ESTIMATION OF IMPLIED VOLATILITY SURFACE AND ITS DYNAMICS: EVIDENCE FROM S&P 500 INDEX OPTION IN POST-FINANCIAL CRISIS MARKET

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PROJECT SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE IN FINANCE

In the Master of Science in Finance Program
of the
Faculty
of
Business Administration

SIMON FRASER UNIVERSITY

Fall 2015

Approval

Name:	**Shouting Sun and Sijia Ji**
Degree:	**Master of Science in Finance**
Title of Project:	**Estimation of Implied Volatility Surface and its Dynamics: Evidence from S&P 500 Index Option in Post-Financial Crisis Market**

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Abstract

There is now an extensive literature on modeling the implied volatility surface (IVS) as a function of options' strike prices and time to maturity. The polynomial parameterization is one of these approaches and it provides a simple and efficient way for practitioners to estimate implied volatility. This project tests the predictive capability of this methodology in the post-financial crisis market. Using data for the period from July 1st, 2012 to June 30th, 2015 for European puts and calls of the S&P 500 index options, we estimate a vector autoregressive model to capture the dynamics of the IVS. Our results show that this methodology has better predictive capability on IVS of index options in post-financial crisis market than on IVS of equity options in pre-financial crisis period.

Keywords: Volatility surface; Volatility dynamics

Acknowledgement

We would like to express our sincerest gratitude to everyone who supported us throughout this program. Thank Professor Atanasova, our senior supervisor, for her guidance and helpful advice through the whole project. Also, we would like to thank our second reader, Professor Gatev, for sharing his illuminating views on several issues. In addition, we would like to thank our classmates, Abhishek Rokade and Sara Soltanizerdele, for proofreading and giving us useful suggestions regarding the paper. Finally, we are sincerely grateful to every SFU faculty for providing much needed support and facilitating the presentation.

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1. Introduction

Volatility has always been a central topic for measuring risk in financial market. Accurate estimation can contribute to good performances in speculation and hedging. The introduction of implied volatility surface (IVS) is one of the methods to analyse volatility across options' strike prices and time to maturity. Thus, various models of surface construction and its dynamics were developed and have been improved over time.

A recent paper named "Can we forecast the implied volatility surface dynamics of equity options? Predictability and economic value tests" (Bernales and Guidolin, 2014) examined whether dynamics of the IVS of individual equity options contains exploitable predictability patterns. They used a polynomial deterministic model to construct IVS and fitted a vector autoregressive model to estimate dynamics of IVS from January 4, 1996 to December 29, 2006. They did not include the data during financial crisis, which is an outlier by historical standards. Their result indicated that this methodology is effective in estimating IVS of equity options and predicting its dynamics. As proved by Goncalves and Guidolin (2006), this methodology also works for IVS of index options before financial crisis.

However, five years after financial crisis, the market has changed in a number of ways as US economy kept growing but slowed down, US Federal Reserve cut down interest rate and kept it at a low level, and S&P 500's bull market continued for four years. Therefore, the predictability of IVS in post-financial crisis market and predictive capability of this approach need to be tested and that is what we aim to achieve in this paper.

This project uses the data of index options quoted in American market after financial crisis, from July 1st, 2012 to June 30th, 2015. To model IVS of index option and its dynamics, we follow the same methodology adopted by Bernales and Guidolin (2014). There are also some alternative ways of modeling volatility surface, such as non-parametric model (Burke, 1988), semi-parametric model (Borovkova and Permana (2009) and Heston's SV model (1993). However, according to Goncalves and Guidolin (2006), the polynomial deterministic approach introduced by Dumas et al. (1988) yields a better fit to option data. Another advantage of this methodology is that "all the explanatory variables are fully observable and correspond to simple transformations of key contract parameters." (Bernales and Guidolin, 2014) Nevertheless, such parametrizations has limited performance in estimating IVS of options with extremely long or short time to maturity as well as options deep in the money or deep out of the money, so before fitting the surface we filter the data to eliminate those options.

Our results show that the polynomial deterministic model adequately fits observed data after financial crisis and the vector autoregressive model provides effective prediction about dynamics of IVS, which is consistent with existing literatures.

