

WHY THE FIRST AND THE SECOND LAW OF ILLUMINATION SHOULD BE REPLACED WITH THE NEW UNIVERSAL LAW OF ILLUMINATION?

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Abstract For many years, from the theoretical and practical aspects, the first and second laws of illumination have been in use. Both mentioned laws were applied in various scientific disciplines, such as physics, ergonomics, architecture, horticulture, as well as in many engineering disciplines. The first law of illumination is called the Inverse Square Law. The second law of illumination is called Lambert's Cosine Law. Conceptually, both of these laws are related to surface illumination. The first law of illumination determines the illumination of the surface on which the light ray falls at a right angle. The second law of illumination determines the illumination of a surface on which light rays fall at a certain angle different from 90°. In this paper, it is shown why both of these laws are not adequate in conceptual terms. The presented examples additionally show what inaccuracies in illumination calculation can occur based on the application of these laws, especially the second law of illumination. The paper presents a new concept of illumination based on the illumination of the point of space. In addition, a new, universal law of illumination has been formulated here, which from a theoretical and practical point of view is more precise than the previously mentioned laws of illumination. This new law of illumination fully reflects the nature of the light emitted by a light source, which does not depend on the orientation of any surface upon which the light beam encounters. Previously formed first and second laws of illumination can be completely replaced from the theoretical and the practical aspect by the newly formed universal law of illumination, which opens up new possibilities for the advancement of the science of illumination and other scientific disciplines that use this knowledge.

Keywords: Illumination; illuminance; lighting; the first law of illumination; the second law of illumination; the inverse square law of illumination; Lambert's cosine law; the universal law of illumination.

1. INTRODUCTION

Quality and level of illumination of the living and working environment have a significant effect on the safety of people, the number of errors in work, the performance, comfort and the state of the sense of sight. The issue of lighting includes, in addition to the mentioned, many other aspects, such as consideration of methods for measuring illumination, determination of criteria for assessing the convenience of the introduced lighting system, practical application of photometric quantities for quantification and study of illumination in a given environment, and the like. All these aspects are mutually interpenetrating, so that in many situations they cannot be considered separately. For this reason, there is sometimes a need to pay special attention to some of them, so that all the segments covered by this issue can be improved in parallel. In this paper, the subject of study is illumination as a physical quantity, its determination and definition, as well as the problems that arise when solving certain tasks in which it inevitably figures. The focus of the paper is on presenting the shortcomings of the first and second laws of illumination, as well as on formulating a new, universal law of illumination that is more precise than the previously mentioned ones.

1.1. Initial consideration of illumination

A number of scientific fields and subfields involve determining illumination. It is studied in physics, ergonomics, architecture, horticulture, as well as in many engineering and scientific disciplines. Theoretical settings related to illumination will be presented in this section, as is done in the various literature dealing with photometric quantities. In a broader sense, illumination is the act of projecting light into space from a light source and illuminating an object. Illuminance refers to the amount of light that falls on a surface or task from ambient or local lighting [1]. Light incident on a surface (illumination or illuminance, which are terms used equally for the same phenomenon) is the most widely used lighting design criterion [2].

The first law of illumination states: "When illuminated by a point source of light, the illumination of a surface is inversely proportional to the square of its distance from the light source (R^2)" [3, 4]. In other literature, definitions may be found that differ in certain details. Thus, in [5], it is stated the following definition: "The illuminance of a surface by a point source on which light falls at the right angle is directly proportional to the intensity of the light from the source, and inversely proportional to the square of the distance from the source to the illuminated surface". When the above is expressed by the formula [6-8], the following expression is obtained

$$\text{Footcandles on the surface which is normal to the direction of the light ray} = \frac{\text{Luminous intensity of the source in the direction of the beam}}{(\text{Distance from the light source to the plane})^2} \quad (1)$$

When d is five times the maximum dimension of the source (or luminaire) as viewed from the point on the surface, this equation holds true within 1% [9]. It should be noted that in the above formula, illumination is expressed in units called footcandle. This unit of illumination is used in the USCS system of units used in America. Within the SI system of units that is in use in the rest of the world, the basic unit for illumination is lux (lx).

