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Chao Huang, Fernando Moreira, Thomas Archibald, Kaidong Yu, Xuan Zhang

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The Impact of a Systemic Tax on Bank Capital Holdings, Optimal Capital Requirements and Social Welfare

Chao Huang

Beijing Jiaotong University <u>hchao@bjtu.edu.cn</u>

Fernando Moreira

University of Edinburgh Fernando.Moreira@ed.ac.uk

Thomas Archibald

University of Edinburgh <u>T.Archibald@ed.ac.uk</u>

Kaidong Yu

City University of Macau <u>kdyu@cityu.edu.mo</u>

Xuan Zhang*

Nanjing University of Aeronautics and Astronautics <u>xuanzhang.nanking@gmail.com</u>

*Corresponding Author

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Abstract

Existing studies suggest levying a systemic risk tax on systemically important banks to cover the costs of governmental interventions in (bailing out) these banks in the case of their bankruptcies. We develop a static model to investigate how this tax would affect the banks' equilibrium capital holdings and its impacts on banks' optimal capital regulations in terms of social welfare. We find that this tax would not only result in a safer banking system but would also help to mitigate the pro-cyclical effects of banking capital requirements. However, these merits would come at the cost of an increase in loan rate. Moreover, the improvements of the tax are less pronounced when the capital requirements are relatively strict as, for example, in Basel III. Regarding welfare, Basel II is closer to the optimal level. Although Basel III results in a safer banking system, this improvement compromises social welfare. Our findings also suggest that regulators should set higher capital requirements for systemically important banks, which is similar to the rules in Basel III.

JEL Codes: G21, G28, E44

Keywords: Systemic risk tax, optimal bank capital requirements, social welfare

Highlights:

- 1. We estimate the optimal capital requirements for systematically important banks and nonsystematically important banks.
- 2. We investigate the impacts of levying a systematic tax on banks and the real economy.
- 3. We provide numerical evidence to support the validity of Basel III in maintaining financial stability.
- 4. We suggest that the government could also regulate (systematically important) banks through appropriate taxation, in addition to the implementation of Basel Accords.

1. Introduction

Banking capital requirements play a role in mitigating banks' insolvency, which can otherwise cause externalities (contagion effects) to the rest of the economy. The recent financial crisis has shown that the systemic risk could also impair financial institutions due to macro-prudential effects in the event of failure of some institutions regarded as Too-Big-To-Fail or Too-Interconnected-To-Fail (Yan et al., 2023; Li et al., 2022; Ma & Nguyen, 2021). Basel I and Basel II Accords are designed to mitigate the micro-prudential effects of financial institutions, but neglect the interconnections between these institutions. Basel III has considered this impact of global systemically important financial institutions (SIFIs) and aims to mitigate greater risks (Triki & Abid, 2023), which they might pose to the financial system (Rubio & Yao, 2020). These SIFIs are, accordingly, stipulated with higher holdings at the ratio of 1% to 3.5% as an additional capital requirement (Suh, 2019). The Basel III Accord also aims to mitigate the negative impact of cyclical effects of the banking regulation by introducing a capital conservation buffer and requires an additional 0–2.5% countercyclical capital buffer in booms, during which time a systemic risk might build up (Basel Committee on Banking Supervision: BCBS, 2011).

How much optimal capital requirement should be set, and should SIFIs be regulated with a higher capital ratio? If so, how much additional capital should be required? Is there any room for the introduction of additional tools that can be used by governments (Saha & Dutta, 2023), such as systemic tax, for regulating SIFIs? Although extant literature discussed some tools to regulate SIFIs, such as using TARP (Berger, *et al.*, 2020) and permitting innovation (Boot, *et al.*, 2021), little is discussed on taxation. To answer these questions, we use this paper to estimate optimal capital requirements for both SIFIs and non-SIFIs and to test the effectiveness of a systemic tax, proposed by Freixas and Rochet (2013) and Acharya, Philippon, and Richardson (2017), to be levied on systematically important banks (SIBs) to cover the expected

costs of bailing out these banks if they go bankrupt. Our contributions relate to 1) evaluating the aforementioned systemic tax and its impacts on bank capital holdings and social welfare. Existing literature propose such tax, but haven't investigated its impacts on banks and other participants, such as firms. This would discount the creditability of the proposal for this tax. 2) Estimating the optimal capital requirements for the SIBs and non-SIBs. Although few papers, such as Repullo and Suarez (2013) have estimated optimal capital requirements for a representative bank across booms and recessions, they haven't investigated optimal capital requirements for SIBs and non-SIBs. Our paper helps to answer this question. 3) Testing the performance/improvements of Basel III and thus revealing that Basel III enhances the stability of the banking system at the sacrifice of social welfare. Few studies consider both the Basel II and Basel III and compares their differences and the improvement introduced by Basel III. Our paper helps to conduct such comparison. In addition, although prior studies, for example Freixas and Rochet (2013), provide a mathematical proof to support the effectiveness of the systemic tax, as far as we are aware, some open questions such as its impacts on banks' (procyclical) capital holding behaviours and its influences on loan borrowers still need to be answered. We find that the systemic tax would help to foster social welfare and mitigate the pro-cyclical effects of banks' capital regulation (especially for Basel II). However, the merits of the systemic tax would come at the cost of an increase in loan rate, thus adding the costs to the loan borrowers, the entrepreneurs. Moreover, the systemic tax is less effective in the context of higher capital requirements, as in the case of Basel III compared to the previous regulations. We also find that Basel II is the closest regulatory regime to the optimal level of capital requirements that would maximise social welfare, while Basel III results in a safer banking system at the sacrifice of social welfare. We identify that regulators should set higher capital requirements for SIBs, which is in line with the current Basel III regime. We have also collected empirical data to compare the difference in capital ratios between large and small banks from

1988 to 2020. The empirical data supports our theoretical results in that large banks should be regulated with a higher capital ratio, especially in recent years, when the capital ratios of large banks are lower than small banks. The empirical data is shown in Figure 1. Thus, this leads to our contribution to future empirical analysis: investigating the difference in capital ratios and optimal capital requirements (using squared terms) between large and small banks and across booms and recessions.

<Insert Figure 1 here>

Our paper relates to the following literature. Miles, Yang, and Marcheggiano (2012) reveal that optimal bank capital structure could be introduced to maximise social welfare, which suggests the existence of the optimal capital requirement. Repullo and Suarez (2013) consider a dynamic equilibrium model and reveal that optimal capital requirements seem to be cyclically varying, but less cyclical for high social costs of bank failure. They also maintain that Basel II is more cyclical than Basel I, by resulting in a higher credit rationing in recessions, while Basel II could make banks safer and would be superior in social welfare. Freixas and Rochet (2013) propose levying a systemic tax and establishing a system risk authority to lessen managers' risk-taking behaviours. They show that capital regulation may have a very limited role in protecting banks from bankruptcy and confirm that a systemic tax might help to solve managers' excessive risktaking. Behn, Haselmann, and Wachtel (2016) study the effect of pro-cyclical capital regulations on banks' lending and argue that a 0.5% increase in capital charge could result in a 2.1–3.9 percentage decrease in loan lending, suggesting cyclical capital regulation can have sizeable effects. Gordy and Howells (2006) suggest a counter-cyclical indexing to change business mix for Basel II, and similarly, Repullo and Saurina (2009) suggest a through-thecycle Probabilities of Default or a Gross Domestic Product (GDP)-growth-based multiplier to mitigate the pro-cyclicality of the Basel II. Our paper, in a way, suggests the systemic tax could also be a solution to deal with this pro-cyclicality.

Our paper reveals some policy implications. First, the proposed systemic tax might not be as effective as it was originally proposed, due to its externality to the increase in borrowing costs of firms. Second, such tax might be less effective in current periods, during which time the capital requirements are exceptionally high. Third, capital requirements should be set differently for large and small banks for the promotion of bank lending and social welfare, thus revealing the limitation of the one-size-fits-all assumption. Fourth, current Basel III seems in the right direction in capital regulations, as in our results, we support the proposal that large banks should be regulated with an additional capital ratio of around 2.5%.

The rest of this paper is organised as follows. Section 2 introduces the participants of our model, and Section 3 describes the model. Section 4 presents the social welfare analysis and compares the optimal capital requirements under different scenarios. Section 5 concludes our paper. The Appendix includes proofs of propositions.

2. Participants

2.1 Banks

We consider two banks: one SIB and one non-SIB. Since banks with large market share are generally considered to be systemically important, and to simplify terminology, we refer to these as the large bank and the small bank respectively in the remainder of our analysis. The banks are operated by their shareholders whose annual required return rate is δ . Shareholders provide the banks with equity and depositors finance the banks with deposits; banks lend the money they raise to the entrepreneurs in the form of loans. The balance sheet of the banks can thus be shown as *loans* = *deposits* + *equity*. Both banks are regulated by the government and are required to comply with minimum capital requirements. To distinguish the large bank's systemic importance, we assume that if this bank fails, the government will bail it out by paying its recovery costs (regarding the rescue of the bank, including honouring the deposits in full),

and by paying the associated contagion effect costs (regarding the expenses of tackling the negative effects caused to the rest of the banking system, i.e. the small bank.¹ This assumption implies that the bankruptcy cost of the large bank would be higher than the sum of the aforementioned costs in the absence of a bailout. Upon the bailout, all the debt (deposits) of the large bank will be paid off.

2.2 Entrepreneurs

Entrepreneurs borrow loans from the banks to undertake their projects. However, the projects faces the danger of failure. The projects' return is realised at the end of each period. There are two periods, denoted by T = t, t + 1 respectively. For each period, if the projects are successful, each investment unit will yield a pledge-able return 1 + a. If the projects fail, it will return $1 - \lambda^2$ where $0 < \lambda < 1$. λ denotes the Loss Given Default (LGD) per unit of the projects. The Probability of Default (PD) of the projects is independent across the economic situations and is denoted by p_M , where M = l, h, representing low default state (boom) and high default state (recession) respectively. For each period T, the economic situation will take either of these two states, and we denote $M \in \{m, m'\}$ as the realisation of the economic states for the period t and t + 1. Defaults of the projects determine the performance of the loans. Suppose the fraction of the nonperforming loans, for each period T, is $x_T \sim [0,1]$, the distribution of x_T is:

$$F_{M}(x_{T}) = \Phi\left[\frac{\sqrt{1-\rho}\Phi^{-1}(x_{T}) - \Phi^{-1}(p_{M})}{\sqrt{\rho}}\right]$$
(1)

Equation (1) is set up by value-at-risk foundation to the capital requirement. $\Phi(\cdot)$ is the cumulative distribution function of a standard normal variable and ρ is a parameter that

¹ As in Dungey and Gajurel (2015), Greenwood, Landier, and Thesmar (2015) and Piccotti (2017), the contagion effects of the large bank, which is systemically important, will negatively affect other banks (in our analysis, the small bank), if the large bank fails. Thereby, this negative effect should be paid if a bailout is conducted.

² We implicitly assume if the projects fail all the remaining value of the projects $1 - \lambda$ will be paid to the banks.

measures the dependence of individual defaults of projects on the common risk factor (see Repullo and Suarez, 2004). Equation (1) implies the probability distribution of loan default rate x_T is governed by a single common risk factor of the projects (Vasicek, 2002). We present a detailed proof of (1) in the Appendix. The economic state may change from period t to t + 1, and the transition of economic situations follows a Markov chain:

$$s_{mm'} = \Pr(m_{t+1} = m' | m_t = m), \text{ for } m, m' = l, h.$$

2.3 Government

The government sets optimal capital requirements in order to maximise social welfare. The government is also responsible for supervising the banks to ensure that they abide by the capital requirements. The government will rescue the large bank in the case of its bankruptcy, and it will levy a systemic risk tax on that bank, which will be discussed in Section 3.1.1, to cover the expected costs of future interventions (bailouts). However, the government will not bail out the small bank if it fails. Thereby, in this case, only a portion of the deposits will be guaranteed given the limited coverage of the deposit insurance, which we will discuss in Section 2.4.

