

**INFORMATIONAL CONTENT OF IMPLIED AND HISTORICAL
VOLATILITY DURING SUB-PRIME CRISIS**

by

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Approval

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Title of Project: **INFORMATIONAL CONTENT OF IMPLIED AND
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CRISIS**

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Abstract

In this paper, we try to test the informational content of implied volatility versus historical volatility during periods of high volatility. In this paper, we have taken the time-period of subprime financial crisis. Canina and Figlewski (1993) first did similar research using time-period of 1983 to 1987. We have extended this research to S&P 100 options during the time of financial crisis (January 2007-December 2010).

The initial paper had concluded that implied volatility is not a good predictor of the realized volatility, and instead historical volatility does a better job of explaining the realized volatility. Based on our findings from the data during the subprime crisis, we observe that both implied volatility and historical volatility are not efficient predictors of future volatility, but when compared, implied volatility does a better job than historical volatility. Our findings differ from the original research as we used a different time-period, and the findings are in line with logical reasoning as during periods of high volatility, historical volatility does not give any prediction of future volatility as circumstances change drastically.

Keywords: Volatility; Implied Volatility; Historical Volatility; Sub-Prime Crisis

Dedication

To my family, who are my foundation and inspiration - Deepanshu

I would like to dedicate this work to my Parents, Brother and all my friends
- Rupinder

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Glossary

AR	Autoregressive Model
ACF	Auto-correlation Function
ARMA	Autoregressive–Moving-average Model
IV	Implied Volatility
MA	Moving Average Model
OEX	Index Option on S&P's 100 Index.
OLS	Ordinary Least Squares regression
PACF	Partial Auto-correlation Function
S&P 100	Standard & Poor's 100 Index
VIX	CBOE Volatility Index

1: Introduction

Implied volatility of an option is widely accepted as the market's forecast of option's future realized volatility over the time remaining to expiration. Considering that option markets and markets in general are efficient, implied volatility should be able to efficiently predict option's future volatility. Various studies have been done in this regard. In this paper, we discuss about the informational content and predictive power of implied volatility, and then compare it with that of historical volatility in the period of market crisis. For this research, we have taken S&P 100 options in the period of financial crisis (2007-2010). Our focus is to test the hypothesis in previous researches that whether implied volatility, which is preferred to historical volatility in the financial industry, has truly more predictive power over historical volatility. As per our tests and analysis, we found that both implied volatility and historical volatility failed the rationality test i.e. both are not a good predictor of future realized volatility. However, when we compared implied vs historical volatility, implied volatility did a better job in predicting future realized volatility than historical volatility.

1.1 Background

Volatility is the measure of dispersion of returns of a security or an index. Volatility is generally measured as the standard deviation of returns. In intuitive terms, it is the amount of uncertainty in price of security. Generally, higher the volatility, higher the risk associated with a security, as higher volatility means that price of a security can change by a bigger amount in upward or downward direction. On the other hand, lower volatility means the price of the underlying won't change much and would be more or less stable.

The term 'volatility' can refer to various types of volatility:

- a. Historical Volatility: Historical volatility refers to the volatility exhibit by the underlying prices in the past.
- b. Current Implied Volatility: It refers to the implied volatility exhibit by the current prices of options observed in the market.
- c. Future Realized Volatility: It refers to the actual volatility of underlying over pre-specified period starting at current time and ending at a future date.

As mentioned, volatility might refer to historical or implied volatility. Implied volatility is a topic of concern for option traders, rather it is one of the most important concepts for option traders. In a broad measure, it is the estimated volatility of the underlying.

In theoretical terms, implied volatility is the value of volatility which when input in an Option-pricing Model (e.g. Black Scholes), would give the option price currently observed in the market. Implied volatility gives an estimate of the future value of the option, and it is reflected in the current price of the option. It is important to understand that implied volatility is essentially a probability and gives an estimate of the deviation of stock prices, but does not give any idea of the direction of movement.

The value of implied volatility is not directly observable in the market, and can be calculated using an Option-pricing Model. The Black Scholes model is among the most widely used, and it takes into account the option price, time to expiration (in terms of years), strike price, and the risk free rate. Black Scholes model works well with the European options, as there is no case of early exercise. However, for American options, binomial mode works better as it takes into account the case for early exercise.

1.2 Literature Review

Studying the informational content of implied volatility has been very popular topic among the academic researchers in finance. Several studies have focused on whether implied volatility is an efficient predictor of future realized volatility or not. These include Harvey and Whaley (1992), Poterba and Summers (1986) and Sheikh (1989). There has been a ton of research related to implied volatility, and its behaviour in certain conditions such as stock splits. French and Dubofsky (1986) found that implied volatility increases after stock splits. Some researches contradict this observation as well, such as Sheikh (1989), and Klein and Peterson (1988). Several other studies further explored this question and used various different approaches. One of them was Latane and Rendleman(1976) and they focused on static cross-sectional tests. One of the findings of these studies was that the stocks that have high implied-volatility tend to have higher ex-post realized volatility.

Researching on implied volatility in various situations, Madura and Tucker (1992) studied how implied volatility of currency options changes with U.S trade deficit announcements. Levey and Yoder (1993) studied how implied volatility behaves around merger and acquisitions announcements.

With improvement in data availability and models, various studies have focused on implied volatility in dynamic conditions. One of such papers, Jorion(1995) observed that in case of foreign currency futures, implied volatility efficiently predicts futures' realized volatility. On the other hand Day and Lewis(1992) did a similar research on S&P 100 index options expiring in 1985 to 1989 and found that, implied volatility is biased and inefficient; and historical volatility does a better job in forecasting future volatility. Lamoureux and Lastrapes (1993) concur with the above finding in case of 10 stocks expiring from 1982 to 1984. Christensen and Prabhala (1998) used monthly overlapping data, examined the informational content of implied volatility and observed that implied volatility is informationally efficient in predicting future realized volatility. They also mentioned that predictive

power of implied volatility increased significantly after the 1987 market crash. In another study, Szakmary et al. (2003) observed that implied volatility outperformed historical volatility in predicting future volatility for majority of 35 future options markets in US. Among recent studies, Becker et al(2007) researched whether implied volatility provide additional information beyond that captured in model-based volatility forecasts. They came with the conclusion that no additional information relevant for forecasting volatility is provided by VIX index.

