Physica A 391 (2012) 2491–2499

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

New features of quantum discord uncovered by q-entropies

A.P. Majtey [a], A.R. Plastino [a,b,*], A. Plastino [a,c]

[a] Instituto Carlos I de Física Teórica y Computacional and Departamento de Física Atómica Molecular y Nuclear, Universidad de Granada, Granada 18071, Spain
[b] National University La Plata UNLP-CREG-Conicet, C.C. 727, 1900 La Plata, Argentina
[c] National University La Plata UNLP-IFLP-Conicet, C.C. 727, 1900 La Plata, Argentina

ARTICLE INFO

Article history:
Received 29 October 2011
Received in revised form 28 November 2011
Available online 8 December 2011

Keywords:
Quantum information
Quantum discord
Tsallis entropy
Mutual information

ABSTRACT

The notion of quantum discord introduced by Ollivier and Zurek [Phys. Rev. Lett 88 (2001) 017901] (see also Henderson and Vedral [J. Phys. A 34 (2001) 6899]) has attracted increasing attention, in recent years, as an entropic quantifier of non-classical features pertaining to the correlations exhibited by bipartite quantum systems. Here we generalize the notion so as to encompass power-law q-entropies (that reduce to the standard Shannon entropy in the limit $q \to 1$) and study the concomitant consequences. The ensuing, new discord-like measures we advance describe aspects of non-classicality that are different from those associated with the standard quantum discord. A particular manifestation of this difference concerns a feature related to order. Let D_1 stand for the standard, Shannon-based discord measure and D_q for the $q \neq 1$ one. If two quantum states A, B are such that $D_1(A) > D_1(B)$, this order-relation does not remain invariant under a change from D_1 to D_q.

1. Introduction

The degree of understanding of quantum correlations (QC) underlies our current picture of Nature [1,2]. It has been recently found that there exist important manifestations of the quantumness of correlations in composite systems that are different from those of entanglement-origin (EO) and that may be relevant in quantum information technologies [3–6]. The quantifier of these non-EO correlations is called the quantum discord (QD) D_1 and is based, for a bipartite system, on Shannon's mutual information. We are thus speaking of an information-theoretic (IT) tool. For pure states, QD does not add any QCs, but that is not the case for mixed states. The D_1-concept, advanced in the pioneer paper by Ollivier and Zurek [4], quantifies: (i) the minimum change in the state of the system and (ii) the information on one of its parts induced by a measurement of the other one. If the state has only classical correlations, D_1 vanishes, which implies that the quantum discord concept somehow quantifies the "correlational-quantumness". It has been evaluated for several families of states both in its original form and in variously altered versions and generalizations. A particularly compelling instance expresses the QD-notion in terms of conditional density operators [7]. Interesting operational QD-interpretations have also been advanced [6]. Evaluating QD requires a rather involved optimization procedure, analytical expressions being known in just a few instances [8–14]. D_1 is built up as an entropic difference, the difference between a quantum entropic measure and its classical counterpart, which is derived from local measurements on one or both of the participant subsystems. Its amount is a new feature $D_1(A)$ of the quantum state A, which in turn induces a D_1-amount "ordering" for states of the form $D_1(A) > D_1(B)$, for instance.

* Correspondence to: Departamento de Física Atómica Molecular y Nuclear, Instituto Carlos I de Física Teórica y Computacional, 18071 Granada, Spain.
E-mail addresses: anamajtey@ugr.es (A.P. Majtey), arplastino@ugr.es, plastino@fisica.unlp.edu.ar (A.R. Plastino), angeloplastino@gmail.com (A. Plastino).

0378-4371/$ – see front matter
doi:10.1016/j.physa.2011.11.062

Now, IT-tools come in many distinct varieties. Given the immense body of literature that has been generated in the past two decades concerning physically motivated statistical formalisms based on generalizations of Shannon's information measure (see Refs. [15–18] and references therein), it seems both natural and necessary to tackle the QD issues from this generalized angle, in the hope of gaining interesting insights, and, in particular, so as to establish the invariance or not of the discord-induced order under a change of the prevailing statistics, from Shannon's to its many rivals.

To show that this is indeed a fruitful endeavor is the aim of the present paper, in which a generalization of the quantum discord *concept*, in the context of generalized statistics, will be advanced and the "ordering-question" answered. In Section 2 we introduce our conceptual QD-extension, discussing its main properties in Section 3. Next we present some results for general bipartite states, focusing attention on analytical results. We also perform numerical simulations for random bipartite states (Section 4). Finally, some conclusions are drawn in Section 5.