This paper is organised as follows: Section 2 reviews the literatures of IVS and its dynamics modeling. Section 3 describes the data and summary statistics. Section 4 introduces the parametric model used for IVS modeling and results of estimation. Section 5 presents the vector autoregressive model used to capture the dynamics of IVS and results of back testing. Section 6 gives conclusions and future work.

2. Literature Review

The creation of Black-Scholes model (1973) contributes to the option pricing method, which is widely used by traders in financial markets. However, the assumptions of this model are so unrealistic, making it difficult to satisfy the real market conditions. These assumptions include a continuous-time economy, same lending and borrowing rates, no taxes and no short selling, etc. It has been criticised that the BS model can be biased in predictable ways, owing to the constant volatility and lognormal distribution assumptions (Hull and White, 1987). This leads to the constant underestimation of the option value.

Other than pricing procedure, practitioners also use BS model to derive the implied volatility (IV) for trading purpose. Similarly, as stock returns are likely to show skewness and fat tails rather than following a normal distribution, it is dangerous to have a blind trust in this implied volatility. This is in line with early study (Canina and Figlewski, 1993), stating that bias occurs when utilising IV to predict future volatility, as their empirical evidence shows no correlation between IV of S&P 100 index options and future realised volatility, thus IV provides little incremental information beyond historical volatility. In contrast, Jorion (1995) and Fleming (1998) believe that volatility implied from option price offer relevant information in terms of future volatility, and practitioners can use it as a good estimate of future volatility. The capability of predicting IV is particularly critical in hedging with complex derivatives because IV predictions do not rely on the historical price or volatility due to instantaneous adjustment of new information in option price. In addition, Fengler (2005) proves that predicting IV is more effective than time-series based method in predicting future volatility. Recent paper also agrees with the predictability of IV in foreign exchange, stock and bond market (Busch, Christensen, and Nielsen, 2011).

Furthermore, when plotting IV against strike price with a fixed maturity, a smile pattern can be observed, reflecting fat tails in the return distribution. This IV smile also varies across different time to maturity and over different periods (Homescu, 2011). The variation of IV across different strike prices and time to maturity is widely referred as the implied volatility surface (IVS). Unlike estimates based on historical data, IVS is broadly accepted as a variable that is assumed to offer the current insight of market risk (Bakshi et al., 2000). Therefore, IVS becomes a useful tool for trading analysis. Traders can use actively traded options with the full range of the strike prices and time to maturity to value a specific option that is not easily visible in the market, and they can assure their option price is consistent with the market, without violating no-arbitrage constraint. They also use it to hedge against potential changes in the IVS or changes in the underlying prices (Daglish, Hull and Suo, 2007). Therefore, market makers continuously screen and bring their IVS up-to-date, and risk management reviews the movement of the IVS to learn the effect of a large market movement on entire portfolio (Fengler, Hardle and Mammen, 2007).

In order to implement the IVS for application in a realistic way, a suitable construction model is needed. One widely accepted way is to build the volatility surface using stochastic volatility model (Heston 1993). This is a closed-form solution to price options, assuming volatility is a stochastic process. It aims to overcome the weakness regarding skewed return in the Black-Scholes model. So one can describe the option model in terms of the first four moments of the spot return, and it can be extended to price stock options, bond options, and currency options. Further extension of Heston model to a multifactor volatility process, which considers correlations between assets' returns and their volatility, better fits the smile effect and offers more flexibility (Fonseca, Grasselli and Tebaldi, 2008).

While the stochastic volatility calibrates the longer time to maturity well, volatility surface based on Levy process handles short-term skew better. Nevertheless, it is challenging to calibrate a Levy-based model in practice. It has been shown that spontaneous calibration of Levy process with several maturities produce less precise result of implied volatility smile, compared with that of single maturity (Cont and Tankov, 2006). Konikov and Madan (2002) also suggest imposing strict conditions on homogeneous Levy processes, such as constant risk-neutral variance and inverse relationship between the term and kurtosis. Therefore, considering the implied volatility of options with different time to maturity, it is difficult to maintain the empirical success of Levy process.

