DEPARTMENT
OF GEOGRAPHY
DISCUSSION
PAPER SERIES

DRAINAGE BASIN MORPHOMETRY AND RIVER NETWORK ANALYSIS: A REVIEW AND SYNTHESIS

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River Studies: Part I

Drainage basin morphometry and river network analysis: a review and synthesis

by

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September, 1980

Discussion Paper No. 9

Preface and Acknowledgements

This paper is the first of a four-part review and discussion of the basic principles and theories of river behaviour. Part II is a review of drainage-basin hydrology while Parts III and IV respectively provide discussions of the fluid mechanics of open-channel flow and of sediment transport theory in rivers.

This discussion paper is a review of drainage basin morphometry and river network analysis. It is essentially an assemblage of techniques which have become both an important part of geomorphology and a useful set of morphometric inputs to hydrologic models of river basins.

The paper also provides a discussion of the ideas which collectively have become known as the theory of dynamic equilibrium. It is interesting that many of the concepts and techniques which grew out of this theory have been incorporated into the conventional discipline while the original conceptual framework has become somewhat suspect!

I would like to acknowledge the benefit of criticism from the many S.F.U. students who used earlier drafts of this paper as class notes. I would also like to express my gratitude to my colleagues and friends, Gerald Nanson and Ken Page, for our helpful discussions about the ways of rivers.

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1.1: The drainage basin - some definitions

A drainage basin is the entire land surface which contributes streamflow to its river system. It is therefore a three dimensional surface rising from the river channel to the ridge tops of the basin perimeter. The boundary of a drainage basin, termed the drainage-basin divide or watershed, usually coincides with the topographic divide forming the rim of the basin.

If we were to project the three-dimensional watershed at right angles to a horizontal plane (as it would be shown on a map or aerial photograph) it would enclose the drainage-basin area; this area is also known simply as the drainage area or catchment.

Although the mouth of a drainage basin is usually taken as the sea, a lake, or a stream confluence, drainage areas are commonly defined for any point on a river channel. Drainage basins may be as small as the few square feet maintaining a rill on a spoil heap or as large as the 2.7 million square miles maintaining the Amazon River in South America. Most drainage basins form part of a nested system in which large basins consist of several sub-basins which in turn consist of still smaller component basins, and so on. The smallest drainage basin in any such nested system, that associated with a single unbranched tributary, is commonly termed a first-order drainage basin.

Although we have noted that the drainage-basin divide generally corresponds with the topographic divide between adjacent rivers, this correspondence may not always be perfect in specific cases. For example, in Figure 2.1A there is shown an example of this type of discrepancy. Here dipping geological strata give rise to a topographic divide (TD) and a groundwater or phreatic divide (PD) which are not coincident; the true drainage-basin area includes some of what, on the basis of topography, we would consider to be part of the adjacent basin. The same effect is produced wherever the phreatic surface is asymmetric with respect to the topographic surface (Figure 1.1B).

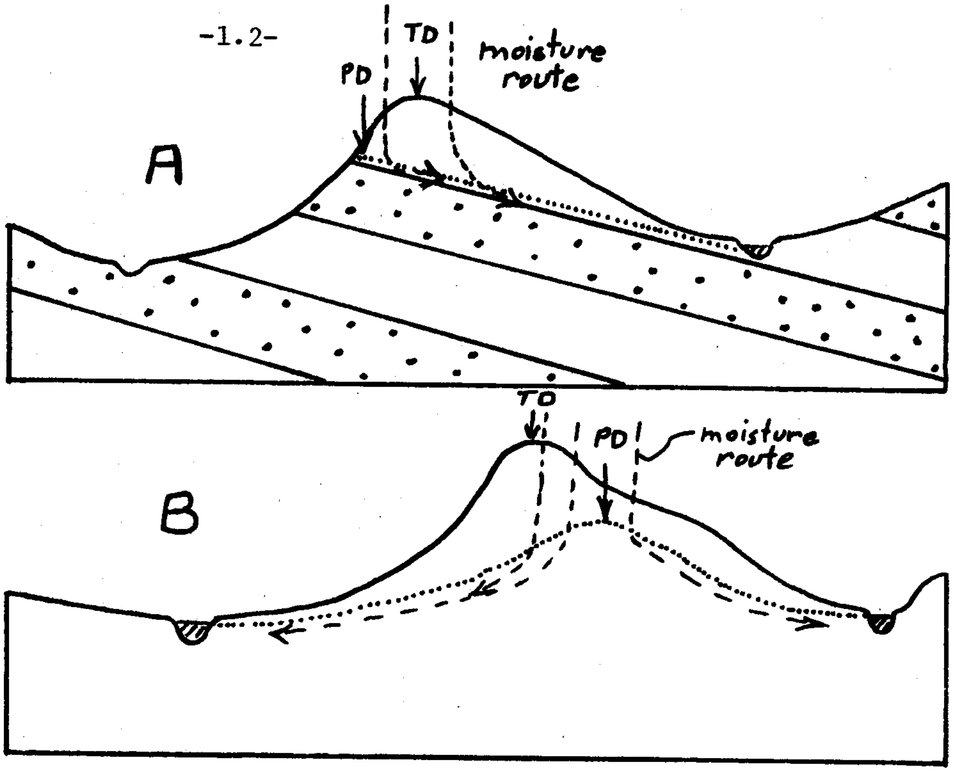


Figure 1.1

Nevertheless, it is more usually the case that the topographic and phreatic divides are sensibly coincident so that the watershed can be defined as the locus of highest points between adjacent drainage basins. Thus the watershed can be obtained from any good quality contour map, as in Figure 1.2. It is obvious that the accuracy of the watershed location depends on map scale; the larger the map scale and smaller the contour interval, the greater will be the accuracy of the watershed location.

Once the watershed has been located it is a simple mapping exercise using a planimeter or squared graph paper to determine the area of the enclosed drainage basin (see Monkhouse and Wilkinson, 1963 for a discussion of area measurement techniques).

We shall soon see that drainage-basin area is probably the most important single parameter describing the character of a drainage basin; it is one of the primary determinants of streamflow magnitude and the consequent size of rivers.

