Injection Amplitude Guidance for Impedance Measurement in Power Systems

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***Abstract*—Impedance measurement techniques based on small-signal injections in AC power system have been well-developed but a gap in the analysis is that the selection of an injection amplitude is based only on experience. This letter develops an analytical process to analyse the noise existing in power systems and thereby determine the minimum injection amplitude according to an allowable error. The process is based on knowledge of stochastic processes and statistics, where the Monte Carlo method is also employed to simplify the process. A power hardware-in-the-loop system with a grid-following inverter is employed in experimental verification of the results.**

***Index Terms*—Impedance measurement, admittance measurement, noise analysis, injection amplitude, Monte Carlo method, power hardware-in-the-loop**

I. INTRODUCTION

The emerging inverter-based resources (IBRs) in power systems are causing new stability issues which means that practical solutions for stability analysis of IBR-dominated systems is urgently needed [1]. The lack of detailed models of IBRs due to confidentiality of proprietary control software hampers conventional state-space stability analysis but impedance models of power systems have been proved effective in such situations [2], [3]. One major advantage of impedance models is that they can be established via data-driven methods based on online small-signal injections, i.e., injecting current perturbations and measuring voltage responses to acquire the impedance model [4], or injecting voltage perturbations and measuring current responses to acquire the admittance model [5]. The injection topologies and the injection waveforms have been well-studied but a critical factor, the injection amplitude, is rarely analysed in detail. On the one hand, the amplitude should be set as large as possible for higher signal-to-noise ratio (SNR), while on the other hand, the amplitude should be as small as possible to minimise its influence on the operation of the system and its stability. The existence of this conflict has been noted by researchers but has usually been solved by a trade-off based on experience or numerous trial-and-error attempts. For example, [6] considers that 5% of AC voltage and 10% of AC current are the proper range for small-signal perturbations without discussing the reason behind this. [7] states that a signal with 1% to 5% of nominal power is required for satisfactory results based on experience at the author's institution. In [5], a set of measurements under perturbations of 0.5%, 5% and 10% of the steady-state value were performed, showing that the measurement results can be affected by noise if the injections are too small, but no overall conclusion on a suitable amplitude was drawn. Researchers have noticed the importance of noise analysis for impedance measurement and tried to assess the value of the SNR [8], but the value is only provided for adding context and aiding intuitive sense rather than being used to determine the injected amplitude. Although an experience-based method can sometimes offer a quick solution, the requirement of human input can increase the cost of measurement, and the selection of injection amplitude is highly dependent on the researcher's own knowledge. Such a method cannot be widely applied in practical situations.

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This letter aims to address the issues by developing a method which can determine the minimum necessary injection amplitude based on an allowable error. Alongside this, a method is needed to determine what the true value of the impedance of a physical systems is so that measurements can be judged for accuracy [9].

II. PRELIMINARIES

To begin with, three premises are introduced:

Premise 1. *The system is considered to be time-invariant during the impedance measurement period since the measurement period is around 30 s for frequency sweep, which is short compared with variations of operating point of a grid system.*

Premise 2. *The noise is additive noise because the injecting source is an independent source, and considered as ideal.*

Premise 3. *During a 30 s measurement period, the noise is considered to be a wide-sense stationary (WSS) stochastic process. This is because when the system is time-invariant as mentioned in premise 1, the noise mainly consists of white Gaussian noise (WGN), and harmonics in the system which are sine waves with random phases, which are all WSS processes.*

It is worth mentioning that the impedance measurement discussed in this letter is applied under steady-state operating conditions since the impedance spectrum is mainly used for small-signal analysis at a steady operation point.

A. Metric for Measurement Error

So far, it is not clear how to estimate the error present in an impedance measurement taken from an experiment because a reference, or true, value is not available. Hardware devices will not be strictly the same as their analytical representation because of the presence of nonlinear behaviors and therefore the plot derived from a linearised model or simulation cannot be considered as the true value for the hardware.