Volatility surface based on parametric method has been also extensively used in the literatures. Ncube (1996) once used OLS regression to perform an empirical estimation of time-varying volatility, indicating that this technique is superior to benchmark method for estimation when analysing FTSE 100 Index European options. One paper also examines implied volatility of the FTSE options by fitting a parametric model to forecast the surface across moneyness, and suggests improving this model for more precise estimation of the recent volatility expectation (Alentorn, 2004). Practitioners also designed a model of stochastic volatility inspire (SVI) parametrization to study the surface (Gatheral, 2006). However, arbitrage may exist in the SVI and many recent papers attempt to study how to build a non-arbitrage interpolation of implied volatilities (Fengler 2009; Glaser and Heider 2012). For example, Gatheral and Jacquier (2014) calibrate the SVI parameterization through analysing recent S&P 500 option data, aiming to avoid butterfly and calendar spread arbitrage. Bloch (2012) also introduces a new parametric model using weighted shifted lognormal distributions in generating volatility surface. Specifically, instead of modeling volatility itself, Bloch decided to model the probability distribution function (PDF) which could be converted to option price and then implied volatility.

Many research papers study how to use non-parametric representations for surface modelling as well. In this method, researchers use interpolation technique to smooth option prices and keep the shape of price function. It works well whether the sample satisfies the convexity and decreasing constraints or not, and empirical study on S&P 500 index option shows that this method is accurate and robust (Wang, Yin and Qi, 2004). Rather than focusing on prices, directly smoothing implied volatility with the help of constrained local quadratic polynomial function provides another perspective for nonparametric modelling (Benko et al, 2007). In order to overcome the estimation bias of non-parametric design due to the large bandwidth in the time-to-maturity dimension, Fengler, Hardle and Mammen (2007) introduce a semi-parametric factor model that approximates the IVS in a finite dimensional function space is introduced and studied using DAX index option data.

The attractiveness of IVS is that its movement can be predictable, which facilitates the process of forecasting in the trading market. Practitioners use option prices to derive the forward-looking facts regarding the asset returns and realised volatility. Likewise, time variation in the IVS can be successfully captured using statistical model, and the predictability may provide economic benefits (Gonçalves and Guidolin, 2006). Researchers discuss several models that describe dynamics of IVS in their papers. One of them is a simple vector autoregressive model that explains information in past dynamics. Bernales and Guidolin (2014) use this model as a benchmark to study the dynamics of equity and index options IVS. The ad-hoc "Strawman model", which is a simple random walk process for the parameters of deterministic IVS, uses current values to predict the shape of IVS. Another powerful technique for forecasting is Principal Components Analysis (PCA), which is a non-parametric method to describe the dynamics of a number of variables, and often used in a linear regression as predictors (Konstantinidi et al., 2008). A recent paper also presents an innovated model assuming most traders build their expectation on implied volatility based on the spot price. Then spot price could drive ATM volatility, skewness, and curvature and these three parameters could explain IVS (Bloch, 2012).

3. Data and Summary Statistics

As mentioned before, we are trying to check if the methodology used by Bernales and Guidolin (2014) in modeling IVS and its dynamics will work for index options in post-financial crisis market. Therefore, we go through almost the same procedure as they do in data collection and cleaning but make a few adjustments because of the limitation of our data source.

Firstly, we download the data of all the Put and Call options of S&P 500 traded in CBOE from OptionMatrix of WRDS database, which includes Price Date, Expiration Date, Strike Price, Highest Ask Price, Lowest Bid Price and Implied Volatility. Bernales and Guidolin (2014) took both American and European style while we only take European style. In addition, they calculate the implied volatility with binomial model for American options and BS model for European options, while we directly use the implied volatility provided by WRDS. The period of their data is between 1996 and 2006 because financial crisis is considered as outliers in the historical time series. Our data period is from July 1st, 2012 to June 30th, 2015. The reason we choose this period is that we attempt to examine the predictability of IVS in post-financial crisis market and we have to collect enough data for back testing. Usually, we need at least 300 observations to conduct regression and one day forecast. With rolling horizon technique, we need 600 observations to implement one-day forecast for 300 times. That is why we get the option data of last three years. Bernales and Guidolin (2014) also get the data of 150 most frequently traded equity options as they focus on providing the dynamics of implied volatility of equity options taking into account the cross-section effect from Index options.

