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How to Derive the Other 37 Valid Modal Syllogisms from the Syllogism ◇A□I◇I-1

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Abstract

Syllogistic reasoning plays an important role in natural language information processing. In order to provide a consistent interpretation for Aristotelian modal syllogistic, this paper firstly proves the validity of the syllogism ◇A□I◇I-1, and then takes it as the basic axiom to derive the other 37 valid modal syllogisms on the basis of some reasoning rules in classical propositional logic, the transformation between any one of Aristotelian quantifiers and its three negative quantifiers, the symmetry of the Aristotelian quantifier *some* and *no*, and some relevant definitions and facts. In other words, there are reducibility between the modal syllogism ◇A□I◇I-1 and the other 37 valid modal syllogisms. There are infinitely many modal syllogism instances in natural language corresponding to every valid modal syllogism, thus this study has important practical significance and theoretical value for natural language information processing in computer science.

Keywords: validity, Aristotelian modal syllogisms, Aristotelian syllogisms, generalized quantifier theory

1. Introduction

Syllogistic reasoning plays a significant role in natural language information processing (Long, 2023). The common syllogisms in natural language are Aristotelian syllogisms (Hui, 2023) and Aristotelian modal syllogisms (Cheng, 2023). So far, the study of the former is relatively mature and complete, while the study of the latter is still inconsistent or even wrong (Xiaojun, 2018). Therefore, this paper focuses on the latter.

There are many research results of modal syllogisms, such as McCall (1963), Thomason (1993, 1997), Johnson (1989, 2004), Triker (1994), Nortmann (1996), Brennan (1997), Malink (2006, 2013), Xiaojun (2020a, 2020b), and so on. Smith (1995) summed up the previous achievements and claimed that Aristotelian modal syllogistic is incoherent. This view is prevailing as usual (Cheng, 2023). This paper tries to overcome this shortcoming. More specifically, the first step in this article is to demonstrate the validity of the syllogism ◇A□I◇I-1, and then takes it as the basic axiom to deduce the other 37 valid modal syllogisms with the help of modern modal logic and generalized quantifier theory.

2. Preliminaries

In this paper, we use the letters *B*, *C* and *D* as the lexical variables in syllogisms, and *U* the universe of lexical variables. Aristotelian syllogisms involve sentences of the following forms: All *B*s are *D*, No *B*s are *D*, Some *B*s are *D*, Not all *B*s are *D*. The four sentences can be formalized as *all*(*B*, *D*), *no*(*B*, *D*), *some*(*B*, *D*), and *not all*(*B*, *D*), and abbreviated as the proposition A, E, I and O, respectively. What is obtained is an Aristotelian modal syllogism, by means of adding one to three non-overlapping necessary operator (i.e.■) or/and possible operator (i.e.＋) to an Aristotelian syllogism.

An Aristotelian modal syllogism can be explained in the form of the following example:

Major premise: All the birds in this tree are possibly frozen to death.

Minor premise: Some crows are necessarily birds on this tree.

Conclusion: Some crows are possibly frozen to death.

Let *B* be the set of all the crows in the universe, *C* be the set of all the birds in the universe, and *D* be the set of all individuals frozen to death in the universe. Thus the formalization of this example is that ＋*all*(*C*, *D*)→(■*some*(*B*, *C*)→＋*some*(*B*, *D*)). This modal syllogism is the first figure, and abbreviated as ＋A■I＋I-1. Other syllogisms can be similarly formalized.

On the basis of generalized quantifier theory (Peters & Westerståhl, 2006) and modal logic (Chellas, 1980), the following definitions, rules and facts can be obtained:

Definition 1:

(1) *all*(*B*, *D*) is true when and only when *B*⊆*D* is true.

(2) *no*(*B*, *D*) is true when and only when *B*∩*D*=∅ is true.

(3) *some*(*B*, *D*) is true when and only when *B*∩*D*≠∅ is true.

