PROCEEDING

THE 2nd INTERNATIONAL CONFERENCE ON SCIENCE AND TECHNOLOGY

Science and Technology for Nation Prosperity

Bengkulu, 6th-7th July 2019

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"SCIENCE AND TECHNOLOGY FOR NATION PROSPERITY"

Bengkulu, Indonesia
6th-7th July 2019

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FOREWORD

All praises be to the Almighty God, for all His grace and guidance, proceeding of the 2nd International Conference on Science and Technology with the theme "Science and Technology for Nation Prosperity" can be completed. This proceeding is a collection of papers held by the Mathematics and Natural Science, University of Bengkulu on 6th – 7th July 2019 at GRAGE Hotel Bengkulu.

Our highest gratitude and appreciation goes to the presenters and authors of the papers, as well as the executive committee who have worked hard so that this proceeding can be published. We also thank the Reviewer Board for reviewing all papers so that the quality of the contents of the paper can be maintained and accounted for. Do not forget to all parties who have provided support for the holding of the international conference and the preparation of this proceeding, we thank you.

We do hope that this conference would bring a great opportunity for all of us to strengthen our contribution to the advancement of our nation.

Finally, I hope this proceeding can provide benefits for all.

Bengkulu, June 2020

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Dr. Nampiah Sukarno (Bogor Agricultural University, INDONESIA)

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Biology

The Properties of Matrix in Group from Kronecker Product on The Representation of Quaternion Group Using Partitioned Matrix

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The 2nd International Conference on Science and Technology (ICST)

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Abstract. This paper discusses about the properties of matrices which are element of a group derived from the application of Kronecker product to the representation of the quaternion group (this group is called by author with *Kronecker quaternion group*). The properties the new matrix that constructed by matrices from the Kronecker quaternion group as submatrix in partitioned matrix are discussed based on transpose and determinant matrix. It's known that theconstruction of the partitioned matrix implies that product of partitioned matrix and the transpose of the matraix is commute.

Keyword: *partitioned matrix, transpose matrix, determinant matrix*

1. Introduction

In this paper, G denotes a finite non-abelian group with 32 orders. Group G was obtained by applying Kronecker product on the representation quaternion group. Thus, the elements of G are 4×4 matrices [1]. There are some specific properties of these matrices, that is:

a. Symmetric matrix (20 symmetric matrices).
b. Non-symmetric matrix (12 non-symmetric matrices)
c. For every $A \in$ G, $A^T = A^{-1}$ (orthogonal matrix)
d. For every $A \in$ G, $|A| = 1$.
e. For every $A \in$ G, $AA^T = A^T A$.
f. For every $A, B \in$ G, $AB^T = A^T B$ A and B non-symmetric matrix.

Let A is an arbitrary $m \times n$ matrix. A matrix A can be divided or partitioned into submatrices by drawing horizontal or vertical lines between various of its rows or columns, in this case the matrix is called a partitioned matrix. Meanwhile, a submatrix of a matrix A is a matrix that can be obtained by striking out rows and/or columns of A [2].

In this paper, we construct a new matrix that entries are matrices in G. Thus, the new matrix can be seen as a partitioned matrix. New matrix properties are arranged in the form of matrix partitions, where the submatrices are matrices derived from [1], using some properties in the partitioned matrices related to transpose (in Part 2: Theorems 2.1, 2.2, 2.3, and 2.4) and the determinant matrix (in Section 3: Theorems 3.3, 3.4 and 3.5).

2. Properties Matrix From G related to Transpose Matrix

It's known that if A is symmetric matrix, then the product $A^T A = AA^T$. Thus the properties given below are in the following theorems for non-symmetric matrix. We refer [3] and [4], to show the following theorems:

Theorem 2.1

Let $\mathbf{A} = \begin{bmatrix} U & 0 \\ V & W \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} U & V \\ 0 & W \end{bmatrix}$ where U, V, W non-symmetric matrices in G and 0 is a 4×4 zero matrix. Then $\mathbf{A}^T\mathbf{A} = \mathbf{B}\mathbf{B}^T$.

Proof.

Noted that $\mathbf{A}^T = \begin{bmatrix} U^T & V^T \\ 0 & W^T \end{bmatrix}$ and $\mathbf{B}^T = \begin{bmatrix} U^T & 0 \\ V^T & W^T \end{bmatrix}$. Based on e. and f. in Section 1, we have

$$\mathbf{A}^T\mathbf{A} = \begin{bmatrix} U^TU + V^TV & V^TW \\ W^TV & W^TW \end{bmatrix}$$

$$= \begin{bmatrix} UU^T + VV^T & VW^T \\ WV^T & WW^T \end{bmatrix}$$

$$= \mathbf{B}\mathbf{B}^T$$ ■

Theorem 2.2

Let $\mathbf{A} = \begin{bmatrix} U & V \\ W & X \end{bmatrix}$ where U, V, W, X are non-symmetric matrices in G. Then $\mathbf{A}\mathbf{A}^T = \mathbf{A}^T\mathbf{A}$.

