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Load-sharing with degradation management in a compressor station

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***Abstract*—Management of compressor degradation is often considered from the perspective of maintenance of the compressor, but most frameworks for the operation of compressors do not take degradation into account.**

This paper proposes a method for operation of compressors that takes into account the current level of degradation in order to manage further degradation. The algorithm can be used in maintenance planning frameworks, in particular if the timings of maintenance activities are fixed. The algorithm can extend the lifetime of a compressor by mitigating its degradation, or, conversely, can intensify the degradation to reach the maximum level in time for planned maintenance.

The performance of the algorithm has been demonstrated in a case study for five compressors. A comparison with equal load approach shows that the new algorithm improves the operation of the system by managing the degradation of selected compressors. Explicit management of degradation allows an extension of the lifetime of selected compressors before maintenance must be performed. Conversely, by ensuring that the desired level of degradation is attained before pre-planned maintenance actions, it contributes to increased efficacy of maintenance actions.

***Index Terms*—Compressors, degradation, process control**

Note to Practitioners: The paper presents a new framework for load-sharing in a compressor station with compressors subject to degradation. The main innovation of the framework is the use of relationships between custom degradation indicators to manage degradation of the compressors. The results in the paper prove that it is possible to manage degradation in an industrial setting by adjusting the load of each compressor.

From a practical perspective, the framework allows more degraded compressors to follow the less degraded compressors (called leaders). The simplicity of the proposed framework enables an intuitive choice of leaders, in particular in compressor stations with more than two compressors. Focused directly on the load sharing, the framework also avoids adjusting lower level control structures, such as speed control or surge protection. The simplicity and the fact that the load-sharing algorithm affects only the loads make the new framework quickly implementable on a high level of the operating system in a compressor station.

The limitation of the framework is that the parameters of the framework must be adjusted manually to ensure that the compressors stay within their operating ranges. For instance, the parameters of the algorithm can be chosen in such a way that the load-sharing aims to strike a balance between multiple compressors. The next step would be to give a demonstration of the method on a high-fidelity industrial simulator that acts as a proxy for a real compressor station.

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I. Introduction

DEGRADATION is defined by [1] as a "detrimental change in physical condition, with time use or operation". Traditionally, degradation has been managed by maintenance actions. On the other hand, some control frameworks attempt to mitigate degradation. They make use of a model of degradation in the controller, or use the model to predict degradation for decision support for maintenance [2]. Degradation-mitigating control approaches were explicitly proposed by [3] and [4] as *life-extending control* or *damage mitigating control*. Using a detailed model of degradation in mechanical structures, they designed a control algorithm to keep degradation of a single rocket engine below a threshold. An analysis of how models of degradation can be included in control systems was done by [5].

The current paper proposes a new operating strategy that allows management of degradation in systems with multiple pieces of equipment contributing to the same objective. The results of the algorithm are shown in a simulation of a compressor station.

A compressor station is typically a part of a gas transport network and provides a boost for transporting the gas to the receivers [6]. Improving the operation of a compressor station would thus improve the performance of the whole network. Typically, a compressor station consists of two or more compressors and the overall flow of gas is shared among the compressors. The flow of gas through each compressor is called a *load*. Several approaches have been used to share the load among compressors working in parallel. An unequal sharing of load can be better than an equal split because it considers existing dissimilarities between the compressors.

The load-sharing problem for a compressor station was analysed among others by [7] and recently by [8]. They approximated the characteristics of a compressor using polynomial functions and updated the parameters on-line to match the approximation to the real system by solving a nonlinear optimisation problem. The updating allowed satisfaction of the required demand despite the changes in the characteristics, but the effects of load-sharing were not analysed. The authors of [9] and [10] presented a load-sharing approach based on mitigation of degradation by adjusting the operating conditions. They assumed that degradation depended on the load in the compressor and its speed. They included the degradation as constraints in the optimisation problem, but without considering the influence of degradation on the compressors.

The approaches from [9], [10], [7] propose load-sharing strategies that take varying characteristics into account by repeatedly solving nonlinear optimisation problems. The current paper bypasses the need for solving nonlinear optimisation problems focussing on mitigating further degradation by exploiting relative degradation levels between compressors.

The paper is structured as follows. Section II introduces background information on compressors and compressor stations and presents the influence of degradation on these systems. Section III presents the degradation models used in this work. The new load-sharing algorithm is introduced in Section IV. A case study for a compressor station with two compressors is described in Section V and Section VI presents the case study for five compressors. The paper ends with conclusions in Section VII.

II. Compressor stations and the behaviour of a compressor

A. Compressor station

Typically, a compressor station consists of two or more compressors. Figure 1 shows a diagram of a single compressor station, where:

- The suction pressure p_{s} is fixed upstream and is the same for all the compressors;
- The discharge pressure p_{d} is controlled by a pressure controller, which adjusts the outlet valve opening;
- The external pressure p_{out} is fixed downstream;
- The mass flows m_i are controlled by flow controllers, each of which adjusts the speed of the compressor to reach the set-point.

The system from Fig. 1 includes flow controllers FC which receive their set-points from the load-sharing module. The load-sharing module allocates the loads to each compressor so that the demand for the gas is satisfied. The demand might come from the final customers or from other compressor stations, as indicated by [11]. The flow controllers then provide set-points to the respective speed controllers SC. The objective of this paper is to propose a load-sharing algorithm to provide set-points to the mass flow controllers taking into account degradation of the compressors. In particular, the paper will propose a method for load-sharing taking into account how the speed N_1 and N_2 affect degradation of compressors. The effects of the speed on degradation of compressors will be described in Section II-D.

B. Dynamic behaviour of a compressor

The behaviour of a single compressor connected to a discharge tank is captured by a set of dynamic equations from [12]:

$$\dot{p}_{\mathrm{d},i} = \frac{a_{01}^2}{V_i}(m_i - m_{\mathrm{out},i}) \tag{1a}$$

$$\dot{m}_i = \frac{A_1}{L_c}(\Psi_i(m_i, N_i)p_{\mathrm{s}} - p_{\mathrm{d},i}) \tag{1b}$$

$$\dot{\omega}_i = \frac{1}{J_i}(\tau_{\mathrm{t},i} - \tau_{\mathrm{c},i}(m_i, \omega_i)) \tag{1c}$$

where p_{s} is the suction pressure, $p_{\mathrm{d},i}$ is the discharge pressure, m_i denotes the mass flow rate through the compressor, ω_i is the compressor speed in rad s^{-1} and $\omega = \frac{2\pi N}{60}$ where N is the speed in rpm. The variable $\tau_{\mathrm{c},i}$ is the reaction torque of the compressor. The mass flow through the outlet $m_{\mathrm{out},i}$ valve is a function of the valve opening $u_{\mathrm{out},i}$ and external pressure p_{out}. The variable $\tau_{\mathrm{t},i}$ denotes the external torque delivered to the compressor by the motor. The geometric parameters are defined by [13]: V_i is the volume of the tank, A_1 is the area of the cross-section of compressor duct, L_{c} is the length of a compressor, and a_{01} is the inlet velocity of the gas, whereas J_i denotes the inertia of the shaft. The parameter Ψ_i from (1b) is called a *compressor map* and shows the pressure ratio across a compressor as a function of compressor flow and speed.

C. Compressor maps

Each compressor is usually characterized by a *compressor map* Ψ_i, which expresses the relationship between the suction and discharge pressures, compressor speed, mass flow, and compressor efficiency. The map is normally provided by the manufacturer, or it may be derived from first principles and measured data [14].

Figure 2 shows the compressor map used in this work. The black lines shows the compressor map if there is no degradation and the compressors are in *up state*, whereas the grey lines show the compressor map in *degraded state*. A compressor is considered to be in *up state* if there is no degradation and in *degraded state* if degradation is present [1]. The red horizontal line shows the operating line if the pressure ratio is fixed, for instance due to the upstream and downstream requirements. The red triangle denotes the value of Ψ_i in up state and the cross shows the value in degraded state. The circle and the asterisk show the minimal and the maximal value for the mass flow. The data for the compressor map in up state were obtained from [15] using the software from [16].

D. Influence of degradation on the behaviour

The operation of a compressor is affected by degradation, for example due to fouling, as described by [17].

Degradation of the i-th compressor is described by a *degradation indicator* d_i. The influence of degradation will appear both in the compressor maps and in the dynamics. A degradation indicator for industrial compressors has been developed by [18].

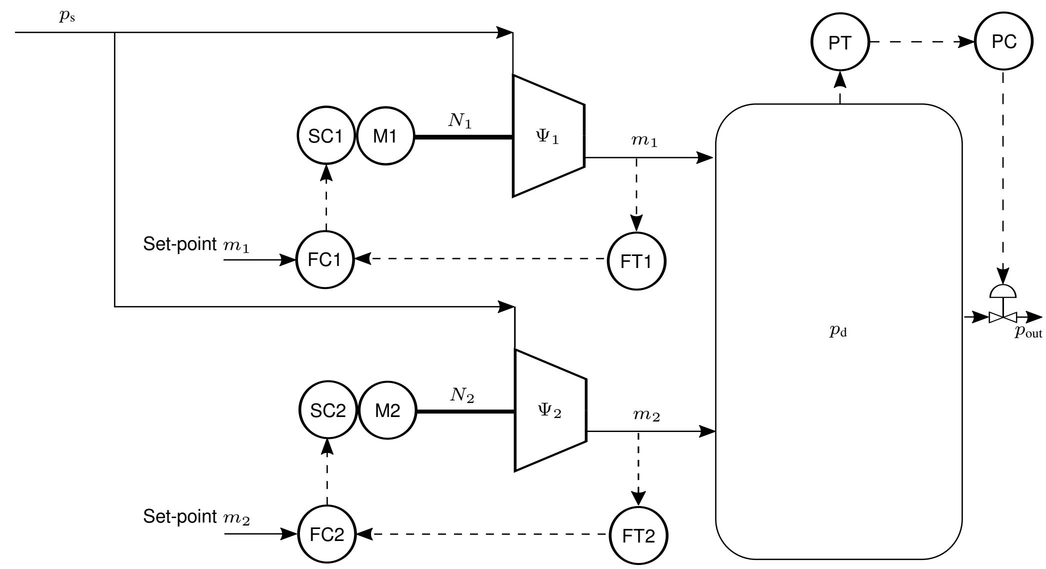


Fig. 1. Compressor station with two compressors connected to the same discharge tank

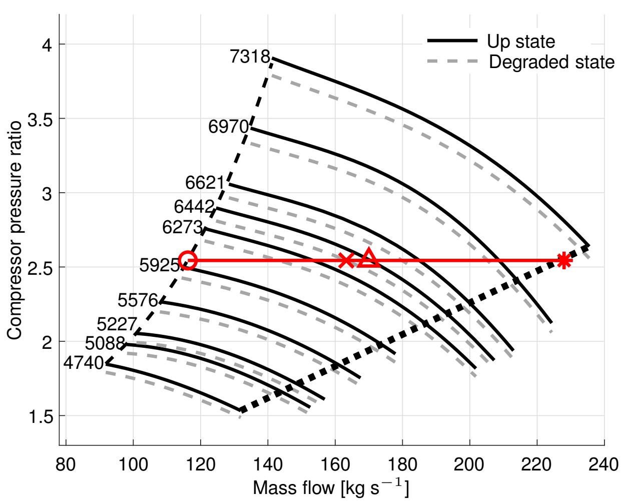


Fig. 2. A compressor map with the pressure ratio as a function of the mass flow and compressor speed in up state and in degraded state. The labels indicate compressor speed in rpm. The data for the compressor map were obtained from [15]

1) Influence on pressure ratio: Degradation due to fouling will affect the pressure ratio [19]. The compressor map in degraded state, Ψ_D, can be obtained from the map in up state Ψ as:

$$\Psi_D = h(d)\Psi \tag{2}$$

with a *degradation function* $h(d)$, which captures how the pressure ratio changes with degradation indicator d. The function $h(\cdot)$ used in this paper will be further described in Section III-D.

