Mathematics Learner Profiling Using Behavioral, Physiological and Self-Reporting Methods

by

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M.Sc., Marmara University, 2003

Thesis Submitted in Partial Fulfillment of the
Requirements for the Degree of
Doctor of Philosophy

in the
Mathematics Education Program
Faculty of Education

SIMON FRASER UNIVERSITY

Summer 2013

Approval

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Degree: **Doctor of Philosophy**

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Abstract

This exploratory case study aimed at investigating learner profiles when participants are studying, self-reporting, restudying content and answering questions related to the division theorem in mathematics. It includes surveys aiming to measure participants' epistemological beliefs, metacognitive strategies and the levels of mathematics anxiety; behavioral data including audio-visuals and screen capture embedded with eye-tracking, and physiological data including heart, respiration and eye blink rates. It uses 'learner profiling framework' built on previous literature and defined with a new perspective. The data are analyzed using mixed research methodology cross validating self-report, behavioral and physiological data. The results from four participants provide contributions to the literature in four aspects. First, learner profiling framework offers a new methodology to educational research with numerous benefits. Second, CUR (Calculation-Understanding-Reasoning) framework offers a new way of categorizing mathematical cognition and corresponding content. Third, qualitative approach in investigating learner motivations indicates motivational constructs are much more nuanced than previously thought. Fourth, single case approach for studying learner behavior and physiology provides successful links to underlying cognitive and affective processes. The investigations are followed by learner profiles that involve assessments from teacher's perspective and recommendations for future work.

Keywords: learner profiling, learner profile; mathematics education, educational neuroscience; CUR framework; motivational constructs; embodied cognition; behavioral and physiological methods

I dedicate this dissertation to my family:

Fehmi Cimen, my dear father

Nevin Cimen, my dear mother

Baha Cimen, my dear brother

Burak Cimen, my dear brother

Elif Tugba Cimen, my dear sister

Acknowledgements

I would like to thank the following people for their support and guidance:

My dear family, for your great support throughout my education and my life.

Dr. Stephen Campbell, for the privilege of having you as my mentor who was always ready to enlighten me with his great knowledge, for being a friend whom I could chat about almost everything and play guitar with, and for your heart of gold.

Dr. Rina Zazkis, for you valuable feedback that helped me to improve my thesis greatly, and for your nice smile that made my day when I catch.

Dr. Peter Liljedahl, for always being there for listening to me, for your fast replies and constructive feedback that helped to observe things from different angles.

Table of Contents

List of Tables

List of Figures

Chapter 1.

Introduction

The center of education is the *learner*. Understanding the learner opens up ways for knowing how to organize lessons, how to develop curriculum, how to train teachers, how to build schools or classes. In other words, even a small achievement towards understanding learner better has potential for giving birth to important contributions to all subfields of education. As a teacher I have always been searching for ways to gain better understanding of the cognitive processes manifested in the minds of learners, and how those can be better observed so that I could shape my teaching based on these data from my students. Although interaction with students is somewhat helpful for knowing them better, and helped improving my senses about their emotional and cognitive states in the classroom, I knew that it is a much deeper issue, thus getting into the minds of the students in real sense has always been problematic. These questions had driven me to pursuing a PhD in Education where I could find some convincing answers to my questions, which unfold in this dissertation.

I believe that most of the problems in education are due to the gap between how learners think, and how the rest of the players (such as teachers, principals, policy makers, researchers etc.) in educational world think. Therefore, the key to augmenting education in the 21st century is being able to understand the learners in real sense, notice how nuanced their personalities are, and customize our approaches, policies, textbooks, and pedagogies with this perspective. Qualitative educational research attempts to accomplish this task. For the last few decades, qualitative studies made strong contributions to educational research by helping us better understand the individual, and interpret quantitative literature from this perspective.

The evolution of research methodologies finds its place in time. Using field notes left its place to the use of tape recordings, and then video recordings started to become more popular. Thanks to the recent technological developments, many research tools normally not well known to most educational researchers became available and more practical to use, such as eye-tracking, computer-based monitoring, facial expression analysis, and physiological measures (Campbell, with the ENL Group, 2007; Campbell, Cimen, & Handscomb, 2009). These methods have strong potential to extend what was unknown to us so far. For example with the help of eye tracking, we can precisely know where learners focus most on a study material, what is attractive and what is not for them to grasp the material. Just like upgrading from tape recordings to video recordings, these new methods have very strong potential to determine the next step of the educational research methodology.

Learner profiling aims to maximize understanding the learner with the help of these new methods. It helps understand how learners conceptualize information, how they interact with material, and how they react to specific content and questions. Therefore, it has strong potential to improve education by providing custom pedagogies for learners from different backgrounds and motivational orientations. In addition, learner profiling can be used to determine the factors that help improving with conceptual thinking by investigating how effective thinking is manifested behaviorally and physiologically. Accordingly it also has potential use for training teachers with regards to how these groups of learners from different levels behave, interact with the subjects, and what type of behavioral manifestations refer to what underlying cognitive or affective constructs, consequently can help developing new ways of instruction in this sense.

The current study aims to unfold learners' interaction with mathematical content and their motivational, cognitive and affective states depending on different sections of the content with the help of behavioral, physiological and self-reporting data. For this purpose the experiments were designed to include studying of mathematical content (the division theorem) that was classified into three categories: Calculation, Understanding and Reasoning (CUR). The experimental design involved a pre-questionnaire that was also CUR-classified to observe the attitudes and beliefs of the participants towards the content prior to the experiments, as well as a post-test and a post-questionnaire (that were again designed based on the CUR classification) to see how the experiments affected participants' knowledge and attitudes towards the content material as a whole, and separately towards each CUR category. In addition, restudying of the same content by the participants included into the experiments to observe the effects of restudying, if there is any. The experiment also involved multiple non-verbal surveys based self-reporting content where participants reflected their attitudes towards the learning material before the experiment, and their level of learning of the material during the experiment. Self-reporting content also involved multiple questionnaires, such as demographics questionnaire, tests and questionnaires related to content material, and some other surveys that are commonly used in the literature (such as the motivated strategies for learning, the epistemic beliefs inventory, the metacognitive awareness inventory, and the math anxiety rating scale-revised version) to gain better insight into learners.

My thesis involves three aspects of validity that should not be conflated or confused. The first concerns calibration that is properly synchronizing different data sets to ensure that the combined data sets are not misrepresented and misinterpreted. The second involves triangulation regarding the use of multiple data sets and sources to substantiate, or cross-validate, a given interpretation. The third concerns ecological validity, which refers to the design, conduct, and limitations of the experiments in providing a more normal or natural kind of study environment for the participants.

Next chapter (Chapter 2) of the dissertation provides a review of related literature. The review first focusses on the literature relating the content material (the division theorem). Secondly, literature on the methodological advances relating learner profiling is reviewed. After this, literature on self-reporting, restudying and motivational constructs are reviewed. Chapter 2 also provides a summary of previous findings for the use of behavioral and physiological methods used in this study. This chapter ends with the introduction of the research questions, those aim at investigating the efficiency of CUR (Calculation-Understanding-Reasoning) classification of mathematical cognition and content, of the restudying of same material, of using proposed learner profiling framework for determining behavioral and motivational aspects of learning process, of using eye-tracking technology for spotting how learners build connections within the study material, and of using the learner profiling methodology potentially for testing validity of self-report data.

Chapter 3 details the methodology, which includes the theoretical framework (where I discuss embodied cognition, learner profiling, CUR and 2 x 2 Achievement-Goal frameworks) and methods (where I discuss the methods and phases of the study) sections.

The methodology is followed by the results chapter (Chapter 4) where findings from four cases are exhibited in detail supported with multiple figures and graphs. Results section was structured by sub sectioning the data and results from each participant separately, and providing data from each participant based on the sections of the experiment, followed by final remarks for each experiment. Results chapter further articulates results by providing learner profiles for each participant and with the inclusion of a cross-case analysis.

The discussion chapter (Chapter 5), explains how the results are significant in alignment with the newly defined methodology and theoretical framework and discusses how they provide important contributions to the literature by revisiting the research questions.

Lastly conclusion chapter (Chapter 6) provides an evaluation of the study followed by recommendations for future research and concluding remarks.

Chapter 2.

A Review of the Study and Related Literature

This chapter will summarize the views of and findings from the previous literature in relation to the content, theories and methods involved in the current study. It will start with the introduction of the content material and related previous literature. Next, recent methodological advances and use of eye-tracking in mathematics education research will be reviewed. A review of previous findings related to the methods used in the current study will be followed by a review of the literature relating learner profiling. Learner motivation literature will also be reviewed as it is a part of theoretical framework. Because this study involves restudying of the same content material by participants and a phase where they self-report their understanding of it, literature on restudy and self-reporting will be reviewed. Each subsection will be accompanied by explanations on how these findings from previous literature are significant in relation to the current study. Note that all sections of the study as well as the theories and methods involved will be investigated further in the methodology chapter.

2.1. The Division Theorem[1] (TDT) and Division Related Mathematics Education Literature

This research study uses TDT and related concepts (such as division, multiplication, prime factorization, divisibility rules) as the content material. TDT can be defined as follows:

> Consider any two whole numbers, A and D, where $D \neq 0$. Given such an A and D, there exist unique whole numbers Q and R, where $A = QD + R$ and $R < 0$. In this definition; A is the dividend, Q is the quotient, D is the divisor and R is the remainder.

[1] As suggested by Campbell (1998; 2002), the term *the division theorem* (TDT) will be used in lieu of *the division algorithm* to prevent possible confusion of it with the procedure of the long division algorithm.

Although the literature referring to TDT in mathematics education is very limited, TDT can be considered a potential focal point in the curriculum that might link many related concepts. Some of these concepts and topics that TDT relates are division, long division algorithm, multiplication, divisibility rules and prime factorization, that make implicit or iterative use of TDT, and are widely studied in mathematics education literature (Ambrose, Baek & Carpenter, 2003; Campbell, 1998; 2002; 2006; Fang, Lee & Yang, 2011; Hickendorff, van Putten, Verhelst, & Heiser, 2010; Kaasila, Pehkonen & Hellinen, 2010; Lee, 2007; Martin, 2009; Sellers, 2010; Tirosh, 2000; Tirosh & Graeber, 1989; Zazkis & Campbell, 1996a).

While the curriculum focuses on the division of whole numbers in Grades 3-5, students are expected to learn division of fractions (rational numbers) in Grades 6-8. According to some literature (Campbell, 2002; Campbell & Fonthal, 2000; Mack, 1995; Silver, 1992) at this point of the curricula, students are having difficulties in the transition from whole number division to rational number division, and understanding connections and differences between the two. TDT is evaluated having an important potential role in Grades 6-8 to introduce rational number division, helping students to enhance their procedural and conceptual understanding of whole number division into rational number division more smoothly (Campbell, 1998; 2006). For more advanced topics, the applications of TDT carry potential use expanding into teaching modular arithmetic and the ring of congruence classes (Smith, 2006) in later grades or at the college level. It is important to state that TDT, long division algorithm or related examples of these also has been widely used as a study material for doing research on beliefs and understandings of related concepts (Ball, 1990; Simon. 1993; Tirosh, 2000; Tirosh & Graeber, 1989). Below, some of the literature referring to the TDT and related concepts are reviewed.

Early literatures prior to focusing on TDT have been done aiming at investigating learners' understanding of division and related concepts. Ball (1990) worked with 19 prospective elementary and secondary teachers to study their understanding of division. Her qualitative study involved interviews of the participants where they were asked questions about division in three contexts (division with fractions, division by zero, and division with algebraic equations) then asked to explain their understanding of division. The interviews were tape-recorded, and then transcribed. According to the results, even though all participants gave answers to the question of division with fractions correctly, only a few of them could provide correct representations explaining their answers, all of whom are participants with mathematics majors. As for the division by zero, less than half of the participants were able to provide correct rule by telling why. The question of division with algebraic equations was successfully solved by most participants, yet only one of them could provide the conceptual meaning behind the solution, while other participants' solutions were procedural. The results indicated insufficient content knowledge background of the prospective teachers for the conceptual understanding of division. This study is important to recognize the weak background of prospective teachers in division and related concepts.

