On wall-ice accretion or melting in shear flow

INI presentation 30th September 2022

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1. Background

2. Pre-impact behaviour

3. Droplets, impacts and ice growth

4. Melting wall-ice in shear flow

Collaborators:

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Manolo Quero – Spain
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Phil Wilson – New Zealand
John Elliott – Hull
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Samire Yazar – Turkey
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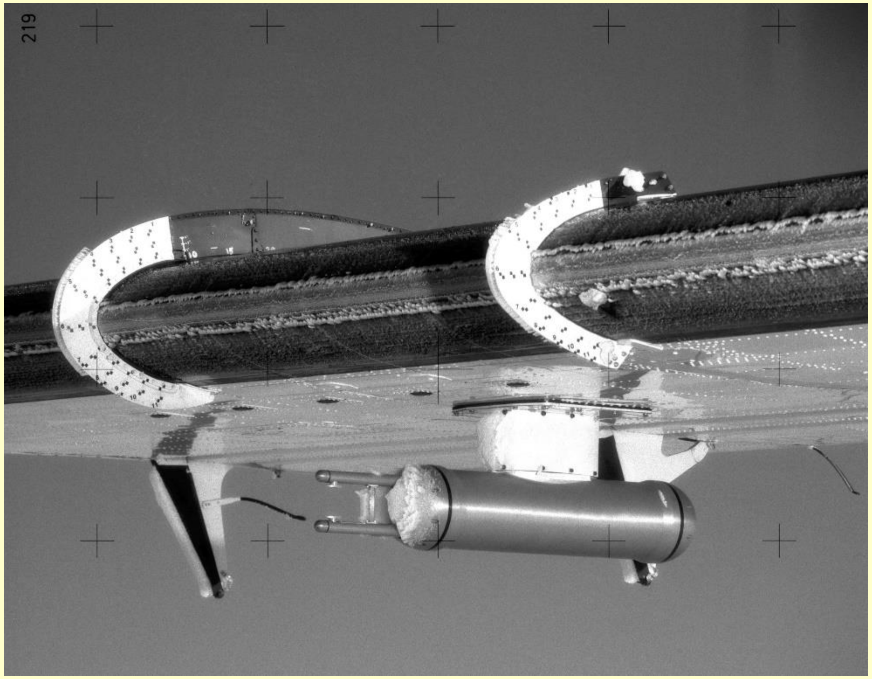
And current UCL PhD students:

Ellen Jolley
Nat Henman
T D Dang
Qingsong Liu

*Thanks to EPSRC, UCL, Totalsim, **Aerotex***

1. Background / aircraft icing



•Icing occurs during flight through clouds at or below freezing

•Supercooled water droplets impinge upon forward facing parts of the vehicle

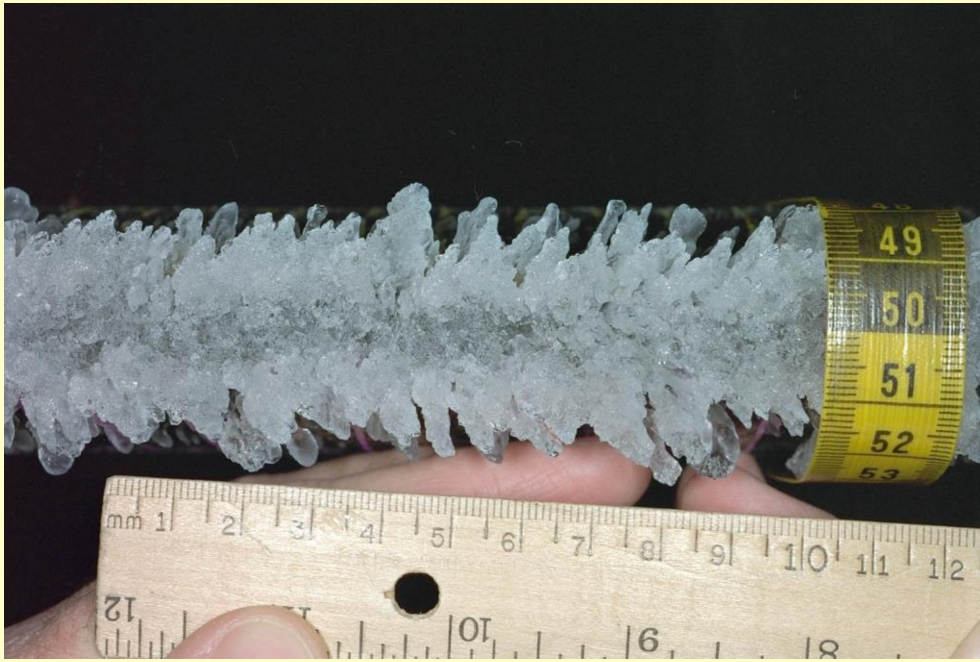
•This water then freezes, either immediately on impact or after spreading back along the wing

Engine icing

Modelling icing

There are many important parameters including:

- Droplet size (or range of sizes)
- Air Temperature
- Air Speed
- Water content of the cloud
- Local geometry

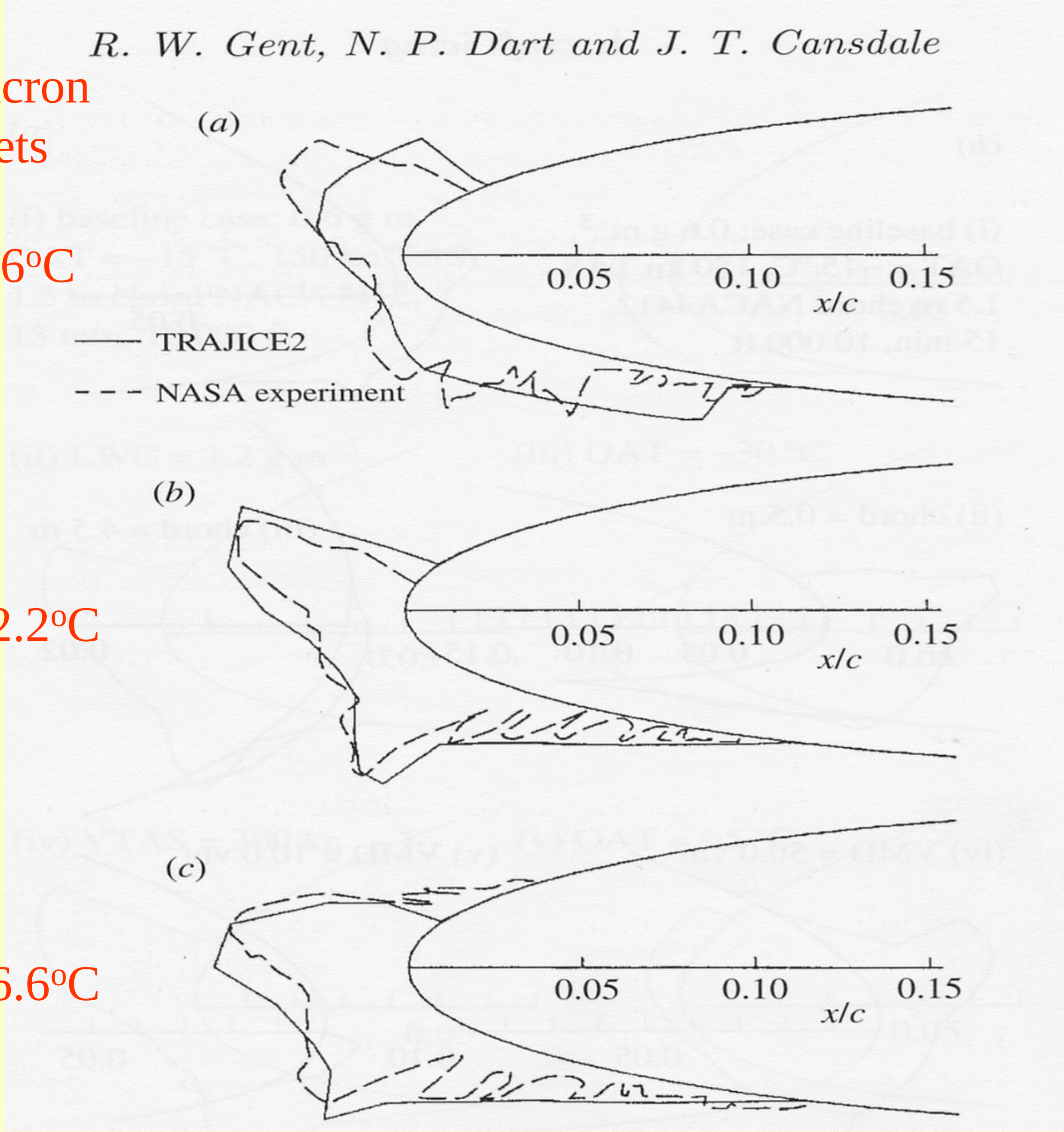


20 micron droplets

T=-6.6^{o}C

T=-12.2^{o}C

T=-16.6^{o}C

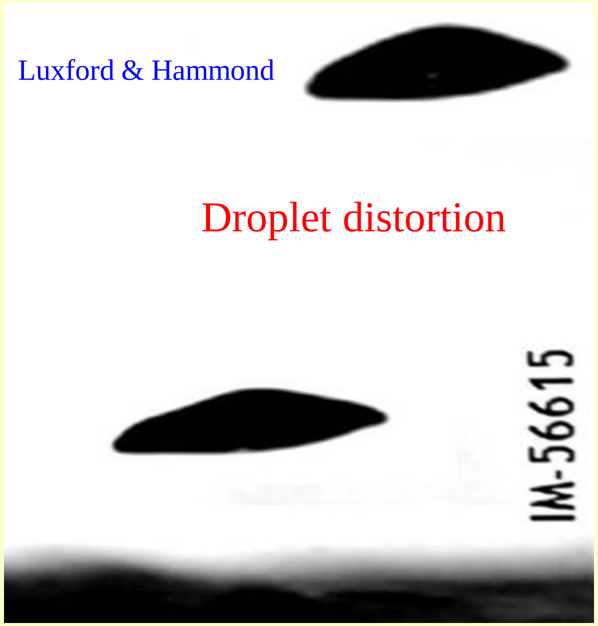


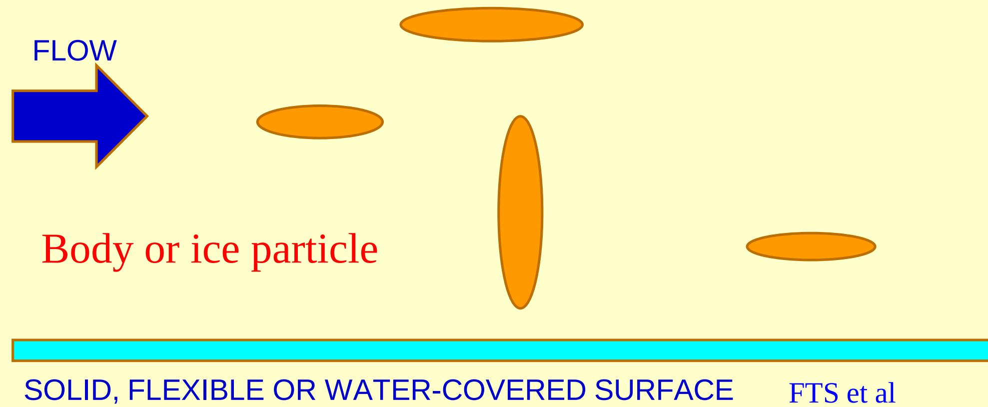
Existing models work well for small droplets (<40 microns)