The grey dashed lines in Fig. 2 show how a compressor map changes for degradation indicator $d = 0.03\%$ and $h(d) = 1-d$. The value chosen for d corresponds to a 3% loss of pressure ratio, as compared to a compressor in up state, and is a typical value encountered in the industry. The grey lines correspond to the same speeds as the black lines in up state. The red line shows the constant pressure ratio with the operating limits for the mass flow marked with a circle (lower limit, on the dashed surge line) and an asterisk (upper limit, on the dotted choke line). The operating point in up state is marked with a triangle, whereas the cross indicates the operating point in degraded state if there was no flow controller. The speed and the pressure stayed the same, because there was no change in the external operating conditions which would require adjustments of the speed. Figure 1 indicated that the flow controller is a part of the system and to keep the compressor at the operating point marked by the red triangle, it is necessary to increase the speed of the compressor.

2) Influence on efficiency: The need for increasing the speed for the same operating pressure ratio and mass flow results in a loss of efficiency, which is described by a degradation indicator d:

$$\eta_{\mathrm{D}} = (1-d)\eta \tag{3}$$

where η is the efficiency in up state and η_{D} is the efficiency in degraded state.

For a given load, the power consumption in degraded state W_{D} is inversely proportional to the efficiency in degraded state η_{D}

$$W_{\mathrm{D}} = \frac{Hm}{\eta_{\mathrm{D}}} = \frac{Hm}{(1-d_\eta)\eta} = \frac{1}{(1-d_\eta)}W \tag{4}$$

where H denotes the head of a compressor calculated according to [20] and W indicates the power consumption of the compressor if there is no degradation.

III. Factor-based model of compressor degradation

The authors of [21] indicated that a common way of modelling degradation of turbomachinery is to treat degradation as a function of time. At the same time, it is necessary to include factors that affect degradation in the model of degradation to be able to mitigate degradation [22]. An *influencing factor* is an "observable qualitative or measurable quantitative item that affects a system property" [23]. The speed N of the compressor is one of the main factors contributing to the degradation [9]. A further review of models of degradation in turbomachinery is provided in [18].

The model of degradation proposed in this paper will have two elements:

- A factor-free component that depends on time;
- A factor-based multiplier that depends on the speed of the compressor.

A. Factor-free component of the model of degradation

The degradation models from [19] and [21] depend solely on time and can be linked with the fouling phenomenon. Both models are captured by an exponential function, common in industry to express cumulative degradation [24]. For the i-th compressor degradation is modelled as an exponential function of form:

$$d_{\mathrm{T},i}(t) = \alpha(1 - \exp(-\gamma_i t)) \tag{5}$$

where $\alpha \in \mathbb{R}$, and $\gamma_i \in \mathbb{R}_+$ are constant parameters defining the maximal degradation level and the rate of change. The subscript T emphasises the time-dependence of the degradation function. The parameters in (5) can be identified in the way described by [18].

B. Factor-based multiplier of the model of degradation

To include the influence of the speed on the degradation in (7), a factor-based multiplier $F_{\mathrm{N},i}(N)$ is introduced:

$$F_{\mathrm{N},i}(N) = a_i \left(\frac{N}{r_i}\right)^3 \tag{6}$$

where a_i and r_i are constant parameters. The subscript N emphasises the dependence on the speed N in rpm. The values of a_i and r_i used in this work are given in Table II. They were chosen so that $F_{\mathrm{N},i}(N) = 1$ for $N = 5848$ rpm. The form of (6) was derived from [9] who indicated that degradation of a compressor depends on the third power of the compressor speed based on experimental results from [25]. The limitations of the model are discussed in Section VII-B.

C. Factor-based model of degradation

A factor-based model of degradation is now introduced to capture the degradation with compressor speed as an influencing factor:

$$d_i(t) = d_{\mathrm{T},i}(t) \cdot F_{\mathrm{N},i}(N_i(t)) \tag{7}$$

where N_i denotes the speed of the i-th compressor and d_i is the degradation. The function $d_{\mathrm{T},i}(t)$ represents a factor-free component, and $F_{\mathrm{N},i}(N)$ represents the factor-based multiplier. Focusing on the speed as the main factor influencing the degradation allows a more direct analysis of the influence of control on degradation as it bypasses the dynamic effect introduced by the inertia of the compressor and allows (1c) to be omitted in the model used for simulation. If not specified, 'degradation indicator' in this paper refers to d_i in (7).

The model from (7) ensures that the compressors are in up state at time $t = 0$. The compressors are in up state because the exponential formula of the factor-free component from (5) is equal to unity for $t = 0$. The exponential formula of (5) indicates also that for a fixed value of compressor speed N in (6), the model from (7) will saturate. The exponential part of the factor-free component from (5) tends to zero for t increasing:

$$\lim_{t\to\infty} d_i(t) = a_i \left(\frac{N}{r_i}\right)^3 \alpha \tag{8}$$

D. Degradation function

The degradation function $h(d)$ captures how degradation affects a selected variable describing the behaviour of a unit. In this paper, it is defined as:

$$h(d) = 1 - d \tag{9}$$

where d is a variable that represents degradation. For instance, if $d = d_i(t)$, the degradation function $h(d)$ from (9) represents the degradation of pressure ratio. It is obtained from (8) that the degradation indicator d is bounded. As a result, the degradation function $h(d)$ is also bounded. The degradation function $h(d)$ is maximal when $d = 0$, i.e. when there is no degradation. Conversely, when d reaches its upper limit, the value of $h(d)$ is minimal.

The degradation function $h(d)$ from (9) can be interpreted as *health* of a compressor with degradation d. For instance, if it is assumed that Compressor 1 is more degraded than Compressor 2 i.e.:

$$d_1 \geq d_2 \tag{10}$$

where d_i is given by (7), (9) yields:

$$h(d_1) \leq h(d_2) \tag{11}$$

with an interpretation that Compressor 2 is healthier than Compressor 1.

The inequality (11) is a basis for the analysis of the relationships between the compressors in this paper.

IV. Load-sharing with degradation management

The new load-sharing strategy is now introduced for a compressor station with two compressors. Assuming that the compressors are similar, [20] indicates that the common approach to load-sharing is to assign the loads using the distance to surge Q_i:

$$m_2 = m_1 \frac{Q_2}{Q_1} \tag{12}$$

where m_i is the load assigned to i-th compressor. The distance to surge can be obtained from Fig. 2 by calculating the difference between the operating point (triangle if there is no degradation, cross if there is degradation) and the lower limit (circle). If the compressors are identical, then $Q_2 = Q_1$ and $m_1 = m_2 = \frac{M}{2}$ where M is the desired overall flow from the station. The authors of [20] indicate that it is almost impossible to find identical compressors in practice. Minor flow variations can lead to imbalances in the loads processed by each compressor, and in consequence, to changes in compressor characteristics. The resulting differences in compressor characteristics lead further to different degradation functions $h(d)$ in (2).

Formula (12) does not take into account the possible changes due to degradation. For instance, it does not consider that to keep the same distance to surge, it is necessary to increase the speed of a compressor.

A. Load-sharing algorithm for two compressors

To address the changes due to degradation, this paper proposes to assign the loads m_1 and m_2 according to (13):

$$m_1 = \frac{M}{2} \exp\left(q\left(1 - f\left(d_1, d_2\right)\right)\right) \tag{13a}$$
$$m_2 = M - m_1 \tag{13b}$$

where M is the required overall demand for the gas from the station, d_i denotes the current degradation level of the i-th compressor, and $q > 0$ is a *scaling factor*. The function $f(d_1, d_2)$ captures the relationships between the compressors, i.e. which compressor is more degraded as indicated by inequality (10) and (11). Depending on the value of $f(d_1, d_2)$, it is possible to manage the degradation of the compressors. Suitable forms for $f()$ are presented in Section IV-B.

The formula (13) depends on the current degradation levels d_1 and d_2 of the two compressors. In practice, the algorithm will adjust the loads when degradation levels become available. For instance, the new data may be available once per day in off-shore natural gas compressors [18].

B. Degradation management

It is now assumed that the two compressors are identical in up state and the default load assignment is done by diving the demand in two. When the compressors degrade, each compressor is characterised by its own value of degradation indicator, calculated according to (7). The management of degradation refers to changes in the value of the degradation indicator d_i from (7) relative to the value obtained in the equal load approach. The management of degradation of the i-th compressor has one of the objectives:

- To decrease degradation, if the degraded compressor gets less load than in the equal load approach. Load sharing strategy that decreases degradation d_i relative to an equal load strategy is called *mitigation of degradation*;
- To increase degradation, if the degraded compressor gets more load than in the equal load approach. Load sharing strategy that increases degradation d_i relative to an equal load strategy is called *intensification of degradation*.

Degradation management is done by choosing the functional form of $f(d_1, d_2)$. The functional forms of $f(d_1, d_2)$ will be analysed for:

- Mitigation of degradation:

$$f_{\mathrm{M}}(d_1, d_2) = \frac{h(d_2)}{h(d_1)} \tag{14}$$

- Intensification of degradation:

$$f_{\mathrm{I}}(d_1, d_2) = \frac{h(d_1)}{h(d_2)} \tag{15}$$

TABLE I
DEGRADATION MANAGEMENT AS INFLUENCED BY THE FUNCTIONAL FORM OF $f(d_1, d_2)$

	Compressor 1 degrades more quickly	Compressor 1 degrades more slowly
f_{M}	Mitigation of degradation of Compressor 1, intensification of degradation of Compressor 2	Intensification of degradation of Compressor 1, mitigation of degradation of Compressor 2
f_{I}	Intensification of degradation of Compressor 1, mitigation of degradation of Compressor 2	Mitigation of degradation of Compressor 1, intensification of degradation of Compressor 2

The results of the degradation management based on the relationships between the degradation functions are gathered in Table I and will be shown in Section V.

C. Scaling factor

The scaling factor q influences what part of the load will be assigned to each compressor and therefore contributes to the degradation management described in Section IV-B. From (13a):

$$\exp(q(1 - f(d_1, d_2))) = m_1 \Big/ \frac{M}{2} \tag{16}$$

Assuming that the inequality (10) is fulfilled, increasing q will allow a larger mitigation of the degradation of Compressor 1 using (14). Conversely, increasing q will also intensify the degradation of Compressor 1, if (15) is used.

