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Abstract: Neutrosophic quadruple numbers are the newest field studied in neutrosophy. Neutrosophic quadruple numbers, using the certain extent known data of an object or an idea, help us uncover their known part and moreover they allow us to evaluate the unknown part by the trueness, indeterminacy and falsity values. In this study, we generalized Hamming similarity measures for the generalized set-valued neutrosophic quadruple sets and numbers. We showed that generalized Hamming measure satisfies the similarity measure condition. Also, we generalized an algorithm for the generalized set-valued neutrosophic quadruple sets and numbers, we gave a multi-criteria decision making application for using the this generalized algorithm. In this application, we examined which of the laws established in different situations were more efficient. Furthermore, we obtained different result compared to previous algorithm and previous similarity measure based on single-valued neutrosophic numbers. Therefore, we have shown that generalized set-valued neutrosophic quadruplet sets and numbers, a new field of neutrosophic theory, are more useful for decision-making problems in law science and more precise results are obtained. The application in this study can be developed and used in decision-making applications for law science and other sciences.

Keywords: Neutrosophic quadruple sets, generalized set valued neutrosophic quadruple sets and numbers, Hamming similarity measure, decision-making applications, law applications

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1 Introduction

Smarandache proposed the neutrosophic logic and the neutrosophic set [3] in 1998. Neutrosophic logic and neutrosophic sets have a degree of membership T, a degree of indeterminacy I and a degree of non-membership F. These degrees are defined independently. Thus, neutrosophic theory is generalized of fuzzy theory [4] and intuitionistic fuzzy theory [5]. Also, many researchers have studied neutrosophic theory [6 - 19]. Recently, Smarandache extended the neutrosophic set to refined (n-valued) neutrosophic set, and to refined neutrosophic logic, and to refined neutrosophic probability, i.e. the truth value T is refined\split into types of sub-truths such as T_1, T_2, …, similarly indeterminacy I is refined\split into types of sub-indeterminacies I_1, I_2, …, and the falsehood F is refined\split into sub-falsehoods F_1, F_2, … [20]; Peng et al. obtained multi-parametric similarity measure for neutrosophic set [21]; Ye et al. introduced similarity measures of single-valued neutrosophic sets [22]; Uluçay et al. studied MCDM-problems with neutrosophic multi-sets [23]; Kandasamy et al. studied refined neutrosophic sets [24]; Hashmi et al. obtained m-Polar neutrosophic topology [25]; Aslan et al. studied Neutrosophic Modeling of Talcott Parsons's Action [2].

Decision-making applications and similarity measures are very important in neutrosophic theory. Thus, many researchers studied based on decision-making applications in neutrosophic theory. Recently, Tian et al. obtained a multi-criteria decision-making method based on neutrosophic theory [28]; Saqlain et al. studied single and multi-valued neutrosophic hypersoft set [29]; Roy et al. introduced similarity Measures of Quadripartitioned single-valued bipolar neutrosophic sets [30]; Uluçay et al. obtained decision-making method based on neutrosophic soft expert graphs [31]; Şahin et al. studied interval valued neutrosophic sets and applications [32]; Nabeeh et al. obtained an integrated neutrosophic-TOPSIS approach and its application to personnel selection [41]; Nabeeh et al. studied neutrosophic multi-criteria decision-making approach for IoT-Based enterprises [42]; Abdel-Basset et al. obtained utilizing neutrosophic theory to solve transition difficulties of IoT-Based enterprises [43].

In 2015, Smarandache discussed neutrosophic quadruple sets and neutrosophic quadruple numbers [1]. A neutrosophic quadruple set is a generalized form of a neutrosophic set. A neutrosophic quadruple set is denoted by {(x, yT, zI, tF): x, y, z, t ∈ ℝ or ℂ}. Here, x is referred to as the known part, (yT, zI, tF) as the unknown part and T, I and F are the usual tools of the neutrosophic logic. So, neutrosophic quadruple sets are generalized of neutrosophic sets. Furthermore, researchers have studied neutrosophic quadruple sets and numbers [33 - 36]. Recently, Rezaei et al. studied neutrosophic quadruple a-ideals [38]; Mohseni et al. obtained commutative neutrosophic quadruple ideals [39]; Kandasamy et al. introduced neutrosophic quadruple algebraic codes [40]. Also, Şahin et al. introduced generalized set-valued neutrosophic quadruple sets and numbers [37]. A generalized set-valued neutrosophic quadruple set denoted by

$$G_{s_i} = \{(K_{s_i}, L_{s_i}T_{s_i}, M_{s_i}I_{s_i}, N_{s_i}F_{s_i}): K_{s_i}, L_{s_i}, M_{s_i}, N_{s_i} \in \mathrm{P(X)};\ \mathrm{i} = 1, 2, 3, \dots, \mathrm{n}\}.$$

Where T_i, I_i and F_i have their usual neutrosophic logic; X is a nonempty set, P(X) is power set of X, K_{s_i} is called the known part and $(L_{s_i}T_{s_i}, M_{s_i}I_{s_i}, N_{s_i}F_{s_i})$ is called the unknown part. Thanks to this definition, neutrosophic quadruple sets have become available in the field of decision-making application. Most importantly, this definition, which has a more general structure than neutrosophic sets, will find more application areas and will give more objective results to many problems with the help of the known part, unknown part and K, L, M, N sets.

As in many branches of science, many uncertainties are encountered in terms of application and decision-making in law science. In order to cope with these uncertainties, mostly known classical methods are inadequate or cause wrong decisions to be made. In addition, many criteria should be considered in determining the laws in law science. In addition, it is clear that unknown situations will arise in the implementation of laws prepared for known situations. For all these reasons, in this study, we have prepared an application in order to determine which of the different legal applications with multiple criteria will yield most effective results. For this application, we generalized Hamming similarity measures for the generalized set-valued neutrosophic quadruple sets (GsvNQs) and numbers (GsvNQn) since GsvNQs and GsvNQn are more useful then neutrosophic sets. Also, we generalized an algorithm [2] (based on single valued neutrosophic number (SvNn) and set (SvNs)) for the GsvNQs and GsvNQn. Also, we gave a multi-criteria decision-making application using this generalized algorithm. In this application, we examined which of the laws established in different situations were more efficient. Furthermore, we obtained different result compared to previous algorithm and previous similarity measure based on SvNn thanks to structure of GsvNQs and GsvNQn.

In this paper, in Section 2, we examined neutrosophic sets [3, 8], Hamming similarity measure [22], GsvNQs and properties [33]. In section 3, we defined firstly generalized Hamming similarity measure based on GsvNQn. In Section 4, we firstly generalized an algorithm [2] for GsvNQn. In Section 5, we give a multi-criteria decision making application using the generalized algorithm in Section 4. In Section 6, we compared the results of the generalized algorithm in Section 5 with the results of algorithm (based on single valued neutrosophic set and Hamming similarity measure [22]) [2]. In Section 6, we give conclusions.

