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Ball, Warrick H; Triaud, Amaury H M J; Hatt, Emily; Nielsen, Martin B; Chaplin, William J

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# Projected spin-orbit alignments from Kepler asteroseismology and Gaia astrometry 

Warrick H. Ball ${ }^{\oplus}$, ${ }^{\star}$ Amaury H. M. J. Triaud ${ }^{\oplus}$, Emily Hatt, Martin B. Nielsen and William J. Chaplin<br>School of Physics \& Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK

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#### Abstract

The angle between the rotation and orbital axes of stars in binary systems - the obliquity - is an important indicator of how these systems form and evolve, but few such measurements exist. We combine the sample of astrometric orbital inclinations from Gaia Data Release 3 with a sample of solar-like oscillators in which rotational inclinations have been measured using asteroseismology. We supplement our sample with one binary whose visual orbit has been determined using speckle interferometry and present the projected spin-orbit alignments in five systems. We find that each system, and the overall sample, is consistent with alignment but there are important caveats. First, the asteroseismic rotational inclinations are fundamentally ambiguous and, secondly, we can only measure the projected (rather than true) obliquity. If rotational and orbital inclinations are independent and isotropically distributed, the likelihood of drawing our data by chance is less than a few per cent. Though small, our data set argues against uniformly random obliquities in binary systems. We speculate that dozens more measurements could be made using data from NASA's Transiting Exoplanet Survey Satellite mission, mostly in red giants. ESA's PLAnetary Transits and Oscillations mission will likely produce hundreds more spin-orbit measurements in systems with main-sequence and subgiant stars.


Key words: asteroseismology - binaries: general - stars: rotation.

## 1 INTRODUCTION

Obliquity is the angle between the rotation spin vector of a celestial object and its orbital spin vector. The Sun's obliquity, for example, is about $7^{\circ}$ relative to the invariable plane of the Solar system. Naïvely, we expect orbital and rotational spins to be aligned because a collapsing cloud imparts its angular momentum to a protostar and its protoplanetary disc, but few measurements exist to confirm this. Most measurements have been obtained for transiting exoplanets via the Rossiter-McLaughlin effect (McLaughlin 1924; Rossiter 1924).

A wide range of spin-orbit angles have been measured, with some planets in polar and retrograde orbits (Triaud 2018; Albrecht, Dawson \& Winn 2022). It is, however, unclear whether the systems arrived in these configurations immediately after they formed or whether they evolved into them later. Recently, Christian et al. (2022) showed that binary companions are preferentially inclined relative to transiting planets, and proposed that the binary companion might align the protoplanetary disc, thereby influencing how planets form. If some planetary systems can be inclined (e.g. Hjorth et al. 2021) and have a stellar companion, one should expect spin-orbit alignment as well as spin-orbit misalignment in binary systems.

Hale (1994) studied 86 stars in 73 binary and higher-order systems, and concluded that binary stars with orbital separations $a<30$ au are aligned, while misalignment is common in more widely separated systems. Justesen \& Albrecht (2020) re-analysed Hale's sample and found it insufficient to make any statements

* E-mail: W.H.Ball@bham.ac.uk
about the distribution of spin-orbit angle with orbital separation. Most recent work producing quality spin-orbit measurements has focused on short-period binaries, typically eclipsing, by modelling the Rossiter-McLaughlin effect (e.g. Kopal 1942; Giménez 2006). This includes high-mass eclipsing binaries such as the inclined DI Herculis (Albrecht et al. 2009) and inclined CV Velorum (Albrecht et al. 2014), or low-mass eclipsing binaries such as EBLM J1219-39 (Triaud et al. 2013) and EBLM J0608-59 (Kunovac Hodžić et al. 2020), both of which show alignment.

Instead of the Rossiter-McLaughlin effect, Marcussen \& Albrecht (2022) used apsidal motion to infer obliquity, and found that only 3 out of 51 surveyed binaries have spin-orbit misalignment but most were also short-period binaries (the longest is $\sim 100 \mathrm{~d}$ ). Unfortunately, close binary stars are a tricky sample to handle, since tidal interactions are expected to realign the rotation and orbital spins.