Another study focusing on prospective teachers' understanding of division was collected by Simon (1993) that has given parallel results to Ball's (1990) study. He aimed at understanding the connectedness between the procedural and conceptual understanding of the division by the participants, as well as their knowledge of units. For the study, 33 participants were provided with a written instrument for the questions, and eight of them were interviewed. One of five questions involved long division algorithm to understand participants' ability to establish relationships among division, multiplication, and subtraction. According to the results participants' conceptual knowledge was found weak in areas such as ability to connect with long division algorithm, the relationship between partitive (i.e. finding number of items per a defined number of groups to constitute a larger set) and quotitive (i.e. finding number of groups with a defined number of items to constitute a larger set) division, the relationship between symbolic division and real-world problems, and identification of the units of quantities encountered in division computations. Another result was that all of the participants used their procedural knowledge of long division algorithm and none of them were able to provide a conceptual explanation to their solutions in that regard. They also failed to connect the procedures to other related concepts, such as multiplication and subtraction. The results indicated that although long division had a potential to connect many concepts in elementary number theory, lack of conceptual understanding resulted as lack of connectedness of these key concepts.

The main research associated with TDT was done by Zazkis and Campbell (1996a; 1996b), and Campbell (1998; 2002). This series of publications mainly depends on similar data sets used by the researchers. The focus of their study set was to investigate preservice teachers' cognitive structures in elementary number theory with a lens of divisibility, and in relation with other connected concepts such as division, multiplication, prime factorization, divisibility rules and TDT (Campbell, 1998; 2002; 2006; Zazkis & Campbell, 1996a). Their data collection included two main stages. For the first stage, 58 preservice students were provided with a questionnaire (Campbell, 1998; Zazkis & Campbell, 1996b). Three question sets in this stage involved self-evaluations of the participants for understandings of some concepts in relation to the elementary number theory, such as natural number, rational number, remainder, quotient, and divisor (Campbell, 1998). The second stage of the data collection involved interviews of 21 participants out of the first 58, expanding upon an investigation into the understandings of elementary number theory with questions relating prime and composite numbers, factor trees, prime decomposition, divisibility and divisibility rules (Campbell, 1998; Zazkis & Campbell, 1996a; Zazkis & Campbell, 1996b). In alignment with the results of the previous literature above, the results of their study reflected that even when some degree of conceptual understanding was involved, participants' responses indicated pervasive dispositions toward procedural attachments, and divisibility rules were generally grasped and applied procedurally (Zazkis & Campbell, 1996a). Similarly, they highlighted the lack of conceptual understanding of concepts among prospective teachers; those have direct ties with TDT, such as divisibility rules and division.

Because they have similar content and participants with similar background, these results of the literature reviewed above are significant for the current study. While interacting with TDT, participants of the current study are expected to confront similar challenges triggering cognitive and affective fluctuations, which can be detected through analyzing their behavioral and physiological responses.

2.2. Recent Methodological Advances in Qualitative Mathematics Education Research

Merriam (1998) indicates that qualitative research is concerned with the lived experience, where the researcher experiences instruments from participants' perspective. The researcher should evaluate the data with enormous tolerance for ambiguity from the designing the study, to reporting the results. She states that there are no step-by-step procedures or protocols to follow in data analysis. Just like a detective, the researcher tries to look for important aspects, dig out interesting events. With no specific order, the researcher finds and matches missing pieces, and put the puzzle together with empathy and sensitivity (Merriam, 1998). She indicates that data collection in case study research usually involves multiple ways to gather information, such as interviews, observations and analysis of documents. Patton (1990) highlights that, “Multiple sources of information are sought and used because no single source of information can be trusted to provide a comprehensive perspective…By using a combination of observations, interviewing and document analysis, the fieldworker is able to use different data sources to validate and cross-check findings” (as cited in Merriam, 1998, p. 244). She also indicates that all methods usually will not be equally weighted in terms of usage.

Denzin and Lincoln (2000) define qualitative research as a field of inquiry to explore and reflect upon the routine and problematic moments of individuals’ lives. They highlight the importance for a qualitative researcher to benefit from multiple interconnected interpretative practices and methods. The aim of qualitative research is to gain deeper insights into the subject of study in ways that embrace context and situatedness, and to be open to unique aspects thereof that may or may not generalize to other subjects (hence the need to follow up any resultant hypotheses using more quantitative methods) Originally developed from the research in other social science fields such as sociology, anthropology, qualitative research has been used in mathematics education research widely since the Erlwanger’s (1973) case study of Benny providing deep focus into student learning of mathematical concepts and problem solving activities (Ernest, 1997). Continuing this tradition, as explained in the literature relating to TDT, many mathematics education researchers (e.g. Ball, 1990; Simon 1993) used qualitative methods such as observations, written and verbal self-reporting techniques and observations to study the learner experience in depth. In their analysis of the 710 published research articles in mathematics education from 1995 to 2005, Hart, Smith, Swars, and Smith (2009) indicated 50% of these studies used only qualitative methodology while this percentage was 21% for quantitative-only studies. They also added that the use of mixed methods methodology between these years was 29% and highlighted the emerging recognition of mixed methods research in mathematics education in the recent years, which was also used for the current study.

Case study is one of the most commonly used methods of qualitative research, and it is also used in the current study. Merriam (1998) indicates there are two types of case studies, one is single case study and the other is multiple case study. The latter has two stages of analysis; those are within-case analysis, and cross-case analysis. Firstly for within case analysis, each case is treated as a comprehensive single case. Following the analysis of all cases, the second stage is the cross-case analysis, where researcher seeks, similarities and differences across cases. She indicates that for that reason, multiple case studies often lead to categories, themes, or typologies, where an integrated theoretical framework that covers multiple cases can be necessary. She states that methodologically, multiple case studies might be supported with case surveys (which are generally quantitative in nature) to answer new questions or confirm interpretations, to gain deeper insight. She also points out that using multiple investigations, multiple sources of data, and multiple methods of data collection and analysis strengthens both internal validity (how congruent are the findings with reality?) and reliability (how replicable are the findings?) of the research. She also indicates that an ideal way to improve external validity (how generalizable are the results of the study?) in a qualitative case study is to multiple cases and doing cross-case analysis. She indicates that generalization in a statistical sense is not the objective in qualitative research, thus rather than probability sampling (such as random sampling from a large sample size), nonprobability sampling (one type of which is purposeful sampling where researcher chooses information-rich cases those (s)he can most learn from) should be considered.

In her book, Merriam (1998) also highlights the great capacity of computers for organizing and analyzing massive amounts of data. Three decades ago qualitative researchers started to realize this fact, saying “we are creating new databases that have the potential to be easily accessible and usable for secondary analysis. This could not only increase the reliability of our studies, but allow a whole new level of secondary analysis. Data from several different field projects could be compared easily.” (Conrad & Reinharz, 1984, p. 8, as cited in Merriam, 1998). She also indicates that the capability of multiple levels of coding of the qualitative analysis software gives researchers an advantage for cross-analysis that is particularly important to augment the study from categorical or taxonomic integration to theory development.

When we look at the literature, we notice that the written data in early qualitative studies traditionally depend mostly on the field notes that reflect what the researcher can remember about the participants at the time of data acquisition. Therefore it might not give enough insight to the researchers about some specific details, such as participants' attention on specific parts of the questions, the time they spent on these questions, the areas of the questions they were mostly focused on, the areas they may have avoided attending, or their talk-aloud feedback. Secondly, similar data might not provide enough information about behavioral cues like facial expressions and gestures. Depending on the research questions, these types of additional elements might not be needed. However these additional data may contribute to the observational control and validity of the study.

As an example, student may not pay attention to the questions, randomly fill the blank pages. If a researcher lacks enough information about the level of concentration of the participants in the experiments, it might be difficult to draw any conclusions based on written data.

There are two possible solutions for such problems to eliminate misleading data. The first way (with a quantitative research methodology perspective) would be to increase the sample size to achieve statistical significance, which also involves determination and elimination of outliers in pre-analysis stage. On the other hand, from a qualitative methodology perspective, increasing observational control at the data acquisition stage with the help of more advanced research tools can be most helpful. Audio transcripts provide better insight into participants' mathematical thinking; however, it is still difficult to have enough insight into the other qualitative components of the data, as exemplified above. Audio files can be saved for years for producing additional research articles; yet, audio files alone may not provide enough information about the levels of attention, feelings, emotions, gestures, or facial expressions of the participants. Video recordings thus have been most helpful in the literature providing better insight into data analysis for the last two decades. One can wonder what would be the next step to improve research methodologies in education that will provide researcher better opportunities in terms of better observational control and extended insight into interpreting the data. Some recent literature addressed these issues above and offered some additional approaches in that regard.

Using eye tracking, screen captures and audio-visuals for their research in mathematics education, San Diego, Aczel, Hodgson and Scanlon (2006) discussed the weaknesses of the traditional methods used in mathematics education research and offered alternative methodological enhancements. They discuss possible disadvantages of using only talk-aloud based face-to-face interview methods in mathematics education research. According to them, this kind of methods cause too many distractions in learning or problem solving situations, and prevents the participant from concentrating on the task comfortably. In addition, they discuss that audiovisual data on itself is inadequate to provide enough insight into participants' cognition. They also propose that written data alone do not provide enough insight because in such cases for the most part researcher has to make guesses what participants were thinking about. Instead, San Diego and his colleagues offer audiovisual and hand written data combined with eye-tracking data to maximize researchers' insight for investigating participants' mathematical thinking. Their study was focused on the use of methods and did not clearly provide conclusions regarding mathematical cognition.

Campbell (2003), aimed at augmenting the traditional research methodology, by introducing some advanced tools in time such as eye-tracking and the use of audio-visuals (Campbell, 2003; Campbell & Fonthal, 2000). He first offered a computer based learning environment with a developed observational control, named *Dynamic Tracking* (Campbell, 2003). Later called as Computer Enhanced Mathematics Learner Environments (CEMLEs) (Campbell, Cimen, & Handscomb, 2009), some examples of these environments are Geometer's Sketchpad and Cabri. Campbell and Fonthal (2000) introduced a new CEMLE called *DivFact*. DivFact was developed for focusing on the exploration of TDT, and intended to improve learner experience for the understanding of the connections and differences between whole number division and rational number division.

The first use of Dynamic Tracking as a research tool in mathematics education research was introduced by Campbell (2003). The study included 30 preservice teachers' experiences using CEMLEs, such as Geometer's Sketchpad and DivFact.

He recorded participants' computer screens; video recorded their behaviors, then integrated and synchronized the two. He stated that the use of Dynamic Tracking was intended “to capture a complete record of a learner’s interactions with a computer-based learning environment in real time—a record complete enough, that is, to adequately observe, identify, and study manifestations of cognitive shifts associated with teaching and learning” (Campbell, 2003).

Future work in the following years (Campbell, with the ENL Group, 2007; Campbell, Cimen, & Handscomb, 2009) introduced the next step for the use of Dynamic Tracking in the field of mathematics education. They used a newer version of DivFact (v3.0) to investigate preservice teachers’ understanding of whole number division and rational number division along with the connections and differences between the two with a more improved observational control and research tools including *eye-tracking (ET)*. The study included recordings of 30 preservice teachers using DivFact. Participants were provided 15-20 minutes for exploring DivFact in both whole number and rational number modes. For the first stage of the study, the participants experienced how the chances of the variables in TDT dynamically changes graph on the screen. The second stage of the study included 36 multiple-choice questions. The task for the questions was to identify the graph (visual representation) or the equality (symbolic representation), that is inconsistent with the rest of the selections. This study was important because it was using many methodological advances combined, such as audio-visual recording, keystroke capture, mouse click capture, and ET, those also used on the current study.

These literature above provide an exciting evolution, especially regarding the research methodologies involved. This was achieved with the help of technological advances became available in time and thanks to the endeavors of researchers in mathematics education. The state-of-the-art techniques help and will continue to help further improving qualitative approaches in the field of mathematics education. The results of these literature explain how these sets of advanced methods bring potential into studying learners to gain more insight. Nevertheless they were limited in terms of providing empirical evidence for insights into learner cognition and understanding. The current study aims to fill this gap in the literature at some level by providing empirical evidence in that regard.

2.3. Eye-Tracking Research

Eye movement related research started to get attention of researchers such as Louis Émile Javal in the 19th century and continued to improve to this date. Especially within the last three decades, thanks to the development of eye-tracking technology this field of study became more popular. Numerous recent research studies have been conducted in the fields of cognitive psychology, neuroscience, and psychophysiology providing clear evidence that eye fixation, gaze duration and saccades (rapid eye movements) are strong indications of attention and attentional shifts (Daniel & d'Ydewalle, 1996; Doré-Mazars, Pouget, & Beauv, 2004; Fischer, Castel, Dodd, & Pratt, 2003; Hodgson & Müller, 1995; Hoffman & Subramaniam, 1995; Kowler, Anderson, Dosher, & Blaser, 1994; Liversedge & Findlay, 2000; Moorea, Armstronga, & Schafera, 2007; Peterson, Kramer, & Irwin, 2004; Rayner, 1998).

