboard plate thickness, but from 7 mm to 9 mm the reverse becomes the case at even a steeper rate.

The results also show that the chosen design spaces for the parameters have an influence on the sensitivity indices and the relative order of ranking of the input parameters. This is quite observable in the PRCC sensitivity analysis where two design spaces were used as a result of the truncation carried out. It is observed that both PRCC sensitivity analysis did not give the same values and ranks even for the input parameters whose design spaces were not altered. It has been observed that parameter variation range has an effect on the absolute and relative ranking of a parameter as well as the ranking of the other parameters in the model (Wang *et al.*, 2013), hence, the selection of a parameters design space for a sensitivity analysis should not be done arbitrarily. Choosing the design spaces for the input parameters in a sensitivity study should depend on the goal of the study and the initial available information (Helton, 1993). The parameters design space selected for this study was based on previous work (Sarip, 2013), and the nominal dimensions of the sample vented brake disc used in this study. The assumption of a uniform distribution for the input parameters is not expected to affect the obtained results, as this has been shown by Helton (1993) to have a less pronounced effect on sensitivity analysis than the ranges of the parameters. But then it should be recognised that the results and conclusions derived from this study like all global sensitivity analysis are based on the assumptions made about the input parameters design spaces and their probability distributions (Helton, 1993).

In the design of a front vented brake disc this study has been able to identify five key design features that influence its thermal fatigue life at the hat-friction plate corner. The use of the two sensitivity methods help in this study to achieve two aims. The aims are the identification of the most critical feature in terms of thermal fatigue life at the region of interest, and the relationship of the dimensions of these features to the thermal fatigue life. The study shows the Undercut depth to be the most critical feature in brake disc design based on the use of both sensitivity methods. Variation in the dimensions of the Undercut would impact significantly on the prediction of the life of the vented brake disc. The other parameters such as inboard plate, undercut thickness and effective offset have moderately low impact on the uncertainty contribution to the studied response. The effect of variability of the outboard plate on the thermal fatigue life can be considered negligible. Previous studies on the stress or thermal fatigue life of the brake disc did not look at the uncertainty contribution of the input parameters to the life of brake disc(Huang and Chen, 2006; Okamura and Yumoto, 2006; Sarip, 2013). These studies were more concerned with the response of the brake disc in terms of the studied effect to dimensional changes. Understanding the behaviour of a component due to uncertainties in its design would make for more robust and reliable designs. The undercut of a brake disc and the effective offset are considered key features in designing the brake disc for life at the hat friction plate corner (Tirovic, 2004; Okamura and Yumoto, 2006). The results obtained in this study agree with these studies as it shows these parameters to be among the prominent parameters in the rankings of both methods.

The results obtained highlight that though parameters may have a significant and strong linear relationship with the output, they may only be contributing only relatively little to the uncertainty in the output. This is seen in the design parameters such as the inboard plate, outboard plate, undercut thickness and effective offset that have a significant and strong relationship to the thermal fatigue life output, but contribute only a small percentage to the output's uncertainty. The objectives and end requirements of a sensitivity analysis should determine the selection of the sensitivity analysis scheme which can be a single method or a combination of methods.

7.5 Chapter Summary

In this chapter global sensitivity methods have been used to assess the sensitivity of the thermal fatigue life of a vented brake disc at the hat friction plate corner to selected geometric design features. The study looked at five design features of the vented brake disc which are the inboard plate, outboard plate, undercut depth, undercut thickness, and the effective offset. The two methods of sensitivity analysis used are the PRCC and the eFAST method. Both methods were shown to complement each other in this study as they assess different characteristics. The need to evaluate the model for monotonicity in the use of the PRCC sensitivity method was demonstrated. This is done to ensure if there is a need to truncate the design space so as to make the sensitivity analysis more reliable. Although the rankings differed based on the range of the truncated design spaces and sensitivity method used, the undercut depth was found to be the most critical design feature. The undercut depth is shown to account for over sixty percent of the uncertainty in the disc/ring hat friction area of the vented brake disc. Design effort should thus be concentrated on the undercut majorly. The next chapter based on the obtained results in this chapter shall undertake the optimisation of the brake disc in the presence of uncertainties using the uncertainty quantification methodology developed in Chapter 6.

8 . AN OPTIMISATION FRAMEWORK IN THE PRESENCE OF MIXED UNCERTAINTY

In real life operations of engineering components or systems variability and uncertainties in design parameters can affect the performance of even the best designs. As a result there is often the demand for designs that can be stable, that is are robust to these variabilities or uncertainties. In engineering design optimisation there may be a preference then for designs that are robust than for the optimal designs. The use of a robust design optimisation serves as a tool to achieve the aim of getting designs that are robust to uncertainties in the engineering component or system. This chapter presents the formulation and algorithms of a robust based design optimisation under both aleatory and epistemic uncertainty.

The proposed robust design optimisation formulations deal with the uncertainties due to the inherent variability in the design parameters (aleatory) and the epistemic uncertainty that arises as a result of model prediction error from the use of a surrogate model. The proposed method is derived from the uncertainty quantification method proposed in Chapter 6. The proposed method is demonstrated for the optimisation of the vented brake disc where it is considered that there is adequate information on the distribution of the random design features and only interval data for the surrogate model prediction errors. This chapter consists of several sections to show the development of the formulation and application of the proposed robust design optimisation on the vented brake disc under a mixed uncertainty.

8.1 Performance and Robustness

To perform a robust design optimisation requires the simultaneous maximisation of the mean performance and the minimisation of the performance variance, thus making RDO a multi-objective problem. The simultaneous maximisation of the performance mean and the minimisation of its variance can be conflicting objectives. This makes the RDO problem a multi-criteria decision making process that requires the decision maker to make a choice amongst conflicting options.

To illustrate the robust concept and a designer's likely preference for robust solutions, Figure 8.1 is used to highlight difference between a robust solution and a global optimum solution. Variation in "A" leads to leads perturbation of the performance function in comparison to "B". Though "B" gives a better performance value than "A", "A" is considered more robust to "B". In a multi-objective optimisation for robust design the sensitivity around the vicinity of the solution should be checked for the objective functions (Deb and Gupta, 2005). The robustness of two design can be compared using the pdf plots of their respective performance. Consider two designs A and B having expected performance values μ_1 and μ_2 respectively as shown in Figure 8.2. The curves show the distribution of the frequency of occurrence of the performance function, *F*, of two designs. Though design A has a smaller expected value compared to design B, the design A is considered more preferable to design B from the robustness concept because it has a smaller magnitude of variation around its expected value. Design A is less sensitive to variations in the uncertain model parameters than design B. In designing for uncertainty the quality of a design is measured not only by its optimal value, but also by its response to variability in its parameters.

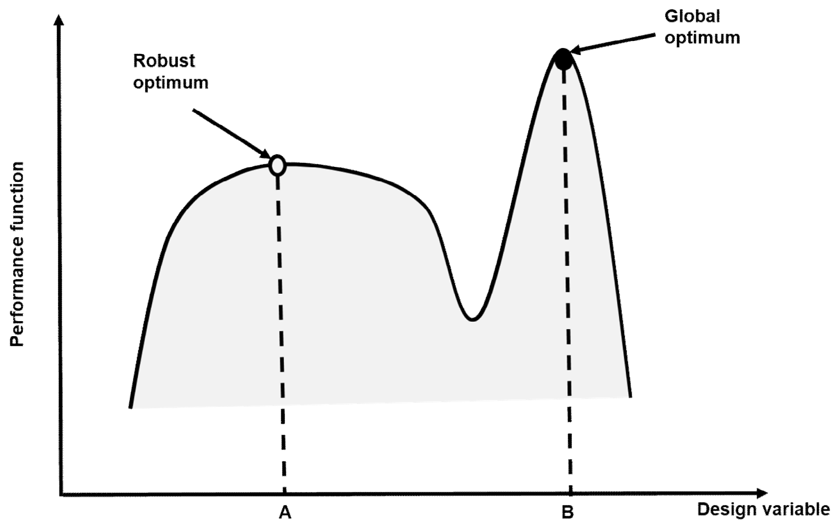


Figure 8.1 A comparison of global and robust optimum solution

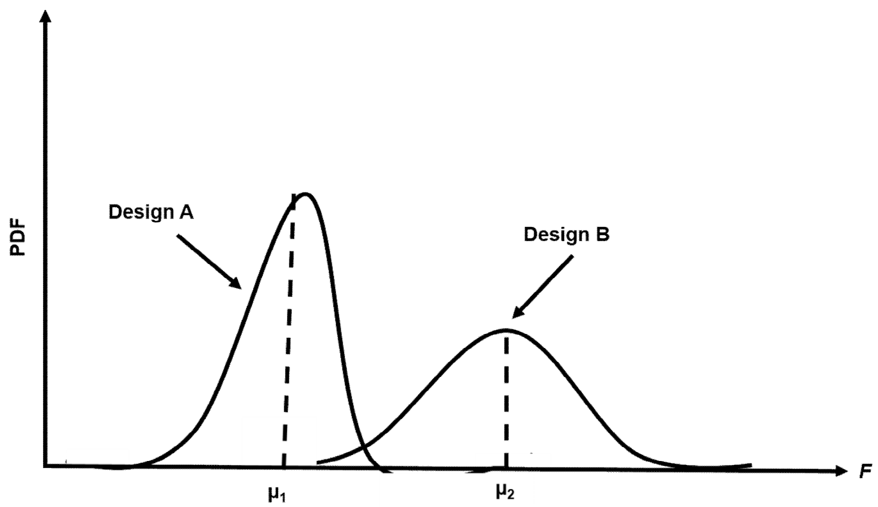


Figure 8.2 Comparison of different designs for robustness using their pdf

8.2 Robustness Assessment

Robust design optimisation involves the determination of the robustness of the design objectives and the related robustness of the design constraints usually through the use of numerical methods. Hence RDO requires the satisfaction of objective robustness and the robustness of the constraints (feasibility robustness). The robustness of the objective functions refers to the degree to which the performance of the objective function is insensitive to variabilities or uncertainties in the functions parameters. Objective robustness can be achieved by minimising the change in the objective function with respect to the variability of the design parameters. While the robustness of the constraints also referred to as feasibility robustness is the satisfaction of the design constraints in the presence of design parameter variability or uncertainty (Chen and Du, 2000). In literature there has been more emphasis on the robustness of the objective function than that of constraints due to the pursuit of designs that are insensitive (Park et al. 2006). The optimisation problem to be solved in this chapter is an unconstrained optimisation so the review on robust assessment methods would focus mainly on methods for evaluating objective robustness. To evaluate objective robustness several measures have been used in literature of which common ones are the variance, coefficient of variation (COV) and percentiles (Huang and Du, 2007; Shimoyama *et al.*, 2009; Raza and Liang, 2011; Wang et al., 2015). In the optimisation problem formulation these measures are introduced in the form of an objective alongside the performance objectives.

The robustness of a design is measured traditionally using the variance of a performance measure around its mean. The use of variance is measuring robustness is easy to implement, and it can be applied to both unimodal and multimodal distributions. Though a limitation of variance is that it only describes the spread around the mean (Huang and Du 2007). The percentile difference method can only be applied to unimodal distributions though it has the advantage of being able to provide more information than variance such as information on tail area distribution probability (Huang and Du 2007). To estimate the variance as a robustness measure several methods have been presented in literature. These methods are; the Taylor series expansion method, point estimate and simulation methods (Huang and Du, 2007). Taylor series expansion methods as has been shown in chapter seven are simple to implement. It can be implemented as a first or second order approximation. The use of Taylor series expansion methods require that the performance function be differentiable. Though the second order Taylor series expansion has better accuracy, variance estimation using it be computationally expensive. An advantage of using Taylor series expansion method is that its uncertainty analysis based variance propagation is straightforward (Helton and Davis, 2003). A shortcoming of the Taylor series expansion methods are that for non-linear models where the variances of the input variables are relatively large, its use could result in significant errors in the performance objective variance estimation. The point estimate methods for variance estimation overcomes the limitation of Taylor series expansion method for dealing with non-linear models, as well as avoid the computation of the gradients of model output with respect to the input variables as required in Taylor series expansion (Mishra, 2000). It uses the first and second moments of each input variable to obtain points and weights for these variables which are then used in evaluating the variance of a performance function. A limitation is that this method may generate points which lie outside the domain of the input variable. Originally developed by

Rosenblueth (1975) it’s been applied in optimisation studies due to its ease of implementation (Evangelopoulos and Georgilakis, 2013; Mohammadi *et al.*, 2013; Saunders, 2014; Hu *et al.*, 2016). The simulation methods make use of randomly generated variables drawn from the distribution of the input variables to estimate variance of the response. Commonly used simulation methods include Monte Carlo sampling, Latin hypercube sampling (LHS) and Hammersly sequence sampling (HSS) (Huang and Du, 2007). The use of the simulation methods which are sampling based can be computationally expensive though the computationally more efficient sampling methods such as LHS can reduce the computational cost.