More evidence is clearly necessary, particularly for binary systems with separations between 1 and 50 au, which have typically been harder to probe because eclipses are less likely and it is difficult to schedule observations with which to model the Rossiter-McLaughlin effect. Here, we show how to measure the projected spin-orbit angle for non-eclipsing binaries that are spatially resolved or not, by combining asteroseismic measurements of the stellar inclination $i_{\text {rot }}$ with astrometric measurements of the orbital inclination $i_{\text {orb }}$. We use Kepler results for the asteroseismology (Hall et al. 2021) and Gaia Data Release 3 (DR3) for the orbital parameters. At the moment, only four measurements are possible but we expect several dozens might eventually be produced once all the Gaia data are released, and thanks to new measurements of $i_{\text {rot }}$ to be produced using asteroseismic data from NASA's Transiting Exoplanet Survey

Satellite (TESS) and ESA's PLAnetary Transits and Oscillations (PLATO) mission. We also include literature orbital data for one system where Gaia provides a spectroscopic solution but not an astrometric one. Our approach is similar to those of Le Bouquin et al. (2009) and Sahlmann et al. (2011).

## 2 METHODS

Most main-sequence solar-like oscillators rotate slowly, in which case a star's pulsations can be described by spherical harmonics, characterized by their angular degree $\ell$ and azimuthal order $m$, multiplied by a radial eigenfunction, characterized by a radial order $n$. For each $\ell$, there are $2 \ell+1$ azimuthal orders $(-\ell \leq m \leq$ $\ell$ ) that pulsate at the same frequency if the star is spherically symmetric. Slow rotation with period $P_{\text {rot }}$ perturbs the frequencies by approximately $m / P_{\text {rot }}$, lifting the degeneracy such that modes of given $n$ and $l$ form multiplets. ${ }^{1}$ This is known as rotational splitting.

Under the standard assumption of energy equipartition between the components of a rotationally split multiplet, the relative amplitudes of the components depend on the inclination angle of the rotation axis $i_{\text {rot }}$ (Gizon \& Solanki 2003). For example, modes with $\ell=1$ and $m \pm 1$ are almost invisible if a star is seen pole on, leaving only the $m=0$ component visible. Conversely, if a star is seen edge on, the $m=0$ component is almost invisible and only the $m= \pm 1$ pair is clearly observed. Thus, one can in principle measure $i_{\text {rot }}$ from high-quality observations of solar-like oscillators, and this method has been widely applied to data from Kepler. For example, Campante et al. (2016) measured the stellar inclination angles of 25 solar-like oscillators that host transiting planets and concluded that the systems are all consistent with alignment.

Hall et al. (2021) created a hierarchical Bayesian model to fit the mode frequencies, including rotational inclination angles, of 91 stars observed by Kepler during its nominal mission. This is the largest sample of asteroseismic rotation inclinations available for main-sequence solar-like oscillators and the one we selected to compare with the Gaia measurements. We used the gaia-kepler.fun cross-match to determine the corresponding Gaia DR3 source IDs and queried the gaiadr3.nss_two_body_orbit table for each star.

Four stars have astrometric solutions - KICs 4914923, 6933899, 9025370, and 12317678 - for which we calculated the orbital inclinations using the method of Binnendijk (1960) as described by Halbwachs et al. (2022). In short, the orbital elements of the system are given in the Gaia data in terms of the Thiele-Innes elements (Thiele 1883; van den Bos 1926)
$A=a\left(\cos \omega \cos \Omega-\sin \omega \sin \Omega \cos i_{\text {orb }}\right)$,
$B=a\left(\cos \omega \sin \Omega+\sin \omega \cos \Omega \cos i_{\text {orb }}\right)$,
$F=-a\left(\sin \omega \cos \Omega+\cos \omega \sin \Omega \cos i_{\text {orb }}\right)$,
$G=-a\left(\sin \omega \sin \Omega-\cos \omega \cos \Omega \cos i_{\text {orb }}\right)$,
where $a$ is the semimajor axis, $\omega$ is the argument of periastron, and $\Omega$ is the longitude of the ascending node; i.e. $a, \omega, \Omega$, and $i_{\text {orb }}$ are the Campbell elements. Halbwachs et al. (2022) gave formulae to convert the Thiele-Innes elements to the Campbell elements. Uncertainties

[^0]are propagated by drawing a sample of $10^{5}$ points from a normal distribution with the mean and covariance given by the Gaia data. We report the means and standard deviations of the derived samples in Table 1. The secondary mass $M_{2}$ is derived from the astrometric mass function under the assumption that the secondary is much fainter than the primary (see equation 15 of Halbwachs et al. 2022). The primary's mass is taken from Hall et al. (2021).