2. Retracing Ollivier and Zurek's path à la Tsallis for getting a quantum q-discord

The Tsallis power-law q-entropy was introduced in Ref. [19] as an extension of the Shannon entropy as follows Ref. [15]

$$H_q(X) = -\sum_x p(x)^q \ln_q p(x), \tag{1}$$

where the q-logarithm is defined by $\ln_q(x) \equiv \frac{x^{1-q}-1}{1-q}$, $p(x) \equiv p(X = x)$ is the probability distribution of the pertinent random variable X, and q is any nonnegative real number. The Tsallis entropy converges to Shannon's in the limit $q \to 1$. H_q plays a fundamental role in recent developments of statistical mechanics [15–18,20]. The generalization has indeed received a lot of attention in the last years, with about 2000 papers containing interesting results and useful applications, many of them in the complex systems' area [15–18,20] but also in connection with a variegated family of quantum mechanical settings (see, for example Refs. [21–32]). In what follows we retrace the developments of Ref. [4] in a Tsallis, q-context. Thus, just by setting $q = 1$ we recover the Ollivier–Zurek quantities. We begin then with the mutual information, defined as

$$I_q(X : Y) = H_q(X) + H_q(Y) - H_q(X, Y), \tag{2}$$

and the following chain rule holds [33]:

$$H_q(X, Y) = H_q(Y) + H_q(X|Y), \tag{3}$$

where the conditional entropy reads

$$H_q(X|Y) = \sum_y p(y)^q H_q(X|y). \tag{4}$$

The chain rule gives the relation between a conditional entropy and a joint entropy. Using this relation we can define another, classically equivalent, expression for the mutual information

$$J_q(X : Y) = H_q(X) - H_q(X|Y). \tag{5}$$

The $I - J$ difference is of the essence for Ollivier–Zurek goals, after expressing the two quantifiers in quantal fashion. Let us then do the same with I_q and J_q. I_q can be easily generalized defining appropriate density matrices for the quantum systems, ρ_A, ρ_B, and $\rho_{A,B}$, and applying then the q-generalization of von Neumann's entropy $S_q(\rho) = -Tr(\rho^q \ln_q \rho)$. One has

$$I_q(A : B) = I_q(\rho_{A,B}) = S_q(\rho_A) + S_q(\rho_B) - S_q(\rho_{A,B}). \tag{6}$$

To generalize the J_q-expression, following Ref. [4], we focus on a perfect measurements of ρ_B defined by a set of projectors $\{\Pi_j^{(B)}\}$ such that $\sum_j \Pi_j^{(B)} = 1$. Accordingly,

$$J_q(\rho_{A,B})_{\{\Pi_j^{(B)}\}} = S_q(\rho_A) - S_q(\rho_A|\{\Pi_j^{(B)}\}), \tag{7}$$

where

$$S_q(\rho_A|\{\Pi_j^{(B)}\}) = \sum_j p_j^q S_q(\rho_{A|\Pi_j^{(B)}}), \tag{8}$$

with the state of A given, once the measurement is performed, by

$$\rho_{A|\Pi_j^{(B)}} = \Pi_j^{(B)} \rho_{A,B} \Pi_j^{(B)} / Tr_{A,B} \Pi_j^{(B)} \rho_{A,B}, \tag{9}$$

and $p_j = Tr_{A,B} \Pi_j^{(B)} \rho_{A,B}$.

The two classical expressions for the standard mutual information we have presented above are identical, but this is not necessarily so in the quantum case Actually, the quantum discord is defined as the *minimum* possible difference between the two possibilities, given by an *optimum* set of $\{\Pi_j^{(B)}\}$ [4]. Thus, we are to be concerned here with what happens to the expressions I_q and J_q. Introduce the quantity

$$C_q(\rho_{A,B}) := \sup_{\{\Pi_j^{(B)}\}} J_q(\rho_{A,B})_{\{\Pi_j^{(B)}\}}. \tag{10}$$

We define now our quantum q-discord as the difference

$$\vartheta_q(\rho_{A,B}) = I_q(\rho_{A,B}) - C_q(\rho_{A,B}). \tag{11}$$

This quantum q-discord is the minimum of the difference between Eqs. (6) and (7). We normalize this measure via a trivial re-scaling in order to compare, in an adequate way, different quantities:

$$D_q(\rho_{A,B}) = \frac{q-1}{1-2^{1-q}}\vartheta(\rho_{A,B}). \tag{12}$$

For similar reasons, log denotes the logarithm of base 2. In what follows $\rho_{A,B} \equiv \rho$. Note that $\lim_{q\to 1} D_q(\rho) = D_1(\rho)$.

3. Properties of the generalized quantum discord

We see that $D_q \geq 0$ for $q \in (0, 1)$, and this might be related to the concavity of the q-conditional entropy $S_q(\rho_{A,B}) - S_q(\rho_B)$ with respect to $\rho_{A,B}$. Indeed, ϑ_q becomes negative if q grows from $1 \to \infty$, negativity increasing with q. Quite convenient is the particular case $q = 2$, since it requires only to compute the traces of ρ^2, ρ_B^2, ρ_k^2, and matrix-diagonalization is avoided, making computations more efficient. Taking the limit $q \to \infty$ of the normalized measure we obtain

$$\lim_{q\to\infty} D_q = \begin{cases} 0, & \text{mixed states,} \\ 1, & \text{pure states.} \end{cases} \tag{13}$$

To study the positivity of the q-discord we consider separately two cases: pure states and mixed states.