Proof.

Noted that $\mathbf{A}^T = \begin{bmatrix} U^T & V^T \\ V^T & X^T \end{bmatrix}$. Based on e. and f. in Section 1, we have

$$\mathbf{A}\mathbf{A}^T = \begin{bmatrix} UU^T + V^TW & UV^T + VX^T \\ WU^T + XW^T & WV^T + XX^T \end{bmatrix}$$

$$= \begin{bmatrix} U^TU + V^TW & U^TV + V^TX \\ W^TU + X^TW & W^TV + X^TX \end{bmatrix}$$

$$= \mathbf{A}^T\mathbf{A}$$

■

In general we have the following theorem:

Theorem 2.3

Let

$$\mathcal{A} = \begin{bmatrix} A_{11} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ A_{21} & A_{22} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ A_{31} & A_{32} & A_{33} & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \mathbf{0} & \mathbf{0} \\ A_{(n-1)1} & A_{(n-1)2} & A_{(n-1)3} & \cdots & A_{(n-1)(n-1)} & \mathbf{0} \\ A_{n1} & A_{n2} & A_{n3} & \cdots & A_{n(n-1)} & A_{nn} \end{bmatrix}$$

and

$$\mathcal{B} = \begin{bmatrix} A_{11} & A_{21} & A_{31} & \cdots & A_{(n-1)1} & A_{n1} \\ \mathbf{0} & A_{22} & A_{32} & \cdots & A_{(n-1)2} & A_{n2} \\ \mathbf{0} & \mathbf{0} & A_{33} & \cdots & A_{(n-1)3} & A_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & A_{(n-1)(n-1)} & A_{n(n-1)} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & A_{nn} \end{bmatrix}$$

where $A_{ij} \in G$ and A_{ij} non-symmetric matrix and 0 is an 4×4 zero matrix. Then $A^TA = BB^T$.

Theorem 2.4

Let $A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}$ where $A_{ij} \in G$ and A_{ij} non symmetric matrix. Then $AA^T = A^TA$.

Proof of Theorem 2.3 and 2.4 are analogues with proof of Theorem 2.1 and 2.2 respectively.

3. Properties Matrix from G Related to Determinant Matrix

Noted that, if an $n \times n$ matrix $A = [a_{ij}]$ is (upper or lower) triangular then the determinant of a triangular matrix equals the product of its diagonal elements. Furthermore, we have the following properties:

Theorem 3.1 [4]
Let P be an $m \times m$ matrix, Q be an $n \times m$ matrix and R an $n \times n$ matrix. Then,

$$\begin{vmatrix} P & 0 \\ Q & R \end{vmatrix} = \begin{vmatrix} R & Q \\ 0 & P \end{vmatrix} = |P||R|.$$

■

The repeated application of Theorem 3.1 leads to the following formulas for the determinant of an arbitrary (square) upper or lower block-triangular matrix (with square diagonal blocks):

$$\begin{vmatrix} A_{11} & 0 & 0 & \cdots & 0 & 0 \\ A_{21} & A_{22} & 0 & \cdots & 0 & 0 \\ A_{31} & A_{32} & A_{33} & & \vdots & \vdots \\ \vdots & \vdots & \vdots & & 0 & 0 \\ A_{(n-1)1} & A_{(n-1)2} & A_{(n-1)3} & \cdots & A_{(n-1)(n-1)} & 0 \\ A_{n1} & A_{n2} & A_{n3} & \cdots & A_{n(n-1)} & A_{nn} \end{vmatrix} = |A_{11}||A_{22}||A_{33}| \ldots |A_{(n-1)(n-1)}||A_{nn}|$$

and

$$\begin{vmatrix} B_{11} & B_{12} & B_{13} & \cdots & B_{1(n-1)} & B_{1n} \\ 0 & B_{22} & B_{23} & & B_{2(n-1)} & B_{2n} \\ & 0 & B_{33} & \cdots & & \\ & \vdots & \vdots & & & \\ 0 & 0 & 0 & & & \\ & 0 & 0 & & & \end{vmatrix}$$

$$0 \quad \cdots \quad B_{3(n-1)} \quad B_{3n} \quad = |B_{11}||B_{22}||B_{33}| \dots |B_{(n-1)(n-1)}||B_{nn}|$$

$$\vdots \qquad \vdots$$

$$B_{(n-1)(n-1)} \quad B_{(n-1)n}$$

$$\cdots \quad 0 \quad B_{nn}$$

The following theorems give formulas for the determinant of a partitioned matrix:

Theorem 3.2 [4]
Let P an $m \times m$ matrix, Q an $n \times m$ matrix, R an $n \times n$ matrix and S an $m \times n$. If P is nonsingular, then

$$\begin{vmatrix} P & S \\ Q & R \end{vmatrix} = \begin{vmatrix} R & Q \\ S & P \end{vmatrix} = |P||R - QP^{-1}S|.$$

■

We continue with the matrix from group G. Based on Theorem 3.1 and 3.2, we have the following theorems:

Theorem 3.3
Let $\mathbf{A} = \begin{bmatrix} U & 0 \\ V & W \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} U & V \\ 0 & W \end{bmatrix}$, where $U, V, W \in G$ and 0 is a 4×4 zero matrix. Then $|\mathbf{A}| = |\mathbf{B}| = 1$.
Proof.