However, in the case of incoming light rays falling at an angle other than 90^0 on a surface, the first law of illumination ceases to apply. Instead, another law of illumination applies, called Lambert's cosine law. Before stating how this law reads, it is necessary to say something about its creator, when it was created and how this law was created.

The creator of the second law of illumination was Johann Heinrich Lambert (1728-1777). Lambert was born in 1728 in the Republic of Mulhouse (currently Alsace, France). In addition to mathematics and physics (particularly optics), he also contributed to philosophy, astronomy, and map projections [10]. A non-SI unit for luminance (but not for illumination) was named Lambert for his contribution to photometry. In connection with that, he published a book on photometry, "Photometria" [11], in 1760. In that book, Lambert presented the concept (see Figure 1) that led to the second law of illumination.

Here is Lambert's exposition of the above-mentioned concept that gave rise to the first and the second law of illumination (attributed to Lambert, as mentioned earlier). This concept will be presented as in the original, in order to be discussed later in this paper, and so that it will be possible to point out the shortcomings of this concept. "On plane AB parallel rays are incident between the parallels CA and

DB from angle CAF = DBF. Let us take these same rays to be intercepted by plane AE normal to the direction of the rays. It is clear that the same number of rays is intercepted by AE, which were previously intercepted by the larger plane AB. So it is necessary that they be more dense on AE than on AB. Therefore, since the density is the number of rays divided by the space on which they are incident, this same number of rays must be divided in the first case by AB and in the second by AE, and therefore the density on AB will be to the density on AE reciprocally as the lines themselves, or directly as AE to AB. But if AB is taken as a radius or total sine, AE will be the sine of the angle of incidence. Therefore, the perpendicular illumination will be to the oblique illumination as the total sine is to the sine of the angle of incidence. So it will be less to the degree that the sine of the angle of incidence is less." [12]

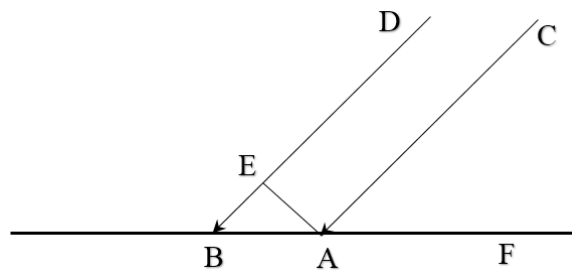


Figure 1. A graphic representation referring to the basic assumptions of the first and second law of illumination presented by Lambert in Photometria [11].

Based on the above consideration, the second law of illumination was formed which states: "The illumination of a surface is proportional to the cosine of the angle of incidence of the light ray" [4]. A similar definition is given in [13]. As mentioned, this law is called Lambert's cosine law. The second law of illumination can be expressed by the formula [6, 9, 14, 15]

Footcandles on
a surface at an angle = $\frac{\text{Luminous intensity of the source in the direction of the beam}}{(\text{Distance from the light source to the point in the plane})^2} \cos \beta = \frac{I}{R^2} \cos \beta$ (2)
other than 90°

In the previous formula, β denotes the angle between the light beam and the normal to plane at the observed point. However, in explaining the concept itself, Lambert mentions the sine function. DiLaura [12] explains this by the fact that unlike modern practice Lambert used the angle between the surface and the direction of a light ray entering, so that the sine function is mentioned in the concept, whereas in the formula of the second law of illumination, the cosine function is used (therefore, there is no essential difference). It should be noted here that DiLaura translated from Latin into English [12] the book Photometria [11], having written a detailed and extensive introduction where it is presented photometry in that period, with an emphasis on Lambert's work in that field (before translation of the book itself).

2. PROBLEM

The basic problem that arises when determining illumination will be illustrated by two examples and the process of solving them.

2.1. Example 1

Determine the illumination at points 1 and 2 belonging to the same surface, if the point light source Z emits light of luminous intensity $I = 1000$ cd. The distance from the light source to point 1 is 3m, while the angle between the surface normal and the direction of the incoming light ray at point 2 is $\beta = 30^\circ$. The mutual position of the points and the source is shown in Figure 2.