(4) *not all*(*B*, *D*) is true when and only when *B*⊈*D* is true

(5) ■*all*(*B*, *D*) is true when and only when *B*⊆*D* is true in any possible world.

(6) ■*no*(*B*, *D*) is true when and only when *B*∩*D*=∅ is true in any possible world.

(7) ■*some*(*B*, *D*) is true when and only when *B*∩*D*≠∅ is true in any possible world.

(8) ■*not all*(*B*, *D*) is true when and only when *B*⊈*D* is true in any possible world.

(9) ＋*all*(*B*, *D*) is true when and only when *B*⊆*D* is true in at least one possible world.

(10) ＋*no*(*B*, *D*) is true when and only when *B*∩*D*=∅ is true in at least one possible world.

(11) ＋*some*(*B*, *D*) is true when and only when *B*∩*D*≠∅ is true in at least one possible world.

(12) ＋*not all*(*B*, *D*) is true when and only when *B*⊈*D* is true in at least one possible world.

Let *Q* be any one of the four Aristotelian quantifiers (that is, *all*, *no*, *some* and *not all*), *Q*¬ and ¬*Q* be the inner and outer negation of *Q*, respectively.

Definition 2: *Q*¬(*B*, *D*) $=_{def}$ *Q*(*B*, *U*–*D*).

Definition 3: ¬*Q*(*B*, *D*) $=_{def}$ It is not that *Q*(*B*, *D*).

Fact 1: (1) *all*(*B*, *D*)=*no*¬(*B*, *D*); (2) *no*(*B*, *D*)=*all*¬(*B*, *D*);

(3) *some*(*B*, *D*)=*not all*¬(*B*, *D*); (4) *not all*(*B*, *D*)=*some*¬(*B*, *D*).

Fact 2: (1) ¬*all*(*B*, *D*)=*not all*(*B*, *D*); (2)¬*no*(*B*, *D*)=*some*(*B*, *D*);

(3) ¬*some*(*B*, *D*)=*no*(*B*, *D*); (4)¬*not all*(*B*, *D*)=*all*(*B*, *D*).

Fact 3: (1) *some*(*B*, *D*)↔*some*(*D*, *B*); (2) *no*(*B*, *D*)↔*no*(*D*, *B*).

Fact 4: (1) ⊢*all*(*B*, *D*)→*some*(*B*, *D*); (2) ⊢*no*(*B*, *D*)→*not all*(*B*, *D*).

The above facts from Fact 1 to Fact 4 are the basic facts in generalized quantifier theory, so their proofs are omitted. In the light of modal logic, it can be seen that ■*Q*(*B*, *D*)$=_{def}$¬＋¬*Q*(*B*, *D*) and ＋*Q*(*B*, *D*)$=_{def}$¬■¬*Q*(*B*, *D*), the following three facts can be obtained:

Fact 5: (1) ¬■*Q*(*B*, *D*)=＋¬*Q*(*B*, *D*); (2) ¬＋*Q*(*B*, *D*) =■¬*Q*(*B*, *D*).

Fact 6 : ⊢■*Q*(*B*, *D*)→*Q*(*B*, *D*).

Fact 7 : ⊢■*Q*(*B*, *D*)→＋*Q*(*B*, *D*).

Let *a*, *b*, *c* and *d* be propositional variables, the following classical propositional logic rules will be used later:

Rule 1 : From ⊢(*a*→(*b*→*c*)) and ⊢(*c*→*d*) infer ⊢(*a*→(*b*→*d*)).

Rule 2 : From ⊢(*a*→(*b*→*c*)) infer ⊢(¬*c*→(*a*→¬*b*)) or ⊢(¬*c*→(*b*→¬*a*)).