Nonetheless, a method to evaluate the measurement error is required for further assessment. Since the measurement result is a frequency spectrum $Y(j\omega)$, it is difficult to select a single metric to represent the error of the whole curve. Consequently,

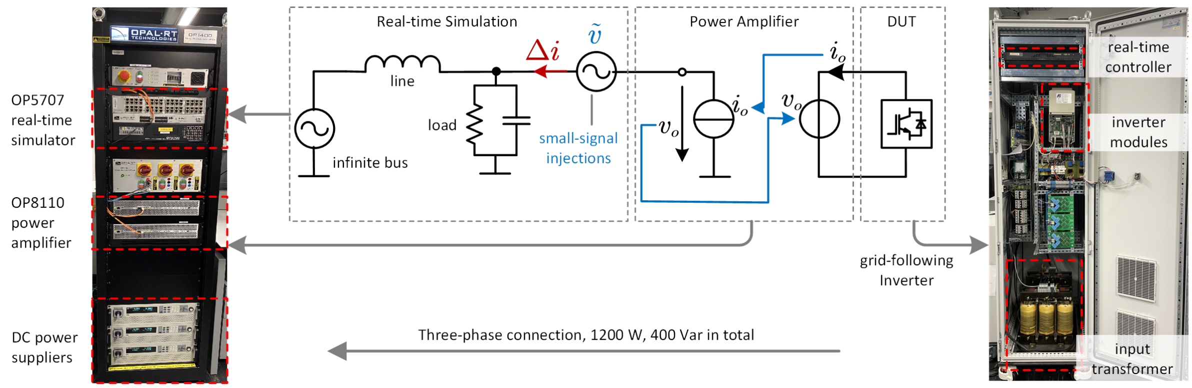

Fig. 1. Experiment platform for the test system: a PHIL system based on an OP1400 series power amplifier (PA). The system rated voltage is 200 V. The bandwidth of the PA extends to 10 kHz, and the total harmonic distortion (THD) for the range 0-1 kHz is maintained below 0.5%, which guarantees a nearly ideal small-signal injection. The real-time simulation runs inside an OP5707 simulator with an update rate of 20 kHz which is also the current and voltage sample rate. The device under test (DUT) is an inverter configured in grid-following mode, exporting 1200 W and 400 Var to the PA.

the error should be analysed at each frequency point instead of the whole spectrum. Consider a frequency point f_c, the injected voltage is

$$\tilde{v}(t) = V_c \cos\left(2\pi f_c t\right), \tag{1}$$

where the injected amplitude is V_c and phase is 0. The measured small-signal current can then be expressed as

$$\Delta i_c(t) = I_{c0} \cos\left(2\pi f_c t + \theta_{c0}\right) + n\left(t\right), \tag{2}$$

where I_{c0} is the true amplitude of output current, θ_{c0} is the true phase, and $n(t)$ refers to the noise which is a stochastic process and is additive noise as set out in premise 2. To extract the amplitude and phase of the current from the time-domain signal, a simple convolution is applied as

$$I_c = \frac{2}{k_c T_c} \int_0^{k_c T_c} \Delta i_c\left(t\right) e^{j\omega t} dt = I_{c0} + N(f_c), \tag{3}$$

where I_c is the current output in complex form, k_c is number of cycles of the injected perturbation, and $T_c = 1/f_c$, I_{c0} is the current true value and $N(f_c)$ is the measurement error caused by the noise, where

$$N(f_c) = \frac{2}{k_c T_c} \int_0^{k_c T_c} n\left(t\right) e^{j\omega t} dt. \tag{4}$$

The relative error η at frequency f_c is then defined as the distance between true value and measured value divided by the true value, i.e.,

$$\eta\left(f_c\right) = \frac{|N(f_c)|}{|I_{c0}|}. \tag{5}$$

Here $\eta\left(f_c\right)$ is also the relative error of the measured admittance $Y(j2\pi f_c)$. The term $|N(f_c)|$ is the absolutely error of the measured current caused by the noise, which is recognised as the noise impact on the measurement, an important factor to evaluate the noise for impedance measurement.

