Simulate Cavitation Bubble with Single Component Multi-Phase Lattice Boltzmann Method

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Abstract

Cavitation occurs when the pressure drops below a critical value at which point it can cause great damage to the machines such as propellers. In this study, a two-dimensional single bubble with different pressure differences between the boundary and the bubble will be studied based on the single component Shan-Chen model with the Carnahan-Starling (C-S) Equation of State (EOS) incorporated, which is similar to the model in [1-2]. Firstly, the model with the C-S EOS will be validated based on Maxwell's equal area construction. The equilibrium density of liquid and vapor is obtained using a flat interface simulation according to [3]. It was demonstrated that the model has great thermal consistency according to this validation. Furthermore, we show results for a single bubble case for which its growth and collapse can be validated against the Rayleigh-Plesset (R-P) equation with various pressure differences. Results show good agreement with the R-P equation and literature.

Key words: *Lattice Boltzmann Method (LBM); Cavitation; Single Bubble, Multiphase*

1. Introduction

Cavitation is a phenomenon that includes liquid-to-vapour transition because of the decrease of pressure, which forms the number of vapor bubbles among liquids. It occurs in many hydrodynamic machines, the collapse of the cavitation bubble can cause high pressure and high temperature which is the reason for noise, vibration, and erosion. However, cavitation can be taken advantage of for enhancing heat transfer and drilling petroleum well because of micro jet induced by the collapse of the bubble [2].

Many numerical attempts have been made to simulate the cavitation bubble. Compared to the traditional macroscopic method based on the Navier-Stokes equation, LBM (Lattice Boltzmann Method) has its advantages in simulating the multiphase model for its simplicity and locality. In particular, the multiphase LBM method is a diffusive interface method that is not necessary to track the interface explicitly. Three are normally three models of multiphase LBM, which are the colour-gradient model, the Shan-Chen model, and the free energy model [4].

Among all the multiphase LBM, the Shan-Chen model is quite popular because of its simplicity and certain accuracy in bubble simulation. It is a method based on pseudo potential which is incorporated into additional Shan-Chen force in the Lattice Boltzmann equation. To increase the accuracy of the model, many attempts have been made, such as a new Equation of State (EOS) [5], a new force expression [6], and a new force incorporation scheme [7]. The new Equation of state method can greatly increase the density ratio from the original model which is widely used in the Shan-Chen model.

In this project, two-dimensional single bubble cavitation was simulated based on the Shan-Chen model with the C-S equation of state incorporated. In addition, two validations were conducted based on Maxwell's area construction and the Rayleigh-Plesset equation.

2. Problem description

In this project, a two-dimensional $D2Q9$ model was implemented based on Shan-Chen multiphase model. The additional force was considered based on velocity shifting method by change the equilibrium velocity without any modification of Lattice Boltzmann Equation, which can be expressed as [8]

$$f_i(\boldsymbol{x}+\boldsymbol{e}_i\Delta t, t+\Delta t) - f_i(\boldsymbol{x}, t) = -\frac{1}{\tau}\left[f_i(\boldsymbol{x}, t) - f_i^{eq}(\boldsymbol{x}, t)\right], \quad (1)$$

where f_i is the density distribution function in different directions, f_i^{eq} is the equilibrium density distribution function, τ is the relaxation time, $\boldsymbol{e}_i$ denotes the particle velocity, $\boldsymbol{x}$ and t are the space and time variables.

The equilibrium density distribution function can be expressed as [9]

$$f_i^{eq}(\boldsymbol{x},t) = \omega_i \rho(\boldsymbol{x})\left[1 + \frac{3\boldsymbol{e}_i \cdot \boldsymbol{u}}{c^2} + \frac{9(\boldsymbol{e}_i \cdot \boldsymbol{u})^2}{2c^4} - \frac{3\boldsymbol{u}^2}{2c^2}\right], \tag{2}$$

where the weights ω_i equals 4/9 (i = 0), 1/9 (i = 1-4), 1/36 (i = 5-9). According to Shan and Chen [10] the additional force can be expressed as

$$F(\boldsymbol{x},t) = -G\psi(\boldsymbol{x},t)\sum_{i=0}^{8} \omega_i \psi(\boldsymbol{x} + \boldsymbol{e}_i t, t) e_i, \tag{3}$$

where G indicates the interaction strength between particles and ψ is the effective density. In addition, the velocity shifting method was implemented because of stability [9] as

$$\boldsymbol{u}^{\mathrm{eq}} = \frac{1}{\rho}\left(\sum_i f_i \boldsymbol{c}_i + \tau \boldsymbol{F}\right). \tag{4}$$