3. Reduction between the Syllogism ◇A□I◇I-1 and the Other Modal Syllogisms

Theorem 1 shows that the syllogism ＋A■I＋I-1 is valid. While the following theorems from Theorem 2 to Theorem 11 mean that there are reducibility between the syllogism ＋A□I＋I-1 and the other 37 valid modal syllogisms. For example, '(2.1) ＋A■I＋I-1⇒＋A■I＋I-3' in Theorem 2 indicates that the validity of ＋A□I＋I-3 can be deduced from the validity of ＋A■I＋I-1, and sheds light on the reducibility between the two syllogisms. The meanings of other theorems are similar.

Theorem 1(＋A■I＋I-1): ＋*all*(*C*, *D*)→(■*some*(*B*, *C*)→＋*some*(*B*, *D*)) is valid.

Proof: The syllogism ＋A□I＋I-1 is the abbreviation of ＋*all*(*C*, *D*)→(■*some*(*B*, *C*) →＋*some*(*B*, *D*)) which is the first figure syllogism. Suppose that ＋*all*(*C*, *D*) and ■*some*(*B*, *C*) are true, then *C*⊆*D* is true at least one possible world according to Definition 1 (9), and *B*∩*C*≠∅ is true at any possible world according to Definition 1 (7). Thus it is easily seen that $B \cap D \neq \varnothing$ is true in at least one possible world. It is reasonable to say that ＋*some*(*B*, *D*) is true in line with Definition 1 (11). It follows that ＋*all*(*C*, *D*)→(■*some*(*B*, *C*)→＋*some*(*B*, *D*)) is valid, exactly as desired.

Theorem 2: The validity of the following three syllogisms can be inferred from ＋A■I＋I-1:

(2.1) ＋A■I＋I-1⇒＋A■I＋I-3

(2.2) ＋A■I＋I-1⇒■I＋A＋I-4

(2.3) ＋A■I＋I-1⇒＋A■I＋I-3⇒■I＋A＋I-3

Proof: For (2.1). In line with Theorem 1, it follows that ＋A■I＋I-1 is valid, and whose expansion is that ＋*all*(*C*, *D*)→(■*some*(*B*, *C*)→＋*some*(*B*, *D*)). According to the Fact 3 (1), it can be seen that ■*some*(*B*, *C*)↔■*some*(*C*, *B*). Therefore, it follows that ＋*all*(*C*, *D*)→(■*some*(*C*, *B*) →＋*some*(*B*, *D*)). That is to say that ＋A■I＋I-3 can be deduced from ＋A■I＋I-1. The others can be similarly proved.

Theorem 3: The validity of the following three syllogisms can be inferred from＋A■I＋I-1:

(3.1) ＋A■I＋I-1⇒＋A■E＋E-2

(3.2) ＋A■I＋I-1⇒■E■I■O-3

(3.3) ＋A■I＋I-1⇒＋A■I＋I-3⇒■I＋A＋I-3⇒■E＋A＋E-1

Proof: For (3.1). Theorem 1 has proved that ＋*all*(*C*, *D*)→(■*some*(*B*, *C*)→＋*some*(*B*, *D*)) is valid. And then it can be derived that ¬＋*some*(*B*, *D*)→(＋*all*(*C*, *D*)→¬■*some*(*B*, *C*)) according to Rule 2. Thus one can obtain that ■¬*some*(*B*, *D*)→(＋*all*(*C*, *D*)→＋¬*some*(*B*, *C*)) in line with Fact 5. Then it follows that ¬*some*(*B*, *D*)=*no*(*B*, *D*) and ¬*some*(*B*, *C*)=*no*(*B*, *C*) in terms of the Fact 2 (3). Hence, it can be seen that ■*no*(*B*, *D*)→(＋*all*(*C*, *D*)→＋*no*(*B*, *C*)), i.e. ＋A■E＋E-2 can be derived from ＋A□I＋I-1, as required. The proofs of other cases follow the same pattern as that of (3.1).