Secondly, we make a few calculations using the data we extract for the purpose of filtering and regression later. Time to maturity is the number of days between price date and expiration date divided by 360. The option price is the average of highest ask price and lowest bid price and the spot price of index is downloaded from Bloomberg. We also get the policy rate of each day in the past 3 years from Federal Reserve System and match it with the price date of all the options we have. For some price dates, the interest rate information is missing. Therefore, we assume it equal to the last available interest rate. For policy rate of each day, the format we get is the spot rate of 1 month, 3 months, 6 months, 1 year, 2 years, 3 years, 5 years, 7 years, 10 years, 20 years and 30 years. Then, we fit a spot curve for each sequence and find the spot rate of the expiration day on the spot curve. Because the spot rates we get from the Federal Reserve System are calculated with the yields of government bonds, we assume the spot rate to be semi-annually compounded. Then the discount factor between option price date and expiration date would be:

$$discount\ factor = 1/(1 + r/2)^{2*T}$$

where r is spot rate and T is time to maturity in years.

Lastly, we filter the data according to four exclusionary criteria, as same as what Bernales and Guidolin (2014) did in their research. First, we eliminate all the options for which the prices fall outside of the no-arbitrage bounds, which are calculated by put-call parity relationships:

$$For\ Call\ Options: max\ (0, S_t - K/discount\ factor) \leq C \leq S_t$$

$$For\ Put\ Options: max\ (0, K/discount\ factor - S_t) \leq P \leq K/discount\ factor$$

where K is strike price and S_t is spot price of the index. Secondly, we exclude the option contracts expiring in less than 6 days and more than one year because the IVS does not capture implied volatility of options with either very long or very short time to maturity well. Then, we eliminate option contracts whose moneyness is less than 0.9 or more than 1.1, since the prices of deep ATM or OTM options are usually outliers in observations. The last exclusionary criteria is option price under \$0.375. Same with Bernales and Guidolin (2014), we keep the option contracts with zero trading volume because as long as the quotes are shown on trader's monitor, they will affect traders' expectations of the market.

Table 1

Summary statistics of implied volatility across moneyness and time to maturity for S&P500 options

	Short (6-120 days)			Medium (120 -240 days)			Long (240-360 Days)		
K/S	Frequency	Mean (IV)	SD (IV)	Frequency	Mean (IV)	SD (IV)	Frequency	Mean (IV)	SD (IV)
[0.9,0.94]	17.10%	20.82%	4.11%	1.76%	18.42%	1.74%	1.46%	18.68%	1.62%
(0.94,0.98]	20.36%	16.88%	2.92%	1.80%	16.53%	1.72%	1.46%	17.32%	1.63%
(0.98,1.02]	21.01%	13.15%	2.49%	1.91%	14.59%	1.66%	1.44%	15.86%	1.59%
(1.02,1.06]	17.50%	11.33%	2.49%	1.81%	13.00%	1.66%	1.44%	14.60%	1.66%
(1.06,1.1]	7.91%	12.67%	4.61%	1.61%	11.71%	1.76%	1.44%	13.46%	1.66%

Table 1 is the summary of our filtered data and it shows the differences between implied volatility across moneyness and time to maturity. To give a big picture of the data we use, we add the measurement of frequency, which is defined by:

$$Frequency = \frac{number\ of\ options\ in\ this\ group}{number\ of\ options\ in\ total}$$

We find the most frequently quoted options are the ATM options with short time to maturity.