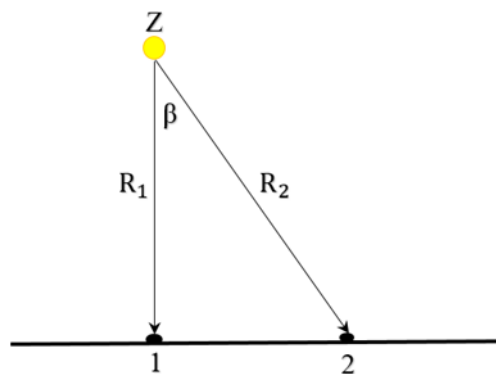


Figure 2. Picture related to Example 1.

2.2. Example 2

Determine the illumination at point 1 located on the line representing the intersection of planes I and II. The light ray falling on that point is normal to the plane I. The distance R from the point source of light Z to point 1 is $R=3$ m, where the source emits light of luminous intensity $I = 1000$ cd. The angle between the normal to plane II and the direction of the light ray falling on point 1 is $\beta = 30^\circ$ (see Figure 3).

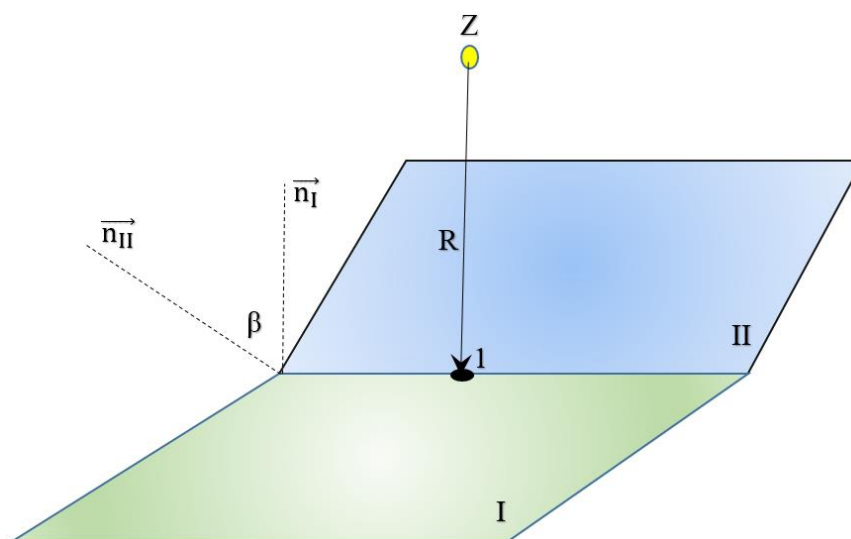


Figure 3. Picture related to Example 2.

3. RESULTS

Each of the two examples has two different solutions, so the end results are different. However, all solutions have a corresponding theoretical basis in existing definitions and formulas relating to illumination. The solutions will be explained in detail.

3.1. Solutions relating to example 1

3.1.1. The first solution

Since the light beam falls at a right angle to point 1, the illumination will be calculated by the formula (1). Substitution of values gives

$$E_1 = \frac{I}{R_1^2} = \frac{1000}{9} = 111.1 \text{ lx}$$

Bearing in mind that a light beam to the surface at point 2 falls at an angle that differs from 90° , the formula (2) will be applied to calculate the illumination. By replacing starting values it is obtained

$$E_2 = \frac{I}{R_2^2} \cos^3 \beta = \frac{I}{R_1^2} \cos^3 \beta = \frac{1000}{9} (0.86)^3 = 72.1 \text{ lx}$$

This approach to solving the problem is common and can be found in different literature [16, 17].

3.1.2. The second solution

The illumination at point 1 is determined in an identical way, so the solution procedure will be omitted. However, the illumination in point 2 can be calculated in a different way in relation to the previously stated theory. For the purpose of clarification, it is necessary to have in mind Figure 4, which is a modification of Figure 2.