Theorem 4: The validity of the following two syllogisms can be inferred from ＋A■I＋I-1:

(4.1) ＋A■I＋I-1⇒＋A■I＋I-3⇒■I＋A＋I-3⇒■O＋A＋O-3

(4.2) ＋A■I＋I-1⇒＋E■I＋O-1

Proof: For (4.1). In line with (2.3) ＋A■I＋I-1⇒＋A■I＋I-3⇒■I＋A＋I-3, it indicates the validity of ■I＋A＋I-3, and whose expansion is that ■*some*(*C*, *B*)→(＋*all*(*C*, *D*)→＋*some*(*D*, *B*)). In terms of Fact 1 (3), it is clear that *some*(*C*, *B*)=*not all*¬(*C*, *B*) and *some*(*D*, *B*)=*not all*¬(*D*, *B*) hold. Then it follows that ■*not all*¬(*C*, *B*)→(＋*all*(*C*, *D*)→＋*not all*¬(*D*, *B*)). According to Definition 2, one can obtain that *not all*¬(*C*, *B*)=*not all*(*C*, *U*–*B*) and *not all*¬(*D*, *B*)=*not all*(*D*, *U*–*B*). Therefore, it can be derived that ■*not all*(*C*, *U*–*B*)→(＋*all*(*C*, *D*)→＋*not all*(*D*, *U*–*B*)) i.e. the syllogism ■O＋A＋O-3 can be inferred from ＋A■I＋I-1. The other case can be similarly demonstrated.

Theorem 5: The validity of the following three syllogisms can be inferred from ＋A■I＋I-1:

(5.1) ＋A■I＋I-1⇒＋A■E＋E-2⇒＋A■E＋O-2

(5.2) ＋A■I＋I-1⇒＋A■I＋I-3⇒■I＋A＋I-3⇒■E＋A＋E-1⇒■E＋A＋O-1

(5.3) ＋A■I＋I-1⇒＋A□E＋E-2⇒＋A■E＋O-2⇒■E■A■O-3

Proof: For (5.1). According to (3.1) ＋A■I＋I-1⇒＋A■E＋E-2, it follows that ＋A■E＋E-2 is valid, and its expansion is that ＋*all*(*C*, *D*)→ (■*no*(*B*, *D*)→＋*no*(*B*, *C*)). It is not difficult to see that *no*(*B*, *C*)→*not all*(*B*, *C*) on the basis of clause (2) in Fact 4. Hence, ＋*all*(*C*, *D*)→(■*no*(*B*, *D*) →＋*not all*(*B*, *C*)) is valid. In other words, the syllogism ＋A■E＋O-2 can be derived from ＋A■I＋I-1. The proof of (5.2) is similar to that of (5.1).

For (5.3). In the light of (5.1) ＋A■I＋I-1⇒＋A■E＋E-2⇒＋A■E＋O-2, it follows that ＋A■E＋O-2 is valid, and its expansion is that ＋*all*(*C*, *D*)→(■*no*(*B*, *D*)→＋*not all*(*B*, *C*)). According to Rule 2, it can be derived that ¬＋*not all*(*B*, *C*)→(■*no*(*B*, *D*)→¬＋*all*(*C*, *D*)). Then it can be inferred that ■¬*not all*(*B*, *C*)→(■*no*(*B*, *D*)→■¬*all*(*C*, *D*)) by Fact 5. One can obtained that ¬*not all*(*B*, *C*)=*all*(*B*, *C*) and ¬*all*(*C*, *D*)=*not all*(*C*, *D*) on the basis of Fact 2 (4) and (1). Therefore, what is obtained is the validity of □*all*(*B*, *C*)→(■*no*(*B*, *D*)→■*not all*(*C*, *D*)), i.e. the syllogism ■E■A■O-3 can be deduced from ＋A■I＋I-1.

