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Topological design of cellular structures for maximum shear modulus using homogenization SEMDOT

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ABSTRACT

This paper incorporates the homogenization theory into non-penalized Smooth-Edged Material Distribution for Optimizing Topology (SEMDOT) algorithm to conduct the design of cellular structures with the maximum shear modulus. The parametric study and comparison with existing results obtained by BESO are carried out. The numerical examples in 2D and 3D demonstrate the effectiveness of non-penalized SEMDOT in generating smooth cellular structures with the maximum shear modulus. Compared to BESO, SEMDOT can achieve comparable results and smoother boundaries. Smooth boundaries obtained by SEMDOT can facilitate the manufacturing of obtained cellular structures in 3D or 4D printing.

1. Introduction

Engineering and industrial applications are becoming sustainable in the era of industry 4.0 as the design and manufacturing of products pursue less material usage, simpler manufacturing processes, and less energy consumption while satisfying the functional requirements of products [1]. The light-weight design plays a crucial role in the development of industry 4.0, and cellular structure design is an important branch of the light-weight design. Compared to solid structures, cellular structures have superior mechanical and thermal properties, such as low density and high energy absorption [2]. Some examples of cellular structures are honeycomb, foam, trabecular bone, and wood [1]. Cellular structures and materials have potential applications in the aerospace industry, civil engineering, biomedical sensors, optics, and semiconductors. In addition, 3D and 4D printed cellular structures are extensively used in the vibration isolation, buckling control and energy absorption [3–6]. Topology optimization (TO) is a powerful virtual tool that can find the optimal material distribution and geometry within a predefined design domain, and it is widely used to automatically obtain unit cell designs [7–9].

This work uses a typical elemental volume fraction-based algorithm named Smooth-Edged Material Distribution for Optimizing Topology (SEMDOT), which is proposed by Fu et al. [10,11], to conduct the topological design of cellular structures. Here, the non-penalization version of SEMDOT in [12] is adopted to yield more reasonable topological layouts and to reduce the total number of optimization iterations required for convergence. In addition, the advantages of the non-penalization material model in material design problems are comprehensively discussed by Li and Huang [7], which motivates the use of non-penalized SEMDOT for this research. Compared to traditional element-based algorithms such as Solid Isotropic Material with Penalization (SIMP) [13] and Bi-directional Evolutionary Structural Optimization (BESO) [14], the obvious advantage of using SEMDOT is that smooth topological boundaries can be directly formed, and hence the smoothing post-processing method is not needed before manufacturing.

Cellular structures and materials with high shear modulus are of particular interest since the shear sliding under pressure is one of their main failure modes [15]. Therefore, this work takes the shear modulus maximization problem as an example to show the capability of SEMDOT in generating cellular structures.

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2. Homogenization SEMDOT and optimization problem

The homogenization theory presented in [16,17] is incorporated into non-penalized SEMDOT to design topological configurations with periodic boundaries and repetition of representative unit cells. This work aims to maximize the shear moduli of cellular structures. It is noted that the design variable in SEMDOT is the element volume fraction, X_e, instead of the elemental density in SIMP and BESO. The homogenized elasticity tensor is formulated as [18]

$$E_{ijkl} = \frac{1}{|V|}\int_V E_{pqrs}(\varepsilon_{pq}^{0(ij)} - \varepsilon_{pq}^{(ij)})(\varepsilon_{rs}^{0(kl)} - \varepsilon_{rs}^{(kl)})dV \tag{1}$$

where $|V|$ is the unit cell volume fraction, E_{pqrs} is the locally varying stiffness tensor, $\varepsilon_{pq}^{0(ij)}$ is the predefined macroscopic strain field, and $\varepsilon_{pq}^{(ij)}$ is the locally varying strain field induced by imposing $\varepsilon_{pq}^{0(ij)}$ on the boundaries of the unit cell.

Because of the discretization for FEA in SEMDOT, the homogenized elasticity tensor can be rewritten as

$$E_{ijkl} = \frac{1}{|V|}\sum_{e=1}^{N}\left(\boldsymbol{u}_e^{(ij)}\right)^T \boldsymbol{k}_1 \boldsymbol{u}_e^{(kl)} \tag{2}$$

where e is the number of the element, N is the total number of elements in the unit cell, $\boldsymbol{u}_e^{(kl)}$ is the elemental displacement solution to the unit test strain field, and $\boldsymbol{k}_1$ is the elemental stiffness matrix.

