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Research article

Non-parametric accelerated life testing estimation for fuzzy life times under fuzzy stress levels

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Abstract: Uncompleted developments in the fields of measurement sciences are categorically agreed on the fact that measurements obtained from continuous phenomena cannot be measured precisely. Therefore, these measurements cannot be considered precise numbers but are nonprecise or fuzzy. For this purpose, it is compulsion of the time that such estimators need to be developed to cover both the uncertainties. The classical accelerated life testing (ALT) approaches are based on precise life times and precise stress levels, but in fact, these are not precise numbers but fuzzy. In this study, the nonparametric procedure of ALT is generalized in such a manner that in addition to random variation, fuzziness of the lifetime observations and stress levels are integrated in the developed estimators. The developed generalized nonparametric estimators for accelerated life time analysis utilize all the obtainable information that is present in the form of fuzziness in single observations and random variation among the observations to make suitable inferences. On the other hand, classical estimators only deal with random variation, which is a strong reason to conclude that the developed estimators should be preferred over classical estimators.

Keywords: accelerated life time; characterizing function; fuzzy life time; non-precise
Mathematics Subject Classification: 62N05, 94D05

1. Introduction

Statistics is the science of decision making based on the obtained data. Obtained observations are generally recorded in the form of numbers, sets, vectors, or functions, containing measurements of some phenomena. Countless techniques, e.g., stochastic models, are available to model variation among precise observations for inferences.

Survival analysis is the group of techniques for analysing lifetime data and can be defined as "the time to the occurrence of a specified event". Lifetime is also known as survival time, failure time, or event time and can be measured in seconds, hours, days, weeks, months, or years.

Any particular or specified event in survival analysis depends on the field of study; if the field of study is medical science, it may be death or recovery of a patient from disease, if its engineering sciences failure of mechanical equipment, in social sciences, i.e., in sociology divorce, change of residence in demography, in education time until degree completion, in business workers compensation claims, etc. [1].

These lifetime data techniques were in use for centuries but received attention approximately a few decades ago, especially during World War II, which sparked interest in the reliability of military equipment [2].

It has been observed that with the advancement of technology, the lifetime of manufactured items and human average life have increased. For reliability estimation techniques in engineering and biological fields, a large number of observations are usually needed. Therefore, it is truly a time-concentrated process to obtain complete information about the life times of the components under study. Furthermore, in real-life practices, it is not possible to have identical conditions for the lifetime units. The idea to measure the lifetime of units under various conditions was presented in "Accelerated life testing of capacitors" by [3], which was an attired motive to test electronic equipment under dissimilar environmental situations.

For the reliability and survival estimation of units in different environmental conditions, lifetime tests are performed under various environmental conditions, stress levels or dose levels, which are more severe or different than usual situations.

To make these inferences more precise, all the information available in sampled observations needs to be addressed. For this purpose, the classical techniques cover variation among the sampled observations, but in real-life situations, particularly dealing with continuous variables, the obtained data have several uncertainties: variation among the observations and imprecision of single observations. It is a point of prime consideration that variation among observations is different from the second kind of uncertainty, i.e., imprecision, which is also called *fuzziness* [4].

In the following sections, some important concepts of fuzzy theory are explained. Section 1.1 shows the nonparametric approach for classical data that need to be generalized. In Section 2, generalized parameter estimators are suggested, and estimates are presented.

From [4] some selected theories of the fuzzy set theory are explained as:

Fuzzy Number

A fuzzy number f^* is a member of special subset of the real number $\mathbb{R}$, represented by *characterizing function* $\Psi(\cdot)$, contains one real variable with conditions:

(1) $\Psi : \mathbb{R} \to [0, 1]$.

(2) The δ-cut $C_\delta(f^*) := \{f \in \mathbb{R} : \Psi(f) \geq \delta\}$ $\quad \delta \in (0, 1]$ is a finite union of non-empty compact intervals, i.e.,
$C_\delta(f^*) = \bigcup_{j=1}^{k_\delta} [a_{\delta,j}, b_{\delta,j}] \neq \emptyset$.