ALSO ALLOW FOR WATER LAYERS AND ROUGH SURFACES

Luxford & Hammond

Droplet distortion

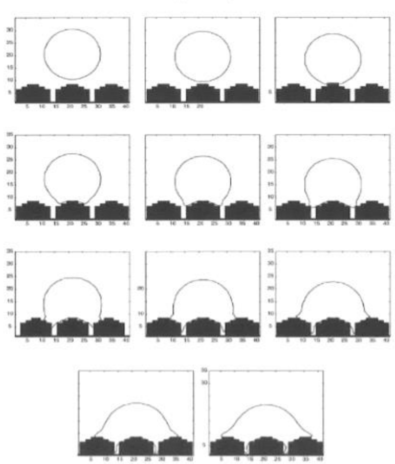




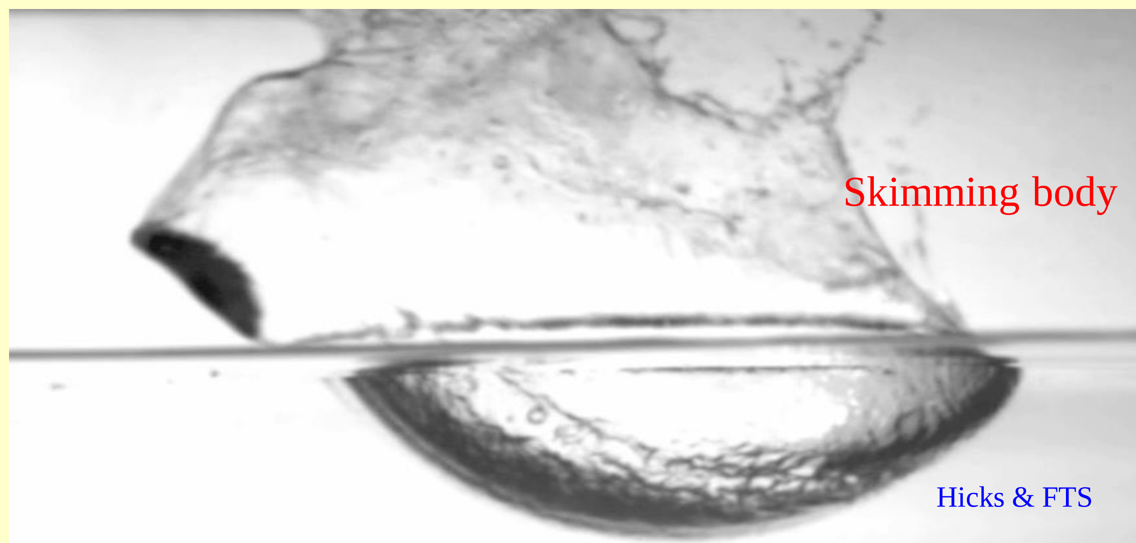
Body or ice particle

FTS et al

Droplet onto roughness



Ellis & FTS



Skimming body

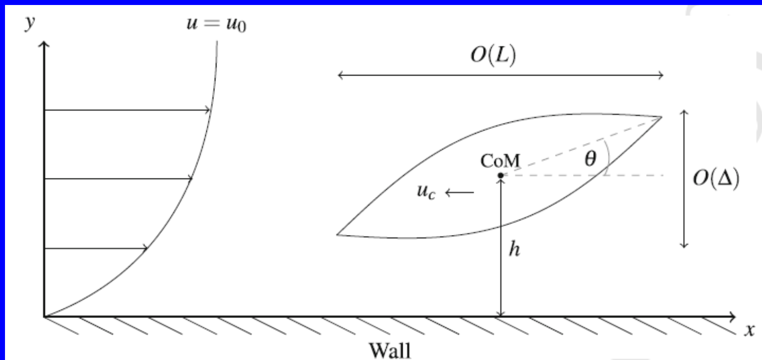
Hicks & FTS

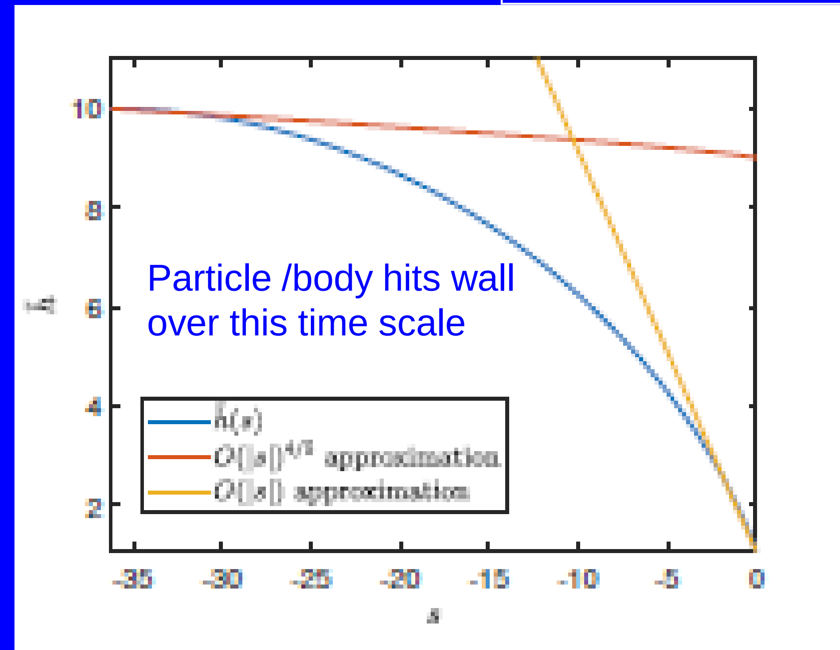
2. Pre-impact behaviour -- e.g. particles in boundary layers

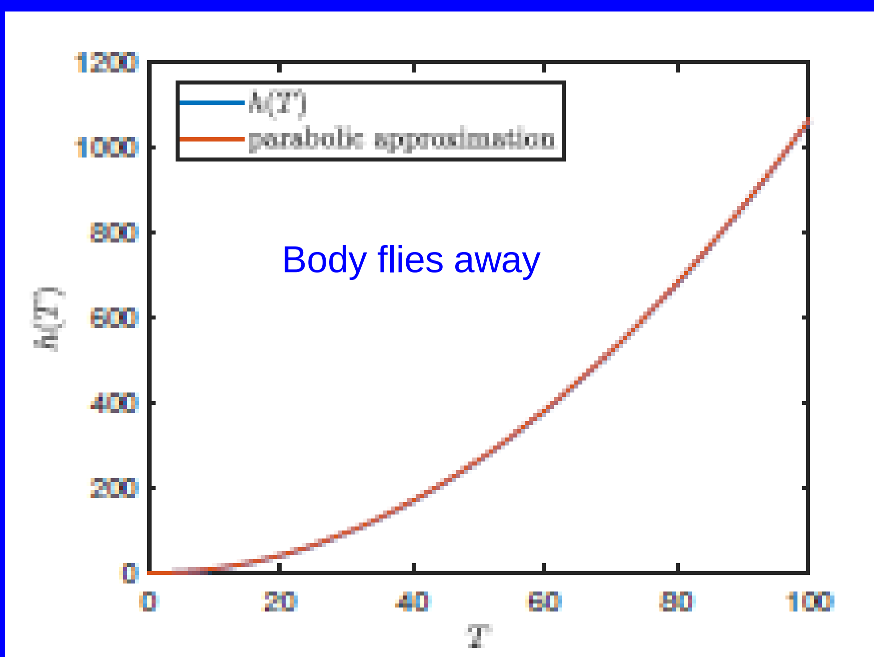
➔ Showing some recent and ongoing work on particles (Ellen Jolley, Ryan Palmer) and droplets (Nat Henman, Manish Tiwari)

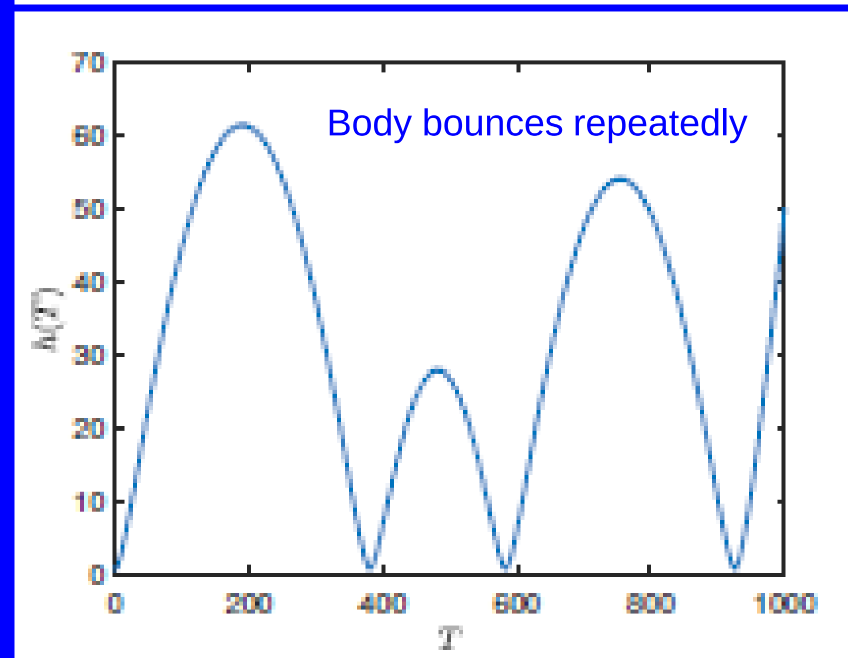
Thin particle travelling through boundary layer