3.1. Pure states

For pure states, that is $\rho = |\Psi\rangle\langle\Psi|$, the q-discord takes the form $\vartheta(\rho) = S_q(\rho_A)$ and the quantum q-discord coincides with the reduced (quantum) Tsallis entropy. We can easily verify this fact by casting $|\Psi\rangle$ in its Schmidt decomposition form $|\Psi\rangle = \sum_i \lambda_i |ii\rangle$. Thus, since the q-entropy is positive for all q,

$$\vartheta_q(|\Psi\rangle\langle\Psi|) \geq 0, \quad \forall q. \tag{14}$$

3.2. Mixed states

In the case of mixed states our q-discord is positive only for values of q in (0, 1). In order to demonstrate the positivity of the q-discord for mixed arbitrary states we follow [4] and consider the proposition: $S_q(\rho_A|\{\Pi_j^{(B)}\}) = S_q(\rho_{A,B}^{(D)}) - S_q(\rho_B^{(D)})$, with $\rho_{A,B}^{(D)} = \sum_j p_j\rho_j$. Now, $\rho_{A,B}^{(D)}$ is block diagonal and as in the $q = 1$ case, doing a block by block analysis the proposition can be proved. Now we need to verify the inequality:

$$S_q(\rho_A|\{\Pi_j^{(B)}\}) \geq S_q(\rho_{A,B}) - S_q(\rho_B). \tag{15}$$

By recourse of the previous preposition we can establish the following relation for any measurement $\{\Pi_j^{(B)}\}$

$$S_q(\rho_A|\{\Pi_j^{(B)}\}) = S_q(\rho_{A,B}^{(D)}) - S_q(\rho_B^{(D)}), \tag{16}$$

and by the (conjectured) concavity (see below) of the conditional entropy ($S_q(\rho) - S_q(\rho_B)$) with respect to ρ for $q \in (0, 1)$ we are led to

$$S_q(\rho_{A,B}^{(D)}) - S_q(\rho_B^{(D)}) \geq S_q(\rho_{A,B}) - S_q(\rho_B). \tag{17}$$

3.3. Random generation of states in an N-dimensional Hilbert space

The set of states in an N-dimensional Hilbert space can be regarded as a product-space of the form [35,34],

$$\mathcal{H} = \mathcal{P} \times \Delta,$$

where $\mathcal{P}$ stands for the family of all complete sets of ortho-normal projectors $\{\hat{P}_i\}_i^N$, $\sum_i \hat{P}_i = \mathbb{I}$ ($\mathbb{I}$ the identity matrix), and Δ is the convex set of all real N-tuples of the form $\{\lambda_1, \ldots, \lambda_N\}$; $\lambda_i \in \mathbb{R}$; $\sum_i \lambda_i = 1$; $0 \leq \lambda_i \leq 1$. Any state in $\mathcal{H}$ takes the form $\rho = \sum_i \lambda_i \hat{P}_i$.

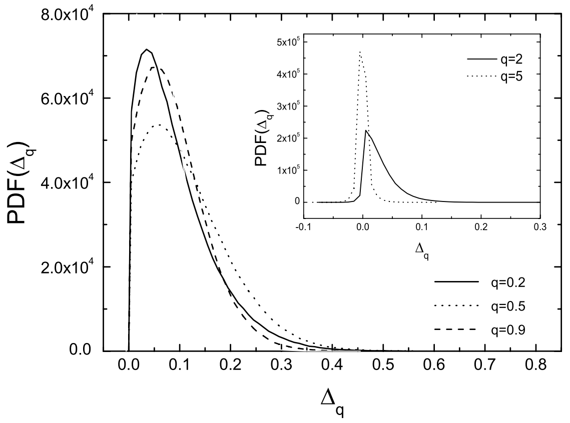

Fig. 1. Probability distribution for Δq for different values of $q < 1$ for which the concavity is verified. Inset: $PDF(\Delta_q)$ for different $q > 1$. The curves are left-shifted. All curves were constructed using the order of 10^6 (numerically generated) states. All depicted quantities are dimensionless.