For a fixed pressure ratio the mass flow assigned to each compressor has to satisfy the inequality:

$$m_i^{\min} \leq m_i \leq m_i^{\max} \tag{17}$$

The values of $m_i^{\min}$ and $m_i^{\max}$ in the inequalities (17) are obtained for a fixed value of pressure ratio from the compressor map bounded by the surge line m_i^{s}, minimal speed line m_i^{Nmin}, maximal speed line m_i^{Nmax} and choke line m_i^{c}:

$$m_i^{\min} = \max\{m_i^{\mathrm{s}}, m_i^{\mathrm{Nmin}}\} \tag{18a}$$
$$m_i^{\max} = \min\{m_i^{\mathrm{c}}, m_i^{\mathrm{Nmax}}\} \tag{18b}$$

It is assumed that the compressors have the same operating ranges $m_1^{\min} = m_2^{\min} = m^{\min}$ and $m_1^{\max} = m_2^{\max} = m^{\max}$. For the pressure ratio $\Psi = 2.52$ marked in Fig. 2, the values are:

$$m^{\min} = 116 \text{ kg s}^{-1} \tag{19a}$$
$$m^{\max} = 228 \text{ kg s}^{-1} \tag{19b}$$

and are marked with a circle and an asterisk in Fig. 2.

The constraints from (17) will affect the possible values of q depending on the objective: the mitigation or intensification of degradation.

The following inequalities are obtained from (13) and (18):

$$\frac{M}{2} \exp(q(1 - f(d_1, d_2))) \geq m_1^{\min} \tag{20a}$$
$$\frac{M}{2} \exp(q(1 - f(d_1, d_2))) \leq m_1^{\max} \tag{20b}$$
$$M - \frac{M}{2} \exp(q(1 - f(d_1, d_2))) \geq m_2^{\min} \tag{20c}$$
$$M - \frac{M}{2} \exp(q(1 - f(d_1, d_2))) \leq m_2^{\max} \tag{20d}$$

TABLE II
PARAMETERS OF DEGRADATION MODEL FROM (7) FOR TWO COMPRESSORS

	a_i	r_i in rpm	α_i	γ_i
Compressor 1	4×10^{-5}	200	0.05	2×10^{-6}
Compressor 2	4×10^{-5}	200	0.05	6.25×10^{-7}

Inequalities (20a) and (20d) are considered to find the value of q for mitigation of degradation for Compressor 1, whereas inequalities (20b) and (20c) are taken into account if the objective is to intensify the degradation of Compressor 1. A further discussion on staying within the operating ranges will be done in Section VI-A where an extension to more than two compressors is presented.

This paper assumes $q = 2$, which keeps the compressors within the operating range.

V. ANALYSIS FOR TWO COMPRESSORS

To evaluate the algorithm proposed in Section IV a scenario with compressors degrading at different rates is analysed. The scenario is analysed both in terms of degradation mitigation using f_{M} from (14) and intensification of degradation with f_{I} from (15). Section VI will present a generalisation of the algorithm.

A. Two compressors degrading - analysis of degradation

This section analyses the load-sharing algorithm from (13) if the compressors degrade at different rates for a fixed value of demand $M = 340$ kg s^{-1}.

It is assumed that the degradation of Compressor 1 is larger than the degradation of Compressor 2, $d_1 > d_2$. This assumption preserves the mitigating and intensifying character of the function from (14) and (15), as indicated in Table I.

The values of the parameters of the models of degradation are presented in Table II. The values of the parameters were chosen to simulate the overall degradation level of 7% over a period of 90 days.

The influence of load-sharing on the compressors will be done by comparing the times when the degradation function from (5) reaches 93%. According to the interpretation of the degradation function from Section III-D, this comparison takes into account when the health of the compressors falls to 93% of the condition in up state.

B. Two compressors degrading - results

1) Degradation mitigation: The results of the new operating strategy for two compressors are shown in Fig. 3. The left column in Fig. 3 shows the mass flows (Fig. 3a), the speed (Fig. 3c), and the power (Fig. 3e) if a mitigation strategy was used, with f_{M} from (14). The results are compared with the equal load approach (green lines in Fig. 3). The degraded compressor gets less load at the beginning (dashed line in Fig. 3a). This is due to the fact that Compressor 1 degrades more quickly and therefore the value of $f(d_1, d_2)$ increases. After ten days, however, the ratio $\frac{d_2}{d_1}$ starts to decrease. This is due to the fact that the quickly degrading compressor reaches its lower bound of degradation after approximately 10 days, whereas the slowly degrading compressor still decreases. From day 60, the compressors become identical again and get assigned the same value of the load.

A low flow has a direct impact on the power necessary to run the compressors subject to degradation. As shown in Fig. 3e for two degrading compressors, the power necessary to run the compressors does not differ from equal load assignment. This is because now the algorithm takes into account the degradation of the second compressor and does not assign too much load to it. Thus, as soon as the ratio is close to unity, the algorithms reverts to equal load. Assigning a lower load from day 0 to day 60 the more degraded compressor improved its health as compared with the equal load approach, shown with a degradation function in Fig. 4a. The degradation function with load-sharing reached 93% ten days later compared with equal-load. At the same time, the compressor with a slower degradation rate reached 93% only two days earlier.

2) Degradation intensification: The results show that it is possible to mitigate degradation by sharing the load using f_{M}. Conversely, the right column of Fig. 3 indicates that it is also possible to intensify degradation of a compressor using the functional form f_{I}. Intensification of degradation comes in useful if a maintenance action has been already scheduled. By intensifying degradation of a compressor, it is ensured that the maintenance is performed on a fully degraded compressor. The degraded compressor is made to work as hard as possible in the run-up to its maintenance. This mitigates the degradation of other compressors and hence the following maintenance interval is longer than it would have been otherwise, hence saving maintenance costs. The ratio $\frac{d_1}{d_2}$ is shown in Fig. 4f. Assigning a larger load to the more degraded compressor (dashed line in Fig. 3b) results in an increased compressor speed, and in consequence, in faster degradation. The degradation function of the degraded unit reaches 93% five days earlier than if the loads are assigned equally. The degradation function of the slowly degrading unit is slowed down and the threshold is reached two days later.

The intensification of degradation affects the power necessary to run the compressors. Until day 50, the power necessary to run the two compressors is larger than in the equal load case. This indicates that the degraded compressor requires more power than can be saved by slowing down the degradation of the slowly degrading unit. After the degradation ratio becomes small, the power becomes equal to the equal load case.

VI. APPLICATION TO MULTIPLE COMPRESSORS

It is now assumed that a compressor station has K units, which are identical in up state. The number of units depends on the application. For instance, a compressor station for natural gas pipelines can include six units [11], whereas an air separation plant can include over 10 units [26]. Each unit has a different degradation rate γ_i in (5) and the units are ordered from the most degraded to the least degraded:

$$d_1 \geq d_2 \geq \ldots \geq d_K \tag{21}$$

It is assumed that the inequalities from (21) persist over time.

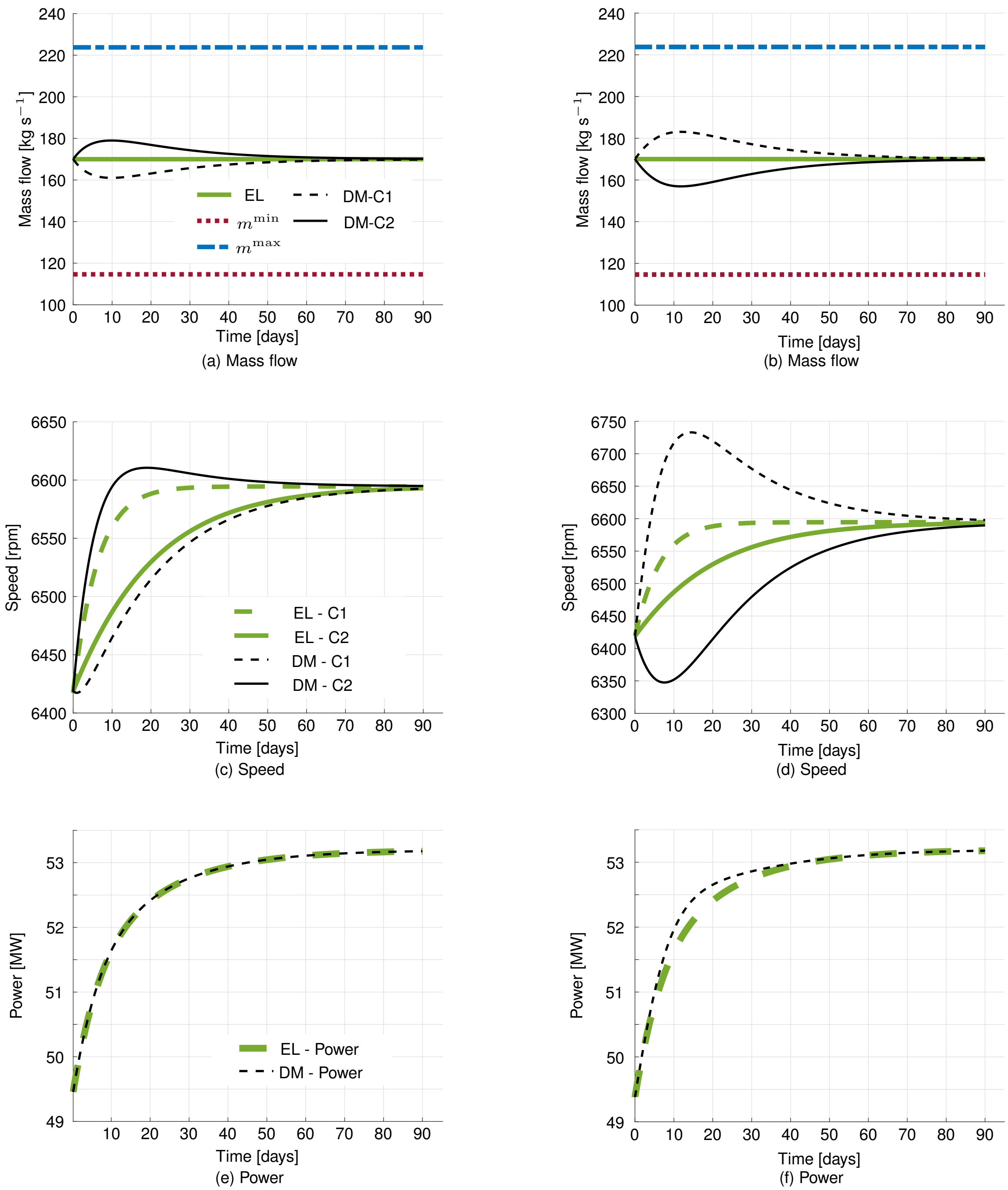


Fig. 3. Process variables for degradation management (DM) in the case of mitigating degradation using f_{M} (left column) and intensifying degradation with f_{I} (right column) if Compressor 1 (C1) degrades more quickly than Compressor 2 (C2), compared with equal load approach (EL)

The algorithm from (13) is expanded to:

$$m_i = \frac{M}{K} \exp\left(q\left(1 - \frac{h(d_{i+1})}{h(d_i)}\right)\right) \tag{22a}$$

$$m_K = M - \sum_{i=1}^{K-1} m_i \tag{22b}$$

where m_i is the mass flow assigned to the i-th compressor, $i = 1, \ldots, K-1$. Equation (22) is used with (1) to continuously update the desired flows for each compressor. The algorithm from (22) compares the degradation indicator of the i-th compressor d_i with the compressor with the most similar, but smaller degradation d_{i+1}. The approach from (22) will be called a *collaboration strategy*, because the loads are assigned to compressors based on the ratios of degradation functions of each two compressors with most similar degradation.