with Ellen Jolley 2022 JFM

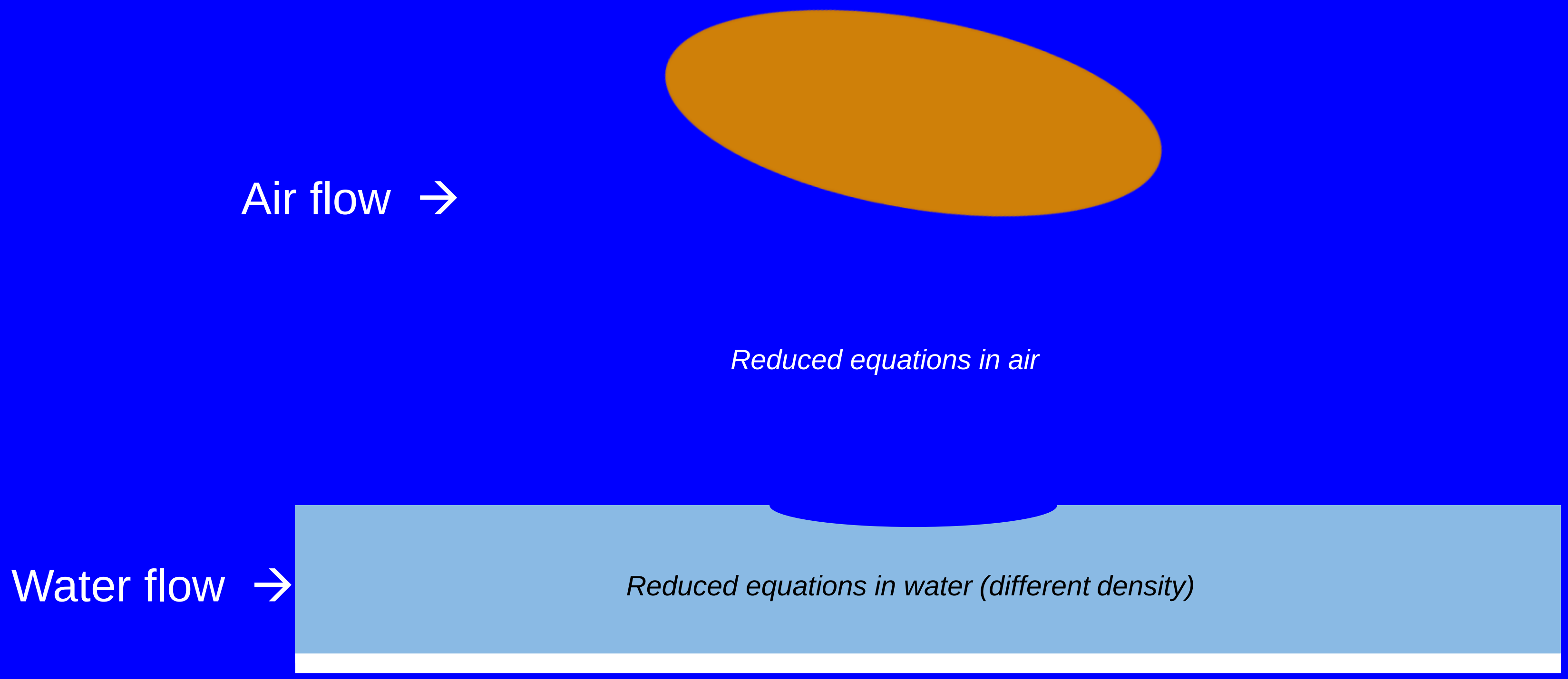








Allowing for thin water layer

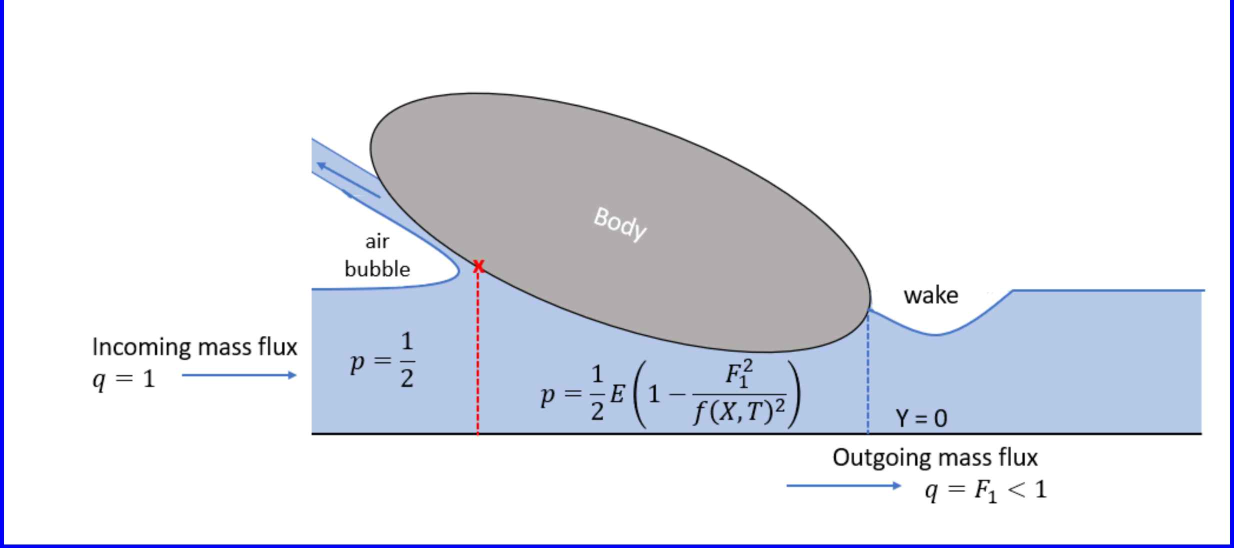


The body may fly away (as seen previously)

Or it may skim (as below)

Or it may hit the substrate and bounce

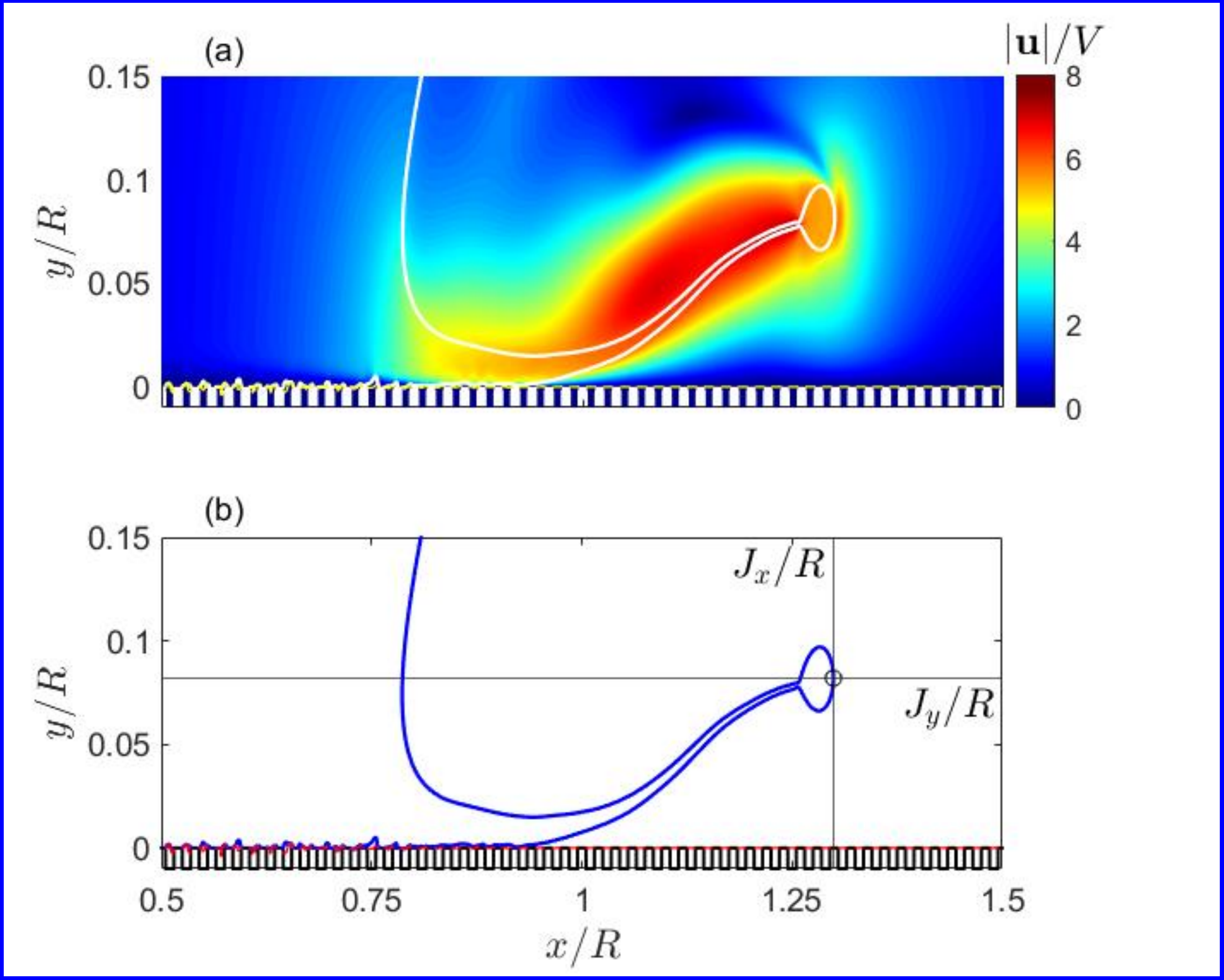
Or it may sink



Works with Ellen Jolley and Ryan Palmer

Free-surface solution for a droplet impact onto a surface

(a) droplet free-surface (white), lubricant free-surface (yellow), velocity magnitude field (colour).
(b) droplet free-surface (blue), lubricant free-surface (red), surface textures (black), with splash jet tip coordinates.



With Nat Henman and Manish Tiwari

3. Droplets, impacts and ice growth

- WATER:-

$div\ \underline{u} = 0$

$\underline{u}_t + (\underline{u}.grad)\underline{u} = -\ gradp/\rho + \underline{g} + \nu(T)\,\Delta\,\underline{u}$

$T_t + (\underline{u}.grad)T = k\,\Delta\,T$

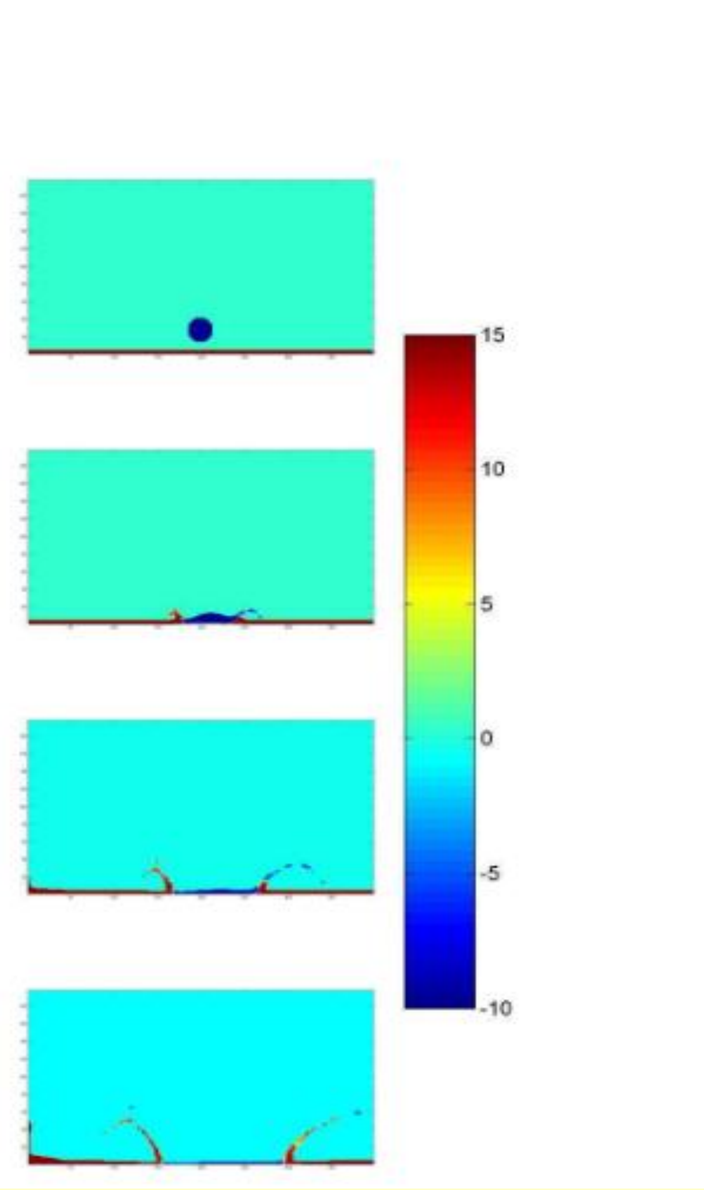
- FREEZING:-

Move ice boundary: change of state involving latent heat

Allow a ‘nucleation layer’ of ice roughness at the bottom of the water layer

- NUMERICAL METHOD:-

Simulations use VOF / PLIC (piecewise linear interpolation calculation)