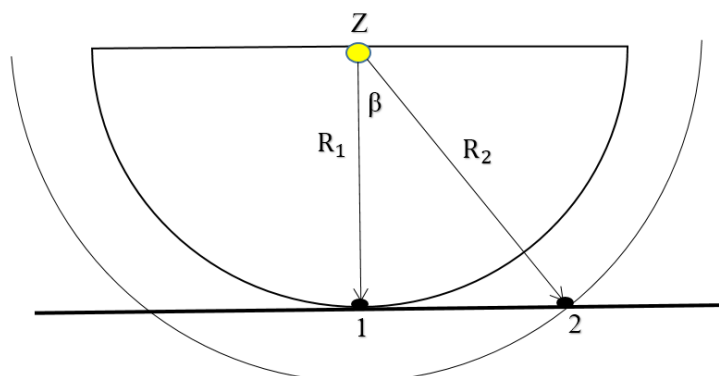


Figure 4. The second solution of example 1.

It should be noted first of all that through formula (1), illumination can be calculated at any point on the surface of the sphere of radius R , which has at its center a point source whose luminous intensity

is I [13]. This is in line with what has been said earlier, whereby the derivation of this relation and its clarification will be carried out a little later.

It is obvious that point 1 belongs to a sphere whose radius is R_1 , so its illumination is calculated using relation (1). However, point 2 belongs to a sphere of radius R_2 in the center of which is the same point source. For this reason, equality (1) can also be applied in point 2. Calculating the illumination at point 2 of a sphere of radius R_2 , we get

$$E_2 = \frac{I}{R_2^2} = \frac{I}{R_1^2} \cos^2 \beta = \frac{1000}{9} (0.86)^2 = 83.3 \text{ lx}$$

It can be easily seen that the values for the illumination at point 2 in the first and the second solution differ from each other. An additional reason for the justification of applying equation (1) in the case of point 2 is that the total illumination of any point in space on which the light falls directly from a point light source does not depend on the orientation of the surfaces that include that point. In other words, we can place an infinite number of surfaces through one point in space, the orientation of which will not have any effect on the total illumination of that point. This will be clarified through the solution of the second example.

3.2. Solutions relating to example 2

3.2.1. The first solution

The illumination at point 1 will first be determined relative to plane I, on which the light ray falls at the right angle. In other words, we will assume that point 1 in this case belongs to the surface I. Given the direction of the light beam relative to that plane, the ratio (1) will be used to determine illumination, so that

$$E_1 = \frac{I}{R^2} = \frac{1000}{9} = 111.1 \text{ lx}$$

3.2.2. The second solution

Considering the fact that point 1 is at the intersection of two planes, we can also assume that it belongs to plane II. With this in mind, the illumination will be determined in point 1 relative to plane II. As the light beam falls at the angle β with respect to the normal of this plane, it will in this case apply the formula (2), so that

$$E_2 = \frac{I}{R^2} \cos \beta = \frac{1000}{9} \cos 30 = 96.2 \text{ lx}$$

It is obvious that the first and the second solution differ from each other also in example 2.

4. ANALYSIS OF RESULTS

Although the solution procedures that have been applied have their theoretical justification, we have seen that the results differ from one another. An absurd result is obtained that the illumination in one

and the same point of space is different when it is adopted that the point belongs to the various surfaces that contain it.

An adequate explanation can be given for each individual result obtained for the illumination, in connection to the reason for the inconsistency with the result obtained by the other method of solving. However, such an approach to analysis seems unnecessary, because the essence of the explanation lies in the very concept of illumination. Through its analysis, it will become clear why such results have been obtained, as well as how each of the set illumination tasks should be solved.

The root cause of the problem consists in that the existing concept of illumination being bound to the surface. First of all, it should be pointed out that the notion of illumination of a surface that figures in Lambert's cosine law does not present anything concrete in the case when we observe a flat surface. This is because each point on a flat surface has a different illumination, depending on its distance from the light source, especially if light rays fall on a surface at an angle other than 90° .