2 Preliminaries

Definition 1: [3] Let E be the universal set. For $\forall x \in E, 0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$, by the help of the functions $T_A: E \to]^-0, 1^+[$, $I_A: E \to]^-0, 1^+[$ and $F_A: E \to]^-0, 1^+[$ a neutrosophic set A on E is defined by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in E\} .$$

Here, $T_A(x), I_A(x)$ and $F_A(x)$ are the degrees of trueness, indeterminacy and falsity of $x \in E$ respectively.

Definition 2: [8] Let E be the universal set. For $\forall x \in E, 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$, using the functions $T_A: E \to [0,1]$, $I_A: E \to [0,1]$ and $F_A: E \to [0,1]$, a SvNs A on E is defined by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in E\} .$$

Here, $T_A(x), I_A(x)$ and $F_A(x)$ are the degrees of trueness, indeterminacy and falsity of $x \in E$ respectively.

Definition 3: [22] Let

$$A_1 = <T_{A_1}(x), I_{A_1}(x), F_{A_1}(x)> \text{ and } A_2 = <T_{A_2}(x), I_{A_2}(x), F_{A_2}(x)>$$

be two SvNns, $S: A_1 \times A_2 \to [0,1]$ be a function. The Hamming similarity measure between A_1 and A_2 denoted by $S(A_1, A_2)$ such that

$$S(A_1, A_2) = \frac{1}{3}[\left|T_{A_1}(x) - T_{A_2}(y)\right| + \left|I_{A_1}(x) - I_{A_2}(y)\right| + \left|F_{A_1}(x) - F_{A_2}(y)\right|]$$

Theorem 1: [22] Let A_1 and A_2 be two SvNns, $S: A_1 \times A_2 \to [0,1]$ be a Hamming similarity measure. $S(A_1, A_2)$ satisfies below properties.

i. $0 \leq S(A_1, A_2) \leq 1$,
ii. $S(A_1, A_2) = 1$ if and only if $A_1 = A_2$,
iii. $S(A_1, A_2) = S(A_2, A_1)$,
iv. If $A_1 \subseteq A_2 \subseteq A_3 \in E$, then $S(A_1, A_3) \leq S(A_1, A_2)$ and $S(A_1, A_3) \leq S(A_2, A_3)$.

Definition 4: [1] Neutrosophic quadruple number is a number of the form

(k, lT, mI, nF)

Here, T, I and F are used as the ordinary neutrosophic logical tools and k, l, m, n ∈ ℝ or ℂ. For a neutrosophic quadruple number (k, lT, mI, nF), k is named the known part and (lT, mI, nF) is named the unknown part where k represents any asset such as a number, an idea, an object, etc. Also,

NQ = {(k, lT, mI, nF): k, l, m, n ∈ ℝ or ℂ}

is defined by neutrosophic quadruple set.

Definition 5: [33] Let X be a set and P(X) be power set of X. A GsvNQs is a set of the form

$$G_{s_i} = \{(A_{s_i}, B_{s_i}T_{s_i}, C_{s_i}I_{s_i}, D_{s_i}F_{s_i}): A_{s_i}, B_{s_i}, C_{s_i}, D_{s_i} \in \mathrm{P(X)}; \mathrm{i} = 1, 2, 3, \dots, \mathrm{n}\}$$

Where, T_i, I_i and F_i have their usual neutrosophic logic means and GsvNQn defined by

$$G_{N_i} = (A_{s_i}, B_{s_i}T_{s_i}, C_{s_i}I_{s_i}, D_{s_i}F_{s_i}).$$

As in neutrosophic quadruple number, for a GsvNQn (A_{s_i}, $B_{s_i}T_{s_i}$, $C_{s_i}I_{s_i}$, $D_{s_i}F_{s_i}$), representing any entity which may be a number, an idea, an object, etc.; A_{s_i} is called the known part and ($B_{s_i}T_{s_i}$, $C_{s_i}I_{s_i}$, $D_{s_i}F_{s_i}$) is called the unknown part.

Definition 6: [33] Let

$$G_{N_1} = (A_{s_1}, B_{s_1}T_{s_1}, C_{s_1}I_{s_1}, D_{s_1}F_{s_1}) \text{ and } G_{N_2} = (A_{s_2}, B_{s_2}T_{s_1}, C_{s_2}I_{s_2}, D_{s_2}F_{s_2})$$

be two GsvNQns. $A_{s_1} = A_{s_2}$, $A_{s_1} = A_{s_2}$, $A_{s_1} = A_{s_2}$, $A_{s_1} = A_{s_2}$ and $T_{s_1} = T_{s_2}$, $I_{s_1} = I_{s_2}$, $F_{s_1} = F_{s_2}$ if and only if we say G_{N_1} is a equal to G_{N_2} and denote it by $G_{N_1} = G_{N_2}$.

Definition 7: [33] Let

$$G_{N_1} = (A_{s_1}, B_{s_1}T_{s_1}, C_{s_1}I_{s_1}, D_{s_1}F_{s_1}) \text{ and } G_{N_2} = (A_{s_2}, B_{s_2}T_{s_1}, C_{s_2}I_{s_2}, D_{s_2}F_{s_2})$$

be two GsvNQns. $A_{s_1} \subset A_{s_2}$, $A_{s_1} \subset A_{s_2}$, $A_{s_1} \subset A_{s_2}$, $A_{s_1} \subset A_{s_2}$ and $T_{s_1} \le T_{s_2}$, $I_{s_1} \le I_{s_2}$, $F_{s_1} \le F_{s_2}$, if and only if we say G_{N_1} is a subset of G_{N_2} and denote it by $G_{N_1} \subset G_{N_2}$.

3 Generalized Hamming Similarity Measure for Generalized Set-Valued Neutrosophic Quadruple Numbers

Now, we define generalized Hamming similarity measure for GsvNQn. Also, we assume that T, I, F ∈ [0, 1], as in SvNn, in this paper.

Definition 8: Let X be a non – empty set,

$$G_{N_1} = (A_{s_1}, B_{s_1}T_{s_1}, C_{s_1}I_{s_1}, D_{s_1}F_{s_1}) \text{ and } G_{N_2} = (A_{s_2}, B_{s_2}T_{s_1}, C_{s_2}I_{s_2}, D_{s_2}F_{s_2})$$

be two GsvNQns, $S_H : G_{N_1} \times G_{N_j} \to [0, 1]$ be a function. Then,

$$S_H(G_{N_1}, G_{N_2}) = 1 - \frac{1}{2}\left[\frac{|T_1 - T_2| + |I_1 - I_2| + |F_1 - F_2|}{3} + \frac{4 - \left[\frac{s(K_1 \cap K_2)}{\max\{s(K_1 \cup K_2),1\}} + \frac{s(L_1 \cap L_2)}{\max\{s(L_1 \cup L_2),1\}} + \frac{s(M_1 \cap M_2)}{\max\{s(M_1 \cup M_2),1\}} + \frac{s(N_1 \cap N_2)}{\max\{s(N_1 \cup N_2),1\}}\right]}{4}\right]$$

is called generalized Hamming similarity measure for GsvNQns.