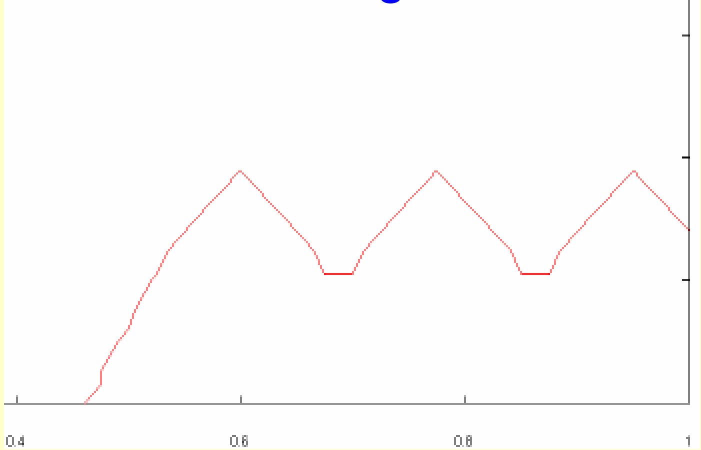
← Simulation for temperature

Quero et al (2006)

Water droplet at – 10 degrees C
Water film at 1 degree C
50 ice crystals at – 0.1 degrees C

The crystals grow into each other and the ice front smooths

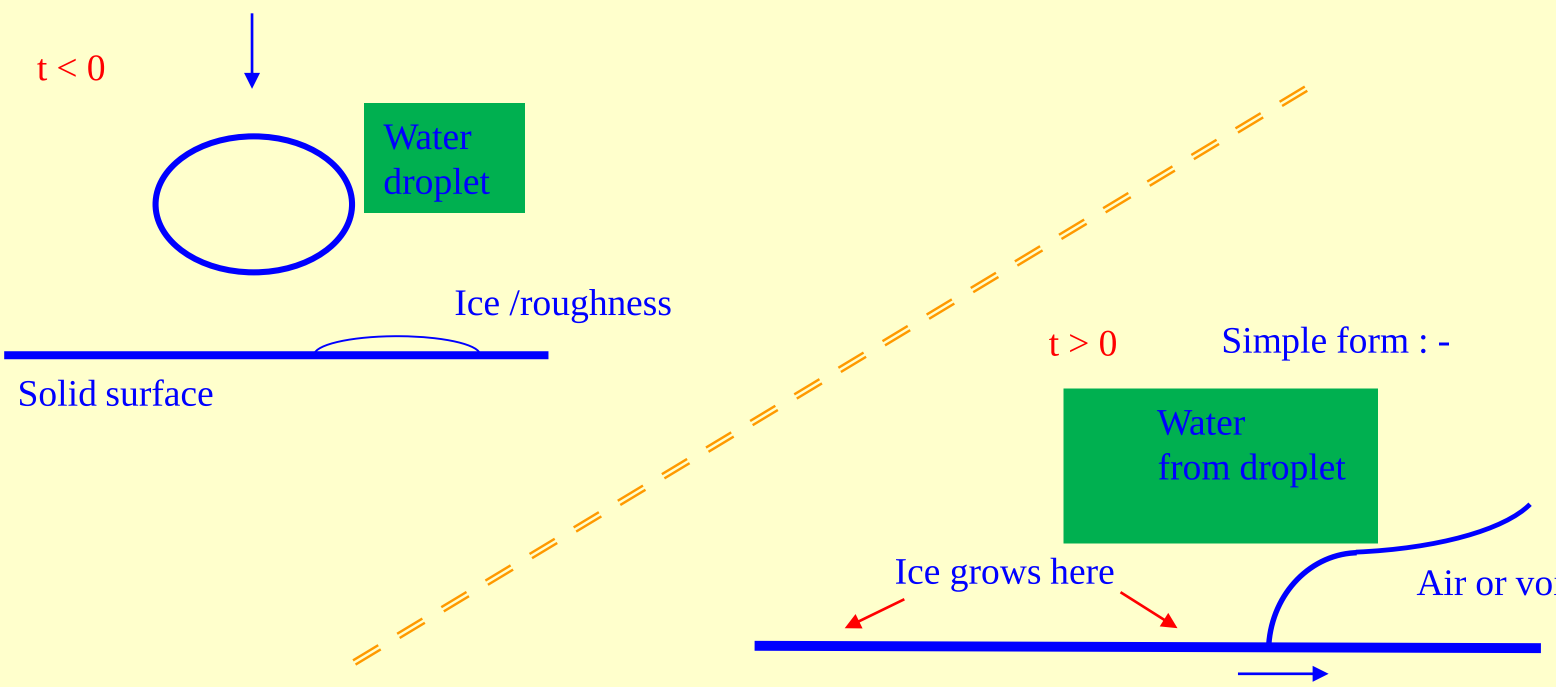
The ice growth : -



More analytical approach :

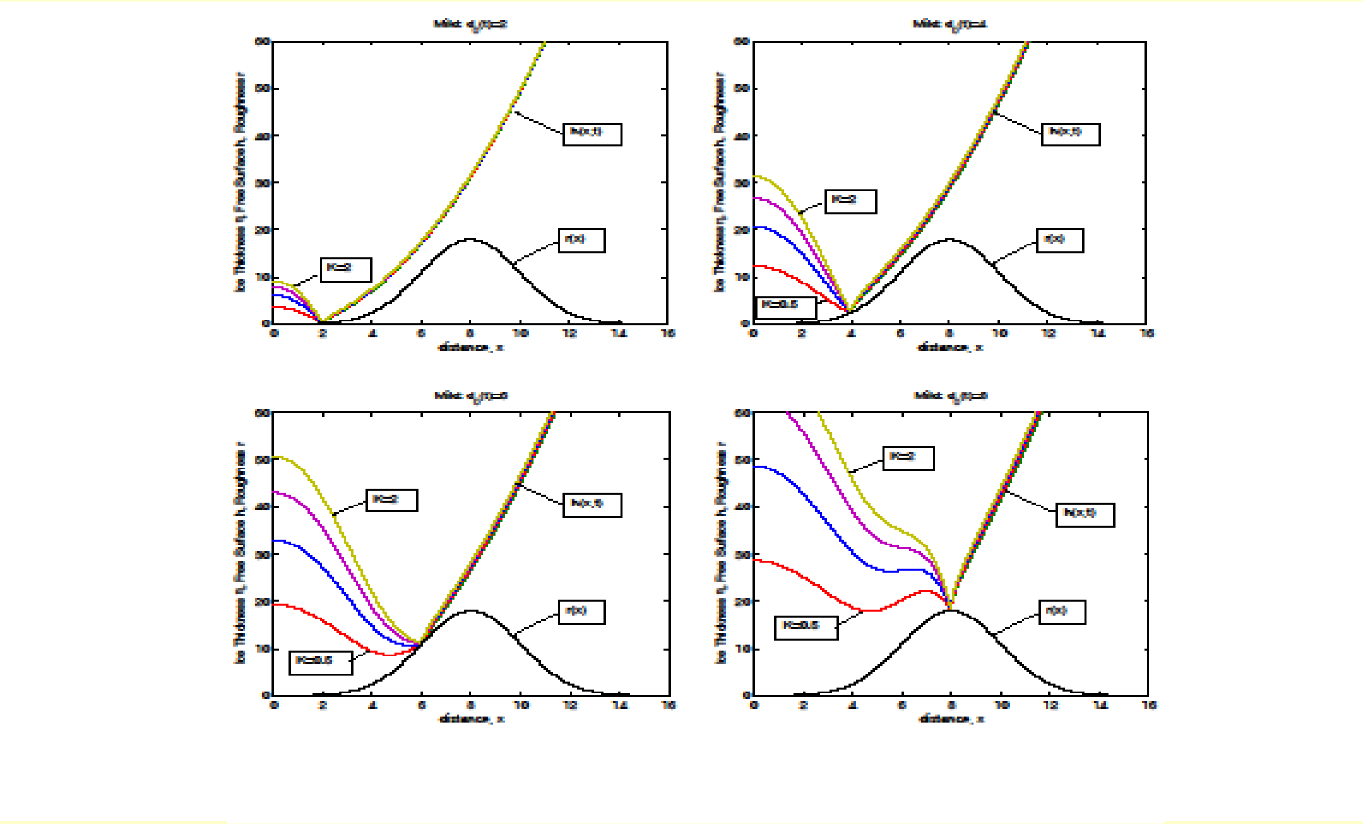
John Elliott & FTS

Ice growth on a cold surface (post-impact)
Small-time analysis as per Wagner et al, with viscous and thermal effects