4. Modeling Volatility Surface

The model we use to construct implied volatility surface is the same with the model adopted by Bernales and Guidolin (2014), proved to be effective in capturing characters of IVS in different asset classes and periods. This model assumes that the implied volatility can be explained by moneyness and time to maturity and its advantage is that all the explanatory variables are observable in the market, which is beneficial in forecasting and simulation. This deterministic linear function is

$$\sigma_i(M,T) = \beta_{0t}^i + \beta_{1t}^i M + \beta_{2t}^i M^2 + \beta_{3t}^i T + \beta_{4t}^i (M * T) + \varepsilon_{it}$$

where ε_{it} represents the random error term that is assumed to be white noise. $\sigma_i(M,T)$ is the observed implied volatility of the option with respect to moneyness M and time to maturity T. $\beta_{0t}^i, \beta_{1t}^i, \beta_{2t}^i, \beta_{3t}^i, \beta_{4t}^i$ are the parameters we want to estimate. "β_{0t}^i is the intercept coefficient in the Black and Scholes world, where volatility is constant. The moneyness slope of IVS is characterised by the coefficient β_{1t}^i. β_{2t}^i captures the curvature of the IVS in the moneyness dimension. β_{3t}^i reflects the maturity slope and β_{4t}^i describes the possible interactions between the moneyness and time to maturity dimensions."(Bernales and Guidolin, 2014). In previous researches, there are different ways in describing the moneyness and the one we use here is the time-adjusted moneyness:

$$M = \frac{\ln K/(S_t/discount\ factor)}{\sqrt{T}}$$

Then, we estimate the IVS for each trading day in the past 3 years and we get the time series of all coefficients as well as R-Square and RMSE (root-mean-square error). The statistics are calculated and listed in Table 2. The mean of R-square is 89.5% with a standard deviation of 5.5% and the mean of RMSE is 0.012 with standard deviation of 0.003.