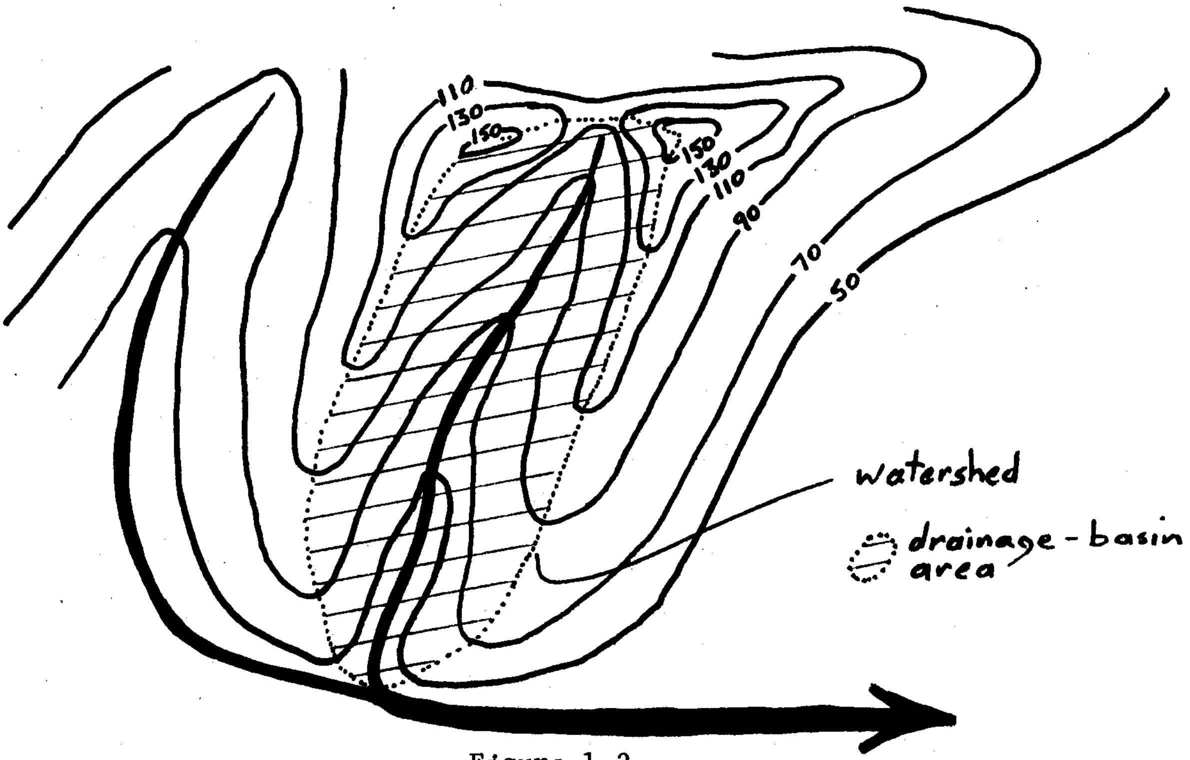


Figure 1.2

1.2: Drainage-basin shape

The shape of drainage basins is probably more important to the fluvial geomorphologist than it is to the geomorphologist concerned with the general form of the land surface. We will see in the next Chapter that drainage-basin shape partly determines the flow regime and thus the channel morphology of rivers.

Unfortunately, meaningful definition of the three-dimensional form of a drainage basin in terms of a simple index is a very difficult if not impossible task. Nevertheless, a number of attempts have been made and these are of three types:

(a) those concerned with basin planform

(b) those concerned with the vertical dimension of basin shape (relief aspects), and

(c) those designed to summarise the three-dimensional basin form (hypsometry)

The more widely accepted of these shape indices are summarised in Table 1.1.

Horton (1941), in a discussion of sheet erosion processes in drainage basins, noted that the outline of a "normal" basin of any size is a pear-shaped ovoid. Hack (1957) quantitatively confirmed the existence of a "normal" basin shape in his now classic study of the rivers in the Shenandoah Valley and adjacent mountains in Virginia. He was able to show that, for this area, successive measurements of distance (L_d) from the headwaters along a watercourse, and the total drainage area (A_d) at each of these points, when graphed one against the other (Figure 2.3), yield a power function of the form:

$$L_d = 1.4\ A_d^{0.6} \qquad (1.1)$$

Similar data analysed by Hack from the northeast of the United States (from Langbein, 1947), together with those from various other large rivers of the world (see Leopold, Wolman and Miller, 1964), are also adequately described by equation 1.1.

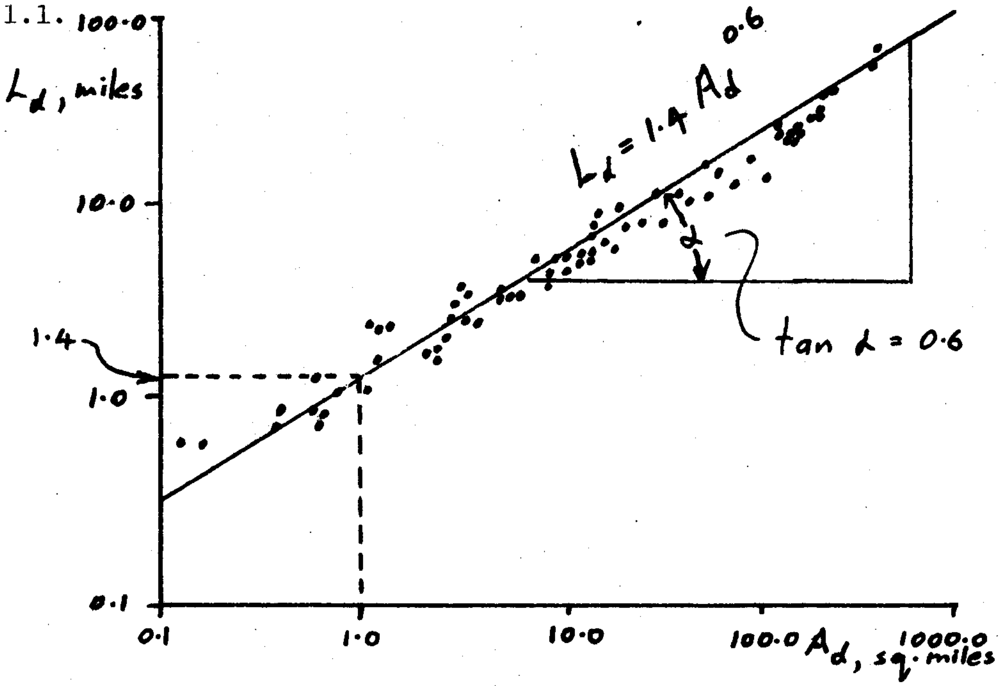


Figure 1.3 (after Hack, 1957, p. 64)