More technical and advanced part of this literature is beyond the scope of this study. As a researcher in mathematics education I will focus on what will be necessary and sufficient to be able to interpret my data. Based on this perspective, what can be gleaned from this literature in a sentence is that, *our eyes focus on where our attention is focused* and *our eye gaze shifts to where our attention shifts*, especially while reading (see Rayner, 1998, for a review). The duration of fixation is longer on the areas that are the most interesting (Hoffman & Subramaniam, 1995; Liversedge & Findlay, 2000), and mathematically more complicated (De Corte, Verschaffel, & Pauwels, 1990; Rayner, 1998).

Due to the existing scientific evidence supporting the relation between eye behavior and attention, I will mainly discuss how attention relates to mathematical thinking and learning, based on mathematics education literature.

2.3.1. Mathematics Education Research Using Eye Tracking

The mathematics education research literature associated with eye tracking (ET) is surprisingly very limited. However available previous studies show some benefits of using eye tracking in studying mathematical thinking and learning.

One of them was conducted by Epelboim and Suppes (2001). The researchers used eye tracking to study performances of experts and novices in solving geometry problems. By interpreting the eye tracking data, they discovered significant differences between the cognitive processes of novices and experts. According to the results, the fixations of experts indicated that they were able to imagine the needed additional element for solving the problem, which was not present in the figure provided to the participants. On the other hand, novices only looked at the existing parts of the shape.

Another study was conducted by Andrà et al. (2009). They collected eye-tracking data from 46 undergraduate level students to study their mathematics performances and reading strategies. 24 of the participants had no recent mathematics training (named as 'novices' by the authors) whereas other 22 had mathematics courses recently (named as 'experts' by the authors). The participants were given three sets of questionnaires. The questionnaires included multiple-choice mathematics questions. The results showed that novices frequently switch their focus among the question part and all of the choices, while experts put more attention on important parts of questions and on less number of choices. Secondly, the results indicated that experts were searching for specific cues, while novices spent more time in reading. The third preliminary result was that, eye-tracking data collected from experts indicated repeated patterns in their shifts of attention among question and choices, whereas there was no specific order of attentional shifts for novices. This finding indicated that experts were more systematic and strategic in reading mathematical text. Based on the eye tracking data, the researchers also highlight the importance of teaching learners how to read problems effectively, as an important issue to consider for future research (that is beyond the scope of this study). As a critique, their approach for naming learners as novices and experts based on their course experience can be further discussed. This classification could be used based on learners' cognitive levels and abilities rather than based on the number of courses they have taken. Although expert learners' systematic thinking can be found out using traditional methods such as interviews, using eye tracking technology to provide parallel evidence could be considered important.

Both studies above indicate the importance of using eye tracking to spot learners attentional behaviors concerning their cognitive orientation. They also show that eye tracking is an effective tool to interpret and analyze data for determining specific characteristics of learners with different cognitive orientations.

Another mathematics education researcher Schwank (2001), used eye tracking to identify participants' systematic eye behaviors in problem solving situations to investigate the type of thinking (predicative/functional) they had. The term "*predicative* was used to characterize a problem solving behavior highly orientated at and sensible for features, relations and judgment, whereas the label *functional* was used to characterize a problem solving behavior highly orientated at and sensible for courses, modes of actions and effects" (Schwank, 2001, p. 1). In this study, participants were shown numerous slides containing different sets of shapes. Each slide included eight figures. The participants were asked to find the ninth figure that matches with the sequence. The author, based on the eye tracking data, was able to identify the way of thinking (predicative/functional) that the participants held. This study was also important to show how cognitive orientations of mathematics learners can be studied using eye tracking.

2.3.2. NCTM's Connection Standard: Potential Use of Eye-Tracking

NCTM documents (2000; 2006) pay significant importance to the connections between concepts. Connection Standard proposes that "instructional programs from prekindergarten through grade 12 should enable all students to recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics" (NCTM 2000, p.64). As evidenced from the findings above, eye-tracking technology has strong potential to be used to investigate students' uses of connections between concepts. In that regard, one of the research questions (as detailed at the end of this chapter) in the current study will target this potential aiming to investigate findings in that parallel.

2.4. Interpreting Psychophysiology Data and Related Literature

The field of psychophysiology is concerned with the relationships between psychological and physiological phenomena. According to the psychophysiology literature, behavioral methods (such as facial expressions and gesture analyses), and physiological methods (such as eye blink, heart and respiration rate analyses) have shown effective use for interpreting reactions of the body throughout a learning process and spotting psychological phenomena underlying these responses.

Harrigan and O'Connell (1996) studied facial displays of anxiety as well as eye blink rates, body movements in low and high anxious states of participants. In their review of the related literature, they indicated that frequent eye blinks are shown relating to anxiety and stress in many previous studies (Argyle & Cook, 1976; Harris, Thackray & Shoenberger, 1966; Jankovic, 1987; Stern, Walrath, & Goldstein, 1984). Frequent eye blinks are evaluated as an "involuntary response that increases when people are emotionally aroused" and often as a result of emotional discomfort and anxiety (Ekman, 1985, p.142, as cited in Harrigan & O'Connell, 1996).

They videotaped 37 participants, and asked them to talk about the most anxious event from their past, then to self-rate their state anxiety during the experiment. Participants' facial expressions were coded in facial action units, which are based on muscle innervation and functional anatomy (Friesen & Ekman, 1984). One of their findings was that eye blink rate increases as anxiety level of the participants increase. They also found that there were more body movements in high anxious state vs. low anxious state. In addition, negative facial expressions occurred more frequently in high anxiety state vs. low anxiety state.

It is important to note that the authors distinguish the nonenjoyment (nervous) smiles from enjoyment smiles. They indicate that former increases with anxiety, whereas latter is exhibited when someone experiences enjoyment, pleasure and amusement. According to the authors, increased rate of eye blinks and nonenjoyment smiles are associated with participants' attempts to suppress and mask their felt emotion, and mislead others by appearing positive or to conceal negative emotions.

A previous study describing the specifications of nonenjoyment smile was done in 1990 by Ekman, Davidson and Friesen, that was based on the work of French anatomist Duchenne de Boulogne in 1862. Ekman and his colleagues describe the *Duchenne smile* activating the muscle orbiting the eyes, and the muscle pulling up the lip corners, whereas in a non-Duchenne smile, muscles orbiting the eyes are not active. In addition, Ekman (2003) distinguishes Duchenne smile where the cheeks get higher and the level eyebrows change all due to the action of the outer part of the muscle that orbits the eye. Duchenne smile is considered as enjoyment smile, whereas non-Duchenne smile is a non-enjoyment smile.

Ekman et al. (1990) intended to test the correlation between level of happiness and type of smile. They recorded the facial expressions, EEG, and self-report of subjective emotional experience from 37 subjects who watched pleasant and unpleasant films. The results validated that the Duchenne smile was related to enjoyment and positive emotions, whereas other smiles were not.

As another significant behavioral cue for anxiety that was frequently observed in the current study, swallowing was linked to emotional arousal in the literature. Fonagy and Calloway (1985) discussed that intense emotional states (such as anxiety, grief or excitement) induce changes in salivation rates and they might also lead to nausea. As it was replicated in their study both of these cases caused by emotional arousal induces increased rate of swallowing reflex. Similarly, Cuevas, Cook, Richter, McCutcheon and Taub (1995) reported almost three times swallowing rate in high arousal conditions in comparison to low arousal conditions. He cross validated this finding with forehead EMG (indicating eye related muscle activity) rate and heart rate, and found both of these measures were also significantly higher in high arousal conditions.

Previous literature relates the effects of cognitive activity and memory recall on the rapid eye movements. Lorens Jr. and Darrow (1962) studied the effects of mental multiplication on eye movements, EKG, GSR (Galvanic Skin Response) and EEG. They studied with 10 participants who were asked to do some multiplications (such as 17x12) in their heads in eyes-closed state with no time limitation. According to the results for all participants eye movement rates (EMR) were more than doubled during mental calculation. For that reason, authors concluded that rapid ocular movement is a consistent outcome of mental multiplication. They could not verify similar effect on other instruments; those are EEG, EKG and GSR.

Ehrlichman and Micic (2012) reviewed previous literature on the relation between non-visual saccadic eye movements (those do not serve visual processing, such as eye movements in eyes closed state) and thinking. They state that people move their eyes about twice as often while accessing long-term memory with compare to tasks do not require long term memory recall. They also discuss that these rapid eye movements occur even in dark rooms or when eyes are closed. According to the authors, the literature that was not available a decade ago, now provide enough evidence that this type of movements are systematically related to cognitive processes those are independent from vision. Some other studies done by same research group (Ehrlichman, Micic, Sousa & Zhu, 2007; Micic, Ehrlichman, & Chen, 2010) suggested that tasks involving minimal long-term-memory search and active working memory produced low EMRs, while tasks requiring high use of long term memory produced high EMRs.

Heart and respiration rates are positively correlated because heart requires oxygen to pump the blood to the body (viz. the more heart beats, the more breathing occurs). Respiration rate measures the number of breaths per given period of time (generally minutes) and heart rate is the number of heart beats per given period of time (generally minutes). For a healthy adult, normal average respiration rate is between 8-18bpm (Fesmire & Luten, 1989) and average heart rate is between 60-80bpm (Hart & Gandhi, 1999). In their review of previous literature, Cacioppo at al. (2000) and Kreibig, Wilhelm, Roth and Gross (2007) separately summarize numerous research results providing evidence that heart rate and respiration rate has a positive relation with fear and anger. Heart rate is known as a significant indicator for the level of stress and anxiety in general (Kelly, 1980; Dishman et al., 2000), and in mathematics learning settings (Dew, Galassi, & Galassi, 1984). The interpretations for the variations of heart and respiration rates in the current study will be based on this previous literature.

2.5. Learner Motivation and Related Literature

Since the early 20th century, *motivation* has been defined in many of different ways in a variety of disciplines based on different theoretical approaches in the literature. Etymologically, the word ‘motivate’ comes from the Latin *movere* that means to *move* (Eccles & Wigfield, 2002). In its most general form, motivation can be defined as any driving force behind behavior (Madsen, 1974, as cited in Kleinginna Jr & Kleinginna, 1981). From an educational perspective, motivation can be defined as a psychological feature that is aroused when acquiring desired knowledge.

In modern literature, researchers introduced some main aspects of motivation in educational settings. One aspect depends on whether learning is motivated by external causes, such as reward/punishment, competence to others, pleasure of teachers/parents, grades or internal causes, such as one’s self-pleasure or appreciation of the new knowledge, connecting it to personal experience. The former is defined as *extrinsic motivation* while the latter is defined as *intrinsic motivation*. The intrinsic and extrinsic dichotomy has been discussed since the dawn of Western Philosophy in ancient Greece. According to Zimmerman (2010), Socrates claimed that *pleasure* is better when accompanied by intelligence, but not always a good thing. In his dialogue *Protagoras*, Plato expanded his mentors’ view by considering pleasure as a motivating factor towards a goal, and said, when people condemn pleasure, they do so not because they take pleasure as a bad thing, but because of the bad consequences it might lead to, thus it is not the highest good to be achieved. Like Plato, his student, Aristotle did not consider pleasure and pain to be the only things that are intrinsically good and bad, although many other philosophers of his time (such as Epicurus) took an opposite stance.

Motivation has been widely studied in the educational psychology literature. In their article reviewing the literature on motivation, Ryan and Deci (2000) emphasized the importance of the distinction between intrinsic and extrinsic motivation mentioning how this contributed to developmental and educational practices. They mention that the research that has been done for the last three decades provides evidence that the quality of experience and performance differs highly between these two types of motivation. They also distinguish the level of motivation from its quality, discussing that two different individuals can have the same level of motivation, while one is extrinsically oriented and other is intrinsically oriented. They discuss that intrinsic motivation results in creativity and high quality learning. Their thesis offers further classification of extrinsic motivation that is beyond the scope of this study. For the last few decades, the terms *mastery* and *performance* motivation have been used interchangeably with intrinsic and extrinsic motivation (Dweck, 1986; Nicholls, 1984; Hannula, 2006). In the current study, this new terminology will be used.

Motivation has recently started gaining increasing attention of scholars in mathematics education. Hannula (2006) mentions the hidden characteristic of motivation by proposing that, in a general sense, motivation could not be directly observed except for affective and cognitional manifestations, which can be measured through studying behaviors of individuals. He points out that it is also necessary to know the motives of students to understand their behaviors. His specific emphasis to the two way relationship between motivation and behaviors, beliefs or emotions in mathematics educational settings demonstrates the importance of studying both aspects (as is done in the current study) to better understand factors relating students' mathematical cognition and understanding. He also mentions that although positive emotions (such as joy, relief and interest) and negative emotions (such as anger, sadness and frustration) are partially observable through the analysis of facial expressions and body language, an important portion of emotions and cognition are hard to observe. Also described as overt and covert behaviors respectively in Campbell (2010) both classes of behaviors are studied in the current study. In addition to analyzing the overt behaviors such as facial expressions and gestures, this study attempts to reveal covert behaviors relating to motivations, emotions and cognition of the participants by analyzing physiological measures such as eye related behaviors in addition to cardiovascular and respiratory responses.