(3) Support of the $\Psi(\cdot)$, defined as $supp[\Psi(\cdot)] := [f \in \mathbb{R} : \Psi(f) > 0]$
$\subseteq [a, b]$ is bounded.

Fuzzy intervals are considered as special cases of fuzzy numbers, for which all δ-cuts are non-empty closed bounded intervals.

Remark: A family $(C_\delta(f^*);\ \delta \in (0\,,\,1])$ is nested, i.e. for $\delta_1 < \delta_2$ we have $C_{\delta_1}(f^*) \supseteq C_{\delta_2}(f^*)$.

Lemma: Let $\mathbb{1}_A(\cdot)$ is representing the indicator function, then for any characterizing function $\Psi(\cdot)$ of a fuzzy number the following holds:

$$\Psi(f) = \max\left\{\delta{\cdot}\mathbb{1}_{C_\delta(f^*)}(f) :\ \delta \in [0\,,\,1]\right\} \qquad \forall f \in \mathbb{R}.$$

Remark: It is a point of prime consideration that it is not necessary all nested families $(A_\delta\,;\ \delta\ \in\ (0\,,\,1])$ of finite unions of compact intervals are the δ-cuts of a fuzzy number. But the following construction lemma holds:

Construction Lemma: Let $(A_\delta\,;\ \delta\ \in\ (0\,,\,1])$, is representing a nested generating family of intervals, that are non-empty subsets of $\mathbb{R}$, then the characterizing function of the generated fuzzy number can be obtained by:

$$\Psi(f) = \sup\left\{\delta{\cdot}\mathbb{1}_{A_\delta}(f)\ :\ \delta \in [0\,,\,1]\right\} \qquad \forall f \in \mathbb{R}.$$

See [5].

Fuzzy data is used in almost every field like, risk assessment in chemical industry [6], clinical decision making [7], Waiting time for electric vehicles [8], efficient power management [9], decentralized charging approach [10], power distribution [11], operational constraints of power grid [12].

Furthermore, in [13] it has already been presented that the obtained measurements on life time of units under study in accelerated life testing approaches are fuzzy instead of precise numbers.

Since then some references like, accelerated life testing [14], for step-stress experiment [15], for empirical acceleration functions [16], accelerated life testing under Weibull distribution [17], machinery product with accelerated testing data [18], are found that integrates the fuzziness of the observations in ALT approaches for inferences.

For any unit that is tested under different stress levels, environmental conditions or dose levels, it is not possible to exert the specified stress level or apply the desired environmental condition or the dose level at one. However, it starts from the initial point and increases to the required situation. This shows that stress levels or different environmental conditions or dose levels are not precise but fuzzy.

Therefore, the said imprecision or fuzziness cannot be ignored for a suitable inference; by doing so, we may lose some important information and will lead to inappropriate conclusions. In this work, a generalized estimator is proposed to cover the fuzziness of the observations as well as random variation among the observations.

Non parametric estimation for ALT

The classical statistics estimation and test theories are based on the assumptions that the sample obtained follows certain assumptions about the parameters. These estimation and testing techniques are called parametric statistics. If the sample does not satisfy or violates the assumptions of parametric procedures, one has to look for an alternative; the most common are nonparametric statistics. Dealing with life time analysis, in classical life time approaches or in ALT approaches, the life times follow some probability distributions such as exponential, Weibull, lognormal, etc. If the assumption of parametric techniques violates one of the popular nonparametric estimation techniques is proposed in [19].

Let the selected stress levels are denoted by $S_1, S_2, \cdots, S_k$ and $y_{i,l}$, $l = 1(1)n_i$, $i = 1(1)k$ are the obtained life times under these accelerated stress levels.

