# An On-Line Energy Management Strategy for Plug-in Hybrid Electric Vehicles Using an Estimation Distribution Algorithm

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***Abstract*--Plug-in hybrid vehicles (PHEVs) have great potential in reducing energy consumption and pollutant emissions, due to the use of electric batteries as another energy source. One of the critical considerations in PHEV development is the design of its energy management strategy, which determines how energy flows in a hybrid powertrain should be managed in response to a variety of system parameters. In this paper, we propose a generic framework of real-time energy management for PHEVs, where an Estimation Distribution Algorithm (EDA) is used for on-line (i.e., real-time) optimization of the power-split strategy. Different methods for controlling the battery pack's State of Charge (SOC) are proposed and sensitivity analyses are conducted to evaluate their performance. Study results validate the effectiveness of the proposed methods and show promise for further field implementation.**

## I. Introduction

In recent years, transportation-related energy consumption and air quality degradation problems have gained an increasingly amount of public concern. According to [1], the total energy consumption by the transportation sector in the United States was estimated to be as high as 26.63 Quadrillion BTU in 2012, and the U.S. Environmental Protection Agency (EPA) reported that nearly 28% greenhouse gas (GHG) emissions resulted from fossil fuel combustion for transportation activities in 2012 [2]. Numerous technologies have been developed to address these issues. Among those, innovative powertrain technologies, such as hybrid electric vehicles (HEVs), are very promising in improving fossil fuel efficiency and reducing exhaust emissions. As one type of HEVs, plug-in hybrid electric vehicles (PHEVs) can be plugged into the electrical grid to charge their batteries, thus achieving an even higher overall energy efficiency [3].

It is important to note that the fuel economy of a PHEV significantly depends on its energy management strategy, characterized by the SOC profile of the battery pack during the entire trip. Thus far, the charge depleting/charge sustaining (CD/CS) binary mode operation strategy is still the most widely implemented [4], where the PHEV consumes the electricity (i.e., charge depletion) as soon as possible before it switches to a charge sustaining (CS) mode. However, many studies have shown that a blended mode strategy, where the internal combustion engine (ICE) operates in conjunction with the electric motor(s) in response to the power demands, may result in better fuel economy for PHEVs. Typical blended mode strategies include Equivalent Consumption Minimization Strategy or ECMS [4], Dynamic Programming or DP [5], and Pontryagin's Minimum Principle or PMP [6]-[7]. For PHEVs, however, the literature is limited. Cao et al. validated a control strategy in terms of engine on/off frequency and engine operating condition [8]. Karbowski et al. applied a Bellman principle based strategy to the energy flow optimization in PHEVs [9]. Sharer et al. compared different charge depleting strategy options [10]. Banvait et al. studied energy management for PHEVs and developed a rule-based controller [11]. Wu et al. proposed an efficient off-line optimization strategy based on Mixed Integer Linear Programming (MILP) [12].

Despite these efforts, few studies have focused on the implementation of real-time energy management for PHEVs. The difficulties of real-time implementation lie in obtaining a priori knowledge of the system states (e.g., speed profile) as well as the time delay of consecutively intensive computation tasks. A rule-based real time controller was developed by extracting the patterns from the power train operation under optimal control solutions [13]. Another real-time suboptimal controller for PHEVs was proposed and compared to an off-line global optimization using Particle Swarm Optimization (PSO) [14].

In this paper, a framework of on-line energy management for PHEVs is proposed. An estimation distribution algorithm (EDA) based strategy is developed to optimize the ICE energy use. To fit in the proposed framework, optimization is conducted on a segment basis rather than on an entire trip basis, which may lead to a sub-optimal solution. To evaluate the performance of the proposed framework, synthesized trip information was used in the numerical studies. Moreover, the impacts of prediction window and

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update interval were investigated via sensitivity analyses.