Furthermore, the C-S equation of state is incorporated by changing the effective density [1]

$$\psi = \sqrt{\frac{2}{Gc_s^2}(p - \rho c_s^2)}, \tag{5}$$

with different pressure. The C-S equation of state [2]

$$P = \rho RT \frac{1 + b\rho/4 + (b\rho/4)^2 - (b\rho/4)^3}{(1 - b\rho/4)^3} - a\rho^2 \tag{6}$$

is implemented in this project with a = $0.4963R^2{T_c}^2/P_c$, b = $0.18727RT_c/P_c$. When a = 1, b = 4, R = 1, the critical temperature, pressure and density are T_c = 0.09433, P_c =0.00441644, and ρ_c = 0.13044. In this project, T/T_c is set to 0.75, and the simulation is assumed to be isothermal.

To validate the thermal consistency of the C-S equation of state, liquid and gas coexistence curve was shown compared with Maxwell's area construction. A flat interface simulation was performed to obtain the equilibrium density of liquid and gas according to Huang et al. [11].

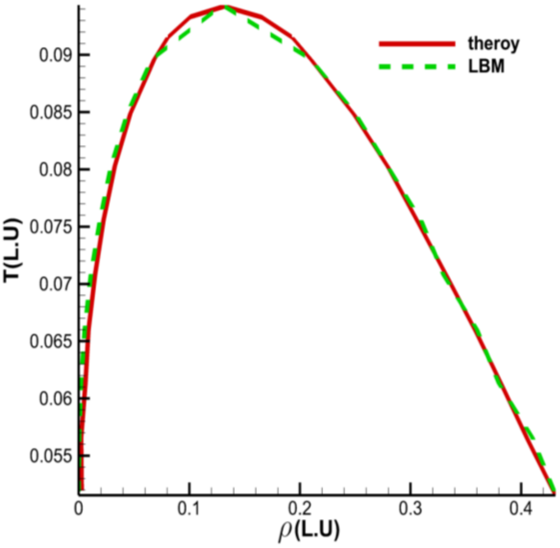
Furthermore, the evolution of bubble is also validated based on R-P equation. Because of the existence of boundary, the original infinite R-P equation is revised according to Peng et al. [12] as

$$\ln(R_{\mathrm{bound}}/R)(\dot{R}^2 + R\ddot{R}) - \frac{1 - (R/R_{\mathrm{bound}})^2}{2}\dot{R}^2 + \frac{2v}{\rho_{\mathrm{liquid}} R}\dot{R} + \frac{\sigma}{\rho_{\mathrm{liquid}} R} = \frac{P_{\mathrm{vapor}} - P_{\mathrm{bound}}}{\rho_{\mathrm{liquid}}}, \tag{7}$$

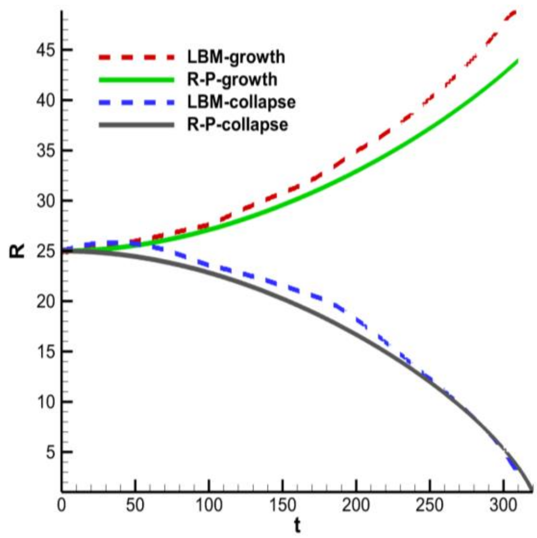
where R is the radius of the bubble, R_{bound} is the distance between the centre and the boundary. $\rho_{\mathrm{liquid}}, v, \sigma, P_{\mathrm{vapor}}, P_{\mathrm{bound}}$ are the density of liquid, kinematic viscosity, surface tension, vapour pressure inside the bubble and pressure on the boundary respectively. In addition, $\dot{R}$ and $\ddot{R}$ are the first order derivative and second order derivative of the bubble radius. The solution of the R-P equation is based on fourth order Runge-Kutta method. The surface tension is obtained by simulating single bubble with periodic boundary condition based on Young Laplace law.