In SEMDOT, elemental Young's moduli are estimated using a linear interpolation between the two phases of solid and void:

$$E(X_e) = (1 - X_e)E_{\min} + X_e E_1 \tag{3}$$

where E_1 is the elastic modulus of the solid material and $E_{\min}$ is the elastic modulus of the void material, which is a small value (for example, 0.001).

Based on Eqs. (2) and (3), the sensitivity of the homogenized elasticity tensor in non-penalized SEMDOT is calculated by

$$\frac{\partial E_{ijkl}}{\partial X_e} = \frac{1}{|V|}((1 - X_e)E_{\min} + X_e E_1)\left(\boldsymbol{u}_e^{(ij)}\right)^T \boldsymbol{k}_1 \boldsymbol{u}_e^{(kl)} \tag{4}$$

Here, periodic boundary conditions and square symmetry are taken into account for the design of cellular structures. Therefore, the following relationships exist: $E_{1111} = E_{2222}$ and $E_{1122} = E_{2211}$ for 2D cases and $E_{1111} = E_{2222} = E_{3333}$, $E_{1122} = E_{2211} = E_{2233} = E_{3322} = E_{3311} = E_{1133}$, and $E_{2323} = E_{3131} = E_{1212}$ for 3D cases.

In terms of 2D cases, the shear modulus related to the material response to the shear strain is

$$G = E_{1212} \tag{5}$$

The sensitivity of the shear modulus with respect to X_e for 2D cases can be easily calculated by

$$\frac{\partial G}{\partial X_e} = \frac{\partial E_{1212}}{\partial X_e} \tag{6}$$

For 3D cases, the shear modulus is

$$G = \frac{1}{3}(E_{2323} + E_{3131} + E_{1212}) \tag{7}$$

The sensitivity of the shear modulus with respect to X_e for 3D cases is

$$\frac{\partial G}{\partial X_e} = \frac{1}{3}\left(\frac{\partial E_{2323}}{\partial X_e} + \frac{\partial E_{3131}}{\partial X_e} + \frac{\partial E_{1212}}{\partial X_e}\right) \tag{8}$$

3. Numerical examples

The parameters of SEMDOT in [10–12] are used for all numerical cases. The Method of Moving Asymptote (MMA) proposed by Svanberg [19] is used as the optimizer, and the move limit in MMA is set to 0.5 for both 2D and 3D cases. In addition, a mesh of 100×100 and filter radius of $r_{\min} = 5$ are used to investigate the effects of different volume fractions (i.e., V=0.5, 0.45, 0.4, 0.35, 0.3, and 0.25) on optimized 2D cellular structures and the maximum shear moduli. In terms of the initial design for all 2D cases, the four elements in the middle of one cell are defined as voids, as schematically illustrated in Fig. 1. It should be noted that the initial design is not limited to the one presented in Fig. 1. The selection of the initial design should depend on the optimization problem.

The resulting 2D cellular structures and their maximum shear moduli are shown in Fig. 2 where different topological configurations are obtained, and reducing the volume fraction results in a lower value of the maximum shear modulus. The results shown in Fig. 3, which was obtained by Huang et al. [8] using BESO, are used for comparison. It is noted that the same parameter setting (i.e., a mesh of 100×100 and $r_{\min} = 5$) is used in [8], which can guarantee the fairness of comparison. Fig. 3 shows that the maximum shear moduli subjected to V = 0.45, 0.35, and 0.25 are 0.124, 0.093, and 0.065, respectively. Fig. 2 shows that the maximum shear moduli subjected to V = 0.45, 0.35, and 0.25 are 0.1231, 0.0935, and 0.0657, respectively. It can be concluded that SEMDOT and BESO can obtain close results in terms of the maximum shear modulus.

Another case subjected to a mesh of 120×120 and $V = 0.25$ is taken into account for the further comparison of the solutions obtained by SEMDOT and BESO presented in [20]. As shown in Fig. 4, a higher value of the shear modulus (0.0659) is obtained by SEMDOT compared to the one (0.0651) obtained by BESO in [20], and similar cellular structures are obtained. Zigzag boundaries generated by BESO are observed in Figs. 3 and 4b, which requires extra efforts to smooth the boundary before fabrication. By contrast, cellular structures obtained by SEMDOT have smooth boundaries, which makes the obtained cellular structures more manufacturable.