In robust design optimisation variance is usually used as the measure of uncertainty even when modelling epistemic or mixed uncertainty (Erfani and Utyuzhnikov, 2010; Zaman and Mahadevan, 2013; Zhang and Hosder, 2013). According to Ferson and Tucker (2006), variance is not the only measure of uncertainty, and that it is often a not very useful measure of uncertainty if it is exceedance risks or tail probabilities that are of concern. Uncertainty can also expressed as intervals (Ferson and Tucker, 2006; Limbourg *et al.*, 2007). Intervals are used because in real life applications, the uncertainty distribution cannot always be stated with precision. So intervals are used to provide a range in which the uncertain parameters are expected to lie. Table 8.1 gives a comparison of some common robustness measures that have found use in robust design optimisation in the presence of uncertainties.

Table 8.1 A comparison of common robustness measures

Robustness measure	Uncertainty type	Advantage	Limitation
Variance/Standard deviation	Aleatory and epistemic	It can be appled to both unimodal and multi-modal distributions It is easy to implement	Standard deviation does not give information about the skewness of the data.
Percentile difference	Aleatory	Percentile difference can provide additional information such as the skewness of the data. Percentile difference is able to provide information as to what extent the design robustness is achieved.	This method cannot be applied to distributions that are multi-modal It is relatively less straight forward to implement
Interval	Epistemic	It does not require precise or complete information	Its application to non-monotonous models can be computationally expensive

In evidence theory there are two types of uncertainty which are due to a lack of knowledge, randomness (or discord) and non-specificity (Abellan and Moral, 2000). There are two classical measures of uncertainty, Hartley measure and Shannon entropy, which are used to measure non-specificity and randomness respectively (Jousselme *et al.*, 2006). Randomness focuses on sets with empty intersections, whereas non-specificity focuses on sets with cardinality greater than one. According to Klir and Wierman (1999) the generalised measures of uncertainty must reduce to a Hartley measure and/or a Shannon entropy whenever the general framework of evidence theory reduces either to a classical sets theory or to a probability theory. In the classical theory of sets, confidence values are not involved, so the only measurable uncertainty is the non-specificity of a set, directly referring to its cardinality (Jousselme *et al.*, 2006). In this study the uncertainty is taken as that contributed by the width of the intervals (Limbourg *et al.*, 2007). Based on Klir and Wierman (1999) requirements the measure of uncertainty to determine robustness to be used is the non-specificity measure as the use of intervals reduces it to the classical sets theory. Several non-specificity measures for the basic probability assignment (BPA) have been proposed in literature, but it is the generalised Hartley measure by Dubois and Prade (1985) that have been proven by (Ramer) to be the only non-specificity measure that satisfies all the requirements for non-specificity, and is given as:

$$GH(m) = \sum_{A \in X} m(A) log_2 |A| \qquad \textbf{(8-1)}$$

But the generalised Hartley measure does not readily extend to the real line as it takes an infinite value for masses on points $(\overline{A} = \underline{A})$ value if $log_2|A|$ is replaced with $log_2(\overline{A} - \underline{A})$, and also becomes a problem when $log_2(\overline{A} - \underline{A})$ is equal to one . Limbourg et al. (2007) thus recommends the use of a more intuitive non-specificity measure which is the aggregated width of all nested intervals.

8.3 Proposed Methodology

8.3.1 Mixed uncertainty Based Multi-Objective Robust Design Optimisation (MURDO)

As has been discussed in chapter two most of the current methods in uncertainty quantification and optimisation under uncertainty that are based on evidence theory have only been concerned with the propagation of input parameters uncertainty into the model output. These methods have not taken into consideration other sources of uncertainties such as model form and model prediction error uncertainty. These methods as applied in optimisation under uncertainty use the belief or plausibility measure as the robust measures, and these are computationally challenging for a practical implementation of evidence based design optimisation. There is a need for an effective robust design optimisation method that can handle the presence of uncertainty in the model input parameters and also those due to other sources of uncertainty such as the uncertainty due to model error. An efficient robust design optimisation method that can handle aleatory and epistemic uncertainty without recourse to the use of the computationally challenging determination of the belief and plausibility functions is thus desirable. As well as a robust optimisation method for a mixed uncertainty problem in which the value of the optimised design is not depended on the selected value for the robust measure. It was also shown that in previous evidence based optimisation methods subjectivity is introduced as the designer had to use a pre-determined threshold value for the robust measure, thus introducing further uncertainty into the analysis.

In this chapter a mixed uncertainty based robust design optimisation (MURDO) method that can handle a mixed uncertainty problem without the use of the computationally intensive belief and plausibility measures is proposed. This formulation would be able to handle both aleatory uncertainty in the input parameters and the epistemic uncertainty due to model prediction error, as well as remove the dependence of the optimised design on a pre-determined threshold robustness measure. The proposed robust optimisation method in this chapter is demonstrated with the design of a vented brake disc for robust fatigue life at the disc/ring hat friction area. The fatigue life of the vented brake disc is estimated through the use of a surrogate model. In this study both deterministic optimisation and the proposed robust optimisation are performed, and the results compared to demonstrate the capabilities of the proposed robust optimisation method for a mixed uncertainty problem. Figure 8.3 demonstrates the modelling process for developing the MURDO. The components of the MURDO modelling process is described in the following sections.

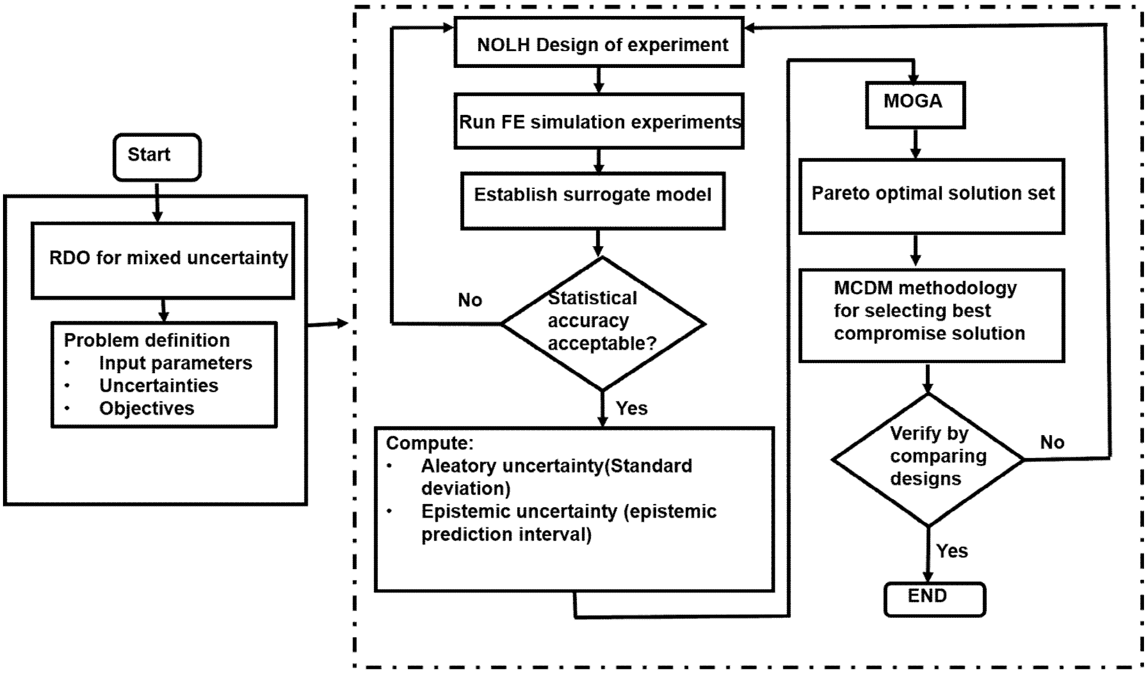


Figure 8.3 MURDO modelling process for optimising in presence of mixed uncertainty

8.3.2 Robustness assessment measures

To model the uncertainties in robust design optimisation, robustness measures are used to assess the system. The use of these robustness measures provide an assessment of the extent to which objective robustness is achieved. Because there are two different sources of uncertainty being handled in this problem, two uncertainty measures treating both sources as purely aleatory and purely epistemic respectively are used as the robustness measures. The use of two different robust measures to simultaneously account for the two types of uncertainty provides a means for handling the different sources of uncertainty in the model, so that one source may not unduly mask the influence of the other. In this section the robustness measures used for assessing the aleatory and epistemic components respectively are described.

Aleatory robustness measure

Aleatory uncertainty is usually measured using the standard deviation which is derived from the variance. Based on the first order Taylor series, approximation of the mean and variance are obtained as follows:

Given that $F = f(X_1, X_2, \dots \dots \dots \dots, X_n)$

Expected value or mean of F:

$$E(F) \approx f(\mu_{X_1}, \mu_{X_2}, \dots \dots \dots, \mu_{X_n}) \qquad \textbf{(8-2)}$$

Variance of *F*:

$$\sigma_F^2 \approx \sum_{i=1}^{n} \left(\frac{\partial f}{\partial X_i}\right)^2 \sigma_{x_i}^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial f}{\partial X_i} \frac{\partial f}{\partial X_j} Cov(X_i, X_j) \qquad \textbf{(8-3)}$$

Assuming the variables are uncorrelated then the correlated covariance term can be neglected in the variance formulation. Then the estimate for the variance becomes:

$$\sigma_F^2 \approx \sum_{i=1}^{n} \left(\frac{\partial f}{\partial X_i}\right)^2 \sigma_{X_i}^2 \qquad \textbf{(8-4)}$$

The standard deviation is then given as:

$$\sigma_F \approx \sum_{i=1}^{n} \left(\frac{\partial f}{\partial X_i}\right) \sigma_{X_i} \qquad \textbf{(8-5)}$$

Epistemic robustness measure

This section describes the development of the epistemic robustness measure used in this study. The epistemic robustness measure as used in this thesis is the aggregated width of all the nested intervals which is obtained using the method of alpha-cuts. The aggregated width of all intervals which is represented as $DSSAvg_{width}$ is as given in Eqn. 8-6

$$DSSAvg_{width} = \sum_{A \in X} m(A_\alpha)(\overline{A_\alpha} - \underline{A_\alpha}) \quad \textbf{(8-6)}$$

where m is the basic probability assignment (BPA) of the set of focal elements A, and $\underline{A}$ and $\overline{A}$ are referred to as the lower and upper bounds of the alpha cut (α-cut) of the fuzzy number respectively. The BPA as used in this thesis is obtained using the method proposed by (Ali and Dutta, 2012) which is given in Eqn.8-7 as:

$$m(A_\alpha) = \frac{1-\alpha_i}{\sum_{i=1}^{n}(1-\alpha_i)} \quad \textbf{(8-7)}$$

where α_i are the respective alpha-cuts used to discretise the fuzzy variable in order to give a family of nested intervals. The nested intervals are obtained by converting a Gaussian fuzzy variable with a possibilistic mean, M_r and standard deviation, σ_E as assumed in this study for the epistemic uncertainty (see chapter 6), to $\underline{A}$ and $\overline{A}$ the lower and upper bounds of the alpha cut (α-cut) of the fuzzy number respectively. The alpha-cuts for the lower and upper bounds are obtained from the possibilistic mean, M_r and the possibilistic standard deviation, σ_E of the Gaussian fuzzy variable using the expressions:

$$\overline{A_\alpha} = M_r + \sigma_E\sqrt{-ln\alpha} \quad \textbf{(8-8)}$$

$$\underline{A_\alpha} = M_r - \sigma_E\sqrt{-ln\alpha} \quad \textbf{(8-9)}$$

The Gaussian fuzzy variable as used in this study to determine the epistemic robustness measure, that is the aggregated width of all nested intervals ($DSSAvg_{width}$) is obtained through the following procedure:

(i) Determine the estimate of the surrogate model error bound in interval form from the prediction interval which is then expressed as [0, a]. From the obtained fuzzy interval determine the possibilistic variance of the error term as σ_ε^2 based on the assumption that the error term follows a Gaussian distribution with mean and variance given as $(0,\sigma_\varepsilon^2)$, respectively.

(ii) Express the model output as a triangular fuzzy number with the predicted value "A" as the mode and the left and right width obtained using the upper bound "a" obtained from the error bound. The triangular fuzzy number can then be expressed as [A-a, A, A+a]. The possibilistic mean and variance of the obtained triangular fuzzy number is then obtained as M_r and σ^2_r, respectively.