2.4 Depositors

Depositors are restricted to equity investment and only has access to deposit investment. All the depositors are risk neutral. All the banks' depositors are under partial deposit insurance and the insured portion is q. However, the large bank's depositors will be able to reclaim all their deposits if that bank fails. This is due to the government bailout policy, an assumption which we have made in Section 2.1. On the other hand, the government will not bail out the small bank and depositors are subject to a partial repayment of their deposits when that bank fails. Accordingly, the depositors of the small bank will require a higher deposit rate to compensate for their potential loss in case of the bankruptcy.

3. Model Setup

At the beginning of period *t*, the large bank and small bank in-elastically lend the loans to entrepreneurs, at the size of $\frac{Q}{Q+1}$ and $\frac{1}{Q+1}$, respectively. To finance the loans, the large bank raises $1 - k_{L,m}$ unit of deposits $(1 - k_{S,m}$ for the small bank); $k_{L,m}$ capital ($k_{S,m}$ for the small bank) to satisfy the capital requirements. It is clear that $k_{L,m} \ge \gamma_{L,m}$ and $k_{S,m} \ge \gamma_{S,m}$, and they will possibly keep a capital buffer $k_{L,m} - \gamma_{L,m} > 0$ or $k_{S,m} - \gamma_{S,m} > 0$ to cope with potential shocks, where $\gamma_{L,m}$ and $\gamma_{S,m}$ are capital requirements for the large and small banks, respectively. The loan rate at period *t*, denoted by r_m , is determined by the large bank. Its calculation is discussed in Section 3.1.3. This assumption implies the entrepreneurs will keep a return of $a - r_m$ from each investment unit in their successful projects. The loan rate in period t + 1 is fixed at the value of *a*, which means all the pledgeable return from the projects will be paid to the banks. The increase in loan rate from r_m to *a* reflects entrepreneurs' dependence on banks for period t + 1 (Repullo and Suarez, 2013; Zaheer et al., 2023).

Banks raise no capital buffers and set their capital holdings at $\gamma_{L,m}$, and $\gamma_{S,m}$, respectively for period t + 1. The intuition for assuming this is based on there being no further periods, so the bank might find it unprofitable to hold any excess capital to secure deposits.

3.1 Large Bank Analysis

At the end of period t, the large bank obtains the return from its loans, which is captured by the performance of the projects by the entrepreneurs. The bank yields $1 + r_m$ from the fraction of performing loans $1 - x_t$, and $1 - \lambda$ from the fraction of the defaulted loans x_t . It will pay for a setup cost μ to manage deposits and pay for the related internal costs. This cost will not be incurred in period t + 1, because depositors are less likely to change bank to deposit, due to

switching costs³. After paying the deposit holders at the amount of $1 - k_{L,m}$, the net worth of the large bank, $k'_{L,m}(x_t)$, is

$$k'_{L,m}(x_t) = k_{L,m} + r_m - (r_m + \lambda)x_t - \mu$$
⁽²⁾

There exist three possible outcomes of the large bank's activities. First, if $k'_{L,m}(x_t) \leq 0$, the bank fails. In this case, it will be liquidated and taken over by the government. Second, if $0 < k'_{L,m}(x_t) \leq \gamma_{L,m'}$, the bank will reduce the amount of lending for period t + 1 as it cannot meet the capital requirement of $\gamma_{L,m'}$ for that period (the portion of loans that will be cut down is $1 - k'_{L,m}(x_t)/\gamma_{L,m'}$). Third, if $k'_{L,m}(x_t) > \gamma_{L,m'}$, the bank is eligible to finance the loans in full and will pay dividend to the shareholders at the amount of $k'_{L,m}(x_t) - \gamma_{L,m'}$, so that the ratio of its equity holdings is exactly $\gamma_{L,m'}$ at the beginning of period t + 1. We can summarise these outcomes as follows:

1) The bank fails when $k'_{L,m}(x_t) < 0$, equivalent to $x > \hat{x}_m$, where

$$\hat{x}_m = \frac{k_{L,m} + r_m - \mu}{\lambda + r_m} \tag{3}$$

2) The bank has insufficient lending capacity when $0 \le k'_{L,m}(x_t) < \gamma_{L,m'}$, equivalent to $\hat{x}_{mm'} \le x < \hat{x}_m$, where

$$\hat{x}_{mm'} = \frac{k_{L,m} + r_m - \mu - \gamma_{L,m'}}{\lambda + r_m}$$

3) The bank has excess lending capacity when $x < \hat{x}_{mm}$.

3.1.1 Systemic Tax

In this section, we introduce the calculation of the systemic tax, proposed by Freixas and Rochet (2013), to be charged on the large bank to cover the costs of interventions (bailouts).

³ For simplicity, we neglect the switching costs in our model, but we assume the depositors will find it is unprofitable to change bank in period t + 1.

To be consistent with the assumption of the management $\cot \mu$, we assume the tax is levied only for period *t*, and this tax will be paid to the government at the beginning of period t^4 . In the event of the bankruptcy of the large bank, the government needs to pay for a proportional recovery cost at *d*, including the payment of the deposits in full⁵, which means the recovery fee is:

$$\lambda_m = \frac{dQ}{Q+1} [1 - F(\hat{x}_m)]$$

In addition, the government will need to pay the expected costs of the contagion effects, to cover the expenses of tackling the negative effects incurred to the small bank, which is:

$$\vartheta_m = \frac{\varphi}{Q+1} [1 - F(\hat{x}_m)],$$

where \hat{x}_m is defined in Equation (3). Thus, the systemic tax T_m is as:

$$T_m = \lambda_m + \vartheta_m = \frac{dQ + \varphi}{Q + 1} \left[1 - F(\hat{x}_m) \right] \tag{4}$$

3.1.2 Large Bank's Shareholder Net Present Value

The net present value of the shareholders of the large bank will be:

$$v_{L,m}(k_{L,m}, r_m) = \frac{1}{1+\delta} E[v_{mm'}(x_t)] - k_{L,m} - T_m$$
(5)

where

⁴ We have another interpretation for this consideration. As banks' ratio of capital holding will be at the level of the capital requirement in period t + 1, an implementation of a systemic tax will not introduce any capital buffer for that period. We thereby only consider the systemic tax for period t.

⁵ In our calibration, we set 1 - q < d to ensure that the recovery costs included in the systemic tax, levied from the large bank, are high enough to cover the repayment of the portion of deposits not covered by the partial deposit insurance.

$$v_{mm'}(x_t) = \begin{cases} \pi_{m'} + k'_{L,m}(x_t) - \gamma_{L,m'} & \text{if } x_t < \hat{x}_{mm'} \\ \pi_{m'} \frac{k'_{L,m}(x_t)}{\gamma_{L,m'}} & \text{if } \hat{x}_{mm'} \le x_t < \hat{x}_m, \\ 0 & \text{if } x_t \ge \hat{x}_m \end{cases}$$
(6)

and

$$\pi_{m'} = \frac{1}{1+\delta} \int_0^1 \max\{\gamma_{L,m'} + a - x_{t+1}(\lambda + a), 0\} dF_{m'}(x_{t+1}).$$
(7)

Equation (7) is the expected gross return that equity earns on each unit of loans made in period t + 1. Equation (5) presents the expected net worth of the bank at period t, and T_m is defined by (4). Equation (6) summarises three outcomes based on the realisation of the loans at the end of period t, where $k'_{L,m}(x_t)$ is defined in Equation (2). Credit rationing is defined as the portion of the loans that cannot be lent for period t + 1 due to bankruptcy and insufficient lending capacity, which can be written as:

$$CR_{L,mm'} = [1 - F(\hat{x}_m)] + \int_{\hat{x}_{mm'}}^{\hat{x}_m} \left[1 - \frac{k'_{L,m}(x_t)}{\gamma_{L,m'}} \right] dF_m(x_t)$$
(8)

The first term of Equation (8) is the large bank's probability of failure, while the second term is the expected reduction in lending due to insufficient capital for period t + 1.

3.1.3 Equilibrium

To determine the value of r_m^* , we assume shareholders have zero net worth. If the net worth is positive, banks will expand lending. If the worth is negative, banks will reduce the lending. Hence, in equilibrium:

$$v_{L,m}[k_{L,m}^*, r_m^*] = 0$$

for

$$k_{L,m}^* = \arg \max_{k_{L,m} \ge \gamma_m} v_{L,m} [k_{L,m}, r_m^*]$$

As the income is strictly increasing with r_m^* and the income is negative for low r_m , there should exist a unique r_m^* that satisfies the above equations, and the large bank will thus charge this loan rate as the equilibrium rate (Randl et al., 2023; Repullo and Suarez, 2013). Regarding the equilibrium choice for the capital of the large bank, we have Proposition 1:

Proposition 1: There cannot be a solution at the corner $k_{L,m} = 1$, while it is possible to obtain a corner solution at $k_{L,m} = 0$. If a solution is interior, namely $0 < k_{L,m} < 1$, and the probability of the default of the large bank is strictly positive, the implementation of the systemic tax will result in a higher loan rate r_m if a higher capital holding is formed by this tax.

We give the proof in the Appendix. The intuition of Proposition 1 is that the large bank would transfer their increased costs (due to the introduction of the tax) by charging higher loan rates to firms, thereby causing an effect external to the banking system.

3.1.4 Baseline Parameters

Table 1 presents our baseline parameters of the model.

<Insert Table 1 here>

Our parameters are mainly adopted from empirical data studies (mainly Repullo and Suarez, 2013) or relevant policy regimes of the US economy. Following Repullo and Suarez (2013), we adopt the rate of return *a* as 0.04, which is approximately calculated by estimating the *Total Interest Income* of the banks minus the *Total Interest Expense* and the *Total Deposits Income*. Parameter $\lambda = 0.45$ denotes the LGD that failed projects cause. This value is based on the Basel II foundation Internal Ratings-Based approach. The value $\mu = 0.03$ is adopted to match the average loan spreads of 100 basis points in booms, from Federal Deposit Insurance Corporation (FDIC) Statistics 2004–2007. The required return $\delta = 0.08$ is from Van den Heuvel (2008), who estimates the value of 3.16% as the lower bound for the cost of Tier 1

capital. Others like Iacoviello (2005) estimate this value at around 4%. We also follow Repullo and Suarez (2013) to double the required return to consider both the Tier 1 and Tier 2 capital set up by shareholders, and thus the value is at $\delta = 0.08^6$. To keep consistency of the parameter values source, we use the value of p_l , p_h and ρ from Repullo and Suarez (2013). De Nicolo, Gamba, and Lucchetta (2014) give the estimated baseline bankruptcy cost at the level of 0.104, and they view this value as a lower bound for bankruptcy costs, because the estimate is based on the nonfinancial sector, while Repullo (2013) sets the social cost of bank failure at 0.2. Thus, we adopt the value of 0.20, nearly double what De Nicolo, Gamba, and Lucchetta (2014) suggest for the lower bound for bankruptcy cost. The recovery cost of the large bank is set at b = c to reflect the fact the cost of the recovery of the large bank is at least costly as the bankruptcy cost of the small bank. We thus adopt the value of c as a lower bound for that of the recovery cost b. We follow Dungey and Gajurel (2015), Greenwood, Landier and Thesmar (2015) and Piccotti (2017) using the average of their estimated value of φ and thereby set $\varphi =$ 0.40. From the FDIC data 1969–2004, the transition probabilities of the Markov process are set at $q_{ll} = 0.80$ and $q_{hh} = 0.64$. Following Repullo and Suarez (2013), we weighted the unconditional probabilities of each state *M* ($\phi_l = 0.643$ and $\phi_h = 1 - \phi_l = 0.357$), which corresponds to the average duration of 5 years for economic booms and 2.8 years for economic recessions.