Theorem 6: The validity of the following four syllogisms can be inferred from ＋A■I＋I-1:

(6.1) ＋A■I＋I-1⇒□E■I■O-3⇒□E■I■O-4

(6.2) ＋A■I＋I-1⇒＋A■E＋E-2⇒＋A■E＋O-2⇒■E■A■O-3⇒■E■A■O-4

(6.3) ＋A■I＋I-1⇒＋A■I＋I-3⇒■I＋A＋I-3⇒■E＋A＋E-1⇒＋A■E＋E-4

(6.4) ＋A■I＋I-1⇒＋A■I＋I-3⇒■I＋A＋I-3⇒■E＋A＋E-1⇒■E＋A＋E-2

Proof: For (6.1). In line with (3.2) ＋A▪I＋I-1⇒▪E▪I▪O-3, it indicates that ▪E▪I▪O-3 is valid, and whose expansion is that▪*no*(*B*, *D*)→(▪*some*(*B*, *C*)→□*not all*(*C*, *D*)). What is obtained is ▪*no*(*B*, *D*)↔▪*no*(*D*, *B*) by Fact 3 (2). Hence, one can infer that ▪*no*(*D*, *B*)→ (▪*some*(*B*, *C*)→▪*not all*(*C*, *D*)) is valid. In other words, ▪E▪I▪O-4 can be derived from ＋A▪I＋I-1, just as desired. With the help of the above facts and rules, one can similarly demonstrate other cases.

Theorem 7: The validity of the following two syllogisms can be inferred from ＋A▪I＋I-1:

(7.1) ＋A▪I＋I-1⇒＋A▪I＋I-3⇒▪I＋A＋I-3⇒▪E＋A＋E-1⇒▪A＋A＋A-1

(7.2) ＋A▪I＋I-1⇒＋A▪E＋E-2⇒＋A▪E＋O-2⇒▪E▪A▪O-3⇒▪A▪A▪I-3

Proof: For (7.1). According to (3.3) ＋A▪I＋I-1⇒＋A▪I＋I-3⇒▪I＋A＋I-3⇒ ▪E＋A＋E-1, it follows that ▪E＋A＋E-1 is valid, and its expansion is that ▪*no*(*D*, *B*)→ (＋*all*(*C*, *D*)→＋*no*(*C*, *B*)). It can be seen that *no*(*D*, *B*)=*all*¬(*D*, *B*) and *no*(*C*, *B*)=*all*¬(*C*, *B*) on the basis of the Fact 1 (2). Then one can infer that ▪*all*¬(*D*, *B*)→(＋*all*(*C*, *D*)→＋*all*¬(*C*, *B*)). One can obtain that *all*¬(*D*, *B*)=*all*(*D*, *U*–*B*) and *all*¬(*C*, *B*)=*all*(*C*, *U*–*B*) in terms of Definition 2. Hence, it follows that □*all*(*D*, *U*–*B*)→(＋*all*(*C*, *D*)→＋*all*(*C*, *U*–*B*)) is valid. That is to say ▪A＋A＋A-1 can be deduced from ＋A▪I＋I-1, as desired. The proof of (7.2) is similar to that of (7.1).

Theorem 8: The validity of the following two syllogisms can be inferred from ＋A▪I＋I-1:

(8.1) ＋A▪I＋I-1⇒▪E▪I▪O-3⇒▪E▪I▪O-1

(8.2) ＋A▪I＋I-1⇒▪E▪I▪O-3⇒▪E▪I▪O-4⇒▪E▪I▪O-2

Proof: For (8.1). In line with (3.2) ＋A▪I＋I-1⇒▪E▪I▪O-3, it shows the validity of ▪E▪I▪O-3, and its expansion is that ▪*no*(*B*, *D*)→(▪*some*(*B*, *C*)→▪*not all*(*C*, *D*)). Then, what is obtained is ▪*some*(*B*, *C*)↔▪*some*(*C*, *B*) according to Fact 3 (1). Hence, it can be proved that ▪*no*(*B*, *D*)→(▪*some*(*C*, *B*)→▪*not all*(*C*, *D*)) is valid. In other words, the syllogism ▪E▪I▪O-4 can be derived from ＋A▪I＋I-1. The proof of (8.2) is along a similar line to that of (8.1).