$|\mathbf{A}| = |\mathbf{B}|$
$= \begin{vmatrix} U & 0 \\ V & W \end{vmatrix}$
$= \begin{vmatrix} U & V \\ 0 & W \end{vmatrix}$
$= |U||W|$
$= 1.$

■

In general, we have

Theorem 3.4
Let $\mathbf{A} = \begin{bmatrix} A_{11} & 0 & 0 & \cdots & 0 & 0 \\ A_{21} & A_{22} & 0 & \cdots & 0 & 0 \\ A_{31} & A_{32} & A_{33} & & \vdots & \vdots \\ \vdots & \vdots & \vdots & & 0 & 0 \\ A_{(n-1)1} & A_{(n-1)2} & A_{(n-1)3} & \cdots & A_{(n-1)(n-1)} & 0 \\ A_{n1} & A_{n2} & A_{n3} & \cdots & A_{n(n-1)} & A_{nn} \end{bmatrix}$ where $A_{ij} \in G$ and A_{ij} non-symmetric matrix and 0 is an 4×4 zero matrix and

$\mathbf{B} = \begin{bmatrix} B_{11} & B_{12} & B_{13} & \cdots & B_{1(n-1)} & B_{1n} \\ 0 & B_{22} & B_{23} & \cdots & B_{2(n-1)} & B_{2n} \\ 0 & 0 & B_{33} & \cdots & B_{3(n-1)} & B_{3n} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \end{bmatrix}$ where $B_{ij} \in G$ and B_{ij} non-symmetric

$$\begin{matrix} 0 & 0 & 0 & & B_{(n-1)(n-1)} & B_{(n-1)n} \\ 0 & 0 & 0 & \cdots & 0 & B_{nn} \end{matrix}$$

matrix and 0 is an 4×4 zero matrix. Then $|A| = 1$ and $|B| = 1$

Proof.

$$|A| = \begin{vmatrix} A_{11} & 0 & 0 & \cdots & 0 & 0 \\ A_{21} & A_{22} & 0 & \cdots & 0 & 0 \\ A_{31} & A_{32} & A_{33} & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & 0 & 0 \\ A_{(n-1)1} & A_{(n-1)2} & A_{(n-1)3} & \cdots & A_{(n-1)(n-1)} & 0 \\ A_{n1} & A_{n2} & A_{n3} & \cdots & A_{n(n-1)} & A_{nn} \end{vmatrix} = |A_{11}||A_{22}||A_{33}| \ldots |A_{(n-1)(n-1)}||A_{nn}|$$

$= 1$

and

$$|B| = \begin{vmatrix} B_{11} & B_{12} & B_{13} & \cdots & B_{1(n-1)} & B_{1n} \\ 0 & B_{22} & B_{23} & \cdots & B_{2(n-1)} & B_{2n} \\ 0 & 0 & B_{33} & \cdots & B_{3(n-1)} & B_{3n} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & & B_{(n-1)(n-1)} & B_{(n-1)n} \\ 0 & 0 & 0 & \cdots & 0 & B_{nn} \end{vmatrix} = |B_{11}||B_{22}||B_{33}| \ldots |B_{(n-1)(n-1)}||B_{nn}|$$

$= 1$

Theorem 3.5

Let $\mathbf{A} = \begin{bmatrix} U & V \\ W & X \end{bmatrix}$ where U, V, W, X are non-symmetric matrices in G. Then

$$|\mathbf{A}| = \begin{vmatrix} U & V \\ W & X \end{vmatrix} = |X - WU^{T}V|$$

Proof.

Since all of matrices in G are nonsingular, so we have U is nonsingular. We can apply Theorem 3.2 and 3.3 to prove this theorem.

Consider that

$\begin{bmatrix} U & V \\ W & X \end{bmatrix} = \begin{bmatrix} I & 0 \\ WU^{-1} & X - WU^{-1}V \end{bmatrix} \begin{bmatrix} U & V \\ 0 & I \end{bmatrix}$, thus

$$|\mathbf{A}| = \begin{vmatrix} U & V \\ W & X \end{vmatrix}$$

$$= \left| \begin{bmatrix} I & 0 \\ WU^{-1} & X - WU^{-1}V \end{bmatrix} \begin{bmatrix} U & V \\ 0 & I \end{bmatrix} \right|$$

$$= \begin{vmatrix} I & 0 \\ WU^{-1} & X - WU^{-1}V \end{vmatrix} \begin{vmatrix} U & V \\ 0 & I \end{vmatrix}$$

$= |I||X - WU^{-1}V||U||I|$

$= |X - WU^{-1}V||U|$

$= |U||X - WU^{-1}V|$

$= |X - WU^{-1}V|$

$= |X - WU^T V|$

■

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