3.1.5 Basel Regulation Regimes

⁶ One can assume that Tier 2 capital will be in a same amount as the Tier 1 capital (as in Basel II), thus the multiplication of the required return by 2 is justified. Although Basel III defines that Tier 1 capital (6%) should be higher than the Tier 2 capital (2%), other capital buffers introduced by Basel III, such as conservations buffer and countercyclical buffer would make the total capital potentially up to twice of the Tier 1 capital, at least in our calibration. Moreover, to make our results comparable across different regulatory regimes, we fix the value of δ to the analysis of Basel III by assuming the average required rate of bank capital is $\delta = 8\%$.

In our analysis, we discuss the requirements for Tier 1 capital⁷. We consider the following four capital regulation regimes: *Laissez-faire regime*, *Basel I regime*, *Basel II regime* and *Basel III regime* and *Basel III regime*. Under the *Laissez-faire regime*, the capital requirements are set at $\gamma_l = \gamma_h = 0$. Under the *Basel I regime* we set $\gamma_l = \gamma_h = 0.04$, following the Basel Accord of 1988. The capital requirements for Tier 1 capital in the *Basel II regime* for corporate loans of one year⁸ maturity are determined⁹ as:

$$\gamma_M = \frac{\lambda}{2} \Phi \left[\frac{\Phi^{-1}(p_M) + \sqrt{\rho(p_M)} \Phi^{-1}(0.999)}{\sqrt{1 - \rho(p_M)}} \right]$$
(9)

where

$$\rho(p_M) = 0.12 \left(2 - \frac{1 - e^{-50p_M}}{1 - e^{-50}} \right) \tag{10}$$

Equations (9) and (10) are based on BCBS (2004). Recall that the notation *M* includes two economic states, i.e. M = l, h. The term $\rho(p_M)$ is equivalent to ρ in (1), and explicitly indicates that the correlation (ρ) between the common risk factor and default of individual projects is negatively related to the PD of the projects (p_M). The reason behind this assumption is that riskier projects are normally held by smaller and riskier firms whose defaults are less correlated with the common risk factor. We present a rationale and a proof of (9) and (10) in the Appendix. Using Equation (9), $\gamma_l = 3.2\%$ and $\gamma_h = 5.5\%$. The Basel III Accord increases the capital requirements for the Tier 1 capital ratio to 6%. To reflect the changes in Basel III, we revise Equation (9) to make the capital requirements for the Tier 1 capital as:

⁷ We focus on the Tier 1 capital because, compared to other tiers, it provides banks with more protection against insolvency (BCBS, 2011). This treatment is also adopted by Repullo and Suarez (2013).

⁸ In our model, the maturity of loans is one year, and thus a multiplier to adjust maturity mismatches, as in Basel II and Basel III, will not affect our calculation.

⁹ According to the rationale behind Basel II and Basel III, the expected losses, represented by the probability of default $p_{m'}$, should be covered by provisions, while a remaining part, i.e. unexpected losses, will be covered by capital. However, as done in Repullo and Suarez (2013), our main objective is to investigate banks' equity holding behaviours among business cycles and under various requirement regimes. Thus, the distinction between provisions and capital is immaterial to our calculations. Accordingly, in our analysis, capital aims at absorbing the total (expected plus unexpected) losses incurred by banks.

$$\hat{\gamma}_M = \frac{3\lambda}{4} \Phi \left[\frac{\Phi^{-1}(p_M) + \sqrt{\rho(p_M)} \Phi^{-1}(0.999)}{\sqrt{1 - \rho(p_M)}} \right]$$
(11)

where $\rho(p_M)$ is defined as in (10). The rationale behind (11) is that Basel III raises the minimum requirement for Tier 1 capital from 4% (Basel II) to 6%. We provide a detailed interpretation in the Appendix. Using (11), the capital requirement for Tier 1 capital requirements are $\hat{\gamma}_l = 4.7\%$ and $\hat{\gamma}_h = 8.2\%$. The *Basel III regime* also introduces a countercyclical buffer, ranging 0-2.5\%, in the form of common equity (BCBS 2011), to be added when the credit growth is high. To reflect it into our analysis, we add this buffer for booms, during which time the credit growth is normally high. Although this buffer is not part of the requirements for Tier 1 capital, the buffer is closely linked to business cycles effects (Li & Xu, 2022), which is our main focus, and thus we consider it into our analysis¹⁰. To add the countercyclical buffer, we adopt the middle value (1.3%) of the suggested range, which means the capital requirements for *Basel III regime* are $\gamma_b = 6.0\% = (4.7\% + 1.3\%)$ and $\gamma_r = 8.2\%$.

3.2 Quantitative Results

We set Q at different levels to identify the effect of bank size on the bank's capital decisions. However, for the ease of comparison, in Table 2, we only report the scenario when Q = 5. The results in brackets denote the case under the systemic tax regime, while the figures without brackets denote the case without the systemic tax regime.

<Insert Table 2 here>

3.2.1 Loan rates

¹⁰ Basel III also introduces a conservation buffer at the ratio of 2.5% (in the form of common equity), however, this buffer is not part of the Tier 1 capital requirement and it does not fit for the characteristics of our model and we exclude it from our model. We also consider the scenario where the conservation buffer is added, making the capital requirements at $\gamma_b = 8.5\%$ and $\gamma_r = 10.7\%$. The main results remain unchanged and are presented in the Online Appendix.

As shown in Table 2, the equilibrium loan rates are higher in recessions than in booms because in recessions the PD increases and thus a higher rate is desirable to compensate the bank shareholders. As in Proposition 1, the loan rates r_m (with the systemic tax) are higher than the r'_m (without the systemic tax) under all regulation regimes, indicating that the banks transfer the cost of being levied the systemic tax on to entrepreneurs. Thus, the systemic tax will be more likely to affect the overall economy by resulting in a lower loan demand or a lower investment payoff (due to an increased loan rate)¹¹.

3.2.2 Capital Buffer and Net Capital Buffer Increase

Without the systemic tax regime, banks hold more capital buffers $(\Delta'_{L,m})$ in booms than in recessions. With the systemic tax regime, the results are opposite; the capital buffers are higher in recessions than in booms (except for *Basel III*). Interestingly, the bank under *Basel III* could effectively hold a higher capital buffer (3.6%=2.3%+1.3%) in booms because of a countercyclical buffer required at 1.3%, while under other capital regimes the buffers are higher in recessions. Moreover, the net capital buffer increase $\alpha_{L,m}$ suggests that the systemic tax could effectively help to make the bank increase its capital holdings in both economic situations, which will result in a safer banking system, although this effect is less pronounced in booms. When in Basel III, the increase in capital holdings with the implementation of the systemic tax is also less significant, being only 1.4% in recessions and 0.0% in booms. This finding indicates that the systemic tax is less effective when the capital requirements are too strict, such as in Basel III.

3.2.3 Capital Holdings

¹¹ Although we consider an inelastic demand of loans in our model, a further study to relax this assumption deserves consideration.

We find that in the case without the systemic tax regime, the capital holdings are 4.2% (3.4%), 6.7% (6.1%), 7.0% (6.5%) and 8.3% (8.6%) respectively for *Laissez-faire, Basel I, Basel II and Basel III regimes* when in booms (recessions). Except for under the *Basel III regime*, capital holdings are all higher in booms than in recessions, due to a higher profitability of holding capital in order to proceed with lending in period t + 1. For the *Basel III regime*, because of a higher capital requirement in booms (at 6.0%), the cost of raising additional capital exceeds the profitability, making the capital holding lower than that of the recessions. When considering the systemic tax, the bank's capital holdings change significantly. The bank's capital holdings are at 6.3% (9.4%), 6.9% (9.5%), 7.3% (9.3%) and 8.3% (10.0%) for the *Laissez-faire*, *Basel I, Basel II and Basel III regimes* in booms (recessions), respectively. We observe that the systemic tax will help to make the bank safer as the capital holdings are all higher in both economic situations. However, this improvement is less significant for Basel III, where the capital requirements are too strict.

3.2.4 Credit Rationing, PD and Social Welfare

After the implementation of the systemic tax, credit rationing reduces from 8.2% to 0.6% for the *Laissez-faire regime*, while reductions are from 4.9% to 1.7%, 5.6% to 1.8% and 7.2% to 5.3% for the *Basel I, Basel II* and *Basel III regimes*. The pro-cyclical effects of capital regulation (especially for the *Basel II*, revealed by Repullo and Suarez (2013)) is also lessened as the credit rationing is reduced from 13.7% and 9.8% to 5.4% and 2.8%, where the economic states in t + 1 are recessions¹².

Regarding PD, after the implementation of the systemic tax, the annual probabilities of bank failure reduces from 5.86% to 2.21%, 0.87% to 0.42%, 0.62% to 0.29% for *Laissez-faire*, *Basel*

¹² Repullo and Suarez (2013) show that the Basel II is pro-cyclical because banks will produce a much lower supply of credit (due to a higher credit rationing) when the economy is followed by recessions, namely when t + 1 is a recession. According to our findings, these credit rationings are reduced after the introduction of the systemic tax, which means the pro-cyclical effects of the capital requirements (Basel II) are lessened.

I, Basel II and Basel III respectively. However, the reduction is less pronounced for *Basel III*, from 0.19% to 0.13%. In terms of social welfare, we can see that the systemic tax increases the welfare, with the *Basel II regime* reaching the highest level (7.99 and 7.72 with and without the systemic tax). Social welfare in the context of the *Basel III regime* is at the value of 7.63 and 7.53 with and without the systemic tax respectively, which suggests Basel III reduces bank failure at the sacrifice of social welfare.

3.3 Small Bank Analysis

3.3.1 Deposit Rate Premium

Due to the low confidence of the small bank depositors over reclaiming full deposits in the case of bankruptcy, they will request a rate premium for the deposits. This premium is payable to the depositors at the end of period t only if the small bank does not fail. Depositors determine the value of the premium according to the capital holdings of the small bank $k_{s,m}$. In order to distinguish the large bank from the small bank, we denote the fraction of the defaulted loans held by the small bank as y_t and y_{t+1} for period t and t + 1, respectively. The value of the small bank's shareholders, in period t, $k'_{s,m}(y_t)$ is as follows:

$$k'_{S,m}(y_t) = k_{S,m} + r_m^*(k_{L,m}) - [r_m^*(k_{L,m}) + \lambda]y_t - \mu - r_{d,m}$$

where $k_{S,m}$ is the capital holding of the small bank and $r_{d,m}$ is the deposit rate premium; both are a function of the economic situation m. Note the small bank takes the loan rate determined by the large bank as given and thus we denote the rate as $r_m^*(k_{L,m})$ for the small bank to reflect it is an exogenous value.