From the discussion above, it can be seen that the value of the maximum shear modulus obtained by SEMDOT is generally higher than that obtained by BESO. There are some reasons that cause the difference between the results of SEMDOT and BESO methods. The first and the most important reason is that the new version of SEMDOT does not use the material penalization scheme, whereas BESO needs to use the penalty coefficient. The second reason is that SEMDOT can use the MMA optimizer, whereas BESO has to use the Optimality Criteria (OC)-like optimizer. Generally, MMA has better performance than OC. At last, SEMDOT is a continuum method

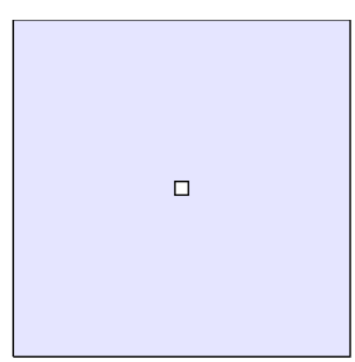

Fig. 1. Initial design for 2D cases.

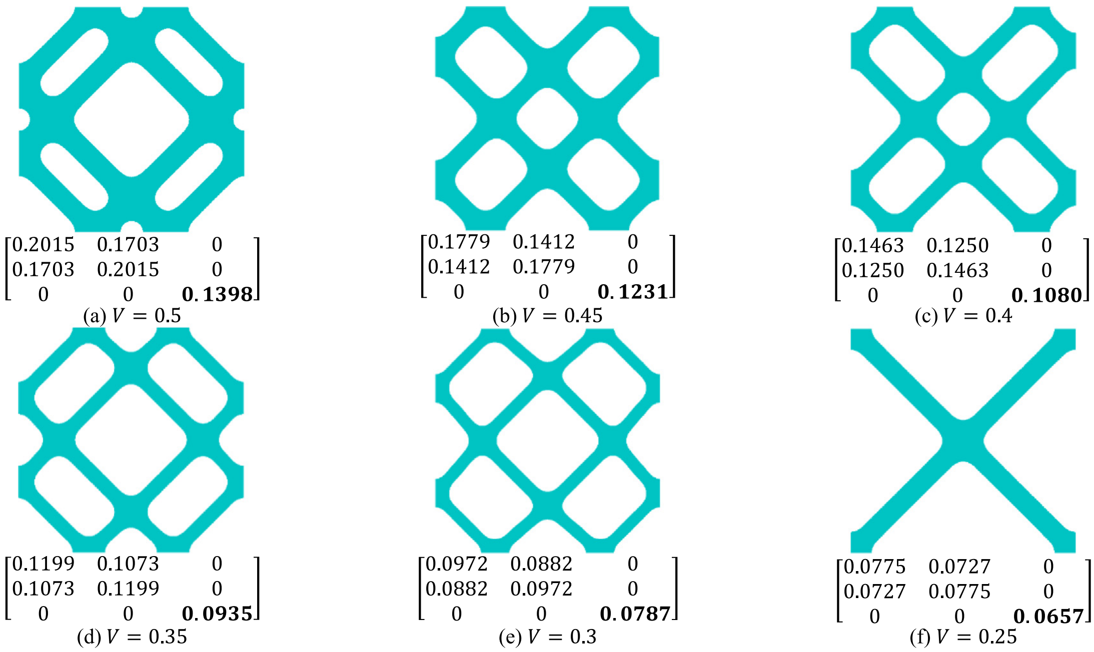

Fig. 2. Resulting unit cells under different volume fractions.

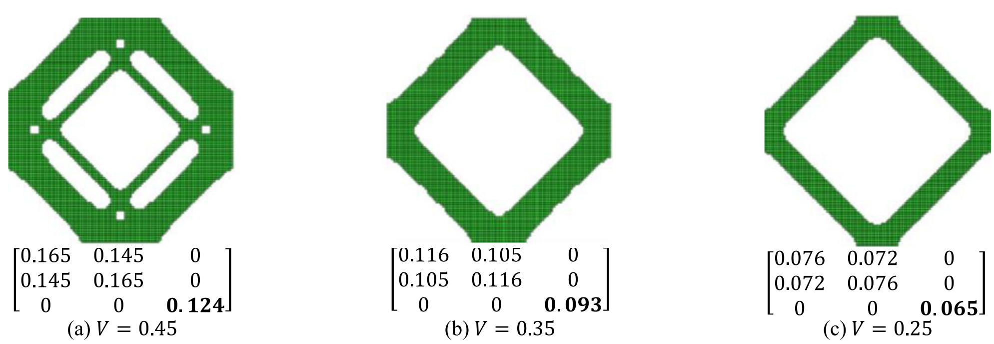

Fig. 3. Unit cells under different volume fractions in [8].