Where, s(A) is the number of element of A ∈ X.

Theorem 2: Let X be a non – empty set;

$$G_{N_1} = (A_{s_1}, B_{s_1}T_{s_1}, C_{s_1}I_{s_1}, D_{s_1}F_{s_1}),\ G_{N_2} = (A_{s_2}, B_{s_2}T_{s_1}, C_{s_2}I_{s_2}, D_{s_2}F_{s_2}) \text{ and } G_{N_3} = (A_{s_3}, B_{s_3}T_{s_3}, C_{s_3}I_{s_3}, D_{s_3}F_{s_3})$$

be three GsvNQns, $S_H : G_{N_1} \times G_{N_j} \to [0, 1]$ be generalized Hamming similarity measure in Definition 8. Then, S_H satisfies the below conditions.

i) S_H (G_{N_1}, G_{N_2}) ∈ [0, 1]

ii) S_H (G_{N_1}, G_{N_2}) = 1 ⇔ $G_{N_1} = G_{N_2}$

iii) S_H (G_{N_1}, G_{N_2}) = S_H (G_{N_1}, G_{N_2})

iv) If $G_{N_1} \subset G_{N_2} \subset G_{N_3}$, then

S_H (G_{N_1}, G_{N_3}) ≤ S_H (G_{N_1}, G_{N_2}) and S_H (G_{N_1}, G_{N_3}) ≤ S_H (G_{N_2}, G_{N_3}).

Proof:

i) Let G_{N_1} = G_{N_2}. Then,

$S_H(G_{N_1}, G_{N_1})$ =

$$1-\frac{1}{2}\left[\frac{|T_1-T_1|+|I_1-I_1|+|F_1-F_1|}{3}+\frac{4-\left[\frac{s(K_1\cap K_i)}{\max\{s(K_1\cup K_1),1\}}+\frac{s(L_1\cap L_1)}{\max\{s(L_1\cup L_1),1\}}+\frac{s(M_1\cap M_1)}{\max\{s(M_1\cup M_1),1\}}+\frac{s(N_1\cap N_1)}{\max\{s(N_1\cup N_1),1\}}\right]}{4}\right]$$

$$= 1-\frac{1}{2}\left[\frac{0+0+0}{3}+\frac{4-[1+1+1+1]}{4}\right] \quad (1)$$

Thus, $\max\{S_H(G_{N_1}, G_{N_1})\}$ = 1.

Now, let $K_1 \cap K_2 = \emptyset$, $L_1 \cap L_2 = \emptyset$, $M_1 \cap M_2 = \emptyset$, $N_1 \cap N_2 = \emptyset$, $|T_1 - T_2| = 1$, $|I_1 - I_2| = 1$ and $|F_1 - F_2|$ = 1. Then,

$$S_H(G_{N_1}, G_{N_2}) = 1-\frac{1}{2}\left[\frac{|T_1-T_2|+|I_1-I_2|+|F_1-F_2|}{3}+\frac{4-\left[\frac{s(K_1\cap K_2)}{\max\{s(K_1\cup K_2),1\}}+\frac{s(L_1\cap L_2)}{\max\{s(L_1\cup L_2),1\}}+\frac{s(M_1\cap M_2)}{\max\{s(M_1\cup M_2),1\}}+\frac{s(N_1\cap N_2)}{\max\{s(N_1\cup N_2),1\}}\right]}{4}\right]$$

$$= 1-\frac{1}{2}\left[\frac{1+1+1}{3}+\frac{4-[0+0+0+0]}{4}\right]$$

$$= 0.$$

Thus, $\min\{S_H(G_{N_1}, G_{N_1})\}$ = 0. Hence, we obtain

$$S_H\,(G_{N_1}, G_{N_2}) \in [0, 1].$$

ii) Let G_{N_1} = G_{N_2}. From (1), we obtain $S_H(G_{N_i}, G_{N_j})$ = 1. We assume that

$$S_H(G_{N_i}, G_{N_j}) = 1-\frac{1}{2}\left[\frac{|T_1-T_2|+|I_1-I_2|+|F_1-F_2|}{3}+\frac{4-\left[\frac{s(K_1\cap K_2)}{\max\{s(K_1\cup K_2),1\}}+\frac{s(L_1\cap L_2)}{\max\{s(L_1\cup L_2),1\}}+\frac{s(M_1\cap M_2)}{\max\{s(M_1\cup M_2),1\}}+\frac{s(N_1\cap N_2)}{\max\{s(N_1\cup N_2),1\}}\right]}{4}\right]$$

$$= 1.$$

Where, it must be

$$\frac{1}{2}\left[\frac{|T_1-T_2|+|I_1-I_2|+|F_1-F_2|}{3}+\frac{4-\left[\frac{s(K_1\cap K_2)}{\max\{s(K_1\cup K_2),1\}}+\frac{s(L_1\cap L_2)}{\max\{s(L_1\cup L_2),1\}}+\frac{s(M_1\cap M_2)}{\max\{s(M_1\cup M_2),1\}}+\frac{s(N_1\cap N_2)}{\max\{s(N_1\cup N_2),1\}}\right]}{4}\right] = 0.$$

Thus,

$$|T_1 - T_2| + |I_1 - I_2| + |F_1 - F_2| = 0$$

and

$$\left[\frac{s(K_1\cap K_2)}{\max\{s(K_1\cup K_2),1\}}+\frac{s(L_1\cap L_2)}{\max\{s(L_1\cup L_2),1\}}+\frac{s(M_1\cap M_2)}{\max\{s(M_1\cup M_2),1\}}+\frac{s(N_1\cap N_2)}{\max\{s(N_1\cup N_2),1\}}\right] = 4.$$
(2)

From (2), we obtain that

$$|T_1 - T_2| = |I_1 - I_2| = |F_1 - F_2| = 0$$

and

$$\frac{s(K_1\cap K_2)}{\max\{s(K_1\cup K_2),1\}} = \frac{s(L_1\cap L_2)}{\max\{s(L_1\cup L_2),1\}} = \frac{s(M_1\cap M_2)}{\max\{s(M_1\cup M_2),1\}} = \frac{s(N_1\cap N_2)}{\max\{s(N_1\cup N_2),1\}} = 1.$$