The small bank fails if $k'_{S,m}(y_t) < 0$, equivalent to $y_t > \hat{y}_m$, where

$$\hat{y}_m = \frac{k_{S,m} + r_m^*(k_{L,m}) - \mu - r_{d,m}}{\lambda + r_m^*(k_{L,m})}$$
(12)

To determine the value of $r_{d,m}$, we assume the depositors are risk-neutral and they would request $r_{d,m}$ to cover their expected loss. Thus, we can formulate:

$$r_{d,m}F(\hat{y}_m) + [1 - F(\hat{y}_m)](q-1) = 0$$
(13)

where the function $F(\cdot)$ is defined in (1). The first part of Equation (13) is the depositors' income due to the deposit rate premium if the small bank does not fail, and the second part is the depositors' (negative) income if the small bank fails. It is not possible to give an explicit solution to Equation (13) because $F(\hat{y}_m)$ also depends on $r_{d,m}$. However, we can present the following proposition for $r_{d,m}$:

Proposition 2: Equation (13) has at most two solutions for $r_{d,m}$, and under some circumstances, there can be one or no solution. If there are two solutions, the smaller value is adopted as the solution. If there is no solution, the value $r_{d,m} = k_{S,m} + r_m^*(k_{L,m}) - \mu$ is the maximum feasible rate the small bank could offer to depositors.

We give the proof in the Appendix.

3.3.2 Quantitative Results

As in the analysis of the large bank, the results for the small bank are computed in a similar way except that the small bank takes the loan rates as given and its net worth might not be zero. To be consistent with the analysis of the large bank, we also report the case when Q = 5 and present the results in Table 3.

<Insert Table 3 here>

Table 3 shows that except for the *Laissez-faire*, capital holdings are generally higher in recessions than in booms. Credit rationing is lowered compared with that of the large bank; for

example, under the *Basel II regime* credit rationing reduces from 5.6% to 2.9%. Without the systemic tax regime, PD is generally lower in the small bank than in the large bank due to the higher ratios of capital holdings, especially during recessions. We therefore conclude on the limitation of a *one-size-fits-all* principle, and that the non-SIBs should be regulated with a lower capital requirement. In addition, one can notice that *Basel III* leads to the lowest probability of default (at 0.16%) by sacrificing social welfare, which reduces from 1.59% (*Basel I* and *Basel III*) to 1.52%. This finding is in line with the analysis of the large bank.

4. Social Welfare and Optimal Capital Requirement

We estimate the optimal capital requirements to maximise social welfare with the large and small bank, respectively. The net welfare analysis of the small bank is similar to that of the large bank, and is reported in the Online Appendix; the results of the large bank are presented below. Details of the social welfare calculation are also presented in the Appendix alongside the results of the small bank (required for the analysis).

4.1 Optimal Capital Requirements for the Large and Small Banks

4.1.1 Large Bank Capital Requirement

<Insert Figure 2 here>

Figure 2 shows the optimal capital requirements for the large bank (with and without systemic tax) as a function of bank size (Q). The calculation of the optimal capital requirements is based on the weighted unconditional probabilities of booms and recessions, and the results are a set of optimal capital requirements (γ_l^*, γ_h^*) that maximises *SW*, the definition of which is in (A3). When there is no systemic tax, the optimal capital requirements for the large bank are around 5.7% and 3.6%, for recessions and booms respectively. These results are very close to the calibrations in the *Basel II regime*, and are similar to those of Repullo and Suarez (2013),

although we set c = 0.20 in our calibration¹³. This result implies that the *Basel II regime* might be the closest to the optimal in terms of the contribution to social welfare, similar to our conclusion made in Section 3.2.3.

With the implementation of the systemic tax, the optimal capital requirements (for different bank sizes) during booms range from 1.2% to 1.6% while the requirements vary from 3.8% to 4.3% for recessions. This indicates that after the implementation of systemic tax, the optimal capital requirements can be lowered, both for booms and recessions, although the requirements are more cyclically varying. The variation increases from 2.1% (without the systemic tax) to 2.8%, after the implementation of the tax.

<Insert Table 4 here>

The PD rises in booms, due to the reduced ratio of capital (from 7.2% to 6.6%). However, in recessions, the unconditional PD slightly decrease from 0.51% to 0.47%, because of the higher capital ratios formed. The (unconditional) credit rationing drops from 4.90% to around 1.41% because of the lowered (optimal) capital requirements for booms and recessions and the increased capital holdings in recessions. The reduction in credit rationing contributes to higher values of the (unconditional) social welfare.

4.1.2 Small Bank Capital Requirement

We finish this section by discussing the optimal capital requirements for the small bank¹⁴. The requirements are around 0.2% and 3.4% for booms and recessions, respectively. Compared with the results of the large bank, we can notice that the small bank might need a lower capital

¹³ In Repullo and Suarez (2013), the pairs of optimal capital requirements in recessions and booms are around (4.8%, 2.5%) and (5.7%, 3.6%) when bankruptcy costs are set at 0.20 and 0.30, respectively. Our estimations of (5.7%, 3.6%) are coincidently same as their estimation when the costs are 0.30. The reason behind this result is the fact that we consider the contagion effects of the large bank, which makes the actual costs of the large bank's bankruptcy very close to 0.30.

¹⁴ Note that the bank size Q has a marginal effect on the small bank's optimal capital requirement; we thus present the detailed results in the Online Appendix (Table OA1) and interpret the results in this sub-section.

requirement, which means the regulators should set higher capital requirements for the large bank (the SIBs). This result is in line with the current Basel III Accords, which requires an additional capital buffer of 1%–2.5% for SIBs.

4.1.3 Large and Small Bank Capital Requirement

Combining the results from Section 4.1.1 and Section 4.1.2, we find that small banks, i.e., small commercial banks should be regulated with lower (optimal) capital requirements; this is in line with the current Basel III Accords. We also find that large banks might need a less time-varying optimal capital requirement (from 3.6% to 5.7% for situations changes from booms to recessions, with difference 2.1%) than small banks (from 0.2% to 3.4% for situations changes from booms to recessions, with difference 3.2%). The reason might be a noticeably higher requirement set for booms (3.6%) on large banks to ensure their healthy financial condition even in expansionary periods. Thus, a less time-varying capital requirement is needed for large banks to ensure their stability across all economic situations.

5. Concluding Remarks

In this paper, we analyse the impact of a systemic tax on banks' cyclical capital holding behaviours and on social welfare, and estimate the optimal capital requirements for SIBs and non-SIBs. We find merits of the systemic tax in reducing the probability of banks' failure, mitigating pro-cyclical effects of capital regulation and contributing to social welfare , although the use of this tax could result in a higher loan rate, causing a negative effect for borrowers. Moreover, these effects of the systemic tax are less significant when the capital requirements are stricter as in Basel III compare to the earlier regulatory rules. In addition, we analyse the capital requirements under different Basel Accords and present a set of optimal capital requirements in terms of the maximisation of social welfare. We conclude that Basel II rules might be the closest to the optimal level of capital requirements, while the enhanced

capital requirement, introduced by Basel III, would result in a safer at the sacrifice of social welfare. Our findings also indicate that regulators should set higher capital requirements for SIBs. This is in line with Basel III, which requires an additional capital buffer for SIBs. Policy-wide, the systemic tax can be seen as an additional tool available to policy makers to enhance financial stability, although it might be ineffective during current period when the capital requirement levels are high enough and alleviate the negative effects of the excessive risk-taking of financial institutions and mitigate the procyclicality of capital regulation. However, this does not necessarily imply that the government should impose such taxes as other revealed influence might arise, such as higher borrowing costs of firms. Rather, the government should consider the tax as a complementary instrument to other policy measures aimed at maintaining financial stability and protecting households and firms from financial crisis-induced losses.

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Appendix

Proof of Equation (1)

The use of (1) involves a single risk factor which governs the default of each project. Suppose the project undertaken by entrepreneur *i* fails if $y_i < 0$, where

$$y_i = \psi_M + \sqrt{\rho}u + \sqrt{1 - \rho}\varepsilon_i \tag{A1}$$

where ψ_M depends on the economic state of T = t, t + 1 and determines the mean value of y_i . u is the single common risk factor, and ε_i is an idiosyncratic risk factor. Both of u and ε_i are standard normal random variables, and are independent of each other, and across time and across projects. We can rewrite (A1) as $y_i = \psi_M + \sqrt{\rho u} + \sqrt{1 - \rho}\varepsilon_i = \psi_M + Z_{u,\varepsilon_i}$, where $Z_{u,\varepsilon_i} \sim N(0,1)$. Thus, the expected probability of default of the project of entrepreneur i in state M is $p_M = \Pr(y_i < 0) = \Pr(\psi_M + Z_{u,\varepsilon_i} < 0) = \Phi(-\psi_M)$. This implies $\psi_M = -\Phi^{-1}(p_M)$. By the law of large numbers, the effects of idiosyncratic factors ε_i will be diversified away, which means the fraction of the defaulted loans x_T equals the probability of default of a representative project, which depends on the economic state (M) in period T and the common risk factor u. We can thus express x_T as:

$$x_T = f_M(u) = \Pr\left[-\Phi^{-1}(p_M) + \sqrt{\rho}u + \sqrt{1-\rho}\varepsilon_i < 0|u\right] = \Phi\left[\frac{\Phi^{-1}(p_M) - \sqrt{\rho}u}{\sqrt{1-\rho}}\right].$$

If we solve for $F_M(x_T) = \Pr[f_M(u) \le x_T]$, we can obtain:

$$F_M(x_T) = \Pr[f_M(u) \le x_T] = \Pr\left[\Phi\left(\frac{\Phi^{-1}(p_M) - \sqrt{\rho}u}{\sqrt{1-\rho}}\right) \le x_T\right] = \Pr\left[\frac{\Phi^{-1}(p_M) - \sqrt{\rho}u}{\sqrt{1-\rho}} \le \Phi^{-1}(x_T)\right] = \Pr\left[\frac{\Phi^{-1}(p_M) - \sqrt{1-\rho}\Phi^{-1}(x_T)}{\sqrt{\rho}} \le u\right]$$

Use the characteristics of the standard normal distribution, we have:

$$F_M(x_T) = \Pr\left[u \le \frac{\sqrt{1-\rho}\Phi^{-1}(x_T) - \Phi^{-1}(p_M)}{\sqrt{\rho}}\right] = \Phi\left[\frac{\sqrt{1-\rho}\Phi^{-1}(x_T) - \Phi^{-1}(p_M)}{\sqrt{\rho}}\right]$$

which is Equation (1).

Rationale and Proof of Equation (9), (10) and (11)

The proof of (9) requires the use of Equation (1)**Error! Reference source not found.**, which we have proved before. According to Basel II, the capital requirements in state m' is $\lambda F_{m'}^{-1}(0.999)$, where λ is the loss given default and $F_{m'}^{-1}(0.999)$ is the 99.9% quantile of the default rate. This assumption implies that the ratio of the Basel II-style capital requirements should be set up to cover the loss due to the default of the loans, with a confidence interval of 99.9%. However, this requirement is set for the requirements at the ratio of 8% (including Tier 1 and Tier 2 capital), thus we divide it by 2 to reflect the fact that in our analysis the capital requirements are set for Tier 1 capital (at 4%) only, and banks are assumed to hold an equal amount of Tier 1 and Tier 2 capital. Therefore, the requirements should be set at $\lambda F_{m'}^{-1}(0.999)/2$ To find $F_{M}^{-1}(0.999)$, we should find the \tilde{x}_{T} that would make $F_{M}(\tilde{x}_{T}) = 0.999$, which means:

$$\Phi\left[\frac{\sqrt{1-\rho(p_M)}\Phi^{-1}(\tilde{x}_T)-\Phi^{-1}(p_M)}{\sqrt{\rho(p_M)}}\right] = 0.999$$

After a rearrangement of above equation, we can obtain:

$$\tilde{x}_{T} = \Phi\left[\frac{\Phi^{-1}(p_{M}) + \sqrt{\rho(p_{M})}\Phi^{-1}(0.999)}{\sqrt{1 - \rho(p_{M})}}\right]$$

Therefore, the capital requirements $\gamma_M = \lambda F_M^{-1}(0.999)/2$ should be expressed as:

$$\gamma_{M} = \frac{\lambda}{2} \Phi \left[\frac{\Phi^{-1}(p_{M}) + \sqrt{\rho(p_{M})} \Phi^{-1}(0.999)}{\sqrt{1 - \rho(p_{M})}} \right]$$

which is Equation (9).

