Supplementary File

A Fuzzy Decision Variables Framework Based on Directed Sampling for Large-scale Multi-objective Optimization

1 EXPERIMENTAL DETAILS

In order to make a fair comparison, all MOEAs in this paper use the parameter settings suggested in their original papers.

Algorithm's optimizer: Simulated binary crossover [4] and polynomial mutation [2] were used in the NSGA-II [3] and MOEA/D [5] as the reproduction operators. The crossover probability p_c and mutation probability p_m were set to 1 and $1/D$ (D indicates the dimension of the decision variables). The distribution indices of crossover and mutation were both set to 20. The LMOCSO adopted a competitive swarm optimizer and polynomial mutation.

Algorithm's parameter: MOEA/D adopted the boundary intersection approach (PBI) [5] as the aggregate function. For WOF, the optimization number of the transformed problem was set to 500 evaluations, whereas the optimization number of the original problem was set to 1000 evaluations, and the weight optimization task had chosen $M + 1$ solutions. The number of groups was four in the WOF. In LSMOF, the population size of the transferred problem $SubN$ was set to 30, and the number of reference solutions was set to 10. In DGEA, the number of reference vectors for offspring generation is 10. In LMOEADS, the clustering number Nw is 10, and the sampling solution number Ns is 30. In FDV, the fuzzy evolution rate $Rate$ is 0.8, and the step acceleration Acc is 1.2. The FDVDS proposed in this paper uses the same parameter settings as LMOEADS and FDV.

Problem's setting: We adopted the LSMOP [1] as the test problem to verify the effectiveness of our proposed algorithm. Table S1 contains all the settings of LSMOP. N represents the number of solutions. M and D represent the number of the objective function and the number of the decision variable, respectively. The function evaluation numbers, which is also called termination criteria, is denoted as FEs.

Table S1: The parameter settings of LSMOP.

Problems	N	M	D	FEs
LSMOP 1-9	100	2	500	250000
			1000	300000
	150	3	2000	400000
			5000	500000

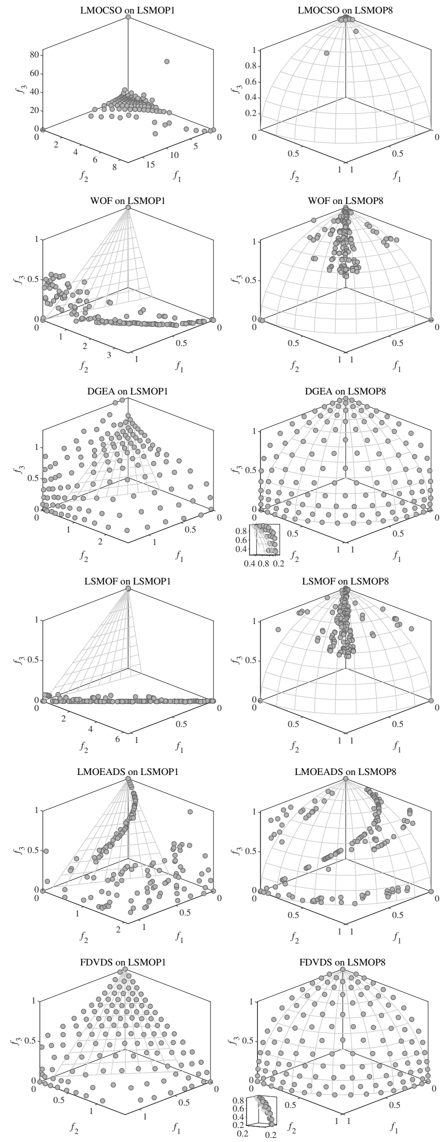

Figure S1: Non-dominated solutions of six large-scale MOEAs on the 3-objective, 1000-dimensional decision variables LSMOP1 and LSMOP8.

Table S2: The mean IGD statistics for the six algorithms on the 72 test problems of LSMOP, with the top-ranked results in each of the two columns, are highlighted.