Hannula (2006) also points out the goal factor affecting motivation. Elliot (1999) offers that the goal factor can be characterized based on whether a learner is motivated to approach to the task (to be successful on it), or motivated to avoid it (to escape from failure). The mastery/performance and approach/avoidance aspects of motivation were integrated as a new achievement goal framework by Elliot and McGregor (2001) titled '2 x 2 Achievement Goal Framework'. This framework comprises four achievement goals: mastery-approach, mastery-avoidance, performance-approach, and performance-avoidance. According to the 2 x 2 framework, students motivated to learn and master a task have *mastery-approach* goal orientation, while students avoid learning that is already known have a *mastery-avoidance* goal orientation. On the other hand, students who are motivated to learn a task to please others have a performance-approach goal orientation, whereas students that avoid learning a task not to show incompetence are thought as having performance-avoidance goal orientation (Elliot & McGregor, 2001). The current study uses 2 x 2 achievement goal framework for analyzing learner motivation, and offers some new insights into it as it will be detailed in the discussion and conclusion sections.

2.6. Learner Profiling and Related Literature

Previous research on learner profiling in the literature are mostly quantitative studies that are done based on determining and extracting some specific types of components of the samples (such as abilities and strategies) out of the data using statistical methods such as cluster analysis (Alexander & Murphy, 1999; Csizér & Dörnyei, 2005), and they are generally related to research on e-learning systems and data base architectures. Because of the limited previous literature that is mostly quantitative, this study required a unique definition for a learner profile.

2.6.1. Defining a Learner Profile

Before discussing the related literature I will give my definition of 'learner profiling', and a 'learner profile' to clarify my view beforehand. Learner profiling is a method used for outlining individual characteristics of learners based on their behavioral, physiological and cognitive traits; such as their abilities, learning strategies, motivation orientations, and personalities (e.g. demographics, general mood, and emotional responses to the content) in the form of narrative inquiry in combination with the analysis of the qualitative and quantitative data. A learner profile is the compact outcome of learner profiling that summarizes overall approaches, motivations and strategies of a learner in relation to the subject followed by pedagogical recommendations for instruction. In other words, rather than being technical and research oriented, a learner profile is a more user friendly, succinct and pedagogically oriented version extracted from learner profiling. It is something that parents and teachers of a student can take home, read and benefit from.

As is will be detailed in Chapter 3 under ‘Theoretical Framework’, a working hypothesis of the current study is the classification of mathematical cognition in three; those are *calculation*, *understanding*, and *reasoning*. Defined as Calculation-Understanding-Reasoning (CUR) framework, this framework presupposes that mathematical cognition and corresponding content can be classified under these three categories matching the abilities and motivational orientations of learners (Note that as it will be exposed at the end of this chapter, one of the research questions targets at the level of success of the CUR classification).

This mixed methods case study focuses on participants’ motivation orientations based on Elliot and McGregor’s (2001) achievement-goal theory framework with the help of eye-tracking methods to provide insight into participants’ cognitive abilities (calculation, understanding and reasoning), in combination with measurements of their physiological response using eye, heart and respiration data. The learner profiling in this study also constitute behavioral data, such as participants’ facial expressions, body movements, and their verbal expressions. Other components of learner profiling here are the analyses of self-reports and tests to gain deeper insight into participants’ levels of understanding the content and their strategies while interacting with it. A similar study in the future using learner profiling with the same principles might involve various methods depending on the needs and resources of the researcher. For example, EEG might be another tool to be used in future research. Because this is a unique definition of learner profile and learner profiling, the literature provide no similar examples in this sense. This study aims to provide four examples of learner profile, and explain how a learner profile is generated in detail in the results section.

Learner profiling, by definition, is structured in the form of narrative inquiry accompanied by the analysis of relevant data that are integrated whenever relevant and meaningful. Therefore learner profiling combines results and data analysis of each case, and includes the data only when harmonious, meaningful and significant with the narratives. In other words, learner profiling does not necessarily involve all data available for a learner, and attempts to interpret them regardless of relevance.

Narrative inquiry has been widely used as a method of qualitative educational research. In their review of literature on narrative inquiry and constructing their view of its place in educational research, Connelly and Clandinin (1990) refer study of narrative to the ways humans experience the world. In educational sense, they introduce narrative as storytelling by the researcher about learners' or teachers' experiences. They use 'storytelling' carefully by highlighting the fact that researchers only describe and reflect upon the lived experience, thus 'narrative' is considered differently from 'story' in that sense. They rephrase 'narrative inquiry' and state that 'inquiry into narrative' could be used interchangeably for a more clear insight. They discuss that narrative inquiry is a qualitative method by nature due to its focus on the qualities of lived experience. They also indicate that for narrative inquiry researcher first should focus on the practitioner's story and then restructure it based on his own observations.

Learner profiling in that sense listens to participants' stories told, not necessarily only by their tongues, but by other parts of their bodies (such as faces, lungs, hearts, eyes, or brains) in the course of learning experience. Therefore by nature this type of narrative inquiry is accompanied by the analysis of the qualitative and quantitative data based on how learners' minds and bodies speak reflectively, behaviorally and physiologically, just like providing transcriptions of verbal data. Also, just like providing only relevant sections of transcriptions, learner profiling involves only relevant data from other measures mentioned above. In that sense, learner profiling should not be thought as traditional narrative inquiry commonly used in the literature, should be rather seen as more of a multiple case study that involves building of narratives for each case.

It is important to state that as a qualitative dominant mixed multiple case study, this research neither intents to make general claims, such as describing all characteristics pertaining to a specific learner profile, nor tries to identify all possible profiles that exist when a learner confronts with a mathematics text. Rather, this study aims at providing an expanded insight into individual learner behavior and physiology, and possible characteristics of such learners while studying and restudying a mathematical text, using comprehensive observational methods such as eye-tracking, heart rate and respiration analysis, and a comprehensive self-report data that constitute multiple surveys and self-judgment of learning of the content material.

The type and number of methods used in mixed methods profiling might vary depending on time and resources. As technology develops, future research will offer more proven techniques those are easier and inexpensive to utilize. For example in the close future, developments in EEG technology might offer more portable, reliable and cost efficient systems those can easily be adapted to learners in classrooms; while EEG research will be providing much precise understanding of neural correlates underlying all emotional and cognitive aspects of human thinking. As technology and science continue offering more proven techniques, the number of methods can be integrated into mixed methods profiling will continue to increase.

2.6.2. Literature on Learner Profile Generation

Limited previous research for learner profiling showed some benefits. Most of the already limited literature for learning profiling research related to the e-learning systems and data base architectures, which does not provide that much of educational research insight. Almost all of the educational research literature on learner profiling pertains to the second language learning, although there are a few exceptions. Some of these studies include achievement-goal scenarios as one component for profiling learners. In one of these studies Harris, Bonnett, Luckin, Yuill and Avramides (2009) aimed to investigate participants' (35 fifth grade students) help seeking behaviors while using an interactive learning environment. For generating learner profiles, they combined participants' help seeking behaviors with the possible achievement-goal scenarios. For investigating help-seeking behaviors, students were asked to complete a questionnaire investigating their attitudes and opinions toward learning situations where they require help. As the second component of learner profiling, where researchers aimed at identifying participants' motivation orientations, students were presented different scenarios of storyboarding. In an interactive environment using third person view, students rated learner reactions with mastery/performance motivations. Researchers also combined transcripts from student interviews to achieve better identification for their profiling.

Pellow, Smith, Beggs and Fernandez-Canque (2005) used an online questionnaire to explore different learning styles of 172 undergraduate students. This questionnaire they used for learner profiling included 44 questions pertaining to 11 categories, those are Academic Expectations, Subject Interest, Understanding Ability, Exam Nervousness, Mathematical Ability, Visual Learning Dimension, Verbal Learning Dimension, Kinesthetic Learning Dimension, Global / Sequential Dimension, Academic Risk, and Personal Risk.

Categories 1 – 9 were graded based on ‘strongly disagree, disagree, unsure, agree and strongly agree’ responses, category 10 requiring ‘yes/no’ responses while 11th category offered a choice from a range of values. After completing the questionnaire, participants received immediate feedback on their own learner profile, including current learning style. The authors also indicated that learners can have more than one learning style, and learning styles may overlap.

May, Taylor, Peat, Barko and Quinnell (2006) aimed at learner profiling of 285 first year undergraduate students by assessing their approaches to study and conceptions of biology. For their study, the researchers used cluster analysis on a data based on some three surveys (Conceptions of Biology Questionnaire, Study Process Questionnaire, and Unit Evaluation Questionnaire). As a result, the students were clustered in groups of two ‘positive’ (deep achievers and enthusiastic achievers) and two ‘negative’ (surface strategists and neutral) levels. Based on the results, the authors also aimed to identify the characteristics of students of four groups.

After six years, same research group (Quinnell, May, & Peat, 2012) published another report entitled “Conceptions of biology and approaches to learning of first year biology students: Introducing a technique for tracking changes in learner profiles over time”. This article was based on the same data as the previous study as described in the previous paragraph. By comparing the survey answers collected at the beginning and end of the semester, this time researchers aimed to investigate if students were able to maintain their incoming approach to study and conception of biology, and whether certain learner profiles were more constant than others. They found that around half of the students remained at the same learner profile. In addition, students with disengaged approach (those have given the most negative answers to survey questions) stood at and did not try hard enough to climb over the low profile they belonged to.

In summary, the literature pertaining to learner profiling is very limited and most of the studies relate to technically improving e-learning data structures. Rest of the literature relating educational research uses quantitative methodology with large sample sizes and statistical cluster analysis based mostly on survey data. Previous literature gives some insight on the need of determining certain characteristics of each learner profile those prevails across the learners within this group.

2.7. Restudying and Related Literature

The literature on restudying the same material and retesting based on the same content provides some evidence for enhanced learning and retention after restudy or retest. For his research, R. E. Mayer (1983) selected study content from scientific passages related to radar and Ohm's Law. His subjects listened to the study materials either once, twice or three times. One finding indicated that overall amount of knowledge recalled was increased with the number of presentations of the study material. This article is specifically important due to its introduction of qualitative and quantitative effects of repetition on learning as a framework. According to this framework, there are two general accounts of repetition. He proposes the quantitative hypothesis stating that repetition provides more information stored in the memory of a learner, in other words, it increases "how much is learned", therefore is most helpful, for "rote strategy" where the learner focuses on memorizing numerical facts or formulas themselves, not the relations or logic behind them. The qualitative hypothesis on the other hand states that repetition helps learners to develop more sophisticated strategies to gather the conceptual understanding of the material presented, in other words, repetition increases "what is learned", therefore is most helpful, for "assimilative encoding strategy", where the learner refocuses attention to understand relations between the key concepts, organize, reword and acquire them as a whole. R. E. Mayer (1983) concluded that recall of conceptual principles and related information was increased by repetition, while formal parts of the content, such as equations, did not (Arguably, mathematics avoidance of the subjects might be the phenomenon responsible for this result. Due to the lack of more detailed empirical evidence, such as eye-tracking data, this hypothesis remains unanswered). Also by repetition, problem solving performance was increased, while verbatim recognition declined. Results also indicated that subjects tend to use different qualitative reading strategies between first and third presentations of the content material, this finding requires more supportive results by future research.

Bromage and R. E. Mayer (1986) aimed to expand the previous research of R. E. Mayer (1983) by investigating the effects of repetition in recall, both measuring the quantity (amount of knowledge gathered in total) and quality (amount of 'important knowledge' gathered) of the learned material. By 'important knowledge', authors refer to the key terms and essential parts of the material that are expected to be acquired at the end of a study period. The subjects were presented the same material three times and were given tests after each study period to measure the effects of repeated study. The results suggested that repeated presentation of the material provided a significant increase in recalling both the quantity of the information acquired, and the quality of the information acquired (such as the number of key concepts could be recalled). The authors also discuss that qualitative characteristics of restudy should be manifested in the use of different strategies. One strategy called 'list processing strategy' is used when the passage is first presented to a subject, in which the reader views the passage as an "undifferentiated list of facts". This strategy is thought to be used when learners study the first and last pieces of the material. Another strategy called 'structural processing strategy' is thought to be used when learners restudy the material. Using a hierarchically structured outline and a conceptual organization of material are some examples of using a structural processing strategy. Authors discuss that using this latter strategy will lead to superior retention of the structurally and conceptually important information, as a result, provides higher quality learning (which eventually provides better transfer of important facts). Because the current study does not intend measuring long-term retention, I will not use this classification of strategies as lenses for my data analysis. The researchers used tests to measure the quantitatively and qualitatively recalled information. However they discuss that their method does not give full insight for the qualitative processes in reading, and more detailed future research is necessary. Their research can be seen as limited due to the lack of tools, such as eye-tracking (which was in its early stages of development when this paper was published), to specifically analyze the learner strategies of study and restudy. These strategies, as discussed above, are strong indicators of the benefits of restudy on learning, both qualitatively and quantitatively.