Let n_i is denoting total number of units tested under considered stress level S_i, where $i = 1(1)k$ and $N = \sum_{i=1}^{k} n_i$ is denoting all units tested.

The data will be transformed by the form $\overline{Y}_i = \frac{1}{n_i}\sum_{l=1}^{n_i} y_{il}$, $i = 1(1)k$.

The scale factor between the distribution functions is denoted by α_{ij}, where

$$\alpha_{ij} = \left(S_i/S_j\right)^{\gamma}, i \neq j$$

$$\gamma = \ln(\alpha_{ij})/\left(\ln S_i - \ln S_j\right).$$

The estimator of α_{ij} is given by

$$\hat{\alpha}_{ij} = \overline{Y}_i/\overline{Y}_j \quad \text{for} \quad i \neq j,$$

and an overall estimator of γ can be obtained as weighted average of the $\hat{\gamma}_{ij}$'s:

$$\hat{\gamma} = \frac{\sum_{i=1}^{k}\sum_{j=i+1}^{k} \ln(\frac{S_j}{S_i})\ln(\frac{\overline{Y}_i}{\overline{Y}_j})}{\sum_{i=1}^{k}\sum_{j=i+1}^{k}(\ln(\frac{S_j}{S_i}))^2}. \quad (1.1)$$

For detail see [19].

The progression of technology and material sciences resulted in a rise in the life times or reliability of units. Therefore, experts must draw inferences about the aggregate of units based on only a few observations. Subsequently, it is essential to combine all the obtainable information in the best possible method to draw true inferences.

2. Generalized non-parametric estimation for ALT obtained from fuzzy life times under fuzzy stress levels

For any unit that is tested under different stress levels, environmental conditions or dose levels, it is not possible to exert the specified stress level or apply the desired environmental condition or the dose level at one. However, it starts from the initial point and increases to the required situation. This shows that stress levels or different environmental conditions or dose levels are not precise but fuzzy.

The Figure 1 below explains the algorithm how the estimation procedure will work:

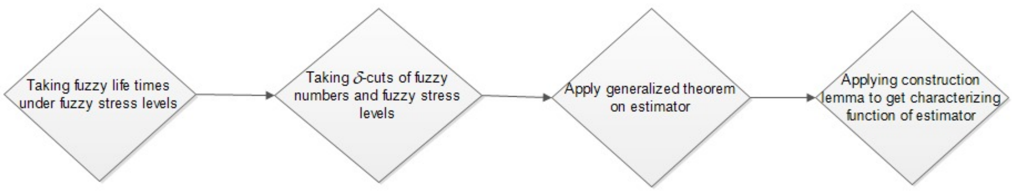

Figure 1. A frame diagram for the analysis.

Figure 1 explains the algorithm of the estimation. First, the lifetime observations were considered under fuzzy stress levels. After that, δ-cuts are taken of the fuzzy life times and fuzzy stress levels, and then using the generalized theorem, the upper and lower ends of the generating family of intervals for the nonparametric estimator $\hat{\gamma}$ are obtained. From these obtained intervals using the construction lemma, the required characterizing function of the generalized estimator is constructed.

The generalized non-parametric estimator $\hat{\gamma}^*$ is for the parameter γ, having δ-cuts

$$C_\delta(\hat{\gamma}^*) = \left[\underline{\gamma}_\delta\,,\, \overline{\gamma}_\delta\right] \quad \forall\, \delta \in (0\,,\, 1].$$

If we take fuzzy life times $y^*_{i,l}$, $l = 1(1)n_i$ and $i = 1(1)k$ having δ-cuts $C_\delta(y^*_{i,l}) = [\underline{y}_{i,l,\delta}\,,\, \overline{y}_{i,l,\delta}]$ $\forall \delta \in (0\,,\, 1]$, under fuzzy stress levels S^*_1, S^*_2, ..., S^*_k, then the corresponding δ-cuts of the mean of these life times is obtained as

$$C_\delta(\overline{Y}^*_i) = \left[\frac{1}{n_i}\sum_{l=1}^{n_i}\underline{y}_{i,l,\delta}\,,\, \frac{1}{n_i}\sum_{l=1}^{n_i}\overline{y}_{i,l,\delta}\right] \quad \forall\, \delta \in (0\,,\, 1].$$