In order to explore $\mathcal{H}$ we introduce an appropriate measure μ on this space. Such a measure is required to compute volumes within $\mathcal{H}$, as well as to determine what is to be understood by a uniform distribution of states on $\mathcal{H}$. An arbitrary state ρ of our N-dimensional Hilbert space can always be expressed as a product of the form

$$\rho = UD[\{\lambda_i\}]U^\dagger. \tag{18}$$

Here U is an $N \times N$ unitary matrix and $D[\{\lambda_i\}]$ is an $N \times N$ diagonal matrix whose diagonal elements are, precisely, our above defined $\{\lambda_1, \ldots, \lambda_N\}$. The group of unitary matrices $U(N)$ is endowed with a unique, uniform measure, known as the Haar measure, ν [34]. On the other hand, the N-simplex Δ, consisting of all the real N-uples $\{\lambda_1, \ldots, \lambda_N\}$ appearing in (18), is a subset of a $(N-1)$-dimensional hyperplane of $\mathbb{R}^N$. Consequently, the standard normalized Lebesgue measure $\mathcal{L}_{N-1}$ on $\mathbb{R}^{N-1}$ provides a measure for Δ. The aforementioned measures on $U(N)$ and Δ lead to a measure μ on the set $\mathcal{S}$ of all the states of our quantum system [34,35],

$$\mu = \nu \times \mathcal{L}_{N-1}. \tag{19}$$

If one needs to randomly generate mixed states, this is to be done according to the measure (19).

3.4. *Concavity of the conditional entropy in the interval* $(0 < q < 1)$

Here we attempt a numerical verification of the concavity of the conditional entropy for $q \in (0, 1)$, that is,

$$\Delta_q = S_q(\rho) - S_q(\rho_A) - \{t[S_q(\sigma) - S_q(\sigma_A)] + (1-t)[S_q(\xi) - S_q(\xi_A)]\} \geq 0, \tag{20}$$

where $\rho = t\sigma + (1-t)\xi$, ρ_A is the reduced density matrix corresponding to ρ, σ_A (ξ_A) the reduced density matrices of σ (ξ) and, finally, $0 \leq t \leq 1$.

The concavity of the standard conditional entropy ($q = 1$) was proved in Ref. [36] by assuming the validity of a lemma by Lieb [37]. The proof is rather difficult even in this case [36]. As a first step we evaluate numerically the inequality (20) by generating random states in an N-dimensional Hilbert space. In order to assess, for these randomly generated states, how the concavity-requirement is satisfied, we evaluate (20) for a large enough number of simulated states (σ and ξ). We set $N = 4$ for the dimension of the state-space in all simulations and we randomly generate $t \in [0, 1]$.

We investigate the positivity of Δ_q, upon which the concavity of the conditional q-entropy is based, by constructing the probability distributions for the values of Δq. The corresponding distributions, for different values of q, are depicted in Fig. 1. In the inset we plot the probability distribution of Δ_q for $q = 2$ and $q = 5$. The curves are constructed using of the order of 10^6 states. These simulations provide us with strong evidence about the validity of the conjecture advanced above on the concavity of the conditional q-entropy and, consequently, on the positivity of the quantum q-discord for $0 < q < 1$.

4. Relation between *q*-discord and orthodox discord

Let us now investigate the relation between the q-discord and its original counterpart for different sorts of states.

We begin with Bell diagonal states. These are two-qubit states with maximally-mixed reduced-density matrices and have the form

$$\rho_{A,B} = \rho = \frac{1}{4}\left(I + \sum_{j=1}^{3} c_j \sigma_j \otimes \sigma_j\right) \tag{21}$$

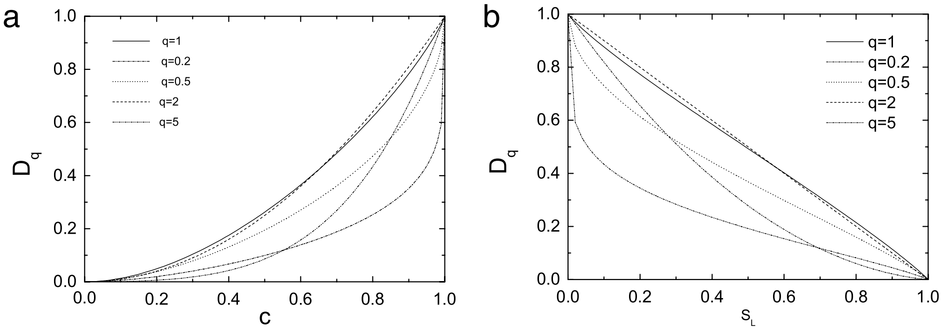

Fig. 2. (a) D_q for the Werner state, as a function of c, for different values of q. (b) D_q for Werner states, as a function of the degree of mixedness measured by the linear entropy, for different values of q. All depicted quantities are dimensionless.

where c_j are real constants constrained by certain conditions (in order to have a well defined density operator ρ) and σ_j's are the Pauli operators. Let $\lambda_i = \lambda_i(c_j) \in [0, 1]$, $(i = 0, \ldots, 3)$ be the eigenvalues of ρ

$$\lambda_0 = \frac{1}{4}(1 - c_1 - c_2 - c_3),$$
$$\lambda_1 = \frac{1}{4}(1 - c_1 + c_2 + c_3),$$
$$\lambda_2 = \frac{1}{4}(1 + c_1 - c_2 + c_3),$$
$$\lambda_3 = \frac{1}{4}(1 + c_1 + c_2 - c_3). \tag{22}$$