In the case of a spherical surface, Lambert's cosine law also does not work. If the point light source is located in the center of the sphere, then the light beam from the point source falls at any point of the sphere directly, not at any angle β . Everything previously mentioned indicates that the second law of illumination, i.e. Lambert's cosine law has neither theoretical nor practical justification. This means that the application of formula (2) in illumination calculations leads to imprecise and inadequate results.

In other words, the physical quantity called surface illumination has no theoretical or practical justification and cannot be calculated in any way for a surface of any shape other than spherical (in the case of a sphere, the illumination of the spherical surface is also calculated over the illumination of any point of that sphere). This is one of the basic reasons why the notion of surface illumination should be omitted from the concept of illumination. An additional reason, as could be inferred from the examples presented, is that the orientation of any surface in the space encompassing the observed point has no bearing on its overall illumination. In the further part of the text, the concept of illumination will be presented, which must be applied instead of the existing one, in order to obtain unequivocal and precise results during calculations.

5. THE CONCEPT OF POINT-OF-SPACE ILLUMINATION

In the case where there is a point source of light with the luminous intensity I in the space, the total illumination at an arbitrary point in the space at a distance R from the source can be determined as follows. Dividing the luminous flux of the light source F_z by the total number of points located at an equal distance R from it, which is S , we get the total illumination at a point in space at a distance R as

$$E = \frac{F_z}{S} = \frac{F_z}{4\pi \cdot R^2}$$

Luminous intensity is defined as the luminous flux per unit solid angle. A spherical surface has a solid angle of 4π . With this in mind, the total luminous flux of a point light source can be determined from the relation

$$F_z = 4 \pi I$$

By substituting this into the previous expression, after shortening, the illumination at an arbitrary point in space, located at a distance R from the point light source, is obtained as

$$E = \frac{I}{R^2} \quad (3)$$

It can be easily seen that the illumination at an arbitrary point in space cannot depend on the incident angle of the light beam when it comes to a point light source. Although equations (1) and (3) have the same form, they are not the same, because equation (3) defines the total illumination at an arbitrary point in space, while (1) determines the illumination at a point on the surface that is at the right angle to the light ray. Once again, it should be emphasized that using equation (3) the total illumination at a point in space is determined regardless of the angle between the normal to the surface at the observed point and the direction of the light ray falling on the same point, i.e. regardless of the angle at which the light ray falls on the surface which contains the observed point. Therefore, equation (2) is unusable for determining the total illumination at an arbitrary point in space.

If we observe a point on a surface where the light beam from a point source falls at an angle (see Figure 5), then only the horizontal E_H , and vertical E_V component of the total illumination vector at the observed point can be calculated. In Figure 5, β is denoted as the angle between the vertical component of the total vector of illumination and the total vector of illumination at the point on the surface. With this in mind, the vertical component of the total vector of illumination at a point on the surface is determined from the expression

$$E_v = E \cos \beta = \frac{I}{R^2} \cos \beta$$

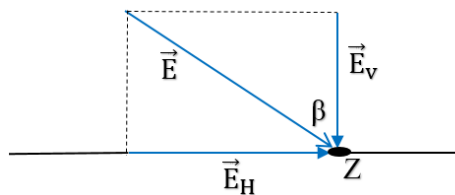


Figure 5. The horizontal and the vertical component of the total vector of illumination at a point in space.

The horizontal component of the total vector of illumination at the point of space is determined by the expression

$$E_H = E \sin \beta = \frac{I}{R^2} \sin \beta$$

The similarity of expression (2) to the vertical component of the total vector of illumination at the observed point of the surface can be observed, while it should be borne in mind that these are not the same expressions as they define different quantities.

If we know the total illumination at a point in space and the distance from that point to the point light source, we would make a mistake if we wanted to use expression (2) to calculate the luminous

intensity of the source. However, this is possible using equation (3). All this is valid under the assumption that there are no obstacles in the path of the light beam to the observed point.