## II. PHEV MODELING

In this study, we use the PHEV model from our previous work [12]. The dynamic equations that govern the mechanical path of a power-split PHEV (e.g., Toyota Prius) are summarized as follows [12]:

$$\omega_{MG1}S + \omega_{MG2}R = \omega_{ICE}(S + R) \quad (1)$$

$$\dot{\omega}_{MG1}(I_S + I_{MG1}) = F_{int}S + T_{MG1} \quad (2)$$

$$\dot{\omega}_{ICE}(I_C + I_{ICE}) = T_{ICE} - (F_{int}S + F_{int}R) \quad (3)$$

$$\dot{\omega}_{MG2}\left(R_t^2/N_f\, M_r + I_{MG2}N_f + I_R N_f\right) = (T_{MG2} + F_{int}R)N_f - T_{br} - \left[M_{veh}g\cos(\alpha)C_r - M_{veh}g\sin(\alpha) - 0.5\rho A C_d\left(\omega_{MG2}R_t^2/N_f\right)^2\right]R_t \quad (4)$$

where, $S$ and $R$ are the radii of the sun gear and ring gear; $\omega_{MG1}$, $\omega_{MG2}$ and $\omega_{ICE}$ denote the angular velocities of one motor/generator (MG1), the other motor/generator (MG2), and internal combustion engine (ICE); $I_S$, $I_C$, $I_R$, $I_{MG1}$, $I_{MG2}$, and $I_{ICE}$ are the inertias of the sun gear, carrier gear, ring gear, MG1, MG2 and ICE, respectively; $F_{int}$ is the internal force on the pinion gears; $T_{MG1}$, $T_{MG2}$, $T_{ICE}$ and $T_{br}$ are the torques applied to MG1, MG2, ICE and the brake; $R_t$ is tire radius; $N_f$ represents final transmission ratio; $M_r = M_{veh} + J_r/R_t^2$ is the effective mass of the vehicle, given that $M_{veh}$ is vehicle mass and $J_r$ is the equivalent moment of inertia of the rotating components in the vehicle; $g$ is gravitational acceleration (9.81 m/s$^2$); $\alpha$ is road grade; $C_r$ is rolling resistance coefficient; $\rho$ is the density of air; $A$ is vehicle frontal area; and $C_d$ is aerodynamic drag coefficient.For more details about model derivation including the electrical path and parameter selection, please refer to [15].

## III. ON-LINE OPTIMIZATION FRAMEWORK

As aforementioned, most of the existing optimal strategies for PHEV energy management are off-line. In this work, a framework for *on-line* energy management strategy based on an evolutionary algorithm is proposed. As shown in Figure 1, the framework is a closed loop system which comprises information acquisition (from external sources), prediction, optimization, and power split control. Regarding the optimization, evolutionary algorithm (EA) based strategies may be applied. However, one of the critics of real-time implementation of EAs is the high computational overhead [16]. The time complexity of EAs is normally worse than $\Theta(m^2 * \log(m))$ where $m$ is the size of the problem [17]. Therefore, we divide the full trip into small segments or time windows, and employ the EA-based optimization over each short time window to reduce computational complexity. For on-line implementation, the sliding time window technique is used, where the optimal solution for the next time window is calculated within the current time window (see Figure 2). It is noted that the length of prediction window, $L_w$(e.g., 150 seconds in Figure 2), should not be shorter than the length of the update interval, $L_m$ (e.g., 50 seconds in Figure 2), to guarantee real-time performance. In other words, the optimal power split for the time window between 50 seconds and 200 seconds can be obtained within the first 50 seconds. A sensitivity analysis on these two parameters is presented in Section V.