TABLE 1.1

SOME MEASURES OF DRAINAGE-BASIN SHAPE

INDEX	FORMULA	SOURCE
	PLANFORM INDICES	
Form Factor, R_f	$R_f = \frac{A_d}{L_d^2}$	Horton (1932)
Circularity Ratio, R_c	$R_c = \frac{A_d}{A_c}$	Miller (1953)
Elongation Ratio, R_e	$R_e = \frac{D_c}{L_m}$	Schumm (1956)
Lemniscate Coefficient, K	$K = \frac{L_m^2 \pi}{4A_d}$	Chorley et al (1957
	RELIEF INDICES	
Maximum Basin Relief, H_m	$H_m = Z_{max} - Z_{min}$	Strahler (1952)
Maximum Basin Relief, H_p	(see text)	Schumm (1956)
Maximum Basin Relief, H_d	(see text)	Maxwell (1960)
Relief Ratio, R_h	$R_h = \frac{H_p}{L}$	Schumm (1956)
Relative Relief, R_{hp}	$R_{hp} = \frac{H_m}{P}$	Melton (1957)
Relative Relief, R_{hd}	$R_{hd} = \frac{H_m}{D}$	Maxwell (1960)
Relative Relief, R_{ha}	$R_{ha} = \frac{H_m}{\sqrt{A_d}}$	Melton (1965)
	COMPLEX THREE-DIMENSIONAL INDICES	
Ruggedness Number, N_R	$N_R = H_m D_d$	Strahler (1958)
Geometry Number, N_G	$N_G = \frac{H_m D_d}{S_g}$	Strahler (1958)
Hypsometric Integral, HI	$HI = \int_0^1 a(h')\, dh'$	Strahler (1952)

All symbols are defined in the text

The coefficient of this equation (indicating that, on average, one square mile of drainage area supports 1.4 miles of channel) ranges between 1.0 and 2.5 for the northeastern United States; the exponent generally varies between 0.6 and 0.7 for a wide range of regions in the United States.

Equation 1.1 also indicates that, as the size of a drainage basin increases, the basin shape tends to elongate. If the shape remained the same as size increased (geometric similarity) the average basin width and length would have to increase in the same proportion and length would thus vary with the square root of the basin area. The fact that the exponent is 0.6 and not 0.5 indicates that length increases somewhat faster than does area (and width).

Although the length/area relationship of equation 1.1 confirms the general uniformity of drainage-basin shape, it is rather insensitive to certain types of variation in watershed planform.

Horton (1932) proposed a basin-shape index termed a form factor, R_f, which is the ratio of drainage-basin area (A_d) to the square of the basin length (L_d). Clearly, as R_f increases from low values (about 0.40) to high values (about 0.80), the basin shape changes from being markedly elongated to being markedly squat and rotund. This index has been applied (in the form $1/R_f$) to hydrologic problems by the U.S. Army Corps of Engineers (1949).

Another basin-shape index, proposed by Miller (1953) is termed the circularity ratio, R_c, which is the ratio of drainage area (A_d) to the area of a circle (A_c) with a circumference equal to the basin perimeter. Low values of R_c (about 0.40) indicate strong elongation, and the upper mathematical limit is unity; Miller found that basins not influenced by structural controls usually displayed values of R_c between 0.6 and 0.7.

A second index of basin shape based on a circle reference is that proposed by Schumm (1956) in his study of badland topography in New Jersey. He defined an elongation ratio, R_e, as the ratio of the diameter of a circle (D_c) with the same area as the basin, to the maximum basin length (L_m). R_e varies between 0.60 and the upper mathematical limit of unity for basins in a variety of climate and geology; high values are associated with areas of low relief and low values are common in steep, high-relief topography.

Chorley, Malm and Pogorzelski (1957) argued that it was inconsistent with the tear or pear-shape of the average drainage basin to use a shape index based on a circle as a reference form. They instead chose the pear-shaped lemniscate curve (see Figure 1.4) as a reference form and from its

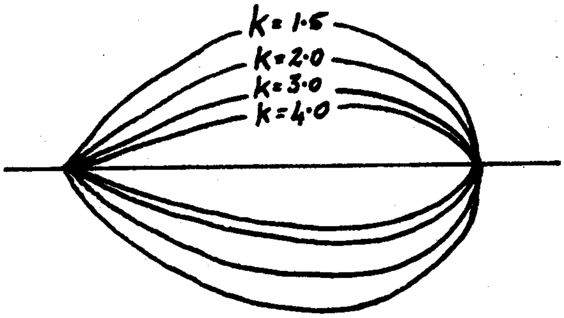


Figure 1.4

geometric properties derived a lemniscate coefficient, K, equal to the ratio of the product of pi and the maximum basin length (L_m) squared, to four times the basin drainage area (A_d). The rotundity of the lemniscate loop or petal depends on the value of K which is simply determined by substitution of the measured values of basin length and area in the lemniscate formula shown in Table 1.1. From Figure 1.4 we can see that a drainage basin with $K > 3.0$ is markedly elongated. Similarly, a basin with $K < 1.5$ is rather rotund; when $k = 1.0$ the basin outline is a circle.

The degree of correspondence between the actual drainage-basin shape and that of its ideal lemniscate counterpart can be obtained by comparing the measured perimeter of the drainage basin to that of the ideal lemniscate having the calculated value of K.

The planform indices in Table 1.1 are general expressions of basin outline. There are, of course, many other planform parameters of a more specific or special-purpose nature; a review of some of these parameters, and a discussion of problems relating to the measurement of basin length, may be found in Gregory and Walling (1973) and Chorley (1969).

Table 1.1 also lists a number of basin relief measures. The most basic of these is that proposed by Strahler (1952) as the elevation difference between basin mouth (E min) and the highest point (E max) on the basin perimeter (maximum basin-relief, H_m). Similar measures have been proposed by Schumm (1956) and Maxwell (1960), the first measuring maximum relief (H_p) along the longest dimension of the basin parallel to the principal drainage line (L), and the second measuring maximum relief (H_d) along a basin diameter (D_b).

Since the area of drainage basins varies, a number of geomorphologists have determined dimensionless indices of relief by dividing a measure of relief by some other linear dimension of the basin. The latter include basin length, L, as defined above (Schumm, 1956), basin perimeter, P, (Melton, 1957), basin diameter, D_b (Maxwell, 1960) and the square root of basin area, A_d (Melton, 1965).

A number of shape indices combine the horizontal and vertical dimensions in order to express the three-dimensional character of drainage basin form. An important measure of this type is slope.