In terms of Equation (10), we refer to the calculation of the paragraph 272 of BCBS (2004). Following Repullo and Suarez (2013), we also parameterise $\rho(p_m)$ as a constant ρ by setting it equal to the average of $\rho(p_m)$, with the weights as the unconditional probabilities of each state m.

To align our analysis with Basel III, which requires Tier 1 capital be at least at the ratio of 6%, we apply a multiplier of 1.5 in Equation (9) to make the requirements equal to 6% in the calibration. After applying the multiplier in Equation (9), we have Equation (11).

Proof of Proposition 1

The large Bank shareholders' net worth function is

$$v_{L,m}(k_{L,m}, r_m) = s_{ml}v_{L,ml}(k_{L,m}, r_m) + s_{mh}v_{L,mh}(k_{L,m}, r_m)$$

After using Equations (4), (5), (6) and (7), and making necessary rearrangements, we obtain:

$$v_{L,m}(k_{L,m}, r_m) = \frac{1}{1+\delta} \left\{ \int_0^{\hat{x}_{mm'}} \left[\pi_{m'} + k'_{L,m}(x_t) - \gamma_{L,m'} \right] dF_m(x_t) + \frac{1}{\gamma_{L,m'}} \int_{\hat{x}_{mm'}}^{\hat{x}_m} \pi_{m'} k'_{L,m}(x_t) dF_m(x_t) \right\} - k_{L,m} - \frac{dQ + \varphi}{1+Q} \left[1 - F(\hat{x}_m) \right]$$

Using the definition of terms above in their corresponding equations, we can derive the following properties of $v_{L,m}(k_{L,m}, r_m)$ with respect to $k_{L,m}$.

1) For $k_{L,m} \le \mu - r_m$, we have $\hat{x}_{mm'} < \hat{x}_m \le 0$, which means the large bank will fail, so

$$\frac{\partial v_{L,m}}{\partial k_{L,m}} = -1 < 0 \tag{A2}$$

Equation (A2) indicates that there might be a corner solution at $k_{L,m} = 0$ but the solution will not be at $k_{L,m} = \mu - r_m$.

2) For $\mu - r_m < k_{L,m} \le \mu + \gamma_{L,m'} - r_m$, we have $\hat{x}_{mm'} \le 0 < \hat{x}_m$, which means the large bank might fail but will be subject to credit rationing, so

$$\frac{\partial v_{L,m}}{\partial k_{L,m}} = \frac{1}{1+\delta} \left[\frac{\pi_{m'}}{\gamma_{L,m'}} F(\hat{x}_m) + \frac{(1+\delta)(dQ+\varphi)}{(1+Q)(\lambda+r_m)} f(\hat{x}_m) \right] - 1 \le 0$$
(A3)

Equation (A3) indicates that there might exist an interior solution within $k_{L,m} \in (\mu - r_m, \mu + \gamma_{L,m} - r_m]$; however, comparing (A3) with the case with no systemic tax, that is $d = \varphi = 0$, we can notice that (A3) is larger than $\pi_{m'}F(\hat{x}_m)/[(1+\delta)\gamma_{L,m'}] - 1$ due to the fact that $f(\hat{x}_m) > 0$. Hence, to make (A3) zero, keeping all parameters constant, $F(\hat{x}_m)$ will be lower compared with the case without systemic tax. Because F(x) is an increasing function, the lowered $F(\hat{x}_m)$ means a lowered value of \hat{x}_m . To maintain a higher level of $k_{L,m}$, the loan rate r_m will be raised to satisfy the decrease in \hat{x}_m because other parameters are constant throughout our analysis.

3) For $\mu + \gamma_{L,m'} - r_m < k_{L,m} \le \mu + \gamma_{L,m'}$, we have $0 \le \hat{x}_{mm'} < \hat{x}_m < 1$, which means the large bank might fail and could be subject to credit rationing, so

$$\begin{split} \frac{\partial v_{L,m}}{\partial k_{L,m}} &= \frac{1}{1+\delta} \left\{ f(\hat{x}_{mm'}) \frac{\pi_{m'}}{\lambda + r_m} + F(\hat{x}_{mm'}) - f(\hat{x}_{mm'}) \frac{\pi_{m'}}{\lambda + r_m} + \frac{\pi_{m'}}{\gamma_{L,m'}} \left[F(\hat{x}_m) - F(\hat{x}_{mm'}) \right] + \frac{dQ + \varphi}{1 + Q} f(\hat{x}_m) \frac{1 + \delta}{\lambda + r_m} \right\} - 1 \\ &= \frac{1}{1+\delta} \left\{ F(\hat{x}_{mm'}) + \frac{\pi_{m'}}{\gamma_{L,m'}} \left[F(\hat{x}_m) - F(\hat{x}_{mm'}) \right] + \frac{(1 + \delta)(dQ + \varphi)}{(1 + Q)(\lambda + r_m)} f(\hat{x}_m) \right\} - 1 \le 0 \end{split}$$

Similar to 2), there might exist an interior solution within the interval of $k_{L,m} \in (\mu + \gamma_{L,m'} - r_m, \mu + \gamma_{L,m'}]$. Due to the fact that $f(\hat{x}_m) > 0$, the implementation of systemic tax regimes will result in lowered values of $F(\hat{x}_m)$ and $F(\hat{x}_{mm'})$ for an interior solution. As a result, the loan rate r_m will be raised to satisfy this change.

4) For $\mu + \gamma_{L,m'} < k_{L,m} \le \mu + \gamma_{L,m'} + \lambda$, we have $0 \le \hat{x}_{mm'} < 1 < \hat{x}_m$, which means the large bank will not fail but might be subject to credit rationing, so

$$\begin{aligned} \frac{\partial v_{L,m}}{\partial k_{L,m}} &= \frac{1}{1+\delta} \left\{ f(\hat{x}_{mm'}) \frac{\pi_{m'}}{\lambda + r_m} - \frac{\pi_{m'}}{\lambda + r_m} f(\hat{x}_{mm'}) + F(\hat{x}_{mm'}) + \frac{\pi_{m'}}{\gamma_{L,m'}} [1 - F(\hat{x}_{mm'})] \right\} - 1 \\ &= \frac{1}{1+\delta} \left\{ F(\hat{x}_{mm'}) + \frac{\pi_{m'}}{\gamma_{L,m'}} [1 - F(\hat{x}_{mm'})] \right\} - 1 \leq 0 \end{aligned}$$

There can exist an interior solution within the interval of $k_{L,m} \in (\mu + \gamma_{L,m'}, \mu + \gamma_{L,m'} + \lambda]$. However, the implementation of systemic tax regimes will not result in lowered values of $F(\hat{x}_{mm'})$ for an interior solution as there is no factor of the systemic tax to affect the choice of the interior solution.

5) For $k_{L,m} \ge \mu + \gamma_{L,m'} + \lambda$, we have $1 \le \hat{x}_{mm'} < \hat{x}_m$, which means the large bank will not fail and is not subject to credit rationing, so

$$\frac{\partial v_{L,m}}{\partial k_{L,m}} = \frac{1}{1+\delta} - 1 < 0$$

We can notice that there will be no corner solution at $k_{L,m} = 1$, assuming $\mu + \gamma_{L,m'} + \lambda < 1$ (in our calibration the maximum value of $\mu + \gamma_{L,m'} + \lambda = 0.56$).

We can conclude that, there cannot be a solution at the corner $k_{L,m} = 1$, while might have a possibility of obtaining a corner solution at the corner of $k_{L,m} = 0$. If the solution is interior, namely $0 < k_{L,m} < 1$, and the probability of default of the large bank is strictly positive, the implementation of a systemic tax will result in a higher loan rate r_m if a higher capital holding is formed due to the systemic tax.

Proof of Proposition 2

Equation (13) shows that $r_{d,m}F(\hat{y}_m) + [1 - F(\hat{y}_m)](q - 1) = 0$. After rearranging this equation we can obtain

$$F(\hat{y}_m) = \frac{1-q}{r_{d,m}-q+1}$$

From Equations (1) and (12), we can show that

$$\Phi\left[\frac{\sqrt{1-\rho}\Phi^{-1}\left(\frac{k_{S,m}+r_m^*(k_{L,m})-\mu-r_{d,m}}{\lambda+r_m^*(k_{L,m})}\right)-\Phi^{-1}(p)}{\sqrt{\rho}}\right] = \frac{1-q}{r_{d,m}-q+1}$$

Adding $\Phi^{-1}(\cdot)$ to both sides of the above equation, we obtain

$$\frac{\sqrt{1-\rho}}{\sqrt{\rho}} \Phi^{-1} \left(\frac{k_{S,m} + r_m^*(k_{L,m}) - \mu - r_{d,m}}{\lambda + r_m^*(k_{L,m})} \right) = \Phi^{-1} \left(\frac{1-q}{r_{d,m} - q + 1} \right) + \frac{\Phi^{-1}(p)}{\sqrt{\rho}}$$

Next, we assume the function $X(r_{d,m})$ as

$$X(r_{d,m}) = \Phi^{-1}\left(\frac{1-q}{r_{d,m}-q+1}\right) + \frac{\Phi^{-1}(p)}{\sqrt{\rho}} - \frac{\sqrt{1-\rho}}{\sqrt{\rho}} \Phi^{-1}\left(\frac{k_{S,m}+r_m^*(k_{L,m})-\mu-r_{d,m}}{\lambda+r_m^*(k_{L,m})}\right)$$

Thus, our aims turn to find the solutions to make $X(r_{d,m}) = 0$. Making differentiation to $X(r_{d,m})$ in terms of $r_{d,m}$, we can show that

$$\frac{dX(r_{d,m})}{dr_{d,m}} = \frac{1}{a+\lambda} \frac{\sqrt{1-\rho}}{\sqrt{\rho}} \frac{d\Phi^{-1} \left(\frac{k_{S,m} + r_m^*(k_{L,m}) - \mu - r_{d,m}}{\lambda + r_m^*(k_{L,m})}\right)}{d\left(\frac{k_{S,m} + r_m^*(k_{L,m}) - \mu - r_{d,m}}{\lambda + r_m^*(k_{L,m})}\right)} - \frac{1-q}{(r_{d,m} - q + 1)^2} \frac{d\Phi^{-1} \left(\frac{1-q}{r_{d,m} - q + 1}\right)}{d\left(\frac{1-q}{r_{d,m} - q + 1}\right)}$$