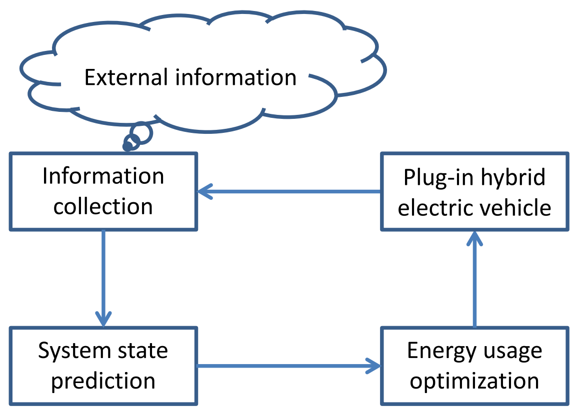


Fig. 1. Framework of on-line PHEV energy management

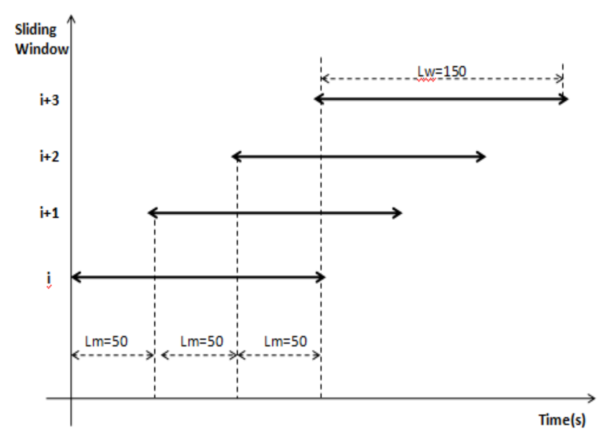


Fig. 2. Time flows of prediction, optimization and power control

## IV. EDA-BASED OPTIMAL CHARGE-DEPLETING STRATEGY

### A. Problem Formulation

As described in [12], the optimal charge-depleting control problem for a power-split PHEV can be formulated as a 0-1 Binary Mixed Integer Nonlinear Programming (MINP) as follows:

$$\min \sum_{k=1}^{T}\sum_{i=1}^{N} x(k,i)\, P_i^{eng}/\eta_i^{eng} \quad (5)$$

subject to:

$$\sum_{k=1}^{j} f\left(P_k - \sum_{i=1}^{N} x(k,i) P_i^{eng}\right) \leq C \quad \forall j = 1, \dots, T \quad (6)$$

$$\sum_{i=1}^{N} x(k,i) = 1 \qquad \forall k \qquad (7)$$

$$x(k,i) = \{0,1\} \qquad \forall k,i \qquad (8)$$

where $T$ is the time span of the entire trip; $N$ is the number of discretized power level for the engine; $k$ is the time step index; $i$ is the engine power level index $C$ is the gap of the battery pack's state of charge (SOC) between the initial and the minimum; $P_i^{eng}$ is the $i$-th discretized level for the engine power and $\eta_i^{eng}$ is the associated engine efficiency; and $P_k$ is the driving demand power at time step $k$.

If the change in SOC ( $\Delta SOC$ ) for each possible engine power level at each time step is pre-calculated, (this task is done by prediction part shown in Fig 2) then constraint (6) can be replaced by

$$SOC^{ini} - SOC^{max} \leq \sum_{k=1}^{j} x(k,i)\Delta SOC(k,i) \leq SOC^{ini} - SOC^{min} \qquad \forall i, j = 1, \dots, T \qquad (9)$$

where $SOC^{ini}$ is the initial SOC; and $SOC^{min}$ and $SOC^{max}$ are the minimum and maximum of SOC, respectively. Therefore, the problem is turned into a Mixed Integer Linear Programming (MILP) whose objective is to select the optimal power level for each time step given the predicted information to achieve the best energy efficiency along the entire trip.