KIC 7510397 is an exception. The Gaia data only contain an entry for the system as a single-lined spectroscopic binary, so we have used the orbital parameters from Appourchaux et al. (2015), who analysed the solar-like oscillations detected in both stars in the binary. Their results included an orbital fit to speckle interferometry that extended previous data presented by Horch et al. (2012) and we have included this value in Table 1.

## 3 RESULTS

Fig. 1 shows a comparison of the inclination angles of the rotational and orbital axes. The asteroseismic measurement cannot distinguish between angle $i_{\text {rot }}$ and $180^{\circ}-i_{\text {rot }}$, so both are shown. The measurement for each system is what we would observe if they were aligned; i.e. alignment is not ruled out in any of the five systems. We cannot, however, conclude that the systems are truly aligned but return to the significance of our result in Section 4.

Three further stars are identified in the cross-match as spectroscopic binaries with measured orbital periods: KICs 7206837, 7510397, and 9098294. The reported orbital periods for KICs 7206837 and 9098294 are consistent with the asteroseismically measured rotation rates. It is unclear if this implies tidal-locking with a close companion or that the rotation rate has been mistaken for an orbital period. The orbital period for KIC 7510397 in the Gaia data of $61.63 \pm 0.47$ d differs significantly from the period of $13.8_{-0.5}^{+0.6}$ yr given by Appourchaux et al. (2015), though the Gaia eccentricity of $0.515 \pm 0.029$ is only mildly inconsistent ( $\lfloor 2 \sigma$ ) with their value of $0.583_{-0.025}^{+0.016}$.

Two further stars - KICs 1435467 and 8379927 - have measured radial-velocity trends but not complete orbital solutions. They appear in the gaiadr3.nss_non_linear_spectro table but not gaiadr3.nss_two_body_orbit. KIC 8379927 is known to be a spectroscopic binary with an orbital period of about 4.8 yr (Griffin 2007). With more data, Gaia might determine a spectroscopic orbit and perhaps astrometric solution for this system.

Pertinent data from both data sets for all seven stars with nonsingle solutions are listed in Table 1, supplemented by the orbital inclination of KIC 7510397 by Appourchaux et al. (2015). We note that KIC 6933899 has a relatively long and very eccentric orbit, with $P_{\text {orb }}=11.13 \pm 1.25 \mathrm{yr}$ and $e=0.917 \pm 0.008$. The orbital period is several times longer than the 34-month duration of the Gaia data in DR3 but the goodness-of-fit statistic (1.5) and significance of the parameters suggest that this is a genuine solution.

## 4 DISCUSSION AND CONCLUSION

We have presented here only a preliminary study of the projected spin-orbit alignments in systems with rotation inclinations measured through asteroseismology and orbital inclinations through astrometry. The sample of astrometric solutions will only increase as Gaia steadily takes more data, but the results are already significant.

For each of the five systems in Fig. 1, the measurements of $i_{\text {rot }}$ and $i_{\text {orb }}$ are what one would expect if they were aligned, but we cannot rule out misalignment for two reasons. First, the asteroseismic measurement of $i_{\text {rot }}$ cannot distinguish between measurements of $i_{\text {rot }}$

Table 1. Table of pertinent properties for the stars that appear in both the asteroseismic sample by Hall et al. (2021) and Gaia's tables of non-single stars. The first four stars are shown in Fig. 1, as is KIC 7510397, for which we list the orbital inclination from Appourchaux et al. (2015). Symmetric uncertainties are indicated in parentheses for that many final digits of the relevant number.