The marginal states of ρ are $\rho_A = \mathbb{I}/2$ and $\rho_B = \mathbb{I}/2$. Thus, the quantum q-mutual information of ρ is

$$I_q(\rho) = -4\left(\frac{1}{2}\right)^q \ln_q \frac{1}{2} + \sum_i \lambda_i^q \ln_q \lambda_i \tag{23}$$

and

$$C_q(\rho) = 2\left(\frac{1}{2}\right)^q \left[-\ln_q \frac{1}{2} + \left(\frac{1-c}{2}\right)^q \ln_q \frac{1-c}{2} + \left(\frac{1+c}{2}\right)^q \ln_q \frac{1+c}{2}\right], \tag{24}$$

where $c := \max\{|c_1|, |c_2|, |c_3|\}$. We find, for a general (Bell-diagonal) two-qubit state,

$$\vartheta(\rho) = -2\left(\frac{1}{2}\right)^q \left[\ln_q \frac{1}{2} + \left(\frac{1-c}{2}\right)^q \ln_q \frac{1-c}{2} + \left(\frac{1+c}{2}\right)^q \ln_q \frac{1+c}{2}\right] + \sum_i \lambda_i^q \ln_q \lambda_i. \tag{25}$$

Let us specialize (25) to the particular instance $c_1 = c_2 = c_3 = -c$, i.e., the celebrated Werner states,

$$\rho = (1-c)\frac{\mathbb{I}}{4} + c|\Psi^-\rangle\langle\Psi^-|, \quad c \in [0, 1], \tag{26}$$

with $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. By following Ref. [9] one easily obtains

$$\vartheta(\rho) = -2\left(\frac{1}{2}\right)^q \left[\ln_q \frac{1}{2} + \left(\frac{1-c}{2}\right)^q \ln_q \frac{1-c}{2} + \left(\frac{1+c}{2}\right)^q \ln_q \frac{1+c}{2}\right]$$
$$+ 3\left(\frac{1-c}{4}\right)^q \ln_q \frac{1-c}{4} + \left(\frac{1+3c}{4}\right)^q \ln_q \frac{1+3c}{4}, \tag{27}$$

and, as seen in Fig. 2, positivity prevails for the prototype-mixed state. In Fig. 2 we plot the (normalized) q-discord as a function of the state-parameter c for different values of q and also as a function of the mixedness-degree as given by the linear entropy

$$S_L = \frac{4}{3}[1 - Tr\rho^2],$$

trivially related to the purity γ of a state via $S_L = 1 - \gamma$. As expected, an inverse relationship between mixedness-degree and quantum correlations is displayed. We remark on the single-valuedness of the Werner-relation between q-discord and mixedness, even for $q = 1$

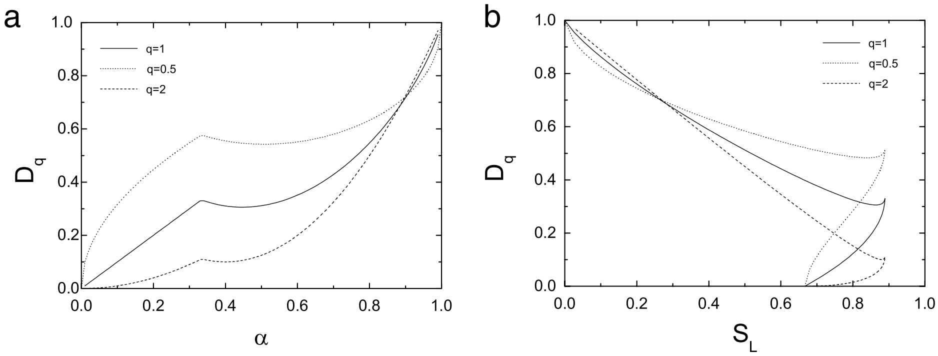

Fig. 3. (a) D_q for α states, as a function of α, for different values of q. (b) D_q for α states, as a function of the degree of mixedness, for different values of q. All depicted quantities are dimensionless.

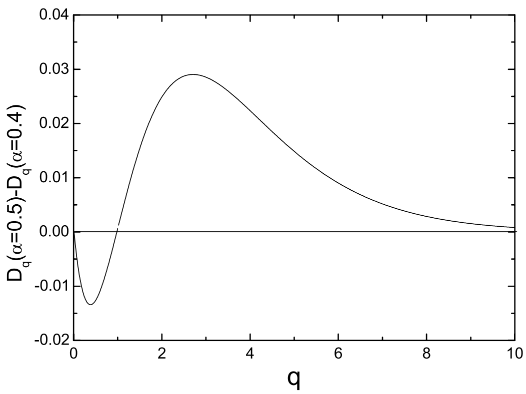

Fig. 4. Difference between D_q of two α states as a function of the parameter q. A non trivial ordering relation is found. All depicted quantities are dimensionless.