(iii) From the quantified uncertainty components of each identified uncertainty source, calculate the combined uncertainty using the method of quadrature. The following expression gives the combined possibilistic standard deviation is then given as:

$$\sigma_E = \sqrt{\sigma_\varepsilon^2 + \sigma_r^2} \qquad \textbf{(8-10)}$$

This results in a Gaussian fuzzy variable with a possibilistic mean, $M_{r,}$ and the possibilistic standard deviation, σ_E, which are used in obtaining the alpha-cuts for the lower and upper bounds of the fuzzy variable.

8.3.3 Proposed Mixed Uncertainty Robust Design Optimisation (MURDO) Formulation

Conventionally the RDO problem can be formulated as a multi objective optimisation problem which can be represented mathematically as (Zang, Friswell and Mottershead, 2005):

$$Minimise: \mu(f(X), \sigma^2(f(X))\}$$

$$Subject\ to: g_j(X) \leq 0\ for\ (j = 1,2, \ldots., m)$$

$$X^L \leq X \leq X^U \tag{8-11}$$

where μ and σ^2 represent the system mean performance and variance respectively. And X^L and X^U being the lower and upper limits of the design variables, and are regarded as the side constraints. The mean and the standard deviation of the model's output can be estimated for the given input variables if the joint probability density function (PDF) of the design (input) variables are known. But in most real life applications the joint PDF is not known, and so the assumption that the variables are independent and normally distributed to simplify the analysis. Based on this the joint PDF is obtained as the product of the PDF of the individual design (input) variables. But evaluating the variance term, σ^2 in Eqn.8-11 can be computationally expensive, so a Taylor series approximation can be used to obtain its estimate. In robust design optimisation the constraints also have to be satisfied. The constraint is not always satisfied for a stochastic analysis, and hence, the probability that the constraints are satisfied has to be chosen a priori. The constraint is then represented as:

$$g_j(X) + \beta\sigma_{gj}(X) \leq 0 \qquad \textbf{(8-12)}$$

where σ_{gj} is gives the approximated standard deviation of the *j*th constraint, $\beta\sigma_{gj}$ the constraint feasibility index, and β is a constant that indicates the probability that the constraint will be satisfied. For instance, if β is set to be 2, this means that the original constraint requirement is satisfied 95.46% of the time if the constraint distribution can be assumed to be normal. The assumption of a normally distributed performance measure in RDO is considered reasonable from engineering perspective. Thus in the mathematical formulation of the RDO the feasibility index, $\beta\sigma_{gj}$ can be considered an appropriate measure of the robustness with respect to the design constraints. To obtain the Pareto optimal solutions of the RDO being a multi-objective optimisation several approaches have been proposed in literature. The most common approach that has been used is the weighted sum (WS) methods which has been shown to have limitations. To overcome the limitations of the weighted sum method, Sahai, Messac and Sundararaj (2000), demonstrated the superiority of using the bi-objective approach in robust design optimisation. Fang et al. (2015) also in a comparison of four robust design optimisation formulations showed that it is the bi-objective formulation that is able to generate a well distributed Pareto front over the entire design space. Mathematically the bi-objective formulation for the robust design optimisation can then be expressed as:

$Maximise: \mu(f(X))$

$Minimise: \sigma(f(X))$

$Subject\ to: g_j(X) + \beta\sigma_{gj}(X) \leq 0$

$$X^L \leq X \leq X^U \quad \textbf{(8-13)}$$

where σ_{gj} can be taken to be the constraint variance for a linear Taylor series and is given as:

$$\sigma_{gj} = \sum_i^n \frac{\partial g_j}{\partial X_i} \Delta X_i \quad \textbf{(8-14)}$$

The implementation of Eqn.8-14 requires that the variances of the design variables be precisely known. In cases like this the robustness measure, which is the response variance (or standard deviation, σ) is due purely to aleatory uncertainty. But in design optimisation under uncertainty this may not always be the case as real life engineering design modelling applications usually includes aleatory and epistemic uncertainties. Hence, an optimisation in the presence of uncertainties should be able to take both types of uncertainty into consideration. Using such an approach would make the design robust to both types of uncertainty. To achieve this the adoption of the aggregated average width of all intervals is proposed to account for the epistemic uncertainty due to modelling prediction error due to the use of a surrogate model in place of the actual FE experiments. While the aleatory uncertainty component which is due to the inherent variability of the design parameters is accounted for using the standard deviation of the model response from its expected value. In the optimisation problem handled in this chapter there are no imposed constraints, hence, the optimisation formulation using the robustness measures defined in the previous sections for the MURDO becomes:

$Maximise: Fatigue\ life, F\ (X_1, X_2, X_3, X_4, X_5)$
$Minimise$: Standard deviation, σ_F
$Minimise: DSSAvg_{width}\ (X_1, X_2, X_3, X_4, X_5)$
Subject to:

$$5 \leq X_1 \leq 9; 5 \leq X_2 \leq 9; 1 \leq X_3 \leq 3; 2 \leq X_4 \leq 6; 15 \leq X_5 \leq 40 \qquad \textbf{(8-15)}$$

The objective functions shown in Eqn.8-15 are expressed fully as shown in Equations 8-16 – 8-18.

Thermal fatigue life:

$$Mean\ fatigue\ life = 5.5983 - 0.14095x_1 + 1.1816x_2 - 1.7708x_3 - 0.47759x_4 - 0.16741x_5 - 0.098692x_2^2 + 0.0020047{x_5}^2 + 0.024294x_1 \cdot x_2 + 0.072232x_1 \cdot x_3 - 0.045315x_2 \cdot x_3 + 0.023179x_2 \cdot x_4 + 0.075855x_3 \cdot x_4 + 0.016073x_3 \cdot x_5 \qquad \textbf{(8-16)}$$

Standard deviation:

$$Std. = \sum_{i=1}^{n} \left(\frac{\partial(Mean\ fatigue\ life)}{\partial X_i}\right) \sigma_{X_i} \qquad i = 1,2,3,4,5. \qquad \textbf{(8-17)}$$

$\mathbf{DSSAvg_{width}}$:

$$DSSAvg_{width} = ((2 * 0.0677\sigma_E * Sqrt(-log(0.8))) + (2 * 0.133\sigma_E * Sqrt(-log(0.6))) + (2 * 0.2\sigma_E * Sqrt(-log(0.0.4))) + (2 * 0.267\sigma_E * Sqrt(-log(0.6))) \qquad \textbf{(8-18)}$$

The variables x_1-x_5 are the design variables and σ_E is the possibilistic standard deviation (variance) of the prediction error, and σ_x, the input design parameter standard deviation. The standard deviations of the design inputs are obtained using the assumed coefficient of variation (COV) of 0.02 (See Chapter 6).

8.4 Initial Conditions for the Multi-Objective Genetic Algorithm Optimisation

To identify the optimum designs in the presence of the aleatory uncertainty due to the input parameters and the epistemic uncertainty due to use of a surrogate prediction model, a multi-objective genetic algorithm (MOGA) was utilised in the optimisation. The robust design formulation for the mixed uncertainty RDO in Eqn.8-15 was solved by adopting an existing Non-dominated Sorting Genetic Algorithm-II (NSGA-II) procedure as implemented in Matlab® for the multi-objective optimisation. The NSGA-II algorithm was utilised in this study as it has been shown to maintain a good spread of solution on the Pareto front, and also has a better performance in converging near the true Pareto-optimal set when compared to most other contemporary algorithms for solving multi-objective optimisation problems (Deb *et al.*, 2002).

A population size of 100 was found to result in a sufficient spread of the Pareto set of solutions based on experimental comparisons of different population size runs while keeping the number of generations fixed to 1000. This was done to determine the population needed to guarantee convergence with the minimal computational time. The MOGA optimisation was done using a crossover fraction of 0.95, and the constraint dependent option as implemented in Matlab®

is adopted for the mutation function. This option is considered best when dealing with optimisation problems that have constraints or side constraints in the use of MOGA in Matlab®. Being that MOGA is a stochastic optimisation method, 10 runs are performed using the selected population size of 100 for 1000 generations with different random number seeds. This is done to ensure consistency in the results. In this study vectors of real numbers are used for the chromosome representation. Real value coded genetic algorithms are recommended for optimisation problems where the parameter space is continuous (Michalewicz, 2013)

8.4.1 Multi-Objective GA Results

A set of initial robust solutions were produced with the optimisation run with parameters as outlined in Section 8.4. Simulation experiments were carried out using the proposed mix uncertainty robust design optimisation (MURDO) framework for the multi-objective problem. A Pareto set of solutions considered robust optimal are obtained using an NSGA-II based multi-objective optimisation. A post multi-objective GA search space reduction method, based on the integration of Grey relational analysis and TOPSIS, presented in Section 8.6.2 is used to identify the best compromise solution from the population of solution sets identified by the multi-objective GA. A sample of seven selected results out of the 350 population of solutions identified by the multi-objective GA are listed in Table 8.2 and Table 8.3. The results as shown in Table 8.2 are a sample of the robust optimal design solution set in the design space, giving the values that guarantee the minimum uncertainty (optimal robustness) at the Pareto frontier design points in estimating the optimal fatigue life for the vented brake disc. While Table 8.3 lists the optimum fatigue life at the corresponding optimal robustness measure for the objectives in the objective space. The problem is made up of three objectives so the Pareto plot is presented as a surface plot as shown in Figure 8.4.

Table 8.2 Sample Pareto frontier design points

Inboard plate (mm)	Outboard plate (mm)	Undercut depth (mm)	Undercut thickness (mm)	Effective offset (mm)
5.35	7.24	1.18	2.75	35.98
8.87	6.78	1.31	2.76	18.73
7.55	6.15	1.93	3.83	20.58
5.44	7.47	1.66	4.04	33.44
5.89	7.44	1.81	3.70	33.96
8.93	7.71	1.19	2.72	18.39
8.33	7.53	1.21	2.75	18.93

Table 8.3 Sample of robust design solutions found by the multi-objective optimisation GA

Thermal fatigue life (Log life repeats)	Std. (Log life repeats)	$DSSAvg_{width}$ (Log life repeats)
4.5667	0.0190	0.2379
5.5563	0.0445	0.2332
4.6893	0.0546	0.2060
4.2290	0.0240	0.2131
4.3305	0.0248	0.2090
5.7158	0.0385	0.2472
5.5844	0.0375	0.2336

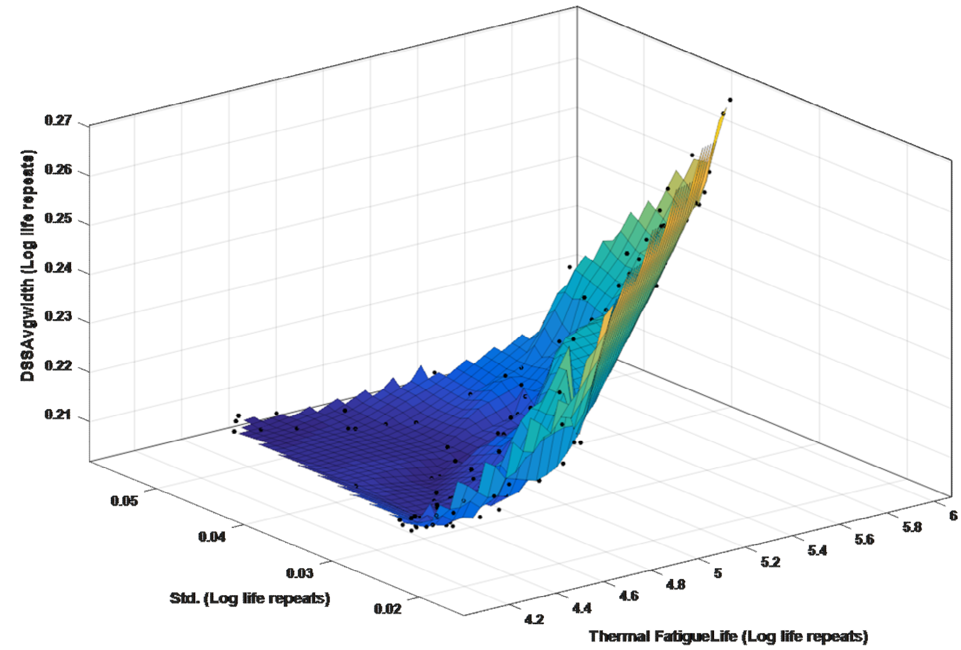


Figure 8.4 Pareto frontier surface plot

8.4.2 A Post Analysis of Multi-objective GA Optimisation Results

A multi-objective optimisation usually involves conflicting objectives leading to asset of Pareto optimal solutions from which the decision maker has to make a choice. The post analysis of multi-objective optimisation GA results involves processing of the obtained solution of the objective space to choose the best possible compromise by ranking the solutions in the Pareto frontier. The post processing is done by integrating two methodologies that have found application in multi-criteria decision making, TOPSIS and Grey relational analysis (GRA) (Chen and Tzeng, 2004). The steps involved in the use of this integrated approach is presented as follows:

1. Normalisation of the obtained Pareto set of solution data: In situations where the performance units or ranges are widely different for different performance measures, the influence of some performance measures

may be overridden by other performance measures. Aside from this where the goals and direction of the performance measures are different, this can lead to incorrect results in the multi-criteria decision analysis, requiring pre-processing of the data by normalising the data according to the goal of the performance measure (Huang and Liao, 2003). This pre-processing which is required for the use of Grey relational analysis is referred to as grey relational generating. Three types of data normalisation are used:

(i) Larger the better

$$x_{ij}^{*} = \frac{x_{ij} - \min x_{ij}}{\max x_{ij} - \min x_{ij}} \quad \textbf{(8-19)}$$

(ii) Smaller the better

$$x_{ij}^{*} = \frac{\max x_{ij} - x_{ij}}{\max x_{ij} - \min x_{ij}} \quad \textbf{(8-20)}$$

(iii) Nominal the best

$$x_{ij}^{*} = 1 - \frac{\left|x_{ij} - x_{obj(j)}\right|}{\max x_{ij} - x_{obj(j)}} \quad \textbf{(8-21)}$$

where $x_{obj(j)}$ is the desired value of entity *j*, $\max x_{ij}$ and $\min x_{ij}$ are the respective maximum and minimum values of entity *j*.