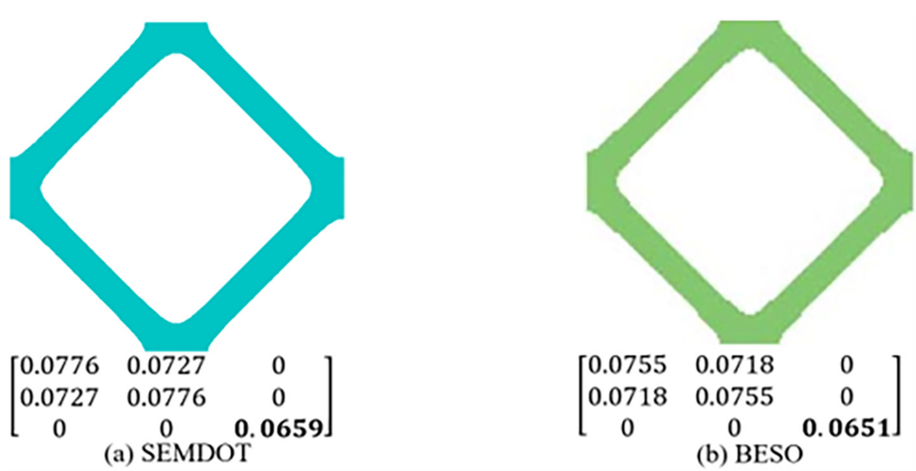

Fig. 4. Comparison of solutions obtained by SEMDOT and BESO [20] under a mesh of 120×120 and $V = 0.25$.

with the design variables from 0 to 1, however BESO is a discrete method with the design variables of 0 and 1. This means that SEMDOT has more options of the optimal solution compared to BESO.

The effect of the mesh size (i.e., 60×60, 80×80, 120×120, and 160×160) on optimized cellular structures and the maximum shear moduli is investigated. The results are shown in Fig. 5 where different cellular structures are achieved, and a coarse mesh of

$\begin{bmatrix} 0.1497 & 0.1184 & 0 \\ 0.1184 & 0.1497 & 0 \\ 0 & 0 & \mathbf{0.1065} \end{bmatrix}$
(a) 60 × 60

$\begin{bmatrix} 0.1467 & 0.1244 & 0 \\ 0.1244 & 0.1467 & 0 \\ 0 & 0 & \mathbf{0.1077} \end{bmatrix}$
(b) 80× 80

$\begin{bmatrix} 0.1448 & 0.1268 & 0 \\ 0.1268 & 0.1448 & 0 \\ 0 & 0 & \mathbf{0.1084} \end{bmatrix}$
(c) 120× 120

$\begin{bmatrix} 0.1452 & 0.1231 & 0 \\ 0.1231 & 0.1452 & 0 \\ 0 & 0 & \mathbf{0.1090} \end{bmatrix}$
(d) 160× 160

Fig. 5. Resulting unit cells under different mesh sizes subjected to $V = 0.4$.

60×60 results in a lower value of the maximum shear modulus (0.1065) compared to fine meshes (0.1077 for 80×80, 0.1084 for 120×120, and 0.1090 for 160×160). Taking the case with a mesh of 160×160 and $V = 0.4$ as an example, the convergence process and obtained 2×2 base cell are shown in Fig. 6. It can be seen from Fig. 6a that although there is a sudden drop during optimization, a steady state is reached after 100 iterations. In Fig. 6a, the topological boundary error indicates the accuracy of the smooth boundary, and generally a value lower than 0.001 is acceptable. The cellular structure in Fig. 6b can be further extended by repeating the unit cell in horizontal and vertical directions.

A 3D case with a mesh of $20 \times 20 \times 20$, $r_{\min} = 1.5$, and $V = 0.3$ is used to evaluate the effectiveness of SEMDOT in designing 3D cellular structures. The convergence process and 3D initial and resulting unit cells are shown in Fig. 7. Fig. 7a shows that the optimization process converges after 105 iterations at the maximum shear modulus of 0.06. Although the 3D initial unit cell shown in Fig. 7b is selected, other initial designs can be used. In terms of maximizing the shear moduli of 3D cellular structures, the initial design depicted in Fig. 7b is sufficient. Despite using a coarse mesh, the smooth 3D unit cell is still obtained by SEMDOT, as shown in Fig. 7c.

4. Concluding remarks and future work

This work extended general SEMDOT to a homogenization version for cellular structure design considering periodic boundary conditions and square symmetry. The effects of the volume fraction and mesh size on optimized cellular structures and the maximum shear moduli were investigated. Comparison with the results obtained by BESO was conducted. The numerical results show the capability of SEMDOT in forming smooth 2D and 3D cellular structures. Smooth boundaries formed by SEMDOT facilitate the manufacturing of topologically optimized cellular structures in 3D or 4D printing technologies.