4.1. α states

We also will study the quantum q-discord for the following one-parameter states

$$\rho_\alpha = \begin{pmatrix} \frac{\alpha}{2} & 0 & 0 & \frac{\alpha}{2} \\ 0 & \frac{1-\alpha}{2} & 0 & 0 \\ 0 & 0 & \frac{1-\alpha}{2} & 0 \\ \frac{\alpha}{2} & 0 & 0 & \frac{\alpha}{2} \end{pmatrix} \tag{28}$$

where $0 \le \alpha \le 1$. Let $\xi = \max\{|\alpha|, |2\alpha - 1|\}$. The q-discord becomes

$$\vartheta(\rho) = -2\left(\frac{1}{2}\right)^q \left[\ln_q \frac{1}{2} + \left(\frac{1-\xi}{2}\right)^q \ln_q \frac{1-\xi}{2} + \left(\frac{1+\xi}{2}\right)^q \ln_q \frac{1+\xi}{2}\right] + 2\left(\frac{1-\alpha}{2}\right)^q \ln_q \frac{1-\alpha}{2} + \alpha^q \ln_q \alpha. \tag{29}$$

In Fig. 3 we depict the quantum q-discord as a function of the state's parameters for different values of q and also plot it as a function of the linear entropy. Positivity again prevails. The single-valuedness between discord and mixedness is lost for these states. This was already noted in [38] for the case $q = 1$.

4.1.1. Discord-differences for two α-states

In Fig. 4 we display the difference between the q-discord of two α-states (corresponding to $\alpha = 0.4$ and $\alpha = 0.5$, respectively), as a function of q.

This difference takes negative or positive values depending on the range of q. This is indeed a novel feature. A relation of order for quantum states based on the discord-concept cannot univocally be established, because it depends on the entropic quantifier one chooses to employ. In other words, the quantal correlations that the discord quantifies are seen in different manners by distinct entropic quantifiers. *This lack of uniqueness is the leitmotif of the present considerations.*

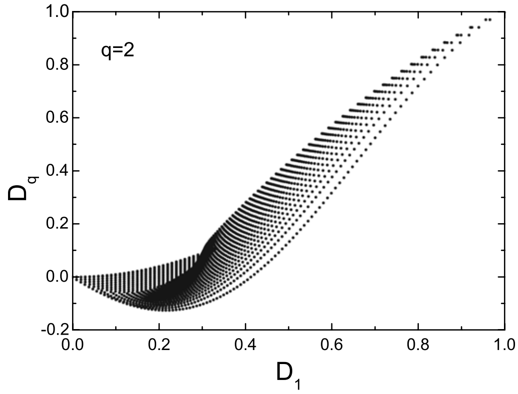

Fig. 5. D_q $(q = 2)$ vs D_1 for α, β-like states. All depicted quantities are dimensionless.

4.2. (α, β) state

As a last particular kind of special state to be analyzed, consider the two-parameters state

$$\rho_{\alpha,\beta} = \frac{1}{2}\begin{pmatrix} \alpha & 0 & 0 & \alpha \\ 0 & 1-\alpha-\beta & 0 & 0 \\ 0 & 0 & 1-\alpha+\beta & 0 \\ \alpha & 0 & 0 & \alpha \end{pmatrix}, \tag{30}$$

where $0 \leq \alpha \leq 1$ and $\alpha - 1 \leq \beta \leq 1 - \alpha$. We display the $q = 2$-discord versus the $q = 1$-discord for this state in Fig. 5. A strong correlation is exhibited between the two q-measures. This could be taken as evidence that changing q from its original $q = 1$-value does not per se modify the overall manner in which q-discord quantifies quantum correlations.

4.3. Arbitrarily mixed two-qubit states

Here, we focus our discussion on general (pure or mixed) states of two qubits. For such system we can parametrize the basis of the measurement by θ and ϕ,

$$\begin{aligned} |\psi\rangle &= \cos(\theta)|0\rangle + e^{i\phi}\sin(\theta)|1\rangle \\ |\psi_\perp\rangle &= e^{-i\phi}\sin(\theta)|0\rangle - \cos(\theta)|1\rangle. \end{aligned} \tag{31}$$

We numerically search the $\theta - \phi$ space for the set of values that maximize Eq. (10). The resultant density operator $\rho_{A|\Pi_j^{(B)}} = \rho_j$, when such measurements are performed on subsystem B, is

$$\rho_j = \frac{1}{p_j}(I \otimes \Pi_j^{(B)})\rho(I \otimes \Pi_j^{(B)}), \tag{32}$$

where each complete set, composed of two elements, of possible measurements is defined as follows,

$$\begin{aligned} \Pi_1^{(B)} &= |\psi\rangle\langle\psi| \\ \Pi_2^{(B)} &= |\psi_\perp\rangle\langle\psi_\perp|. \end{aligned} \tag{33}$$

We randomly generate states uniformly distributed according to the measure μ and by recourse of the previously described optimization procedure. Of course, we numerically search for θ and ϕ and compute the q-discord for these states. In Fig. 6 we display the correlation between the discord and the q-discord for different values of q. Negative values of the q-discord are depicted in the plot for the case $q > 1$ $(q = 2)$. Fig. 7 depicts the probability distribution of finding a given value of q-discord in the whole space of two-qubit states for $q = 0.5, 1, 2$.