A widely adopted method for determining the average slope of an area was proposed by Wentworth (1930). The number (N) of intersections between a set of traverse lines and the contours in a basin is counted and the total length of the traverse lines (L) is measured. The mean slope ($\tan \alpha$) is estimated by

$$\tan \alpha = I(N.L)/0.6366 \quad (1.2)$$

where I is the contour interval in the same units as L.

Point measurements of slope within drainage basins appear to be normally distributed in many areas (Strahler; 1950, 1956) although Speight (1971) reports that in some cases a long-normal distribution seems to be more appropriate.

An examination of the many sampling problems and techniques of slope measurement is beyond the scope of our present discussion. Anyone interested in pursuing the analysis of slope form should consult the reviews by Carson and Kirkby (1972) and Mark (1975).

Closely related to mean slope is the ruggedness number (N_R) proposed by Strahler (1958) as the product of maximum relief and drainage density (D_d). Drainage density is the ratio of total channel length to drainage area and is a measure of channel spacing in a basin (see Section 1.4). It can be easily shown (Horton, 1945) that,

$$\tan \alpha = 2H_m D_d \quad (1.3)$$

Thus if D_d increases while H remains constant, the average horizontal distance from divides to adjacent channels is reduced and slopes consequently steepen. Similarly, if H increases while D remains unchanged, the elevation difference between divides and adjacent channels will also increase so that slopes consequently steepen. Values of N_R range from as low as 0.06 in areas of subdued relief (coastal plains) to over 1.0 in areas of long steep slopes (mountain ranges and badlands).

Strahler (1958) also introduced average slope into the ruggedness number, producing the geometry number, N_G (see Table 1.1). From equation 1.3 it can be seen that the geometry number should equal 0.5. In fact, Strahler found that although the drainage density of his test basins ranged over two orders of magnitude, N_G remained between 0.4 and 1.0. It seems that, because of its conservative nature, the geometry number is of little value as an indicator of shape change.

The last index of basin form listed in Table 1.1, the hypsometric integral, is one of a number which can be derived from an analysis of the basin area/elevation relationship: the hypsometric curve (see Figure 1.5).

The hypsometric curve is a plot of relative basin-area (a') above a given

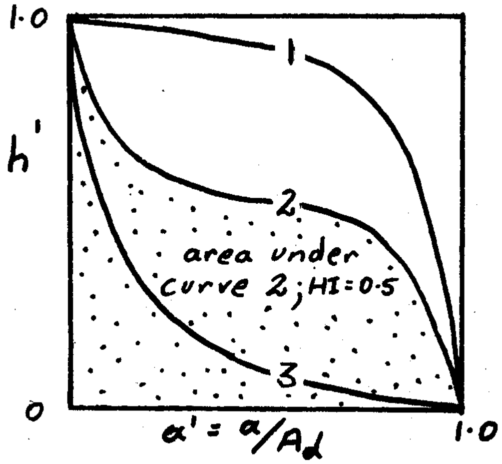


Figure 1.5

height (Z) against relative height (h') and is the graph of the hypsometric function a(h') where

$$h' = \frac{Z - Z\,min}{H_m} \qquad (1.4)$$

In other words, if we consider a drainage basin to be bounded by vertical sides with a horizontal base plane passing through the mouth, the relative height (h') is the ratio of height (Z) above the base plane of a given contour to total basin height (relief, H_m). Relative area (a') is the ratio of horizontal cross-sectional area (a) to the total basin area (A_d). The continuous function relating h' to a' is the hypsometric curve and the area beneath it is the hypsometric integral (HI).

Curves 1 (HI = 0.75), 2 (HI = 0.50) and 3 (HI = 0.25) in Figure 1.5 would respectively correspond to a slightly dissected upland, a severely eroded surface of many valleys and peaks, and a low-relief plain with a few upstanding plateau remnants.

Although the hypsometric integral is essentially a tool of physiographic classification, it is potentially useful in estimating space-averages of elevation-dependent climatic variables (such as snow-cover depth) in drainage basins.

A number of other hypsometric parameters are reviewed and evaluated by Mark (1975).

Many of the techniques outlined in this Section involve the rather tedious measurement of drainage basin properties from maps and aerial photographs. However, the rapid development of computer cartographic techniques during the last decade (for examples, see Peucker, 1972, Mark, 1977) has allowed the possibility of much more extensive use of complex basin-shape indices in hydrologic modelling.

1.3 Types of river-channel networks

For purposes of description and comparison, river channels are often classified according to the shape of the channel network - the drainage pattern. Although there are many ways of categorising drainage patterns, the most widely adopted classification, developed by Zernitz (1932), has seven major components (see Figure 1.6): dendritic, trellis, rectangular, radial, annular, parallel, and irregular.

Dendritic drainage is the term, first used by Russell (1898) and subsequently defined by Cleland (1916), to describe the branching or tree-like pattern of valleys that commonly develop on rocks of uniform resistance, such as horizontal sediments and massive igneous and metamorphic complexes (see Figure 1.6A). It is the drainage pattern which is typical of areas