Overlearning is defined as "deliberate overtraining of a task past a set criterion" (Driskell, Willis, & Copper, 1992). Its effects have been studied since the 1920s. In their meta-analysis of previous research for the effects of overlearning on retention, Driskell et al. (1992) suggest that it produces a significant improvement on retention, and this effect is in parallel with the degree of overlearning. In a recent study, on defining this 'set criterion' and examining the effects of overlearning in more detail, Rohrer and Pashler (2007) examined if retention is affected by the duration of a study session and the distribution of the time across multiple study sessions. According to their findings, a single session should be long enough for mastery learning to happen, and an immediate restudy session is not effective for retention. They discuss that overlearning increases performance in short term, however its benefit fades away over time. Their results suggest that instead of non-stop overlearning of the same content, separating two sessions by an intersession interval is a more effective strategy for both short-term and long-term retention, that is called the 'spacing effect'. The authors also discuss the effects of overlearning and spacing, on mathematics learning. They gave a permutation task to different groups of subjects, one group was applied overlearning while other was applied spacing method. The overlearning effect on retention diminished after one week and spacers outscored overlearners in the long term. My data will not measure the long term effects of overlearning or repeated retesting/restudying, therefore investigating long term retention is beyond the scope of my thesis. However these results are significant in terms of showing the importance of spacing when designing experiments those involve restudy sessions for observing benefits of restudying.

In their study, Barnett and Seefeldt (1989) investigated the effect of rereading the same material on recalling its content. The researchers examined two groups of subjects. The first group read the material once, while the second group was asked to reread the same material after the first trial. The subjects were also grouped with respect to their reading abilities. The results indicated that recall of the information and transfer of the concepts were superior for high skilled subjects who read the material twice, whereas rereading indicated no positive effect for low skilled subjects. One factor improving recall is discussed as reduced anxiety as a result of restudy, although there is no empirical evidence supporting this hypothesis in the paper. Therefore their research findings can be improved with future research using some more advanced empirical methods, such as analyzing participants' heart rates and respiration rates; because these methods are strong indicators for the level of stress (Kelly, 1980; Dew et al., 1984). The authors interpret the results also based on the processing strategies introduced in R. E. Mayer (1983). According to the results, poor readers benefit from rereading only quantitatively, while good readers benefit both quantitatively and qualitatively. The researchers also presume that good readers focus more on the most important information, organize and integrate these information into cognitive structures, while low skilled readers put less effort on the key information, and spend less time in rereading in general. Similar to the previous paper, due to the lack of supporting empirical evidence to this claim (such as eye-tracking data), future research is needed to clarify what strategies readers use while studying and restudying material.

For his research, Butler (2010) conducted four experiments to find out the effect of repeated studying and testing on retention and transfer of concepts. In the experiments, the subjects studied prose passages, then either restudied the same content or were given tests based on the same material. They were given final tests before and after a restudy/retest period to measure if there were any improvements with their retention. According to the results, both restudying the same material and retesting on the same material produced superior retention, and both methods were effective for the transfer of concepts while retesting was more effective for the transfer of concepts compared to restudying. Note that the current research will cover the transfer of learning over shorter periods of time and studying retention over longer periods of time is beyond its scope.

While all studies above focus on restudy, retest and recall in general, Kornell, Hays and Bjork (2009) specifically investigated the effect of unsuccessful retrieval attempts on future learning. They applied a series of experiments investigating the successful and unsuccessful attempts in answering a series of fictional and nonfictional general-knowledge questions. For some experiments the questions were presented with answers, while for the rest of the experiments the questions were presented without answers. For measuring the retention in testing, some test questions included some cues and feedback relating to each question, while some others had no cues or feedback. The researchers eliminated the successful attempts in their analysis aiming to measure only the effect of making unsuccessful attempts in learning. According to the results, unsuccessful performance in tests was evidenced to provide better retention and learning.

A study aiming at investigating cognitive mechanisms underlying the positive effects of restudying and retesting on retention was conducted by Pyc and Rawson (2012). According to their ‘mediator shift’ hypothesis, a test-restudy practice is beneficial for retention because throughout the learning process, mainly due to retrieval failures, learners revaluate the effectiveness of mediators they use, and shift from less effective mediators to more effective ones. Their results substantiate that this hypothesis indeed takes effect in practice, and its impact is more visible when test-restudy is applied together versus only restudy, and within retrieval-failure trails versus retrieval-success trials.

Although there are a few broad overviews, e.g. Byers and Erlwanger (1985), there is no publication that involves empirical research focusing on restudying and retesting effects in a mathematical text learning and retention. This gap in the literature provides a good opportunity for future research in this area. The current study aims at filling this gap by investigating the effects of repeated testing/studying on the transfer of learning over shorter periods of time, and generating individual cases of learner profiles, but not investigating effects on long term memory and retention.

Another gap in the literature is the lack of empirical research for restudy-retest effects on anxiety levels and stress. Although they did not use any physiological methods, such as the EKG or respiration analysis for their research, in their study, Tse and Pu (2012) examined the effects of restudying and retesting on subjects with different levels of working memory capacity (WMC) and trait test anxiety (TA). TA is defined as "behavioral and physiological responses that occur in individuals who are concerned about negative outcomes in evaluative situations". WMC is defined as "the ability to maintain or process task-relevant information and inhibit task-irrelevant information simultaneously". Their definition of WMC in the paper is somewhat similar to R. E. Mayer's (1983) qualitative retention. Participants were provided with Swahili–English word pairs. Half of subjects were exposed to repeated studying while the other half to repeated testing, and all of them were given a delayed cued-recall test for all pairs, as well as other tests to measure their WMC and TA. According to the results, subjects with lower WMC and higher TA made more errors in recall. Retesting effect had no correlation with TA for higher WMC participants and its effect was negatively correlated with TA for lower WMC participants. The current research will not target measuring TA or WMC but will involve reflections on anxiety in a more general sense with a qualitative approach.

2.8. Self-Reporting Methods and Related Literature

Self-report is defined as 'a report by the self on the self' or 'any report by the self' (J. D. Mayer, 2004). Self-report data might be acquired using different methods, such as questionnaires or interviews that provide insight into a person's feelings, emotions, beliefs, or opinions about the self. This study does not involve qualitative self-reports during the experiment (i.e. think-aloud or interview). Such methods are considered to be potentially distracting while more importance was given understanding participants' cognitive and affective processes manifested naturally while they study the material as they would do alone. Talk-aloud methods during experiments could also cause too many artifacts in the physiological data, thus affecting the reliability of such data negatively. Future studies with a different experimental design could benefit from the advantages of such methods. Below literature relating questionnaires and Judgments of Learnings self-reporting techniques will be reviewed.

2.8.1. Questionnaires and Related Literature

Below, the literature on the questionnaires used in the current study will be reviewed.

The Motivated Strategies for Learning Questionnaire (MSLQ) (Pintrich, Smith, García & McKeachie, 1991; 1993) was developed to measure the motivation and cognition of learners (Duncan & McKeachie, 2005). In their review of the literature relating to the development of the MSLQ, Duncan & McKeachie (2005) introduced the MSLQ as a reliable and useful tool to assess motivation and cognitive strategies of learners. They discuss that the MSLQ was developed with a social-cognitive view of motivation where the learner represents an active processor of information, whose beliefs and cognitions are mediated motivationally by the context, while his/her learning strategies can be self-developed. They discuss that learners can use these strategies differently depending on the nature of the task (such as multiple choice or essay exams). According to this article, validity and reliability of the MSLQ was tested and proven reliable in numerous studies across the world in different languages.

The authors describe the MSLQ having 81 items scored on a 7-point scale from 1 (not at all true of me) to 7 (very true of me). The score of each scale is calculated by taking the mean of all item scores within that scale. The items aim at scoring different motivation and learning strategies scales. The motivation section (used in the current study) consists of 31 items in six subscales where learners self-evaluate their goals in a context, beliefs for this context, skills to succeed in it, and their test anxiety in the context. This section is based on three motivational constructs; expectancy, value, and affect. The expectancy scales aim at measuring learners' self-efficacy. The value scales are based on achievement-goal theory. Three subscales measure value beliefs relating: intrinsic goal orientation (focus on mastery in learning content), extrinsic goal orientation (focus on performance, grades and approval from others), and task value beliefs (judgments on the importance and usefulness of the content). The third component in the MSLQ motivation scales is affect that aims at measuring test-anxiety.

The Epistemic Belief Inventory (EBI) includes 28 items and participants rate their epistemological beliefs from 1 (strongly disagree) to 7 (very strongly agree). Similar to the MSLQ, the score of each scale is calculated by taking the mean of all item scores within that scale. Schraw, Bendixen and Dunkle (2002) define these scales as certain knowledge, simple knowledge, quick learning, omniscient authority, and innate knowledge. In this book chapter, the authors also describe a study in which they tested the validity and reliability of the EBI with 160 undergraduate students. They used the 28-items version of the EBI for their research and compared it with Schommer's (1990) Epistemological Questionnaire (EQ) which has been widely used in the literature. According to the results, the EBI has been proven to have superior predictive validity and test-retest reliability.

The Metacognitive Awareness Inventory (MAI) (Schraw & Dennison, 1994) contains 52-items where participants rate their answers on a 7-point scale from 1 (not true at all of you) to 7 (true of you). The MAI measures two metacognitive variables: knowledge of cognition (17 items), and regulation of cognition (35 items). Knowledge of cognition have three subcategories: declarative knowledge (that is the knowledge about learner's own skills, learning abilities and strategies), procedural knowledge (that is the knowledge about how to implement these strategies), and conditional knowledge (that is the knowledge about when and why to use these strategies). Regulation of cognition contains the subcategories of planning, information management strategies, comprehension monitoring, debugging strategies, and evaluation. Schraw and Dennison (1994) discuss previous literature evidences that learners with higher metacognitive awareness are more strategic in the process of learning, are better problem solvers, and perform better than metacognitively less aware learners perform. The authors designed and tested the MAI.

The results suggested that the MAI is a highly valid and reliable technique for assessing the metacognitive awareness of learners, and provides valuable predictive information about learners' subsequent cognitive performance.

The Math Anxiety Rating Scale Revised (MARS-R) was introduced by Hopko in 2003. It involves items where individuals rate their level of math anxiety in numeracy situations and their attitude towards mathematics in general. The MARS was originally developed by Richardson and Suinn (1972) to spot and provide solutions to math anxiety. In this paper, they also included results of a research study in which they tested validity and reliability of the MARS with 397 college students, and the scale was evidenced as a valid and reliable tool to measure math anxiety. This first version consisted of 98-items in total, and used a 5-point scale in which participants rate their math anxiety from 1 ("not at all" anxious) to 5 ("very much" anxious), while sum of all item scores reflect the total score of the MARS (higher score reflects higher math anxiety). In their review of the MARS, Rounds and Hendel (1980) explain the MARS consisting two factors (as cited in Yucedag-Ozcan & Brewer, 2011). The first factor, Mathematics Anxiety Scale aimed to investigate math anxiety manifested in learning situations such as studying or test taking, while the second factor, Numerical Anxiety, focused on math anxiety manifested in daily life situations such as daily computations. Ten years after its first introduction, Plake and Parker (1982) introduced a revised version of the MARS with only 24-items while evidencing high validity and reliability of this revised version. They also evidenced this improved version was very highly correlated with original the MARS in terms of measurement of math anxiety. They also renamed the two factors Mathematics Learning Anxiety and Mathematical Evaluation Anxiety, respectively. Hopko (2003) reassessed this revised version and aimed to improve it while keeping its validity and reliability high. Using a large sample scale study (n=815), he tested and successfully evidenced that a 12-items version of the MARS can be safely used to measure mathematics anxiety. He also suggested that more proper naming of the MARS factors should be the Learning Mathematics Anxiety (8-items; item numbers 1, 2, 4, 5, 6, 8, 9, and 12) and the Mathematics Testing Anxiety (4-items; items numbers 3, 7, 10 and 11). Just like the original version, also in this version participants rate their math anxiety on a 5-point scale from 1 (not at all) to 5 (very much).