| Solution type | KIC | $i_{\text {orb }}\left({ }^{\circ}\right)$ | $i_{\text {rot }}\left({ }^{\circ}\right)$ | $P_{\text {orb }}(\mathrm{d})$ | $P_{\text {rot }}(\mathrm{d})$ | $e$ | $a(\mathrm{au})$ | $M_{1}\left(\mathrm{M}_{\odot}\right)$ | $M_{2}\left(\mathrm{M}_{\odot}\right)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AstroSpectroSB1 | 4914923 | $113.9 \pm 1.8$ | $46.6_{-8.1}^{+13.3}$ | $99.2443(664)$ | $21.40_{-3.53}^{+5.39}$ | $0.212(9)$ | $1.301(31)$ | $1.06_{-0.05}^{+0.06}$ | 0.51 |
|  | 6933899 | $74.6 \pm 0.6$ | $64.3_{-14.0}^{+16.1}$ | $4065(455)$ | $28.91_{-4.83}^{+3.70}$ | $0.917(8)$ | $12.15(88)$ | $1.13_{-0.03}^{+0.03}$ | 0.56 |
| Orbital | 9025370 | $51.8 \pm 1.0$ | $67.5_{-19.1}^{+5.2}$ | $239.124(454)$ | $24.71_{-4.67}^{+3.67}$ | $0.271(28)$ | $1.417(22)$ | $0.97_{-0.03}^{+0.03}$ | 0.18 |
|  | 12317678 | $128.6 \pm 1.9$ | $35.3_{-5.2}^{+10.1}$ | $80.8435(599)$ | $5.20_{-0.92}^{+1.80}$ | $0.393(36)$ | $1.019(30)$ | $1.34_{-0.01}^{+0.04}$ | 0.68 |
|  | 7206837 |  | $31.7_{-2.8}^{+3.2}$ | $4.05012(7)$ | $3.97_{-0.45}^{+0.55}$ | $0.002(10)$ | $1.30_{-0.03}^{+0.03}$ |  |  |
| SB1 | 7510397 | $\left[14_{-10}^{+11}\right]$ | $19.9_{-2.0}^{+2.0}$ | $61.6302(4664)$ | $6.13_{-0.64}^{+0.67}$ | $0.515(29)$ | $1.37_{-0.02}^{+0.02}$ |  |  |
|  |  | $58.2_{-16.3}^{+1.0}$ | $20.1013(26)$ | $27.21_{-7.00}^{+5.75}$ | $0.018(10)$ | $0.97_{-0.03}^{+0.02}$ |  |  |  |
| Radial velocity | 1435467 |  | $63.4_{-6.6}^{+10.2}$ |  | $6.54_{-0.62}^{+0.76}$ |  | $1.32_{-0.05}^{+0.03}$ |  |  |
| trend |  |  |  | $9.3_{-2.3}^{+2.5}$ |  |  | $1.12_{-0.04}^{+0.04}$ |  |  |



Figure 1. Comparison of rotation inclination angles by Hall et al. (2021) and orbital inclination angles from Gaia (circles) or for KIC 7510397 (Appourchaux et al. 2015, square). The rotation angles $i_{\text {rot }}$ are ambiguous, so both $i_{\text {rot }}$ and $180^{\circ}-i_{\text {rot }}$ are plotted. The filled points indicate which value of $i_{\text {rot }}$ is closer to the one-to-one line.
and $180^{\circ}-i_{\text {rot }}$, so, far from $i_{\text {rot }}=90^{\circ}$, each system is roughly as likely to be aligned as misaligned. Secondly, the true obliquity $\psi$ can lie anywhere between $\left|i_{\text {rot }}-i_{\text {orb }}\right|$ and $i_{\text {rot }}+i_{\text {orb }}$. True alignment can only be confirmed using the projected obliquity if $i_{\text {rot }}=i_{\text {orb }}=0^{\circ}$.

The fact that our results do not rule out alignment in any system is nevertheless striking and we can compute the probability of measuring such data - five data points consistent with $i_{\text {rot }}=i_{\text {orb }}$ or $180^{\circ}-i_{\text {orb }}$ - under the assumption that rotational and inclination axes are distributed isotropically and independently. In this case, the underlying joint distribution of $i_{\text {rot }}$ and $i_{\text {orb }}$ is $\propto \sin i_{\text {rot }} \sin i_{\text {orb }}$. For simplicity, we have integrated the region where $i_{\text {rot }}$ is within some range $\pm \sigma$ of either $i_{\text {orb }}$ or $180^{\circ}-i_{\text {orb }}$. Fig. 2 shows the likelihood of finding different numbers of stars in this region, as a function of the parameter $\sigma$. The curve for five stars corresponds to our sample. If we take $\sigma=20^{\circ}$ as a representative width for the observed uncertainties, the likelihood of measuring our data in a population of isotropically distributed inclinations is about 2.5 per cent. Although