Defining

$$\underline{\overline{Y}}_{i,\delta} = \frac{1}{n_i}\sum_{l=1}^{n_i}\underline{y}_{i,l,\delta} \qquad \text{and} \qquad \overline{\overline{Y}}_{i,\delta} = \frac{1}{n_i}\sum_{l=1}^{n_i}\overline{y}_{i,l,\delta}.$$

Similarly the δ-cuts for fuzzy stress level S^*_j can be defined as

$$C_\delta(S^*_i) = \left[\underline{S}_{i,\delta}\,,\, \overline{S}_{i,\delta}\right] \quad \forall\, \delta \in (0\,,\, 1].$$

Devised on the δ-cuts of fuzzy stress levels and fuzzy life times, $\left([\underline{\gamma}_\delta\,,\, \overline{\gamma}_\delta];\quad \forall\, \delta \in (0\,,\, 1]\right)$ are denoting the inferior and superior ends of the generating family of intervals for the generalized non-parametric estimator of the parameter γ is attained as mentioned in Eqs (2.1) and (2.2):

$$\underline{\gamma}_\delta = \min\left[\frac{\sum_{i=1}^k\sum_{j=i+1}^k\left(\ln\left(\frac{\underline{S}_j}{\overline{S}_i}\right)\right)\left(\ln\left(\frac{\underline{\overline{Y}}_{i,\delta}}{\overline{\overline{Y}}_{j,\delta}}\right)\right)}{\sum_{i=1}^k\sum_{j=i+1}^k\left(\ln\left(\frac{\underline{S}_j}{\overline{S}_i}\right)\right)^2}\,,\quad \frac{\sum_{i=1}^k\sum_{j=i+1}^k\left(\ln\left(\frac{\underline{S}_j}{\overline{S}_i}\right)\right)\left(\ln\left(\frac{\overline{\overline{Y}}_{i,\delta}}{\underline{\overline{Y}}_{j,\delta}}\right)\right)}{\sum_{i=1}^k\sum_{j=i+1}^k\left(\ln\left(\frac{\underline{S}_j}{\overline{S}_i}\right)\right)^2}\right] \tag{2.1}$$

$$\overline{\gamma}_\delta = \max\left[\frac{\sum_{i=1}^k\sum_{j=i+1}^k\left(\ln\left(\frac{\overline{S}_j}{\underline{S}_i}\right)\right)\left(\ln\left(\frac{\underline{\overline{Y}}_{i,\delta}}{\overline{\overline{Y}}_{j,\delta}}\right)\right)}{\sum_{i=1}^k\sum_{j=i+1}^k\left(\ln\left(\frac{\overline{S}_j}{\underline{S}_i}\right)\right)^2}\,,\quad \frac{\sum_{i=1}^k\sum_{j=i+1}^k\left(\ln\left(\frac{\overline{S}_j}{\underline{S}_i}\right)\right)\left(\ln\left(\frac{\overline{\overline{Y}}_{i,\delta}}{\underline{\overline{Y}}_{j,\delta}}\right)\right)}{\sum_{i=1}^k\sum_{j=i+1}^k\left(\ln\left(\frac{\overline{S}_j}{\underline{S}_i}\right)\right)^2}\right]. \tag{2.2}$$

These Eqs (2.1) and (2.2) give the lower and upper ends of the generating family of intervals for the characterizing function of the generalized non-parametric estimator $\hat{\gamma}^*$. Construction lemma is used to obtain Characterizing function from the generating family of intervals, using the said lemma required characterizing function for the fuzzy estimator $\hat{\gamma}^*$ is obtained.