2. Weights determination: The importance of a criterion is dependent either on the decision maker's subjective preference or the objective

characteristics of the criteria themselves (Deng et al., 2000). In this study an objective weighting process is used, as it is particularly applicable when it is not possible to obtain reliable subjective weights as is the case in this study. Assigning weights to the performance measures presents a means through which the criteria for different performance can be brought together. The weights assigned to the performance measures are obtained using Shannon's entropy concept (Deng et al., 2000; Shannon and Weaver, 2002) for this study. The entropy of the normalised performance measures for the *j*th entity is obtained as follows:

$$E_j = -k \sum_{i=1}^{N} r_i(j) \log r_i\,(j) \tag{8-22}$$

where $r_i(j) = \frac{x_i(j)}{\sum_{i=1}^{N} r_i(j)}$, and $k = \frac{1}{logN}$

The weight w_j for the *j*th entity is then defined as:

$$w_j = \frac{1 - E_j}{\sum_{j=1}^{K}(1 - E_j)} \tag{8-23}$$

3. Determine the ideal solution and the negative ideal solution: The ideal solution is designated as x^+ and indicates the most preferable alternative, while the negative ideal solution is designated as x^- and indicates the least preferable alternative. The ideal and the negative ideal solutions are obtained using Eqn.8.24 and 8.25 respectively.

$$x^{+} = (\max x_{i1}^{*}, \max x_{i2}^{*}) \quad \textbf{(8-24)}$$

$$x^{-} = (\min x_{i1}^{*}, \min x_{i2}^{*}) \quad \textbf{(8-25)}$$

4. Determination of the grey relation coefficient: The grey relation coefficient, γ, of each alternative to the ideal solution, $\gamma(x_j^{+}, x_{ij}^{*})$ and the negative ideal solution, $\gamma(x_j^{-}, x_{ij}^{*})$ are determined by taking the ideal, x^{+} and the negative ideal solution, x^{-} as the referential sequence and each of the alternatives to be the comparative sequence.

$$\gamma(x_j^{+}, x_{ij}^{*}) = \frac{min_i min_j \left|x_j^{+} - x_{ij}^{*}\right| + \xi max_i max_j \left|x_j^{+} - x_{ij}^{*}\right|}{\left|x_j^{+} - x_{ij}^{*}\right| + \xi max_i max_j \left|x_j^{+} - x_{ij}^{*}\right|} \quad \textbf{(8-26)}$$

$$\gamma(x_j^{-}, x_{ij}^{*}) = \frac{min_i min_j \left|x_j^{-} - x_{ij}^{*}\right| + \xi max_i max_j \left|x_j^{-} - x_{ij}^{*}\right|}{\left|x_j^{-} - x_{ij}^{*}\right| + \xi max_i max_j \left|x_j^{-} - x_{ij}^{*}\right|} \quad \textbf{(8-27)}$$

5. Determination of the grade of grey relation: The grade of grey relation of each alternative to the ideal, x^{+} and the negative ideal, x^{-} solutions are calculated using Eqn.8.28 and 8.29 respectively as:

$$\gamma(x^{+}, x_i^{*}) = \sum_{j=1}^{2} w_j \ \gamma(x_j^{+}, x_{ij}^{*}) \quad \textbf{(8-28)}$$

$$\gamma(x^{-}, x_i^{*}) = \sum_{j=1}^{2} w_j \ \gamma(x_j^{-}, x_{ij}^{*}) \quad \textbf{(8-29)}$$

$\sum_{j=1}^{N} w_j = 1.$

6. Determine the relative closeness C_i of distance that an alternative is close to the ideal solution, and this is defined as:

$$C_i = \frac{\gamma(x^+, x_i^*)}{\gamma(x^-, x_i^*)} \quad \textbf{(8-30)}$$

7. Rank the priority: The priority of the alternatives are ranked in a descending order of C_i, and the best possible compromise solution is selected.

In this integrated method the conventional TOPSIS general distance is replaced with the grey relation coefficient from the Grey relation model, and the conventional Grey relation modified to reflect the impact of decision making theory for the preference of performance measure weight (Chen and Tzeng, 2004). A satisfactory compromise solution can then be found based on this. The Grey-TOPSIS integrated MCDM model is selected for the post analysis as it has been shown to provide a best compromise design amongst several competing alternatives from a MOGA optimisation (Fang *et al.*, 2015).

8.5 Post Multi-Objective GA Result Analysis

In this section the analysis of the results obtained in the post multi-objective GA processing by using the strategy outlined in the previous section is presented. The use of the hybrid multi-criteria decision making method of TOPSIS integrated with GRA assisted in ranking the designs in terms of priority to obtain the most preferred robust design solution, which in this study is referred to as the best compromise design. The best compromise design is determined from all the unique points in the Pareto-fronts consisting of 350 Pareto optimal solutions obtained from the ten MOGA runs using this hybrid method by ranking the Pareto solutions. The robust optimal results of the best compromise design are compared to those of the deterministic optimisation design configuration. The comparison is based majorly on the robust assessment of the designs, as the aim of the MURDO is to provide a design that is less sensitive to uncertainties. In Table 8.4 the top ten ranked Pareto solutions are listed. Table 8.5 presents the comparison of these different design configurations, and Table 8.6 a comparison of their performance measures. The robustness measure is presented in both standard deviation and the average width of the intervals for the aleatory and epistemic uncertainty respectively.

Table 8.4 Top ten ranked MCDM Grey-TOPSIS designs

X_1 (mm)	X_2 (mm)	X_3 (mm)	X_4 (mm)	X_5 (mm)	FATIGUE (Log life repeats)	Std. (Log life repeats)	$DSSAvg_{width}$ (Log life repeats)	Rank
5.53	7.16	1.04	2.26	38.12	4.7378	0.017	0.2592	1
6.72	7.58	1.83	4.07	32.76	4.4185	0.028	0.2060	2
6.52	7.44	1.88	3.74	32.39	4.4282	0.028	0.2065	3
6.93	7.44	1.72	3.25	33.56	4.5949	0.027	0.2080	4
5.70	7.29	1.27	2.71	36.21	4.5985	0.020	0.2306	5
5.35	7.24	1.18	2.75	35.98	4.5667	0.019	0.2379	6
6.90	7.86	1.96	3.99	21.73	4.7453	0.037	0.2050	7
7.04	7.81	2.02	4.18	22.10	4.7112	0.038	0.2050	8
6.53	7.39	1.98	4.04	31.79	4.3679	0.030	0.2065	9

5.89	7.46	1.76	3.25	33.82	4.4158	0.024	0.2105	10

Table 8.5 A comparison of MURDO best compromise design points vs deterministic optimised design points

Description		Best compromise	Deterministic
Design Variables	X_1	5.53	9.00
	X_2	7.16	7.56
	X_3	1.04	1.00
	X_4	2.26	2.00
	X_5	38.12	15.00

Table 8.6 A comparison of robustness measures for best compromise design vs deterministic optimised design

Objective	Best compromise	Deterministic
Expected life	4.7378	6.2249
Aleatory component (Standard deviation)	0.017	0.037
Epistemic component ($DSSAvg_{width}$)	0.2592	0.2885

The deterministic design gives a better performance that is fatigue life than the best compromise design obtained from the optimisation under uncertainty. Based on the PRCC sensitivity analysis in Chapter 7 it can be understood why the deterministic design has a higher fatigue life than the best compromise design. Fatigue life increases with inboard and outboard plate thickness dimensions, and for this features the deterministic design has larger dimensions than the best compromise design. Also for undercut depth, undercut thickness and effective offset, the fatigue life improves with a decrease in the dimensions of these features. The deterministic design has smaller dimensions of these features as compared to the best compromise design. Thus in general the deterministic design has feature designs that make it most favourable to a better fatigue life than the best compromise design. The use of sensitivity analysis can then be used as a tool to explain difference in performance for competing designs as has been shown in this study. But since the aim of the thesis is to obtain a design that is least sensitive to the uncertainties in the system, the emphasis here is on the robustness of the design. Table 8.5 shows a significant difference between the value of the design variable, X_5, for the deterministic optimal versus that of the best compromised solution. This is reflected in Fig. 7.2 where multimodal solution for the X_5 design variable value is demonstrated. Further analysis of this X_5 value is conducted next in the section to determine the robustness of the design points using sensitivity analysis. The best compromised design is more robust compared to the deterministic optimised design as it has lower aleatory and epistemic robustness values than the deterministic design (see Table 8.6). The application of this approach is able to obtain an optimised solution that gives not only a more robust design by minimising the uncertainty in the design response, but also provides a means for minimising the uncertainty in estimating the response performance as a result of the use of a surrogate model.

8.5.1 Robustness validation of the design points using sensitivity analysis

In the previous section it has been shown that the best compromised design obtained through the use of the proposed MURDO approach gives a more robust solution to that of the deterministic optimised design. To validate this a comparison of the sensitivity of the optimisation objectives to infinitesimal changes in the design points of both designs is undertaken. In the context of this thesis, the design whose design points give the smaller sensitivity values is regarded as having less uncertainty, and hence more robust. Given that the optimisation objectives can be represented using quadratic response surface model (RSM):

$$f = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} b_{ii} x_{ii}^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} b_{ij} x_i x_j \quad \textbf{(8-31)}$$

where *f* is the optimisation objective, b_0 is a constant term, b_i the coefficients of the linear term, b_{ii} and b_{ij} the coefficients of the quadratic terms. Sensitivity is estimated by taking the partial derivative of the response with respect to each variables. This gives the gradient of the estimated response surfaces which is used as the sensitivity measure, and is given as:

$$\frac{\partial f}{\partial x_j} = b_i + \frac{\partial}{\partial x_j}\left(\sum_{j=1}^{n} \sum_{k=j}^{n} b_{jk} x_j x_k\right) \quad \textbf{(8-32)}$$

For a better comparison of the sensitivities against each other, the obtained gradient for each parameter has to be normalised:

$$\left(\frac{\partial f}{\partial x_j}\right)_{norm} = \frac{\partial f}{\partial x_j} \frac{x_m}{f_m} \quad \textbf{(8-33)}$$

where $\mathrm{x_m}$ and $\mathrm{f_m}$ are the mean values of the input parameters and their respective response. To carry out the sensitivity analysis quadratic response surface models were created to emulate the response of each objective. A D-optimality criterion design of experiment was used to create the response surface using the mean values of the design points in both the best compromise and deterministic optimised designs as centre points, and the boundary taken as three standard deviations from the mean values. The partial derivatives of the objectives were derived using the obtained response surface models for the optimisation objectives. The response surface models used for the sensitivity analysis for each of the objectives are as shown in Eqn.8-33 to Eqn.8-38 for the best compromise and the deterministic optimised designs respectively.