In reality, I_{c0} cannot be known. However, since the noise $n(t)$ is considered as a WSS process, the expected value of $N(f_c)$ can then be easily known because it is zero, a characteristic of WSS process. Therefore, the expected value of the measured current is equal to its true value. According to *the law of large numbers*, when there is a large number of repetitions of the same test, the expected value will be very close to the mean value of tests. As a result, I_{c0} can be estimated by the mean value of a large number of tests.

B. Experimental Platform

Because the focus is on measurement error, which is a problem caused by hardware devices, a hardware test bench is required for further analysis. In this letter, a power hardware-in-the-loop (PHIL) system emulating a real inverter connected to an infinite bus is adopted, as shown in Fig. 1, with detailed descriptions in the caption. A small-signal voltage injection source $\tilde{v}$ is added at the point of common coupling (PCC) and the current flows through the PCC Δi is sampled, therefore, the measured result is an element of the so-defined whole-system admittance [3], i.e.,

$$\Delta i = Y(j\omega) \cdot \tilde{v}. \tag{6}$$

Although the measurement is carried out on both d and q axes, the analysis is only performed on the d-d axis, i.e., Y_{dd}. The same analysis can be extended to any of the axes. For the sake of brevity, Y is used in the following discussion to represent Y_{dd}. Frequency sweep is chosen as the injection method for its high accuracy, and the range covered is from 1 Hz to 1 kHz. Each round of sweep consists of 81 frequency points and takes around 30 seconds.

III. Methodology

The proposed method of noise analysis and injection determination for impedance or admittance measurement is depicted in Fig. 2. The method contains 4 steps. For a better understanding, each step is introduced in turn together with the relevant experimental results.

A. Noise Record

To evaluate the noise, the first step is to record a long period of noise from the output port, i.e., the d-axis current, without any injection present. Since the frequency sweep period is expected to be 30 s, the sampling period for this step is set as

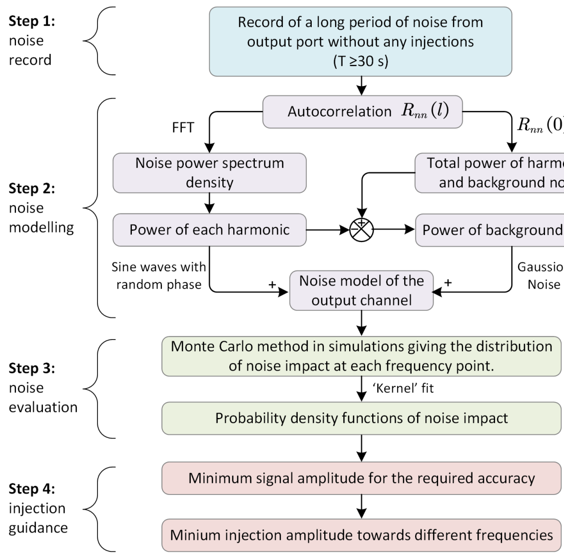

Fig. 2. Proposed method of noise analysis and injection determination for impedance or admittance measurement.

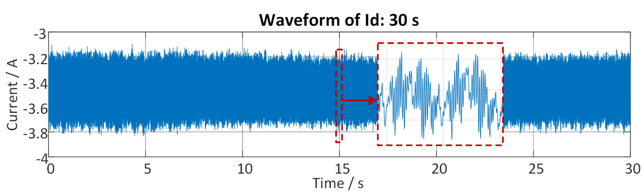

Fig. 3. Step-1: 30 s record of d-axis current with noise, on two timescales, at a sample rate of 20 kHz.