It is straightforward to show that $\frac{d\Phi^{-1}(z)}{dz}$ is always positive because $\Phi^{-1}(z)$ is an increasing function. Additionally, we notice $r_{d,m}$ can only range from 0 to $k_{S,m} + r_m^*(k_{L,m}) - \mu$ because the definition domain of $\Phi^{-1}(x)$ is from 0 to 1. When $r_{d,m}$ increases from zero, $\frac{dX(r_{d,m})}{dr_{d,m}}$ is negative infinity as $\frac{d\Phi^{-1}(z)}{dz}$ is positive infinity when z approaches 1, and when $r_{d,m}$ approaches $k_{S,m} + r_m^*(k_{L,m}) - \mu$, $\frac{dX(r_{d,m})}{dr_{d,m}}$ is positive infinity because $\frac{d\Phi^{-1}(z)}{dz}$ is also infinity when z approaches 0. We can conclude that when $r_{d,m}$ changes from 0 to $k_{S,m} + r_m^*(k_{L,m}) - \mu$, $\frac{dX(r_{d,m})}{dr_{d,m}}$ changes from 0 to $k_{S,m} + r_m^*(k_{L,m}) - \mu$, $\frac{dX(r_{d,m})}{dr_{d,m}}$ changes from 0 to $k_{S,m} + r_m^*(k_{L,m}) - \mu$, $\frac{dX(r_{d,m})}{dr_{d,m}}$ changes from negative infinity to positive infinity. Thus, the function $X(r_{d,m})$ is a U-shaped curve and it reaches its minimum level when $\frac{dX(r_{d,m})}{dr_{d,m}} = 0$. It is also easy to notice that when $r_{d,m} = 0$ and $r_{d,m} = k_{S,m} + r_m^*(k_{L,m}) - \mu$, $X(r_{d,m})$ is positive infinity. Namely, when $r_{d,m}$ increases from 0 to $k_{S,m} + r_m^*(k_{L,m}) - \mu$, $X(r_{d,m})$ starts from positive infinity; decreases to its minimum; increases back to positive infinity. Thus, for appropriate value sets, the minimum of $X(r_{d,m})$ can be negative, resulting in two solutions, and we choose the smaller value of $r_{d,m}$ for the deposit rate premium. However, if q or $k_{S,m}$ is too small, making $X(r_{d,m})$ high above zero, there will exist no solutions to make $X(r_{d,m})$ zero. Under this circumstance, we will let $r_{d,m} = k_{S,m} + r_m^*(k_{L,m}) - \mu$; because if q or $r_{d,m}$ is too small, the depositors will find the banks are under larger exposure and thus they will require the highest feasible deposit rate premium from the bank.

Small Bank's shareholder net present value

For the small bank's analysis, due to it lower systemic importance, it will not be levied a systemic tax, and thus the small bank's shareholder net present value is as follows:

$$v_{S,m}[k_{S,m}, r_m^*(k_{L,m})] = \frac{1}{1+\delta} E[v_{mm'}(y_t)] - k_{S,m}$$

where we denote y_t and y_{t+1} as the fraction of failed loans held by the small bank to distinguish it from that of the large bank. Note that following our assumption the small bank take the loan rate which is determined by the

large bank as given, and we denote $r_m^*(k_{L,m})$ as the loan rate to indicate that it is an exogenous value to the small bank. The term $v_{mm'}(y_t)$ can be summarised as:

$$v_{mm'}(y_t) = \begin{cases} \pi_{Sm'} + k'_{S,m}(y_t) - \gamma_{S,m'} & \text{if } y_t < \hat{y}_{mm'} \\ \pi_{Sm'} \frac{k'_{S,m}(y_t)}{\gamma_{S,m'}} & \text{if } \hat{y}_{mm'} < y_t < \hat{y}_m \\ 0 & \text{if } y_t > \hat{y}_m \end{cases}$$

where

$$\pi_{Sm'} = \frac{1}{1+\delta} \int_0^1 \max\{\gamma_{S,m'} + a - (\lambda + a)y_{t+1}, 0\} dF_{m'}(y_{t+1})$$

and

$$k'_{S,m}(y_t) = k_{S,m} + r_m^*(k_{L,m}) - [r_m^*(k_{L,m}) + \lambda]y_t - \mu - r_{d,m}$$

where $r_{d,m}$ denotes the deposit rate premium in state *m*. Additionally, we can get

$$\hat{y}_m = \frac{k_{S,m} + r_m^*(k_{L,m}) - \mu - r_{d,m}}{\lambda + r_m^*(k_{L,m})}$$

and

$$\hat{y}_{mm\prime} = \frac{k_{S,m} + r_m^*(k_{L,m}) - \mu - \gamma_{S,m\prime} - r_{d,m}}{\lambda + r_m^*(k_{L,m})}$$

The small bank is to choose the optimal ratio of the capital holding $k_{s,m}$ to maximise $v_{s,m}[k_{s,m}, r_m^*(k_{L,m})]$. The credit rationing of the small bank is as follows

$$CR_{s,mm'} = [1 - F(\hat{y}_m)] + \int_{\hat{y}_{mm'}}^{\hat{y}_m} \left[1 - \frac{k'_s(y_t)}{\gamma_{s,m'}}\right] F_m(y_t)$$

Social welfare analysis

In our model, social welfare can be measured by the sum of the expected net present value gained from the participants of the economy. The overall social welfare, $SW_{mm'}$, can be written as

$$SW_{mm'} = E_{mm'} + GI_{mm'} + FC_{mm'}$$
(A4)

where

$$E_{mm'} = \frac{Q}{Q+1} \{ (1-p_m) \left[a - r_m^* (k_{L,m}) + b \right] + \left(1 - CR_{L,mm'} \right) (1-p_m) b \} + \frac{1}{Q+1} (1-p_m) \left[a - r_m^* (k_{L,m}) \right]$$
(A5)

Equation (A5) presents the payoffs to the entrepreneurs over the two periods, and the term GI_{mm} , denotes the payoffs to the governments and the depositors in case of the bank failure (note the deposits are partially insured). The term $GI_{mm'}$ can be written as follows:

$$GI_{mm'} = T_m + \frac{Q}{Q+1} \left\{ \int_{\hat{x}_m}^1 k'_{L,m}(x_t) dF_m(x_t) + (1 - CR_{L,mm'}) \int_{\hat{x}_{m'}}^1 [\gamma_{L,m'} + a - (\lambda + a)x_{t+1}] dF_{m'}(x_{t+1}) \right\} \\ - \frac{dQ + \varphi}{Q+1} \left[1 - F(\hat{x}_m) + (1 - CR_{L,mm'}) [1 - F(\hat{x}_{m'})] \right] + \frac{1}{Q+1} \{r_{d,m}F(\hat{y}_m) + (q-1)[(1 - k_{S,m})[1 - F(\hat{y}_m)] + (1 - CR_{S,mm'})(1 - \gamma_{S,m'})[1 - F(\hat{y}_{m'})]] \} \\ + \frac{1}{Q+1} GS_{mm'}$$
(A6)

The first term of (A6) represents the (positive) payoff to the government as the tax levier. The second and third terms of (A6) is the (negative) payoff to the government, as a deposit insurer and a rescuer in the bailout, and depositors, for the failure of the large bank. Observe that $\hat{x}_{m'} = (\gamma_{L,m'} + a)/(\lambda + a)$ indicates the critical value of the default of the large bank in period t + 1. The fourth and fifth terms of (A6) denote the (negative) payoff to the government (insurer of the (partial of) deposits) and the depositors (for being under a partial deposit insurance) for the failure of the small bank. It includes the payoff to the small bank's depositors: deposit rate premium $r_{d,m}$ if the bank succeeds after period t and q - 1 if the bank fails in period t and period t + 1 respectively and the payoff $GS_{mm'}/(Q + 1)$ to the government as the insurer of the (partial of) deposits, where:

$$GS_{mm'} = \int_{\hat{y}_m}^{1} [k_{S,m} + r_m^*(k_{L,m}) - [r_m^*(k_{L,m}) + \lambda]y_t - \mu] dF_m(y_t) + (1 - q)(1 - k_{S,m})[1 - F(\hat{y}_m)] + (1 - CR_{S,mm'}) \{\int_{\hat{y}_{m'}}^{1} [\gamma_{S,m'} + a - (a + \lambda)y_{t+1}] dF_{m'}(y_{t+1}) + (1 - q)(1 - \gamma_{S,m'})[1 - F(\hat{y}_{m'})]\}$$

where $\hat{y}_{m'} = (\gamma_{S,m'} + a)/(\lambda + a)$ denotes the critical value of the default of the small bank in period t + 1. The detailed calculation of $GS_{mm'}$ and a simplification of (A6) will be presented in Proposition 3.

Additionally, in Equation (A4):

$$FC_{mm\prime} = -\frac{c}{Q+1} \Big[1 - F(\hat{y}_m) + \big(1 - CR_{s,mm\prime} \big) \Big[1 - F\big(\hat{\hat{y}}_{m\prime}\big) \Big] \Big]$$

is the negative payoff to the social cost due to the small bank's failure. Note that FC_{mm} , does not include the bankruptcy cost of the large bank, which is covered by the government in the bailout. The parameter *c* indicates the proportional bankruptcy cost, the value of which is reported in Table 1.

Proposition 3

After simplifying Equation (A6) we get obtain the following

$$GI_{mm'} = \frac{Q}{Q+1} \left\{ \int_{\hat{x}_m}^1 k'_{L,m}(x_t) dF_m(x_t) + (1 - CR_{L,mm'}) \int_{\hat{x}_{m'}}^1 [\gamma_{L,m'} + a - (\lambda + a)x_{t+1}] dF_{m'}(x_{t+1}) \right\} - \frac{dQ + \varphi}{Q+1} (1 - CR_{L,mm'}) [1 - F(\hat{x}_{m'})] + \frac{r_{d,m}F(\hat{y}_m)}{Q+1} + \frac{1}{Q+1} \left\{ \int_{\hat{y}_m}^1 \tilde{k}'_S(y_t) dF_m(y_t) + (1 - CR_{S,mm'}) \int_{\hat{y}_{m'}}^1 [\gamma_{S,m'} + a - (\lambda + a)y_{t+1}] dF_{m'}(y_{t+1}) \right\}$$
(A7)

where

$$\tilde{k}'_{S,m}(y_t) = k_{S,m} + r_m^*(k_{L,m}) - [\lambda + r_m^*(k_{L,m})]y_t - \mu$$

Proof of Proposition 3

We have assumed that when the small bank fails, the government will take over it, and repay the depositors the promised proportion of q of their deposits. Thus, in period t, the government's payoff for taking the failed bank is

$$GS_m^t = \int_{\hat{y}_m}^1 \{k_{S,m} + r_m^*(k_{L,m}) - [r_m^*(k_{L,m}) + \lambda]y_t - \mu + (1 - k_{S,m})(1 - q)\} dF_m(y_t)$$

Notice that in the case of bankruptcy, the small bank is not responsible for paying the deposit rate premium $r_{d,m}$, and it is thus dropped out. The above equation shows the negative payoff to the government for period t. It is clear that when the small bank fails but the loss is not significant, i.e. when y_t ranges from \hat{y}_m to \bar{y}_m where $\bar{y}_m = [k_{S,m} + r_m^*(k_{L,m}) - \mu + 1 - q]/[r_m^*(k_{L,m}) + \lambda]$, the bank still has some positive revenues due to the partial deposit insurance regime. We can simplify the above equation and yield the following:

$$GS_m^t = \int_{\hat{y}_m}^1 \{k_{S,m} + r_m^*(k_{L,m}) - [r_m^*(k_{L,m}) + \lambda]y_t - \mu\} dF_m(y_t) + (1 - q)(1 - k_{S,m})[1 - F(\hat{y}_m)]$$