Best compromise design

$$Thermal\ fatigue\ life = 4.8757 + 0.10371 * x_1 - 0.2625 * x_3 - 0.2427 * x_4 + 0.002642 * x_5 \quad \textbf{(8-34)}$$

$$Std. = 1.1962 + 0.01456 * x_1 - 0.24934 * x_2 - 0.0101 * x_3 + 0.04066 * x_4 - 0.020111 * x_5 + 0.001041 * x_1^2 + 0.020385 * x_2^2 + 0.000242 * x_5^2 - 0.004049 * x_1 * x_2 + 0.00353 * x_1 * x_3 - 0.008209 * x_2 * x_3 - 0.004574 * x_2 * x_4 - 0.00596 * x_3 * x_4 + 0.001791 * x_3 * x_5 \quad \textbf{(8-35)}$$

$$DSSAvgwidth = 1.9008 - 0.09759 * x_1 - 0.09970 * x_2 - 0.3611 * x_3 - 0.0723 * x_4 - 0.039348 * x_5 + 0.005323 * x_1^2 + 0.007549 * x_2^2 + 0.12266 * x_3^2 + 0.01044 * x_4^2 + 0.000578 * x_5^2 - 0.000897 * x_1 * x_2 + 0.019121 * x_1 * x_3 - 0.000835 * x_1 * x_4 + 0.000305 * x_1 * x_5 - 0.001414 * x_2 * x_3 - 0.002118 * x_2 * x_4 + 0.00136 * x_2 * x_5 + 0.01822 * x_3 * x_4 - 0.002696 * x_3 * x_5 + 0.000198 * x_4 * x_5 \quad \textbf{(8-36)}$$

Deterministic optimised design

$$Thermal\ fatgue\ life = 7.523 + 0.11437 * x_1 - 0.496 * x_3 - 0.278 * x_4 - 0.09347 * x_5 + 0.055 * x_3 * x_4 \quad \textbf{(8-37)}$$

$$Std. = 0.4889 + 0.01207 * x_1 - 0.14903 * x_2 - 0.03034 * x_3 + 0.01920 * x_4 - 0.000373 * x_5 + 0.00028 * x_1^2 + 0.012142 * x_2^2 - 0.002384 * x_1 * x_2 + 0.000883 * x_1 * x_3 + 0.000447 * x_{1*} x_4 - 0.004506 * x_2 * x_3 - 0.002991 * x_2 * x_4 \quad \textbf{(8-38)}$$

$$DSSAvgwidth = 1.5180 - 0.07987 * x_1 - 016255 * x_2 - 0.0841 * x_3 - 0.02423 * x_4 - 0.02875 * x_5 + 0.04677 * x_1^2 + 0.011202 * x_2^2 + 0.0642 * x_3^2 + 0.000528 * x_5^2 - 0.002770 * x_1 * x_2 - 0.018887 * x_1 * x_3 + 0.001893 * x_1 * x_4 + 0.000624 * x_1 * x_5 - 0.00533 * x_2 * x_3 - 0.004292 * x_2 * x_4 - 0.000164 * x_2 * x_5 + 0.01923 * x_3 * x_4 + 0.001906 * x_3 * x_5 \quad \textbf{(8-39)}$$

The response surface models for both designs all had R-squared (R^2) and the R-squared adjusted (R^2 adj.) values that are greater than 0.9 for both measures.

Table 8.7 gives the sensitivity comparisons of the best compromise design and the deterministic optimised design based on their design points for each of the objectives in the MURDO. Table 8.7 show also that the X_5 design variable for the best compromised design is a more design robust point than that of the deterministic optimised design. This provides further proof for the conclusions reached in the previous section with respect to the significant difference in the corresponding values of X_5 for the best compromise and deterministic optimal solutions. And in Figures 8.5-8.7 the graphical comparison of the absolute sensitivity values of the objective functions to the design parameters for both the best compromise and deterministic optimised design are shown. The graphical comparison is done using absolute values of the sensitivities so as to best visualise any difference in these values for both designs. The best compromise design and the deterministic optimised design are represented with the numerals 1 and 2 respectively. Design points with sensitivities closer to zero produce responses that are less sensitive to small changes in these design points, and so are considered more desirable points in optimising in the presence of uncertainties. From Figure 8.5 it can be concluded that the design points for the best compromise design are more robust than those of the deterministic optimised design as they have lower sensitivity values with respect to the thermal fatigue life. Figure 8.6 and 8.7 also indicate the same trend for the aleatory and epistemic uncertainties respectively. Though from the respective plots it can be observed that the undercut depth influences the objectives more than the other parameters. This is in agreement with the sensitivity results presented in chapter 7. Observation of the sensitivity plots also show that the for both design configurations, the design parameters have relatively negligible influence on the aleatory and epistemic uncertainties

Table 8.7 Sensitivity comparison of best compromise design and deterministic optimised design

Parameter	Thermal fatigue life (Log life repeats)		Standard deviation (Log life repeats)		$DSSAvg_{width}$ (Log life repeats)	
	Best	Deterministic	Best	Deterministic	Best	Deterministic
X_1	0.0887	0.0789	2.26E-06	3.49E-06	-7.23E-04	6.26E-04
X_2	0	0	3.03E-06	1.26E-05	8.52E-05	5.87E-04
X_3	-1.1935	-2.3964	8.68E-05	1.53E-04	-0.0178	-0.0286
X_4	-0.5078	-0.6922	1.26E-05	1.11E-05	-0.0021	-0.0029

Parameter	Thermal fatigue life (Log life repeats)		Standard deviation (Log life repeats)		$DSSAvg_{width}$ (Log life repeats)	
	Best	Deterministic	Best	Deterministic	Best	Deterministic
X_5	3.28E-04	-3.87E-02	8.77E-08	9.03E-07	3.40E-05	-1.28E-04

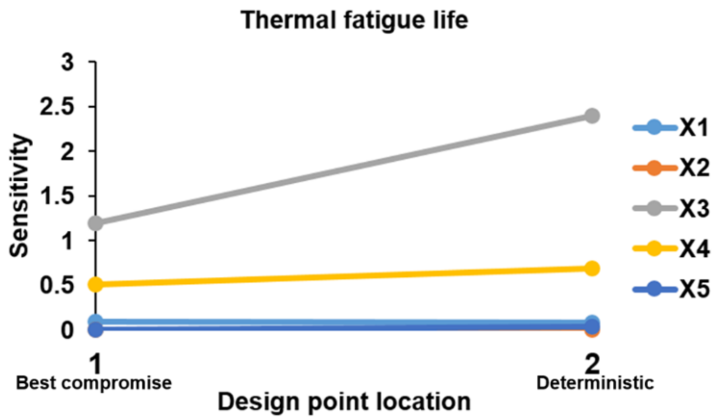


Figure 8.5 Thermal fatigue life sensitivity comparison

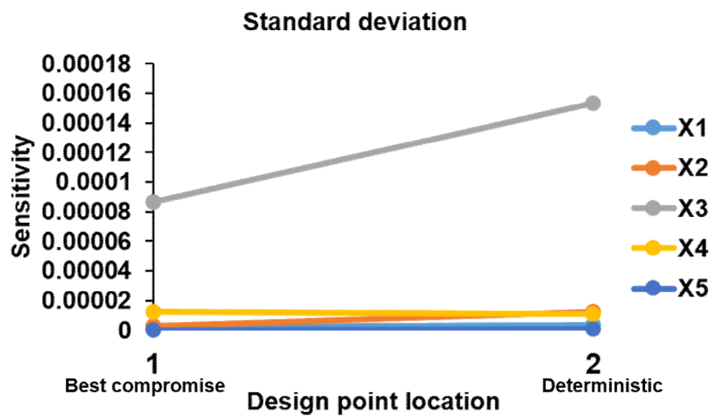


Figure 8.6 Aleatory uncertainty sensitivity comparison

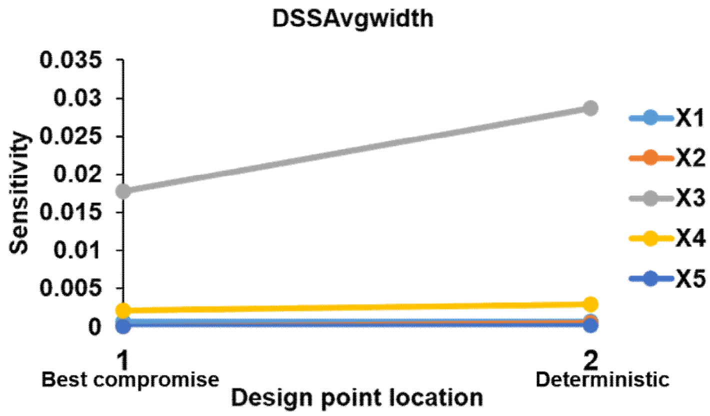


Figure 8.7 Epistemic uncertainty sensitivity comparison

8.6 MURDO Model Validation Based on Use of a Surrogate Model

To validate the results of the robust design optimisation in the presence of uncertainties solution an operational black box validation is carried out using the actual FE simulations. Operational validation for simulation models involves determining whether a surrogate model's output behaviour has the required accuracy for the model's intended purpose over the region of the model's intended application (Sargent, 2013). In the use of operational validation the simulation/predictive model output behaviour is compared with the system output behaviour or some other valid models using graphical visual displays or statistical tests and procedures (Sargent, 2013). According to Sargent (2013) this can be achieved by exploring the model behaviour which involves the use of a variability-sensitivity analysis. The surrogate model output behaviour can be qualitatively or quantitatively explored. The qualitative exploration involves analysing the direction of the output behaviours and also possibly whether the magnitudes are reasonable, while the quantitative analysis involves both the direction and precise magnitude of the output behaviour (Sargent, 2013). In this study a qualitative comparison of the predictive surrogate model behaviour is compared to the behaviour of the actual FE model behaviour. The comparison is done with the FE model based on conclusions reached as a result of informal interviews and discussions with brake disc modelling experts in the course of the model development. According to the opinion of the experts the brake disc thermal fatigue cracking is a complex phenomenon and so cannot be adequately modelled through the use of analytic models. Also brake disc come in different designs and configurations, as well as the mode of application of the braking pads on the disc. All these make one brake disc life analysis different from another. The modelling of brake discs thermal behaviour involves the making of several limiting assumptions and as such estimating a precise life or range may not be feasible. But according to the experts even at a heavy duty braking cycle a well design brake disc is not expected to crack at the disc/hat ring friction area until several thousands of cycles. In this study the thermal fatigue life of the best compromise design is 4.4638 Log life repeats which is equivalent to 14546 cycles to crack initiation, several thousand of cycles. Though limiting in terms of their prediction of the disc thermal behaviour these models assist in giving insight on the efficiency of design configurations as they can be used to compare what is expected amongst different design configurations. Based on this a qualitative exploration of the behaviour of the actual FE simulation and the surrogate model predicted thermal fatigue life is made.

The sensitivity trend in varying a design parameter for both the actual FE simulation and the surrogate model are compared. The undercut depth is selected as the design feature to use for the parameter sensitivity analysis. The undercut depth is selected as it contributes over sixty percent to the uncertainty in the model output. Its large contribution to model uncertainty implies that a change in its value would likely have an observable impact on the thermal fatigue life of the brake disc. The value of the undercut depth is varied in steps of three to six standard deviations of its baseline value for the best compromise design while keeping the other design parameters fixed. The qualitative exploration involves comparing the directions of the output of the behaviours of the respective outputs to determine if both exhibit the same trend. The behaviours are both depicted in Figure 8.8. It is observed that both outputs of the actual FE simulation and the surrogate model follow the same trend in the comparison of their sensitivity to variations in the dimension of the undercut depth.

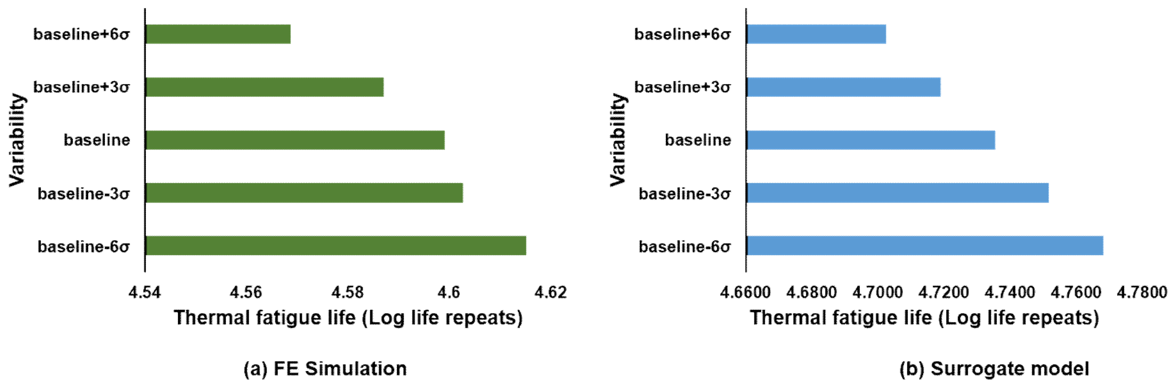


Figure 8.8 Operational validation comparison of FE simulation and surrogate model

A comparison of the respective percentage change in the thermal fatigue life estimation of both the actual FE and the surrogate model to the varying of the baseline undercut depth in steps of three to six standard deviations is also carried out. This is done to determine if the surrogate model can be used in place of the actual FE simulation for the optimisation of the brake disc in the presence of the uncertainties that the proposed method is able to treat. The results of the comparison are as presented in Table 8.7. The analysis of Table 8.7 indicate that the absolute difference in the percentage changes in the response due to the variations in the undercut depth baseline value between the actual FE simulation and the surrogate model is less than 1%. This error margin being less than 5% is accepted conventionally to be within statistically acceptable error margin, and can be considered as negligible. The surrogate model can thus be used in place of the actual FE simulations for the MURDO as it provides solutions within acceptable error margins.