*B. EDA-based Optimization strategy*

In this paper, we propose a real-time optimal charge-depleting (ROCD) strategy based on the estimation distribution algorithm (EDA). EDA is a population-based and iterative evolutionary algorithm which has been successfully applied to many different engineering domains [18]. The algorithm starts searching with a randomly generated population as initial candidate solutions. The population is then updated at each generation according to the strategy presented in Figure 4. More specifically, each individual (encoded as a row vector) in the algorithm is a candidate solution. The length of the vector is the number of time steps within the trip segment. The value of the $i$-th element is the ICE power level chosen for that step. In the following example (see Figure 3), the ICE power level is 3 (or 3 kW) for the 1st time step and 0 kW (i.e., the ICE is off and only the battery pack supplies power) for the 2nd time step.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **3** | **0** | **1** | **4** | **…………** | **1** | **2** | **0** | **5** |

Fig. 3. Example of an individual

In this study, we assume the value of each element in a good individual follows a univariate Gaussian distribution. This assumption has been proven to be effective in many engineering applications [19], although there could be many other options [20]. As another critical issue in EDA implementation, the fitness function herein is defined as the summation of total ICE energy consumption for the trip segment (given in objective function (5)) and a penalty term (the largest possible amount of energy that can be consumed in this trip segment). The penalty is added to guarantee the feasibility of solution, satisfying Constraint (6) which means the SOC always falls within the required range at each time step. The proposed ROCD algorithm is given below.

**Algorithm** ROCD algorithm

1: Initialize parameters: M (number of time steps); N (population size); α(top α% of the current population)
2: $P_{current}$ <= Generating initial population randomly

3: Evaluate each individual in the population as following
4: For 1 to M
5: Calculate SOC at each time step using based on (9).
6: If Constraint (6) is violated at any time step
7: Fitness=total ICE consumed energy + penalty
8: Else
9: Fitness=total ICE consumed energy
10: End If
11:End For
12: Rank $P_{current}$ in ascending order based on fitness

13: While iteration_number ≤ Max_iterations, do
14: $P_{top}$ <= Select top α% individuals from $P_{current}$
15: E <= Estimate a new distribution from $P_{top}$
16: $P_{new}$ <= Sample N individuals from built model E
17: Evaluate each individual in $P_{new}$ using line 3 to 11
18: Mix $P_{current}$ and $P_{new}$ to form 2N individuals
19: Rank the 2N individuals in ascending order
20: $P_{current}$<= Select top N individuals
21: Iteration_number ++
22: End While

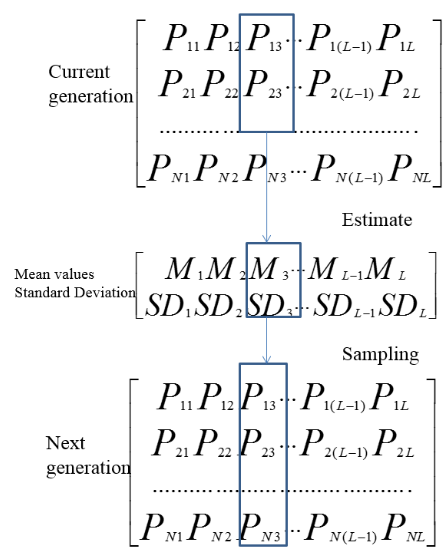


Fig. 4. Estimation and Sampling Process

## V. NUMERICAL STUDIES

### A. Synthesized predicted trajectory

The proposed strategy is validated using the same dataset as in [12]. Figure 5 presents the synthesized velocity trajectory of a trip. It should be pointed out that in practice, the synthesized velocity trajectory cannot be known a priori. But in this study, we treat it as a predicted trajectory and use it to illustrate the proposed on-line energy management strategy for PHEVs.

