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Research Article

Numerical Solution for Time Period of Simple Pendulum Under Magnetic Field

Viyan Jamal Jalal [1], Bawar Mohammed Faraj [2,*], Dana Taha Mohammed Salih[2], Sarkew Salah Abdulkareem[1], Bnar Hama Zaman Hama Ali [1], Bahadin Muhammad Hussien[1], Sarkhel Akbar Mahmood[1], Shewa Abid Hama[1]

[1] Department of Physics, College of Science, University of Halabja, Halabja, 46018, Iraq

[2] Computer Science Department of Physics, College of Science, University of Halabja, Halabja, 46018, Iraq

*Corresponding Author: Bawar Mohammed Faraj, E-mail: bawarm.faraj@uoh.edu.iq

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Abstract

In the present study, the numerical solution of the time period of a Simple Pendulum under a magnetic field investigated. The analytical solution presented for the given problem. The numerical solution for the problem achieved by using two numerical quadrature methods, namely, Simpson's 3/8 and Boole's method. The period of a simple pendulum with a large angle is presented. The results of the numerical quadrature have been compared to the exact solution. Absolute and relative mistakes of the problem have been presented. The Matlab program 2013R has created a numerical method to analyze the outcome. Moreover, it is established that the comparison results guarantee the present method's ability and accuracy.

1. Introduction

In last decades, Differential equations are used in a wide variety of disciplines, Partial differential Equation(PDE) and Ordinary Differential Equations(ODE) are play a major role in other fields such as medical, chemistry, physics, engineering, finance, physics and seismology [1-7]. They have several approximation methods which are different from each other [8-12]. Many numerical methods have been applied for solving linear and non-linear differential equations [13-16]. One of the most popular physical models encountered in undergraduate courses is the simple pendulum and the differential equation describing its motion [14, 15, 17-25]. Historically, the equation arises when studying the oscillations of a pendulum clock, but also appears in various other areas of physics, since problems often can be reduced to a differential equation similar to that describing the pendulum [16, 21, 22, 26, 27]. The exact solution to the equation of motion of the undamped pendulum is well known in the literature and involves the Jacobi elliptic functions [15, 21, 22, 28].

He et al., presents a Periodic property and instability of a rotating pendulum system [27], Moatimid and Amer presents an analytical solution and stability analysis for pendulum in [25]. Li et al., studies Theoretical, numerical, and experimental in a vibrator–pendulum coupling system [28]

Simple pendulum is a simple mechanical system in terms of setup, but it is difficult to calculate the factors that act on its motion, such as time period, amplitude, angle of oscillation, acting forces, and energy [23]. This simple mechanical system oscillates with a symmetric force due to gravity acting on it as a restoring force, as illustrated in (Fig. 1)[19]. Its equation of motion is given by:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L} sin\theta = 0 \quad (1)$$

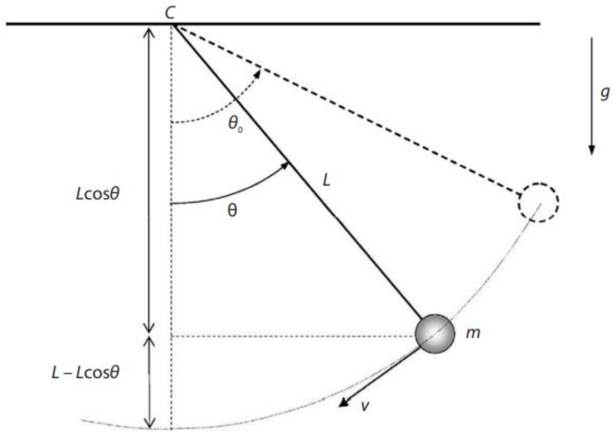

Figure 1. Scheme of a simple pendulum motion [22]

In The present study, we will describe numerical solution for the time period of a simple pendulum under influence of magnetic which presented in [22], as shown in Figure 2 .

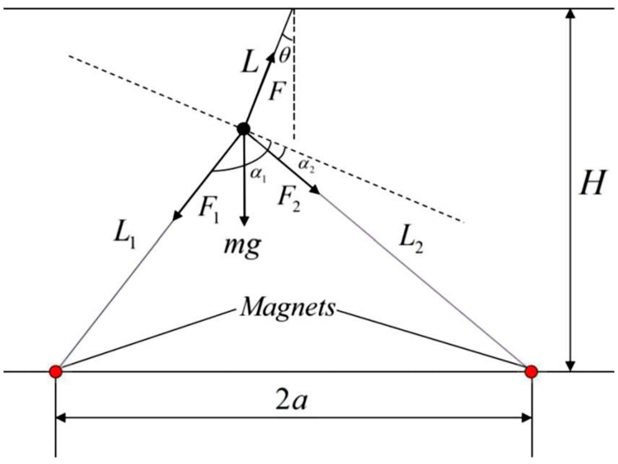

Figure 2. Pendulum system under action of double magnets [22]

2. The Proposed Method

Ma and Zhang in [22], presented a periodic solution for the pendulum under magnetic action[29]. They have modelled pendulum under magnetic action as follow:

$$T = \frac{4}{\sqrt{1-A^2}} \int_0^{\frac{\pi}{2}} (1 + k \sin^2 t\,)^{-\frac{1}{2}}\, dt \tag{2}$$

Where $k = \frac{A^2}{2(1-A^2)}$, The period of integration is stated as $\left[0, \frac{\pi}{2}\right]$ in (Ma & Zhang, 2022).