Problem	M	D	FDV-NSGA-II	FDVDS-NSGA-II	FDV-MOEA/D	FDVDS-MOEA/D	FDV-LMOCSO	FDVDS-LMOCSO
LSMOP1	2	500	6.3682e-1 (9.96e-2) =	6.4017e-1 (1.56e-2)	3.0197e-1 (1.40e-1) -	1.5274e-1 (5.02e-3)	2.3991e-2 (5.38e-3) =	2.1818e-2 (3.67e-3)
		1000	1.7366e+0 (4.30e-1) -	6.5896e-1 (7.21e-3)	3.9921e-1 (1.77e-1) -	2.2945e-1 (2.28e-2)	8.8617e-2 (1.52e-2) =	7.2086e-2 (7.86e-3)
		2000	2.9141e+0 (2.64e-1) -	6.6080e-1 (7.15e-3)	1.1052e+0 (3.06e-1) -	6.5559e-1 (1.58e-2)	1.8650e-1 (1.39e-2) -	1.6031e-1 (2.17e-3)
		5000	4.8720e+0 (1.46e-1) -	6.6509e-1 (4.65e-3)	3.9234e+0 (5.02e-1) -	6.7041e-1 (3.73e-3)	2.6722e-1 (1.25e-2) -	2.4312e-1 (4.46e-3)
	3	500	1.4256e+0 (1.50e-1) -	3.6130e-1 (4.44e-2)	3.2588e-1 (3.53e-2) =	3.5037e-1 (2.27e-2)	2.3549e-1 (2.54e-2) -	1.9198e-1 (8.34e-3)
		1000	3.3858e+0 (3.25e-1) -	5.1288e-1 (2.27e-2)	8.7574e-1 (2.59e-1) =	7.3831e-1 (9.21e-2)	2.9525e-1 (1.44e-2) -	2.4887e-1 (6.45e-3)
		2000	5.4028e+0 (6.59e-1) -	5.6041e-1 (7.89e-3)	2.5176e+0 (8.05e-1) -	8.0036e-1 (1.10e-2)	3.1049e-1 (1.19e-2) -	2.7969e-1 (3.39e-3)
		5000	8.5389e+0 (4.86e-1) -	6.1757e-1 (6.10e-3)	4.7504e+0 (5.08e-1) -	8.0639e-1 (1.78e-2)	3.3518e-1 (8.16e-3) -	3.1470e-1 (2.69e-3)
LSMOP2	2	500	5.7474e-2 (1.05e-3) -	3.3274e-2 (6.81e-4)	5.3605e-2 (1.10e-3) -	2.7378e-2 (4.09e-4)	1.1512e-2 (2.15e-3) -	9.1064e-3 (1.50e-4)
		1000	3.5110e-2 (9.09e-4) -	1.9170e-2 (3.15e-4)	3.4738e-2 (5.86e-4) -	1.4673e-2 (5.15e-5)	7.5091e-3 (8.25e-4) -	6.5164e-3 (1.20e-4)
		2000	2.0860e-2 (2.45e-4) -	1.2619e-2 (1.91e-4)	1.9882e-2 (3.13e-4) -	7.7554e-3 (1.80e-5)	5.8582e-3 (7.66e-4) -	4.8016e-3 (2.17e-5)
		5000	1.0915e-2 (7.85e-5) -	9.7402e-3 (4.42e-4)	9.7252e-3 (2.52e-4) -	3.4817e-3 (2.61e-5)	3.9764e-3 (3.12e-5) =	3.9543e-3 (1.39e-5)
	3	500	7.3986e-2 (2.73e-3) =	7.6165e-2 (1.78e-3)	5.7925e-2 (7.12e-4) -	4.2258e-2 (2.86e-4)	4.0681e-2 (4.04e-4) -	3.9031e-2 (1.99e-4)
		1000	5.8104e-2 (2.47e-3) =	6.0791e-2 (3.25e-3)	4.4319e-2 (1.45e-4) -	2.4398e-2 (2.22e-4)	3.5976e-2 (2.10e-4) -	3.5398e-2 (1.03e-4)
		2000	5.1405e-2 (3.41e-3) =	5.1936e-2 (1.62e-3)	3.7635e-2 (1.98e-5) -	1.4735e-2 (8.08e-5)	3.4102e-2 (1.27e-4) -	3.3899e-2 (2.93e-5)
		5000	4.8296e-2 (2.15e-3) =	5.0725e-2 (2.19e-3)	3.4481e-2 (1.54e-5) -	9.3135e-3 (1.66e-4)	3.3278e-2 (2.24e-5) -	3.3237e-2 (1.65e-5)
LSMOP3	2	500	1.4810e+1 (1.47e+0) -	1.5678e+0 (1.07e-3)	7.5583e-1 (1.38e-2) -	7.1382e-1 (1.20e-2)	7.5279e-1 (1.53e-2) +	1.1055e+0 (7.01e-2)
		1000	1.8117e+1 (1.54e+0) -	1.5775e+0 (5.70e-4)	9.4524e-1 (4.03e-2) -	8.6200e-1 (3.18e-2)	8.7433e-1 (2.72e-2) +	1.3334e+0 (4.12e-2)