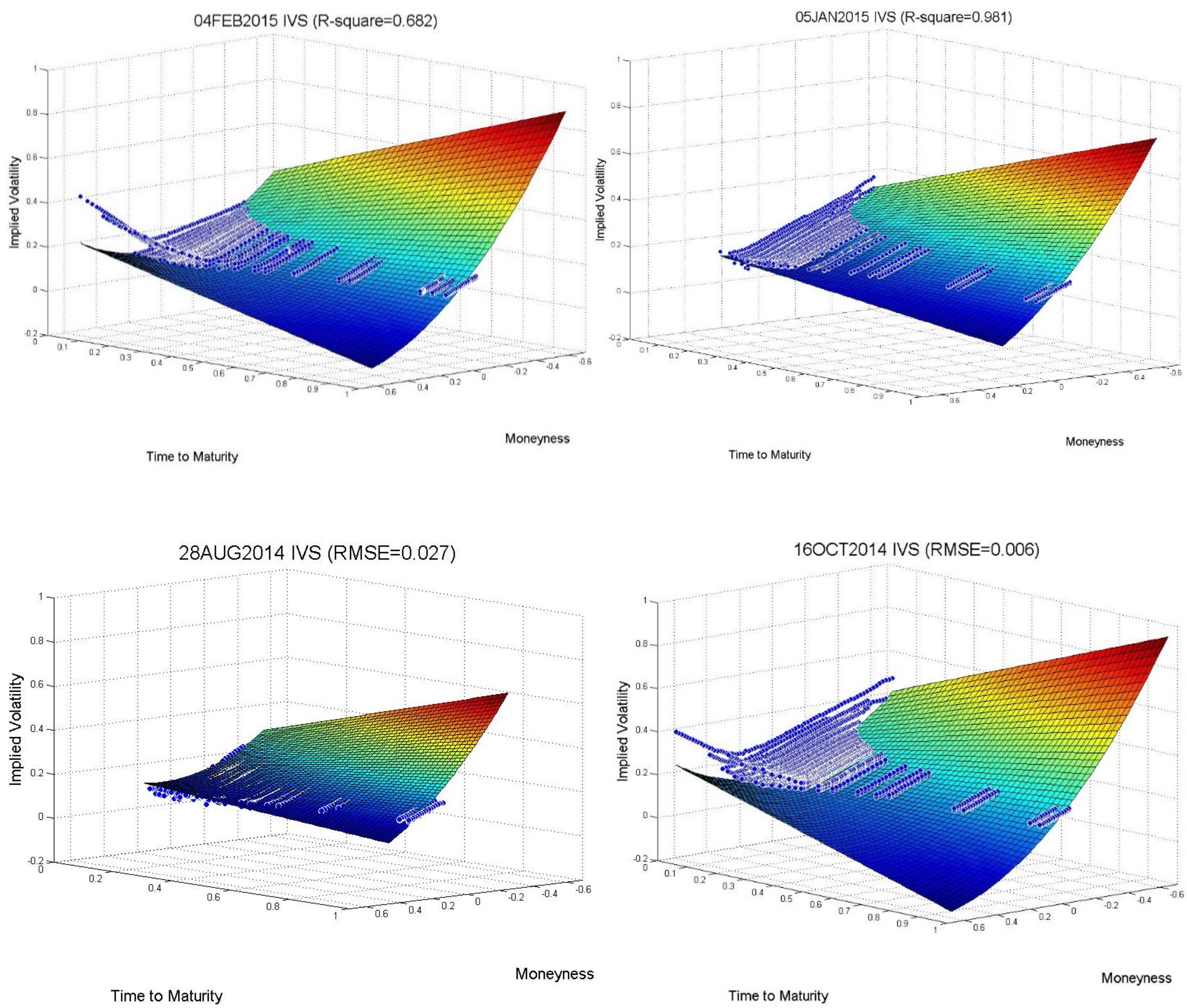

Figure 1. Best and Worst Estimated IVS according to measurement of R-square and RMSE

Compared with the research of Bernales and Guidolin (2014), R-square of our estimation is higher and RMSE is lower. Moreover, the standard deviations of these two measurements are relatively small. It means that the quality of estimation in this model is better and more stable in post-financial crisis market than before. Same with the research of Bernales and Guidolin (2014), the estimation indicates that the implied volatility is decreasing as a function of moneyness as well as the interaction between moneyness and time to maturity while it is increasing as a function of time to maturity and square of moneyness. Using Ljung-Box test with 1 and 3 lags, we find that all the coefficients show significant autocorrelation, which justifies our adoption of VAR model in estimation the dynamics of IVS.

Table 2

Summary Statistics of deterministic IVS model coefficients

Coefficient	Mean	SD	Skewness	Kurtosis	Min	Max	Times of Non-Sig	Lag1	Lag3
beta0	0.13	0.02	1.01	4.09	0.08	0.21	0	505	1221
beta1	-0.12	0.09	-0.47	2.25	-0.34	0.06	33	164	426
beta2	0.04	0.02	-0.49	4.82	-0.05	0.10	15	485	1172
beta3	0.45	0.20	-0.15	2.20	-0.01	0.93	3	164	402
beta4	-0.39	0.21	0.26	2.29	-0.88	0.16	27	151	391
R-Square	0.89	0.05	-0.49	2.58	0.68	0.98		143	392
RMSE	0.01	0.00	0.62	3.82	0.01	0.03		108	250

5. Modeling Dynamics of IVS

In the research of Bernales and Guidolin (2014), they focus on modeling the dynamics of equity options' IVS with VARX model, taking into account the cross section effect from IVS of index options. They also compare the predictive capability of VARX model with three benchmark models, which are VAR model, ad-hoc 'Strawman' model and random walk model. As we are trying to model the dynamics of index option IVS, VAR model is our best choice because it is closest to Bernales and Guidolin's design. The VAR model we use is:

$$\widehat{\mathrm{B}}_t = \Upsilon + \sum_{j=1}^{p} \Phi_j \widehat{\mathrm{B}}_{t-j} + \mathrm{E}_t$$

where $\widehat{\mathrm{B}}_t = \left[\widehat{\beta}_{t0}, \widehat{\beta}_{t1}, \widehat{\beta}_{t2}, \widehat{\beta}_{t3}, \widehat{\beta}_{t4}\right]'$ is the vector of estimated coefficients, Υ is constant vector whose size is the same with $\widehat{\mathrm{B}}_t$ and Φ_j is 5*5 matrix of coefficients. Same as what Bernales and Guidolin (2014) do with IVS of equity options, we adopt 3 as the number of lags in our regression.