Let three fuzzy stress levels are taken, in the below Figure 2 characterizing functions of the considered three fuzzy stress levels S^*_1, S^*_2, and S^*_3, are depicted respectively.

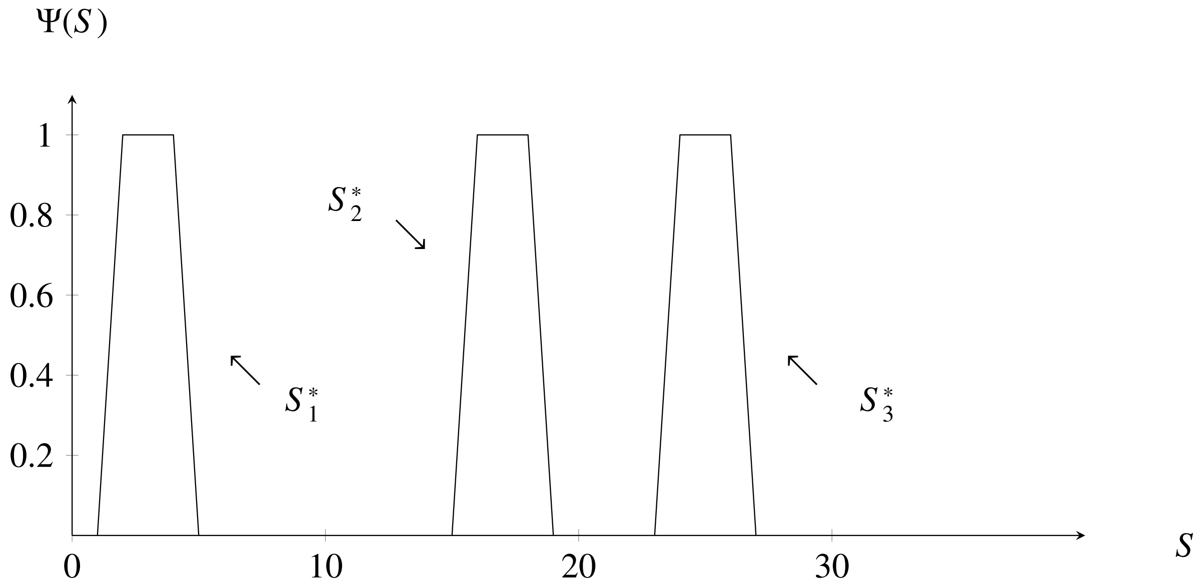

Figure 2. CF of the three fuzzy stress levels $S_1^*,\ S_2^*,\ S_3^*$.

A sample of 6 fuzzy observation is taken, in the below Figure 3 characterizing functions of the considered fuzzy life times under the fuzzy stress level one.