30 s as well. It is important that the sample rate is the same as the rate used for the subsequent admittance measurement, i.e., 20 kHz in this work, because the noise is specific to the sample rate. Since the noise is a WSS process, the start time of the sampling of the noise will not affect the results. Fig. 3 shows an example measured current from the d-axis for 30 seconds, together with a zoomed-in view showing that the noise includes both repetitive harmonic components and random background noise.

B. Noise Modelling

Autocorrelation is first applied to the noise since it is a general approach for power analysis of stochastic process. The autocorrelation of the noise $n(t)$ is

$$R_{nn}(l) = \frac{1}{N_s} \sum_{k=0}^{N_s-1} n(k)\, n(k-l), \tag{7}$$

where l and k are integers that refer to sample points, and N_s is the total number of sample points. Since $n(t)$ is a WSS process, we have

$$R_{nn}(0) = E(n^2(0)), \tag{8}$$

meaning that the total power of the noise equals $R_{nn}(0)$, which is a property of autocorrelation. Because this is a WSS process, the *Wiener-Khinchin theorem* can be applied, from which the power spectral density (PSD) of the noise can

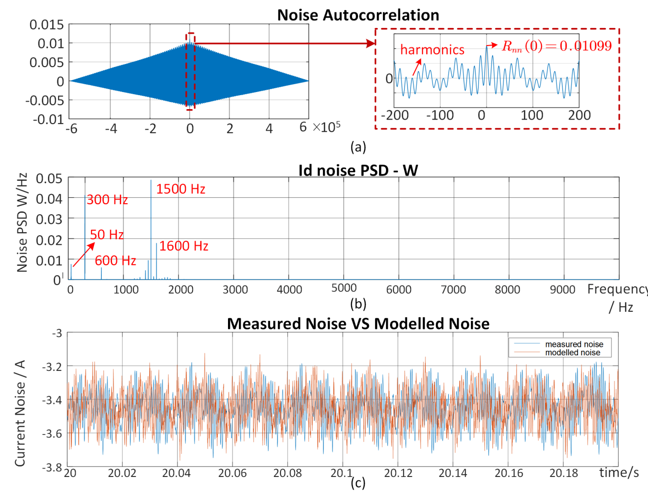

Fig. 4. Step-2: (a) Autocorrelation of the noise. (b) PSD of the noise.

be acquired by performing a Fast Fourier transform (FFT) on $R_{nn}(l)$. Fig. 4 shows the results of autocorrelation and PSD. The diamond shape indicates that the correlation value is decreasing along with the increasing value of interval $|k-l|$, which is in accordance with the feature of WSS process. This also proves that the assumption in premise 3 is correct. Also, the value at 0 is 0.01099 meaning that the total power of the noise is about 0.01099 W. The zoomed-in autocorrelation plot also reveals the presence of harmonics at various frequencies. From the PSD plot, it is obvious that several peaks stand out in the spectrum, which are harmonics as might be expected from an inverter. Since the noise was observed on a d-axis signal, 50 Hz in this spectrum corresponds to DC in the physical signal and is a DC offset. Similarly, 300 Hz corresponds to the 5-th and 7-th harmonics. The terms above 1 kHz could be caused by excitation of resonances in the inverter's LCL output filter. The PSD indicates the power of each harmonic. Taking 300 Hz as an example, the power density at that point is 0.0411 W/Hz, and the frequency resolution in this case is 1/7.5 Hz. The power of the 300 Hz harmonic P_{300} is therefore 0.00548 W. The harmonics appearing in Fig. 4(b) with relative large power are picked out to build a model of the noise. Meanwhile, using (8), the power of WGN, σ^2, can be calculated as the difference between the total power and the power of the harmonics.