2: Volatility Forecast and Implied Volatility

2.1 Historical Volatility, Realized Volatility, Implied Volatility and Volatility Forecast

It is widely believed in the financial services industry that implied volatility is a better estimate and predictor of future prices and volatility compared to historical volatility. In research work as well, implied volatility is perceived to be a better measure of the ex-ante research than the historical volatility.

To get a measure of implied volatility, Black Scholes equation gives a decent benchmark by inputting option prices and other observable parameters. By iterative methods, we get the implied volatility, which gives an estimate of the expectation of volatility of underlying.

However, there is an inherent conflict in this approach, as assumption of Black Scholes is that stock price follows logarithmic process with constant volatility, and to use this method to estimate the future volatility (that changes randomly with time) is not accurate. Some researches deal with this problem and take into account the stochastic nature of volatility e.g. Wiggins (1987), or Hull and White (1987). In such stochastic models, the requirement is not just to calculate the volatility parameter but also the joint probability distribution of asset returns and changes in volatility and even the market price of volatility risk. The difficulty in implementation of such models is increased by these requirements. In spite of these conflicts of the Black Scholes model, traders use the implied volatility, or a combination of implied volatility and historical volatility to get an estimate of the expected volatility and then use this to estimate future option prices, and then make trading decisions based on this analysis. Academic researches also use implied volatility from 'fixed-volatility' models to measure the expectations of future volatility.

Going by that rationale, our study is valid in the sense that many researchers and market players follow the 'fixed-volatility' models in their forecast.

Given a data series, in our case a series of prices $\{X_0, X_1, \dots\}$, realized volatility is calculated as standard deviation (annualized) of continuously compounded returns $\{R_0, R_1, \dots\}$. Here $R_t = \ln(S_t/S_{t-1})$ and K is number of observation intervals in an year. To annualize the standard deviation we multiply by root of K .

$$\sigma = \left[\frac{K}{T-1} \sum_{t=1}^T (R_t - R_m)^2 \right]^{1/2}$$

Taking the case of historical volatility, an assumption can be made that it would continue in the future period as well, and this gives us a way to calculate future volatility of stock. Another way of estimating future volatility is through current option prices. We can observe the parameters in the market such as the option price, time to expiration etc, and there exists a 1-1 correspondence between implied volatility and price of option. The calculated implied volatility gives an estimate of future option volatility, which is generally taken as market's belief.

$$IV = E_{MKT}[\sigma] \tag{1}$$

where, E_{MKT} refers to the market's expectation of implied volatility.

Realized volatility, by definition can be explained as the expected value based on a conditional set of information data ϕ in addition to random error with a mean of 0, which is orthogonal to the given information set ϕ .

$$IV = E[\sigma|\phi] + \epsilon, \quad E[\epsilon|\phi] = 0 \tag{2}$$

This establishment gives us an already well-established test for regression for the rationality of forecast:

$$\sigma = \alpha + \beta F(\phi) + \mu \quad (3)$$

In this equation, $F(\phi)$ refers to the forecast of the σ based on the information set ϕ , and μ is the residual we get from regression. In the case that our forecast is correct and is the true expected value, the hypothesis testing for regression parameters would give us values of 0 and 1 for α and β respectively.

Next we try to regress σ on two forecasts, $F_1(\phi_1)$ and $F_2(\phi_2)$, this would give us a multiple linear regression equation as follows:

$$\sigma = \alpha + \beta_1 F_1(\phi_1) + \beta_2 F_2(\phi_2) + \mu \quad (4)$$

Above approach was first used by Fair and Shiller (1990), in which they used to evaluate forecasting performance of various model. Here depending upon the value we get for both slope coefficients, we can figure which of the forecast is better in predicting actual realized volatility or what combination of both forecast would better predict future realized volatility. Increasing the components normally should improve the accuracy of the predictions. So we can find out whether adding any additional independent variable to the regression improves the accuracy and predictive power.

3: Data and Methodology

As this paper is focused on the volatility during the period of financial crisis, the data period we took includes such a period in financial markets, the subprime crisis of 2007-08. The data sample drawn contains closing prices of S&P 100 options (ticker: OEX), from the period January 3, 2007 – end of 2010. We removed those data points which have less than 7 or more than 127 days to expiration, and those which have less than or more than 60 points in or out of the money. As level of OEX was around 600 during the period options with intrinsic value of 60 were not hugely in the money. We did not consider the options, which were deep in the money, or deep out of the money, as in such cases the impact of volatility on the option price is minimal. We took only American calls in consideration. We averaged the bid and ask price to calculate the option price.

On each trading day, there are several option prices, and hence different implied volatilities. The IV (implied volatility) of OEX options varies across different strike prices and along different maturity periods, where maturity periods contain options that expire in near, second, and third, and fourth month.

Table 3.1 Statistics of current implied volatility categorized by maturity group

Maturity Group (i)	Days to Expiration	Number of Observations	Mean	Standard Deviation
All		26058	0.2417	0.1098
1	7 - 32	5700	0.2583	0.1303
2	32-63	7490	0.2426	0.1155
3	63-98	7838	0.2347	0.0974
4	98-127	5030	0.2327	0.0901

Table 3.1 shows implied volatility of options grouped into different maturities. We observe from Table 3.1 that the mean of implied volatility in our sample decreases with increase in time to option expiration. The mean of all the observations (26058)

is 0.2417, and the mean for the four maturity groups decline from 0.258 for first group (expiring on near month) to 0.2327 for the last maturity group (expiring in 4th month). For the argument, whether mean values of implied volatility at different maturities are significantly different or not, we conducted a T Test between 1st and 4th maturity groups. Based on the result we reject the hypothesis that mean of implied volatility of 1st and 4th groups are same.

Figure 3.1 Plot of Implied Volatility against Time to Expiration.