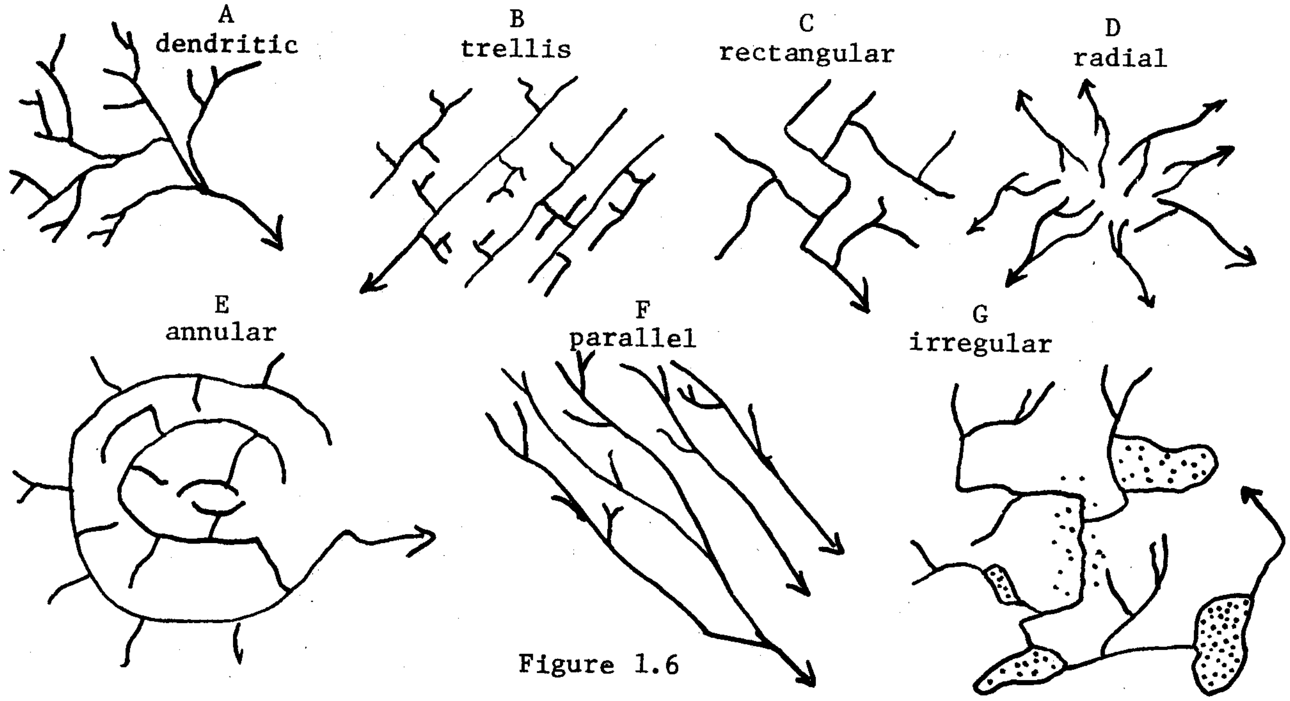


Figure 1.6

which are largely free of structural controls on river erosion (for example, the concentrated channel networks in the badlands of the U.S. southwest and of Alberta; or on a larger scale, the Prairie rivers of southern Canada and northern U.S.A.

There are two important exceptions to this general association of dendritic drainage pattern and the absence of geological structures. The first, termed antecedence, refers to cases where the dendritic pattern existed prior to the formation of the structures. Perhaps the best known example of antecedence is the Brahmaputra River which drains from the Tibetan plateau southward through the Himalayas rather than flowing northward down a gentle gradient to the sea. This lack of adjustment to structure is taken to indicate that the river predates the uplift of the mountains. It simply maintained the original pre-uplift drainage pattern by vigorous downcutting during the orogeny. Many other examples can be found in the late Tertiary mountains around the Pacific Ocean (for example, the Fraser River in southwestern Canada, the Columbia River in northwestern U.S.A. and the Shoalhaven River in eastern Australia).

The second type of discordance between geological structure and river pattern is produced by superimposition (or superposition) of the river on subsurface structures. In this case the dendritic pattern develops on a surface free of structural control and later incision lowers the river network into underlying structures. The subsequent removal of the original surface material by erosion leaves the unaltered dendritic pattern superimposed or overprinted in a discordant manner on the previously buried structures. Examples of superposed drainage are common and include the many gorges of the Rocky Mountains in North America (Atwood and Atwood, 1938).

Trellis drainage was named by Willis (1895) to describe the stream pattern common in the ridge and valley province of the Appalachian Mountains in eastern U.S.A. Here the folded and tilted strata form parallel ridges and valleys which give rise to a right-angular and elongated drainage pattern resembling a garden trellis (see Figure 1.6B).

Trellis drainage is often associated with other types of parallel and linear structures (for example, glacial features such as moraines, windblown sand dunes, and with parallel belts of rock of varying resistance).

Rectangular drainage, characterised by right-angle bends in both tributaries and the master channel, is generally attributed to the influence of rectangular fault or joint patterns (see Figure 1.6c). It is distinguished from the trellis pattern by the equal development of the drainage in both directions (i.e. there is no pronounced elongation of the network). Rectangular drainage is common in areas of strongly jointed rocks such as the Adirondack Mountains in New York in the fiords of Norway, Scotland, and Baffin Island and in the Canadian shield.

Radial drainage commonly develops on domed structures such as volcanoes, exhumed plutons, and circular inselbergs. Channels radiate outward from a common centre at the high point of the structure producing a pattern resembling the spokes of a bicycle wheel (see Figure 1.6D). The term centripetal drainage is given to the radial inward type of internal drainage that commonly develops when channels flow to a common centre from the circular walls of craters, calderas and structural basins.

Annular drainage develops on maturely dissected domes and basins (see Figure 1.6E). The initial radial pattern eventually disappears as stream captures accompanies breaching of the dome. A concentric ring-like arrangement of streams develops on the least resistant formations of the dome strata. Many examples of this type of drainage can be found in the Henry Mountains of Utah (described in a classic monograph by G.K. Gilbert, 1877), and in the Colorado Plateau and Rocky Mountain region of Wyoming, South Dakota and Colorado.

Parallel drainage, as the name implies, consists of parallel master and tributary streams (see Figure 1.6F). This type of drainage pattern can result from pronounced regional slope or structural controls such as alternating exposures of hard and soft rocks, faults, folding, or from other linear landforms such as drumlins and lateral moraines. Examples of structurally induced parallel drainage can be found in Mesa Verde National Park, Colorado, and parallel drainage of both structural and glacial origins, is common in the Canadian Shield.

Irregular drainage is common in recently glaciated areas and is characterised by disrupted and uncoordinated drainage into many local drainage basins and lakes (see Figure 1.6G). Abandoned valleys may occur where the river has been dammed by a moraine; swamps, lakes, pitted outwash, moraines, drumlins and askers all contribute to the pattern of irregular or deranged drainage. This drainage type is best developed in the shield areas of Canada and Siberia but many other examples have been described in glaciated areas elsewhere (for examples, see Dury, 1971).

The Zernitz classification of drainage patterns, and others like it (for example, see Howard, 1967) although useful in qualitative descriptions of river networks, provides a poor basis for objective comparison of river planform. Clearly, many drainage patterns will be complex composites of the types discussed above, and thus may not well fit the classification.

An alternative approach to the qualitative classification of drainage patterns is the description of specific geometric and topologic properties of the drainage network. Measurements of this type have the advantage of allowing precise and objective comparison of drainage network properties; their principal disadvantage is that they provide no general impression of the drainage planform as does a qualitative classification. Some of these types of drainage pattern parameters are discussed below.

1.4 Drainage density

I touched on this very important parameter in the discussion of Strahler's ruggedness number; we now need to consider it in some detail. Drainage density was defined by Horton (1932) as the total length of stream channels per unit area. It is a very important expression of channel spacing which has been found to be closely related to mean stream discharge (for example see Carlston, 1963), to mean annual precipitation (see Chorley and Morgan, 1962) and to sediment yield (Abrahams, 1972). Because of its simplicity and utility drainage density has been widely adopted in geomorphic and hydrologic studies. A related measure is the constant of channel maintenance, defined by Schumm (1956) as the inverse of the drainage density. This constant is a measure of the drainage basin area necessary to maintain a unit length of channel. Many other measures similar to drainage density have been proposed but have not gained wide acceptance (for example, see Horton, 1945; Penck, 1924; Smith, 1950; Faniran, 1969).