2.8.2. Validity of Self-Reporting Literature

Methodologically, self-report data can be acquired quantitatively, where results are generalized based on large sample sizes, or qualitatively, where the researcher tries focusing more on each individual’s experiences and attitudes in the self-reporting process. Ensuring the high validity is an important part of research that relies on self-report data. For quantitative studies validity issues are generally dealt with by improving sample size and some other statistical methods such as outlier elimination. Validity issues in qualitative studies are generally problematic when interviews are involved and when self-report data are collected in a classroom environment. This type of context requires researchers to consider ensuring cognitive validity (Karabenick et al., 2007; Koskey, Karabenick, Woolley, Bonney, & Dever, 2010). This current study does not depend on interviews and was not collected in a classroom environment. Rather it was collected in a fully controlled computer environment where participants’ behavioral data (such as audio-visual recordings) and physiological data (such as eye-tracking) were acquired simultaneously while they study the content, and answer questions in a most natural way as they would normally do when they are alone. This experimental design is expected to help overcome possible validity issues mentioned above.

Perry and Winne (2006) discussed that there are two main issues threatening validity in self-reporting: context and calibration. According to them, the context of the self-reporting should be established carefully in the experimental design of the study to ensure that the participant is self-reporting upon what (s)he is supposed to be thinking, and the self-reporting context should be reflecting and compatible with the present experience of the content. For example, participants should be asked to self-report what they just experience or they are about to experience. Secondly, learners might not self-report what they study accurately, because usually they cannot calibrate their thoughts and actions. Therefore, they might fail to reflect qualities of their past actions (e.g., difficulty or usefulness of a task) due to lack of attention. Both of these issues are related to memory and recall of information. This study was designed to take both of these validation issues of self-report data into consideration by acquiring it in the same context where the participants were exposed to the related content (TDT) all within approximately one hour, thus memory/recall of the information is not considered as a validity issue here.

In summary, all of the self-reporting tools used in this study are proven as valid methods of measurement in the literature. The data are collected in a computer environment where data entry is all computer based and simultaneously recorded. Participants also self-reported their learning of concepts in a computer environment using JOLs as it will be detailed below. For the study/restudy procedures participants were provided with free navigation among pages and flexible study times to help improve ecological validity.

2.8.3. Judgments of Learning (JOLs) and gStudy

Another type of self-reporting that is used in the current study is JOLs where participants rate their learning of some concepts or specific sections within the study material (Nelson, Dunlosky, Graf, & Narens, 1994). In this study, participants self-reported JOLs in a computer environment using a program called gStudy. Developed by Perry and Winne in 2006, gStudy is a computer-based environment where learners can study material and self-report their learning almost simultaneously as they are studying. Important parts of the study material (TDT) are classified under three cognitive levels: calculation, understanding, and reasoning. These parts were identifiable to the participants in the study text as shaded areas where, by right clicking, they could rate their understanding of these parts of the content as 'very well', 'well', or 'not well' understood (See Figure 2.1). It is important to note that self-reporting JOLs period was separated from the actual study period to ensure that participants are not interrupted with the procedure of self-reporting while they study the material, so that they can study with no interruption as they would do in a natural context. This approach also gives the opportunity to reliably analyze both study and self-reporting JOLs periods of the observations separately.

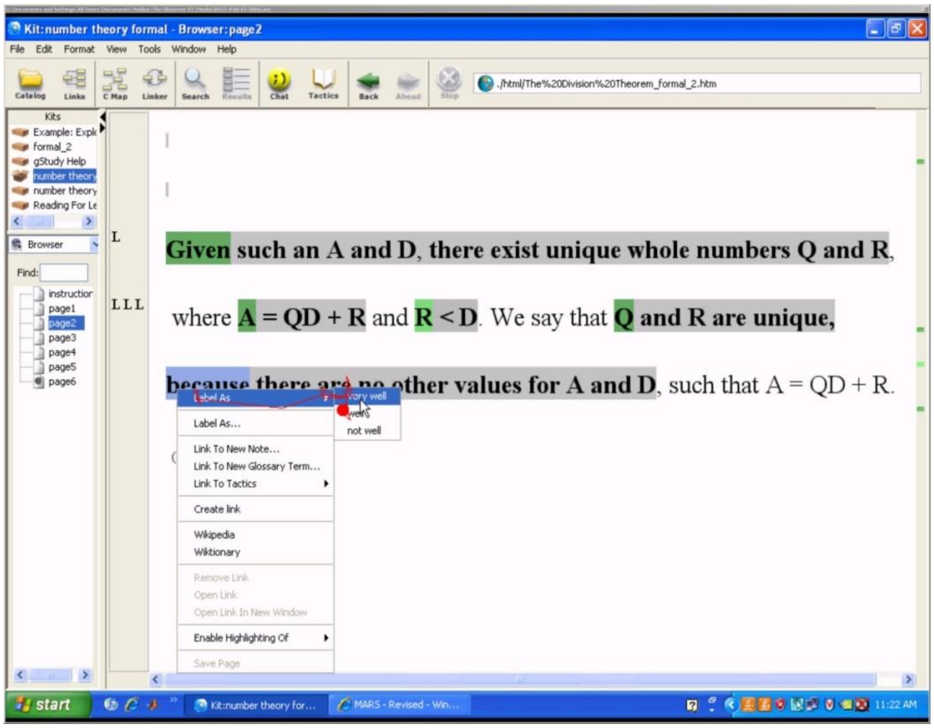


Figure 2.1. Screen capture of study material with participant indicating a JOL on gStudy

2.9. Summary and Research Questions

The review above indicates a growing need for constructing a framework that benefits from multiple theories and methodologies for learner profiling with a qualitative dominant approach to gain better understanding of the learners as individuals with different cognitive and affective profiles. From one angle there is limited research has been done in learner profiling, however this research is mainly oriented towards developing better e-learning systems with a quantitative approach rather than having a more educational driven approach for studying learners. On the other side while qualitative research literature on studying learners provides promising results, does not benefit enough from the recent methodological advances those can be adapted and effectively used to augment our understanding of the learners to develop better educational practices both on a smaller scale in classrooms and on a larger scale for developing better curriculums and policies. The review above provides a summary from previous literature how a successful learner profiling research can be generated and structured with the support of theoretical approaches such as motivational constructs and methodological approaches from Educational Neuroscience and psychophysiology. In that respect this study defines a learner profiling framework that will be discussed in the next chapter, aiming to benefit from these theories and methods to address the following research questions:

1. How beneficial is the calculation/understanding/reasoning (CUR) framework for classifying learners' cognitive orientations and corresponding mathematical content?
2. How beneficial is restudying same material for learners cognitively or affectively?
3. How beneficial is *learner profiling* for determining factors associated with learners' motivational orientations?
4. How beneficial is *learner profiling* for determining kinds of behaviors associated with different types of learners?
5. How beneficial is eye tracking technology for determining connections built by learners among different concepts/sections within the study material?
6. How beneficial is the learner profiling framework for testing the validity of self-report data?

Chapter 3.

Methodology

This chapter will detail the learner profiling methodology starting with the introduction of its theoretical framework that is embodied cognition, and its conceptual framework that involves the 2 x 2 achievement goal framework (Elliot & McGregor, 2001) with regards to motivational constructs, and the Calculation-Understanding-Reasoning (CUR) framework with regards to the classification of the mathematical cognition and corresponding content. After that, the chapter will detail the experimental design, parts of an experiment as well as the methods and tools used to collect, integrate and analyze the data. Lastly, sections of the researcher's screen that were used to generate the figures included in the thesis will be explained to provide more clear understanding of the learner profiling that will be detailed in the following chapter.

3.1. Theoretical Framework

Embodied cognition and learning is the theoretical component of the methodology. Embodiment in mathematics education research has been widely acknowledged, but can also viewed in a variety of different ways (Campbell, 2010). Lakoff and Núñez (2000) introduce their view of embodiment from a metaphorical point of view. According to their view, cognition and learning are grounded and based upon metaphors pertaining to bodily actions. This view provided an important insight into understanding embodied actions of learners and consequently was commonly used as reference by many mathematics education scholars and researchers especially with regards to the uses of language and gestures in mathematics education research (e.g., de Freitas & Sinclair, 2012; Gerofsky, 2010; Radford, 2003).

A more radical view of embodied cognition is used as a theoretical framework, one that provides both a justification for educational neuroscience and the current study. Explaining the role of this radical view of embodied cognition in the development of educational neuroscience methodology, Campbell (2010) states that "capturing embodied manifestations of learners' cognitive and affective processes and states can provide rich and important insights into learners' experiences and behavior and afford exciting new venues for research in mathematics education" (p. 322). In alignment with its introduction in Varela, Thompson and Rosch (1991), this view presupposes that human cognition is embodied and every subjective experience is enacted in some objective and observable embodied behavior (Campbell, 2010). Campbell (2010) also points out that these cognitive and affective processes are manifested through overt and covert behaviors that can be revealed using the behavioral and physiological methods, these were used in this study by taking a mixed research methods approach. The theoretical background behind the mixed methods research methodology will be summarized below by using the terms of mixed methods literature as they apply to the current study.

It is important to state that holding a radical view of embodied cognition, affect, emotion, cognition and learning and the associated physiology, although appearing conceptually distinct, are ontologically inseparable, therefore they were often addressed together or interchangeably throughout my thesis. Hence, in terms of this embodied framework subjective experiences of affect and associated objectively measureable physiological manifestations thereof, such as heart and respiration rates, are considered as one thing that can be referred to and discussed in two different ways.

Mixed methods studies are defined as studies in which "quantitative and qualitative data collection, data analysis and the mixing of quantitative and qualitative approaches within a single study, with data integrated at some stage" (Creswell, Plano Clark, Gutmann, & Hanson, 2003, as cited in Collins & O'Cathain, 2009). Based on the classifications detailed in Johnson, Onwuegbuzie and Turner (2007), this study should be considered as qualitative dominant (i.e. QUAL-quan) mixed methods research. According to Denzin's (1978) classification for triangulation in mixed methods research, this study uses a *methodological triangulation* approach where findings from both qualitative and quantitative methods (i.e. *between-methods triangulation*) were used in combination to interpret the learning experiences, cognitive and affective states and motivational orientations of learners. The details of these methods and triangulation of these methods will be detailed in the methods section. The results from the mixed methods triangulation of the data were found to be *complementary* to each other meaning whether qualitative or quantitative, data from different sources support each other and harmonious when combined. Therefore the methods were successfully *supplementing* each other from a mixed methods research point of view (Erzberger & Kelle, 2003). As noted above, this study also uses a theory of embodied cognition that informs the methodology and the observational methods. Conceptual framework consists of 2 x 2 achievement goal framework for motivational constructs (Elliot & McGregor, 2001) in addition to the CUR framework which will be detailed below.

Using mixed research methods and benefiting from Campbell's view of embodiment and educational neuroscience methodology, this study is a specialized form of this framework in a sense that its focus on individual experiences of learners in the process of learning, and it is an expandable version in the sense that it is adaptable to different disciplines in addition to education. Therefore this study uses the methods of educational neuroscience to the extent that they are sufficient and efficient for gaining results targeted towards the scope of its objective. Consequently this methodology will be differently named under '*learner profiling'* as a subset of '*mixed methods profiling'* as detailed below.

The literature review section provides some theoretical background for generating learner profiles, however due to the very limited number of studies integrating a variety of contemporary methods in learner profiling research; a newly defined theoretical scope is required that I will call 'mixed methods profiling'. I propose that this framework is adaptable to a variety of research with different subject profiling in different disciplines, such as animal behavior research (animals) or human-computer interaction (users). In this educational research study, the subjects of profiling are learners, therefore learner profiling can be considered as a subset of the mixed methods profiling framework adapted to educational research. This study combines qualitative aspects such as individual learning processes of learners, with some other quantitative aspects such as comparisons of physiological data among participants. It is exploratory, thus has no presuppositions to identify and classify learners into predefined structures. It benefits from previous theories, particularly motivational constructs defined by Elliot and McGregor (2001) in their 2 x 2 achievement goal framework, however does not assume that every learner will solely fit into one of four possible motivational orientations. The mixed methods profiling framework requires in depth analysis of the subjects, this entails complementary use of multiple data acquisition and analysis tools (as it will be detailed below) that will combine qualitative and quantitative components whenever required. In that aspect it benefits from the previous research, such as Campbell's Dynamic Tracking and Educational Neuroscience literature, and attempts to offer an organized compact structure for profiling any individual that will be subject of similar future research studies.