		2000	2.1418e+1 (9.25e-1) -	1.5820e+0 (5.49e-4)	1.2766e+0 (5.85e-2) -	1.1146e+0 (2.13e-2)	1.0099e+0 (2.90e-2) +	1.4731e+0 (2.55e-2)
		5000	2.7016e+1 (9.58e-1) -	1.5844e+0 (4.47e-4)	3.2514e+0 (1.61e-1) -	1.5851e+0 (4.06e-4)	1.4314e+0 (4.06e-2) +	1.5877e+0 (5.38e-3)
	3	500	7.2117e+0 (1.69e+0) -	8.3367e-1 (1.68e-2)	6.1628e-1 (1.67e-2) -	5.6126e-1 (1.85e-2)	8.0215e-1 (3.39e-2) =	7.9283e-1 (3.64e-2)
		1000	1.2586e+1 (2.55e+0) -	8.5989e-1 (1.81e-3)	8.0749e-1 (4.85e-2) -	6.7429e-1 (3.54e-2)	8.5058e-1 (3.42e-3) =	8.4763e-1 (1.79e-2)
		2000	1.5425e+1 (1.19e+0) -	8.6070e-1 (2.29e-5)	9.4770e-1 (9.44e-2) -	7.7333e-1 (1.99e-2)	9.4551e-1 (2.10e-1) =	8.6035e-1 (5.85e-4)
		5000	2.1488e+1 (3.25e+0) -	8.6071e-1 (1.18e-5)	1.7674e+0 (2.93e-1) -	8.6172e-1 (2.43e-3)	1.5124e+0 (4.78e-1) -	8.6072e-1 (1.22e-16)
LSMOP4	2	500	8.9877e-2 (1.17e-3) -	7.5917e-2 (2.72e-3)	7.0428e-2 (1.05e-3) -	6.6870e-2 (6.79e-4)	1.3201e-2 (1.38e-3) +	2.0696e-2 (1.16e-3)
		1000	6.0199e-2 (1.22e-3) -	5.3418e-2 (5.98e-4)	4.5446e-2 (1.61e-3) -	4.1117e-2 (6.99e-4)	1.5624e-2 (1.04e-3) =	1.6070e-2 (8.10e-4)
		2000	3.8268e-2 (5.03e-4) -	3.4222e-2 (8.61e-4)	2.8971e-2 (8.41e-4) -	2.3416e-2 (3.02e-4)	1.1105e-2 (2.46e-4) -	1.0779e-2 (1.07e-4)
		5000	1.8998e-2 (2.38e-4) -	1.7328e-2 (3.21e-4)	1.6238e-2 (5.25e-4) -	1.0666e-2 (4.78e-5)	6.2905e-3 (1.07e-4) -	6.0977e-3 (1.95e-5)
	3	500	1.9132e-1 (5.25e-3) +	2.0892e-1 (9.81e-3)	1.4226e-1 (4.14e-3) =	1.3878e-1 (1.32e-3)	8.6603e-2 (3.05e-3) -	8.1966e-2 (5.69e-4)
		1000	1.2095e-1 (2.58e-3) +	1.3364e-1 (4.74e-3)	9.3125e-2 (3.21e-3) -	8.0850e-2 (5.89e-4)	5.6542e-2 (8.18e-4) =	5.6016e-2 (6.31e-4)
		2000	8.1262e-2 (1.15e-3) +	8.4553e-2 (2.70e-3)	6.0325e-2 (7.83e-4) -	4.4762e-2 (2.19e-4)	4.1982e-2 (3.45e-4) =	4.1962e-2 (2.25e-4)
		5000	5.7682e-2 (3.87e-3) =	5.7471e-2 (2.25e-3)	4.1499e-2 (1.18e-4) -	2.0811e-2 (1.40e-4)	3.5403e-2 (5.20e-5) =	3.5382e-2 (7.30e-5)
LSMOP5	2	500	6.6769e-1 (1.09e-1) +	7.4209e-1 (1.22e-16)	4.2483e-1 (2.87e-3) -	1.9486e-1 (1.37e-2)	2.4857e-2 (1.13e-3) -	2.3049e-2 (2.61e-3)
		1000	1.5996e+0 (7.47e-1) -	7.4209e-1 (1.22e-16)	5.0612e-1 (1.24e-1) -	3.2230e-1 (1.28e-1)	5.5273e-2 (5.10e-3) -	4.5725e-2 (2.67e-3)
		2000	3.6370e+0 (2.34e+0) -	7.4209e-1 (1.22e-16)	1.4853e+0 (4.01e-1) -	7.4192e-1 (2.25e-4)	2.2615e-1 (1.26e-1) -	1.0331e-1 (1.58e-2)
		5000	1.0921e+1 (3.73e-1) -	7.4209e-1 (1.22e-16)	7.3509e+0 (1.08e+0) -	7.4198e-1 (6.01e-5)	7.0964e-1 (1.81e-1) -	4.5919e-1 (2.32e-2)