Thus, we have that

$$T_1 = T_2,\ I_1 = I_2,\ F_1 = F_2,\ K_1 = K_2,\ L_1 = L_2,\ M_1 = M_2,\ N_1 = N_2.$$

Therefore, from Definition 6; we obtain

$$G_{N_1} = G_{N_2}$$

iii)

$$S_H(G_{N_1}, G_{N_2}) = 1 - \frac{1}{2}\left[\frac{|T_1-T_2|+|I_1-I_2|+|F_1-F_2|}{3} + \frac{4-\left[\frac{s(K_1\cap K_2)}{\max\{s(K_1\cup K_2),1\}}+\frac{s(L_1\cap L_2)}{\max\{s(L_1\cup L_2),1\}}+\frac{s(M_1\cap M_2)}{\max\{s(M_1\cup M_2),1\}}+\frac{s(N_1\cap N_2)}{\max\{s(N_1\cup N_2),1\}}\right]}{4}\right]$$

$$= 1 - \frac{1}{2}\left[\frac{|T_2-T_1|+|I_2-I_1|+|F_2-F_1|}{3} + \frac{4-\left[\frac{s(K_2\cap K_1)}{\max\{s(K_2\cup K_1),1\}}+\frac{s(L_2\cap L_1)}{\max\{s(L_2\cup L_1),1\}}+\frac{s(M_2\cap M_1)}{\max\{s(M_2\cup M_1),1\}}+\frac{s(N_2\cap N_1)}{\max\{s(N_2\cup N_1),1\}}\right]}{4}\right]$$

$$= S_H(G_{N_2}, G_{N_1}).$$

iv) Let $G_{N_1} \subset G_{N_2} \subset G_{N_3}$. From Definition 7, we obtain that

$T_1 < T_2 < T_3$,

$I_1 < I_2 < T_3$,

$F_1 < F_2 < T_3$,

$K_1 \subset K_2 \subset K_3$,

$L_1 \subset L_2 \subset L_3$,

$M_1 \subset M_2 \subset M_3$,

$N_1 \subset N_2 \subset N_3$. (3)

From (3), we have that

$$\frac{s(K_1\cap K_2)}{\max\{s(K_1\cup K_2),1\}} + \frac{s(L_1\cap L_2)}{\max\{s(L_1\cup L_2),1\}} + \frac{s(M_1\cap M_2)}{\max\{s(M_1\cup M_2),1\}} + \frac{s(N_1\cap N_2)}{\max\{s(N_1\cup N_2),1\}} >$$
$$\frac{s(K_1\cap K_3)}{\max\{s(K_1\cup K_3),1\}} + \frac{s(L_1\cap L_3)}{\max\{s(L_1\cup L_3),1\}} + \frac{s(M_1\cap M_3)}{\max\{s(M_1\cup M_3),1\}} + \frac{s(N_1\cap N_3)}{\max\{s(N_1\cup N_3),1\}}.$$
(4)

Also, from (4), we have that

$$|T_1 - T_2| + |I_1 - I_2| + |F_1 - F_2| < |T_1 - T_3| + |I_1 - I_3| + |F_1 - F_3|.$$
(5)

Thus, from (4) and (5), we obtain that

$$\frac{1}{2}\left[\frac{|T_1-T_2|+|I_1-I_2|+|F_1-F_2|}{3} + \frac{4-\left[\frac{s(K_1\cap K_2)}{\max\{s(K_1\cup K_2),1\}}+\frac{s(L_1\cap L_2)}{\max\{s(L_1\cup L_2),1\}}+\frac{s(M_1\cap M_2)}{\max\{s(M_1\cup M_2),1\}}+\frac{s(N_1\cap N_2)}{\max\{s(N_1\cup N_2),1\}}\right]}{4}\right] <$$
$$\frac{1}{2}\left[\frac{|T_1-T_3|+|I_1-I_3|+|F_1-F_3|}{3} + \frac{4-\left[\frac{s(K_1\cap K_3)}{\max\{s(K_1\cup K_3),1\}}+\frac{s(L_1\cap L_3)}{\max\{s(L_1\cup L_3),1\}}+\frac{s(M_1\cap M_3)}{\max\{s(M_1\cup M_3),1\}}+\frac{s(N_1\cap N_3)}{\max\{s(N_1\cup N_3),1\}}\right]}{4}\right]. \quad (6)$$

Hence, from (6), we have that

$$1 - \frac{1}{2}\left[\frac{|T_1-T_3|+|I_1-I_3|+|F_1-F_3|}{3} + \frac{4-\left[\frac{s(K_1\cap K_3)}{\max\{s(K_1\cup K_3),1\}}+\frac{s(L_1\cap L_3)}{\max\{s(L_1\cup L_3),1\}}+\frac{s(M_1\cap M_3)}{\max\{s(M_1\cup M_3),1\}}+\frac{s(N_1\cap N_3)}{\max\{s(N_1\cup N_3),1\}}\right]}{4}\right] <$$
$$1 - \frac{1}{2}\left[\frac{|T_1-T_2|+|I_1-I_2|+|F_1-F_2|}{3} + \frac{4-\left[\frac{s(K_1\cap K_2)}{\max\{s(K_1\cup K_2),1\}}+\frac{s(L_1\cap L_2)}{\max\{s(L_1\cup L_2),1\}}+\frac{s(M_1\cap M_2)}{\max\{s(M_1\cup M_2),1\}}+\frac{s(N_1\cap N_2)}{\max\{s(N_1\cup N_2),1\}}\right]}{4}\right].$$

Therefore, we obtain $S_H(G_{N_1}, G_{N_3}) \leq S_H(G_{N_1}, G_{N_2})$.

Also, S_H $(G_{N_1}, G_{N_3}) \leq S_H$ (G_{N_2}, G_{N_3}) can be proved similar to $S_H(G_{N_1}, G_{N_3}) \leq S_H(G_{N_1}, G_{N_2})$.