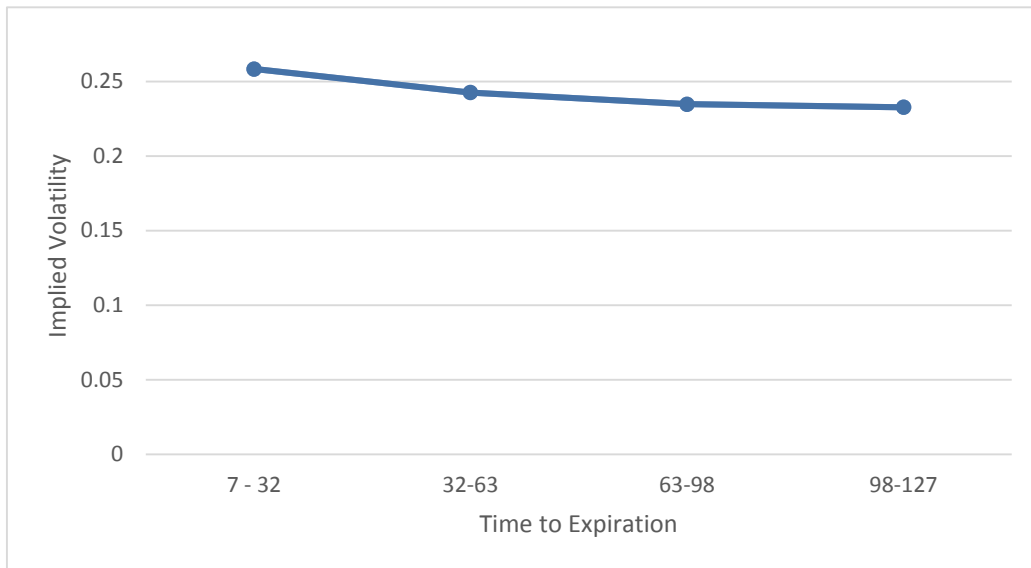


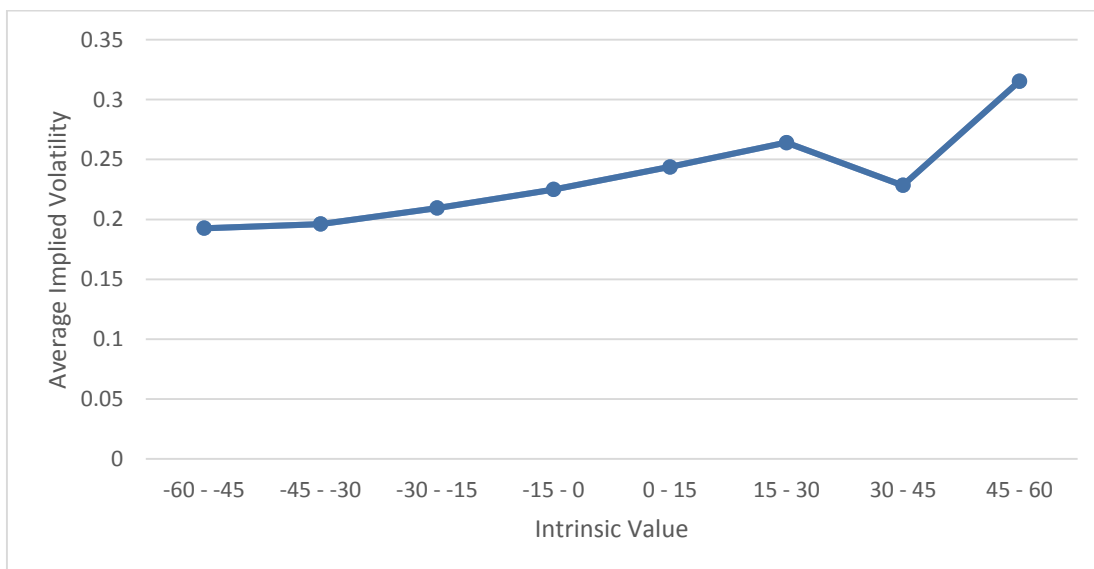
Table 3.2 categorizes the options into different classes of moneyness (where moneyness of an option is the relative position of the strike price to the current price of underlying). For a call, the moneyness is equal to $S-X$ (X refers to strike price). Moneyness is also called the intrinsic value. Figure 3.2 depicts the same thing in a graph.

Table 3.2 Statistics of current implied volatility categorized by intrinsic value group

Group (j)	Intrinsic Value	Number of Observations	IV Mean	Standard Deviation
All		26058	0.2417	0.1098
1	-60 - -45	3134	0.1926	0.0879
2	-45 - -30	3225	0.196	0.0911
3	-30 - -15	3480	0.2094	0.0938
4	-15 - 0	3266	0.225	0.0983
5	0 - 15	3242	0.2438	0.1013
6	15 - 30	3240	0.2641	0.1047
7	30 - 45	3226	0.2284	0.1102
8	45 - 60	3245	0.3154	0.1219

Latane and Rendleman (1976) advocated the approach of calculating weighted-average implied standard deviation using various options on the same stock. This approach would be right if the only reason of difference in volatility is the sampling noise. However, in practice, volatility changes with time and thus options expiring on different dates might be priced taking into account different volatilities.

Figure 3.2 Plot of Implied Volatility against Intrinsic Value.



Coming to observations from Table 3.2, we notice that deep in the money options (intrinsic value: 45-60) have IV 0.3154, which is much greater when compared to IV (0.23) of at the money options (intrinsic value: 0-15). One of the reasons for our

finding could be attributed to sample selection. While selecting the sample, we excluded the calls that had negative IV. As mentioned above, options that are deep in the money have less impact of volatility on prices. Such deep in the money options are illiquid, which means a bigger bid-ask spread and thus introduces noise in the observations. Thus, our sample selection process potentially introduces a bias by including calls whose prices might be artificially high due to noise and eliminating them when their price went below the boundary condition due to noise.

To check the effect of such bias, we included the calls that violated boundary conditions and had negative IVs, but we took IVs of those options as 0. We found out that this changes the mean estimate IVs of deep in the money calls and brings them near to the values observed in other option groups.

When a regular volatility structure is exhibit by the market, which also reflects true systematic factors, we wouldn't know which IV is the true expectation of the market. We have already established that simply averaging the IV would contaminate market volatility's forecast. Thus, we divide the sample into subgroups according to maturity and intrinsic value. This gives us an opportunity to test if any subgroup, gives a better estimate of IV.