	3	500	3.2054e+0 (4.09e-1) -	3.7514e-1 (1.59e-3)	6.4181e-1 (1.50e-1) -	4.8302e-1 (3.82e-2)	2.4933e-1 (2.87e-2) -	2.1299e-1 (1.15e-2)
		1000	8.4070e+0 (3.73e-1) -	3.7912e-1 (3.36e-3)	7.3752e-1 (1.80e-1) -	6.2966e-1 (1.32e-1)	2.8851e-1 (2.34e-2) =	2.9468e-1 (2.56e-2)
		2000	1.3358e+1 (7.29e-1) -	4.2277e-1 (1.11e-2)	1.9185e+0 (3.04e-1) -	9.3729e-1 (1.58e-2)	3.2935e-1 (1.69e-2) =	3.1715e-1 (1.43e-2)
		5000	1.8974e+1 (4.22e-1) -	5.4082e-1 (5.03e-6)	5.6931e+0 (4.05e-1) -	9.4523e-1 (8.15e-4)	4.1890e-1 (6.94e-3) =	4.2858e-1 (1.10e-2)
LSMOP6	2	500	8.0405e-1 (1.06e-3) -	6.9476e-1 (1.00e-3)	7.6964e-1 (4.53e-3) -	6.7859e-1 (4.24e-3)	7.7043e-1 (1.87e-3) -	1.7606e-1 (6.86e-3)
		1000	7.7454e-1 (6.45e-4) -	6.6171e-1 (5.36e-2)	7.7015e-1 (4.88e-4) -	6.8177e-1 (1.24e-3)	7.5850e-1 (2.00e-4) -	1.7168e-1 (1.63e-3)
		2000	7.5730e-1 (8.69e-5) -	6.3167e-1 (1.08e-1)	7.5686e-1 (1.12e-4) -	6.4885e-1 (6.81e-2)	7.1276e-1 (5.98e-2) -	1.6899e-1 (3.33e-3)
		5000	7.4774e-1 (3.31e-13) -	5.7843e-1 (1.45e-1)	7.4774e-1 (4.89e-7) -	6.7070e-1 (3.96e-3)	7.4531e-1 (1.51e-5) -	1.6412e-1 (1.31e-3)
	3	500	6.7915e+0 (2.20e+0) -	1.2824e+0 (1.27e-3)	2.4620e+0 (6.10e-1) -	1.5955e+0 (7.10e-2)	7.6794e-1 (2.59e-1) =	6.1594e-1 (7.94e-2)
		1000	5.9292e+2 (4.51e+2) -	1.3093e+0 (5.60e-4)	2.6136e+0 (3.45e-1) -	1.6553e+0 (4.56e-2)	1.1070e+0 (7.95e-1) =	6.6935e-1 (4.59e-2)
		2000	3.0189e+3 (1.23e+3) -	1.3208e+0 (1.83e-3)	3.0407e+0 (3.37e-1) -	1.6917e+0 (4.15e-3)	6.7907e-1 (5.89e-2) =	6.8158e-1 (5.18e-2)
		5000	1.4844e+4 (2.25e+3) -	1.3285e+0 (1.94e-3)	5.0663e+0 (5.25e-1) -	1.7010e+0 (1.20e-3)	9.5072e-1 (4.61e-1) =	6.8244e-1 (4.94e-2)
LSMOP7	2	500	4.9632e+0 (3.17e-1) -	1.5032e+0 (1.57e-3)	4.1580e+0 (1.60e+0) -	1.5091e+0 (9.02e-4)	1.4785e+0 (5.26e-2) =	1.4999e+0 (6.38e-3)
		1000	8.0386e+0 (6.23e-1) -	1.5119e+0 (1.49e-3)	5.8784e+0 (1.20e+0) -	1.5155e+0 (9.39e-4)	1.7049e+0 (4.78e-1) -	1.5054e+0 (3.40e-3)
		2000	8.0363e+1 (1.05e+1) -	1.5157e+0 (4.11e-4)	1.5139e+1 (1.54e+1) -	1.5175e+0 (9.33e-4)	1.5130e+0 (1.61e-3) =	1.5105e+0 (2.81e-3)
		5000	3.6892e+3 (4.25e+2) -	1.5183e+0 (5.58e-4)	1.8823e+3 (2.19e+3) -	1.5192e+0 (8.45e-4)	1.5182e+0 (3.46e-3) =	1.5153e+0 (1.18e-3)
	3	500	1.2625e+0 (1.65e-2) -	8.7445e-1 (4.63e-3)	9.4671e-1 (9.26e-5) -	9.0983e-1 (1.19e-4)	8.8921e-1 (1.40e-1) =	8.3720e-1 (8.46e-4)
		1000	1.1015e+0 (3.04e-3) -	8.5526e-1 (2.13e-3)	9.4652e-1 (3.08e-5) -	9.1122e-1 (1.90e-4)	9.4648e-1 (5.14e-5) -	8.3833e-1 (8.70e-3)