Similarly, in period t + 1, we can obtain:

$$GS_{mm'}^{t+1} = \left(1 - CR_{S,mm'}\right) \left\{ \int_{\hat{y}_{m'}}^{1} [\gamma_{S,m'} + a - (a+\lambda)y_{t+1} + (1-q)(1-\gamma_{S,m'})] dF_{m'}(y_{t+1}) \right\}$$

which can be simplified to:

$$GS_{mm'}^{t+1} = \left(1 - CR_{S,mm'}\right) \left\{ \int_{\hat{y}_{m'}}^{1} [\gamma_{S,m'} + a - (a+\lambda)y_{t+1}] dF_{m'}(y_{t+1}) + (1-q)\left(1 - \gamma_{S,m'}\right) \left[1 - F(\hat{y}_{m'})\right] \right\}$$

Thus, the value of $GS_{mm'}$ is the sum of GS_m^t and $GS_{mm'}^{t+1}$, which means:

$$GS_{mm'} = GS_m^t + GS_{mm'}^{t+1}$$

Thus, and in addition use the definition of T_m , we can simplify the Equation (A6):

$$GI_{mm'} = T_m + \frac{Q}{Q+1} \left\{ \int_{\hat{x}_m}^1 k'_{L,m}(x_t) \, dF_m(x_t) + \left(1 - CR_{L,mm'}\right) \int_{\hat{x}_{m'}}^1 \left[\gamma_{L,m'} + a - (\lambda + a)x_{t+1} \right] dF_{m'}(x_{t+1}) \right\}$$

$$- \frac{dQ + \varphi}{Q+1} \left[1 - F(\hat{x}_m) + \left(1 - CR_{L,mm'}\right) \left[1 - F(\hat{x}_{m'}) \right] \right] + \frac{1}{Q+1} \{ r_{d,m}F(\hat{y}_m) + (q-1) \left[(1 - k_{s,m}) \left[1 - F(\hat{y}_m) \right] + (1 - CR_{s,mm'}) (1 - \gamma_{s,m'}) \left[1 - F(\hat{y}_{m'}) \right] \right\}$$

$$+ \frac{1}{Q+1} GS_{mm'}$$

to

$$GI_{mm'} = \frac{Q}{Q+1} \left\{ \int_{\hat{x}_m}^1 k'_{L,m}(x_t) dF(x_t) + (1 - CR_{L,mm'}) \int_{\hat{x}_{m'}}^1 [\gamma_{L,m'} + a - (\lambda + a)x_{t+1}] dF_{m'}(x_{t+1}) \right\}$$

$$- \frac{dQ + \varphi}{Q+1} (1 - CR_{L,mm'}) [1 - F(\hat{x}_{m'})] + \frac{r_{d,m}F(\hat{y}_m)}{Q+1}$$

$$+ \frac{1}{Q+1} \left\{ \int_{\hat{y}_m}^1 [k_{S,m} + r_m^*(k_{L,m}) - [\lambda + r_m^*(k_{L,m})]y_t - \mu] dF_m(y_t) \right\}$$

$$+ (1 - CR_{S,mm'}) \left[\int_{\hat{y}_{m'}}^1 [\gamma_{S,m'} + a - (\lambda + a)y_{t+1}] dF_{m'}(y_{t+1}) \right] \right\}$$

If we replace $k_{s,m} + r_m^*(k_{L,m}) - [\lambda + r_m^*(k_{L,m})]y_t - \mu$ with $\tilde{k}'_{s,m}(y_t)$, we can obtain Equation (A7).



Figure 1

US bank credit to total assets for large and small (domestically chartered) commercial banks

The figure shows the US market statistics, which is collected from the Federal Reserve Bank of St. Louis database. The data is calibrated monthly and ranges from January 1988 to January 2020. The shaded areas are the recession periods, defined by National Bureau of Economic Research (NBER) recession data. The recession periods are from July 1990 to March 1991, March 2001 to November 2001 and December 2007 to June 2009.



| Parameter | Description | Value | Source or Target |
|-----------------|--|-------|--|
| а | Annual return of successful projects | 0.04 | FDIC Statistics on US commercial banks |
| λ | Loss given default (LGD) | 0.45 | Basel II on IRB approach |
| μ | Unit setup cost | 0.03 | Loan spreads of 100 basis points in booms |
| δ | Required return of shareholders | 0.08 | Spreads of cost of capital of Tier 1 and 2 |
| p_l | Probability of default in booms | 0.010 | Nonperforming loan ratio in US banks |
| p_h | Probability of default in recessions | 0.036 | Nonperforming loan ratio in US banks |
| ρ | Correlation parameter | 0.174 | Weighted values as in Equation (10) |
| С | Unit bankruptcy cost | 0.20 | Repullo (2013), De Nicolo, Gamba, and Lucchetta (2014) |
| d | Recovery cost of the large bank | 0.20 | Bankruptcy cost of the small bank as a lower bound |
| φ | Contagion effects of the large bank | 0.40 | Spill-over effects of systemically important banks |
| s _{ll} | Transition probabilities of booms | 0.80 | Duration of booms of 5 years |
| S _{hh} | Transition probabilities of recessions | 0.64 | Duration of recessions of 2.8 years |
| ϕ_l | Probabilities of booms | 0.643 | Duration of booms of 5 years |
| ϕ_h | Probabilities of recessions | 0.357 | Duration of recessions of 2.8 years |

| Table | 2 |
|-------|---|
|-------|---|

Large Bank: Loan rate, capital buffers, systemic tax under different regulatory regimes, with and (without) systemic tax (all variables in %)

| | Laissez-faire | Basel I | Basel II | Basel III |
|--|----------------------------|-------------|-------------|-------------------------|
| Bank Size: Q=5 | | | | |
| Loan rate in state <i>m</i> | | | | |
| $r_l(r_l')$ | 1.0 (0.8) | 1.4 (1.4) | 1.4 (1.4) | 1.7 (1.7) |
| $r_h(r'_h)$ | 3.1 (2.5) | 3.5 (3.2) | 3.6 (3.3) | 3.8 (3.7) |
| | | | | |
| Capital holdings in state <i>m</i> | | | | |
| $k_{L,l} \left(k_{L,l}' \right)$ | 6.3 (4.2) | 6.9 (6.7) | 7.3 (7.0) | 8.3 (8.3) |
| $k_{L,h} \left(k_{L,h}' \right)$ | 9.4 (3.4) | 9.5 (6.1) | 9.3 (6.5) | 10.0 (8.6) |
| | | | | |
| Capital buffer in state <i>m</i> | | | | 15 |
| $\Delta_{L,l} = k_{L,l} - \gamma_l \left(\Delta'_{L,l} = k'_{L,l} - \gamma_l \right)$ | 6.3 (4.2) | 2.9 (2.7) | 4.1 (3.8) | 3.6 (3.6) ¹⁵ |
| $\Delta_{L,h} = k_{L,h} - \gamma_h \ (\Delta'_{L,h} = k'_{L,h} - \gamma_h)$ | 9.4 (3.4) | 5.5 (2.1) | 3.8 (1.0) | 1.5 (0.1) |
| | | | | |
| Systemic tax in state <i>m</i> | | | | |
| T_l | 0.08 | 0.04 | 0.07 | 0.01 |
| <i>T_h</i> | 0.16 | 0.13 | 0.14 | 0.09 |
| | | | | |
| Credit rationing in state m | 0.5 (2.0) | 1.7(2.1) | | 16(10) |
| $\frac{CR_{L,ll}(CR_{L,ll})}{CR_{L,ll}}$ | 0.5 (3.2) | 1.7(2.1) | 0.6 (0.8) | 1.6 (1.6) |
| $CR_{L,lh}(CR_{L,lh})$ | 0.5 (3.2) | 1.7 (2.1) | 5.4 (9.8) | 22.9 (22.9) |
| $CR_{L,lm'}$ ($CR'_{L,lm'}$) | 0.5 (3.2) | 1.7 (2.1) | 1.6 (2.6) | 5.9 (5.9) |
| $CR_{L,hl} (CR'_{L,hl})$ | 0.7 (17.2) | 1.6 (9.9) | 1.4 (5.9) | 2.1 (4.5) |
| $CR_{L,hh} (CR'_{L,hh})$ | 0.7 (17.2) | 1.6 (9.9) | 2.8 (13.7) | 5.3 (12.5) |
| $CR_{L,hm'}$ $(CR'_{L,hm'})$ | 0.7 (17.2) | 1.6 (9.9) | 2.3 (10.9) | 4.3 (9.6) |
| Unconditional | 0.6 (8.2) | 1.7 (4.9) | 1.8 (5.6) | 5.3 (7.2) |
| | | | | |
| Net capital buffer with tax in state <i>m</i> | | | | |
| $\alpha_{L,l} = \Delta_{L,l} - \Delta'_{L,l}$ | 1.8 | 0.6 | 0.3 | 0.0 |
| $\alpha_{L,h} = \Delta_{L,h} - \Delta'_{L,h}$ | 5.6 | 3.4 | 3.5 | 1.4 |
| | | | | |
| Probability of Bankruptcy in state <i>m</i> | | | | |
| $1 - F(\hat{x}_{l}) + (1 - CR_{L,lm'})[1 - F(\hat{x}_{m'})]$ | 2.80 | 0.47 | 0.30 | 0.09 |
| $(1 - F'(\hat{x}_l) + (1 - CR'_{L,lm'})[1 - F'(\hat{x}_{m'})])$ | (5.59) | (0.50) | (0.33) | (0.09) |
| $1 - F(\hat{x}_h) + (1 - CR_{L,hm'})[1 - F(\hat{x}_{m'})]$ | 7.31 | 1.52 | 1.10 | 0.54 |
| $\frac{(1 - F'(\hat{x}_h) + (1 - CR'_{L,hm'})[1 - F'(\hat{x}_{m'})])}{1 - F'(\hat{x}_{m'})[1 - F'(\hat{x}_{m'})]}$ | (22.76) | (3.96) | (2.90) | (0.92) |
| Unconditional (Per Year) | 2.21 (5.86) | 0.42 (0.87) | 0.29 (0.62) | 0.13 (0.19) |
| Social walfers in state w | | | | |
| Social wellare in state m | 8 55 (9 24) | 8 62 (8 50) | 8 64 (8 60) | 8 78 (0 70) |
| $\frac{SW_{L,l}(SW_{L,l})}{SW_{L,l}(SW_{L,l})}$ | 0.33 (0.24) 5 90 (2.00) | 0.02 (0.39) | 0.04 (0.00) | 0.20 (0.20) |
| $\frac{SW_{L,h}(SW_{L,h})}{1}$ | 3.80 (3.09) | 0.03 (0.07) | 0.01 (0.14) | 0.45 (0.17) |
| Unconditional | /.57 (6.40) | /.91 (7.69) | 1.92 (1.12) | /.63 (/.53) |

The term m = h stands for the states in recessions; m = l stands for booms, and the variables without and with the brackets denotes results for the cases under the systemic tax regimes and under the non-systemic tax regimes respectively. The terms labelled as *Unconditional* are the expected values for both financial situations when the likelihood of booms $\phi_l = 0.643$ and recessions $\phi_h = 1 - \phi_l = 0.357$ as shown in Table 1. The result marked as *Per Year* indicates the average probability of default of the bank across the two periods to ensure the default rate is for one year. The subscript *L* in the variables *CR* and *SW* refers to the large bank.

¹⁵ Note that 3.6% = 2.3% + 1.3%, where 1.3% is the countercyclical capital buffer required by the Basel III.