As a theoretical assumption, this study also defines three types of mathematical cognition as a working hypothesis; these are *calculation*, *understanding*, and *reasoning*. I will define this view as the Calculation-Understanding-Reasoning (CUR) framework, which is used for the experimental design for classifying study content, JOLs and test items. According to this framework, mathematical ability is not in a single and unique form. It is very organic and depends on the abilities and experiences of learners. According to this framework mathematical ability, cognition and scriptures have three main components. First one is *calculation* that refers to the ability to do arithmetic efficiently and properly, ability to do mental calculations with whole numbers. The level of this ability is very depended on experience and self-discovered strategies, for example street sellers can sometimes develop better strategies for calculation compared to students who receive formal school mathematics training (Nunes, Carraher & Schliemann, 1993). It is a common experience for many of us with mathematics background that sometimes students can do better in calculation than their teachers, and mathematics professors. However this ability of calculation does not necessarily imply that such students are more knowledgeable in mathematics than their professors because ability to do efficient calculations does not require much advanced mathematical knowledge. Second type of mathematical cognition is *understanding*, which stands for being able to understand definitions and to apply what is understood into problem situations. Understanding component, in other words, pertains to reading, recall and comprehension of information. The third and last type is *reasoning*, which stands for thinking mathematically and logically, ability to conceptualize content involving inferences, and being able to build if/then relations. In other words, it is being able to understand relations among concepts and definitions (those are mostly present in a definition of a theorem), to self-bridge and relate them when necessary, and to apply these bridges to problem situations. As an example, solving '65 : 3' uses a learner's calculation skill, knowing the definition of divisor is an understanding skill, and conceptualizing "Where A, B and C are all positive whole numbers, if A divides B and B divides C, then A divides C" is a reasoning skill. I propose that experiences, abilities, motivations and strategies of learners in experiences with mathematical situations might vary based on the CUR classification of mathematical cognition. (See Table 3.1 for the partitioning of the content material corresponding to this classification.)

In the current study, all components of the data were analyzed and written as separate narratives for each participant. Only relevant data are included into the narratives. The data was analyzed in accordance with CUR and 2 x 2 achievement goal frameworks.

Table 3.1. List of JOLs and corresponding CUR categories

JOL#	CUR Type	JOL Content
P1-JOL1	Understanding	The Division Theorem
P1-JOL2	Understanding	Number Theory
P1-JOL3	Understanding	whole numbers
P1-JOL4	Understanding	Infinite set {0, 1, 2, 3, …}
P1-JOL5	Reasoning	Consider any two numbers, A and D
P1-JOL6	Reasoning	$D \neq 0$
P2-JOL1	Reasoning	Given such A and D, there exist unique whole numbers Q and R
P2-JOL2	Reasoning	$A = QD + R$
P2-JOL3	Reasoning	$R < D$
P2-JOL4	Reasoning	Q and R are unique, because are no other values for A and D
P3-JOL1	Calculation	When $A = 26$ and $D = 4$, the only two numbers that will satisfy this formula are $Q = 6$ and $R = 2$
P3-JOL2	Understanding	A is the dividend
P3-JOL3	Understanding	Q is the quotient
P3-JOL4	Understanding	D is the divisor
P3-JOL5	Understanding	R is the remainder
P4-JOL1	Calculation	If $A = 7$ and $D = 3$, then $Q = 2$ and $R = 1$, since $7 = (2)(3) + 1$
P4-JOL2	Calculation	If $A = 27$ and $D = 4$, then $Q = 6$ and $R = 3$, since $27 = (6)(4) + 3$
P4-JOL3	Reasoning	A and D, where A and D are elements of W, and $D > 0$
P4-JOL4	Reasoning	There exist unique elements, Q and R, of W
P4-JOL5	Reasoning	$A = QD + R$, where $R < D$
P5-JOL1	Understanding	Divisibility Relations
P5-JOL2	Reasoning	Given whole numbers A and D, if there is a Q such that $A = QD$ and $R = 0$
P5-JOL3	Understanding	A is divisible by D
P5-JOL4	Understanding	D divides A
P5-JOL5	Understanding	D is a factor of A
P5-JOL6	Understanding	A is a multiple of D
P5-JOL7	Reasoning	If D is a divisor of A, then D divides A
P5-JOL8	Reasoning	If D divides A, then D is a divisor of A
P6-JOL1	Reasoning	If D divides A, then D is a factor of A
P6-JOL2	Reasoning	If D is a factor of A, then D divides A
P6-JOL3	Reasoning	If D is not equal to A, then D is a proper divisor of A
P6-JOL4	Understanding	Prime numbers have no proper divisors
P6-JOL5	Calculation	When $A = 20$ and $D = 2$, then $2 \mid 20$, because $20 = (10)(2)$
P6-JOL6	Calculation	When $A = 15$ and $D = 5$, then $5 \mid 15$, because $15 = (3)(5)$
P6-JOL7	Calculation	17 is a prime number because it has no divisors aside from 1 and itself

3.2. Methods

3.2.1. Participants

All four participants were post-secondary level students. None of them were exposed to the content material before the experiment. All participants were given arbitrary names to ensure confidentiality. Linda, Betty and Susan are respectively 22, 21 and 22 years old female students with Asian background while John is a 45 years old male graduate student with mixed English-German descent. The data were selected from a larger pool (n≈100) of participants with similar profiles; those are pursuing either an undergraduate or a graduate degree, whom have no direct previous exposure to the study/restudy content material. Some of these experiments were pilot studies while some others lack one or more data components (such as video files or eye-tracking data) thus were excluded from evaluation. The four participants of the current study were chosen out of a subset this larger pool of participants that used the exact same experimental design using non probability sampling based on the quality of their data (i.e. availability of multiple measures and high signal to noise ratio), and their unique and rich learner profiles that demonstrated a variety of individual behaviors, characteristics and strategies specific to each case.

3.2.2. Phases of the Experiment

After they arrive to the lab, participants were asked to fill and sign the consent form (Appendix A). After that, participants were prepared with the physiological sensors attached to their body. After they were wired-up, they were invited to the room with the computer they will use during the experiment as well as the eye-tracking monitor. This room also involved microphones and speakers to allow communication with the researcher as well as multiple cameras for video-recording, a table, and a chair. It was ensured that the participants were comfortable in the room, ready and relaxed to start the experiment. They were provided with relaxation periods between phases. Otherwise they were sitting on the chair and completing all sections in the same position while being recorded.

Each observation started with a set of questionnaires those participants were asked to fill out electronically as listed below respectively. The Demographics Questionnaire (Appendix B) provides background information for all participants, including gender, language, education, age, socioeconomic status. The Number Theory Pre-Questionnaire (Appendix C) includes self-evaluations of the participants on their abilities of calculation, reading, comprehension, mathematical thinking, and their attitude towards studying TDT. Pre- and post-questionnaires were developed by Campbell (2007) to gain deeper understanding of participants' attitude towards mathematics and TDT, along with a Number Theory Test to measure their understanding of TDT. The Number Theory Pre-Questionnaire (that was presented as the last chain of questionnaires before participants started studying the material) includes 7-items in which participants rate their confidence with mental calculations involving 1 or 2 digit numbers, reading/recall and comprehension, thinking/reasoning, and their attitude towards studying TDT. This questionnaire has a 5-point scale from 1 ("not at all" comfortable with) to 5 ("completely" comfortable with).

The Motivated Strategies for Learning Questionnaire (MSLQ) (Appendix D) aims to measure the motivational orientations, cognitional orientations, and strategies of participants. The Epistemic Beliefs Inventory (EBI) (Appendix E) aims to measure the epistemological beliefs of participants. The Metacognitive Awareness Inventory (MAI) (Appendix F) aims to measure participants' knowledge of their cognition and regulation. The Math Anxiety Rating Scale Revised (MARS-R) (Appendix G) includes self-ratings of participants on their level of math anxiety in numeracy situations and on their attitudes towards math.

The Number Theory Post-Questionnaire is the last part of each observation in this study, where participants reflect opinions about their performance in the observation (i.e., how well they attended to the tasks), as well as its content (how challenging and interesting it was), and its design. Having 5-questions in total, this questionnaire was mainly developed to gain better insight into the reliability of the study.

After the surveys the participants were exposed to the study material (Appendix H). They were provided with a maximum of 10 minutes to study content explaining TDT, related concepts and examples. They were free to end the period whenever they wanted to. The material was six pages long. They were given the opportunity to study pages in mixed order and multiple times. Note that participants reviewed the material multiple times in this phase.

Following the study phase, participants were asked to rate their understanding of the material. Key sections of the material were highlighted for participants to judge their learning, thus these parts were named under Judgment of Learning (JOL). The instructions for self-reporting JOLs were first provided (Appendix I). Each page involved 4-8 JOLs (35 in total) those were presenting three types of mathematical cognition (calculation, understanding, and reasoning) (Appendix J). They rated their understanding on three levels; those are *not well*, *well* and *very well*. To find their average ratings of JOLs, those ratings were valued as *0*, *0.5* and *1* respectively.

For the next phase, participants were given a maximum of 10 minutes for restudying the same material. They were free to end the period whenever they wanted to. Just like the study phase, participants reviewed the material multiple times during this period.

After they restudied the material, participants answered a Number Theory Test (Appendix K). The Number Theory Test aims to measure their calculation, reasoning, and understanding abilities of TDT and related concepts (i.e., division, divisor, dividend, and divisibility). This test included 10-multiple choice or True/False questions, each of which (to gain better understanding of the level of reliability of these answers) immediately followed by a 10-point confidence scale where participants self-rate their level of confidence with their last answer. (The test was given after the restudy period for all participants, and additionally prior to the restudy period for Susan and John.)

Lastly they answered the Number Theory Post-Questionnaire (Appendix L) that includes information about participants' opinions and attitudes towards the TDT after they studied it. This questionnaire provides the researcher additional insight into the reliability of the data (by asking the level of attention they paid on the task and how well they think they have learnt the concepts). After they completed all sections, participants were thanked and the experiment was ended. Learner motivation could be discussed as being affected not only by the content material but also from some unknown and unaccounted for external factors unrelated to the experiment, such as a negative experience of a participant had just before the experiment (e.g. having an accident). To be able to spot such effect (if exists), at the beginning and at the end of each experiment, baseline data were collected while no stimulus was presented to the participants. It was presumed that an external cause for discomfort could be monitored behaviorally or physiologically at the baseline points as well as throughout the experiments. Having no evidence for any external cause of discomfort that would affect the motivation, the data from the four participants are considered valid in that regard. Moreover, participants indicated different motivational orientations based on CUR content. This is considered another indication that their motivational orientations are dependent on the content and not on any external events.

3.2.3. Study Material

The study material introduces the division theorem in an informal way, followed by some examples and explanations of related concepts. The main definition within the material is as follows:

The Division Theorem

Let's study some Number Theory. We are only concerned here with whole numbers. The set of whole numbers, W, is defined by the infinite set {0, 1, 2, 3, ...} Consider any two numbers, A and D, where $D \neq 0$. Given such an A and D, there exist unique whole numbers Q and R, where $A = QD + R$ and $R < 0$."

3.2.4. Behavioral Data

Audio-Visual Data

The data include audio-visual recordings of the participants from three different angles. Two of the three cameras are infrared; those are capable of making clear recording in dark rooms. The front camera provides information about participants' facial expressions. Thanks to this video from the front facing camera, when there is no gaze data appearing on the ET output, it can be known whether participant's eyes were closed or the participant was looking elsewhere and not attending to the screen. Having this feature prevents misinterpretation of the ET data. The side and leg infrared cameras provide better insight into the body gestures (such as reactions related to anxiety or task engagement). All audio-visual recordings were done using the Noldus Media Recorder program that simultaneously starts and records all these data. It is also fully integrated with the Noldus Observer software which is the main software package used to record, integrate and qualitatively observe all sessions (Figure 3.2). This software is capable of integrating all components of the data. After doing manual synchronizations, the software can play them all together that providing strong insight into observing and spotting important events, and coding them while observing the sessions.