We also compute the difference between the q-discord and the discord between pairs of randomly generated states ρ and σ. In Fig. 8 we plot the resultant differences of the q-discord versus similar differences (for the same pair of states) corresponding to 1-discord. This plot depicts the pertinent results.

Overall, our numerical simulations confirm the conclusions reached by the analysis of special kinds of states.

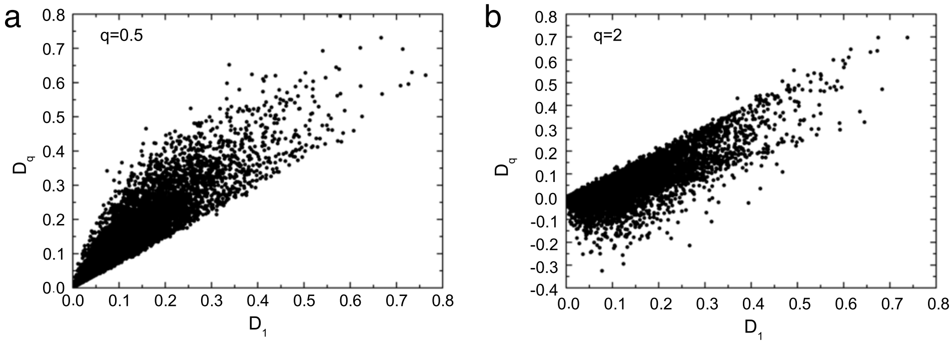

Fig. 6. D_q as a function of D_1 for randomly generated two-qubit states (a) $q = 0.5$, (b) $q = 2$. All depicted quantities are dimensionless.

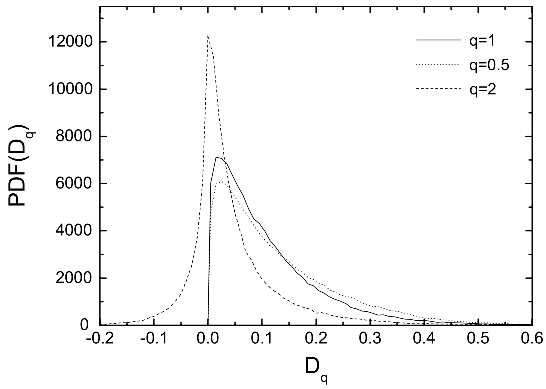

Fig. 7. Probability distribution of finding an arbitrary two-qubit state with a given value of q-discord for different values of q. The curves were constructed using 10^5 generated random states. All depicted quantities are dimensionless.

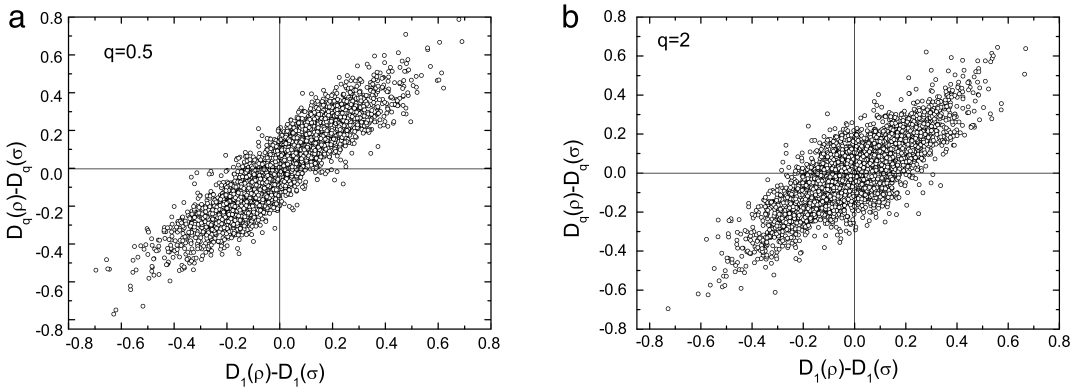

Fig. 8. $D_q(\rho) - D_q(\sigma)$ as a function of $D_1(\rho) - D_1(\sigma)$ (a) $q = 0.5$, (b) $q = 2$. All depicted quantities are dimensionless.

5. Conclusions

We have introduced a new family of quantum discord measures that quantifies quantum correlations based in the chain rule relating the (i) conditional- and (ii) joint-Tsallis entropies. Via two types of study

- of special kinds of quantum states
- arbitrary, randomly generated mixed states,

we have been able to extract the following conclusions:

1. There is a strong correlation between the "new" q-discord and the original one of Ollivier and Zurek.
2. However, an order-relation for quantum states based on discord lacks unity because it definitely depends on the quantifier one chooses to employ. This means that q-discord functionals corresponding to different values of q measure different aspects of the non-classicality (quantumness) of correlations.