		2000	1.0202e+0 (2.26e-3) -	8.4501e-1 (7.56e-4)	9.4636e-1 (2.81e-5) -	9.1184e-1 (2.20e-4)	9.4669e-1 (2.15e-2) -	8.4005e-1 (9.03e-4)
		5000	9.7292e-1 (2.08e-4) -	8.4047e-1 (5.45e-4)	9.4672e-1 (6.93e-5) -	9.1256e-1 (1.47e-4)	9.4592e-1 (2.25e-2) -	8.3901e-1 (7.18e-4)
LSMOP8	2	500	4.0418e-1 (3.92e-2) +	7.4209e-1 (1.22e-16)	2.3927e-1 (2.48e-1) =	1.2669e-1 (1.41e-2)	2.2185e-2 (4.26e-3) -	1.6563e-2 (1.82e-3)
		1000	1.5646e+0 (2.08e-1) -	7.4209e-1 (1.22e-16)	2.9955e-1 (2.24e-1) =	1.9100e-1 (6.08e-2)	3.0757e-2 (4.38e-3) -	2.5390e-2 (2.14e-3)
		2000	3.5721e+0 (2.87e-1) -	7.4209e-1 (1.22e-16)	3.2903e-1 (4.70e-2) =	3.2321e-1 (9.99e-2)	7.0496e-2 (1.35e-2) -	4.7803e-2 (9.21e-4)
		5000	7.9811e+0 (4.28e-1) -	7.4209e-1 (1.22e-16)	2.4399e+0 (3.04e-1) -	7.4182e-1 (3.32e-4)	2.5614e-1 (8.37e-2) -	1.6309e-1 (3.36e-3)
	3	500	5.0989e-1 (3.37e-2) -	3.5299e-1 (1.38e-2)	7.4488e-1 (8.04e-2) -	5.3547e-1 (5.61e-2)	7.3968e-2 (7.38e-3) -	6.2203e-2 (2.68e-3)
		1000	9.5760e-1 (2.37e-4) -	3.5794e-1 (5.75e-5)	7.1884e-1 (9.14e-2) -	5.6512e-1 (5.05e-2)	6.7156e-2 (1.29e-3) -	6.3255e-2 (7.70e-4)
		2000	9.5369e-1 (2.90e-4) -	3.4543e-1 (2.98e-2)	7.1513e-1 (8.25e-2) -	5.3990e-1 (6.26e-2)	6.5976e-2 (1.16e-3) -	6.2790e-2 (6.71e-4)
		5000	9.5106e-1 (1.66e-4) -	3.4959e-1 (1.90e-2)	6.5405e-1 (3.44e-2) -	5.1921e-1 (4.67e-2)	6.7283e-2 (6.24e-4) -	6.5311e-2 (3.51e-4)
LSMOP9	2	500	6.9706e-1 (2.14e-2) +	8.0931e-1 (7.61e-4)	4.2534e-1 (2.13e-2) -	2.1041e-1 (3.87e-3)	3.9199e-2 (1.11e-2) +	8.0688e-1 (2.51e-3)
		1000	6.2639e-1 (4.13e-3) +	8.0786e-1 (9.46e-4)	4.4885e-1 (1.91e-2) -	2.2780e-1 (7.56e-3)	5.0450e-2 (8.51e-3) +	7.9486e-1 (5.68e-3)
		2000	6.1545e-1 (3.23e-2) +	8.0560e-1 (6.66e-4)	5.6148e-1 (4.45e-2) -	2.5924e-1 (3.02e-2)	2.9468e-2 (3.79e-3) +	5.6777e-1 (1.70e-1)
		5000	3.8887e+0 (3.58e-1) -	8.0487e-1 (1.24e-3)	7.5113e+0 (1.73e+0) -	5.3972e-1 (3.72e-2)	1.0108e-1 (1.05e-1) +	5.0815e-1 (1.43e-1)
	3	500	1.7727e+0 (3.09e-1) -	1.5379e+0 (0.00e+0)	4.1230e-1 (5.08e-3) -	3.4994e-1 (3.78e-3)	4.1833e-1 (6.06e-2) +	5.9504e-1 (9.69e-4)
		1000	1.7164e+0 (5.11e-1) =	1.2105e+0 (1.60e-1)	4.3321e-1 (6.08e-3) -	3.5122e-1 (4.18e-3)	4.8310e-1 (2.37e-2) +	5.9627e-1 (9.74e-4)
		2000	5.1610e+0 (5.66e-1) -	1.0519e+0 (2.27e-1)	4.3008e-1 (1.39e-2) -	3.5166e-1 (1.37e-3)	6.0562e-1 (5.07e-2) =	5.9536e-1 (1.08e-3)
		5000	2.4846e+1 (1.30e+0) -	1.0515e+0 (2.27e-1)	9.6500e+0 (1.43e+0) -	3.8270e-1 (1.18e-2)	1.1005e+1 (1.32e+0) -	5.9580e-1 (5.27e-4)
+/ - / =			8/57/7		0/65/7		11/39/22	