Based on the result of the autocorrelation and the PSD, the noise can be modeled as the sum of WGN and a harmonic series:

$$\begin{aligned} n_M(t) &= G_W(t) + \sum_{i=1}^{k_m} H_i(t), \text{where} \\ G(t_0) &\sim \mathcal{N}(0, \sigma^2) \\ H_i(t) &= \sqrt{P_{fi}} \sin(2\pi f_i t + \theta_i), \quad \theta_i \sim U(0, 2\pi), \end{aligned} \tag{9}$$

where $n_M(t)$ is the modelled noise, $G_W(t)$ is the WGN, and H_i is the i-th selected harmonic with uniformly randomly distributed phase in [0,2π]. A comparison of the measured noise and the modelled noise is shown in Fig. 4(c), showing that the two are very close to each other.

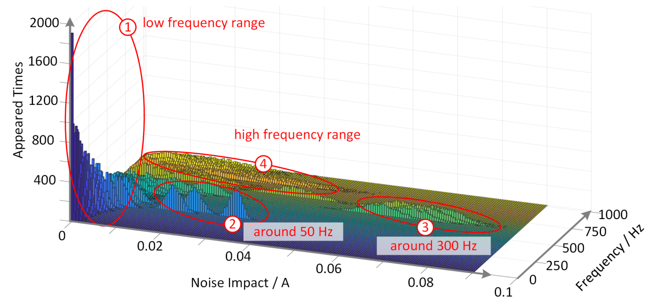

Fig. 5. Step-3: histogram of noise impact from the Monte Carlo simulation.

C. Noise Evaluation

The noise model built in step 2 can be utilised to assess the noise impact on different frequency points, i.e., the probability density function (PDF) of the noise impact. Here, the Monte Carlo method is applied as a simple but effective way to analyse the noise by conducting thousands of tests in simulations from which the PDF can be estimated based on histograms of noise impact.

For each of the 81 frequency points in the present study, 2,000 random tests were performed in simulation so that histograms could be plotted. The histograms of $|N(f_c)|$ for each frequency point are shown as a 3-D bar chart in Fig. 5. Four red ellipses are marked to help identify categories of noise impact in various frequency ranges. One observation is that in ellipse-3, which includes frequencies around 300 Hz, several peaks of impact occur, indicating that the noise impact will be significant and therefore a high amplitude of signal injection will be needed, especially for frequency around 300 Hz. Another observation is that the histogram of noise impact at different frequency points are of different shapes. This means that they can not be represented by one single type of distribution such as a normal distribution. As a result, kernel density estimation (KDE), which is a non-parametric way to estimate the PDF of a random variable, is applied as a generic way to acquire a PDF of noise impact.

D. Injection Guidance

A 95% probability curve of noise impact value was calculated using the PDF of noise impact and is shown in Fig. 6. For each frequency, there is a 95% probability that the absolute error caused by noise is smaller than the corresponding value of the curve. Here the value 95% is chosen as it is a common value in the field of statistics when considering a confidence interval. The curve in Fig. 6 can serve as a guide for choosing the amplitude for signal injection. Taking 300 Hz as an example, the absolute error, at 95% probability, is 0.08 or less. If the measurement is required to have a relative error lower than 12%, then the small-signal current should have an amplitude of $\frac{0.08}{0.12} = 0.67\ A$. Based on the required amplitude of the current, the necessary amplitude of voltage injection can be obtained by using knowledge of the expected range of impedance or through circuit simulations. In practical situations where the system is unknown, the amplitude of voltage can be determined by injecting a set of sine waves

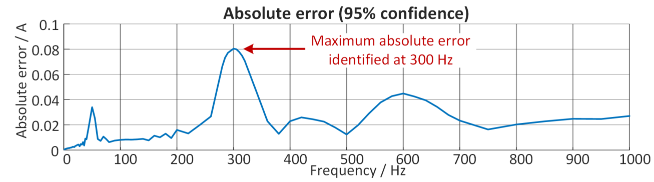

Fig. 6. Step-4: absolute error caused by noise plotted at the 95% confidence level.

at the frequency causing the largest absolute error (300 Hz in this case) with gradually increasing amplitude, and observing the amplitude of current response using with an FFT. This process is required only for the first measurement, because an expected range of the impedance will be known for subsequent measurements. In the experimental system under study, a voltage injection with an amplitude of 15 V is needed to drive 0.67 A of current, i.e., a 15 V amplitude of voltage injection can achieve the goal on accuracy, which is approximately 4.3% of the steady value of v_d.