Theorem 9: The validity of the following seven syllogisms can be inferred from ＋A▪I＋I-1:

(9.1) ＋A▪I＋I-1⇒＋A▪E＋E-2⇒＋A▪E＋O-2⇒▪E▪A▪O-3⇒▪E▪AO-3

(9.2) ＋A▪I＋I-1⇒＋A▪E＋E-2⇒＋A▪E＋O-2⇒▪E▪A▪O-3⇒▪E▪A▪O-4⇒ ▪E▪AO-4

(9.3) ＋A▪I＋I-1⇒▪E□I▪O-3⇒▪E▪IO-3

(9.4) ＋A▪I＋I-1⇒▪E▪I▪O-3⇒▪E▪I▪O-4⇒▪E▪IO-4

(9.5) ＋A▪I＋I-1⇒▪E▪I▪O-3⇒▪E▪I▪O-1⇒▪E▪IO-1

(9.6) ＋A▪I＋I-1⇒▪E▪I▪O-3⇒▪E▪I▪O-4⇒▪E▪I▪O-2⇒▪E▪IO-2

(9.7) ＋A▪I＋I-1⇒＋A▪E＋E-2⇒＋A▪E＋O-2⇒▪E▪A▪O-3⇒▪A▪A▪I-3⇒ ▪A▪AI-3

Proof: For (9.1). In line with (5.3) ＋A▪I＋I-1⇒＋A▪E＋E-2⇒＋A▪E＋O-2⇒ ▪E▪A▪O-3, it indicates that ▪E▪A▪O-3 is valid. It is seen that □O⇒O according to Fact 6. Therefore, it is obvious that ▪E▪AO-3 is valid. The proofs of other cases follow the same pattern as that of (9.1).

Theorem 10: The validity of the following seven syllogisms can be inferred from ＋A□I＋I-1:

(10.1) ＋A▪I＋I-1⇒＋A▪E＋E-2⇒＋A▪E＋O-2⇒▪E▪A▪O-3⇒▪E▪A＋O-3

(10.2) ＋A▪I＋I-1⇒＋A▪E＋E-2⇒＋A▪E＋O-2⇒▪E▪A▪O-3⇒▪E▪A▪O-4 ⇒▪E▪A＋O-4

(10.3) ＋A▪I＋I-1⇒▪E▪I▪O-3⇒▪E▪I＋O-3

(10.4) ＋A▪I＋I-1⇒▪E▪I▪O-3⇒▪E▪I▪O-4⇒▪E▪I＋O-4

(10.5) ＋A▪I＋I-1⇒▪E▪I▪O-3⇒▪E▪I▪O-1⇒▪E▪I＋O-1

(10.6) ＋A▪I＋I-1⇒▪E▪I▪O-3⇒▪E▪I▪O-4⇒▪E▪I▪O-2⇒▪E▪I＋O-2

(10.7) ＋A▪I＋I-1⇒＋A▪E＋E-2⇒＋A▪E＋O-2⇒▪E▪A▪O-3⇒▪A▪A▪I-3 ⇒▪A▪A＋I-3

Proof: For (10.1). In line with (5.3) ＋A▪I＋I-1⇒＋A▪E＋E-2⇒＋A▪E＋O-2⇒ ▪E▪A▪O-3, it follows that ▪E▪A▪O-3 is valid. It is obvious that ▪O⇒＋O in terms of Fact 7. Thus, the validity of ▪E▪A＋O-3 is straightforward, which can be deduced from ＋A▪I＋I-1. The proof of others is similar to that of (10.1).