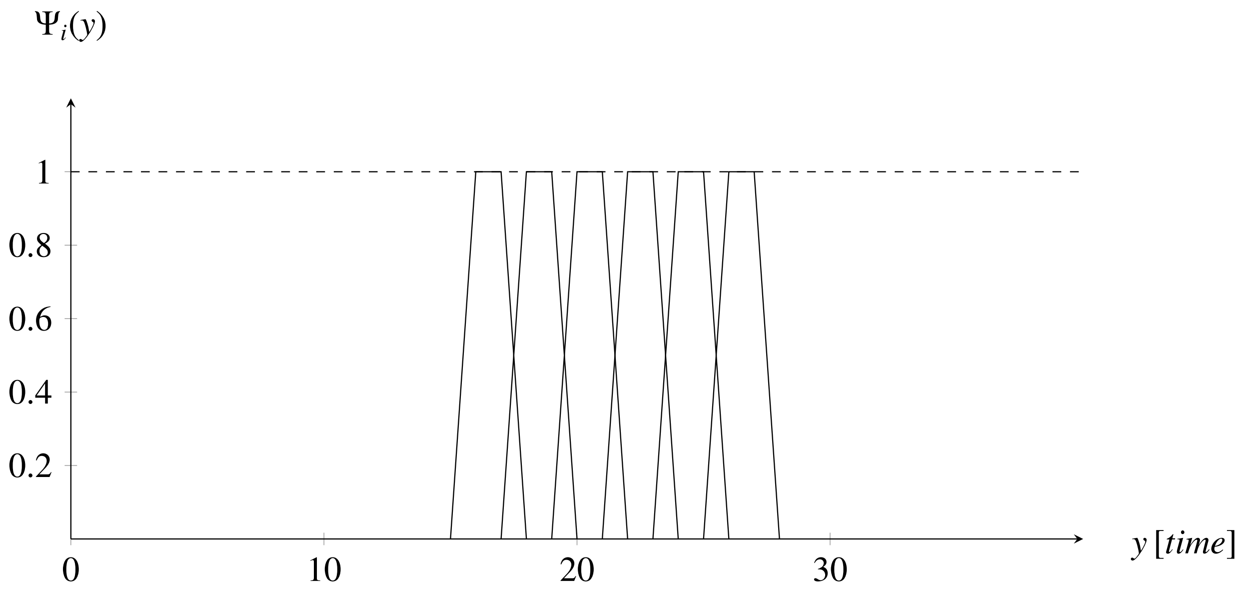

Figure 3. Sample of fuzzy observations under fuzzy stress S_1^*.

A sample of 6 fuzzy observation is taken, in the below Figure 4 characterizing functions of the considered fuzzy life times under the fuzzy stress level two.

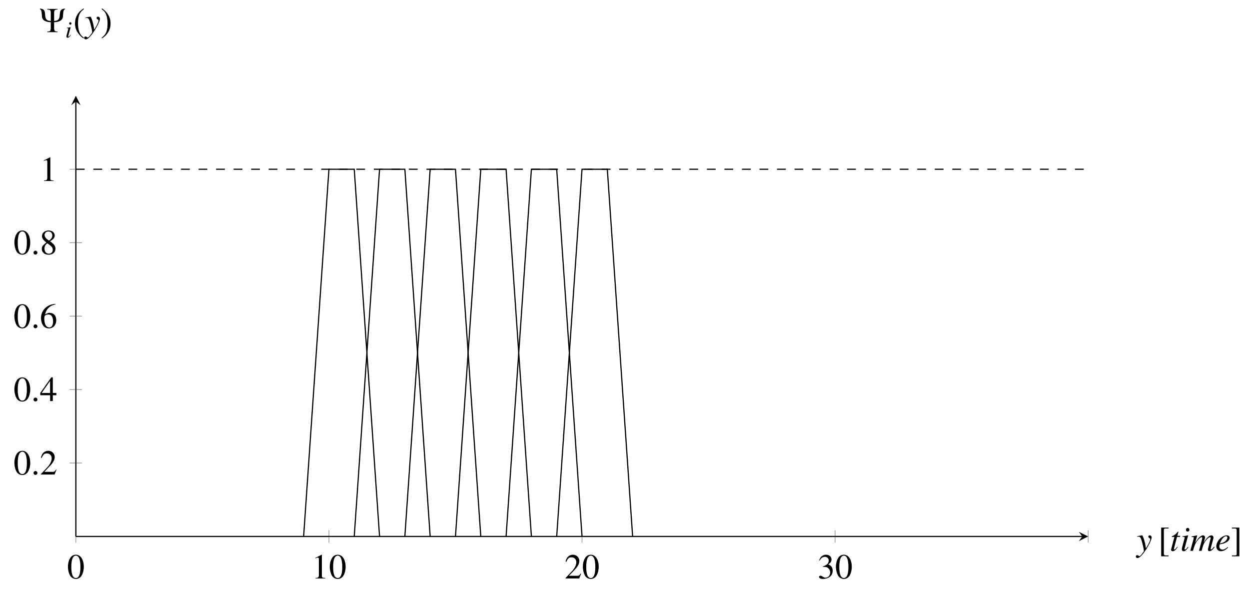

Figure 4. Sample of fuzzy observations under fuzzy stress S_2^*.