A. Interpretation for multiple compressors

1) Collaboration strategy: Since the compressors are ordered according to expression (21), all the ratios are larger

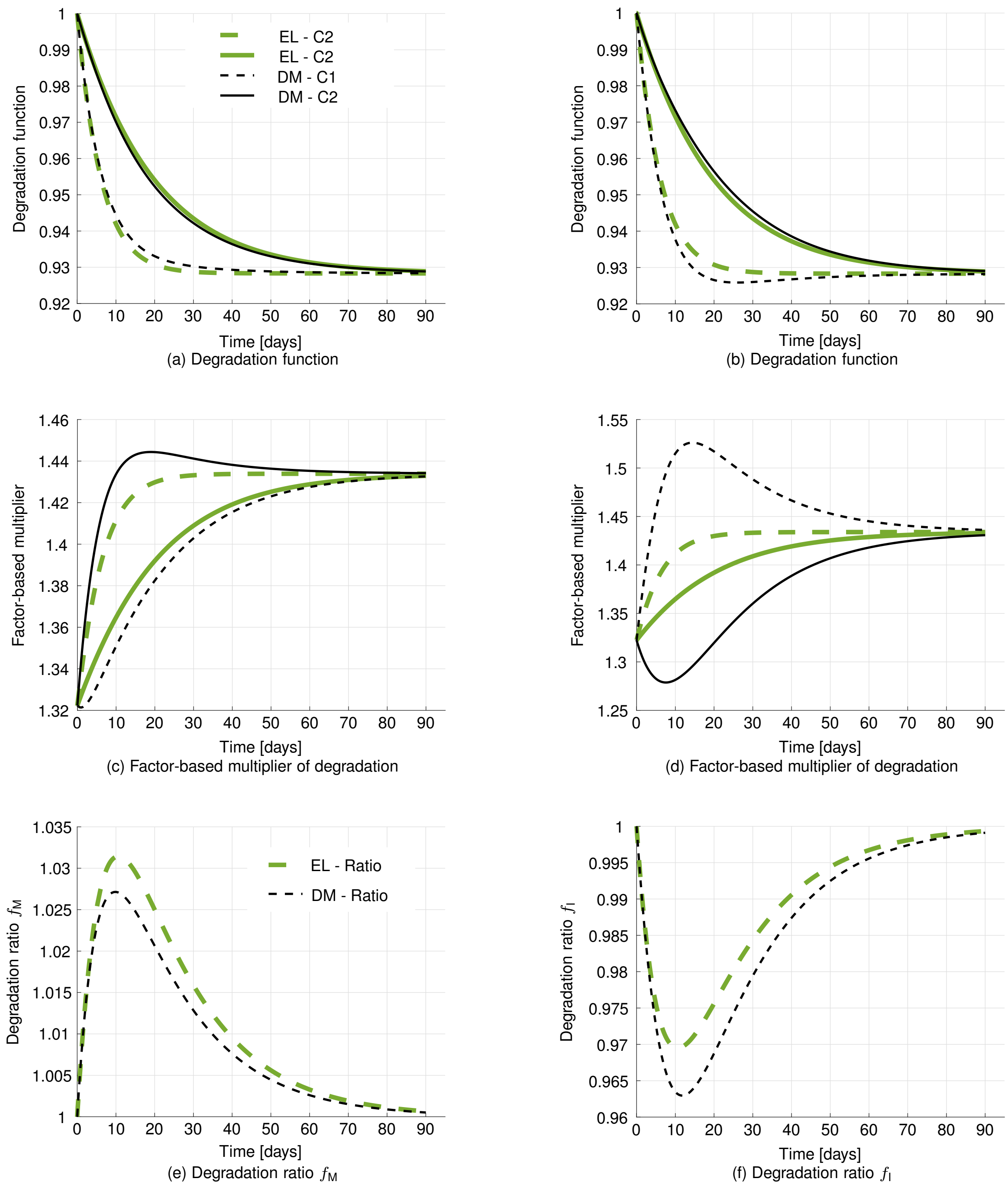


Fig. 4. Degradation variables for degradation management (DM) in the case of mitigating degradation using f_{M} and intensifying degradation with f_{I} if Compressor 1 (C1) degrades more quickly than Compressor 2 (C2), compared with equal load approach (EL)

than or equal to unity:

$$\frac{h(d_{i+1})}{h(d_i)} \geq 1 \tag{23}$$

The case when the ratio from expression (23) becomes unity is a special case and the load-sharing algorithm from (22) reverts to equal load approach. From now on, it will be assumed that the strong inequality is fulfilled in expression (23).

Given the current values of the functions $h(d_i)$, (22) gives a calculation of the loads for all the compressors. The expression (22) indicates that the i-th compressor, $i < K$, is assigned a lower load than in the equal load case. The least degraded K-th compressor with degradation d_K gets the remaining load, which is larger than in the equal load case. How much less each compressor will get depends on the ratio $\frac{h(d_{i+1})}{h(d_i)}$. If the compressors i and $i+1$ have degradation indicators close to each other, the ratio will be close to unity and loads assigned to the i-th and $i+1$-st compressors will be similar. If the ratio $\frac{h(d_{i+1})}{h(d_i)}$ is larger than unity, compressor i will get less load than in the equal load approach.

In consequence, the algorithm makes the degradation of a compressor more similar to that of a compressor with the most similar degradation function, but in better health. If the degradation values are significantly different between the compressors, the algorithm is able to mitigate the degradation of the more degraded units.

2) Leader strategy: The interpretation suggests an alternative to the collaboration strategy from (22). The alternative compares the degradation function of each compressor with the degradation function of the least degraded:

$$m_i = \frac{M}{K} \exp\left(q \left(1 - \frac{h(d_K)}{h(d_i)} \right) \right) \tag{24a}$$

$$m_K = M - \sum_{i=1}^{K-1} m_i \tag{24b}$$

If the strategy from (24) is used, the compressor that is initially least degraded becomes significantly more degraded using this strategy, compared to the equal load strategy. Nevertheless, Compressor K is assumed to remain the healthiest one of all the compressors, so that the assumption about the order of the compressors is fulfilled. The values of degradation of the remaining $K-1$ compressors tend to the degradation of the least degraded unit, if the strategy from (24) is used. The approach from (24) will be called a *leader strategy*, because the degradation functions of the i-th compressor, $i = 1, \ldots, K-1$ are related to the degradation function of Compressor K, which can be considered the *leader*.

3) Comparison of the strategies: The collaboration strategy leads to assigning larger loads to degraded compressors than the leader strategy. Conversely, the collaboration strategy assigns a smaller load to the healthiest compressor. This is because the ratios in the collaboration strategy are based on compressors with most similar degradation, whereas in the leader strategy, the ratios use the healthiest compressor. As a result, the ratios in the collaboration strategy are closer to unity and the loads assigned to degraded compressors according to (22) will be closer to the values obtained from equal load approach. The ratios used in the leader strategy will be smaller than unity and using (24) will yield values of the load that are different from the values in the equal load approach. As a result, the loads assigned to the i-th compressor, $i = 1, \ldots, K-1$, in the leader strategy will be lower than the loads assigned to the same compressor in the collaboration strategy.

4) Demand satisfaction and tuning: Both the collaboration and the leader strategies ensure that the demand is satisfied:

$$\sum_{j=1}^{K} m_K = M \tag{25}$$

Satisfaction of the demand from (25) is ensured by (22b) in the collaboration strategy and (24b) in the leader strategy. Similarly as in the case for two compressors, it is necessary to ensure that the compressors stay within their operating limits, as indicated in (20) for two compressors. Thanks to the assumption from (21), the i-th compressor, $i = 1, \ldots, K-1$, will get a load smaller than if the load was distributed equally. Therefore, it is necessary to ensure that

$$m_i \geq m_i^{\min} \tag{26}$$

for all $i = 1, \ldots, K-1$ in both the collaboration and the leader strategy. In this paper, the inequality (26) is fulfilled by setting $q = 2$ in (22a) and (24a).

Conversely, Compressor K will get a larger value in both the collaboration and the leader strategy than in the equal load case. Thus, it is necessary to ensure that

$$m_K \leq m_K^{\max} \tag{27}$$

where m_K is given by (22b) in the collaboration strategy or by (24b) in the leader strategy. Satisfaction of (27) will depend on the values assigned to compressors from i to $K-1$ as well as on the demand M. Using inequality (26), it is obtained that the flow assigned to compressor K if either of the strategies is used is smaller than if the minimal load was assigned to each compressor from i to K

$$\underbrace{M - \sum_{i=1}^{K-1} m_i}_{m_K \text{ from (22b) or (24b)}} \leq \underbrace{M - \sum_{i=1}^{K-1} m_i^{\min}}_{m_K \text{ if } m_i^{\min} \text{ was assigned}} \tag{28}$$

Therefore, to ensure that $m_K \leq m_K^{\max}$, this paper assumes that the demand M is such that the inequality is fulfilled

$$M - \sum_{i=1}^{K-1} m_i^{\min} \leq m_K^{\max} \tag{29}$$

In a general case, satisfaction of inequality (27) can be ensured by assigning the K-th compressor the smaller of the two values:

$$m_K = \min\{m_K^{\max}, M - \sum_{i=1}^{K-1} m_i\} \tag{30}$$

If $m_K = m_K^{\max}$ and $m_K^{\max} < M - \sum_{i=1}^{K-1} m_i$, then additional steps are needed. Otherwise the overall demand M is not satisfied:

$$\sum_{i=1}^{K} m_i < M \tag{31}$$

The inequality (31) indicates that the healthiest compressor K is unable to compensate for the lower loads assigned to compressors $1, \ldots, K-1$ even if working at its upper limit. Satisfaction of the demand M can then be ensured by increasing the loads assigned to compressors $1, \ldots, K-1$ by adjusting the value of q. The parameter q indicates how much the exponential function in (24a) in the leader strategy and in (22a) in the collaboration strategy decreases the load obtained for equal load assignment M/K. In both strategies decreasing q leads to the equal load assignment:

$$\lim_{q \to 0} m_i = \frac{M}{K} \tag{32}$$

where m_i is obtained from either (24a) in the leader strategy or from (22a) in the collaboration strategy. Therefore, choosing

Algorithm 1: Load-sharing for multiple compressors with ensured demand satisfaction

Input: Initial choice for $q > 0$, demand M, number of compressors K, mass flow limits $m_i^{\min}$, $m_i^{\max}$ for $i = 1, \ldots, K$, choice of strategy from (22a) (collaboration) or (24a) (leader), scaling parameter $0 < \rho < 1$

Output: Loads m_i, $i = 1, \ldots, K$

1 Obtain measurements d_i from all K compressors, $i = 1 \ldots, K$;
2 Evaluate $h(d_i)$;
3 **while** $i \leq K - 1$ **do**
4   Compute the load m_i according to the chosen strategy;
5   **if** $m_i < m_i^{\min}$ **then**
6   Assign $m_i^{\min}$ to Compressor i
7   **else**
8   Assign m_i to Compressor i
9   **end**
10 **end**
11 Compute m_K according to the chosen strategy;
12 **if** $m_K > m_K^{\max}$ **then**
13   Assign $m_K^{\max}$ to Compressor K
14 **else**
15   Assign m_K to Compressor K
16 **end**
17 Compute $\sum_i^K m_i$;
18 **if** $\sum_i^K m_i \neq M$ **then**
19   Set $q \leftarrow q\rho$ and go to 4.
20 **else**
21   Apply m_i, $i = 1, \ldots, K$ to the compressors and go to 2.
22 **end**

a smaller q will lead to assigning larger loads to compressors $1, \ldots, K - 1$, and a smaller load to compressor K. The inequalities to find q have the same form as (20).