We divide the sample into four maturity groups, and each group is further divided into 8 subgroups each based on intrinsic value. This categorizing gives us 32 subgroups in total which would be denoted as subsample (i, j), here i represents maturity value and j refers to intrinsic value. For instance, subsample (2, 3) would represent maturity group 2 and intrinsic value group 3 – subsample would contain options with maturities of 2nd month and are between 30 to 15 points out of the money. The construction of subsamples is also shown on in Table 3.1 and 3-2. As we selected options which were in the money and out of the money by up to 60, this gave us 120 points in total which were divided further into 8 groups with each break point 15 points apart. We construct the subsamples in a way such that each

contains a maximum of one option price per day. We had enough data points in each sub-group to provide reliable statistics.

After we divide the subsamples, we estimate the following equation for each subsample:

$$\sigma_t(\tau) = \alpha + \beta \cdot IV_t(i, j) + u_{t,i,j} \quad (5)$$

This equation is derived from the equation (3) discussed in previous section. In the above equation $\sigma_t(\tau)$ is the actual realized volatility calculated by annualizing standard deviation of daily returns between period t and $t + \tau$ (option's expiry date). We multiplied daily standard deviation by square root 260 to get the annualized number. $IV(i, j)$ represents the implied volatility at time t , calculated from option in subsample whose maturity group is i and moneyness group is j . For daily returns, we used natural log returns of daily prices which is $\ln(S_t/S_{t-1})$.

Each of the 32 samples is tested separately. Estimation is done using equation 5 for each subsample. If the forecast truly predicts the expected value $\sigma_t(\tau)$, regression realized values should give regression results of 0.0 for α and 1.0 for β . Implied volatility will be a biased and inefficient forecaster of actual realized volatility if we don't get above results.

For the least squares estimate of α and β to be consistent, the regressors and disturbances should be uncorrelated with one another. However, when we use the daily data, the disturbances might be serially correlated, as was shown by Cont, R.(2005). The catch here is that the realized volatility involves time period from $t+1$ to expiration and thus $\sigma_t(\tau)$ is known after the expiration time has passed. This means that the forecast errors are correlated for IVs, which are computed by taking pair of options having overlap in their remaining lifetimes.

To deal with the problem of serial correlation in the sample data, we use the following steps to calculate residuals and then perform regression. We use the ARMA filter to remove the problem of serial correlation. It is important to note that we used the ARMA filter for both sides of regression equation.

- Performed hypothesis test for constant mean and constant variance
- Plotted Autocorrelation and Partial Auto Correlation function of the volatility data to determine the order of ARMA filter
- Used an ARMA model to eliminate any serial correlation. The ARMA model can be pure AR(p), MA(q) or ARMA(p,q)
- Until this step, we dealt with the problem of serial correlation, now we would use the residuals for regression and perform OLS
- Regressed the residuals obtained with independent variable as implied volatility/ historical volatility and dependent variable as future realized volatility.

ARMA Filter

ARMA filter is a linear combination of Autocorrelation (AR) and Moving Average (MA) processes. ARMA (p, q) model is generally of the form:

$$x_t = \phi_0 + \sum_{i=1}^p \phi_i x_{t-1} + \sum_{i=1}^q \theta_i a_{t-1} + a_t$$

- The optimal order (p, q) is obtained by trial and error.
- p is the Autoregressive order and q is the order for Moving Average part.
- The order (p, q) of an ARMA processes is different from optimal orders of pure AR (p*) or MA (q*) fit for the same data.
- ARMA (p, q) model is said to be “parsimonious if $p + q < p^*, q^*$.

ARMA (p, q) model:

If given a time series, ARMA model is generally used for understanding, simulating and for predicting future values of the series as well. As mentioned, the model

contains 2 parts: and Autoregressive and a Moving Averages part. The optimization of an ARMA (p, q) model is generally made by conditional maximum-likelihood. ACF and PACF cannot be used to determine order (p, q), and we have to use trial and error to find the optimal order for the ARMA model. In ARMA, the error terms are assumed to be independent identically distributed variables with zero mean.

3.1 Forecasting Performance of Implied and Historical Volatility

As mentioned in the previous section, following regression equation was fitted for each of the 32 subsamples. This equation is same as equation 5.

$$\sigma_t(\tau) = \alpha + \beta \cdot IV_t(i, j) + u_{t,i,j} \quad (6)$$

First, we tested the implied volatility. Table 3.3 shows the regression results of equation 6. Both the dependent and independent variable were tested for serial correlation, stationarity, and ARMA model was fitted to generate the residuals. Next OLS regression was fitted on the residuals. Groups are divided into different maturity groups and intrinsic value groups vertically and horizontally respectively.

Subsample wise results for predictive power of implied volatility are shown in Table 3.3. For instance, for subsample of options that expire in the third month ($i=3$), and that are between 0 and 15 points in the money ($j=5$), the intercept value (α) comes out to be .000016, and the estimated slope coefficient (β) is .01997. The value of R-squared for the regression is .0012. Thus, the hypothesis that $\alpha=0$, and $\beta=1$, which means that implied volatility is unbiased forecaster of future realized volatility is rejected in case of this particular subsample.

The result of this subsample is a representative of other subsamples as well. In every other subsample, implied volatility fails to pass the unbiasedness test. The slope coefficient is significantly different from zero in only 6 out of 32 subsamples at 5 percent significance level (These observations are highlighted in the table). In cases where coefficients were positives, implied volatility coefficient ranged from .00084 to .0998, and value of R-squared varied from .00028 to .017. We can see from these results that implied volatility has statistically no significant correlation with realized volatility, instead of being an efficient and unbiased indicator of volatility.