This last fact should constitute strong stimulus for establishing a more detailed assessment of just what kind of correlations the discord concept quantifies.

Acknowledgments

This work was partially supported by the projects FQM-2445 and FQM-4643 ofthe Junta de Andalucia and the grant FIS2011-24540 of the Ministerio de Innovación y Ciencia (Spain). A.P.M. acknowledges support by GENIL through the YTR-GENIL Program. A.P. acknowledges support from the Senior Grant CEI Bio-Tic GENIL-SPR.

References

[1] L. Amico, L. Fazio, A. Osterloh, V. Vedral, Rev. Mod. Phys. 80 (2008) 517.
[2] I. Bengtsson, K. Zyczkowski, Geometry of Quantum States: An Introduction to Quantum Entanglement, Cambridge University Press, Cambridge, 2006.
[3] B. Dakic, V. Vedral, C. Brukner, Phys. Rev. Lett. 105 (2010) 190502.
[4] H. Ollivier, W.H. Zurek, Phys. Rev. Lett. 88 (2001) 017901.
[5] L. Henderson, V. Vedral, J. Phys. A 34 (2001) 6899.
[6] D. Cavalcanti, L. Aolita, S. Boixo, K. Modi, M. Piani, A. Winter, Phys. Rev. A 83 (2011) 032324.
[7] N. Li, S. Luo, Z. Zhang, J. Phys. A 40 (2007) 11361.
[8] S. Luo, Phys. Rev. A 77 (2008) 022301.
[9] S. Luo, Phys. Rev. A 77 (2008) 042303.
[10] N. Li, S. Luo, Phys. Rev. A 78 (2008) 024303.
[11] V. Vedral, Phys. Rev. Lett. 90 (2003) 050401.
[12] J. Maziero, L.C. Celéri, R.M. Serra, V. Vedral, Phys. Rev. A 80 (2009) 044102.
[13] J. Oppenheim, M. Horodecki, P. Horodecki, R. Horodecki, Phys. Rev. Lett. 89 (2002) 180402.
[14] M. Ali, A.R.P. Rau, G. Alber, Phys. Rev. A 81 (2010) 042105.
[15] C. Tsallis, Introduction to Nonextensive Statistical Mechanics, Springer, Heidelberg, 2009.
[16] M. Gell-Mann, C. Tsallis, Nonextensive Entropy-Interdisciplinary Applications, Oxford University Press, New York, 2004.
[17] C. Beck, Contemp. Phys. 50 (2009) 495.
[18] J. Naudts, Generalized Thermostatistics, Springer, Heidelberg, 2011.
[19] C. Tsallis, J. Stat. Phys. 52 (1988) 479.
[20] J.S. Andrade, Jr., G.F.T. da Silva, A.A. Moreira, F.D. Nobre, E.M.F. Curado, Phys. Rev. Lett. 105 (2010) 260601.
[21] F.D. Nobre, M.A. Rego-Monteiro, C. Tsallis, Phys. Rev. Lett. 106 (2011) 140601.
[22] A. Luis, Phys. Rev. A 84 (2011) 034101.
[23] J.C. Angulo, J. Antolin, S. Lopez-Rosa, R.O. Esquivel, Physica A 390 (2011) 769.
[24] R. Silva, D.H.A.L. Anselmo, J.S. Alcaniz, Europhys. Lett. 89 (2010) 10004.
[25] H. Hasegawa, Physica A 389 (2010) 2358.
[26] A. Hamadou-Ibrahim, A.R. Plastino, A. Plastino, Braz. J. Phys. 39 (2009) 408.
[27] R.K. Niven, Eur. Phys. J. B 70 (2009) 49.
[28] F. Caruso, C. Tsallis, Phys. Rev. E 78 (2008) 021102.
[29] C. Zander, A.R. Plastino, Physica A 364 (2006) 145.
[30] J. Batle, M. Casas, A.R. Plastino, A. Plastino, Phys. Lett. A 296 (2002) 251.
[31] L.C. Malacarne, R.S. Mendes, E.K. Lenzi, Phys. Rev. E 65 (2002) 046131.
[32] U. Tirnakli, F. Buyukkilic, D. Demirhan, Phys. Lett. A 245 (1998) 62.
[33] S. Furuichi, J. Math. Phys. 47 (2006) 023302.
[34] M. Pozniak, K. Zyczkowski, M. Kus, J. Phys. A: Math. Gen. 31 (1998) 1059.
[35] K. Zyczkowski, P. Horodecki, A. Sanpera, M. Lewenstein, Phys. Rev. A 58 (1998) 883.
[36] A. Wehrl, Rev. Mod. Phys. 50 (1978) 221.
[37] E. Lieb, Adv. Math. 11 (1973) 267.
[38] A. Al-Qasimi, D.F.V. James, Phys. Rev. A 83 (2011) 032101.