Example 1: Let X = {k, l, m, n, p, r} be a set, G_{N_1}= ({k, l, m}, {k, l}(0.7), {m, l}(0.4), {n, p, r}(0.1)), G_{N_2}= ({k, l, m, n, r}, {k, l, m, n}(0.8), {n, r}(0.2), {p}(0.2)) be two GsvNQns and $S_H(G_{N_1}, G_{N_2})$ be generalized Hamming similarity measure for GsvNQns. Then,

$$S_H(G_{N_1}, G_{N_2}) = 1 - \frac{1}{2}\left[\frac{|T_1-T_2|+|I_1-I_2|+|F_1-F_2|}{3} + \frac{4-\left[\frac{s(K_1\cap K_2)}{\max\{s(K_1\cup K_2),1\}}+\frac{s(L_1\cap L_2)}{\max\{s(L_1\cup L_2),1\}}+\frac{s(M_1\cap M_2)}{\max\{s(M_1\cup M_2),1\}}+\frac{s(N_1\cap N_2)}{\max\{s(N_1\cup N_2),1\}}\right]}{4}\right]$$

$$= 1 - \frac{1}{2}\left[\frac{|0.7-0.8|+|0.4-0.2|+|0.1-0.2|}{3} + \frac{4-\left[\frac{3}{\max\{5,1\}}+\frac{2}{\max\{4,1\}}+\frac{0}{\max\{4,1\}}+\frac{1}{\max\{3,1\}}\right]}{4}\right]$$

$$= 0.6125.$$

Where,

T_1 = 0.7, I_1 = 0.4, F_1 = 0.1; K_1 = {k, l, m}, L_1 = {k, l}, M_1 = {l, m}, N_1 = {n, p, r};

T_2 = 0.8, I_2 = 0.2, F_2 = 0.2; K_2 = {k, l, m, n, r}, L_2 = {k, l, m, n}, M_2 = {n, r}, N_2 = {p}.

4 Algorithm for Multi-Criteria Decision-Making Application

In this section, we rearranged the algorithm in Aslan et al. [2] for GsvNQns. Also, in this new algorithm, we used generalized Hamming similarity measure in section 3. So, we use the GsvNQns and generalized Hamming similarity measure instead of SvNns and similarity measure in algorithm [2]. Also, we assume that X is a nonempty set.

Step 1: The criteria are determined by considering the application. Let the set of criteria of laws be

$$K = \{k_1, k_2, \dots, k_m\}.$$

Step 2: The weight values of the criteria for the application. Let the set of weight values be

$$W = \{w_1, w_2, \dots, w_m\}.$$

Where,

the weight value of criterion k_1 is w_1,

the weight value of criterion k_2 is w_2,

the weight value of criterion k_3 is w_3,

.

.

.

the weight value of criterion k_m is w_m,

Also, $w_i \in [0, 1]$ and $\sum_{i=1}^{m} w_i = 1$.

Step 3: The ideal object is determined as GsvNQs according to criterias in Step 1 such that

$I = \{k_1:(A_{I_1}, B_{I_1}T_{I_1}, C_{I_1}I_{I_1}, D_{I_1}F_{I_1}), k_2:(A_{I_2}, B_{I_2}T_{I_2}, C_{I_2}I_{I_2}, D_{I_2}F_{I_2}), \ldots, k_m:(A_{I_m}, B_{I_m}T_{I_m}, C_{I_m}I_{I_m}, D_{I_m}F_{I_m}), \quad A_{I_i}, B_{I_i}, C_{I_i}, D_{I_i} \in P(X); i = 1, 2, 3, \ldots, m\}.$

Step 4: The n objects are determined as GsvNQs according to criterias in Step 1 such that

$O_1 = \{k_1:(A_{O_{1_1}}, B_{O_{1_1}}T_{O_{1_1}}, C_{O_{1_1}}I_{O_{1_1}}, D_{O_{1_1}}F_{O_{1_1}}), k_2:(A_{O_{1_2}}, B_{O_{1_2}}T_{O_{1_2}}, C_{O_{1_2}}I_{O_{1_2}}, D_{O_{1_2}}F_{O_{1_2}}), \ldots, k_m:(A_{O_{1_m}}, B_{O_{1_m}}T_{O_{1_m}}, C_{O_{1_m}}I_{O_{1_m}}, D_{O_{1_m}}F_{O_{1_m}}), \quad A_{O_{1_i}}, B_{O_{1_i}}, C_{O_{1_i}}, D_{O_{1_i}} \in P(X); i = 1, 2, 3, \ldots, m\}$

$O_2 = \{k_1:(A_{O_{2_1}}, B_{O_{2_1}}T_{O_{2_1}}, C_{O_{2_1}}I_{O_{2_1}}, D_{O_{2_1}}F_{O_{2_1}}), k_2:(A_{O_{2_2}}, B_{O_{2_2}}T_{O_2}, C_{O_{2_2}}I_{O_{2_2}}, D_{O_{2_2}}F_{O_{2_2}}), \ldots, k_m:(A_{O_{2_m}}, B_{O_{2_m}}T_{O_{2_m}}, C_{O_{2_m}}I_{O_{2_m}}, D_{O_{2_m}}F_{O_{2_m}}), \quad A_{O_{2_i}}, B_{O_{2_i}}, C_{O_{2_i}}, D_{O_{2_i}} \in P(X); i = 1, 2, 3, \ldots, m\}$

.

.

.

$O_n = \{k_1:(A_{O_{n_1}}, B_{O_{n_1}}T_{O_{n_1}}, C_{O_{n_1}}I_{O_{n_1}}, D_{O_{n_1}}F_{O_{n_1}}), k_2:(A_{O_{n_2}}, B_{O_{n_2}}T_{O_{n_2}}, C_{O_{n_2}}I_{O_{n_2}}, D_{O_{n_2}}F_{O_{n_2}}), \ldots, k_m:(A_{O_{n_m}}, B_{O_{n_m}}T_{O_{n_m}}, C_{O_{n_m}}I_{O_{n_m}}, D_{O_{n_m}}F_{O_{n_m}}), \quad A_{O_{n_i}}, B_{O_{n_i}}, C_{O_{n_i}}, D_{O_{n_i}} \in P(X); i = 1, 2, 3, \ldots, m\}$

Step 5: The objects given in Step 4 are stated in the form of table (Table 1).

Table 1. Table of objects

	k_1	k_2	$\dots$	k_m
O_1	$(A_{O_{1_1}}, B_{O_{1_1}}T_{O_{1_1}}, C_{O_{1_1}}I_{O_{1_1}}, D_{O_{1_1}}F_{O_{1_1}})$	$(A_{O_{1_2}}, B_{O_{1_2}}T_{O_{1_2}}, C_{O_{1_2}}I_{O_{1_2}}, D_{O_{1_2}}F_{O_{1_2}})$	$\dots$	$(A_{O_{1_m}}, B_{O_{1_m}}T_{O_{1_m}}, C_{O_{1_m}}I_{O_{1_m}}, D_{O_{1_m}}F_{O_{1_m}})$
O_2	$(A_{O_{2_1}}, B_{O_{2_1}}T_{O_{2_1}}, C_{O_{2_1}}I_{O_{2_1}}, D_{O_{2_1}}F_{O_{2_1}})$	$(A_{O_{2_2}}, B_{O_{2_2}}T_{O_2}, C_{O_{2_2}}I_{O_{2_2}}, D_{O_{2_2}}F_{O_{2_2}})$	$\dots$	$(A_{O_{2_m}}, B_{O_{2_m}}T_{O_{2_m}}, C_{O_{2_m}}I_{O_{2_m}},D_{O_{2_m}}F_{O_{2_m}})$
.	.	.	$\dots$	.
.	.	.	.	.
.	.	.	.	.
O_n	$(A_{O_{n_1}}, B_{O_{n_1}}T_{O_{n_1}}, C_{O_{n_1}}I_{O_{n_1}}, D_{O_{n_1}}F_{O_{n_1}})$	$(A_{O_{n_2}}, B_{O_{n_2}}T_{O_{n_2}}, C_{O_{n_2}}I_{O_{n_2}}, D_{O_{n_2}}F_{O_{n_2}})$	$\dots$	$(A_{O_{n_m}}, B_{O_{n_m}}T_{O_{n_m}}, C_{O_{n_m}}I_{O_{n_m}},D_{O_{n_m}}F_{O_{n_m}})$