Figure 2. Probability of a sample of stars with isotropically distributed and independent rotation inclinations $i_{\text {rot }}$ and orbital inclinations $i_{\text {orb }}$ all having $i_{\text {rot }}$ within $\sigma$ of $i_{\text {orb }}$ (dashed lines) or within $\sigma$ of either $i_{\text {orb }}$ or $180^{\circ}-i_{\text {orb }}$ (solid lines). The blue, orange, and green curves show samples of 1,5 , or 25 stars, respectively. The solid grey lines show where $\sigma=20^{\circ}$; the corresponding probability is 2.5 per cent.
we cannot conclude that all the systems in our sample are aligned, our data are significantly at odds with the assumption of isotropic and independent rotation and orbital inclinations.

Measuring rotational inclinations with asteroseismology has so far required space-based photometry. Most of these results, including those used here, employ data from Kepler, but similar measurements have been made using data from CoRoT (e.g. HD 52265; Gizon et al. 2013). Aside from a sufficient signal-to-noise ratio to detect the solar-like oscillations, asteroseismic measurements of rotation benefit from time series that are several times longer than the stellar rotation rate, which is several weeks for Sun-like stars.

TESS is observing most of the sky in 27.4-d-long sectors. Though most targets are only observed in a few sectors separated by long gaps, some stars are in regions of the sky that TESS observes


Figure 3. Stacked histogram showing the number of stars with orbital solutions from Gaia and solar-like oscillations in their short-cadence TESS light curves, as a function of the number of sectors of TESS data.
continuously for up to 13 sectors (roughly 1 yr). Solar-like oscillators in these regions could potentially have their rotational inclination angles measured, which could increase the sample of spin-orbit measurements.

To explore TESS's potential further, we cross-matched the table of Gaia two-body orbits with the catalogue by Hatt et al. (2023) of stars showing solar-like oscillations in short-cadence TESS data. Fig. 3 shows a histogram of the number of stars in the Gaia table that also shows solar-like oscillations in their short-cadence light curves, as a function of the number of sectors in which TESS has observed them. There are 26 solar-like oscillators with astrometric orbits and at least four sectors of TESS data, though most of these are giants. Of these stars, six have $\log g>3.4$, compared to the minimum $\log g$ $=3.91$ in the sample of Hall et al. (2021). The rotational inclinations of red giants certainly can be measured - Gehan et al. (2021) have measured 1139 in data from Kepler - but we have not analysed any here.

Finally, ESA's upcoming PLATO mission (Rauer et al. 2014) will measure solar-like oscillations in thousands of cool subgiants and main-sequence stars. Even if we only assume a yield of about 5 per cent, like the sample presented here, then PLATO's core sample of $\sim 15000$ stars would add hundreds of spin-orbit measurements. We re-iterate that the sample is currently limited by the number of orbital inclinations that have been measured, which can only increase as Gaia continues its observations. Our results thus demonstrate the enormous potential to assemble a large sample of projected spinorbit angles, with which we will be able to investigate the dynamics of binary stars.

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This work made use of the Gaia-Kepler cross-match data base created by Megan Bedell. ${ }^{2}$

## DATA AVAILABILITY

Data from Gaia are publicly available from the Gaia Archive. ${ }^{3}$ PYTHON scripts to recreate Figs 1 and 2 and the data of which part is presented in Table 1 are available from a public repository. ${ }^{4}$ Other data underlying this article will be shared on reasonable request to the corresponding author.

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[^1]This paper has been typeset from a $\mathrm{T}_{\mathrm{E}} \mathrm{X} / \mathrm{LA}_{\mathrm{E}} \mathrm{X}$ file prepared by the author.


[^0]:    ${ }^{1}$ This is mathematically the same as Zeeman splitting, where the presence of a magnetic field breaks the degeneracy between electron orbitals, which are also described by spherical harmonics.

[^1]:    ${ }^{2}$ https://gaia-kepler.fun
    ${ }^{3} \mathrm{https}: / /$ gea.esac.esa.int/archive/
    ${ }^{4} \mathrm{https}: / /$ gitlab.com/warrickball/kepler-gaia-spin-orbit