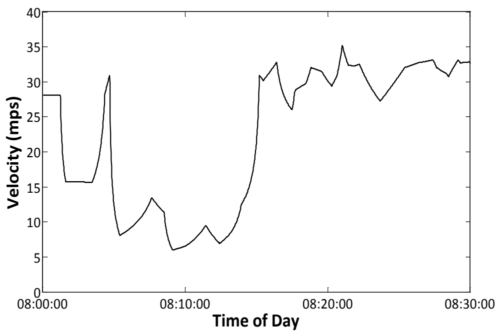


Fig.5. Synthesized velocity trajectory

### B. Validation with off-line optimization

To validate the selection of EDA as the kernel of the proposed algorithm, we first test the proposed algorithm along the entire trip. The results are compared with those obtained from two popular evolutionary algorithms: Genetic Algorithm (GA) and Particle Swarm Optimization (PSO). As shown in Figure 6, the fitness (i.e., total ICE energy consumption) of EDA-based algorithm converges more slowly but obtains better result (4.213x $10^7$ Joule) than the other two (4.416x$10^7$ Joule for GA and 4.583x$10^7$Joule for PSO, respectively). This comparison is under the same computational expense (i.e. same population size and same number of iterations).In addition, the result is quite close to the global optimum (4.198x $10^7$Joule presented in [12]), with the difference being less than 1%. Figure 7 presents the SOC profiles obtained from the different algorithms.

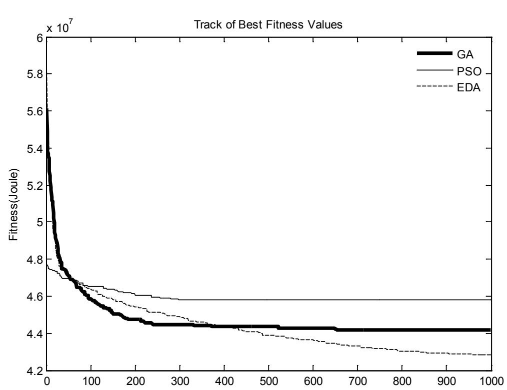


Fig. 6. Fitness track (vs. No. of generations) of different EA-based algorithms (results reflect the best out of 30 runs)

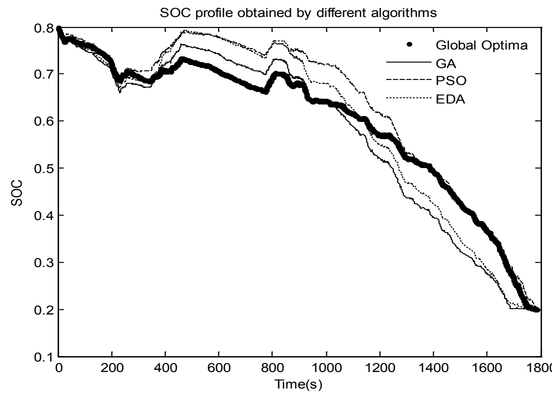


Fig. 7. SOC profile of the best solution obtained by different algorithms

### C. On-line optimization without SOC control

As mentioned previously, the entire trip should be segmented by a prediction window for on-line implementation. To test the proposed algorithm, we divide the entire trip into multiple segments, which can be realized simply by forcing $L_m \leq L_w$ (note that the value of $L_w$ determines the number of segments). Here, we set $L_m = L_w = 200$ seconds. Figure 8 shows the fitness of the optimization during each time window. It is clear that for all the time windows, the optimization process converges to its achievable best solution within 50 generations. Figure 9 presents the SOC profile of the optimization results from the proposed algorithm, The total energy consumption (fitness) is more than 5.5x$10^7$ Joule, which is much larger than the actual global optima (4.198x $10^7$ Joule). Also, the obtained SOC profile is quite different from the ones obtained by off-line full trip optimization (see Figure 7), but looks similar to one typically generated by a binary mode strategy where the SOC drops as fast as it can. A hypothesis is that the ICE energy use is locally optimized within each segment without considering the impacts on the subsequent segment. Therefore, the battery pack is depleted as soon as possible. It can be expected that the smaller the prediction window length (i.e., $L_w$) is, the worse the results would be.