A sample of 6 fuzzy observation is taken, in the below Figure 5 characterizing functions of the considered fuzzy life times under the fuzzy stress level three

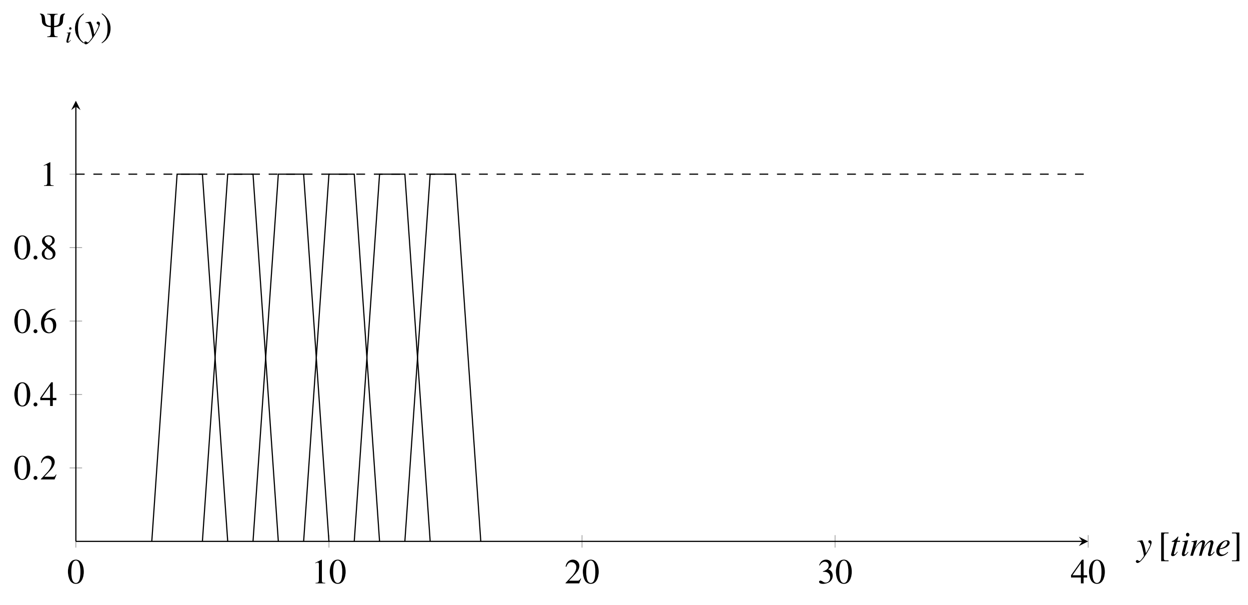

Figure 5. Sample of fuzzy observations under fuzzy stress S_3^*.

Using construction lemma, in the below Figure 6 the characterizing function obtained of the generalized fuzzy parameter estimate is depicted which is obtained from fuzzy life time observations and fuzzy stress levels.