High values of drainage density are common in areas of impervious materials which force most of the precipitation into channelled flow. Similarly, any factor which promotes water storage rather than runoff (vegetation, highly permeable soils) will tend to be associated with low-density drainage. Horton considered drainage density to range between 1.5 to 2.0 miles/square mile in steep impervious basins with high precipitation. He recognised that values of D_d would decline to nearly zero in highly permeable basins. Langbein (1947) suggested that D_d would range from 0.89 to 3.37 in humid regions; average stream density for such an area would be 1.65.

The last few decades of research have revealed somewhat more variable drainage density than was supposed by Horton and Langbein. For example, Schumm (1956) reported drainage densities in the weak claysof Perth Amboy New Jersey to be several orders of magnitude higher than the above values.

However, values of drainage density must be visualised against the methods of analysis. To measure D_d it is necessary to delineate the network and to measure the total length of channel and the drainage area represented on maps, photographs or field surveys. Horton (1945) constructed a drainage net using the watercourses shown as blue lines on topographic maps. Most subsequent researchers have adopted Horton's recommendation of extending the watercourses back to the watershed (for example, see Morisawa, 1957; Gregory, 1968;

Mark, 1975). The principal deficiency of the "blue-line method" is that representation of rivers by these lines is done rather subjectively by mapping agencies. Consequently, some geomorphologists supplement the blue-line network with the finer network implied by contour crenulations (Morisawa, 1957; Bowden and Wallis, 1964; Carlston, 1963; Orsborn, 1970). However, this procedure is obviously also rather subjective. Methods for the rapid estimation of drainage density have been proposed by Carlston and Langbein (1960) and McCoy (1971). The line intersection method for estimating D_d was used in both of these studies. It involves counting the number (N) of intersections with the drainage net per unit length (L) of sample traverse line and multiplying the result by a correction coefficient (K):

$$D_d = K\frac{N}{L} \tag{1.5}$$

Various values have been suggested for the coefficient in equation 1.5 but the most theoretically sound value, derived from the work of Wentworth (1930), is K = 1.571 proposed by Mark (1974).

We will find it necessary on many occasions to return to this concept of drainage density; in many ways it is the link between the shape of a fluvially eroded surface and the processes operating in the river channels.

1.5 The topology and geometry of channel networks

Topology is the study of the way in which sequences of points or lines are structured in space; it is concerned with relative positions or sequences and not with the metric of a system of points and lines. In the context of fluvial geomorphology, stream or channel order is a topological concept which has received a great deal of attention since the seminal paper on the subject by Horton (1945). Although the limitations of the Horton ordering scheme have become more obvious with the passage of time, the introduction of this type of quantitative approach to geomorphology marked the change of drainage basin analysis from qualitative and often subjective studies to rigorous quantitative science.

The concept of stream order was essentially introduced to North America from the European literature by Gravelius in 1914. However, his scheme of ordering was little more than a numbered nominal classification of stream segments (see Figure 1.7A).

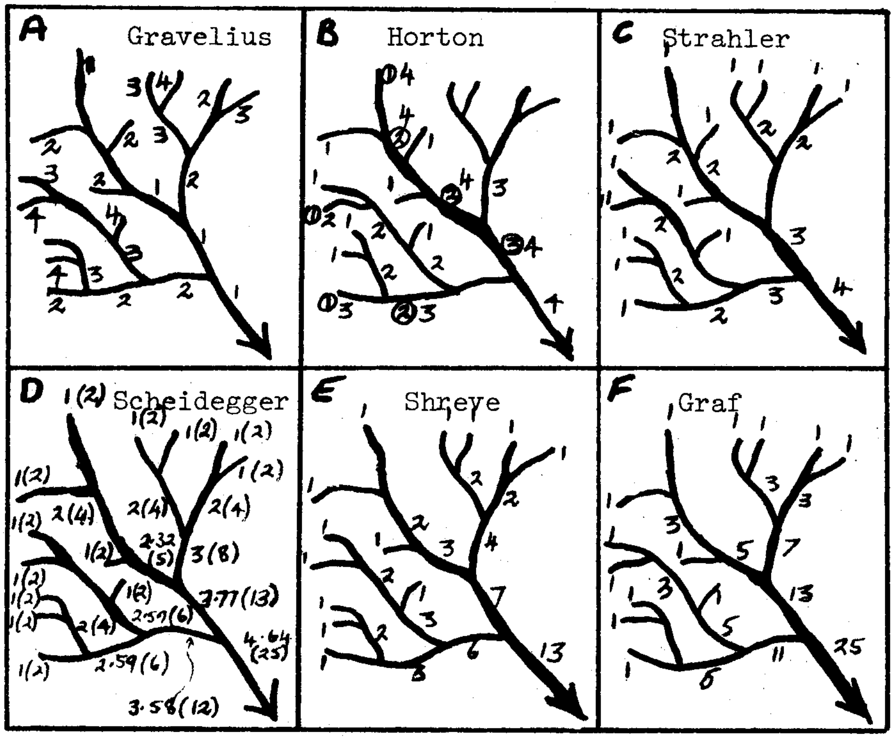


Figure 1.7

It assigned order 1 to the main channel from outlet to source; the main channel was distinguished from secondary channels by its greater width and discharge, or by its more consistent alignment with the average mainstream direction. Immediate tributaries were similarly assigned order 2 from junction to source, and so on for higher orders. Although Gravelius' scheme of ordering yields a quantitative description of the channel network, it does not provide a meaningful basis for comparison of networks. For example, there is no necessary correspondence between streams of a given order and the discharges of water and sediment flowing through them; the Amazon and the smallest coastal rivulet are, both order 1 channels where they meet the sea! Furthermore, the ordering system does not respond to stream combination because the orders do not possess the properties of numbers.

Horton's 1945 system of stream ordering, illustrated in Figure 1.6B, involves the provisional designation of all source tributaries as order 1 streams. The combination of two channels of identical order, U, yields a channel segment of order U + 1; however addition of tributaries of lower order to a channel segment does not alter the order of that segment. Once all the streams are assigned provisional orders, all major trunks are reclassified to extend their assigned order back to their sources (provisional orders subject to reclassification are circled in Figure 1.6B).