6. DISCUSSION

Although the term illumination at a point exists, it is always associated with the surface in the literature [18] (point on a surface), while it exists in parallel with the term illumination of the surface. For this reason, errors and ambiguities when determining illumination are inevitable, if we rely on such an approach and treatment of the problem. In the following, from a conceptual point of view, it will be additionally explained how and why errors occurred during the formation of the first and second laws of illumination.

To that end, let us return to Figure 1 and Lambert's explanation on the basis of which the first and second laws of illumination were formed. The concept of the first and second laws of illumination has both formal and substantive flaws. As can be seen from cited Lambert's explanation, the basic reason why the surfaces AE and AB are illuminated differently is the difference in the density of light rays falling on these two surfaces, and such a difference according to Lambert arises because the area determined by the distance AB is greater than the area determined by the distance AE (Lambert observed the triangle ABE and since the hypotenuse AB is certainly longer than the side AE of a right triangle he concluded that the area determined by the dimension AB is greater than the area which is determined by the dimension AE).

In order to make the shortcomings of this consideration more visually obvious, let's move from the two-dimensional representation of the mentioned explanation of Lambert to a three-dimensional representation. Regarding the first law of illumination, let's look at Figure 6. This figure (which was used in [9] to explain the first law of illumination, but without the Z-a-b-c ray representation added here) shows three surfaces I, II and III located at distances d , $2d$, and $3d$ respectively from a point source of light Z. Let the light ray Z-a-b-c pass through surfaces I, II, and III at right angles at points a, b, and c representing the midpoints of these surfaces. If we wanted to determine the illumination at point e belonging to surface I by applying formula (1), we would make a mistake because the distance Z-e is greater than the distance Z-a. Similarly, the illumination at point f is different from the illumination at point b, although both mentioned points belong to surface II. The same applies to points g and c belonging to surface III. Therefore, the first law of illumination does not make a clear distinction in illumination between points belonging to the same surface.

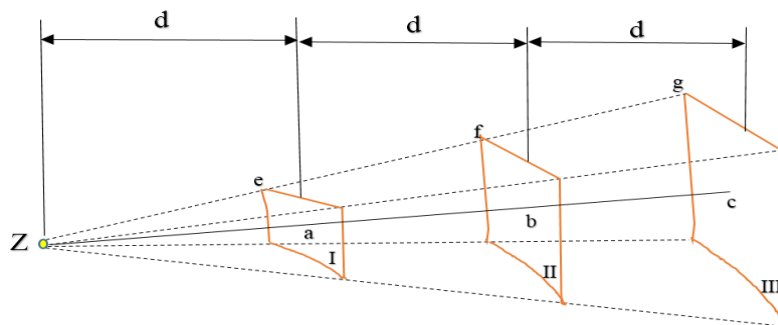


Figure 6. Three-dimensional representation related to the explanation of the first law of illumination (according to [9]).

Let's now look at Figure 7, which will serve to explain the shortcomings related to Lambert's cosine law. The conceptual error of Lambert comes to the fore even more in the case of the second law of illumination. The upper surface in Figure 7 essentially refers to surface AE from Figure 1, while the lower surface in Figure 7 refers to surface AB from Figure 1. Light rays from the light source fall on surface 1 at a right angle, while on surface 2 they fall at some angle different from 90° (β is the angle between the normal to surface 2 and the direction of the light rays). According to Lambert's explanation, the entire area 1 has an illumination E_1 , while the entire area 2 (which is larger than area 1) has an illumination E_2 . The illuminations of these two surfaces are connected by the relation $E_2 = E_1 \cos \beta$ [9].

However, the aforementioned relation $E_2 = E_1 \cos \beta$ is not true in the case of Figure 7 for all points belonging to surface 2. Light rays have traveled a certain distance from surface 1 to surface 2, so the illumination of each point on surface 2 will be less than on surface 1, with the difference that most points on surface 2 have a different illumination. The illumination at the observed point on surface 2 depends on the distance that the light beam falling on the observed point has traveled between surface 1 and that point on surface 2 (i.e., the illumination of the point on surface 2 depends on the distance between the light source and the position of the point on surface 2). Only points on surface 2 at which the light beam from the light source to the observed point has traveled an identical distance can have the same illumination. In other words, the illumination of the points on surface 2 will progressively decrease from the left side to the right side of surface 2.