However, solving inequalities from (20) requires information about the function $h(d_i)$ for all compressors. In particular, if the upper limit of degradation is unknown, finding q may be challenging. To facilitate the choice of q, a positive scaling parameter ρ is introduced. It is assumed that ρ is strictly smaller than unity, $\rho < 1$, with the default value $\rho = 0.999$. Then the value of q can be iteratively decreased by multiplying the current value by ρ until all the compressors work within their limits, $m_i^{\min} \leq m_i \leq m_i^{\max}$, and the demand is satisfied, $\sum_{i=1}^{K} m_i = M$. The performance of the scaling parameter will be discussed in Section VI-B5.

The entire algorithm with the adjustment of q is presented as Algorithm 1.

B. Five compressors degrading - results

The results from the collaboration strategy and the leader strategy will be now presented in simulation for a compressor station with five units. The parameters of the degradation due to speed in (6) are the same for all compressors and are $a_i = 4 \times 10^{-5}$, $r_i = 200$ rpm. The parameter of the degradation due to time in (7) is $\alpha_i = 0.05$, but the rates γ_i are different for each compressor and are presented in Table III. This ensures that all the compressors reach the same value of degradation, but at different times, as well as that the inequality (10) is satisfied over the whole time. The simulation period was extended to 180 days to capture the varying degradation rates. The demand is $M = 850$ kg s^{-1} resulting in 170 kg s^{-1} if equal load is used.

TABLE III
PARAMETERS OF FACTOR-FREE COMPONENT OF DEGRADATION γ_i

Compressor	1	2	3	4	5
$\gamma_i/10^{-5}$	0.2	0.067	0.05	0.04	0.033

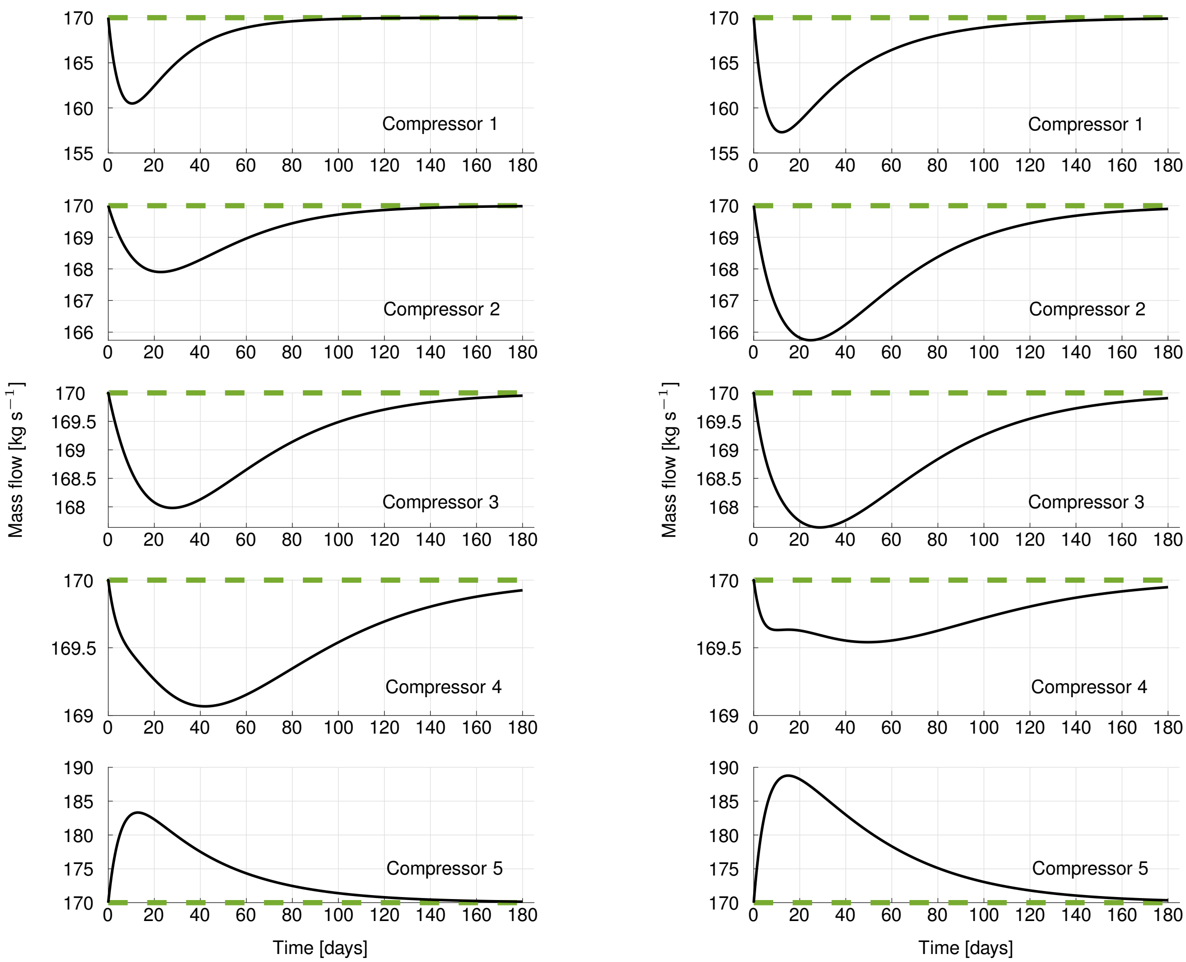
1) Discussion of the collaboration strategy: The results are presented in Figs. 5, 6 and 7, ordered from the top according to the value of degradation. The top plot represents the values for the most degraded Compressor 1 with degradation d_1 and the bottom plot corresponds to the healthiest unit, Compressor 5, with degradation d_5. The results of the collaboration strategy are the flows (Fig. 5a), the speeds (Fig. 6a), and the factor-based multiplier from (6) (Fig. 7a) for the five compressors.

The results show that, compared to the equal load assignment, the algorithm in the form of the collaboration strategy assigns lower loads to four degraded compressors and a higher load to the least degraded compressor. As a result, the speeds of Compressors 1-4 decrease compared to the equal load assignment of loads, and in consequence, the factor-based multiplier $F_{\mathrm{N},i}$ decreases as well (Fig. 7a).

Figure 8a presents the degradation function for all five compressors if the collaboration strategy is used compared to the values obtained in the equal load case. It shows that the overall change of degradation function in this paper is primarily visible for Compressors 1 and 5. The change of degradation function for Compressors 1 is due to the fact that degradation function of Compressors 1 is significantly smaller than degradation function of Compressor 2, which is the closest one in terms of degradation function. Thus, the ratio of degradation function of these two compressors is large and leads to assigning a low load to Compressor 1. The change of degradation function for Compressor 5 is due to the fact that Compressor 5 has to process all the load that is required to satisfy the overall demand, but has not been assigned to any of the more degraded compressors. As a result, the speed of Compressor 5 increases, affecting the degradation-dependent multiplier according to (6) and leading to a lower value of degradation function.

The ratios of degradation functions for the five compressors are in Fig. 9a. In this case, the maximal values of all the ratios can be ordered as:

$$\begin{aligned} &\max\left(\frac{h(d_2)}{h(d_1)}\right) > \max\left(\frac{h(d_3)}{h(d_2)}\right) > \\ &\max\left(\frac{h(d_4)}{h(d_3)}\right) > \max\left(\frac{h(d_5)}{h(d_4)}\right) \end{aligned} \tag{33}$$



(a) Collaboration strategy from (24) (b) Leader strategy from (24)

Fig. 5. Mass flows for five compressors, obtained from the collaboration strategy (━, left) and from the leader strategy (━, right) compared with equal load approach (╍)

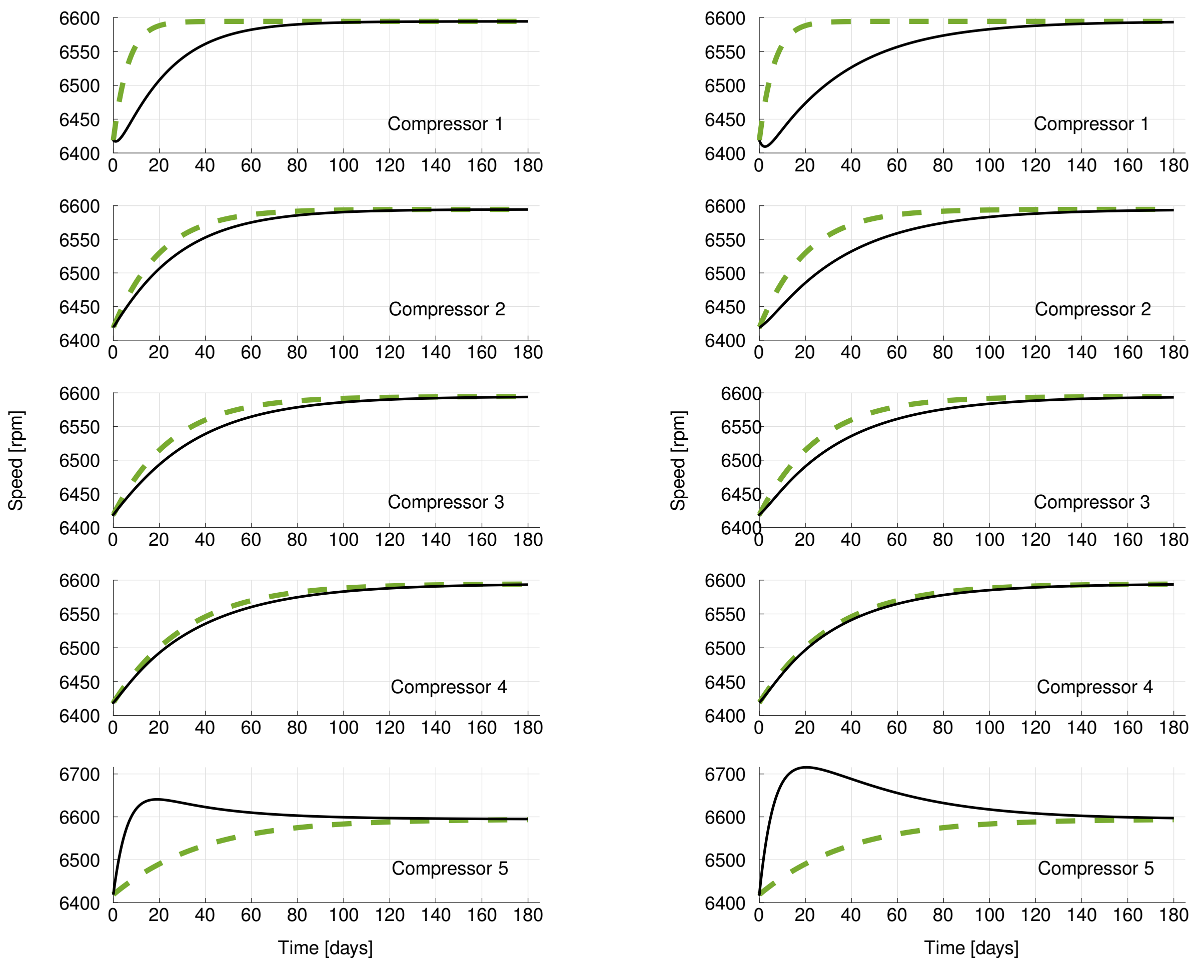
Moreover, all the ratios are greater than unity which means that the more degraded compressor in each ratio will get less load than in the equal load approach. In particular, Compressor 1 gets the smallest part of the equal load assignment at the beginning because the ratio obtained for Compressors 1 and 2 is the largest. In consequence, Compressor 1 reaches the threshold of 93% 10 days later than in the equal load case (Table IV).