Table 8.8 Comparison of change of response to undercut variation for actual FE simulation and surrogate model

Change in undercut depth	Thermal fatigue life (Log life repeats)		% change in response		Abs. Difference
	Actual FE simulation (Log life repeats)	Surrogate model Prediction (Log life repeats)	% Actual FE simulation	% Surrogate model Prediction	
-6σ	4.6152	4.7682	0.35	0.69	0.34
-3 σ	4.6028	4.7518	0.08	0.35	0.27
3 σ	4.5871	4.7190	0.26	0.35	0.09
6 σ	4.5688	4.7026	0.66	0.69	0.03

8.7 Chapter Summary

In this chapter the application of the proposed optimisation of a mechanical component in the presence of mixed uncertainty method was successfully demonstrated. The uncertainty that were handled were uncertainty due to the inherent variability of the dimensions of the design features and the epistemic uncertainty that is due to the prediction error in the use of a surrogate model in place of an actual FE simulation. A three objective RDO problem in the presence of uncertainty for optimising the thermal fatigue life of a vented brake disc at the region of interest for the minimisation of the aleatory and epistemic uncertainties present was solved using a multi-objective GA based optimisation. A post GA hybrid method integrating TOPSIS and Grey relational analysis was used to reduce the search space of the multi-objective GA in order to determine the design that is considered best compromise from the obtained Pareto optimal set.

The results achieved as presented in this chapter confirm that the proposed optimisation method in the presence of mixed uncertainties and the post multi-objective GA result analysis method used were able to identify a best compromise design that can be considered robust to variability in the input design feature parameters. The validation of robustness test carried showed the best compromise design determined from the post optimisation analysis to be more robust, which is less sensitive to parameter changes than the optimised deterministic design. The following were achieved in this chapter:

- The development of a robust optimisation method for handling mixed uncertainties in a design problem.

- The successful application of the developed robust design optimisation method to optimise a mechanical component, the vented brake disc in the presence of aleatory and epistemic uncertainties.
- The use of a post optimisation multi-criteria decision making method to successfully select a design that is less sensitive to uncertainties when compared to the design obtained from deterministic optimisation.
- The successful validation of the developed robust design optimisation in the presence of uncertainties method results using a black box approach.

While in this chapter the successful development and application of the proposed robust design optimisation method in the presence of mixed uncertainties with the validation of the optimisation results is presented, the next chapter presents the discussions of results and the conclusions of the thesis.

9 . DISCUSSIONS AND CONCLUSIONS

This chapter presents the summary of the findings of the research in fulfilment of the research aim and objectives. Section 9.1 provides a summary of the key research findings, and is followed by section 9.2 which provides a summary discussion of the research findings in the application of the proposed methodologies in this thesis. In section 9.3 the research contributions to the body of knowledge are highlighted. Section 9.4 discusses the limitations of the research, and in section 9.5 future work for the extension of the work done in this thesis is presented. Finally section 9.6 presents the research conclusions against the research objectives.

9.1 Key Research Findings and Observations

There has been less attention on the influence of geometric design features on the degradation life of machine components as compared to the influence of material and environmental conditions. In real life engineering the issue of variability in the design parameters and lack of knowledge both lead to non-negligible uncertainty in the parameters of a design model. With such uncertainty being propagated through the model, the output of the model also becomes uncertain. The presence of these uncertainties if not properly accounted for can lead to unreliable designs that are not able to fulfil their design requirements. Therefore this research aims to develop a methodology for studying the influence of geometric design features on the thermal fatigue degradation life of a mechanical component taking into consideration the presence of uncertainties. The research aim relates to making proposals for new methods within well understood research domains, rather than generating a new theory in a new domain. Thus to achieve the aim and objectives of this research a case study research strategy was adopted.

A review of literature and an industry case study selection were undertaken to identify the gaps in the problem domain which motivates the propositions in this research and provide the basis for the selection of a case study degradation mechanism and machine component. The review of literature achieved two major outcomes in this study. The review of literature provided the researcher with an understanding of the domain of this research; the concepts and the models that have been proposed and their applications. The second outcome of the literature review assisted the researcher by providing an appropriate framework within which to place the research. This framework supports the location of the research proposal within the existing body of academic literature and also to identify the gaps in literature the research proposals may contribute to. While the industry case study selection process provided a basis for machine component and degradation mechanism case study selection that has practical applications in real life operations of machines. The industry case study selection achieved this by identifying what constitutes a problem in industry by identifying a component where design features is considered to influence component life, particularly its fatigue life.

The review of literature was carried out in three domains: review of studies on geometric design influence on fatigue degradation life, uncertainty determination and modelling, and design optimisation in the presence of uncertainty. The review of literature on the influence of geometric design influence on fatigue degradation life of components was important as it helped to identify the nature of the researches that have been undertaken in this domain. The review on uncertainty determination and modelling provided definitions of for the different type of uncertainties, current methods for modelling uncertainty, the strengths and limitations of these methods and current approaches in their applications. While the review of literature on optimisation identified various techniques in optimisation under uncertainty.

The review of literature identified the need for additional studies on the influence of geometric design features on a component fatigue degradation life as only relatively fewer studies have looked at this. Moreover, even the limited previous studies have been limited to case studies with their emphasis on specific components and are predominantly deterministic. The review showed that the use of probabilistic methods in design leads to more reliable and realistic designs. To undertake studies on the influence of design influence on the fatigue degradation life based on the probabilistic approach requires the understanding of the concept of uncertainty and its modelling. The literature review undertaken in the uncertainty modelling domain showed that though there exists several methods for uncertainty quantification, Dempster-Shaffer (evidence theory) offers better advantages as it provides a framework to simultaneously account for both aleatory and epistemic uncertainty in a problem, and has also found applications in design optimisation under uncertainty. Based on the review of literature the following gaps were identified:

- The studies on design influence on degradation of a component aside from being limited compared to studies on material and environmental influence, are mostly deterministic and so may not give realistic results.

There is the need for more studies on the influence of design feature on component degradation as previous studies have been case studies that are component specific.

- Existing methods of uncertainty quantification based on evidence theory that are able to handle mixed uncertainties have only been concerned with the propagation of input parameters uncertainty into the model output. These methods have not taken into consideration other sources of uncertainties such as model form and model prediction error uncertainty.
- In the use of evidence theory in design optimisation under uncertainty the use of a predetermined plausibility or belief threshold introduces its own uncertainty into the optimisation due to the subjectivity in choosing these values. The calculation of these measures (plausibility and belief) for use as the robustness measure is also computationally challenging in a practical implementation of evidence based design optimisation.

The combination of literature findings, especially the gaps identified, and the knowledge gained from the industrial study of the relationship between design and degradation led to the proposal for a novel methodology to assess the degradation life of a mechanical component due to geometric design influence in the presence of uncertainties. The proposed methodology also includes the application of the proposed uncertainty quantification method to the optimisation of a component in the presence of uncertainties. The following section presents the strategy and the method development used to address the problems in those domains.

9.2 Uncertainty Analysis Method Development and Sensitivity

This section describes the development of the proposed uncertainty analysis method, its application to optimisation of the components design and the sensitivity analysis. The method would be summarised in this section along with a description of how the research gaps were addressed. Uncertainty analysis usually precedes the sensitivity analysis, so the uncertainty methodology was developed first before carrying out the sensitivity analysis and the subsequent application of the uncertainty methods for component optimisation.

9.2.1 Surrogate Model Development for Uncertainty Analysis

An approximate quantitative surrogate model was built to reduce the complexity of the FE simulation process and also save time. This section describes the surrogate model building framework that is used in place of the actual FE simulations. Experimental determination of the fatigue life of a brake disc at the region of interest is very challenging due to the issue of repeatability of results informed the choice for the use of a quantitative modelling framework. The proposed surrogate model generation framework is based on the use of the statistical design of experiment and response surface methodologies. These methods have found widespread use in literature for generating approximate surrogate models for replacing complex processes. The use of a surrogate model provides the advantage of describing complex simulations with parametric coefficients that are easier to interpret while showing the functional relationship between output responses and the input design variables. It also reduces computational cost and time and also difficulty that would have been experienced in an online design optimisation.

9.2.2 Surrogate Model Validation

The developed surrogate model was validated using statistical measures and methods for a polynomial regression model validation. As shown in Chapter seven the developed prediction models give good predictions. The R^2 and R^2 adj. values of the models are statistically high for the models to be considered statistically acceptable. The P-values are also less than 0.05 which is the desired. A comparison of the models prediction and the actual FE simulation results provided validation that the surrogate models are able to give good predictions and as such suitable for approximating the actual FE simulations for the uncertainty analysis and the design optimisation.

9.2.3 Uncertainty Analysis Methodology

This section discusses the observations made when using the surrogate model for the proposed uncertainty quantification methodology. Uncertainty analysis is important in the performance analysis of engineering system as this helps to assess and control the uncertainties that may be present. This thesis adopted the Dempster-Shafer method for uncertainty analysis to estimate uncertainty. The review of literature indicated that Dempster-Shafer method for uncertainty analysis is more robust for uncertainty determination as it is able to handle uncertain information that is both random and imprecise at the same time. In designing the component by taking uncertainty into consideration, two sources of uncertainty are treated in the proposed method used in this thesis. The sources of uncertainty includes the aleatory uncertainty due to the input parameter variability and the epistemic uncertainty as a result of likely prediction errors that may arise from using the surrogate model. The method used is considered novel because unlike previous application of the Dempster-Shafer method had considered only the propagation of the uncertainties associated with the input variables and their propagation into the output without considering other likely sources of uncertainty, this method is able to treat the aforementioned uncertainties within a single framework. The proposed method was demonstrated using two case studies. The demonstration of the proposed method using the second case study was to show the general applicability of the method. The proposed method was able to give epistemic intervals which were tighter than the probabilistic prediction intervals in using the surrogate model for future prediction. It was demonstrated that for a brake disc the proposed method in giving a tighter prediction bound than the traditional probabilistic interval provided results that are less uncertain thereby improving the confidence in the results.

9.2.4 Sensitivity Analysis Methodology

In the analysis of systems or components to study the effect of parameters on the output, it is recommended to perform a sensitivity analysis after the uncertainty analysis. This study adopted the use of global methods for sensitivity analysis to determine the influence of the design features on the studied performance measure. Previous studies in literature have generally been deterministic in their analysis of design influence on their studied performances. This study used two sensitivity analysis methods to corroborate the obtained results. The partial rank correlation coefficient (PRCC) and eFAST methods were used. In studies were the PRCC has been used the researchers had not taken monotonicity into consideration. But there is an agreement in literature that the use of the PRCC without considering monotonicity could result in unrealistic results. In this study a methodology to take monotonicity into consideration is proposed. The use of this method which takes monotonicity into consideration reduces the uncertainty in the results that would have been generated had this not been considered. The use of the sensitivity framework in this thesis provided results that are in agreement with previous studies on the brake disc. But it should be noted that previous studies of design influence studies on the thermal behaviour of the brake disc did not look at the percentage contribution of a parameter to the uncertainty in the thermal performance, they were more concerned with the directional response of the thermal performance to dimensional changes. But in this study the percentage contributions as well as the directional response of the performance measures to input parameter changes were considered.

9.3 Optimisation in the Presence of Uncertainties

This thesis adopted the use of the proposed uncertainty method used for the uncertainty analysis in the optimisation of the vented brake disc as a case study. In this robust design optimisation for a mixed uncertainty problem as proposed in this thesis, two robust measures were adopted. The robustness measures were introduced to account separately for the aleatory and epistemic uncertainty components the optimisation is expected to handle. This was done to minimise any undue influence an uncertainty component or source may have on the other. For the epistemic component unlike the robustness measures used in previous studies as was shown in the review of literature a robustness measure that is not subjective and also whose selected value does not influence the optimisation solution was adopted. A multi-objective GA technique was adopted in this thesis to search for the robust solutions in the design optimisation of the vented brake disc in the presence of mixed uncertainty. The use of GA optimisation techniques have been shown in literature to have the ability to obtain a good diverse set of solutions that are near optimal. The proposed used mixed uncertainty robust design optimisation results in a population of solutions for the user to choose from, but does not give the most desired design. To overcome this problem and obtain the design that is considered best compromise, a multi-criteria decision making (MCDM) methodology is proposed. The MCDM technique was applied in a post optimisation analysis for identifying the best compromise design from the set of solutions obtained by the multi-objective GA. The MCDM technique was a hybrid integration of Grey relational analysis and TOPSIS. The best compromised design determined from the use of the MCDM is quantitatively compared with a design obtained from the deterministic optimisation of the case study component and the nominal sample brake disc. The purpose of carrying out a robust design optimisation in the presence of uncertainties is to get designs that are robust to the changes in the design parameters or uncertainties in the system. Thus the comparison with the optimised deterministic and nominal sample design were based on the robustness of the designs. The best compromise design obtained from the MCDM after the optimisation is seen to be better in terms of robustness to this other designs. This shows that the objective of proposing this robust design optimisation in the presence of mixed uncertainty was achieved.