Step 6: In this step, the similarity value of the criteria of the ideal object and the criteria of other objects are calculated by using Table 1 with S_H in Section 3. So, $S_H(I_{k_j}, O_{i_{k_j}})$ is calculated for i = 1, 2, …, n; j = 1, 2, …, m. After all calculations, Table 2 is obtained.

Table 2. Similarity of the criterias of object to the criteria of ideal object

	k_1	k_2	$\dots$	k_m
O_1	$S_H(I_{k_1}, O_{1_{k_1}})$	$S_H(I_{k_2}, O_{1_{k_2}})$	$\dots$	$S_H(I_{k_m}, O_{1_{k_m}})$
O_2	$S_H(I_{k_1}, O_{2_{k_1}})$	$S_H(I_{k_2}, O_{2_{k_2}})$	$\dots$	$S_H(I_{k_m}, O_{2_{k_m}})$
.	.	.	$\dots$	.
.	.	.	.	.
.	.	.	.	.
O_n	$S_H(I_{k_1}, O_{n_{k_1}})$	$S_H(I_{k_2}, O_{n_{k_2}})$	$\dots$	$S_H(I_{k_m}, O_{n_{k_m}})$

Step 7: The weight value of each criterion given in Step 2 is multiplied by the similarity values in Table 2. Hence, the weighted similarity of the criterias of object to the criteria of ideal object in Table 3 is obtained.

Table 3. Weighted Similarity of the criterias of object to the criteria of ideal object

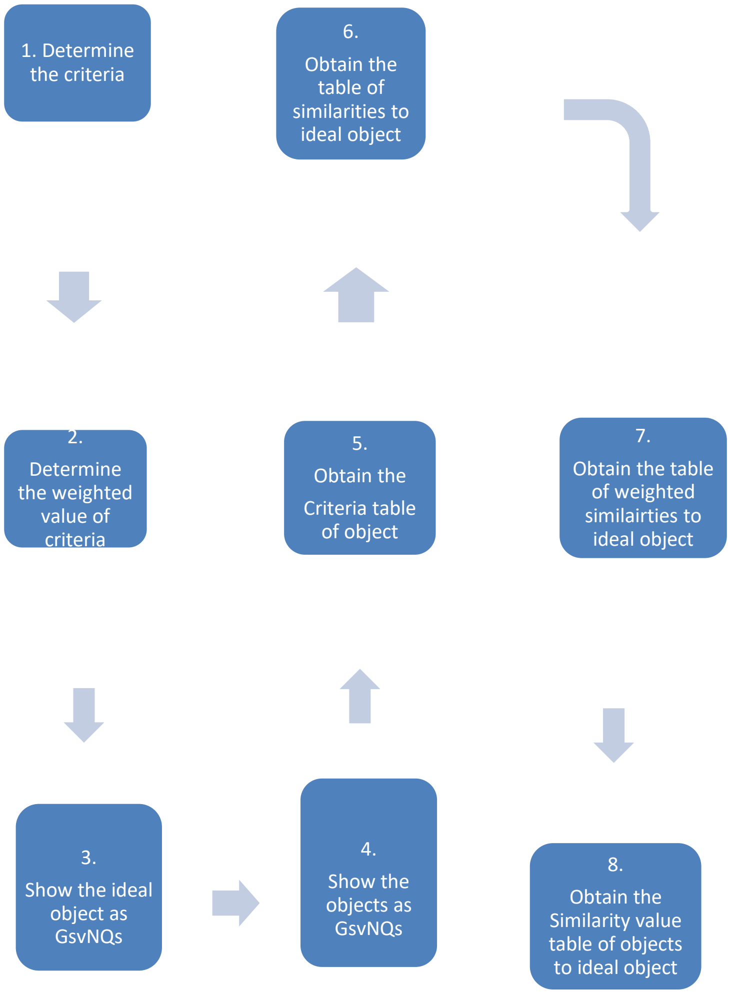
	w_1k_1	w_2k_2	…	w_mk_m
O_1	$w_1.S_H(I_{k_1}, O_{1_{k_1}})$	$w_2.S_H(I_{k_2}, O_{1_{k_2}})$	…	$w_m.S_H(I_{k_m}, O_{1_{k_m}})$
O_2	$w_1.S_H(I_{k_1}, O_{2_{k_1}})$	$w_2.S_H(I_{k_2}, O_{2_{k_2}})$	…	$w_m.S_H(I_{k_m}, O_{2_{k_m}})$
.	.	.	…	.
.	.	.	.	.
.	.	.	.	.
O_n	$w_1.S_H(I_{k_1}, O_{n_{k_1}})$	$w_2.S_H(I_{k_2}, O_{n_{k_2}})$	…	$w_m.S_H(I_{k_m}, O_{n_{k_m}})$

Step 8: In this last step, the weighted similarity values for each objects given in Table 7 are added and the similarity ratio of each law over the ideal law is obtained. So,

S_{H^t} (I, O_t) = $\sum_{t=1}^{m} w_t.\,S_H(I_{k_t}, O_{n_{k_t}})$ is calculated for k = 1, 2, …, m. After all calculations, Table 4 is obtained.

Table 4. The similarity value of the object' to the ideal object

	Similarity Value
O_1	S_{H^1} (I, O_1)
O_2	S_{H^2} (I, O_2)
.	.
.	.
.	.
O_n	S_{H^n} (I, O_n)



Graph 1: Diagram of the algorithm.

5 Multi-Criteria Decision-Making Application

We assume that four different state laws should be created to make use of night watchmen in places where police are inactive at night in four different states. We used the algorithm in Section 4 to find out which law in which state is more effective after a period of time.