2) Discussion of the leader strategy: Figures 5b, 6b, and 7b show the results for the flow, speed, and degradation for the leader strategy. Compressors 1 to 4 get less load, whereas Compressor 5 gets the largest load. In consequence, the speed of Compressors 1 to 4 decrease and the factor-based multiplier of degradation is also smaller than in equal load assignment.

The plot for Compressor 4 in Fig. 5b shows what happens when the ratios are close to each other. As indicated in Fig. 9b, the degradation ratio between Compressor 5 and Compressor 4 is the smallest. As a result, the loads assigned to these compressors according to (24) will be close to the loads assigned in the equal load approach. Moreover, the degradation ratio changes very slowly around day 20. As a result, the load assigned to Compressor 4 also changes very slowly around day 20. When the ratio increases after day 20, the load assigned to Compressor 4 decreases.

Figure 8b presents the degradation function for all five compressors (black) if the leader strategy is used compared to the values obtained in the equal load case (green). It shows that the overall change of degradation function is primarily visible for Compressors 1 and 5. The reasons will be analysed in Section VI-B3 where the two approaches are compared.

3) Comparison between the collaboration and leader strategies: Figure 9 shows the degradation ratios in both the collaboration strategy (Fig. 9a) and the leader strategy (Fig. 9b). The dashed blue line shows the degradation ratio between Compressor 1 and Compressor 2 (Fig. 9a) between Compressor 1 and Compressor 5 (Fig. 9b). Since $d_2 > d_5$, the



(a) Collaboration strategy from (22)

(b) Leader strategy from (24)

Fig. 6. Speeds of five compressors, obtained from the collaboration strategy (—, left) and from the leader strategy (—, right) compared with equal load approach (- -)

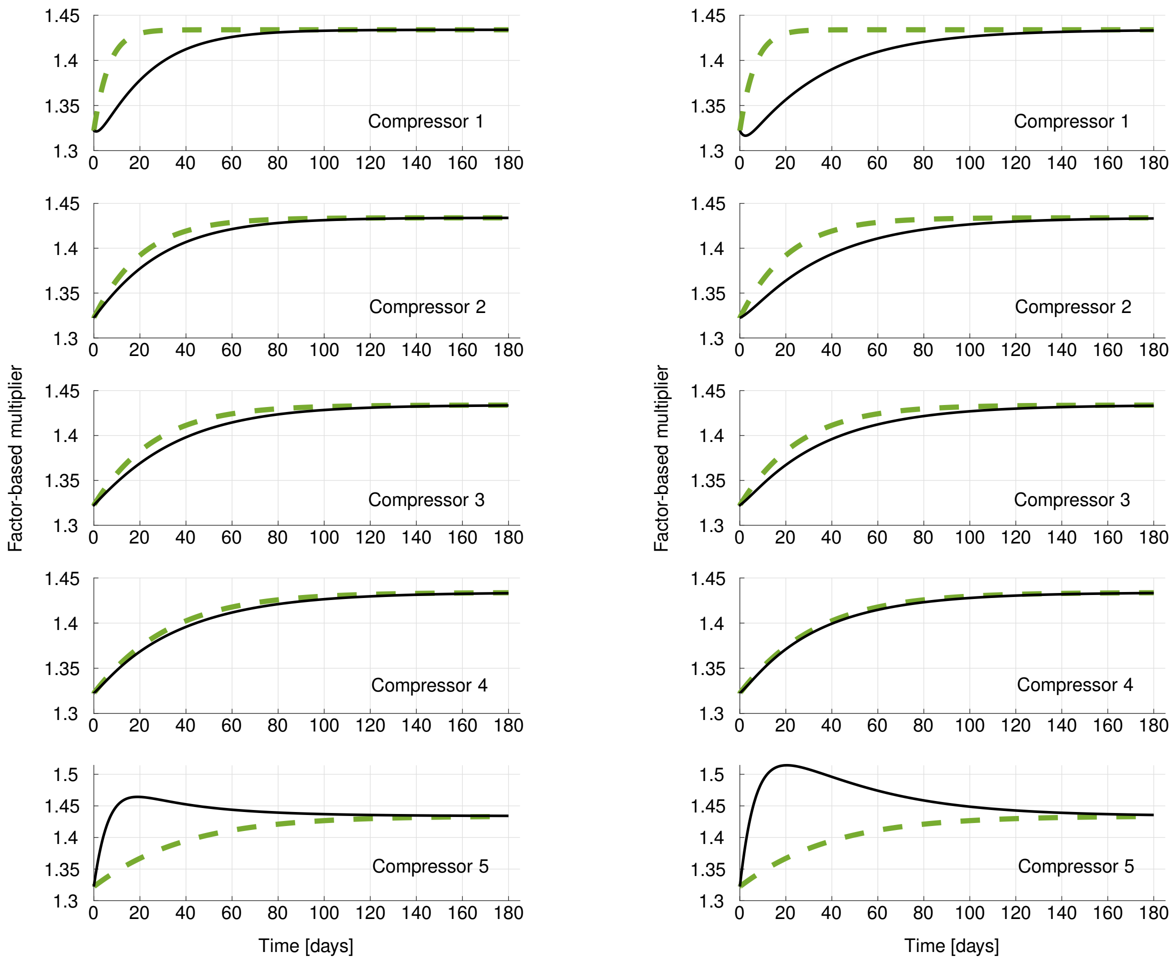
maximal ratio of the blue curve in Fig. 9a is smaller than in Fig. 9b. This means that using the leader strategy enables a lower load for the most degraded Compressor 1. This extended the time before reaching 93% threshold from 10 to 27 days for Compressor 1 (Table IV). The extension is also visible in Fig. 8. The dashed black line in Fig. 8a corresponding to the collaboration strategy, reaches the threshold earlier than in Fig. 8b.

The dotted red line and the dash-dotted yellow line show how the degradation functions of Compressor 2 and 3 interact with each other and with the degradation function of Compressor 5, respectively. The ratio between the degradation functions of Compressor 3 and Compressor 2 (dotted red line in Fig. 9a) is smaller than the ratio between Compressor 5 and Compressor 2 (from the assumption in (21)) and shown with dotted red line in Fig. 9b. Therefore, the load-sharing strategy from the leader strategy helped Compressor 2 and extended the time before it reached the 93% threshold from three to seven days. The leader strategy also enabled a longer time for Compressor 3, but only two days longer. This is due to the fact that the ratio in Fig. 9b is only slightly higher than in Fig. 9a (dash-dotted yellow line).

TABLE IV
NUMBER OF DAYS GAINED (+) OR LOST (-) FOR COMPRESSORS C1, C2, C3, C4, C5 IN THE COLLABORATION AND THE LEADER STRATEGIES

	C1	C2	C3	C4	C5
Collaboration	10	3.5	3	2	-5
Leader	27	7	5	1.5	-10

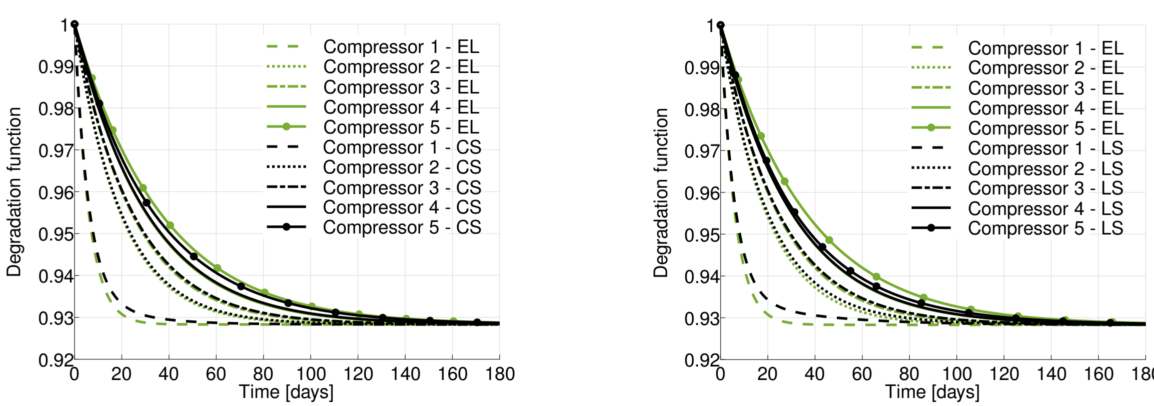
Such behaviour is a result of increased load assigned to Compressor 5. Increasing the load intensifies the degradation of Compressor 5, which reached the threshold 10 days earlier than in equal load case (solid line with dots in Fig. 8b). The column for Compressor 4 in Table IV also shows how the increased degradation of Compressor 5 affected Compressor 4.