Theorem 11: The validity of the following four syllogisms can be inferred from ＋A▪I＋I-1:

(11.1) ＋A▪I＋I-1⇒＋A▪I＋I-3⇒▪I＋A＋I-3⇒▪E＋A＋E-1⇒▪E＋A＋E-2⇒ ▪E＋A＋O-2

(11.2) ＋A▪I＋I-1⇒＋A▪I＋I-3⇒▪I＋A＋I-3⇒▪E＋A＋E-1⇒＋A▪E＋E-4⇒ ＋A▪E＋O-4

(11.3) ＋A▪I＋I-1⇒＋A▪I＋I-3⇒▪I＋A＋I-3⇒▪E＋A＋E-1⇒▪A＋A＋A-1⇒ ▪A＋A＋I-1

(11.4) ＋A■I＋I-1⇒＋A■I＋I-3⇒■I＋A＋I-3⇒■E＋A＋E-1⇒■A＋A＋A-1⇒ ■A＋A＋I-1⇒＋A■A＋I-4

Proof: For (11.1). According to (6.4) ＋A■I＋I-1⇒＋A■I＋I-3⇒■I＋A＋I-3⇒ ■E＋A＋E-1⇒■E＋A＋E-2, it indicates the validity of ■E＋A＋E-2, and whose expansion is that ■*no*(*D*, *B*) →(＋*all*(*C*, *D*)→＋*no*(*C*, *B*)). What is obtained is that *no*(*C*, *B*)→*not all*(*C*, *B*) on the basis of Fact 4 (2). Hence, one can infer that □*no*(*D*, *B*)→(＋*all*(*C*, *D*)→＋*not all*(*C*, *B*)) is valid. In other words, ■E＋A＋O-2 can be derived from ＋A■I＋I-1. The proofs of (11.2) and (11.3) are along similar lines to that of (11.1).

For (11.4). In line with (11.3) ＋A■I＋I-1⇒＋A■I＋I-3⇒■I＋A＋I-3⇒■E＋A＋E-1⇒ ■A＋A＋A-1⇒■A＋A＋I-1, it follows that ■A＋A＋I-1 is valid, and its expansion is that □*all*(*D*, *B*)→(＋*all*(*C*, *D*)→＋*some*(*C*, *B*)). It is clear that ＋*some*(*C*, *B*)↔＋*some*(*B*, *C*) hold by using Fact 3 (1). Therefore, it follows that ■*all*(*D*, *B*)→(＋*all*(*C*, *D*)→＋*some*(*B*, *C*)). In other words, the syllogism ＋A□A＋I-4 can be deduced from ＋A■I＋I-1.

All of the above have completed our deductive proof that the other 37 valid Aristotelian modal syllogisms can be inferred from the validity of the syllogism ＋A■I＋I-1 on the basis of modern modal logic and generalized quantifier theory.

4. Conclusion

This paper firstly proves the validity of the syllogism ＋A■I＋I-1, and then takes it as the basic axiom to derive the other 37 valid modal syllogisms on the basis of some reasoning rules in classical propositional logic, the transformation between any one of Aristotelian quantifiers and its three negative quantifiers, the symmetry of the Aristotelian quantifier *some* and *no,* and some relevant definitions and facts. In other words, there are reducibility between the syllogism ＋A□I＋I-1 and the other 37 valid modal syllogisms. In this way, one can avoid inconsistency in the processes of the above deductions. There are infinite modal syllogism instances in natural language corresponding to every valid modal syllogism, thus this study has important practical significance and theoretical value for natural language information processing in computer science.

Can we use several valid modal syllogisms (e.g. ■A□I■I-1, ■A■I＋I-1, ■A＋I＋I-1, ■A■II-1, ＋AI＋I-1, A＋I＋I-1, ■AI＋I-1, A■I＋I-1, ■AII-1, A■II-1 and AI＋I-1) as the basic axioms, similarly to deduce the remaining valid modal syllogisms? This question needs further study.

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