In this study, we will examine two different numerical quadrature methods for (2), after that we will compare numerical results with exact solution that have been presented in [22] A numerical solution can be found and compared with the results in [22].

There are many numerical integration methods to evaluate composite integrals; in this paper, we use two numerical quadrature methods, Simpsons 3/8 method, and Boole's method [16, 30-34].

If we set, $c = \frac{4}{\sqrt{1-A^2}}$, $f(t) = (1 + k \sin^2 t\,)^{-\frac{1}{2}}$ for the integral in Eq. (2), and applying Simpson's 3/8 method, we obtain:

$$c\int_0^{\frac{\pi}{2}} f(t)\, dt = c\frac{3h}{8}\left[\sum_{i=1}^{n/3} f_{3i-3} + 3\,(f_{3i-2} + f_{3i-1}) + f_{3i}\right] + O(h^4) \tag{3}$$

where $O(h^4) = -\frac{\theta_M}{80} h^4 f^4(\zeta)$, where $0 \le \zeta \le \theta_{\mathrm{M}}$. where θ_{M} is the maximum angle

Similarly, by applying Boole's method, we get:

$$c\int_0^{\frac{\pi}{2}} f(t)\, dt = c\frac{2h}{4}\left[\sum_{i=1}^{n/4} 7(f_{4i-4} + f_{4i}) + 32(f_{4i-3} + f_{4i-1}) + 12 f_{4i-2}\right] + O(h^6) \tag{4}$$

where $O(h^6) = -\frac{2\theta_M}{945} h^6 f^6(\zeta)$, where $0 \le \zeta \le \theta_{\mathrm{M}}$.

3. Numerical Calculation

The present work focused on the time period of simple pendulum under magnetic action as a function of its starting amplitude at a large angle numerically., for both integral equations (3) and (4), the results of Simpson's 3/8 and Boole's method will be compared with the exact results in [22] which counted as analytical solution of the problem, and absolute errors (E_A) and relative errors (R_A) are calculated by the following,:

$$E_A = |Exact\ value - Numerical\ value| \quad and \quad R_A = \frac{E_A}{Exact\ Value}$$

The Matlab program have been implemented for comparison between exact and approximation solutions [16, 21, 22, 35, 36], absolute error and relative error have been calculated, the following Tables shows the comparison between numerical methods in the present study and the results in [22].

Table 1. Numerical results in (3) compared with the exact solution in paper [22], in different values of A, where n=60.

Method	**A=0.1**	**A=0.5**	**A=0.9**
Simpson's 3/8 method in eq. (3)	6.30702032830215	6.98268054764193	10.6604517372438
Results in [22]	6.30688812487383	6.97832699208398	10.61923418562900
Absolute error	1.3220×10^{-4}	4.3536×10^{-3}	4.1218×10^{-2}
Relative error	2.0962×10^{-5}	6.2387×10^{-4}	3.8814×10^{-3}

Table 2. Numerical results in (4) compared with the exact solution in paper [22], in different values of A, where n=60.

Method	**A=0.1**	**A=0.5**	**A=0.9**
Boole's method in eq. (4)	6.30702032829067	6.98268054740341	10.6604517383238
Results in [22]	6.30688812487383	6.97832699208398	10.61923418562900
Absolute error	1.3220×10^{-4}	4.3536×10^{-3}	4.1218×10^{-2}
Relative error	2.0962×10^{-5}	6.2387×10^{-4}	3.8814×10^{-3}

From Table 1. and Table 2. We obtain that the numerical results have good accuracy. Both methods has a same accuracy approximately and have a difference after 9 digits, that errors can be neglected.

Table 3. Numerical quadrature method in (3) and (4) are compared with the exact solution in paper [22], in different values of A, where n=600.

Method	**A=0.1**	**A=0.5**	**A=0.9**
Simpson's 3/8 method in eq. (3)	6.30690135098058	6.97876252252756	10.6233569517113
Boole's method in eq. (4)	6.30690135098058	6.97876252252756	10.6233569517113
Results in [22]	6.30688812487383	6.97832699208398	10.61923418562900
Absolute error	1.3226×10^{-5}	4.3553×10^{-4}	4.1228×10^{-3}
Relative error	2.0971×10^{-6}	6.2412×10^{-5}	3.8824×10^{-4}

In Table 3, Both methods have been applied for eq(2), in differen values of A, From the results we can conclude that both numerical quadrature methods are accurate and suitable for solving simple pendulum integra eqs. (2). Absolute error and relative which gives the results in table 3. these table show the accuracy of the methods. However, the accuracy of the results depends on increase iteration number n. We can notice that while the number of iterations n are increased, then better more accurate have been found, as well as both methods have same results when number of n increased to 600.

4. Conclusion

In this study, we presented approximation solutions for a Simple Pendulum under a magnetic field. Two different numerical quadrature methods have been presented. The result was obtained using a numerical integration technique based on Simpson's and Boole's methods. The analytical solution has been compared with the numerical solution, and the agreement is very good. Matlab software has been implemented for calculation. Absolute error and Relative error have been calculated. The results guarantee the accuracy and applicability of both presented methods.

Declaration of Competing Interest The authors declare that they have no known competing of interest.

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