(a) Collaboration strategy from (22)

(b) Leader strategy from (24)

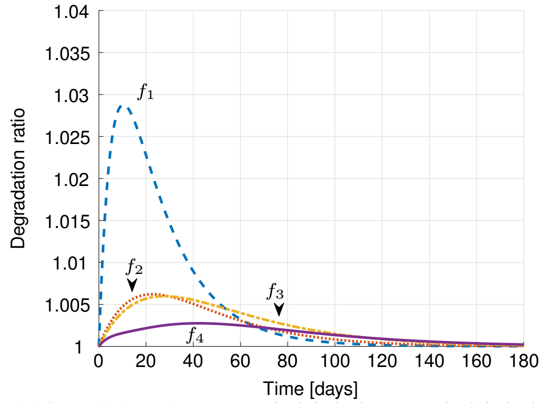
Fig. 7. Factor-based multiplier of compressor degradation for five compressors, obtained from the collaboration strategy (▬, left) and from the leader strategy (▬, right) compared with equal load approach (╍)



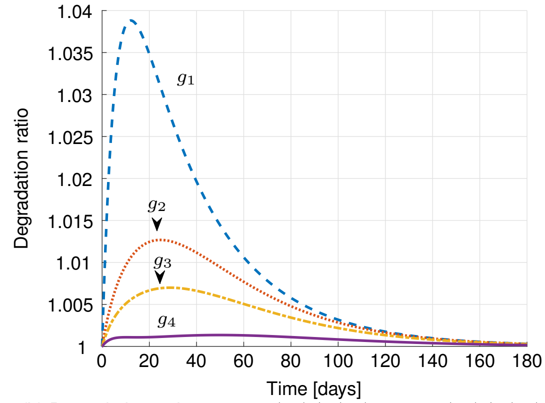
(a) Collaboration strategy from (22)

(b) Leader strategy from (24)

Fig. 8. Degradation functions for five compressors, obtained from the collaboration strategy CS (left) and from the leader strategy LS (right) compared with equal load approach EL



(a) Degradation ratios: $f_1 = h(d_2)/h(d_1)$, $f_2 = h(d_3)/h(d_2)$, $f_3 = h(d_4)/h(d_3)$, $f_4 = h(d_5)/h(d_4)$



(b) Degradation ratios: $g_1 = h(d_5)/h(d_1)$, $g_2 = h(d_5)/h(d_2)$, $g_3 = h(d_5)/h(d_3)$, $g_4 = h(d_5)/h(d_4)$

Fig. 9. Degradation ratios for five compressors, obtained using the collaboration strategy (left) and the leader strategy (right)

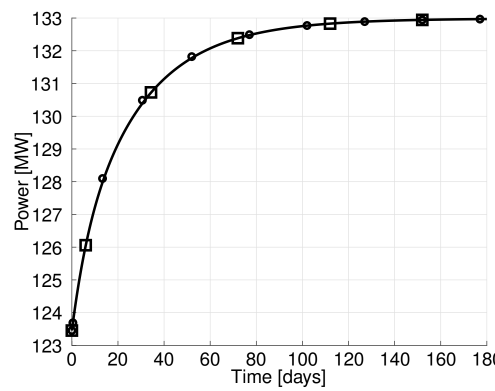
The decrease of load in Compressors 1 to 3 caused an increase of the load for Compressor 5. The leader strategy also resulted in an increase of the degradation function of Compressor 4 to make it closer to the degradation function of Compressor 5. This means that the ratio between the degradation functions of Compressor 5 and Compressor 4 decreased compared to the algorithm from the collaboration strategy (purple line in Fig. 9b and 9a). As a result, Compressor 4 improved only by 1.5 days instead of 2.

At the same time, Compressor 5 reached the threshold five days more quickly. It means that assigning the loads in order of increasing degradation allows a mitigation of quickly degrading compressors. However, the degradation of the healthiest compressor was intensified and the compressor reached its 93% degradation threshold ten days earlier than in the collaboration strategy.

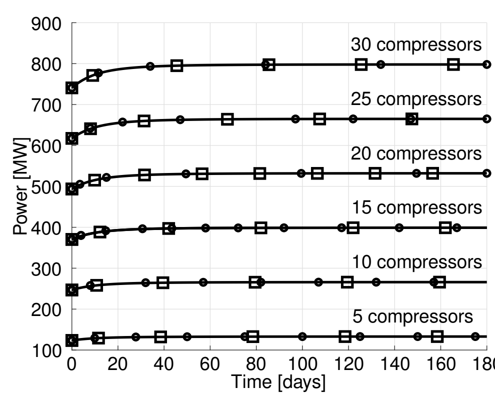
The results from the leader strategy confirm that the largest mitigation of degradation is achieved when the ratio of the degradation functions between two compressors is high. This is why using the leader strategy improved the most degraded compressor the most relative to the equal load strategy, and then the improvements were decreasing. Table IV also shows that the larger improvements were done at the expense of the healthiest unit, which degrades more quickly than in the equal load strategy if the leader strategy is used.

4) Power consumption: Figure 10a shows the power consumption of the whole station in the whole period. The power consumption of the leader (black) and collaboration (blue) strategies is similar to the power consumption obtained in the equal load approach (yellow). This means that by adjusting the loads, the new algorithms did not have a significant impact on the power consumption. At the same time, they mitigated the degradation of Compressor 1 to 4.

The collaboration and leader strategies preserve the power consumption also for compressor stations with more than five compressors, as shown in Fig. 10b. The parameters of both strategies were kept the same for the stations and the demand M was scaled so that $M/K = 170$ kg s^{-1}, where K denotes the number of compressors in the station. The parameters γ_i,



(a) Compressor station with five compressors

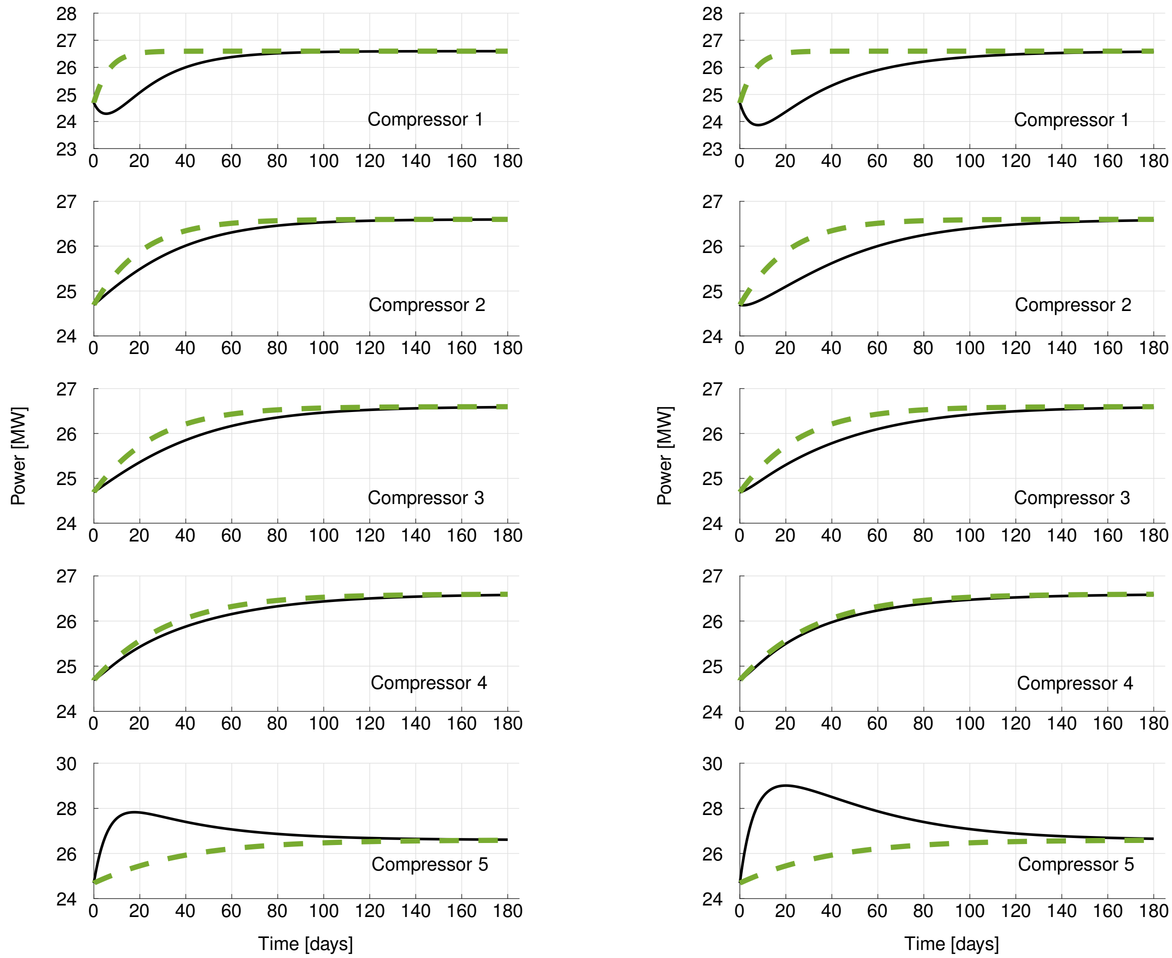


(b) Compressor stations with up to 30 compressors

Fig. 10. Power consumption of the whole station, obtained using the collaboration strategy (squares) and the leader strategy (black), compared with the equal load approach (circles). The differences are minor and cannot be seen within the resolution of the plot

$i = 1, \ldots, K$, were distributed evenly between $0.2{\cdot}10^{-5}$ and $0.033{\cdot}10^{-5}$ in a similar way as in the case in Table III.

The power consumption of five individual compressors is shown in Fig. 11. Both the collaboration and the leader strategy reduced the power consumption for Compressor 1 to 4 in relation to the equal load approach. The black lines



(a) Collaboration strategy from (22)

(b) Leader strategy from (24)

Fig. 11. Power consumption for five compressors, obtained from the collaboration strategy (—, left) and from the leader strategy (—, right) compared with equal load approach (- -)

corresponding to the two strategies are below the green lines corresponding to the equal load approach. In particular, the leader strategy reduced the power consumption of Compressor 1, but at the expense of increased power consumption by Compressor 5. A larger value of the ratio between the degradation functions of Compressor 1 and 5 was obtained in the leader strategy and resulted in a lower load assigned to the most degraded Compressor 1 according to (22). A lower load enabled mitigation of degradation, as shown in Fig. 8. In turn, a smaller value of degradation means that less power is needed to compensate for the degraded characteristics as indicated in (4). Conversely, the leader strategy from (24) assigned a larger load to Compressor 5, which intensified its degradation and increased the power consumption, compared to the collaboration strategy.

5) Influence of the number of compressors on degradation: Figure 10b showed that the power consumption in the proposed leader and collaboration strategies remains unaffected, compared to the equal load strategy, even if the number of compressors in a station increases up to 30. However, the number of compressors in a station will affect how much load is assigned to the healthiest compressor. The proposed algorithm prevents crossing the upper limit by adjusting the parameter q as described in Section VI-A4. In both collaboration and leader strategies, the degrading compressors will get a lower load than in the equal load strategy. Then both the leader and the collaboration strategies assume that the healthiest compressor can take the remaining load to satisfy the overall demand ((22b) and (24b)). In particular in the leader strategy the load assigned to the healthiest compressor could increase to the point of crossing the upper limit $m^{\max}$. This is is because the loads are assigned based on the ratio between degrading compressors and the healthiest compressor.

The performance of the proposed adjustment of the parameter q with $\rho = 0.99$ for the leader strategy for up to 30 compressors is illustrated in Fig. 12. The leader strategy was chosen because this strategy requires the healthiest compressor to take the most load. Therefore, the healthiest compressor is most likely to reach its upper limit in the leader strategy. Figure 12a shows the mass flows through the healthiest compressor (solid lines) and the most degraded compressor (dashed lines) for compressor stations with five (blue), 10 (yellow), 15 (green), 20 (dark red), 25 (orange), 30 (purple) compressors. Regardless of the number of compressors, the load assigned to the most degraded compressors initially decreases, whereas the load assigned to the healthiest compressor increases. Similarly to the compressor station with only five compressors shown in Fig. 5, the mass flows eventually reach the equal load assignment 170 kg s^{-1}.