Table S3: Six large-scale MOEAs were used to conduct experiments on 54 test problems of LSMOP to compare the mean IGD values statistically. The best results are highlighted in each row.

Problem	M	D	LMOCSO	WOF	DGEA	LSMOF	LMOEADS	FDVDS-CSO
LSMOP1	2	1000	1.2274e+0 (6.30e-2) -	6.1493e-1 (3.07e-2) -	3.0235e-1 (1.09e-2) -	6.3406e-1 (2.42e-2) -	3.0054e-1 (5.99e-3) -	7.2086e-2 (7.86e-3)
		2000	1.3890e+0 (6.75e-2) -	5.8679e-1 (4.06e-2) -	3.2492e-1 (1.07e-2) -	6.4328e-1 (1.94e-2) -	3.0225e-1 (5.75e-3) -	1.6031e-1 (2.17e-3)
		5000	1.6225e+0 (3.44e-2) -	5.8480e-1 (5.18e-2) -	3.3247e-1 (1.22e-2) -	6.5339e-1 (8.12e-3) -	3.1974e-1 (5.72e-3) -	2.4312e-1 (4.46e-3)
	3	1000	1.2755e+0 (7.36e-2) -	5.0151e-1 (5.86e-2) -	4.2944e-1 (9.33e-2) -	5.4439e-1 (7.82e-3) -	4.3596e-1 (7.53e-3) -	2.4502e-1 (5.00e-3)
		2000	1.3830e+0 (5.42e-2) -	5.7136e-1 (1.55e-2) -	3.7862e-1 (1.21e-2) -	5.7503e-1 (5.65e-3) -	4.4835e-1 (1.22e-2) -	2.7969e-1 (3.39e-3)
		5000	1.4520e+0 (3.13e-2) -	6.0361e-1 (2.94e-2) -	3.8335e-1 (1.60e-2) -	6.2363e-1 (1.31e-2) -	4.6024e-1 (5.42e-3) -	3.1470e-1 (2.69e-3)
LSMOP2	2	1000	2.4638e-2 (6.70e-4) -	1.8894e-2 (2.11e-4) -	8.2655e-3 (2.82e-4) -	1.8975e-2 (2.78e-4) -	8.1549e-3 (2.32e-4) -	6.5164e-3 (1.20e-4)
		2000	1.4315e-2 (1.65e-4) -	1.2366e-2 (2.19e-4) -	5.4810e-3 (6.80e-5) -	1.2697e-2 (1.51e-4) -	5.7673e-3 (1.78e-4) -	4.8016e-3 (2.17e-5)
		5000	7.5403e-3 (1.38e-4) -	9.4817e-3 (2.25e-4) -	4.1617e-3 (6.90e-5) -	9.8015e-3 (3.50e-4) -	4.5548e-3 (1.85e-4) -	3.9543e-3 (1.39e-5)
	3	1000	4.1215e-2 (3.41e-4) -	6.1112e-2 (1.99e-3) -	3.8495e-2 (4.84e-4) -	6.2553e-2 (3.14e-3) -	3.9441e-2 (1.25e-3) -	3.5448e-2 (6.14e-5)
		2000	3.6689e-2 (1.16e-4) -	5.5258e-2 (4.22e-3) -	3.5114e-2 (1.33e-4) -	5.5552e-2 (3.28e-3) -	3.6832e-2 (6.90e-4) -	3.3899e-2 (2.93e-5)
		5000	3.4427e-2 (2.01e-5) -	5.0996e-2 (3.38e-3) -	3.3810e-2 (7.97e-5) -	5.0916e-2 (1.54e-3) -	3.4866e-2 (8.15e-4) -	3.3237e-2 (1.65e-5)
LSMOP3	2	1000	7.1909e+0 (4.69e+0) -	1.2201e+0 (8.78e-2) =	1.5437e+0 (8.02e-2) -	1.5732e+0 (6.54e-4) -	1.5726e+0 (5.79e-4) -	1.3334e+0 (4.12e-2)
		2000	1.3056e+1 (3.39e+0) -	1.3631e+0 (4.77e-2) +	1.3421e+0 (1.50e-1) =	1.5768e+0 (3.89e-4) -	1.5760e+0 (3.86e-4) -	1.4731e+0 (2.55e-2)
		5000	1.3606e+1 (6.68e+0) -	1.5843e+0 (5.27e-3) =	1.9611e+0 (5.43e-1) =	1.5791e+0 (1.59e-4) =	1.5781e+0 (2.92e-4) +	1.5877e+0 (5.38e-3)
	3	1000	1.2904e+1 (1.76e+0) -	8.5744e-1 (8.04e-3) =	8.4700e-1 (1.29e-1) =	8.4606e-1 (5.97e-3) =	8.6044e-1 (4.58e-5) =	8.4328e-1 (1.07e-2)
		2000	1.2478e+1 (4.73e-1) -	8.6049e-1 (5.68e-4) =	1.4236e+0 (1.43e+0) =	8.5855e-1 (2.31e-3) =	8.6036e-1 (5.42e-4) =	8.6035e-1 (5.85e-4)