Table 3.3 Regression results – Actual Realised Volatility against Implied Volatility

Maturity Group	-60 to -45.01, j=1	-45 to -30.01, j=2	-30 to -15.01, j=3	-15 to -0.01, j=4	0 to -15 j=5	15.01 to -30, j=6	30.01 to -45, j=7	45.01 to -60, j=8
Maturity i=1; t = 7 to 31								
α	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
T-Stat(α)	0.0073	0.0191	0.0374	0.0181	0.0171	0.0172	0.0091	0.0113
β	0.0154	0.0534	0.0111	0.0998	0.0725	0.0336	0.0105	-0.0087
T-Stat(β)	0.4332	1.5246	0.5460	3.0786	2.4082	1.3327	0.5281	-0.5260
R squared	0.0003	0.0034	0.0003	0.0136	0.0084	0.0026	0.0004	0.0004
N	653	689	939	689	688	688	683	671
Maturity i=2; t = 32 to 63								
α	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
T-Stat(α)	0.0428	0.0367	0.0374	0.0382	0.0336	0.0381	0.0374	0.0389
β	0.0107	0.0139	0.0111	0.0170	0.0140	0.0008	-0.0021	-0.0061
T-Stat(β)	0.4736	0.6506	0.5460	0.8610	0.7420	0.0824	-0.2141	-0.7593
R squared	0.0002	0.0005	0.0003	0.0008	0.0006	0.0000	0.0000	0.0006
N	920	939	939	939	939	939	937	938
Maturity i=3; t = 64 to 92								
α	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
T-Stat(α)	-0.0549	-0.0318	-0.0356	-0.0397	-0.0486	-0.0316	-0.0396	-0.0475
β	0.0260	0.0352	0.0840	0.0404	0.0200	0.0238	0.0735	0.0542
T-Stat(β)	1.1957	1.7386	4.1783	2.1798	1.0774	1.4328	4.1903	3.5407
R squared	0.0015	0.0031	0.0175	0.0048	0.0012	0.0021	0.0176	0.0125
N	949	979	980	992	983	983	980	992
Maturity i=4; t = 93 to 127								
α	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
T-Stat(α)	0.0127	-0.0077	-0.0078	-0.0417	-0.0010	-0.0043	-0.0053	0.0162
β	0.0512	0.0093	0.0073	0.0416	-0.0292	0.0049	0.0055	0.0153
T-Stat(β)	0.9713	0.3621	0.8712	1.7758	-1.2217	0.2203	0.2365	0.7660
R squared	0.0065	0.0002	0.0023	0.0049	0.0024	0.0001	0.0001	0.0009
N	612	618	622	646	632	630	626	644

As per our observations from the tests, we observe that the value of intercept is significantly close to 0 in almost all of the subsamples. The values of slope coefficients are very low, and significantly less than the previous research done by Canina and Figlewski(1993). This is in line with the expected results that during

the time of crisis, implied volatility is not an efficient predictor of future realized volatility.

Table 3.4 Regression results – Actual Realised Volatility against Historical Volatility

Maturity Group	-60 to -45.01, j=1	-45 to -30.01, j=1	-30 to -15.01, j=1	-15 to -0.01, j=1	0 to -15 j=1	15.01 to -30, j=1	30.01 to -45, j=1	45.01 to -60, j=1
Maturity i=1; t = 7 to 31								
α	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
T-Stat(α)	0.0089	0.0188	0.0382	0.0188	0.0191	0.0191	0.0083	0.0078
β	-0.4246	-0.4165	0.0345	-0.4165	-0.3770	-0.3770	-0.3776	-0.4169
T-Stat(β)	-4.9571	-4.9855	0.6669	-4.9855	-4.5112	-4.5112	-4.5027	-4.9739
R squared	0.0364	0.0349	0.0005	0.0349	0.0288	0.0288	0.0289	0.0357
N	653	689	939	689	688	688	683	671
Maturity i=2; t = 32 to 63								
α	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
T-Stat(α)	0.0422	0.0382	0.0382	0.0382	0.0382	0.0382	0.0372	0.0393
β	0.0345	0.0345	0.0345	0.0345	0.0345	0.0345	0.0347	0.0345
T-Stat(β)	0.6593	0.6669	0.6669	0.6669	0.6669	0.6669	0.6716	0.6657
R squared	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
N	920	939	939	939	939	939	937	938
Maturity i=3; t = 64 to 92								
α	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
T-Stat(α)	-0.0550	-0.0301	-0.0351	-0.0406	-0.0437	-0.0303	-0.0363	-0.0452
β	-0.0020	-0.0853	-0.1642	-0.1177	-0.0088	-0.0840	-0.1269	-0.1554
T-Stat(β)	-0.0467	-2.0570	-3.8870	-2.9683	-0.2146	-2.0274	-2.8035	-3.6820
R squared	0.0000	0.0043	0.0152	0.0088	0.0000	0.0042	0.0080	0.0135
N	949	979	980	992	983	983	980	992
Maturity i=4; t = 93 to 127								
α	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
T-Stat(α)	-0.0127	-0.0113	-0.0217	-0.0319	-0.0043	-0.0081	-0.0055	0.0157
β	-0.2176	-0.2377	-0.2719	-0.2333	-0.2641	-0.2349	-0.2464	-0.2202
T-Stat(β)	-6.5163	-7.2086	-5.1140	-6.9539	-8.0837	-7.2115	-6.9998	-6.5164
R squared	0.0653	0.0778	0.0952	0.0698	0.0940	0.0765	0.0728	0.0620
N	612	618	622	646	632	630	626	644

As we have mentioned previously, slope coefficients of regression equation is significantly different from 1 in most of the subsamples in both cases of implied

and historical volatility. Thus, we can say that information contained in market is very little and not much useful for predicting volatility, and forecast errors dominate the results.

We can also say that forecasting future volatility is more of an art rather than science. One reason for this could be behavioural biases of derivatives traders.

For that argument, we checked whether if at all, we could predict volatility from the data available in the market. We tried to predict the volatility using historical volatility as the independent variable in the regression equation 7. In equation 7 $\sigma_t(\tau)$ is the actual realized volatility calculated by annualizing standard deviation of daily returns between period t and $t + \tau$ (option's expiry date). We multiplied daily standard deviation by square root 260 to get the annualized number. For daily returns we used natural log returns of daily prices which is $\ln(S_t/S_{t-1})$. Vol60 represents the historical annualized volatility of daily log returns from period $t-60$ to t .

$$\sigma_t(\tau) = \alpha + \beta.Vol60(i, j) + u_{t,i,j} \quad (7)$$

Next, we look at how good is historical volatility at predicting actual realised volatility during period of financial crisis. As mentioned, here the independent variable is the annualized standard deviation of S&P 100 stock index portfolio over 60 day period prior to the date of implied volatility. We used the 60-day sample period for historical volatility, as this time period is approximately the average forecast horizon in the sample we have taken. Similar tests taking time horizon from 30 to 120 days also yield similar results. Table 3.4 shows the result.