Although the Horton ordering scheme represents an improvement over that of Gravelius, it unfortunately retains many of the latter's inadequacies. Networks of a given order can vary considerably in the scale of channel geometry and discharge, and the ordering system ignores the addition of tributaries to a higher order channel; true ordinal scaling is not satisfied by this system although it does satisfy the commutative property for a given basin.

Strahler (1952) proposed a slight modification of the Horton ordering scheme and it has been widely adopted. He argued that the provisional orders of Horton should be accepted as final because they conform more closely to the intuitive concept of an ordered system; the Strahler scheme is illustrated in Figure 1.7C.

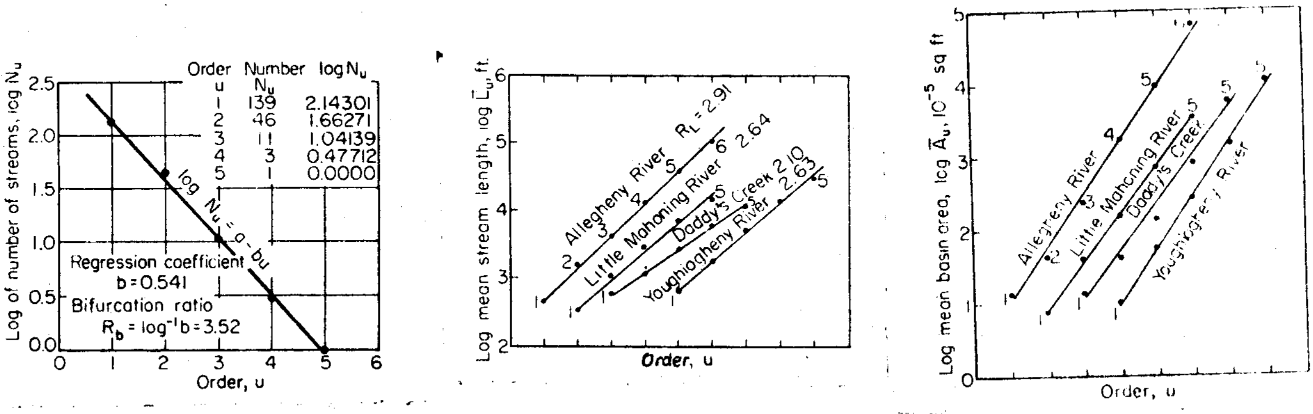
Nevertheless, the order sequence in this scheme is also not a true ordinal scale and it suffers from the same defficiencies as the Horton order numbers. In addition, the order of a network in Strahler's scheme is very sensitive to small changes in the arrangments of first-order channel junctions. Also, because of the importance of the first-order tributaries to the scheme, it becomes essential to standardise the base network for analysis; the computed stream order is dependent on the scale and compilation conventions adopted for the network map or photograph (for example, see Leopold and Wolman, 1956).

Horton developed a system of fluvial morphometry which was supplemented by Langbein(1947) and subsequently extended in detail during the 50's and 60's by Strahler and his Columbia University associates (see Strahler, 1964 and references therein). In its most widely adopted form, this system is largely based on the Strahler concept of stream order and finds expression in Horton's laws of drainage-network composition. The more important of these laws are briefly discussed below.

Horton's law of stream numbers states that the numbers of stream segments of each order form an inverse geometric sequence with order number, or

$$\frac{N_u}{N_{u+1}} = \text{Constant} \qquad (1.6)$$

for any given drainage net. In other words, the ratio of the number of first to second order streams will equal the ratio of second to third, of fourth to fifth order, and so on. Because of this property, the logarithm of the number of streams will plot against order to yield a straight-line relationship (see Figure 1.8A). The constant in equation 1.5 is termed the bifurcation



A B C

Figure 1.8 (from Strahler, 1964, pp. 4-44, 4-46, 4-49

ratio, R_b. The average value of R_b for a drainage net can be determined from the regression line through log N_u versus u. The regression coefficient b (the slope of the regression line) is equal to log R_b. Thus, in the example in Figure 1.8A, b is equal to 0.541 and the antilog yields the bifurcation ratio of 3.52. In other words there are 3½ times as many channel segments of any given order as of the next higher order.

The bifurcation ratio is a very stable parameter and rarely falls beyond the range 3.0 to 5.0 for drainage nets undistorted by geological structure. Structures which give rise to long elongated valleys favouring the development of first order streams and restricting the union of the major trunks are associated with very high values of R_b. The rather flared net shown in Figure 1.7 has a bifurcation ratio of 2.4.

The second of Horton's laws is concerned with the mean lengths of channel segments of given order. As we might expect, the mean length of channel segments of a given order increases as the order number increases. Horton's law of stream numbers states that

$$\bar{L}_u = \bar{L}_1 R_L^{u-1} \tag{1.7}$$

where R_L is a length ratio (analogous to the bifurcation ratio) equal to the ratio of mean length $\bar{L}_u$ of segments of order u to mean length of segments of the next lower order $\bar{L}_{u-1}$. In other words the ratio of mean stream-segment lengths of first order channels to that of second order channels will equal the same ratio of second to third, and of fourth to fifth order, and so on. If the length ratio is constant then the logarithm of mean-segment-length will plot against order to yield a linear relationship as in Figure 1.8B. This law, like that for stream number, has been confirmed in many widely varying drainage basins (for example, see Schumm, 1956; Leopold and Miller, 1956; Morisawa, 1959; Eyles, 1968; and Abrahams, 1971).

As with the bifurcation ratio in the law of stream numbers, the mean length ratio R_L can be obtained as the antilogarithm of the regression coefficient b in the equation relating segment length to stream order (see Figure 1.8B). Values of R_L appear to average about 2.5 for many areas. That is, the length of second order segments in 2½ times that of the first order and about 2/5ths that of the third order.

Horton (1945) reasoned that mean drainage basin areas of progressively greater orders should also increase as a geometric series. Schumm (1956) confirmed this inference and formally expressed it as the law of stream areas. This law states that the mean basin-areas of each order stream tend closely to form a direct geometric sequence in which the first term is the area of the first order basin. This law can be expressed as

$$\bar{A}_u = \bar{A}_1 R_a^{u-1} \tag{1.8}$$

where $\bar{A}_u$ is the mean area of order u basins, $\bar{A}_1$ is that of first-order basins, and R_a is an area ratio analogous to the length ratio, R_L. As before, regression of the logarithm of basin area on order is linear (see Figure 1.8C) and the antilogarithm of the regression coefficient is the mean area ratio for the net. And again, the law has been found to be applicable to drainage networks in a wide range of environments (for example, see Morisawa, 1959; Leopold and Miller, 1956; Eyles 1968; Abrahams, 1971).