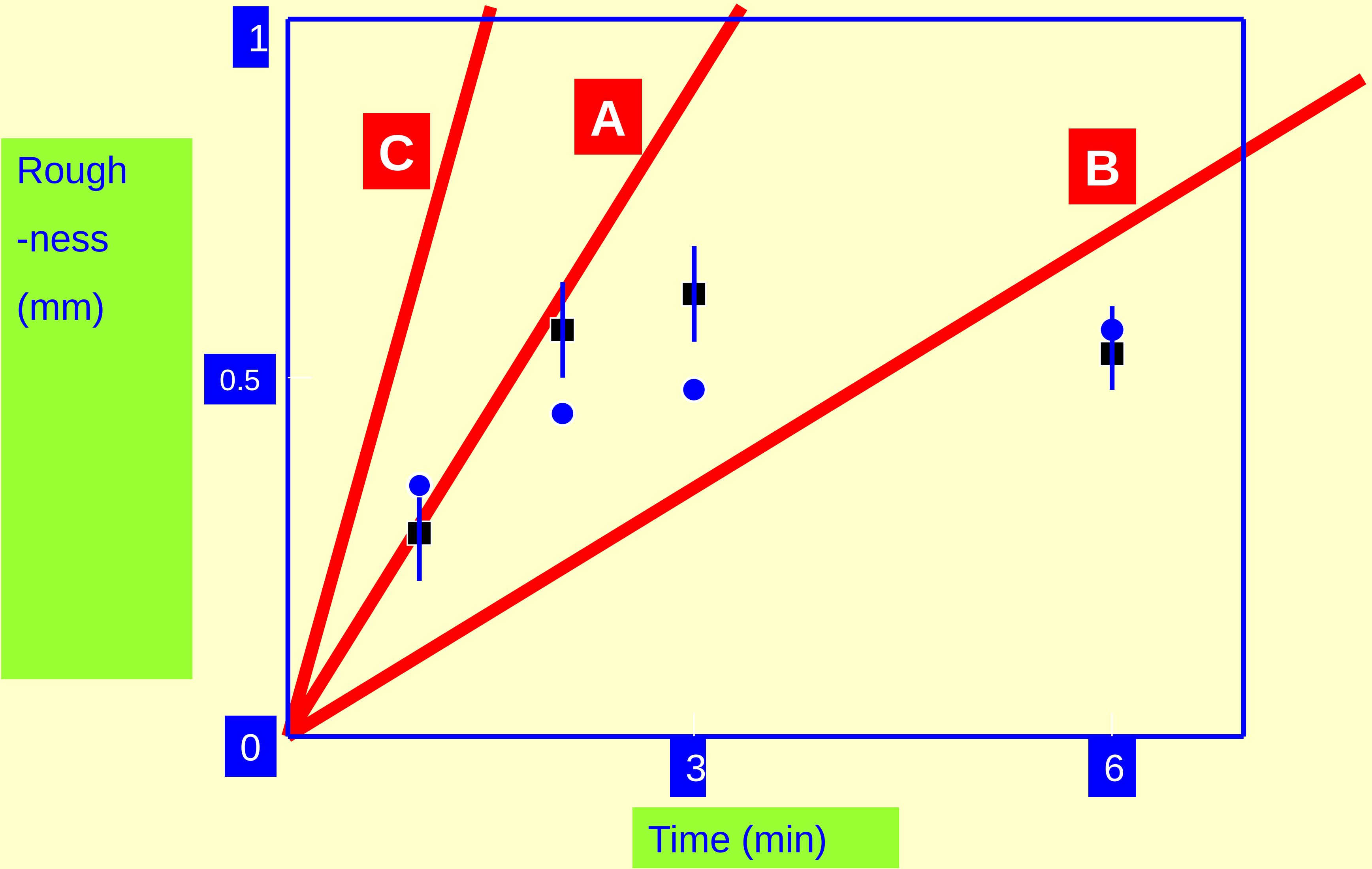


There are many parameters : -
Re (~ $10^{4\text{-}5}$), Fr (~ 10^{7}), We (~ $10^{5\text{-}7}$)
σ (Prandtl, 7-13)
St (Stefan, 10^{-2}-10)
β (kinetic underheating)

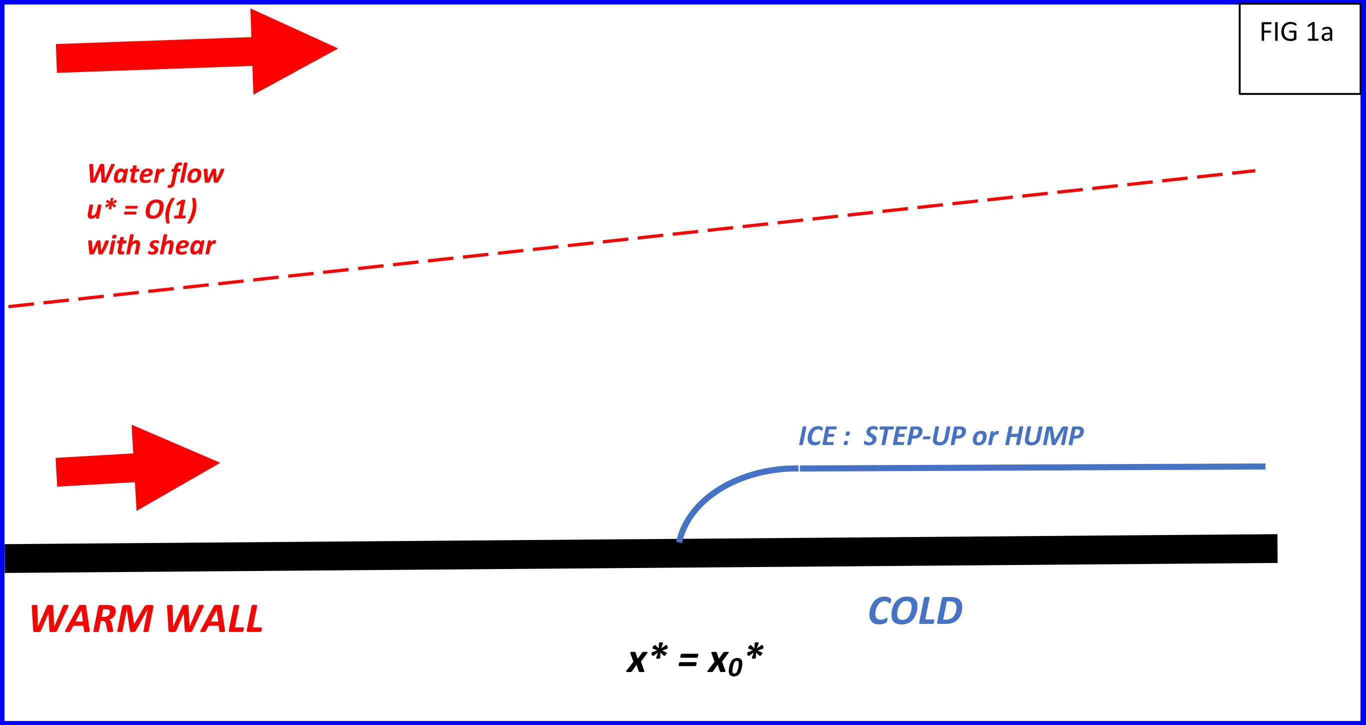
Main findings are : -
- icing can accelerate or decelerate the spreading of the droplet
- multi-structure contains restricted regions of turbulence
- parabolic shape of ice in certain conditions
- connection with experiments



Variation of ice thickness (roughness) with time :
Symbols (experiments); red curves (maths predictions)



4. *Melting or growing wall-ice in shear flow*



This study is on the melting or possible accretion of wall-mounted ice deep inside a boundary layer.

With T D Dang
& Ellen Jolley

The configuration has water flow past a slender ice hump of relatively small finite length on a flat wall (or a step up), with the water being warmer than the ice.

The wall is at the same temperature as the oncoming water except underneath the ice where the temperature of the wall is the same as that of the ice; a temperature-adjustment region may also be present in the wall just upstream and downstream of the ice hump.

The aim is to find out if the whole ice hump melts, in what manner and how long it takes, and also whether accretion may take place instead.

Non-dimensional Navier-Stokes and thermal equations:

$$div\ \mathbf{u^*} = 0,\ D\mathbf{u^*}/Dt^* = -\ grad\ p^* + Re^{-1}\ del^2\ \mathbf{u^*},$$

$$D\vartheta^*/Dt^* = (\sigma\ Re)^{-1}\ del^2\ \vartheta^* .$$

Here *Re* is the Reynolds number and σ is the Prandtl number.

- The boundary conditions on the unknown ice surface $y^* = f^*(x^*, t^*)$ lying between $x^* = x_0^*$ and $x^* = x_1^*$ say are

$$u^* + f^*_{x^*}\ v^* = 0,\ v^* - f^*_{x^*}\ u^* = (1 - r^*)\ f^*_{t^*},$$

$$\vartheta^* = -1$$

Stefan condition on heat transfer ,

for $x_0^* < x^* < x_1^*$.

- On the fixed wall $y^* = 0$ for $x^* < x_0^*$ and for $x^* > x_1^*$ we have the constraints of no slip and prescribed temperature,

$$u^* = v^* = 0,\ \vartheta^* = c^*(x^*) .$$

The factor r^* denotes the ice density relative to the water density.

The prescribed wall temperature c^* is assumed to tend to zero far upstream. It is also assumed to be continuous, implying in particular that $c^* = -1$ at the ends x_0^*, x_1^* of the ice hump.

Far-field conditions on velocity, pressure and temperature associated with matching to the remainder of the flow field are also required in general.

Parameter values:-

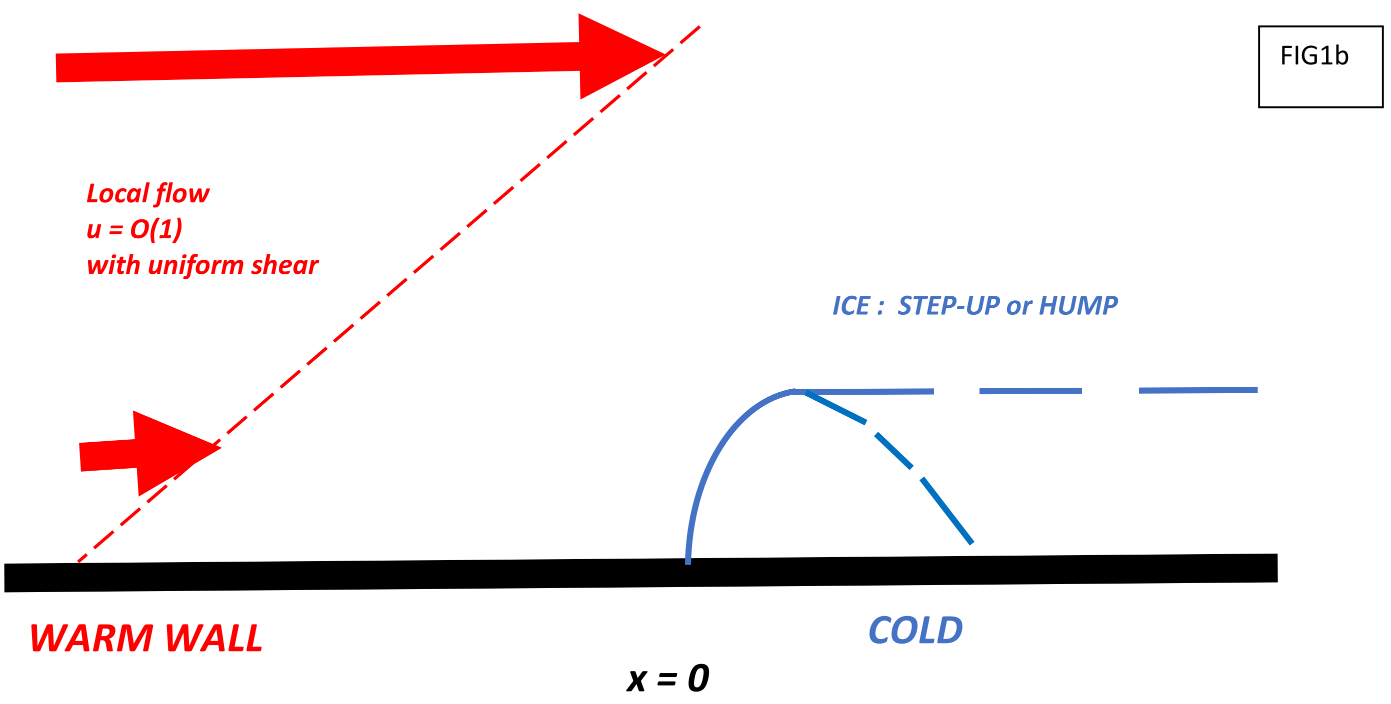
Re (~ 10^{4-5}), Fr (~ 10^7), We (~ 10^{5-7})

σ (Prandtl number, 7- 8 for water at 18 degrees C)

St (Stefan number, 10^{-2}-10)

For large Reynolds numbers the scenario of interest occurs close to the wall within a boundary layer whose length and height scales are of *O(1)* and $Re^{-1/2}$ respectively whereas the length scale of the ice hump (or its front) is small.