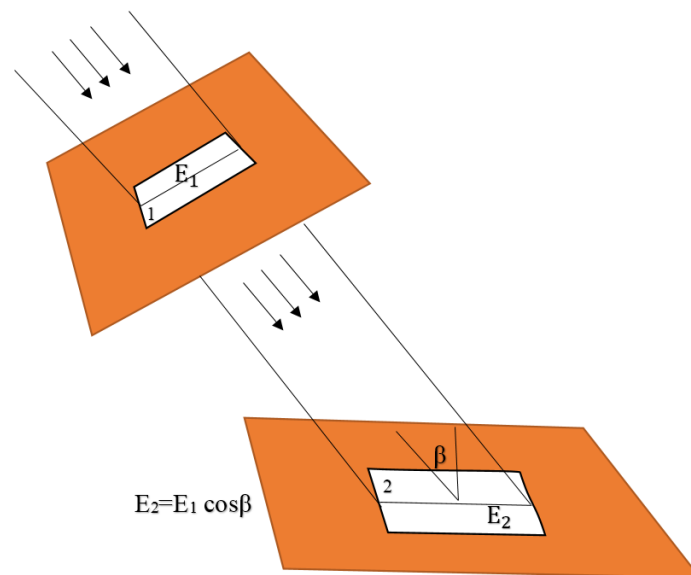


Figure 7. Three-dimensional representation referring to the explanation of the second law of illumination (according to [9]).

As for the concept of illumination that Lambert presented in Figure 1, it is not entirely true that all points of the surfaces AE and AB have different illumination. Given that point A is common to both previously mentioned surfaces, the illumination at that point will be identical, regardless of whether we treat point A as belonging to one or the other surface. In addition, the points belonging to the surface AB will not have identical illumination. It will decrease from point A to point B

progressively, since the light ray from the source passes a greater distance to reach point B than to point A.

DiLaura points out that Lambert has always worked with surfaces. He even considered a point as a two-dimensional surface element [12]. In addition, Lambert arrived at the second law of illumination on the basis of theoretical and geometric considerations rather than experimental research. At that time, there was no device for measuring illumination (lux meter). When the photocell of the lux meter moves on a (for example, flat) surface, the illumination value changes. It is likely that this information would have influenced Lambert to change his concept at that time.

7. CONCLUSION

Illumination (illuminance) is a physical quantity related to the luminous characteristic of the emitted light beam of any light source. For this reason, any surface or orientation of that surface has no effect on the physical properties of light rays falling at a point in space (or a point on a surface in space). The orientation of the surface can only affect the light reflected from the incoming light beam. However, light reflected from a surface is characterized by another physical quantity called luminance. The reflection properties of a surface can also be described by additional physical quantities associated with reflection.

Based on everything that has been presented so far, it can be concluded that the concept of illumination that is related to the surface is unnecessary and wrong, as well as that it can cause errors when calculating the illumination, which is all presented in this paper. It is necessary to replace such a concept with a new concept that has its basis in the illumination of a point of space, regardless of whether a surface passes through it or not. If necessary, at a certain point located on a surface of arbitrary shape, the horizontal and the vertical component of the total vector of illumination of that point can be determined.

The first and the second law of illumination were connected to the surface. If there is no surface, illumination as a phenomenon cannot be explained by the first and the second law of illumination. For example, the illumination of an arbitrarily selected point in space or an atmosphere where no surface exists cannot be adequately explained with these two laws. Both of these laws, because of the inaccuracies they cause in determining illumination, need to be replaced by one universal law of illumination, which is independent of the concept connected with a surface. The universal law of illumination is essentially related to the illumination of any point in space. With the foregoing in mind, here will be defined the **universal law of illumination** which states: **The illumination at an arbitrary point in space is directly proportional to the luminous intensity of the point light source and inversely proportional to the square of its distance to that light source.** The universal law of illumination is represented by the formula (3).

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