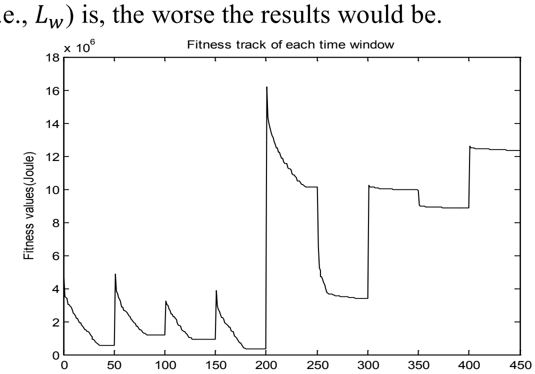


Fig. 8. Fitness track of each time interval ($\mathrm{L_m} = \mathrm{L_w} = 200$)

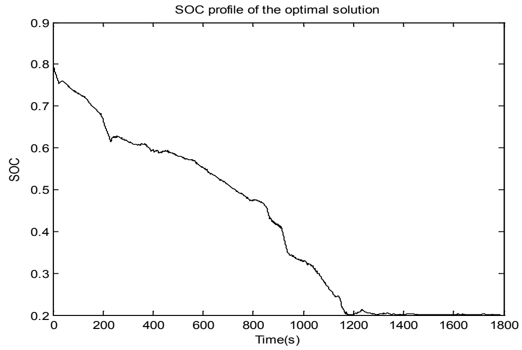


Fig. 9. SOC profile of the obtained best solution ($L_m = L_w = 200$)

*D. On-line optimization with SOC control*

The above results imply that the performance of the proposed algorithm can be improved by controlling the decreasing rate of SOC. One heuristic way is to set a reference SOC profile to prevent the actual SOC from dropping too fast within each trip segment. Such SOC control strategies can be either predetermined or self-adaptive. For the predetermined SOC control strategies, the reference SOC profile may be synthesized by using the pre-trip information, such as predicted arrival time. For example, Eq. 10 depicts a control strategy using a linear reference SOC profile where the minimal allowable SOC level at each trip segment follows a predefined linear function (trend line). If more information (e.g., elevation profile) is taken into account, a more complex reference SOC profile, such as quadratic function (as defined in Eq. 11) can be used.

Linear reference SOC profile:

$$SOC_i^{min} = \frac{-(SOC_i^{min} - SOC^{min})}{L_f} \cdot ((i-1) \cdot L_m + L_w) + SOC_{init} \quad (10)$$

Quadratic reference SOC profile:

$$SOC_i^{min} = a \cdot \left((i-1) \cdot L_m + L_w\right)^2 + SOC_{init} \quad (11)$$

Where $i$ is the index of the trip segment or update window; $L_f$ is the total time span (in seconds) of the entire trip (1786 sec in this study); $SOC_{init}$ is the initial SOC at the starting point (0.8 in this study); $SOC_i^{min}$ is the minimum SOC at the end of $i$-th update window; $SOC^{min}$ is the minimum SOC at the end of entire trip (0.2 in this study); $L_w$ is the length of the prediction window; $L_m$ is the length of update step; $a(< 0)$ is a parameter to govern the decreasing rate of reference SOC profile. In this study, we choose $a = -1.9 \times 10^{-7}$.

For the self-adaptive control strategies, the reference SOC profile within the $i$-th update window can be adjusted by using the information up to the ($i$-1)-th window. Eq. 12 provides an example formulation, where $SOC_{i-1}^{end}$ is the SOC at the end of the ($i$-1)-th update window. Figure 10 provides a depiction of these control trajectories.

Self-adaptive reference SOC profile:

$$SOC_i^{min} = \frac{(SOC_{i-1}^{end} - SOC^{min})}{L_f - (i * L_m)} * L_w + SOC_{init} \quad (12)$$

*E. Sensitivity Analysis*

As stated above, the parameters $L_w$ and $L_m$ may significantly affect the algorithm performance. Figure 11 and figure 12 presents the results of the sensitivity analysis of these two parameters (using Self-adaptive reference SOC control profile). It is noteworthy that in this study when the prediction window is 550 seconds (i.e., $L_w = 550$), the optimization can be completed within 10 seconds (with Intel Core i5 2.7GHz, RAM 4G, and 64bit-Matlab 2012).