Locally the ice surrounds the position $x^* - x_0^* = \Delta x$ close to the wall:-



A local scaling of velocity, pressure, temperature, space and time applies.

Now the governing equations reduce to

$$u_x + v_y = 0\,,$$

$$uu_x + vu_y = -\,p_x(x,\, t) + u_{yy}\,,$$

$$u\vartheta_x + v\vartheta_y = \sigma^{-1}\,\vartheta_{yy}\,,$$

with $p = p(x,\, t)$ being independent of y because of the normal momentum balance.

The boundary conditions become

$$u - \lambda y \to 0,\ \vartheta\ - \varphi y \to 0 \text{ as } y \to \infty\,,$$

$$u = v = 0,\ \vartheta = c(x) \text{ at } y = 0 \text{ for } x < 0 \text{ or } x > 1\,,$$

$$u = v = 0,\ \vartheta = -\,1 \text{ at } y = f(x,\, t) \text{ for } 0 < x < 1\,,$$

$$B\, f_t = -\,\vartheta_y \text{ at } y = f(x,\, t) \text{ for } 0 < x < 1\,.$$

- $B = 1$ without loss of generality.

- The given function $c(x)$ is the scaled form of the wall temperature and is continuous, with $c(0) = c(1) = -1$ and $c(\infty) = c(-\infty) = 0$. The constant factors λ, φ are present because of the locally uniform shear flow and heat transfer from the boundary layer above the local region.

- The far field here has $(u, v, p, \vartheta) = (\lambda y, 0, 0, \varphi y)$ and, although λ is positive for the current fluid flow from left to right, the heat transfer contribution φ can be positive or negative.

- The local flow and thermal balances in (2.5b, c) are quasi-steady due to the assumption of slow erosion of the ice hump. Ours is a basic case: no kinetic underheating, etc.

The aim is to determine the flow and temperature properties and, especially, the evolution of the ice shape and the wall shear.

The lack of upstream influence provided the flow is forward (u positive) somewhat simplifies the task of solving.

On the other hand algebraic decay is found in the temperature field at large y which somewhat complicates the task.

We seek an analytical solution first for small disturbances to the far-field flow.

Linearized solution for small heights.

If the ice height f is small, say $f = hf_1$,

$$(u, v, p) = (\lambda y, 0, 0) + h(u_1, v_1, p_1) + \dots, \vartheta = O(1) .$$

THIS LEADS US TO A LINEAR SYSTEM:-

$$u_{1x} + v_{1y} = 0 ,$$

$$\lambda y\, u_{1x} + v_1 \lambda = - p_{1x} (x, t) + u_{1yy} ,$$

$$\lambda y\, \vartheta_x = \sigma^{-1} \vartheta_{yy} ,$$

subject to

$$u_1 \to 0, \vartheta - \varphi y \to 0 \text{ as } y \to \infty ,$$

$$u_1 = v_1 = 0, \vartheta = c(x) \text{ at } y = 0 \text{ for } x < 0 \text{ or } x > 1 ,$$

$$u_1 = - \lambda f_1, v_1 = 0, \vartheta = - 1 \text{ at } y = 0 \text{ for } 0 < x < 1 ,$$

$$f_{1t'} = - \vartheta_y \text{ at } y = 0 \text{ for } 0 < x < 1 ,$$

with t set to hBt' with t' of $O(1)$, corresponding to faster evolution now.

The solution for the thermal behaviour is obtainable through a Fourier transform in x, giving

$$(\vartheta - \varphi y)^{(FT)} = C^{(FT)} Ai((i \alpha \lambda \sigma)^{1/3} y) ,$$

$$f_{1t'} = RHS ,$$

where (FT) denotes the transform, α is the transform variable and Ai is the Airy function. The function $C(x)$ is $c(x)$ for $x < 0, x > 1$ and -1 for $0 < x < 1$, and

$$RHS = -\varphi - Ai'(0) (\lambda \sigma)^{1/3} \int_{(a, x)} (x - s)^{-1/3} c'(s) ds \ / (Ai(0) \Gamma(2/3)) ,$$

with x restricted to $0 < x < 1$. (For other x values, the function $-STUFF$ determines the scaled heat transfer at the wall.) Hence

$$f(x, t') = f(x, 0) + (RHS) t' \text{ if } f>0, \text{ and } f(x, t') = 0 \text{ otherwise},$$

gives the ice shape, with allowance made for complete melting.

In this linearized regime the background fluid flow determines the coefficients λ, φ and these then determine the thermal response, which then determines the shape evolution, which then determines the flow perturbation.