Using rolling horizon technique, we want to conduct forecast as many times as we can to test the predictability of this methodology. That means we use $\widehat{\mathrm{B}}_1, \widehat{\mathrm{B}}_2, \widehat{\mathrm{B}}_3, \dots \widehat{\mathrm{B}}_{362}, \widehat{\mathrm{B}}_{363}$ for regression and forecast $\widehat{\mathrm{B}}_{364}$ with $\widehat{\mathrm{B}}_{361}, \widehat{\mathrm{B}}_{362}, \widehat{\mathrm{B}}_{363}$. Then, we use $\widehat{\mathrm{B}}_2, \widehat{\mathrm{B}}_3, \dots \widehat{\mathrm{B}}_{363}, \widehat{\mathrm{B}}_{364}$ for regression and forecast $\widehat{\mathrm{B}}_{365}$ with $\widehat{\mathrm{B}}_{362}, \widehat{\mathrm{B}}_{363}, \widehat{\mathrm{B}}_{364}$….With 663 coefficient vectors, we can conduct forecast like this for 300 times. With each forecast, we can predict the IVS of the next day. On the forecast surface, we can find the forecast IV of all the options traded on that day and then, we could get:

$$forecast: \hat{\Sigma}_{364}, \hat{\Sigma}_{364}, \dots \hat{\Sigma}_{662}, \hat{\Sigma}_{663}$$

$$observation: \Sigma_{364}, \Sigma_{364}, \dots \Sigma_{662}, \Sigma_{663}$$

where $\hat{\Sigma}_i = \left[\hat{\sigma}_i^1, \hat{\sigma}_i^2, \hat{\sigma}_i^3, \dots, \hat{\sigma}_i^{n_i}\right]'$ is the vector of forecast IV on i day and $\Sigma_i = \left[\sigma_i^1, \sigma_i^2, \sigma_i^3, \dots, \sigma_i^{n_i}\right]'$ is the vector of observed IV on day i .

We then examine the predictability of IV with two measures: RSME (root-mean-square error) and MAE (mean absolute error) which are defined by:

$$RMSE = \sqrt{\frac{\sum_{i=364}^{663}\left(\sum_{j=1}^{n_i}\left(\hat{\sigma}_i^j - \sigma_i^j\right)^2\right)}{\sum_{i=364}^{663} n_i}}$$

$$MAE = \frac{\sum_{i=364}^{663}\left(\sum_{j=1}^{n_i}\left|\hat{\sigma}_i^j - \sigma_i^j\right|\right)}{\sum_{i=364}^{663} n_i}$$

By comparison, both measurements of prediction in our research are lower than that in Bernales and Guidolin's research.

Table 3

Performance of model on IVS of equity (1996-2006) and SPX (2012-2015) options

Predictability Measurement	**RMSE**	**MAE**
Equity IVS Dynamics(1996-2006)	0.046	0.037
SPX IVS Dynamics(2012-2015)	0.018	0.012

6. Conclusion and Future Work

According to our research, the parametric deterministic model and vector autoregressive model have better predictive capability on IVS of index options in post-financial crisis market than IVS of equity options in pre-financial crisis market, which could be explained by following reasons. The first reason is that, in Bernales and Guidolin's research, polynomial deterministic model fits the IVS of index options better than equity options because a single stock contains specific risk besides systematic risk and its volatility is more difficult to capture. Moreover, equity options are usually less liquid than index options, resulting in larger bid/ask spread, which makes the price of equity options less sensitive to market expectation. The other reason why this approach captures dynamics of IVS better in post-financial market is that the market after financial crisis is much more active than before. The average number of daily bid/ask quotes from 2005 to 2006 is 635, while it is 1962 from 2014 to 2015. More observations help us to capture the features of IVS and its dynamics more precisely, making great contribution to better performance of this methodology. However, this approach provides small error in predicting IVS in post-financial crisis market might also because we only use data of three years, while previous research adopts ten years' data.

Volatility clustering enables model to fit observations better in short period. Moreover, although this project adopts the same approach with previous research, our data source and data cleaning procedure are not exactly the same with theirs. This might also cause the difference in predictive capability of this approach.

In Bernales and Guidolin's research, they adopt the GLS (Generalized Least Square) method in estimating the parametric model, which, according to Hentschel (2003), has better performance in IVS modeling. Therefore, researchers could test the predictability of this methodology under different fitting procedures in the future.

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