IV. Measurement Results

Two sets of experimental tests have been performed in order to verify the preceding analysis. In the first test, the frequency point 300 Hz is measured repeatedly 1,000 times under 15 V injection amplitude. The time interval between each test is set at a random value. Fig. 7(a) shows the voltage and current waveforms during the injection, where $\tilde{v}$ is not obvious in the overall phase voltage v_a, but clearly causes a current perturbation. The system is also stable during the injections. Fig. 7(b) shows the histogram of the relative errors from 1,000 measurements. It is clear that the error is confined within 12% under 15 V injection, which matches the intention of the proposed method. A frequency sweep is then executed so that Bode plots of the whole-system admittance Y^{sys} can be acquired, as shown in Fig. 7(c). It can be seen that under 15 V injection amplitude, the measured result is a smooth curve with very small fluctuations at around 300 Hz. Knowing that 300 Hz is the major frequency point which affects the accuracy of admittance, the good results in this region prove that a 15 V injection amplitude can generate an accurate result.

From these experimental results, it can be remarked that the noise analysis method for impedance measurement developed in this letter is effective for selecting an injection amplitude to obtain results that are not adversely affected by the noise present in the current measurement.

V. Conclusions and Future Work

In this letter, an analytical method which can offer guidance for injection amplitude for impedance measurement is developed and verified through a PHIL experimental system. The method is based on a set of noise analyses and finds the minimum injection amplitude based on a maximum allowable error. Knowledge of stochastic processes, statistics and the Monte Carlo method is employed for the analysis. It is worth noting that the entire method can be fulfilled by automation software and finished within several minutes [9], such that

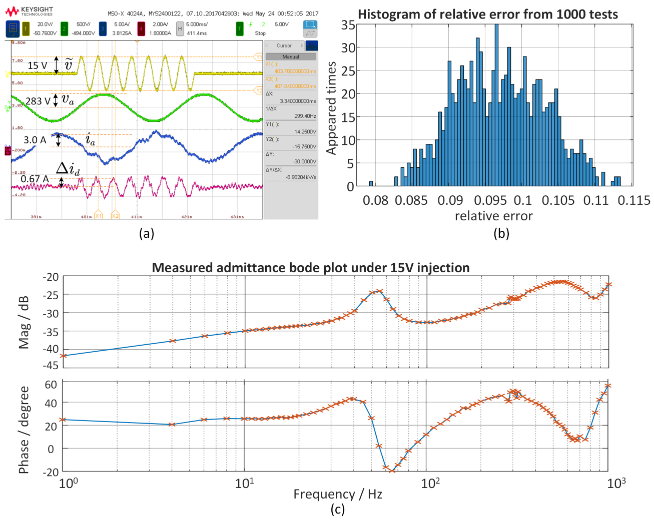

Fig. 7. Experimental results: (a) 300 Hz injection captured as oscilloscope screenshots. (b) Histogram of relative errors at 300 Hz from 1000 tests. (c) Admittance bode plots acquired from frequency sweeps under 15 V injection amplitude.

it can be easily implemented for practical application. Compared with existing methods which select injection amplitude based on experience, the proposed method minimises the need for human input and experience, thus provides a more cost-effective and more reliable solution. Recognising that this letter is taking a first look at this key issue, potential improvements can be envisaged. First, the allowable error 12% was chosen arbitrarily. In practice, such value should be selected based on the purpose of impedance results, e.g., Nyquist analysis or transfer function fitting. Second, according to the results in Fig. 6, the injection amplitude in different frequency ranges can be set differently according to the noise impact, i.e., adaptive injection amplitude can be developed which can minimise the influence of impedance measurement process.

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