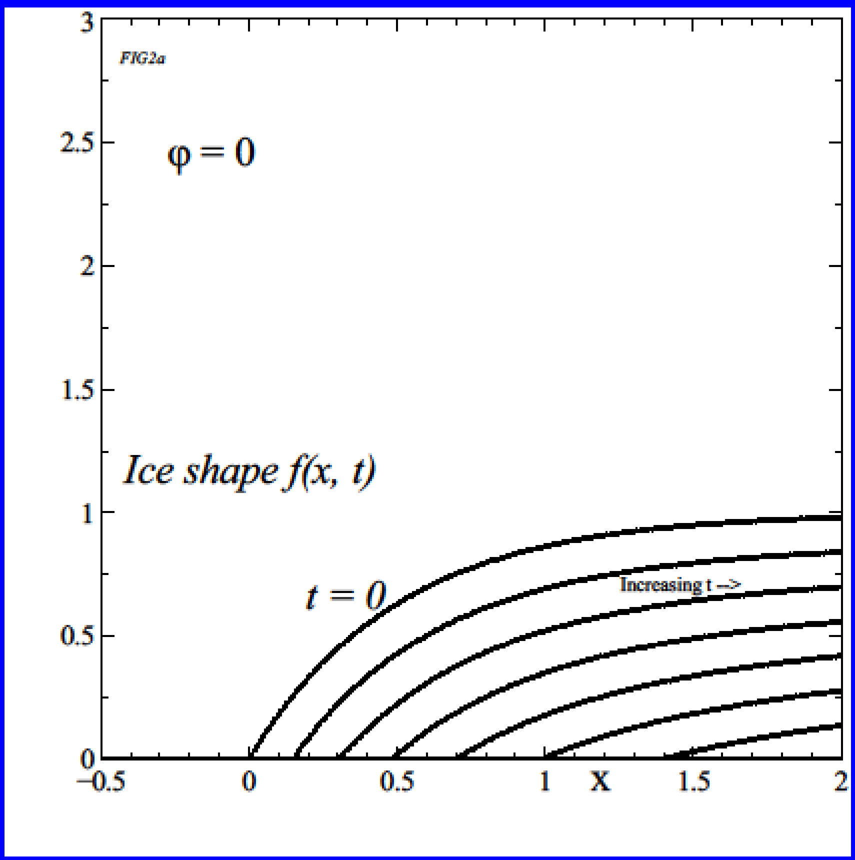


FIG2a
φ = 0
Ice shape f(x, t)
t = 0
Increasing t -->

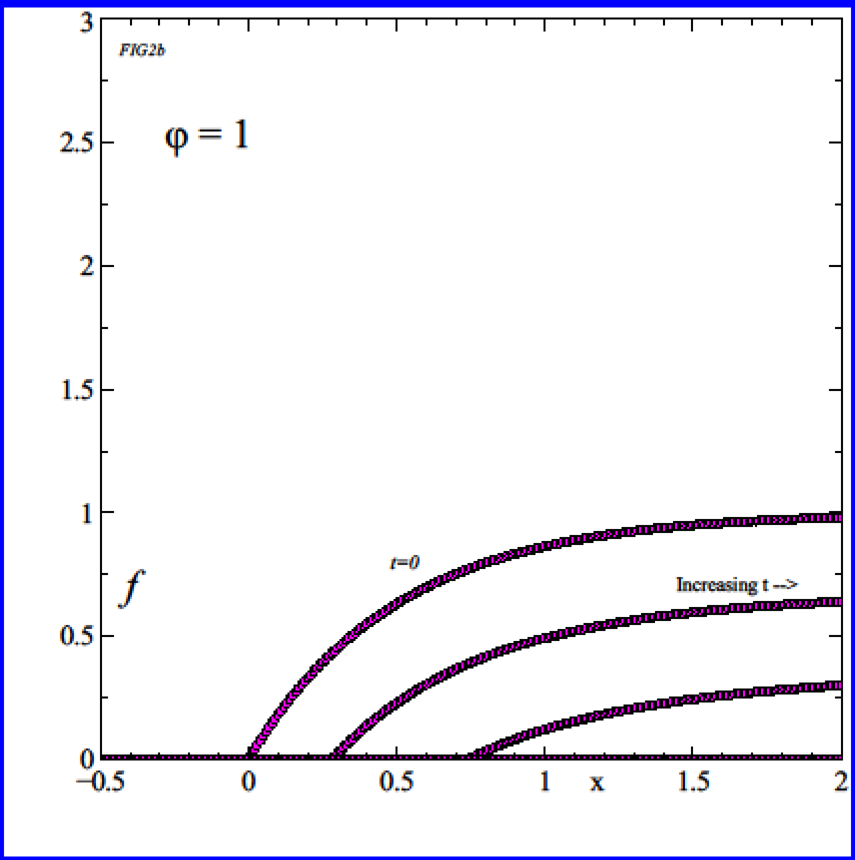


FIG2b
φ = 1
f
t=0
Increasing t -->

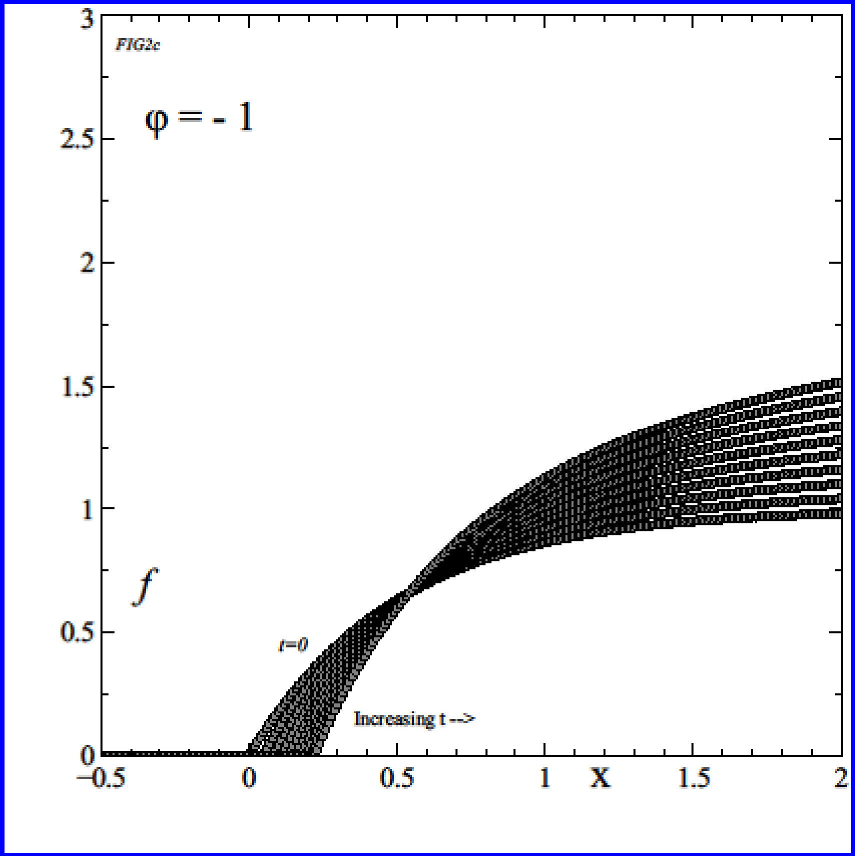


FIG2c
φ = - 1
f
t=0
Increasing t -->
X
0
0.5
1
1.5
2
2.5
3
-0.5
0
0.5
1
1.5
2

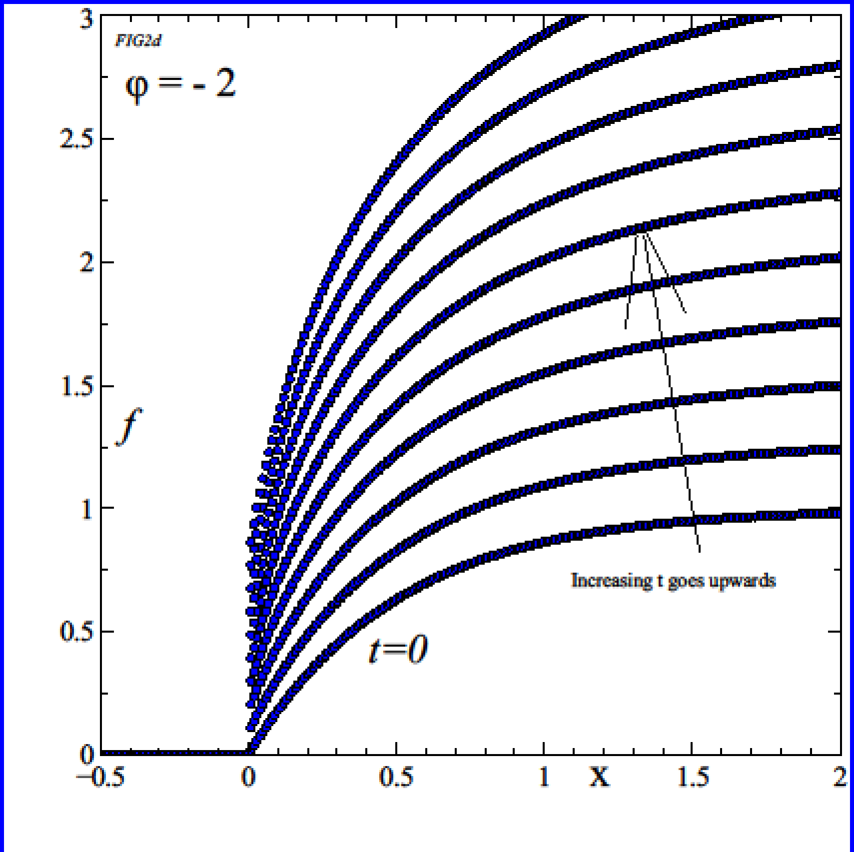


FIG2d
φ = - 2
f
t=0
Increasing t goes upwards
X
0
0.5
1
1.5
2
2.5
3
-0.5
0
0.5
1
1.5
2

Nonlinear solution for medium heights.

Use is made of the Prandtl transposition in which

$$y = f + y^{**},\ v = f_x u + v^{**},$$

leaving (2.5a-c) intact except for the replacement of y by y^{**} and v by v^{**}.

The boundary conditions become (ignoring asterisks)

$$u - \lambda y \to \lambda f,\ \vartheta - \varphi y \to \varphi f \text{ as } y \to \infty\,,$$

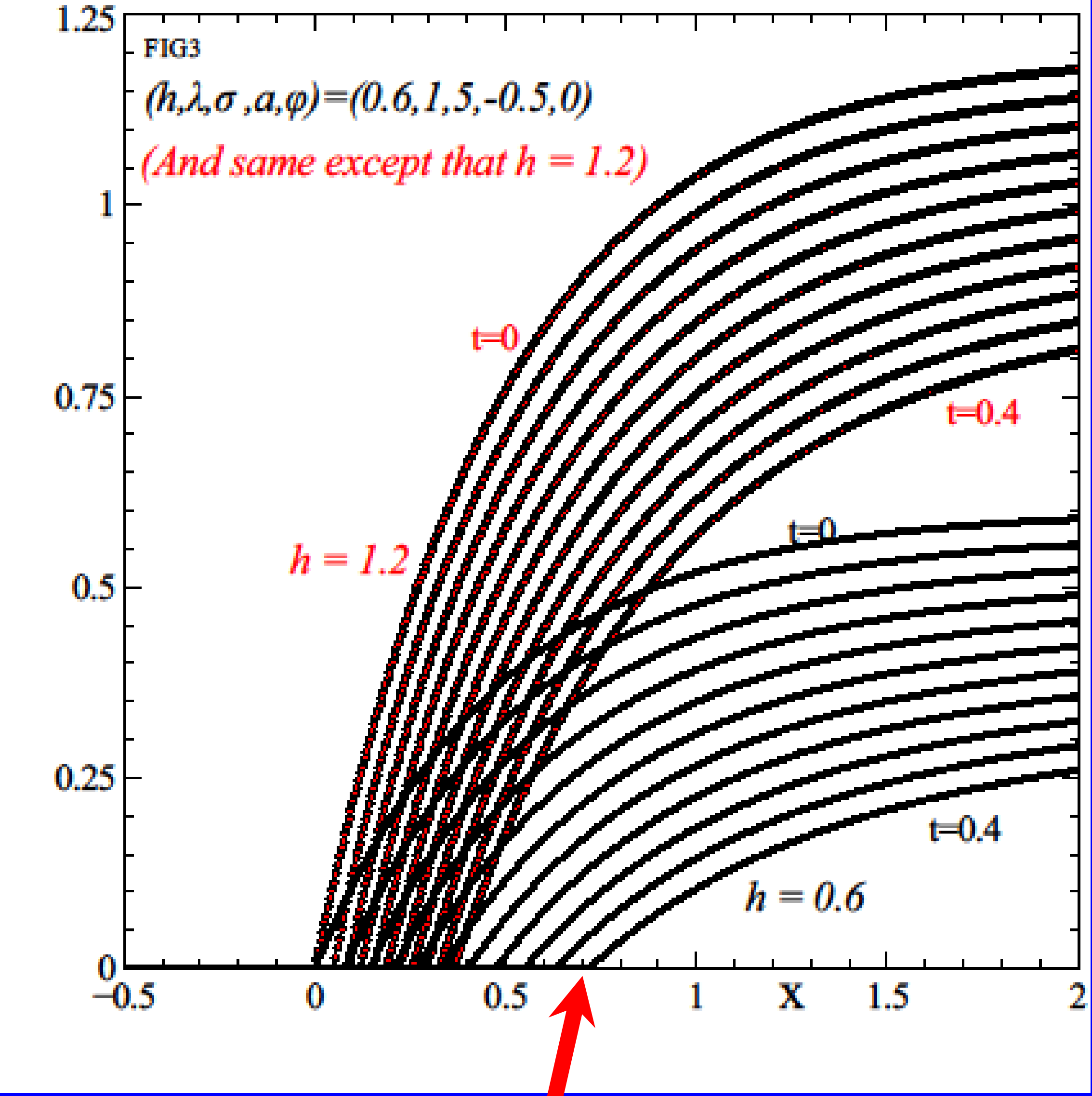
$$u = v = 0,\ \vartheta = c(x) \text{ at } y = 0 \text{ for } x < 0 \text{ or } x > 1\,,$$

$$u = v = 0,\ \vartheta = -1 \text{ at } y = 0 \text{ for } 0 < x < 1\,,$$

$$B\, f_t = -\vartheta_y \text{ at } y = 0 \text{ for } 0 < x < 1\,,$$

Solutions are shown below, with $\sigma = 5$ and $\varphi = 0$.