We can see from Table 3.4 that historical volatility is also not an efficient predictor of OEX future volatility. Slope coefficient is positive in only 9 subsamples. Thus, like the implied volatility, historical volatility also fails the rationality test.

It is important to note that compared to implied volatility, historical volatility is a worse predictor in the time of crisis. We can see from the table that most of slope coefficients in this case are negative. R-squared values in the case are also lower than that of implied volatility. In the previous study (during normal financial period), historical volatility was a better estimate as compared to implied volatility, but at the time of crisis, it is opposite and implied volatility, even though bad predictor is still a better predictor than historical volatility.

In our next test, we try to compare the future realized volatility against both implied and historical volatility through multiple linear regression as per the following equation:

$$\sigma_t(\tau) = \alpha + \beta_1 \cdot IV_t(i, j) + \beta_2 VOL60_t(i, j) + u_{t,i,j} \quad (8)$$

Above equation is derived from equation 4 mentioned earlier. In table 3.5, the estimated slope coefficient on implied volatility is greater than 0 in many cases while the estimated slope coefficient on historical volatility is negative. The values of slope coefficient of historical volatility in table 3.5 are comparable to that in table 3.4 regression. We observe that the IV coefficient is not significantly different than 0 and is negative in 3 out of 32 subsamples. If we look at the historical volatility, the slope coefficient is positive in only 9 of the 32 subsamples. Therefore, the overall message of the tests is – both historical volatility and implied volatility are poor forecasters of future realized volatility but, implied does better job than historical during financial crisis.

Table 3.5 Regression results – Realised Volatility vs Implied Volatility and Historical volatility

Maturity Group	-60 to -45.01, j=1	-45 to -30.01, j=2	-30 to -15.01, j=3	-15 to -0.01, j=4	0 to -15 j=5	15.01 to -30, j=6	30.01 to -45, j=7	45.01 to -60, j=8
Maturity i=1; t = 7 to 31								
α	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
T-Stat(α)	0.0080	0.0177	0.0375	0.0165	0.0153	0.0151	0.0076	0.0070
β_1	0.0431	0.0834	0.0109	0.1255	0.0946	0.0500	0.0218	0.0030
T-Stat(β_1)	1.2217	2.3938	0.5358	3.9131	3.1601	1.9914	1.1013	0.1833
β_2	-0.4410	-0.4491	0.0341	-0.4637	-0.4165	-0.4000	-0.3891	-0.4191
T-Stat(β_2)	-5.0881	-5.3235	0.6584	-5.5494	-4.9603	-4.7511	-4.6050	-4.9448
R squared	0.0386	0.0429	0.0008	0.0560	0.0428	0.0344	0.0306	0.0357
N	653	689	939	689	688	688	683	671
Maturity i=2; t = 32 to 63								
α	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
T-Stat(α)	0.0429	0.0368	0.0375	0.0383	0.0338	0.0382	0.0372	0.0391
β_1	0.0108	0.0138	0.0109	0.0165	0.0136	0.0003	-0.0025	-0.0064
T-Stat(β_1)	0.4765	0.6450	0.5358	0.8381	0.7220	0.0331	-0.2559	-0.7986
β_2	0.0346	0.0343	0.0341	0.0330	0.0334	0.0344	0.0355	0.0369
T-Stat(β_2)	0.6612	0.6614	0.6584	0.6373	0.6446	0.6623	0.6858	0.7104
R squared	0.0007	0.0009	0.0008	0.0012	0.0010	0.0005	0.0006	0.0012
N	920	939	939	939	939	939	937	938
Maturity i=3; t = 64 to 92								
α	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
T-Stat(α)	-0.0549	-0.0321	-0.0370	-0.0407	-0.0487	-0.0318	-0.0406	-0.0490
β_1	0.0261	0.0356	0.0827	0.0414	0.0201	0.0242	0.0737	0.0549
T-Stat(β_1)	1.1958	1.7595	4.1441	2.2435	1.0852	1.4588	4.2191	3.6093
β_2	-0.0026	-0.0859	-0.1614	-0.1193	-0.0104	-0.0847	-0.1278	-0.1573
T-Stat(β_2)	-0.0615	-2.0744	-3.8507	-3.0149	-0.2532	-2.0454	-2.8476	-3.7479
R squared	0.0015	0.0075	0.0322	0.0138	0.0012	0.0063	0.0257	0.0263
N	949	979	980	992	983	983	980	992
Maturity i=4; t = 93 to 127								
α	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
T-Stat(α)	0.0113	-0.0121	-0.0182	-0.0436	-0.0039	-0.0082	-0.0080	0.0152
β_1	0.0378	0.0144	0.0237	0.0457	-0.0294	0.0101	0.0170	0.0224
T-Stat(β_1)	0.7723	0.5826	0.8145	2.0190	-1.2915	0.4770	0.7579	1.1564
β_2	-0.1952	-0.2382	-0.2212	-0.2350	-0.2641	-0.2354	-0.2484	-0.2224
T-Stat(β_2)	-6.6382	-7.2184	-7.1320	-7.0195	-8.0894	-7.2192	-7.0341	-6.5725
R squared	0.6870	0.0783	0.0878	0.0757	0.0964	0.0768	0.0737	0.0640
N	612	618	622	646	632	630	626	644

In order to further check the validity of our results, we applied our procedure to all the data points at once i.e. without dividing it into subsamples. The results in this case were also very similar to previous tests. Intercept and slope coefficient in case of implied volatility were .000036 and 0.0164 respectively. For historical volatility, they were 0.000038 and -0.1870 respectively. Even when we run the regression across all similar maturities or across similar intrinsic values, we get similar results.

4: Conclusion

Implied volatility is widely considered as market's forecast of future volatility. In addition, any variations in implied volatility are viewed as market's response to new information about the underlying. In the previous studies done in this regard, the results have been conflicting. Some of the previous studies observed that during normal periods, historical volatility is a better predictor than implied volatility, and some studies report otherwise. As per our findings, during the period of sub-prime crisis of 2007-08, both implied volatility and historical volatility were poor predictors, but when we compare implied to historical volatility, implied volatility still did a better job.