However, the number of compressors influences when the healthiest compressor reaches its upper limit, $m^{\max} = 228$ kg s^{-1}. The more compressors in the station, the more quickly the healthiest compressor reaches the upper limit (Fig. 12b). For 30 compressors the upper limit was reached after 1.5 days, whereas for 10 compressors the upper limit was reached after 8.5 days. This result is expected because the healthiest compressor must compensate for a larger number of degrading compressors in stations with more compressors. For illustration, two compressor stations are considered, one with five compressors and one with 30 compressors. It is assumed that the demand is such that the equal load approach gives 170 kg s^{-1} for each compressor, which corresponds to the demand 850 kg s $^{-1}$ and 5100 kg s $^{-1}$, respectively. If in the compressor station with five compressors, Compressor 1, 2, 3, and 4 get assigned 165 kg s^{-1}, then Compressor 5 must take $850 - 4 \cdot 165 = 190$ kg s^{-1}. The mass flow assigned to Compressor 5 stays below the upper limit, 190 kg s^{-1} is below 228 kg s^{-1}. Conversely, if in the compressor station with 30 compressors all the compressors except Compressor 30 get assigned 165 kg s^{-1}, then Compressor 30 would have to take $5100 - 29 \cdot 165 = 315$ kg s^{-1}, which exceeds the upper limit. The strategies proposed in this paper prevent exceeding the upper limit of the healthiest compressor. In all the cases, the demand was satisfied throughout the whole simulation time.

Reaching the upper limit of the healthiest compressor triggers the adjustment of parameter q (lines 19-20 in Algorithm 1). The adjustments are shown in Fig. 12c across 180 days and in Fig. 12d over 17 days to show the day when the adjustments began. The larger the number of compressors in a station, the lower the value of q necessary to ensure that the healthiest compressor stays below its upper limit.

6) Days gained: The influence of the number of compressors on their degradation is shown in the form of days gained (+) or lost (-) in the collaboration (circles) and the leader strategy (triangles), compared to the equal load strategy in Fig. 13. The number of days corresponding to the most degraded compressor is marked in green and to the healthiest compressor in orange. Regardless of the number of compressors or the strategy, the healthiest compressor always reached the threshold of 93% more quickly than in the equal load strategy (orange circles and triangles correspond to negative values). The remaining compressors gained up to 17 days, obtained for the most degraded compressor in a compressor station with 10 compressors if the leader strategy was used. In all the cases, the leader strategy enabled extending the days before reaching the threshold in a similar way as in the compressor station with five compressors from Section VI.

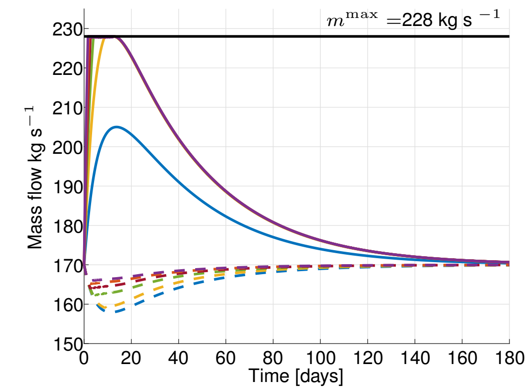
The collaboration strategy also extended the days before degrading compressors reached the threshold, but only up to one day. This is due to the ratios $h(d_{i+1})/h(d_i)$. As the parameters of the models of degradation were chosen from the same interval for all the compressors, a larger number of compressors K resulted in parameters γ_i similar to γ_{i+1}, $i = 1, 2, \dots, K-1$. In consequence, the degradation ratios for collaboration strategy will be smaller than the degradation ratios in the leader strategy, regardless of the number of compressors in the station. As described in Section VI-B3, smaller degradation ratios have less effect on the mitigation of degradation, and as a result the number of days gained decreases. Figure 13 also confirms that the leader strategy enables extending the days before reaching the threshold for degrading compressors at the expense of the healthiest compressor. The healthiest compressor reaches the threshold around 23 days earlier in the leader strategy, compared to four days in the collaboration strategy.

VII. Discussion, Limitations, and Conclusions

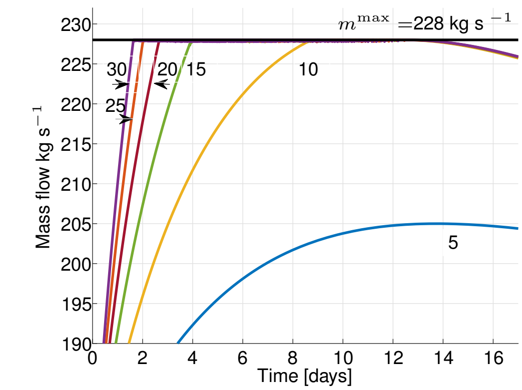
A. Industrial uses

Management of degradation is focused on either mitigation or intensification of degradation. The mitigation of degradation aims to preserve the degrading compressor and extend its lifetime. The counter-intuitive idea of intensifying degradation also has industrial applications. For instance, it might be useful to intensify degradation if a pre-scheduled maintenance event is coming up for the degraded unit. As the timing of the maintenance event has already been fixed, increasing the value of degradation helps to make the most of the maintenance. For instance, if a piece of machinery is to be replaced, it is better to remove it when it is no longer able to perform the required function, instead of replacing a still functioning machine. The results confirm a potential usefulness of the new framework both as an operating strategy and as a decision support tool.

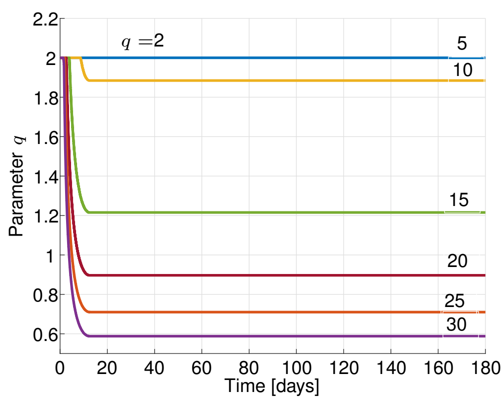
Both leader and collaboration approaches can be seen as mitigation strategies for all the compressors except the healthiest one. At the same time, both strategies correspond to intensification of degradation of the healthiest compressor. The uses of the two strategies depend on parameters of degradation functions for the healthiest compressor. The collaboration strategy compares the degradation functions between two compressors with most similar degradation functions. Due to the similarity of degradation functions, the collaboration strategy can be seen as spreading the load over all the compressors. Conversely, in the leader strategy, the degradation function of degraded compressors is compared with the degradation function of the leader, which is the healthiest. In both strategies, the healthiest compressor will then get assigned the remaining load. As the loads assigned to degraded compressors in the collaboration strategy are lower than the loads assigned in the leader strategy, the remaining load assigned to the healthiest compressor will be smaller. This indicates that the collaboration strategy should be used if the objective is to



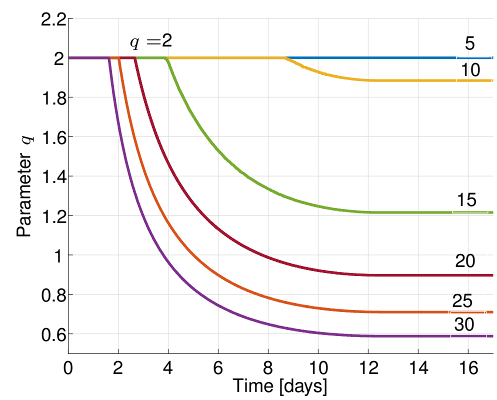
(a) The flows assigned to the healthiest compressor (solid lines) and to the most degraded compressor (dashed lines) for up to 30 compressors



(b) The flows assigned to the healthiest compressor (solid lines) zoomed in to show when the healthiest compressor reaches the upper limit



(c) The adjustment of parameter q



(d) The adjustment of parameter q - zoomed in

Fig. 12. The flows for the healthiest and the most degraded compressor in the leader strategy with adjusting the parameter q depending on the number of compressors: 5 (blue), 10 (yellow), 15 (green), 20 (dark red), 25 (orange), and 30 (purple)

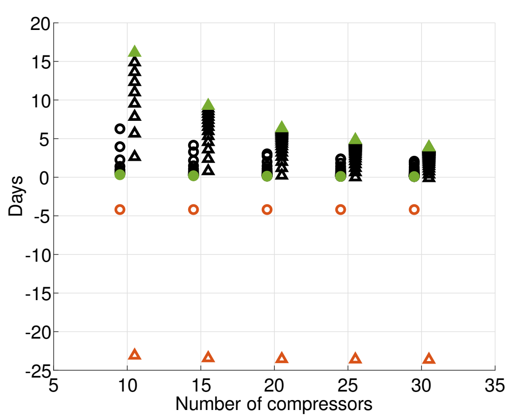


Fig. 13. Number of days gained (+) or lost (-) for compressor stations with 10, 15, 20, 25, and 30 compressors in the collaboration (circles) and the leader strategies (triangles), with the numbers corresponding to the most degraded compressor marked in green and to the healthiest compressor in orange

mitigate degradation of more degraded compressors, without significantly intensifying the degradation of the healthiest compressor. However, the collaboration strategy will lead to less significant mitigation of degradation than the leader strategy. The leader strategy should be used if significant mitigation of degradation of the degraded compressors is sought, as the expense of intensification of degradation of the healthiest compressor.

Degradation management in the form of controlling the effects of the operating conditions on degradation remains a new concept, in particular in the area of the intensification of degradation. The results presented in this work would enable an extension of common load-sharing strategies to include degradation and improve maintenance planning. Thus, this work presents a step forward in integration of planning and operation of industrial systems as called for by [27].

B. Limitations

The presented study is a simulation based on data from the public domain. It has the limitation that it does not describe any specific compressor station. In particular, the simulations presented in the paper are mainly focused on compressor stations with either two or five compressors. The results show the algorithm can extend to larger numbers of compressors and hence is generalizable. However, it needs to be tested in a realistic setting.

A practical implementation would require a real industrial compressor station. Existing experimental rigs, such as described in [28] are unsuitable because their operating conditions differ significantly from industrial compressor stations working with natural gas, or because they are designed for other types of investigations [29]. Moreover, Syverud in [29] explained that simulation of degradation in laboratory settings requires specialized approaches because one does not want to deliberately degrade laboratory equipment. The authors suggest that the next step towards industrial implementation in a specific process would be a demonstration in a proprietary industrial simulator.

A historical limitation has been the lack of availability of degradation indicators. However, degradation indicators are starting to become available from industrial vendors such as ABB [30] and TGM [31], and hence the requirement for a degradation indicator should not pose significant limitation.

C. Conclusions

A new framework has been presented for load-sharing in a compressor station subject to degradation. The performance of the load-sharing strategy has been evaluated for the two objectives of degradation management in a compressor station with two compressors. The main objective of the framework was to use the ratios between the degradation indicators of the compressors to manage the degradation of the compressors.

The performance of the new framework has also been analysed in a larger compressor station with five compressors. It was shown that the new framework allows the more degraded compressor to follow the less degraded compressor in terms of the degradation functions. At the same time, the new strategies do not significantly increase the power necessary to run the compressors, compared to the equal load approach.

The results from the case studies indicate that managing degradation by adjusting the loads leads to improved operation of compressors in a compressor station. Taking degradation into account in load-sharing strategies enables explicit management of degradation of compressors and will lead to improved efficacy of maintenance in a compressor station.

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