3. Numerical results

Firstly, the equilibrium density of liquid and gas with different temperature was show by flat interface simulation based on this model by Figure 1 (a). For simplicity, the Maxwell's area construction was acquired according to Peng et al. [2]. It is demonstrated that the numerical solution and the theory are almost the same with less than 2.5% error. The liquid density of numerical solution is quite close to the analytical one expect the results with very low and high temperature. With very high temperature, the numerical solution have less results which cause minor discrepancy. On the other hand, the gas density on the left side a slightly bigger discrepancy with analytical solution. Considering the density ratio of LBM can simulate and the assumption of R-P equation with large density ratio, the reduced temperature equals 0.75 was chosen in this work.



(a)



(b)

Figure 1 (a). Equilibrium density of liquid and gas compared with Maxwell's area construction (b). Growth and collapse of bubble compared with R-P equation.

Secondly, the bubble growth and collapse is validated with different pressure on the boundary by Figure 1 (b). The density of liquid and gas in this project are 0.33 and 0.011 according to the equilibrium density of flat interface simulation.

As can be seen from Figure 1 (b), the numerical solution has very good agreement with theory one except the late stage of the growth and the early and middle stage of the collapse process. This is because the derived R-P equation is within a circular boundary while the LBM simulation is within a rectangular boundary. Furthermore, the contour plot of bubble growth and collapse is also shown by Figure 2 for better clarification of the validation of R-P.

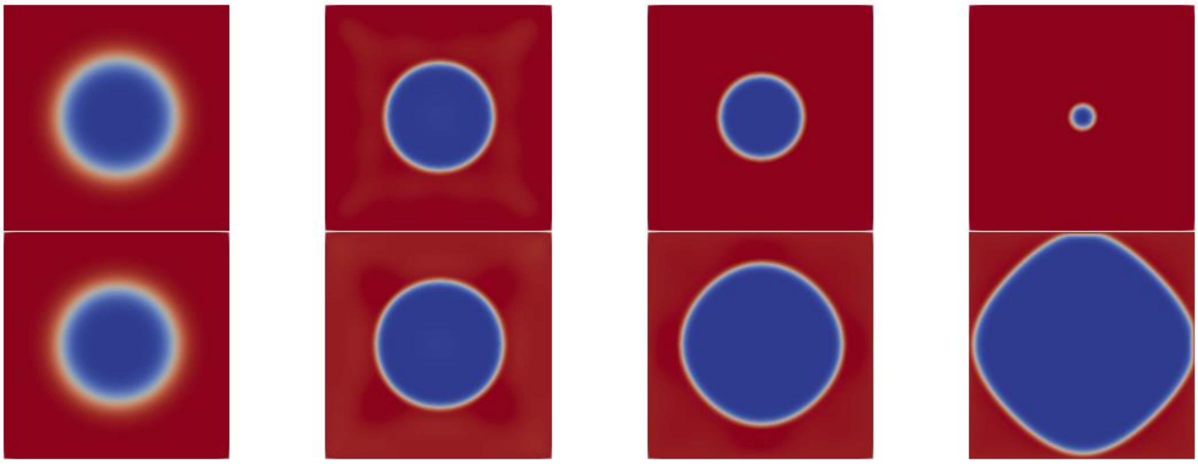


Figure 2. Contour plot of bubble growth and collapse.

4. Conclusions

According to previous results and discussion, the following conclusions can be drawn:

1. The single component Shan-Chen model incorporated with C-S equation have a good thermal consistency compared with the Maxwell's area construction.

2. The elvolution of bubble including growth and collapse has a good agreement with the revised R-P equation except the late stage of growth and early and middle stage of the collapse due to different boundary settings.

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