Much has been written on the regularity of drainage networks and the geometric similarity of basin shape that seems to persist over all scales of drainage area. This similarity was at first taken as an indication that drainage basins quickly achieve a state of dynamic equilibrium, an expression of which is the invariant geometry of the drainage net. This view of drainage basin character is summarised by Strahler (1964, p. 4-41) as follows:

> "Validity of the Horton system of fluvial morphometry depends on the theory that, for a given intensity of erosion process, acting upon a mass of given physical properties, the conditions of surface relief, slope, and channel configuration reach a time-independent steady state in which morphology is adjusted to transmit through the system just the quantity of debris and excess water characteristically produced under the controlling regimen of climate. Should controlling factors of climate or geologic material be changed, the steady state will be upset. Through a relatively rapid series of adjustments, serving to reestablish a steady state, appropriate new values of basin geometry are developed (Strahler, 1958). In brief, steady state manifests itself by invariant geometry;

transient state, by rapid changes in geometry in which new sets of forms replace the old."

The Horton laws of drainage composition are usually regarded as forming a growth model describing the steady-state relationship between the size (or age - see Abrahams, 1972 for a discussion of space/time substitution) and structure of drainage nets.

However, the geomorphic significance of the Horton laws of drainage-network composition, have in recent years been challenged by a number of geomorphologists and mathematicians with a quite different view of the underlying cause of drainage network regularity. The central argument of this alternative school of thought is that a drainage net is simply a randomly branched system representing the most likely arrangement of channel junctions (Leopold and Langbein, 1962; Scheidegger and Langbein, 1966; Shreve, 1966, 1967, 1969; Smart, 1969, 1972). The same approach has also been used to interpret the patterns of ridge lines between drainage basins (see Werner, 1972a, b, and c; Mark, 1977). Shreve (1975) and Smart and Werner (1976) have recently reviewed the many quantitative properties of drainage systems which are amenable to this probabilistic-topologic approach.

The pioneering work in this field was conducted by Leopold and Langbein in 1962. They generated a synthetic drainage network which displayed the same topologic and geometric properties as those for natural channels. The network was generated by selecting random numbers to control the steps to extend the drainage network on a square grid. Each series of steps commenced at a specified source and a step "downstream" or to the left or right occurred with equal probability. If any two series of steps met they would continue as one "drainage line" (see Figure 1.9).

The fact that the Horton laws applied to the Leopold and Langbein synthetic network convinced them that natural channel networks exist in a most probable state and that the laws are of a statistical rather than structural nature. Many other subsequent studies of random-walk networks support this viewpoint. (For examples, see Schenck, 1963; Smart, Surkan and Considine, 1967).

More recent studies of drainage network structure have placed the study of topological randomness soundly in a graph theoretic framework. Topology is of course concerned neither with the drainage pattern nor with measurements of length and area but rather with the way in which the channels are connected. The topological properties of the network are completely specified when it is known how the junctions are connected to each other and to the sources. The relations among stream numbers, bifurcation ratios, and network topology were described in an important paper by Shreve (1966). He suggested that, in the absence of geological controls, a natural population of channel networks is topologically random. That is, all topologically distinct channel networks (TDCN) with a given number of sources (first-order streams) are equally likely to occur. Although there are a very large number of TDCN for networks of more than a few sources, stream ordering combines different TDCN into a much smaller number of categories. Shreve's 1966 study showed that the most probable sets of stream orders have bifurcation ratios in the range 3.0 to 5.0, thus "explaining" Horton's law of stream numbers.

There is considerable support for the view that the findings of the topologists have made Horton's laws irrelevant to geomorphology (see Milton, 1966, 1967). My own view is that the drainage-composition laws remain a useful standard against which to assess the influence of controls (such as geological guidance) on drainage net structure. However, it is also clear that Horton's laws can no longer be taken as confirmation of steady-state drainage development.

The focus on the statistical properties of stream order has led to several suggestions for improving the definition of order number. Scheidegger (1965) introduced the "consistent" law of stream ordering to correct the deficiencies in the combination algebra of existing methods. His law designates the order produced by the combination of two streams of orders n and m (denoted n*m) as

$$n*m = \frac{\log\ (2^n + 2^m)}{\log\ 2} \tag{1.9}$$

Selection of base 2 is dictated by the number of streams in each combination and the use of logarithms reflects the exponential growth of the network. The orders are not integers (see Figure 1.7D) but may be represented by a set of integers obtained by raising the base 2 to a power equal to the order number (the bracketed numbers in Figure 1.7D). The Scheidegger orders, unlike those of Horton and Strahler, form a true ordinal scale and reflect all tributary connections. However, although the scheme is algebraically superior to those of Horton and Strahler, the Scheidegger orders and others like them (see Woldenberg, 1969) have not been widely adopted over the simpler schemes.

Shreve (1967) perceived a drainage network as a sequence of exterior links and nodes; he considered the exterior links to be the fundamental units for analysis. In his ordering scheme, the magnitude of each link in the network was determined by the number n of its contributing exterior links (each assigned order 1). It follows that the number of network links is 2n-1 and the number of forks is n-1. The scheme of ordering, directly related to the physical system, is easily constructed, and unlike the Scheidegger scheme, can be easily adjusted for initial mis-ordering. Stream combination in the Shreve scheme yields simple order sums on an interval scale: $U_{(m*n)} = U_m + U_n$ (see Figure 1.7E).

Another magnitude scheme, proposed by Graf (1975), is a cumulative ordering scheme which assigns source streams order 1, and increments the order of any link one unit for the addition of each new order 1 link in the series. Thus each link has a cumulative order corresponding to its total number of contributory links plus one (see Figure 1.7F). The Graf scheme is most useful in small basins; clearly, it rapidly becomes cumbersome for very large networks.

Although the Shreve and Graf ordering schemes are distinctly superior in terms of versatility, information content, algebraic validity and ease of computation, few studies have made use of them. Part of the reason for this lack of use is the fact that drainage network analysis is an area of declining research activity for geomorphologists. Morphometric analyses of drainage systems held great promise in the 50's and 60's but have failed to provide many insights into the processes of drainage evolution and maintenance. Ordering schemes are descriptions of structure and description for its own sake is a rather sterile exercise. On the other hand, the morphometric analyses have focussed attention on the important random elements of landscape. In addition they have provided a set of drainage basin parameters for hydrologic modelling studies; these are in part the subject of the next Chapter.

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