In the future, accurate FEA will be used to simulate the mechanical behavior of topologically optimized cellular structures, as well as related experimental validations.

CRediT authorship contribution statement

Yun-Fei Fu: Conceptualization, Methodology, Investigation, Writing – original draft, Writing – review & editing, Visualization, Software. **Kai Long:** Writing – original draft, Writing – review & editing, Software. **Ali Zolfagharian:** Writing – original draft, Writing – review & editing. **Mahdi Bodaghi:** Supervision, Writing – review & editing. **Bernard Rolfe:** Supervision, Resources, Writing – review & editing.

Data availability

No data was used for the research described in the article.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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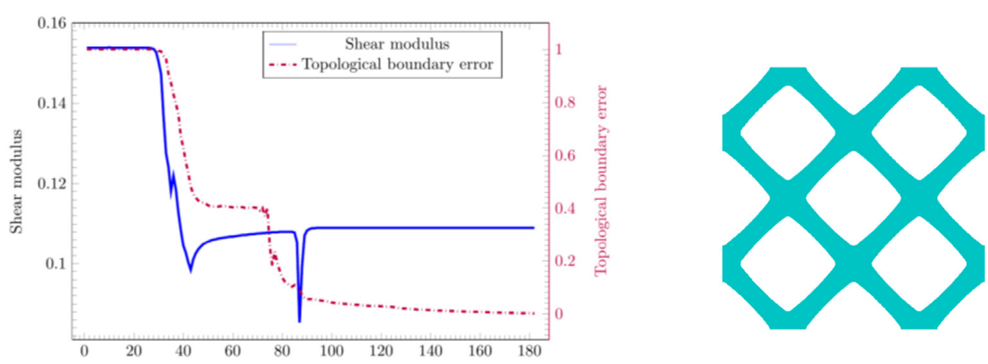
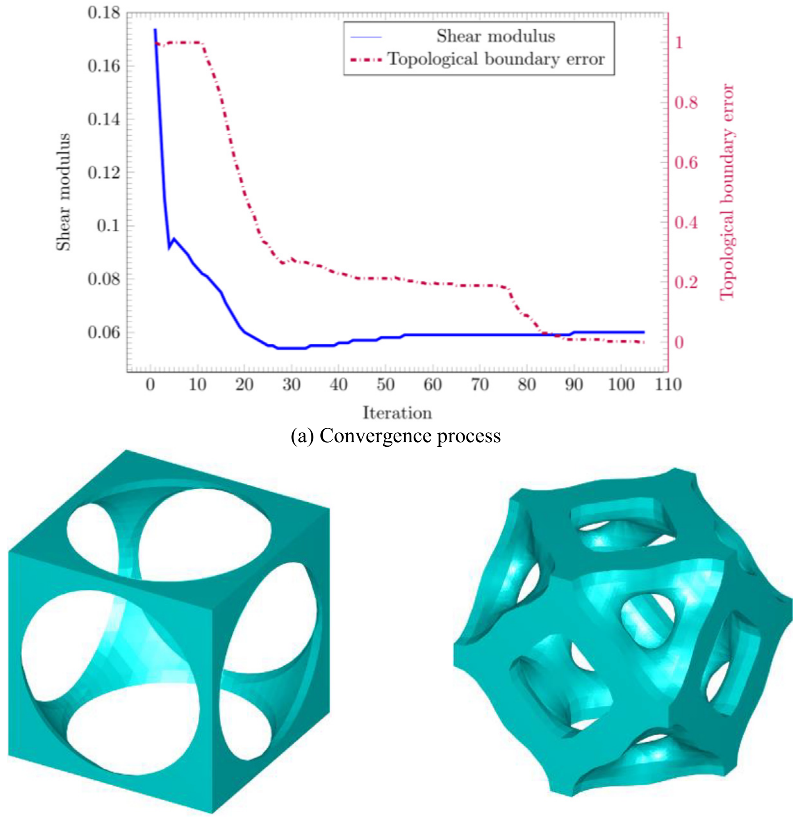


(a) Convergence process

(b) 2 × 2 base cell

Fig. 6. Convergence process and 2×2 base cell subjected to a mesh of 160×160 and $V = 0.4$.



(a) Convergence process

(b) 3D initial unit cell

(c) 3D resulting unit cell

Fig. 7. Convergence process and 3D initial and resulting unit cells.

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