9.4 Research Validations

A number of steps were undertaken to test the validity of the final solutions with respect to the intended research problem. The results obtained from the proposed uncertainty quantification were validated for generalisability for the brake discs components, while the mixed uncertainty robust design optimisation (MURDO) model was validated by comparing the best compromise design obtained using the proposed method with the deterministic optimised design. A black box operational validation method was further used in validating the MURDO results. The validation of procedures in this research are as follows:

Uncertainty quantification method validation

The validation of the proposed uncertainty quantification was carried out by randomly selecting design points for different configurations of the solid and vented brake disc. The same procedure used in the uncertainty analysis of the nominal sample solid and vented brake disc configuration is applied to these randomly generated designs. The validation is done without including the uncertainty due to input parameter variability. The obtained results were similar in characteristics to those obtained in using the nominal sample brake discs. The different design configurations for both types of brake discs gave epistemic prediction bounds that were tighter than their respective probabilistic prediction bounds. This shows that the proposed method is effective and is repeatable.

MURDO method validation

To ensure that the multi-objective optimisation and the use of the MCDM technique to obtain the design that is considered best compromise was able to achieve its intended outcome, a final results validation were undertaken. A robustness validation of the design points was undertaken by carrying out a comparison of the sensitivity of the response to infinitesimal changes in the design points of the best compromise design obtained from the MURDO and the deterministic optimised design. The results indicates the ability of the method to obtain a design with points that produce less aleatory and epistemic uncertainties in the optimisation objectives. The black box operational validation used involved a qualitative exploration of the behaviour of the actual FE simulation and the surrogate model predicted thermal fatigue life is made. The qualitative exploration involved comparing the directions of the output of the behaviours of the respective outputs to determine if both exhibit the same trend in respect to their sensitivity to an input parameter. The validation results showed that both showed similar trend in responding to changes in a significant influencing parameter the undercut depth. This indicates that the surrogate model can be adequately used in place of the actual FE simulations for the multi-objective optimisation. The design points of the best compromise design were used for developing the model for the actual FE simulation test of robustness. The robustness test was carried out by determining the percentage changes in the thermal fatigue life at the region of study for the vented brake disc by varying the undercut depth the most influential parameter up to six standard deviations about the mean value of the undercut depth while the other design features were fixed at their mean values. The absolute difference between the percentage changes in the thermal fatigue life at the region of interest in the vented brake disc of the Actual FE simulation and that of the surrogate model for these changes were found to be negligible. This further indicates that the use of the surrogate model in place of the actual FE simulations can provide results that are within acceptable error margin.

9.5 Contribution to Knowledge

This research has made contributions to the modelling of uncertainty from various sources in the design of a machine component the brake disc and the optimisation of the design in the presence of aleatory and epistemic uncertainties. This research has provided a methodology for modelling quantitative and uncertain information as a result of the use of a surrogate model to replace actual FE simulation. The methodology proposed is able to handle sources of uncertainty from the input parameters of the model and also the uncertainty due to the use of the model in place of the actual experiment itself. The research has also proposed a robust design optimisation in the presence of mixed uncertainty framework for handling design problems in the presence of aleatory input parameter uncertainty and epistemic uncertainty due to the use of a surrogate model in an optimisation. A sensitivity analysis methodology is also proposed that is able to take into account non-monotonicity in the problem. The contributions to knowledge in this thesis can then be summarised as follows:

- The proposal of a unified methodology to handle simultaneously the presence of aleatory and epistemic uncertainty in designing a component for thermal fatigue life. The proposed method was also demonstrated through application to two case studies. The proposed uncertainty analysis method gives a tighter (smaller) prediction interval than the equivalent probabilistic prediction interval. The method thus provides a reduction in the predictive uncertainty.

- Developed a method of robust design optimisation in the presence of mixed uncertainty for addressing aleatory and epistemic uncertainties within a single framework.
- Introduced a technique for carrying out a partial rank correlation coefficient analysis when non-monotonicity is present to obtain reliable results.
- The proposed methodology can assess the influence of geometric design features on the degradation life of a component even in the presence of uncertainties.
- The proposed uncertainty quantification method and optimisation in the presence of mixed uncertainties though demonstrated using a solid and vented brake discs are generalisable to the design of mechanical components.

9.6 Research Limitations

This section outlines the limitations of the research in the use of the approximate modelling frameworks.

- The FE modelling of the brake disc was formulated using some simplifying assumptions to the obtained thermal fatigue life. Also in the brake disc model some intricate design features such as the outboard inner corner radius were omitted due to lack of information on their characteristics. These assumptions make the modelling prone to over-simplification, and thus may influence the quality of the models in capturing the actual thermal behaviour of the brake discs.

- Increasing the number of FE simulation runs reduces the predictive uncertainty in the use of the surrogate model. This can have a significant impact on computational cost of the simulation. This research addressed this problem by using of the NOLH which has fixed designs based on the number of design parameters used in the analysis.
- The approximate model building involves significant handling time as a result of the manual manipulation and transfer of data between different software. The methodology used can be time consuming as it involved the combination of different frameworks, for example as in this thesis finite element analysis and statistics.
- In estimating the epistemic uncertainty due to the use of a surrogate model in place of the actual FE model, the application of the uncertainty quantification method as proposed in this thesis is dependent on being able to determine the traditional probabilistic prediction interval for any new prediction to be made using the surrogate model. The method is hence limited to black box models which require the use of a design of experiments.

9.7 Further Research Work

There are other areas of research within this research problem domain that can be avenues for further research. The recommended future research are presented as follows:

- This research focused on epistemic uncertainty that arises from surrogate model prediction error. Aside from this source there are other sources of epistemic uncertainty such as those due to sparse data points and error due to FE mesh discretisation. The methodology as described in this thesis can be extended to accommodate these other sources of epistemic uncertainty.
- In this thesis the uncertainty propagation and representation methods were developed to handle aleatory uncertainty in the design input parameters. But in some instances a mixture of aleatory and epistemic design input parameters may be present. The methods developed for the uncertainty analysis and optimisation in the presence of mixed uncertainty in this thesis have the capability to also handle epistemic design input parameters, though the developed methods as described in this thesis have not been illustrated to solve such a problem. The implementation of the developed uncertainty and optimisation methods to solve such a problem would be worthwhile in the future.
- In real life situations there may be correlations and inter-variable dependencies may exists among the design variables, and this can have significant impact on the uncertainty analysis as well as on the design optimisation. In this thesis the uncertainty analysis and robust design optimisation are developed on the assumption that there is no dependency or statistical correlation between the input variables. The methods as presented in this thesis can be extended to account for correlations and dependency among the input variables.

- This thesis develops uncertainty analysis and design optimisation methods for a single component systems. In real life systems can exist as multidisciplinary systems which involves the integration and interaction of several components. As several components are integrated the complexity of the model increases, and it becomes more challenging to access the predictive capability of such a multi-disciplinary system model. There can be a future extension of the methods described in this thesis for such multi-disciplinary systems.

9.8 Conclusions

This section presents the research conclusion and also summarised how the research aim and objectives stated in chapter 1 were achieved. The sections that follow outline the conclusions for the research objectives, and are as stated as follows:

- A critical review of the literature on the extent of researches carried out on design feature influence on the degradation life of machine components was presented. The review indicated that only relatively few studies with respect to this has been carried out. It was also seen from the literature review that even in this limited studies, the deterministic approach has been the majorly applied technique compared to the probabilistic techniques which are able to provide more realistic results. The review indicated the need for more studies concerning the influence of design on components degradation life. This research also identified the gaps in carrying out uncertainty analysis based on the use of evidence theory, and the application of evidence theory method to design optimisation. Though studies have been done using evidence theory in uncertainty quantification, these studies were generally restricted to only the uncertainties in the input parameters, without taking into consideration under sources such as those due to the use of a surrogate model.

- This research demonstrated the effectiveness of using complimentary probabilistic methods to study the influence of design features on the degradation life, in this case thermal fatigue life of a component.
- The novel uncertainty quantification developed proves that with the use of evidence theory, uncertainty from different sources in the the use of a model can be accounted. The proposed developed methodology showed that evidence theory can be used to determine the uncertainty in a system as a result of the inherent variation of the input parameters and the uncertainty in using a surrogate model in place of the actual system due to surrogate model prediction error.
- The successful application of the uncertainty quantification method to the optimisation of a component, the vented brake disc was demonstrated. The proposed optimisation methodology in the presence of mixed uncertainties gave results that were more robust, resilient to uncertainties in the system than the conventional deterministic optimisation. The optimisation method showed that component design optimisation can be achieved successfully without the use of the computationally difficult to obtain evidence based belief and plausibility functions.

In conclusion this research has successfully proposed novel approximate modelling methodology for studying the influence of design features on the degradation life of a component. This methodology consists of novel methods for determining the uncertainties, sensitivity and the optimisation of a component in the presence of uncertainties, which makes for a robust analysis of the influence of design on a component’s degradation life. Surrogate models were created and validated to show to the performance of the modelling methodology. The results obtained for the sensitivity analysis identified features for the components under study that are in agreement with previous studies. The robust optimal solutions obtained in the application of the uncertainty quantification methodology to the component optimisation demonstrates that the uncertainty and optimisation techniques developed in this research are quite capable of handling design problems with mixed uncertainty.

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APPENDICES

Appendix A : Interview Questions for Semi-Structured Knowledge Elicitation on Degradation Mechanisms in Real Life Applications.

Name:…………………………………………………………………………………

Job designation:………………………………………………………………………

Role in organisation:…………………………………………………………………

Years of experience in role:…………………………………………………………

Telephone Number:…………………………………………………………………

E-mail:…………………………………………………………………………………

The information provided in this interview would only be used for academic and research purposes. The details of the respondents would be treated with confidence and would not be disclosed. If you agree please tick the box:

I agree: ☐

Purpose of Interview:

The purpose of this study is to understand and characterise the damage evolution in brake discs. The major focus is to access how damage evolution in the brake disc is influenced by geometric design features. The interview shall take a semi-structured format in the knowledge elicitation process. The interview questions consists of a single section to answer in your own words.

Interview questions:

The aim of this set of questions is to capture the experts’ knowledge on the degradation mechanisms that affect the brake disc while in service and the factors that influence the degradation. The section permits the experts to provide their knowledge based on experience they have in maintaining brake discs. The produced transcribed questions and responses are those necessary to fulfil the research aim.

Q1. What are the types of damages that affect the brake discs?

Q2. Can you rank these damage modes in terms of criticality to brake discs life?

Q3. How are the damages identified?

Q4. How is the extent of damage measured?

Q5. Who is responsible for determining the brake discs examination criteria?

Q6. On what basis where these criteria established?

Q7. The root cause(s) of these damages are they known?

Q8. Has there been a formal study to determine the root cause(s) of the damages?

Q9. What is the average service life of the brake discs?

Q10. What are the materials used for the manufacturing of these brake discs?

Q11. Are the brake disc of the same design or they are they different in their design configurations?

Q12. Are particular kind of damages more common to a particular brake discs design?

Q13. In the design of brake discs what are the required design parameters?

Q15. Can these parameters be listed in terms of importance to brake discs design?

Q16. And if yes to the previous question, how would you list them in terms of importance to brake discs life?

Q17. Do these parameters affect brake discs life, and in what manner?

Q18. Are the brake discs repaired?

Q19. Is there any standard for the repair?

Transcripts:

The transcript provides the responses provided by the interviewed experts to the interview questions. The experts are designated as expert 1 and expert 2. Where the answers are similar a summary is provided. The details are as given below:

Q1. What are the types of damages that affect the brake discs?

Answer:

The typical damages are wear, cracking, distortion and corrosion.

Q2. Can you rank these damage modes in terms of criticality to brake discs life?

Answer:

Both experts provided the same ranking.

Cracks due to pad and disc surface friction and wear are the most critical.

Expert B: The brake disc surface is expected to wear, so crack formation is more critical to brake disc life.

Q3. How are the damages identified?

Answer:

Expert A: The damages are identified initially mainly by visual inspection.