		5000	1.2783e+1 (6.95e-1) -	8.6071e-1 (1.51e-5) =	2.7449e+0 (3.14e+0) =	8.6060e-1 (2.98e-5) +	8.6062e-1 (1.83e-4) =	8.6072e-1 (1.22e-16)
LSMOP4	2	1000	5.1924e-2 (4.50e-4) -	4.2272e-2 (2.85e-3) -	2.5741e-2 (2.16e-3) -	3.7251e-2 (9.97e-4) -	2.1847e-2 (3.06e-4) -	1.6070e-2 (8.10e-4)
		2000	3.0019e-2 (1.27e-4) -	3.1604e-2 (2.65e-3) -	1.3684e-2 (7.72e-4) -	2.8784e-2 (1.89e-3) -	1.2622e-2 (2.11e-4) -	1.0779e-2 (1.07e-4)
		5000	1.4888e-2 (7.33e-5) -	1.7581e-2 (4.73e-4) -	6.9911e-3 (1.44e-4) -	1.6440e-2 (6.84e-4) -	6.8898e-3 (1.47e-4) -	6.0977e-3 (1.95e-5)
	3	1000	9.1660e-2 (7.53e-4) -	1.3423e-1 (3.14e-3) -	6.6417e-2 (1.39e-3) -	1.3682e-1 (2.77e-3) -	6.8208e-2 (1.91e-3) -	5.5682e-2 (5.38e-4)
		2000	5.9010e-2 (2.92e-4) -	8.6849e-2 (5.56e-3) -	4.6009e-2 (5.94e-4) -	8.7725e-2 (8.75e-4) -	4.9442e-2 (8.77e-4) -	4.1962e-2 (2.25e-4)
		5000	4.0803e-2 (5.64e-5) -	5.9075e-2 (1.25e-3) -	3.6741e-2 (2.04e-4) -	6.0654e-2 (3.61e-3) -	3.8647e-2 (9.88e-4) -	3.5382e-2 (7.30e-5)
LSMOP5	2	1000	2.6260e+0 (1.72e-1) -	5.5602e-1 (2.04e-1) -	1.7615e+0 (1.51e+0) -	7.4209e-1 (1.22e-16) -	7.4208e-1 (9.72e-6) -	4.5725e-2 (2.67e-3)
		2000	2.9295e+0 (1.21e-1) -	3.4720e-1 (1.33e-2) -	7.4209e-1 (2.98e-7) -	7.4209e-1 (1.22e-16) -	7.4208e-1 (1.11e-5) -	1.0331e-1 (1.58e-2)
		5000	3.3598e+0 (9.42e-2) -	6.6050e-1 (9.55e-2) -	7.4209e-1 (1.22e-16) -	7.4209e-1 (1.22e-16) -	7.4208e-1 (1.75e-5) -	4.5919e-1 (2.32e-2)
	3	1000	2.4248e+0 (1.20e-1) -	5.3568e-1 (4.52e-3) -	5.0643e-1 (1.39e-2) -	5.4068e-1 (3.35e-4) -	5.0481e-1 (8.23e-3) -	2.8716e-1 (2.04e-2)
		2000	2.7348e+0 (8.92e-2) -	5.3493e-1 (5.13e-3) -	5.1538e-1 (6.72e-3) -	5.3994e-1 (2.17e-3) -	5.0649e-1 (6.89e-3) -	3.1715e-1 (1.43e-2)
		5000	3.1583e+0 (7.16e-2) -	5.3590e-1 (2.89e-3) -	5.2403e-1 (6.93e-3) -	5.4082e-1 (9.08e-6) -	5.1745e-1 (1.70e-3) -	4.2858e-1 (1.10e-2)
LSMOP6	2	1000	7.6913e-1 (3.70e-3) -	5.3241e-1 (1.51e-1) -	5.7282e-1 (2.88e-1) -	3.1243e-1 (2.85e-4) -	3.1282e-1 (2.46e-3) -	1.7168e-1 (1.63e-3)
		2000	7.5676e-1 (7.81e-4) -	6.2292e-1 (1.17e-1) -	5.5733e-1 (2.99e-1) =	3.0878e-1 (7.65e-5) -	3.0826e-1 (6.08e-4) -	1.6899e-1 (3.33e-3)
		5000	7.4774e-1 (6.04e-15) -	6.5289e-1 (4.48e-2) -	7.4531e-1 (7.20e-5) -	3.0700e-1 (7.44e-6) -	3.0787e-1 (2.19e-3) -	1.6412e-1 (1.31e-3)
	3	1000	9.7419e+1 (2.62e+1) -	1.3064e+0 (1.16e-3) -	7.6250e+0 (8.27e+0) =	7.3639e-1 (1.05e-2) -	7.6400e-1 (2.29e-2) -	6.8449e-1 (4.07e-2)
		2000	1.4480e+2 (1.92e+1) -	1.3202e+0 (1.56e-3) -	1.8252e+1 (1.64e+1) =	7.3158e-1 (1.27e-2) -	7.6972e-1 (4.16e-2) -	6.8158e-1 (5.18e-2)
		5000	2.8165e+2 (3.69e+1) -	1.3279e+0 (1.77e-3) -	8.2367e-1 (4.38e-1) =	8.1359e-1 (8.09e-2) -	7.7582e-1 (4.96e-2) -	6.8244e-1 (4.94e-2)