Table 3

Small Bank: Loan rate, capital buffers, systemic tax under different regulatory regimes, with and (without) systemic tax (all variables in %)

| | Laissez-faire | Basel I | Basel II | Basel III |
|--|---------------|--------------|--------------|-------------------------|
| Bank Size: Q=5 | | | | |
| Loan rate in state <i>m</i> | | | | |
| $r_l(r_l')$ | 1.0 (0.8) | 1.4 (1.4) | 1.4 (1.4) | 1.7 (1.7) |
| $r_h(r'_h)$ | 3.1 (2.5) | 3.5 (3.2) | 3.6 (3.3) | 3.8 (3.7) |
| | | | | |
| Deposit rate premium in state m | | | | |
| $r_{d,l} \left(r_{d,l}' \right)$ | 0.06 (0.06) | 0.02 (0.02) | 0.01 (0.01) | 0.00 (0.00) |
| $r_{d,h}\left(r_{d,h}'\right)$ | 0.40 (0.40) | 0.11 (0.11) | 0.11 (0.11) | 0.06 (0.06) |
| | | | | |
| Capital holdings in state <i>m</i> | | | | |
| $k_{S,l} (k'_{S,l})$ | 3.4 (3.6) | 6.7 (6.7) | 7.1 (7.1) | 8.3 (8.3) |
| $k_{S,h}(k'_{S,h})$ | 2.9 (3.4) | 8.2 (8.4) | 8.2 (8.4) | 9.1 (9.0) |
| | | | | |
| Capital buffer in state <i>m</i> | | | | |
| $\Delta_{S,l} = k_{S,l} - \gamma_l \left(\Delta_{S,l}' = k_{S,l}' - \gamma_l \right)$ | 3.4 (3.6) | 2.7 (2.7) | 3.9 (3.9) | 3.6 (3.6) ¹⁶ |
| $\Delta_{S,h} = k_{S,h} - \gamma_h \left(\Delta'_{S,h} = k'_{S,h} - \gamma_h \right)$ | 2.9 (3.4) | 4.2 (4.4) | 2.7 (2.9) | 0.6 (0.5) |
| | | | | |
| Credit rationing in state <i>m</i> | | | | |
| $CR_{S,lml}(CR'_{S,lml})$ | 7.3 (7.3) | 1.9 (2.1) | 2.2 (2.2) | 5.9 (5.9) |
| $CR_{Shm}(CR'_{Shm})$ | 23.3 (17.6) | 3.2 (3.3) | 4.1 (4.2) | 7.2 (8.0) |
| Unconditional | 13.0 (10.9) | 2.4 (2.5) | 2.9 (2.9) | 6.4 (6.7) |
| | | | | |
| Net capital buffer with tax in state <i>m</i> | | | | |
| $\alpha_{S,l} = \Delta_{S,l} - \Delta_{S,l}'$ | -0.2 | 0.0 | 0.0 | 0.0 |
| $\alpha_{Sh} = \Delta_{Sh} - \Delta'_{Sh}$ | -0.5 | -0.2 | -0.2 | 0.1 |
| She She She | | | | |
| Probability of Bankruptcy in state m | | | | |
| $1 - F(\hat{y}_{l}) + (1 - CR_{S,lm'}) [1 - F(\hat{y}_{m'})]$ | 9.60 | 0.49 | 0.32 | 0.09 |
| $(1 - F'(\hat{y}_l) + (1 - CR'_{S,lm'})[1 - F'(\hat{y}_{m'})])$ | (9.57) | (0.50) | (0.32) | (0.09) |
| $1 - F(\hat{y}_h) + (1 - CR_{S,hm'}) [1 - F(\hat{y}_{m'})]$ | 23.28 | 2.02 | 1.52 | 0.76 |
| $(1 - F'(\hat{y}_h) + (1 - CR'_{S,hm'})[1 - F'(\hat{y}_{m'})])$ | (23.09) | (2.04) | (1.55) | (0.81) |
| Unconditional (Per Year) | 7.24 (7.20) | 0.52 (0.53) | 0.37 (0.38) | 0.16 (0.18) |
| | | | | |
| Shareholder net worth in state <i>m</i> | | | | |
| $v_{sl}(v'_{sl})$ | 0.11 (-0.09) | 0.15 (0.06) | 0.06 (0.06) | 0.00 (0.00) |
| $v_{sh}(v'_{sh})$ | 0.48 (-0.01) | 0.15 (-0.13) | 0.18 (-0.10) | 0.01 (0.01) |
| Unconditional | 0.24 (-0.06) | 0.15 (-0.01) | 0.10 (0.00) | 0.00 (0.00) |
| | | | | |
| Social welfare in state m | 1 51 (1 51) | 1.72 (1.72) | 1.74 (1.74) | 1.66 (1.66) |
| $SW_{S,l}(SW_{S,l})$ | 1.51 (1.51) | 1./3 (1./3) | 1.74 (1.74) | 1.00 (1.00) |
| $SW_{S,h}(SW_{S,h})$ | 0.65 (0.67) | 1.33 (1.33) | 1.33 (1.33) | 1.28 (1.27) |
| Unconditional | 1.20 (1.21) | 1.59 (1.59) | 1.59 (1.59) | 1.52 (1.52) |

The term m = h stands for the states in recessions; m = l stands for booms, and the variables without and with the brackets denote the results for the cases under the systemic tax regimes and under the non-systemic tax regimes respectively. The subscript *S* in the variables refers to the small bank. The values of *Loan rate* are taken from the corresponding scenario from the large bank to reflect the fact that the small bank is the loan rate taker. The terms labelled as *Unconditional* are the expected values for both financial situations when the likelihood of booms $\phi_l = 0.643$ and recessions $\phi_h = 1 - \phi_l = 0.357$ as shown in Table 1. The result marked as *Per Year* indicates the average probability of default of the bank across the two periods to ensure the default rate is for one year.

¹⁶ Note that 2.5% = 1.2% + 1.3%, where 1.3% is the countercyclical capital buffer required by the Basel III.

Table 4

Large bank under optimal capital requirements: Loan rate, capital buffers, systemic tax and bank size, with and (without) systemic tax (all variables in %)

| | Q=1 | Q=5 | Q=10 | Q=20 | Q=100 |
|---|-------------|-------------|-------------|--------------|---------------|
| Optimal capital requirement in state <i>m</i> | | | | | |
| $\gamma_{L,l}^*\left(\gamma_{L,l}^{\prime*}\right)$ | 1.2 (3.6) | 1.5 (3.6) | 1.6 (3.6) | 1.6 (3.6) | 1.6 (3.6) |
| $\gamma_{L,h}^*(\gamma_{L,h}^{\prime*})$ | 3.8 (5.7) | 4.3 (5.7) | 4.3 (5.7) | 4.3 (5.7) | 4.3 (5.7) |
| | | | | | |
| Loan rate in state m | | | | | |
| $r_l(r_l')$ | 1.2 (1.4) | 1.2 (1.4) | 1.2 (1.4) | 1.2 (1.4) | 1.2 (1.4) |
| $r_h(r_h')$ | 3.4 (3.3) | 3.4 (3.3) | 3.4 (3.3) | 3.4 (3.3) | 3.4 (3.3) |
| | | | | | |
| Capital holdings in state m | | | | | |
| $k_{L,l} \left(k_{L,l}' \right)$ | 6.6 (7.2) | 6.5 (7.2) | 6.6 (7.2) | 6.6 (7.2) | 6.6 (7.2) |
| $k_{L,h} \left(k_{L,h}' \right)$ | 9.8 (7.0) | 9.4 (7.0) | 9.2 (7.0) | 9.2 (7.0) | 9.2 (7.0) |
| | | | | | |
| Capital buffer in state m | | | | | |
| $\Delta_{L,l} = k_{L,l} - \gamma_l^* \ (\Delta'_{L,l} = k'_{L,l} - \gamma'_l^*)$ | 5.4 (3.6) | 4.0 (3.6) | 4.0 (3.6) | 4.0 (3.6) | 4.0 (3.6) |
| $\Delta_{L,h} = k_{L,h} - \gamma_h^* \left(\Delta'_{L,h} = k'_{L,h} - \gamma'_h^* \right)$ | 6.0 (1.3) | 5.1 (1.3) | 4.9 (1.3) | 4.9 (1.3) | 4.9 (1.3) |
| | | | N | | |
| Credit rationing in state m | | | | | |
| $CR_{L,lm'}$ ($CR'_{L,lm'}$) | 0.80 (2.58) | 1.27 (2.58) | 1.24 (2.58) | 1.24 (2.58) | 1.24 (2.58) |
| $CR_{L,hm'}$ ($CR'_{L,hm'}$) | 1.14 (9.08) | 1.53 (9.08) | 1.73 (9.08) | 1.73 (9.08) | 1.73 (9.08) |
| Unconditional | 0.92 (4.90) | 1.36 (4.90) | 1.41 (4.90) | 1.41 (4.90) | 1.41 (4.68) |
| | | | | | |
| Net Capital buffer with tax in state <i>m</i> | | | | | |
| $\alpha_{L,l} = \Delta_{L,l} - \Delta'_{L,l}$ | 1.8 | 0.4 | 0.4 | 0.4 | 0.4 |
| $lpha_{L,h}=\Delta_{L,h}-\Delta_{L,h}'$ | 4.7 | 3.8 | 3.6 | 3.6 | 3.6 |
| | | | | | |
| Probability of Bankruptcy in state m | | | | | |
| $1 - F(\hat{x}_{l}) + (1 - CR_{L,lm'}) [1 - F(\hat{x}_{m'})]$ | 0.72 | 0.65 | 0.61 | 0.61 | 0.61 |
| $(1 - F'(\hat{x}_l) + (1 - CR'_{L,lm'})[1 - F'(\hat{x}_{m'})])$ | (0.29) | (0.29) | (0.29) | (0.29) | (0.29) |
| $1 - F(\hat{x}_h) + (1 - CR_{L,hm'}) [1 - F(\hat{x}_{m'})]$ | 1.62 | 1.54 | 1.55 | 1.55 | 1.55 |
| $(1 - F'(\hat{x}_h) + (1 - CR'_{L,hm'})[1 - F'(\hat{x}_{m'})])$ | (2.36) | (2.36) | (2.36) | (2.36) | (2.36) |
| Unconditional (Per Year) | 0.52 (0.51) | 0.48 (0.51) | 0.47 (0.51) | 0.47 (0.51) | 0.47 (0.51) |
| 7 1 1 10 17 I | | | | | |
| Social welfare ¹⁷ in state <i>m</i> | | | | | |
| $SW_{L,l}(SW'_{L,l})$ | 5.19 (5.11) | 8.78 (8.59) | 9.60 (9.38) | 10.06 (9.83) | 10.44 (10.22) |
| $SW_{L,h} (SW'_{L,h})$ | 3.84 (3.33) | 6.72 (6.17) | 7.36 (6.82) | 7.73 (7.19) | 8.05 (7.51) |
| Unconditional | 4.71 (4.47) | 8.05 (7.73) | 8.80 (8.47) | 9.23 (8.89) | 9.60 (9.26) |

The term m = h stands for the states in recessions; m = l stands for booms, and the variables without and with the brackets denotes the results for the cases that are under the systemic tax regimes and under the non-systemic tax regimes respectively. The term *L* in variables refers to the large bank. The terms labelled as *Unconditional* is the expected values for both financial situations when the likelihoods of booms $\phi_l = 0.643$ and recessions $\phi_h = 1 - \phi_l = 0.357$ as shown in Table 1. The result marked as *Per Year* indicates the averaged probability of default of the bank across the two periods to ensure the default rate is for one year.

¹⁷ Note that the social welfare is adjusted by the size of its credit supply Q/Q + 1.



Figure 2

Optimal capital requirements versus bank size for the large bank, with and without systemic tax regime The term m = h stands for the states in recessions; m = l stands for booms. The term Q refers to the ratio of loan size of the large bank to the small bank. The optimal capital requirement (γ) is the percentage of equity to Risk-Weighted Assets (RWA).