Steps



Erosion point agrees with linear result

Humps

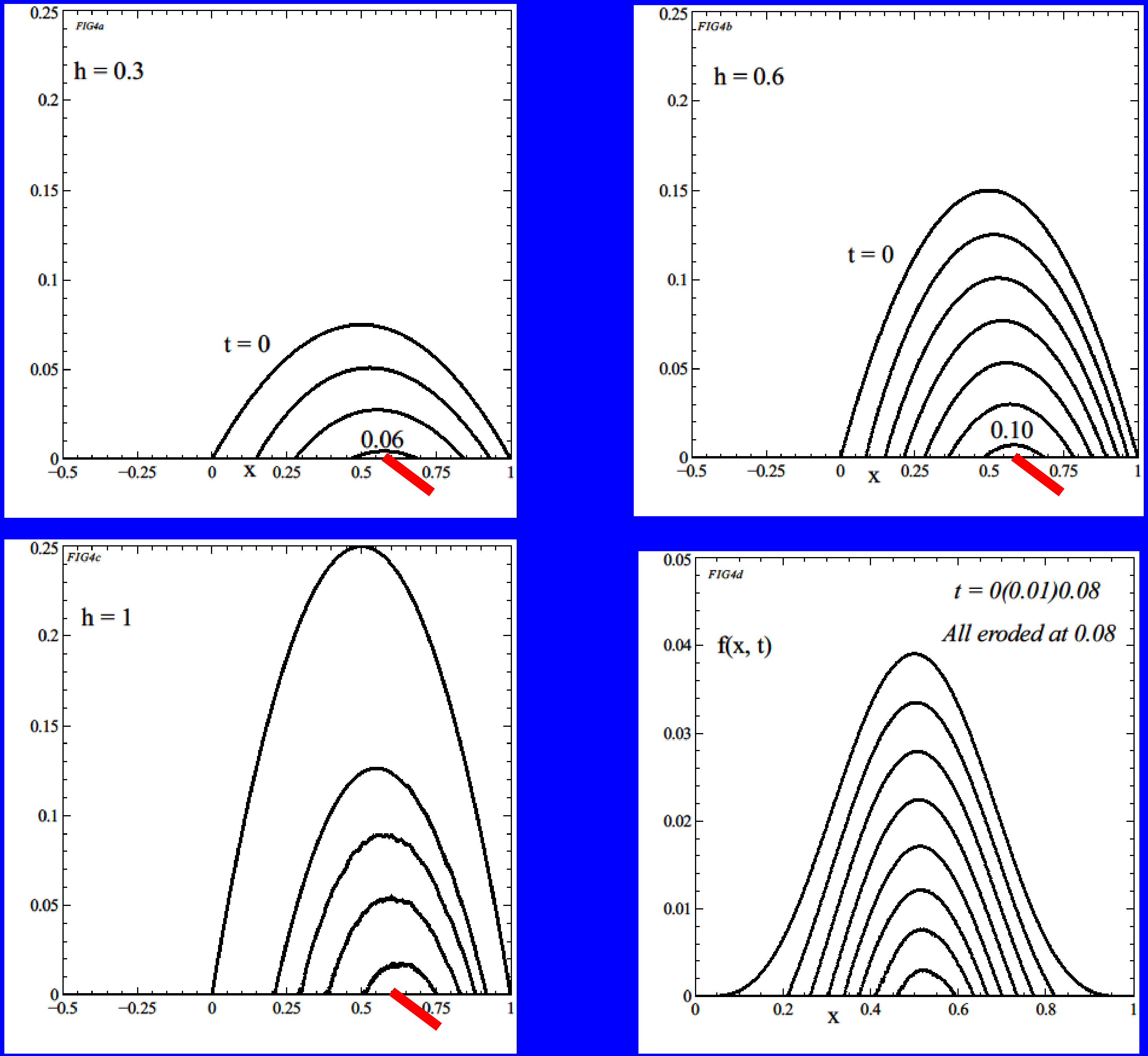
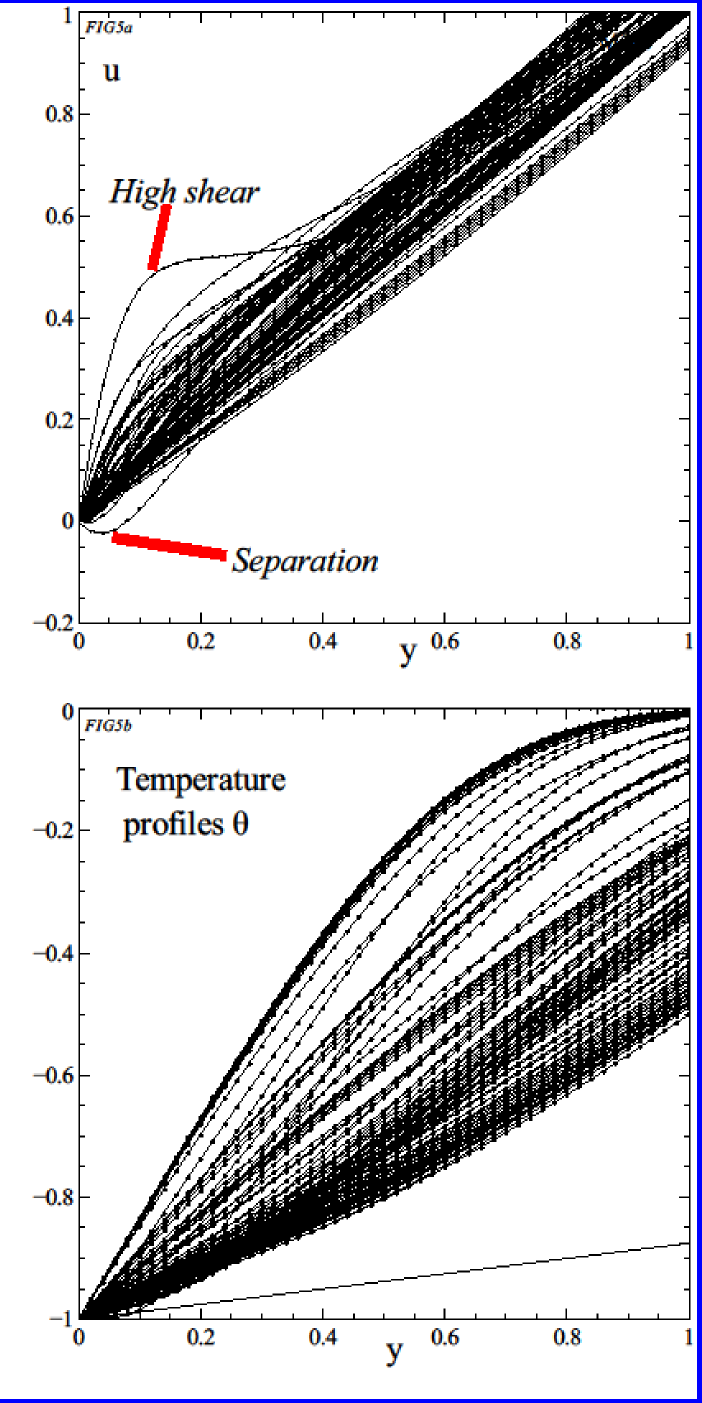


FIG4a
h = 0.3
t = 0
0.06
x
FIG4b
h = 0.6
t = 0
0.10
x
FIG4c
h = 1
FIG4d
t = 0(0.01)0.08
All eroded at 0.08
f(x, t)
x

Velocity and temperature profiles for case *h = 1*

Shows flow reversal and eddy formation



Further analyses

For steps at large times, $x_{erosion} \sim t^3$

Large σ → multi-structure

For any icing shapes, small t → origin shift of erosion point

Final behaviour for step and hump cases

Small-time behaviour.

Consider the early behaviour for the basic case of no background heat transfer:

$$(u, v, \vartheta, p, f) = (u_0, v_0, \vartheta_0, p_0(x), f_0(x)) + t\,(u_1, v_1, \vartheta_1, p_1(x), f_1(x)) + \dots .$$

Here the subscript *0* denotes the initial state of flow and temperature over the initial ice hump $F_0(x)$. The perturbations with subscript unity then satisfy a linear system. In particular

$$f_1 = -\vartheta_{0y} \text{ at } y = 0,$$

confirming that the main thermal properties control the temporal change in the ice shape initially.

Of interest is the response near the beginning of the ice.

There $f_0(x)$ is (say) linear in x whereas typically $f_1(x)$ is $O(x^{-1/3})$, since ϑ is $O(1)$ from the surface condition and y is of order $x^{1/3}$. Hence the ice shape takes the form $O(x) + O(x^{-1/3}\, t)$ locally, implying that there exists a small subzone close to the beginning where x is $O(t^{3/4})$.

In the subzone,

$$x = t^{3/4}\, \tilde{x},\; y = t^{1/4}\, \tilde{y},$$

$$u = \lambda\, t^{1/4}\, \tilde{y} + t^{3/4}\, \tilde{u} + \ldots,\; p = \ldots,\; f = t^{3/4}\, \tilde{f}(\tilde{x}),\; \vartheta = O(1),$$

The thermal equation becomes or remains

$$\lambda y\, \vartheta_x = \sigma^{-1}\, \vartheta_{yy},$$

subject to matching and boundary conditions. A similarity solution holds,

$$\vartheta = q(\eta)\,;\; \eta = \tilde{y} / \tilde{x}^{1/3}$$

➔ $$\tilde{f} = a_0\, \tilde{x} - \tilde{x}^{-1/3}\, q'(0)$$

➔ *Erosion point is :* $\qquad x = (q'(0)/a_0)^{3/4}\, t^{3/4}$

for small times.

Final behaviour (vanishing) of ice lumps

Hump is then small ➔ linear effects

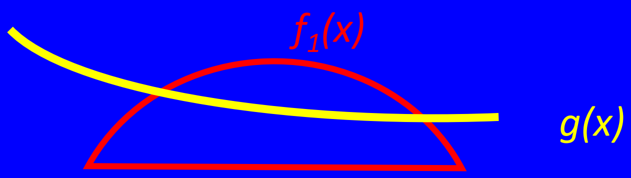
➔ $y \sim x^{1/3}$ over entire hump but history matters

➔ $\vartheta_y\ (y = 0) = g(x) \sim 1$ and $g(x)$ stays the same for all late t.

➔ $f_t = -\ g(x)$ for some late time $t > t_1$.

Hence if $f(x, t)$ has shape $f_1(x)$ at time $t=t_1$ say then

$f(x, t) = f_1(x) - (t - t_1)\ g(x)$:-



➔ ***For parabola, vanishing point x = 4/7.***

Final behaviour (spreading erosion) of ice steps

Find

$$f \sim 1 - A\, t\, x^{-1/3}$$

where

$$A = 3\,(\lambda / 9\sigma)^{1/3} \;/\; \Gamma(1/3)$$

→

Spreading downstream is given by $x \sim t^{3}$

Strongly nonlinear solution for large heights.

This tells us what happens for much thicker ice.

Suppose φ is of order unity.

If $f = hF$ with $h >> 1$, a two-zoned flow structure emerges.

OUTER ZONE: $y^{**} = hY^{+}$ and

$$u = h\,(\lambda Y^{+} + \lambda F),\ \psi = h^{2}\,(\tfrac{1}{2}\,\lambda\, Y^{+\,2} + \lambda F Y^{+}),\ p = -\,\tfrac{1}{2}\,h^{2}\,\lambda^{2}\,F^{2}\ \ .$$

---- an exact solution of the governing equations which satisfies the outermost boundary conditions but leaves a nonzero slip velocity $u \sim h\lambda F$ at $Y^{+} = 0+$. The corresponding thermal solution is

$$\vartheta = h\,\varphi\,(Y^{+\,2}\ + 2\,F\,Y^{+})^{\,\frac{1}{2}}\,.$$

The algebraic decay into the far-field response $\varphi h Y^{+}$ at large Y^{+} is notable. This outer zone has inviscid motion.

INNER ZONE : thin layer on the ice, with $y^{**} = h^{-1/2}\, Y$ and

$$(u, v^{**}, \vartheta) = (hU, h^{1/2}\, V, h^{1/4}\, \Theta) + \dots .$$

The time scaling is $t' = h^{1/4}\, T$, implying that the evolution is relatively slow. Substitution into the governing equations yields

$$U_x + V_Y = 0\,,$$

$$UU_x + VU_Y = \lambda^2\, F\, F_x + U_{YY}\,,$$

$$U\Theta_x + V\Theta_Y = \sigma^{-1}\, \Theta_{YY}\,.$$

The boundary conditions :

$$U \to \lambda F(x, T),\ \Theta \sim \varphi\, (2\, F\, Y)^{1/2},\ as\ Y \to \infty\,,$$

$$U = V = 0,\ \Theta = 0,\ F_T = -\ \Theta_Y,\ at\ Y = 0.$$

- This is a classical boundary-layer problem with the pressure effect prescribed at each time level T, supplemented by the temporal evolution of the scaled ice shape F.
- The ice temperature -1 is negligible to the present order.
- The square-root trend with Y is unusual.

The above formulation holds on the iced part of the surface, not upstream since the scaled temperature ϑ is in effect zero there.

Expect positive background heat transfer $\varphi > 0$ to produce positive heat transfer on the ice at $Y = 0$, thus inducing ice erosion, whereas negative φ might provoke negative heat transfer on the ice, inducing ice growth, but in any case the nonlinearity of the system and the x-dependence need to be taken fully into account. Numerical solutions are being considered.

If φ is small or zero:-

The critical scale here is $\varphi = h^{-1/4}\varphi\hat{}$ because it reinstates the influence of the ice temperature at leading order in the inner zone. It gives

$$U \to \lambda F(x, T),\ \vartheta \sim \varphi\hat{}\ (2\,F\,Y)^{1/2},\ as\ Y \to \infty\ ,$$

$$U = V = 0,\ \vartheta = -1,\ F_T = -\ \vartheta_Y,\ at\ Y = 0.$$

The presence of *'– 1'* is noted.

Conclusions

Pre-impact models have been developed for ice particles passing through an air boundary layer or through air and water.

Post-impact ice accretion due to a droplet spreading on a wall has been described.

Wall-ice melting or accretion in near-wall flow is being studied, beginning with a simple model.

Further work is ongoing. Hot-wall effects; rebounds; viscosity; 3D.