LSMOP7	2	1000	3.3345e+2 (4.18e+1) -	1.5125e+0 (1.69e-3) -	1.2978e+2 (3.14e+2) -	1.5094e+0 (9.93e-4) =	1.5097e+0 (4.28e-4) -	1.5054e+0 (3.40e-3)
		2000	6.4589e+2 (4.11e+1) -	1.5165e+0 (1.07e-3) -	1.5156e+0 (6.33e-4) -	1.5130e+0 (4.44e-4) =	1.5136e+0 (1.88e-4) -	1.5105e+0 (2.81e-3)
		5000	1.2131e+3 (2.62e+2) -	1.5187e+0 (1.18e-3) -	5.8902e+1 (1.41e+2) -	1.5155e+0 (6.37e-4) =	1.5159e+0 (4.92e-5) =	1.5153e+0 (1.18e-3)
	3	1000	1.0268e+0 (3.03e-2) -	8.5319e-1 (2.80e-3) -	9.0748e-1 (5.70e-2) -	8.6190e-1 (2.82e-3) -	8.5635e-1 (1.06e-3) -	8.3832e-1 (7.14e-3)
		2000	9.9122e-1 (1.19e-2) -	8.4286e-1 (8.68e-4) -	8.5298e-1 (1.25e-1) =	8.4853e-1 (1.36e-3) -	8.3591e-1 (1.92e-2) =	8.4005e-1 (9.03e-4)
		5000	9.6547e-1 (3.16e-3) -	8.3811e-1 (4.31e-4) =	8.3943e-1 (1.05e-1) =	8.4143e-1 (4.78e-4) -	8.3768e-1 (3.82e-4) +	8.3901e-1 (7.18e-4)
LSMOP8	2	1000	1.5198e+0 (4.34e-2) -	4.9904e-1 (1.93e-1) -	7.2309e-1 (1.56e-2) -	7.4209e-1 (1.22e-16) -	5.8981e-1 (3.45e-2) -	2.5390e-2 (2.14e-3)
		2000	1.9029e+0 (9.09e-2) -	4.3680e-1 (1.53e-1) -	7.4294e-1 (1.35e-3) -	7.4209e-1 (1.22e-16) -	7.3967e-1 (4.90e-3) -	4.7803e-2 (9.21e-4)
		5000	2.4256e+0 (7.94e-2) -	6.3332e-1 (6.95e-2) -	1.1311e+0 (9.61e-1) -	7.4209e-1 (1.22e-16) -	7.4208e-1 (8.94e-6) -	1.6309e-1 (3.36e-3)
	3	1000	7.5217e-1 (1.55e-1) -	3.2263e-1 (3.30e-2) -	1.2072e-1 (2.76e-2) -	3.3833e-1 (1.63e-2) -	1.7885e-1 (5.46e-2) -	6.3474e-2 (1.27e-3)
		2000	7.1741e-1 (1.88e-1) -	3.1895e-1 (3.56e-2) -	1.0096e-1 (2.10e-2) -	3.3823e-1 (1.75e-2) -	1.4937e-1 (9.20e-3) -	6.2790e-2 (6.71e-4)
		5000	6.8805e-1 (2.01e-1) -	3.1388e-1 (1.97e-2) -	3.7755e-1 (1.65e-1) -	3.4048e-1 (1.29e-2) -	1.4361e-1 (8.22e-3) -	6.5311e-2 (3.51e-4)
LSMOP9	2	1000	4.9352e-1 (1.15e-2) +	8.0782e-1 (1.19e-3) -	7.0568e-1 (8.94e-2) =	8.0645e-1 (3.75e-4) -	5.5764e-1 (6.83e-2) +	7.9486e-1 (5.68e-3)
		2000	8.6194e-1 (1.32e-1) -	8.0558e-1 (1.78e-3) -	2.3057e+0 (1.14e+0) -	8.0494e-1 (5.32e-4) -	3.8837e-1 (1.24e-1) =	5.6777e-1 (1.70e-1)
		5000	4.6975e+0 (1.45e+0) -	8.0140e-1 (9.32e-3) -	8.4812e+0 (1.50e+0) -	8.0304e-1 (6.38e-4) -	5.1362e-1 (1.14e-1) =	5.0815e-1 (1.43e-1)
	3	1000	3.2634e+1 (2.14e+1) -	1.1450e+0 (2.88e-5) -	9.5986e+0 (1.86e+0) -	1.4067e+0 (2.03e-1) -	5.7677e-1 (3.49e-3) +	5.9427e-1 (3.57e-4)
		2000	1.0713e+2 (4.38e+1) -	1.1445e+0 (2.68e-4) -	1.6779e+1 (3.43e+0) -	1.2098e+0 (1.61e-1) -	5.7555e-1 (2.22e-3) +	5.9536e-1 (1.08e-3)
		5000	1.3746e+2 (2.32e+1) -	1.1439e+0 (5.53e-4) -	2.9480e+1 (1.41e+1) -	1.1441e+0 (1.12e-4) -	5.7709e-1 (3.19e-3) +	5.9580e-1 (5.27e-4)
+/ - / =			1/53/0	1/47/6	0/42/12	1/47/6	6/41/7	

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