A potential criticism of our analysis could be some of the assumptions in the methodology. However, we believe that in spite of the potential technical criticisms of the testing procedure, our assumptions and procedures were reasonable and results were black and white, and not in the grey area.

If we look at the information provided by IV and its predictive accuracy, academics consider implied volatility a good forecaster of future volatility, because implied volatility accounts for the information available in the market. One of the reason for this is that academics believe that markets are informationally efficient. On the other hand, option traders use implied volatility of option largely to measure its pricing relative to that of the underlying, and to a large extent are not concerned about the accuracy of IV's predictive powers.

We also reject the hypothesis that option traders are not rational. Although this cannot be ruled out entirely without additional data on market's expectations, but it would be a far stretch to assume that option traders are inferior to other market traders, given ample evidence that financial markets are largely efficient.

Black Scholes Formula is widely used for pricing options around the world. The inputs to the Black Scholes Model are observable. These include strike price, price of the underlying, time to maturity and the risk free rate. One of the important inputs to this model is sigma, which is standard deviation of underlying's log price return.

However, these option-pricing methodologies ignore a number of real world factors and assume completely frictionless markets. Nevertheless, in the real world there are number of other factors that should be considered. These include liquidity of the market, taxes, transaction costs etc. One of the assumptions in frictionless markets is that no arbitrage opportunities exist and if they do appear, they quickly disappear. This is not true in the real world. There might be some arbitrage opportunities between the option and the set of underlying stocks. However, executing arbitrage trade for these options is comparatively difficult. All these factors affect the pricing of the OEX options in the real world, which in turn would affect the implied volatility.

There have been previous studies on implied volatility on stock index futures (Feinstein (1989) and Park and Sears (1985)). These have shown that implied volatilities from stock index futures options does a better job of explaining volatility of futures contracts. It should be noted that arbitrage trades are easier in case of options on futures. Therefore, we can say that the easier it is to execute arbitrage trade, better is informational content of implied volatility.

To conclude, both implied and historical volatility failed to pass the rationality test during subprime crisis, which is logical, as during periods of high volatility historical volatility does not give any prediction of future volatility because circumstances change drastically. Thus, the million-dollar question of how to accurately forecast the future volatility of market remains open.

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Appendix A

Code for Final Project Analysis

Function and Script for Realized Volatility

```
function realVol =
calRealVol(datesOption,expDates,datesPrices,logReturns)
% Function to calculate the realized volatility of a stock index.
%
% Syntax:
% realVol = calRealVol(datesOption,expDates,datesPrices,logReturns)
%
% Inputs: datesOption - Input the dates of all the available options
%         : expDates - Corresponding expiry date for each option
%         : datesPrices - OEX(Underlying Prices)
%         : logReturns - daily log returns%
%
% Output: realVol - This contains actual realized volatility
corresponding
%
%           to each option.
%           Realized volatility is annualized standard deviation of daily
log
%           returns.
%
%
% Author: Rupinder and Deepanshu
% Date : 19/11/2015
% Function to calculate realised volatility

numberOfDates = length(datesOption);           % Total number of
volatility to calculate

realVol = nan(numberOfDates,1);                 % Preallocating memory to
realVol

% This part runs a loop to calculate realised volatility.
for idx = 1:numberOfDates
    startDay = datesOption(idx);
    endDay = expDates(idx);
    endDayIndex = 0;
    [~, startDayIndex] = ismember(startDay,datesPrices);
    while endDayIndex == 0
        endDay = endDay-1;
        [~, endDayIndex] = ismember(endDay,datesPrices);
    end
    realVol(idx,1) = std(logReturns(startDayIndex:endDayIndex)) *
sqrt(260);
end

% This script is for calculating Realized Volatility
```

```

% It uses raw option data and OEX returns to calculate actual realized
% volatility.
% After realized volatility is calculated.
% We filter the options so that there is only one option per day for a
% given expiry.

% Author:
% 1. Rupinder and Deepanshu

% Read OEX Prices. Column 1 - Dates, Column 2 - Prices
% Column 3 - Log Returns
fileName2 = 'OEX.xlsx';
givenOEX = xlsread(fileName2);

% File containing option data.
fileName1 = 'ThesisData.xlsx';

for counter=1:1:32

% Read Data for options. Column 1 - Dates, Column 2 - Expiry Dates,
% Column 3 - Days to expiry, Column 4 - Implied Volatilities

givenOptions = xlsread(fileName1,counter);

% Calculate Realised Volatility
realVol =
calRealVol(givenOptions(:,1),givenOptions(:,2),givenOEX(:,1),givenOEX(:,
4));

% Creating Final Date
% First Eliminate Duplicates from Dates
[uniqueDates, ia, ic]= unique(givenOptions(:,1));

% Extract Corrospounding Expiry Dates;
givenExpDates = givenOptions(:,2);
uniqueExpDates = givenExpDates(ia);

% Extract Corrospounding IVs
givenImpVol = givenOptions(:,3);
uniqueImpVol = givenImpVol(ia);

% Extract Calculated Realized Volatilities
uniqueRealVol = realVol(ia);
daysExpiry = uniqueExpDates - uniqueDates;

% Combining Regression Data
regressionData = [uniqueDates uniqueImpVol uniqueRealVol uniqueExpDates
daysExpiry];

% Write Calculated Data to a worksheet
xlswrite('RegDataLN.xlsx',regressionData,counter);

end