Expert B: Visual inspection is used to identify the damage before a comprehensive assessment of the extent of damage is done.

Q4. How is the extent of damage measured?

Answer:

Expert A: The extent of the damages are measured in the maintenance job floor. There are standards and procedures, but this is done for instance for cracks by measuring the length of the crack using a caliper. The extent of wear is also measured using a measuring gauge for the disc thickness.

Q5. Who is responsible for determining the brake discs examination criteria?

Answer:

Expert A: Being a safety critical component, there are established standards for carrying out the damage inspection.

Expert B: The manufacturers do provide procedures for this.

Answer:

Q6. On what basis where these criteria established?

Expert B: I believe these criteria were developed based on experiments and maybe experience.

Q7. The root cause(s) of these damages are they known?

Answer:

Expert A: The root causes are known. Wear as we know is as a result of abrasive contact between the disc and pad surfaces. For cracks on the surface of the disc aside from the general known cause of thermal fatigue, the brake disc type in terms of its design is also contributory. This is based on experience obtained over the years on brake disc maintenance.

Expert B: Thermal solicitations are the major cause of cracks on the brake disc. As we know brake discs are expected to wear naturally. The concern is basically about the wear rate. The wear rate has to do with the brake material properties and the pad.

Q8. Has there been a formal study to determine the root cause(s) of the damages?

Answer:

Expert A: There has been a formal study to determine the crack formation, but the study was a metallographic analysis. The study could not provide a definite reason for the difference in crack formation between brake discs of different designs.

Q9. What is the average service life of the brake discs?

Answer:

The service life is usually less than the design life. The disc are used on different routes, experience different braking conditions. Hence, it is not possible to give the average service life.

Q10. What are the materials used for the manufacturing of these brake discs?

Answer:

The material used for making brake disc is mainly Grey cast Iron.

Q11. Are the brake disc of the same design or they are they different in their design configurations?

Answer:

Expert A: The brake disc based on manufacturers and also how they are mounted come in different designs.

Expert B: The designs are different, for instance you have completely circular brake discs, semi-circular etc. You also have designs with mounting holes on the surface and those without holes on the brake friction surface.

Q12. Are particular kind of damages more common to a particular brake discs design?

Answer:

Expert A: It's been observed that particular types of damages such as wear and disc cracking are more pronounced on some design configurations than on others.

Q13. In the design of brake discs what are the required design parameters in terms of the geometry?

Answer:

Expert B: The design parameters depends mainly on the type and size of the vehicle the brake disc would be mounted on. Small car designs are quite different from those of say trains.

Q14. Can these design parameters be listed in terms of importance to brake discs design?

Answer:

Expert A: I think the designs impact on the brake disc life differently, so some features in the design of the disc may contribute more to cracking for instance. Though we have not undertaken a formal study of that.

Expert B: Yes the geometric features does affect the disc life, and from experience they do so differently. A particular design may wear faster than another design but you would discover it has a longer life in terms of cracking. A scientific investigation of this should be undertaken.

Appendix B : Transcripts of Unstructured Interview with Participants C, D and E

The transcripts presented in this section reports on the unstructured (informal) interview had with participants C, D and E.

<u>Transcripts of unstructured interview with Participants C and D</u>

Question: What is the nature of your job?

Expert C: I am involved in the maintenance of axles and brake discs

Expert D: I am involved the damage inspection of brake disc

Question: How many years of experience do you have working this job?

Expert C: 5 years

Expert D: 2 years

Question: In your experience what do you think is the major cause of damage for putting a brake disc out of service?

Cracks are the major reasons for putting a brake disc out of service. Most of the discs that have been put out of service still has some life in terms of wear, but as a result of cracks they are taken out of service.

Question: What do you think could be the likely reasons for this?

Expert C: I have no idea.

Expert D: I cannot give an answer to that.

Question: Are all the brake discs of the same design?

They are not of the same designs.

Question: Do you think the difference in design has any effect on the cracking of the brake discs?

Expert C: The brake discs don’t crack at the same area for the different discs designs. So I believe the design has on influence on how the disc cracks.

Transcripts of unstructured interview with Participants E

Question: How is brake thermal behaviour analysis done?

Expert E: Brake thermal analysis can be carried out using either physical experiments by testing the thermal behaviour of the discs using dynamometers or with the use of computer experiments that is finite elements and computational fluid dynamics.

Question: What factors influence the thermal behaviour of the brake discs and of the methods you mentioned which would best be able to model these factors better?

Expert E: There are several factors that influences the thermal response of brake discs. These factors can be the brake disc material, the brake pad material, the type of contact between the brake pad and the brake disc friction surface, the thickness of the brake rotor, the groove in the brake disc. Brake thermal behaviour is in fact a complex process that no single method would be able to adequately represent it. The method of analysis chosen should be dependent on the aim of the study. But a combination of physical experiments and FEA/CFD analysis would provide more reliable results or understanding of brake disc behaviour.

Question: This study aims to study design influence on the thermal life of the brake disc, in your opinion what method do you think would best suit the requirements of this study?

Expert E: This is of course would require a parametric study. For a study of this kind the FEA/CFD approach would provide an alternative but fast and cheaper way to carry out a study of this nature. FEA and CFD have been shown to be effective in the analysis of brake disc thermal behaviour. Performing experiments would be expensive and time consuming.

Question: Can the results obtained from FEA/CFD analysis be considered to be reliable enough?

Expert E: In the use of FEA/CFD certain limiting assumptions have to be made. The making of these assumptions impact on the level of accuracy of these methods. Moreover, there are different approaches to brake disc modelling using FEA/CFD. The type of FEA modelling approach used also has an influence on the results gotten. The purpose of the analysis and the resources available can determine the modelling approach to use. FEA/CFD can be used also for initial exploration, and if need be supported with results from physical brake disc experiments and real life findings.

Appendix C : Grey Cast Iron Fatigue Life Modelling

Cast iron exhibits a markedly different inelastic behavior in tension and compression in comparison with other metals such as steel. Cast iron consists of a microstructure of graphite flakes distributed in a steel matrix. The stress strain response of cast iron particularly grey cast iron is controlled by the properties of the steel matrix, and more importantly the details of the graphite morphology (Downing and Socie, 1982). In tension the graphite flakes act as stress concentrators while in compression they act as stress transmitters. Hence, cast iron is brittle in tension, but in compression has a similar behavior to steel. Based on plots of the stress-strain curve of grey cast iron, it's been generally observed that the stress-strain plots of cast iron exhibits curvatures at relatively low stresses in comparison with 0.2% offset yield strength, and they display different hardening trends in tension and compression. According to James, Richard and John M.T (2003) they show a significant curvature from the initial increment of stress in their stress strain curve. As a result of this the stress strain response of cast iron cannot be ideally represented by the Ramberg-Osgood relationship which is used for wrought metals. Hence the requirement for a model to predict the fatigue life of cast iron based on its peculiar character. Downing, (1984) modified the standard Ramberg-Osgood equation which is given by:

$$\varepsilon = \frac{\sigma}{E} + \frac{\sigma}{(K)^{1/n}} \quad \textbf{(1)}$$

By proposing the Eqn. 1 to account for the curvature in the early part of the stress strain curve. In this modified equation a corresponding component of total strain called the secant strain, ε_s can be determined. The total strain is the sum of the secant strain and the remaining plastic strain as shown below:

$\varepsilon = \varepsilon_s + \varepsilon_r$

and

$$\varepsilon_r = \frac{\sigma}{(E_s)linear}$$

$$\varepsilon_r = \frac{\sigma}{(E_0 + m\sigma)} \qquad \textbf{(2)}$$

Characterising the plastic component by the standard power law, the cyclic stress strain curve becomes:

$$\varepsilon = \frac{\sigma}{E_0 + m\sigma} + \left(\frac{\sigma}{K}\right)^{1/n} \qquad \textbf{(3)}$$

E_0 is the initial secant modulus, the intercept of the initial straight line portion at zero stress, *M* is the slope of the secant modulus versus stress curve and it reflects the amount of the initial stress strain curve, *K* the cyclic strain hardening coefficient and *n* the cyclic strain hardening exponent.

By itself Eqn. 3 is not sufficient for modeling cyclic behavior, hence the requirement for additional information.

The additional information for the modeling the stress strain curve of cast iron can be realized by considering the following components of cast iron behavior;

- Symmetrical bulk effects of cast iron
- Effects of the internal surface of cast iron
- Effects of surface crack closure due to compressive stresses

(Fe-safe, 2002)

Accounting for the symmetrical bulk effects based on certain assumptions Downing (1983) suggested that the stress strain curve will have the same form as the monotonic curves. Therefore, the cyclic stress strain curve can then be represented as:

$$\varepsilon = \frac{\sigma_B}{E_0 + m_B\sigma_B} + \left(\frac{\sigma_B}{K_B}\right)^{1/n_b} \quad \textbf{(4)}$$

where σ_B is the bulk stress at a given strain, ε and m_B, K_B and n_b are material properties. And which on using Massing's hypothesis, the stress strain curve for the bulk stress can now be written as:

$$\Delta_\varepsilon = \frac{\Delta\sigma_B}{E_0 + 0.5m_B\Delta\sigma_B} + 2\left(\frac{\sigma_B}{2K_B}\right)^{1/n_B} \quad \textbf{(5)}$$

where Δ_ε strain is range and $\Delta\sigma_B$ is the bulk stress range.

The surface effects also have to be accounted for. As a result of crack like defects on the gray cast iron surface due to the surface graphite, the stiffness of the material changes. This surface effect becomes obvious when the material is unloaded from a tensile stress such that the unloading modulus, $E_a = \sigma/\varepsilon$ reduces as the peak tensile stress increases. This results in a reduction in the bulk cross-sectional area particularly with increasing peak stress. To account for this a dimensionless parameter is defined, A_{eff} , called the effective area can be estimated by:

$$A_{eff} = E_U / E_0 \quad \textbf{(6)}$$

where E_0 is the conventional elastic modulus.

Describing the cyclic stress-strain response with the functional dependence of the unloading modulus, E_U, on the maximum stress we obtain for each hysteresis loop that:

$$E_U = E_0 + m_U \sigma_{max} \quad \textbf{(7)}$$

Substituting Eqn.6 in Eqn.7 we obtain:

$$A_{eff} = 1 + \frac{m_U \sigma_{max}}{E_0} \quad \textbf{(8)}$$

During unloading as the stress reduces, open surface cracks closes and add compressive stress to the overall response. Based on this a crack closure stress is defined as:

$$\sigma_{cc} = Q\,(\varepsilon_{max} - \varepsilon)^q \quad \textbf{(9)}$$

where Q is the coefficient of crack closure and q is the exponent of crack closure.

The total stress for the stress-strain response of cast iron under cyclic loading is then defined by:

$$\sigma = A_{eff}(\sigma_B + \sigma_{max}) + \left(1 - A_{eff}\right)\sigma_{cc} \quad \textbf{(10)}$$

Downing (1984) in analyzing the stress-strain response of cast iron under cyclic loading proposed eight parameters. These parameters are obtained from the treatment of tension and compression responses of cast iron with two parallel but separate analysis. The parameters are E_0 and E_U, with K_T, m_T and n_T for the tension response and K_c, m_c and n_c for the compression responses.

To account for the role of graphite inclusions in cast iron fatigue life a damage model with a damage parameter based on crack length was proposed (Weinacht and Socie, 1987). The damage parameter is defined as:

$$D = \frac{a}{a_f} \tag{11}$$

where a is the length of the failure crack at given point and a_f the final failure crack length.

The life is then summed using a continuum damage model which is based on the consideration that the rate at which damage occur is not linear, but rather related to the accumulated damage. This model is given in Eqn.12

$$\Delta D = \frac{(1 - D_i)^{P_i}}{(P_i + 1)N_{fi}} \tag{12}$$

Where ΔD is the damage for the cycle

D_i is the accumulated damage

P_i is the damage rate parameter

N_{fi} is the cycle endurance

Pi for a cycle can be correlated with the SWT parameter, $\sigma_{max}\Delta\varepsilon/2$ to give:

$$P_i = 2.55(\sigma_{max}\Delta\varepsilon/2)^{-0.8} \tag{13}$$

Studies have shown that grey cast iron fatigue life based on the Downing method can be adequately modeled based on the use of the biaxial SWT parameter (Weinacht and Socie, 1987). This relationship between fatigue life of gray cast iron and SWT parameter according to (Fash and Socie, 1982) is then expressed as:

$$\frac{\sigma_{max}\Delta\varepsilon_t}{2} = A\left(N_f\right)^b \qquad \textbf{(14)}$$

Where A is the fatigue life coefficient, and b is the fatigue life exponent. The SWT parameter presents a ready mechanism for the inclusion of mean stresses into the fatigue analysis.