Step 1: Let $K = \{k_1,\ k_2,\ k_3\}$ be set of criterias such that

k_1 = life safety

k_2 = property safety

k_3 = cost

Step 2: Let W = {0.6, 0.3, 0.1} be set of the weight values such that

0.6 for the criterion k_1

0.3 for the criterion k_2

0.1 for the criterion k_3

Step 3: Let the ideal law of state be I such that

$$I = \begin{Bmatrix} k_1: \left(\{p_1, \dots, p_4, q_1, \dots, q_4, r_1, \dots, r_4, t_1, \dots, t_4\}, \{p_1, \dots, p_4, q_1, \dots, q_4, r_1, \dots, r_4, t_1, \dots, t_4\}(1),\ \emptyset(0),\ \emptyset(0)\right), \\ k_2: \left(\{p_1, \dots, p_4, q_1, \dots, q_4, r_1, \dots, r_4, t_1, \dots, t_4\}, \{p_1, \dots, p_4, q_1, \dots, q_4, r_1, \dots, r_4, t_1, \dots, t_4\}(1),\ \emptyset(0),\ \emptyset(0)\right), \\ k_3: \left(\{p_1, \dots, p_4, q_1, \dots, q_4, r_1, \dots, r_4, t_1, \dots, t_4\}, \{p_1, \dots, p_4, q_1, \dots, q_4, r_1, \dots, r_4, t_1, \dots, t_4\}(1),\ \emptyset(0),\ \emptyset(0)\right) \end{Bmatrix}$$

Where, $\{p_1, \dots, p_4, q_1, \dots, q_4, r_1, \dots, r_4, t_1, \dots, t_4\}$ is known part and

$\{p_1, \dots, p_4, q_1, \dots, q_4, r_1, \dots, r_4, t_1, \dots, t_4\}(1),\ \emptyset(0),\ \emptyset(0)$ is unknown part for each criteria. Where, T = 1, I = 0 and F = 0. This means that this law gave exactly the desired result. Therefore, this law is the ideal law.

Also,

p_1: Pedestrian police with night watchmen who drive a vehicle from 7.00 p.m to 10.00 p.m

p_2: Pedestrian night watchmen with police who drive a vehicle from 1.00 a.m to 4.00 a.m

p_3: Pedestrian police with pedestrian night watchmen from 7.00 p.m to 10.00 p.m

p_4: Police who drive a vehicle with night watchmen who drive a vehicle from 7.00 p.m to 10.00 p.m

q_1: Police who drive a vehicle with night watchmen who drive a vehicle from 7.00 p.m to 10.00 p.m

q_2: Pedestrian police with pedestrian night watchmen from 1.00 a.m to 4.00 a.m

q_3: Pedestrian night watchmen with police who drive a vehicle from 7.00 p.m to 10.00 p.m

q_4: Pedestrian police with night watchmen who drive a vehicle from 7.00 p.m to 10.00 p.m

r_1: Pedestrian police with pedestrian night watchmen from 7.00 p.m to 10.00 p.m

r_2: Police who drive a vehicle with night watchmen who drive a vehicle from 1.00 a.m to 4.00 a.m

r_3: Pedestrian police with night watchmen who drive a vehicle from 7.00 p.m to 10.00 p.m

r_4: Pedestrian night watchmen with police who drive a vehicle from 7.00 p.m to 10.00 p.m

t_1: Pedestrian night watchmen with police who drive a vehicle from 7.00 p.m to 10.00 p.m

t_2: Pedestrian police with night watchmen who drive a vehicle from 1.00 a.m to 4.00 a.m

t_3: Police who drive a vehicle with night watchmen who drive a vehicle from 7.00 p.m to 10.00 p.m

t_4: Pedestrian police with pedestrian night watchmen from7.00 p.m to 10.00 p.m

Step 4: Let L = $\{L_1, L_2, L_3, L_4\}$ be set of law of states such that

$$L_1=\begin{Bmatrix} k_1: (\{p_1, p_2, p_3, p_4\}, \{p_1, p_2\}(0.8),\ \{p_4\}(0.2),\ \{p_3\}(0.1)), \\ k_2: (\{p_1, p_2, p_3, p_4\}, \{p_3\}(0.8),\ \{p_1\}(0.3),\ \{, p_2, p_4\}(0.1)), \\ k_3: (\{p_1, p_2, p_3, p_4\}, \{p_1, p_2, p_3, \}(0.9),\ \emptyset(0),\ \{p_4\}(0.3)) \end{Bmatrix}$$

$$L_2=\begin{Bmatrix} k_1: (\{q_1, q_2, q_3, q_4\}, \{q_1, q_2, q_3\}(0.8),\ \{q_4\}(0.4),\ \emptyset(0)), \\ k_2: (\{q_1, q_2, q_3, q_4\}, \{q_1, q_2, q_3\}(0.5),\ \emptyset(0),\ \{q_4\}(0.4)), \\ k_3: (\{q_1, q_2, q_3, q_4\}, \{q_3, q_4\}(0.4),\ \{q_1\}(0.1),\ \{q_2\}(0.7)) \end{Bmatrix}$$

$$L_3=\begin{Bmatrix} k_1: (\{r_1, r_2, r_3, r_4\}, \{r_1\}(0.9),\ \{r_2, r_3\}(0.2),\ \{r_4\}(0.3)), \\ k_2: (\{r_1, r_2, r_3, r_4\}, \{r_1, r_2, r_3, r_4\}(0.9),\ \emptyset(0),\ \emptyset(0)), \\ k_3: (\{r_1, r_2, r_3, r_4\}, \{r_1, r_4\}(0.6),\ \{r_2\}(0.4),\ \{r_3\}(0.3)) \end{Bmatrix}$$

$$L_4=\begin{Bmatrix} k_1: (\{t_1, t_2, t_3, t_4\}, \{t_4\}(0.9),\ \{t_1, t_2\}(0.1),\ \{t_3\}(0.1)), \\ k_2: (\{t_1, t_2, t_3, t_4\}, \{t_2, t_4\}(0.7),\ \{t_3\}(0.5),\ \{t_1\}(0.2)), \\ k_3: (\{t_1, t_2, t_3, t_4\}, \{t_1, t_2, t_4\}(0.4),\ \{t_3\}(0.5),\ \emptyset(0)) \end{Bmatrix}$$