```

Function and Script for Historical Volatility

```
function histVol = calHistVol(datesOption,datesPrices,logReturns)
% Function to calculate the historical realized volatility of a stock
index.
%
% Syntax:
% histVol = calHistVol(datesOption,datesPrices,logReturns)
%
% Inputs: datesOption - Input the dates of all the available options
%         : datesPrices - OEX(Underlying Prices)
%         : logReturns - daily log returns%
%
% Output: histVol - This contains actual realized historical volatility
corresponding
%                 to each option.
%                 Historical Realized volatility is annualized standard deviation
of 60 daily log
%                 returns prior to date of option.
%
%
% Author: Rupinder and Deepanshu
% Date   : 19/11/2015

numberOfDates = length(datesOption);           % Total number of
volatility to calculate
%daysToExpiry = expDates - datesOption;       % Days to Expiry for
each option
histVol = nan(numberOfDates,1);               % Preallocating memory to
realVol

% This part uses a for loop to calculate historical realized
volatility.
for idx = 1:numberOfDates
    startDay = datesOption(idx);
    [~, startDayIndex] = ismember(startDay,datesPrices);
    endDay = startDay - 60;
    [~, endDayIndex] = ismember(endDay,datesPrices);
    while endDayIndex == 0
        endDay = endDay+1;
        [~, endDayIndex] = ismember(endDay,datesPrices);
    end
    histVol(idx,1) = std(logReturns(endDayIndex:startDayIndex)) *
sqrt(260);
end

% This script is for calculating historical Realized Volatility
% It uses raw option data and OEX returns to calculate actual realized
% volatility.

% Author:
% 1. Rupinder and Deepanshu

% Read OEX Prices. Column 1 - Dates, Column 2 - Prices
```

```

% Column 3 - Log Returns
fileName2 = 'OEX.xlsx';
givenOEX = xlsread(fileName2);

fileName1 = 'RegDataLN.xlsx';

for counter=1:1:32

% Read Data for options. Column 1 - Dates, Column 2 - Expiry Dates,
% Column 3 - Days to expiry, Column 4 - Implied Volatilities

givenOptions = xlsread(fileName1,counter);

% Calculate Realised Volatility
histVol = calHistVol(givenOptions(:,1),givenOEX(:,1),givenOEX(:,4));

% Combining Regression Data
regressionData = [givenOptions(:,1) givenOptions(:,2) histVol
givenOptions(:,3) givenOptions(:,4) givenOptions(:,5)];

% Write Calculated Data to a worksheet
xlswrite('RegData3LN.xlsx',regressionData,counter);

end

```

Function for Regression

```

function [regParameters, resids] = LinReg(regData)

% Function to regressing Realized Volatolity against Implied and
Historical Volatiltiy.
%
% Syntax:
% [regParameters, resids] = LinReg(regData)
%
% Inputs: regData - This contains the series that have to be regressed.
%
% Output: realVol - Required Regression Parameters and Resids%
%
% Author: Rupinder and Deepanshu
% Date : 19/11/2015
% Function to calculate realised volatility

% Reg Data has 2 column
% Column 1 : Implied Volatility or Historical .i.e Independent Variable
- X
% Column 2 : Realised Volatility .i.e. Dependent Variable - Y

for counter = 1:1:2

% Read Data
Vol = regData(:,counter);

```

```

%%
%% Step 1: Test for White Noise

% Test for constant mean
% Dividing series into two parts
x1 = Vol(1:round(length(Vol)/2),1);
x2 = Vol(round(length(Vol)/2)+1:end,1);
[hTTest2, pValueTTest2] = ttest2(x1,x2);
fprintf('PValue for TTEST2 = %.4f \n', pValueTTest2);
if pValueTTest2 <=0.05
    fprintf ('HYPOTHESIS OF CONSTANT MEAN IS REJECTED. \n\n')
else
    fprintf ('HYPOTHESIS OF CONSTANT MEAN CANNOT BE REJECTED.
\n\n')
end

%%
%% Step 3: Test for Serial Correlations
[hLBQTest,pValueLBQTest] = lbqtest(Vol);
fprintf('PValue for LBQTest = %10.6f \n', pValueLBQTest);
if pValueLBQTest <=0.05
    fprintf ('HYPOTHESIS OF NO SERIAL CORRELATION IS REJECTED.
\n\n')
else
    fprintf ('HYPOTHESIS OF NO SERIAL CORRELATION CANNOT BE
REJECTED. \n\n')
end

%%
%% Step 3: Fitting an ARMA Model
%Finding order for ARMA model
fprintf('FITTING AN ARMA MODEL \n');
maxx = 15;      % Maximum value of P + Q in ARIMA
target = 0;
idx = 1;
for pq = 0:1:maxx
    for p = 0:1:pq
        for q = 0:1:pq
            if target == 0 && (p+q) == pq
                Mdl = arima(p,0,q);
                EstMdl = estimate(Mdl,Vol,'print',false);
                res = infer(EstMdl,Vol);
                [hRes, pValueRes] = lbqtest(res);
                fprintf('P = %.1f ', p);
                fprintf(' Q = %.1f ', q);
                fprintf(' PValue for LBQ Test of Residuals = %.4f
\n', pValueRes);
                %pause(0.5)
                if pValueRes > 0.05;
                    target = 1;
                    break
                end
            end
        end
    end
end
if target == 1

```



```

        break
    end
    end
    if target == 1
        break
    end
end

%%
%% Step 4: Save Residuals for Regression

if counter == 1
    res_imp = res;
    hypConstMean_imp = hTTest2;
    p_imp = p;
    q_imp = q;
    resids_imp = res;
else
    res_real = res;
    hypConstMean_real = hTTest2;
    p_real = p;
    q_real = q;
    resids_real = res;
end

end

%% Linear Regression

resids = [resids_imp resids_real];

Y1 = res_real; % Dependent Variable
X1 = [ones(length(res_imp),1) res_imp ]; % Independent
[Coefficient,ConfidenceInterval,resReg,rint,Stats] = regress(Y1,X1);
Intercept = Coefficient(1);
InterceptLCI = ConfidenceInterval(1,1);
InterceptUCI = ConfidenceInterval(1,2);

Slope = Coefficient(2);
SlopeLCI = ConfidenceInterval(2,1);
SlopeHCI = ConfidenceInterval(2,2);

Rsquare = Stats(1);

fprintf(' SLOPE OF 2nd REGRESSION = %10.4f \n', Slope);
fprintf(' Intercept OF 2nd REGRESSION = %10.4f \n', Intercept);

regParameters(1,1) = Intercept;
regParameters(2,1) = InterceptLCI;
regParameters(3,1) = InterceptUCI;
regParameters(4,1) = Slope;

```

```
regParameters(5,1) = SlopeLCI;  
regParameters(6,1) = SlopeHCI;  
regParameters(7,1) = Rsquare;  
regParameters(8,1) = hypConstMean_imp;  
regParameters(9,1) = hypConstMean_real;  
regParameters(10,1) = p_imp;  
regParameters(11,1) = q_imp;  
regParameters(12,1) = p_real;  
regParameters(13,1) = q_real;  
regParameters(14,1) = length(Vol);
```