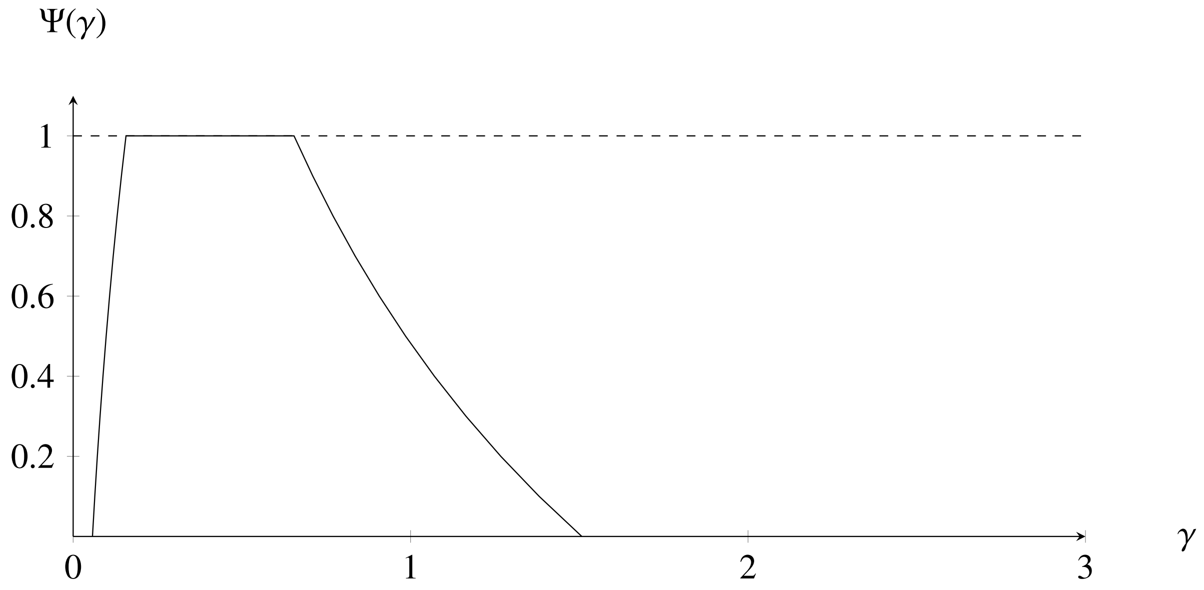

Figure 6. CF of generalized fuzzy parameter estimator $\hat{\gamma}^*$.

Based on the fuzzy life times obtained under the various fuzzy stress levels, the characterizing function of the generalized fuzzy parameter estimate is depicted in Figure 6, from which we can infer about the generalized fuzzy parameter estimate. The amount of parameter γ is completely possible between 0.15 and 0.65 with a membership degree of 1, and it could be from 0.098 to 0.98 with a degree of possibility of 0.5. The same can be found for any degree of membership in interval [0,1].

The above figure shows that for the smaller degree of membership, the estimator has high fuzziness, but as the degree of membership increases, the fuzziness decreases. For trapezoidal fuzzy life times and fuzzy stress levels, the estimator is slightly right skewed.

3. Conclusions

With advancements in measurement sciences, it has been confirmed that measurements based on continuous phenomena cannot be measured accurately. Therefore, these measurements cannot be considered precise numbers but have more or less fuzziness.

For sure life time is of continuous nature and most of the times these observations are dealt as precise numbers, but in real life applications life times are no more precise observations but fuzzy numbers.

Dealing with life time analyses, one cannot ignore the area of accelerated life testing. It has already been presented that the obtained measurements on the life time of units under study in accelerated life testing approaches are fuzzy instead of precise numbers.

In addition, for lifetime observations, the stress level, environmental conditions, dose level, etc., are not the same for all times called accelerated life tests. Furthermore, it is fact that the observation obtained under accelerated life testing, the stress level/environmental conditions cannot be precise numbers but fuzzy numbers.

Therefore, in this study, a nonparametric approach of the ALT is generalized in such a way that in addition to the random variation, fuzziness of the life times and stress levels are integrated in the estimator. On the other hand, classical estimators only deal with random variation and have nothing to do with the fuzziness of the life times and stress levels, which is a strong reason to conclude that the developed estimators should be preferred over classical estimators.

Furthermore, fuzziness of the stress levels needs to be incorporated in parametric approaches and probability distributions obtained under various stress levels.

Conflict of interest

There is no conflict of interest.

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