Step 5: We obtain Table 5 according to Step 4

Table 5. Table of laws

	k_1	k_2	k_3
L_1	$\left(\begin{matrix}\{p_1,p_2,p_3,p_4\},\{p_1,p_2\}(0.8),\\ \{p_4\}(0.2),\{p_3\}(0.1)\end{matrix}\right)$	$\left(\begin{matrix}\{p_1,p_2,p_3,p_4\},\{p_3\}(0.8),\{p_1\}(0.3),\\ \{,p_2,p_4\}(0.1)\end{matrix}\right.$	$\left(\begin{matrix}\{p_1,p_2,p_3,p_4\},\{p_1,p_2,p_3,\}(0.9),\emptyset(0),\\ \{p_4\}(0.3)\end{matrix}\right)$
L_2	$\left(\begin{matrix}\{q_1,q_2,q_3,q_4\},\{q_1,q_2,q_3\}(0.8),\\ \{q_4\}(0.4),\emptyset(0)\end{matrix}\right)$	$\left(\begin{matrix}\{q_1,q_2,q_3,q_4\},\{q_1,q_2,q_3\}(0.5),\emptyset(0\\ \{q_4\}(0.4)\end{matrix}\right.$	$\left(\begin{matrix}\{q_1,q_2,q_3,q_4\},\{q_3,q_4\}(0.4),\{q_1\}(0.1),\\ \{q_2\}(0.7)\end{matrix}\right)$
L_3	$\left(\begin{matrix}\{r_1,r_2,r_3,r_4\},\{r_1\}(0.9),\{r_2,r_3\}(0.2\\ \{r_4\}(0.3)\end{matrix}\right.$	$\left(\begin{matrix}\{r_1,r_2,r_3,r_4\},\{r_1,r_2,r_3,r_4\}(0.9),\emptyset(0\\ \emptyset(0)\end{matrix}\right.$	$\left(\begin{matrix}\{r_1,r_2,r_3,r_4\},\{r_1,r_4\}(0.6),\{r_2\}(0.4),\\ \{r_3\}(0.3)\end{matrix}\right)$
L_4	$\left(\begin{matrix}\{t_1,t_2,t_3,t_4\},\{t_4\}(0.9),\{t_1,t_2\}(0.1\\ \{t_3\}(0.1)\end{matrix}\right.$	$\left(\begin{matrix}\{t_1,t_2,t_3,t_4\},\{t_2,t_4\}(0.7),\{t_3\}(0.5\\ \{t_1\}(0.2)\end{matrix}\right.$	$\left(\begin{matrix}\{t_1,t_2,t_3,t_4\},\{t_1,t_2,t_4\}(0.4),\{t_3\}(0.5),\\ \emptyset(0)\end{matrix}\right)$

Step 6: We obtain similarity of the criterias of law to the criteria of ideal law in Table 6.

Table 6. Similarity of the criterias of law to the criteria of ideal law

	k_1	k_2	k_3
L_1	0.322917	0.306250	0.339583
L_2	0.400000	0.350000	0.266667
L_3	0.400000	0.483333	0.316667
L_4	0.450000	0.333333	0.316667

Step 7: We obtain weighted similarity of the criterias of law to the criterias of ideal law in Table 7.

Table 7. Weighted similarity of the criterias of law to the criterias of ideal law

	$(0.6).k_1$	$(0.3).k_2$	$(0.1).k_3$
L_1	0.19375	0.091875	0.033958
L_2	0.24000	0.10500	0.026667
L_3	0.24000	0.144999	0.031667
L_4	0.27000	0.099990	0.031667

Step 8: We obtain similarity value of the object' to the ideal object in Table 8.

Table 8. The similarity value of the law' to the ideal law

	Similarity value
L_1	S_{H^1} (I, L_1) = 0.319583
L_2	S_{H^2} (I, L_2) = 0.371667
L_3	S_{H^3} (I, L_3) = 0.41666
L_4	S_{H^4} (I, L_4) = 0.31958

From Table 8, the laws that work best are L_3, L_2, L_1 and L_4, respectively.

6 Comparison Method

In this section, we compared the results of the generalized algorithm based on the generalized Hamming similarity measure and GsvNQn with the results of the algorithm [2] based on the Hamming similarity measure and SvNn.

If only the T, I, F components of the GsvNQns are in Section 5, we obtain in Table 9.

Table 9. Table of laws based on only (T, I , F)

	k_1	k_2	k_3
L_1	(0.8, 0.2, 01)	(0.8, 0.3, 0.1)	(0.9, 0.0, 0.3)
L_2	(0.8, 0.4, 0.0)	(0.5, 0.0, 0.4)	(0.4, 0.1, 0.7)
L_3	(0.9, 0.2, 0.3)	(0.9, 0.0, 0.0)	(0.6, 0.4, 0.3)
L_4	(0.9, 0.1, 01)	(0.7, 0.5, 0.2)	(0.4, 0.5, 0.0)

If we used the Hamming similarity measure [22] with algorithm [2] according to Table 9, we obtain Table 10 for choosing the best laws.

Table 10. The similarity value of the law' to the ideal law according to Hamming similarity measure [22] and SvNn

	Similarity value
L_1	S_{H^1} (I, L_1) = 0.826656
L_2	S_{H^2} (I, L_2) = 0.74333
L_3	S_{H^3} (I, L_3) = 0.833333
L_4	S_{H^4} (I, L_4) = 0.80333

From Table 10, the laws that work best are L_3, L_1, L_4 and L_2, respectively. Thus, we obtain different result from Section 5.

7 Discussion and Conclusions

In this study, we firstly generalized Hamming similarity measures for the GsvNQn. We showed that generalized Hamming measure satisfies the similarity measure condition. Also, we firstly generalized an algorithm (based on SvNn) for the GsvNQn and we gave a multi-criteria decision-making application using this generalized algorithm. In this application, we examined which of the laws established in different states were more efficient.

From Table 8, if we use generalized Hamming similarity measure and GsvNQn we obtain the laws that work best are

$$L_3,\ L_2,\ L_1 \text{ and } L_4$$

respectively.

From Table 10, if we use Hamming similarity measure and SvNn, we obtain the laws that work best are

$$L_3,\ L_1,\ L_4 \text{ and } L_2$$

respectively. Thus, we obtain different results according to Hamming similarity measure and SvNn in this paper. In addition, the result we obtained in Table 8 is more valid because the generalized set-valued neutrosophic quadruple numbers contain components (T, I, F) of neutrosophic sets and have more extensive components (known part, unknown part) than neutrosophic sets. As can be seen in this study, it is clear that generalized set-valued neutrosophic structures will give more objective results than both the applications using classical structures and the applications using neutrosophic structures.

Also, using this study or revising this application researchers can also work on other law applications and other science applications for decision-making problems. Furthermore, there are a lot of similarity measure for neutrosophic sets. Researchers can generalize the other similarity measures of neutrosophic set according to GsvNQn. Also, in this paper, we use single-valued neutrosophic component T, I, F ∈ [0, 1] (as in SvNn). Researchers can study generalized set-valued neutrosophic quadruple set according to bipolar neutrosophic component or interval valued neutrosophic component and researchers can use these structures for decision-making applications.

Abbreviations

SvNn: Single valued neutrosophic number

SvNs: Single valued neutrosophic set

GsvNQn: Generalized set valued neutrosophic quadruple number

GsvNQs: Generalized set valued neutrosophic quadruple set

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