As shown in Figure 11, the best parameter combination is $L_w$ =250 and $L_m$ =10. There is also a clear trend in Figure 12 (cross-section of Figure 11 when $L_m$ =10) that if the prediction window is too short, the system performance degrades significantly. The potential reason is that the battery pack is used aggressively within a short window, thus resulting in optimality loss.

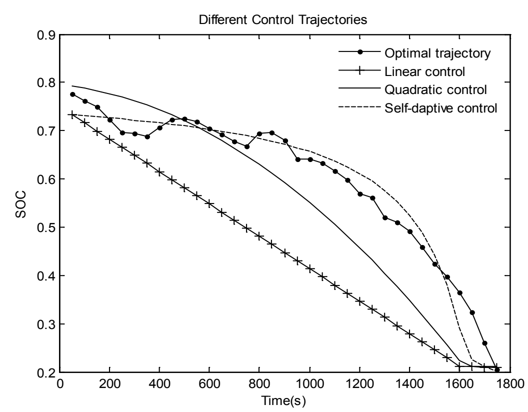


Fig. 10. Different SOC control trajectories ($L_w = 200,\ L_m = 50$)

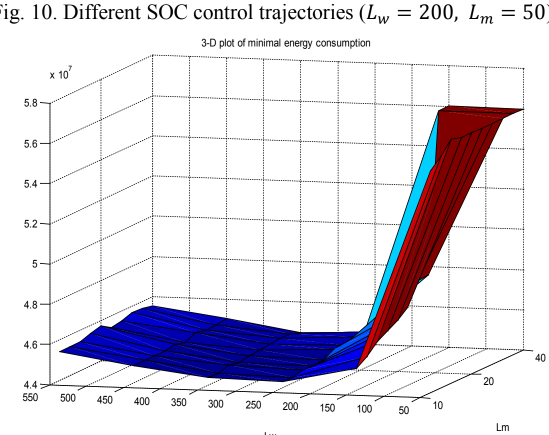


Fig. 11. Obtained optimal results on different window length and step length

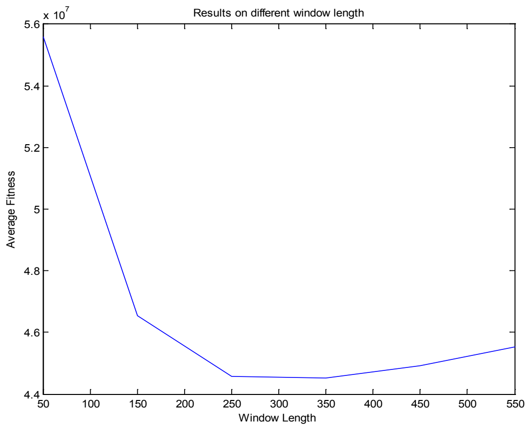


Fig. 12. Obtained optimal results on different window length

When the time window length is too long, the computation time needed for obtaining the optima or near-optima for that time window increases largely. So the quality of the best results for each window decreases noticeably, which also results in a larger loss of optimality.

## VI. CONCLUSIONS & FUTURE WORK

In this paper, we propose a generic framework of on-line energy management strategy for PHEVs. Under this framework, an EDA-based power-split optimization algorithm is developed in order to enable real-time implementation. The results from the numerical studies validate the effectiveness of the proposed algorithm and also show that the EDA-based algorithm outperforms the GA- and PSO-based methods.

One of the future goals is to conduct field operational testing (FOT) of the proposed strategy. But before that, some prerequisite work needs to be done, such as analyzing the impacts of prediction errors on the system performance and investigating an effective prediction model for ICE energy consumption.

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