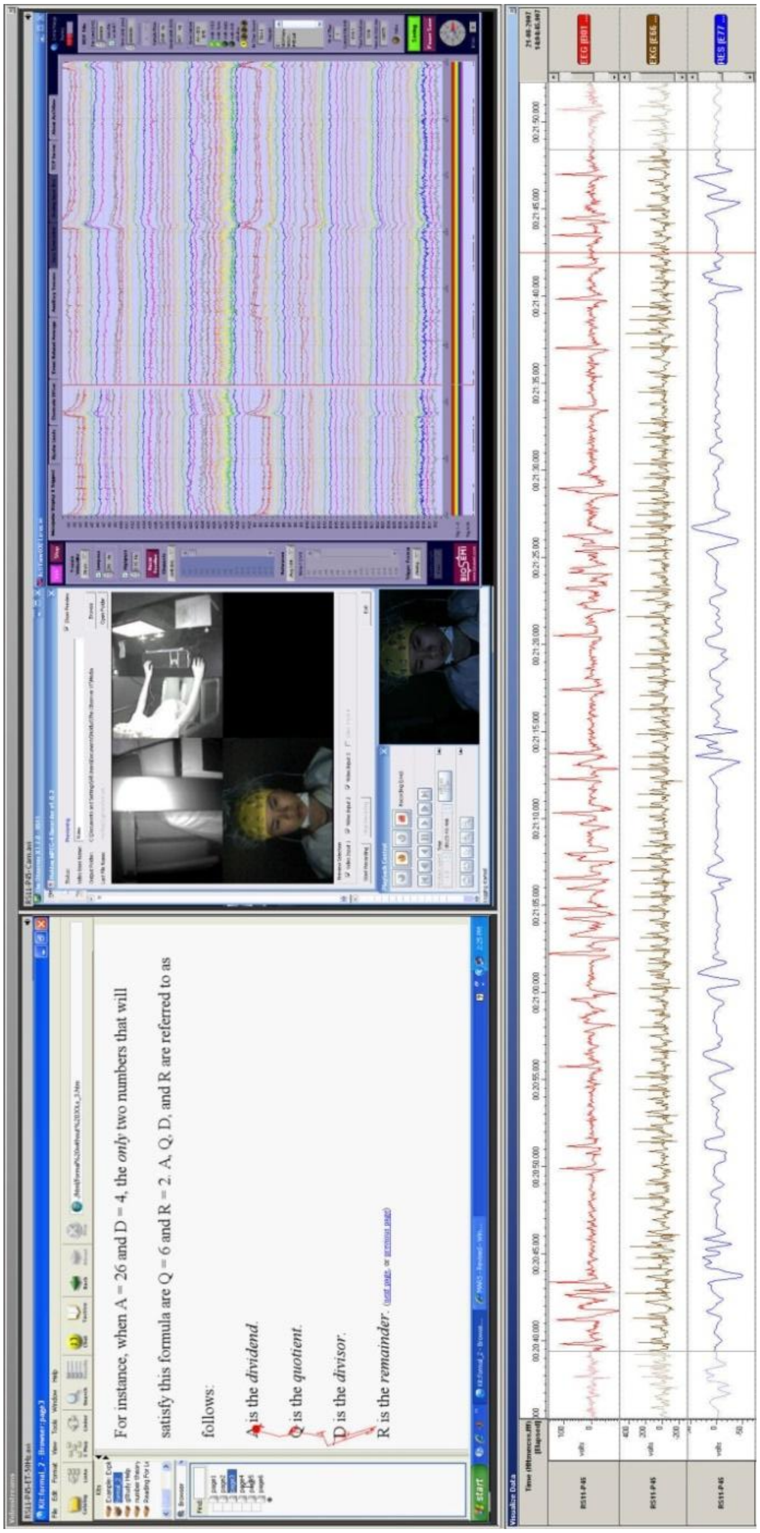


Figure 3.1. The Noldus observer screen

Screen Capture and Eye-Tracking (ET)

The ET data were acquired using the Tobii 1750 eye-tracking monitor. Clearview software was used for ET data collection and analysis. This software is capable of capturing the computer screen used by the participants. Simultaneously, the software records the Cartesian coordinates of the points on the screen that a participant looks at throughout the observation (the gaze points) with a high precision and sampling frequency. After the data are collected, the software integrates the screen recording with these numerical values (coordinates) by converting the data into a qualitatively interpretable visual form (See Figure 3.3 for an illustration).

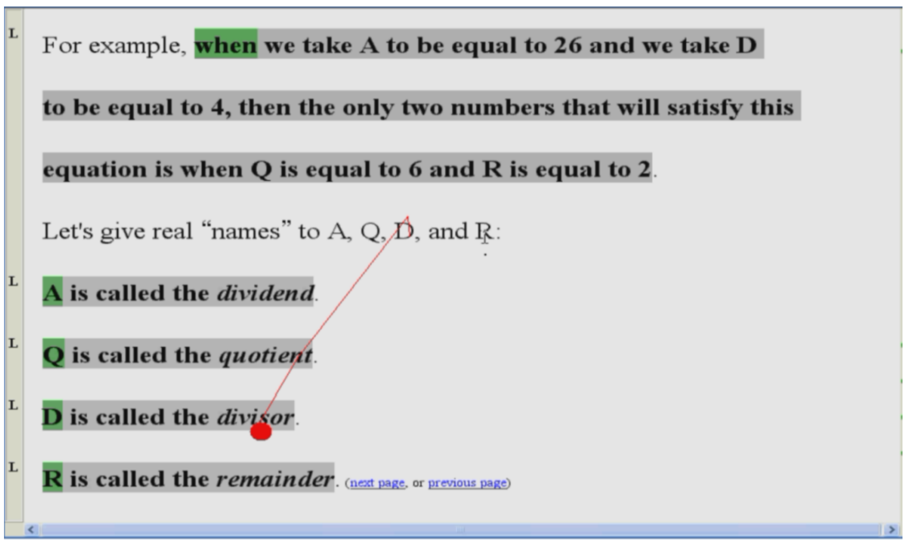


Figure 3.2. Eye-tracking data: Studying TDT

(The participant moves gaze from the letter 'D' to the term 'divisor')

This interpretable visual form illustrates the path that the participant's eye has been following for the last few moments (this time frame is definable by the researcher, for the current study it was fixed at one second) and the point that the eye is focused at that moment. The dot keeps dilating as long as the participant keeps looking at that same point on the screen (fixation), and disappears when the participants close their eyes, blink, or when they are not looking at the screen. These two components of the ET data provide deeper insight into the connections that a participant establishes between concepts while studying the material. In addition, these components of the data inform researcher about the specific parts of the task that a participant pays most attention to.

Another capability of ET technology is to define *areas-of-interest (AOI)* on a task. This means selecting desired parts of the participant screen (e.g. shape, graph, term, sentence, or expression) and letting the software calculate the total times, or number of total fixations on these selected areas.

Additionally, Clearview is capable of producing *heatmaps* of the participant screen, which illustrates the density of eye fixations for a defined time interval. It is also possible to generate *gazeplots*, showing the paths, durations, and sequences of fixations for a desired section of an observation. AOI, heatmap and gaze plot features of the ET data provide deeper insight into what participants mostly attend to, and where they avoid looking (see Figure 3.4 for examples of gaze plot and heat map data).

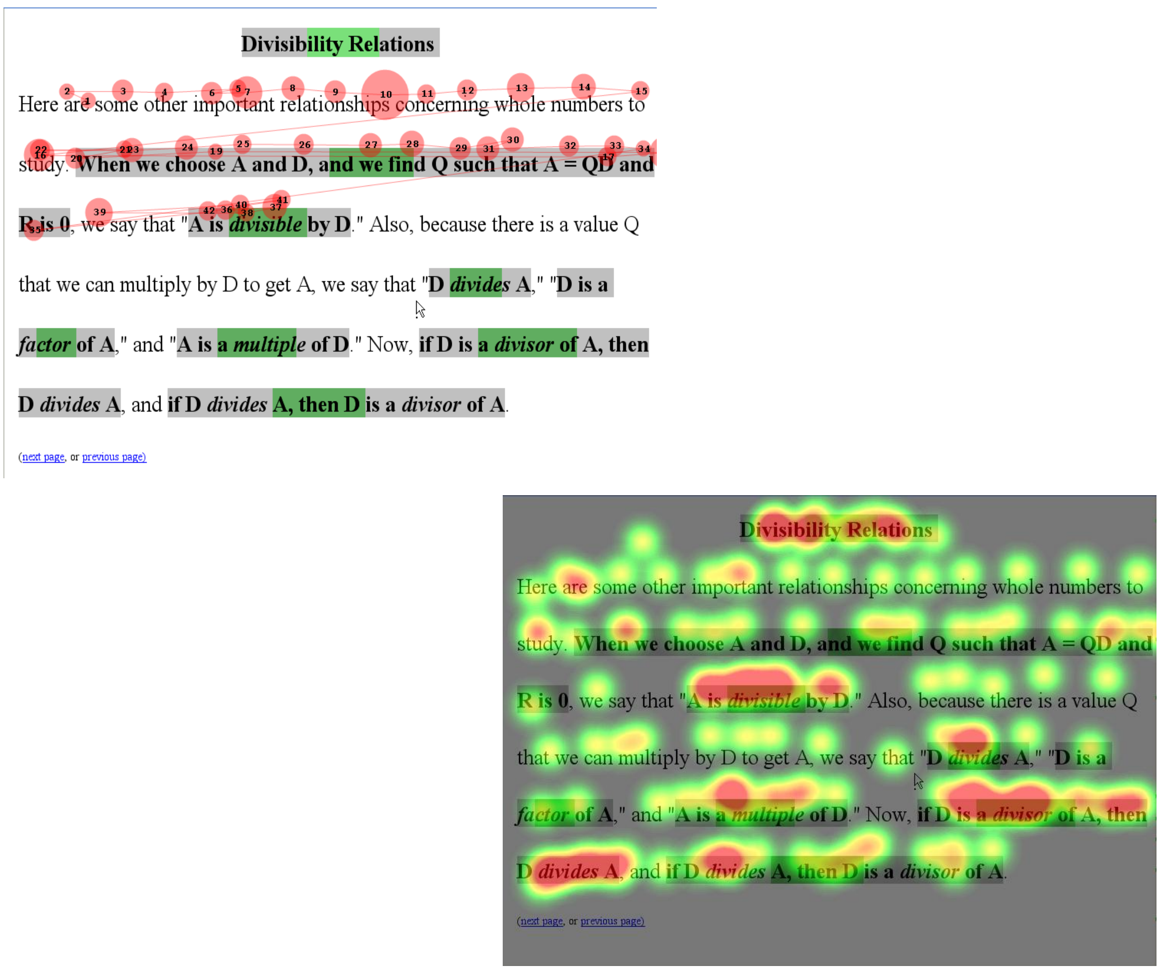


Figure 3.3. Gazeplot and heatmap exports from ET data

(The image on the right indicates that the participant was paying more attention to the verbal explanations with respect to the formulas and variables)

3.2.5. Physiological Data

Physiological data were acquired using Biosemi ActiveTwo hardware and ActiView software. Participants were wired up and prepared on a seat in a booth that is electromagnetically noise-free to ensure highly reliable measurements. The room has microphones, video cameras, an eye tracking monitor, a computer with necessary accessories, a desk, and a comfortable chair in it.

Eye Blinks (B01)

The original data include EEG recordings for all participants. Detailed EEG analysis is beyond the scope of this study, yet, one of the EEG channels, B01 (named under the code of the channel based on 64-channel EEG montage) is included in the analysis, because it is located very close to the eyes, and thus provides important information about the eye-related data, specifically eye blinks and flutters. The importance of these data was explained in the literature review section under subtitle 'Interpreting Psychophysiology Data and Related Literature'.

Heart (EKG)

Heart beats were recorded using participants' pulses. While all four participants in the current study were right handed, the electrodes were placed on their left wrists to acquire better quality signal.

Respiration

The respiration signal was acquired using a respiration belt that is placed around the participant's chest. These sensors are a part of the Biosemi equipment.

3.2.6. Calculation of Rates and Other Pre-analysis Data Processing

After physiology data are collected, it is recorded in .bdf format by the ActiView program that is also compatible with BESA software. As the first step for processing the data, the .bdf files for all participants were opened using BESA, and converted into text files in ASCII format. These text files were processed using MATLAB to extract required channels (eye blink, EKG and respiration channels). These files involving data from the necessary channels were opened in AcqKnowledge software to eliminate unwanted noise, and calculate rates of blink, heart, and respiration in units of beats per minute (bpm).

3.2.7. Steps of Data Integration and Analysis

Qualitative Data

The following components were integrated into the Noldus Observer for qualitative analysis:

1. Screen recording video embedded with eye-tracking data
2. Video file that integrates recordings from 3 cameras and ActiView interface (this video was captured from researcher's computer screen in data acquisition stage using Camtasia software)
3. The visual physiology channels extracted from AcqKnowledge (B01, EKG and Respiration)

After these data were integrated into the Noldus Observer, they were manually time synchronized using reference events present in multiple components (such as an eye blink can be monitored as absence of fixations dot in eye-tracking video, as eyes-closed moment on the front facing camera, and as a positive peak on B01). After data synchronization was complete, the data were played and qualitatively monitored in great detail for significant behavioral and physiological cues providing links underlying cognitive and affective processes. These significant events were transcribed and coded as references with precise times they were manifested (Figure 3.5 and 3.6). They were also exported as Excel documents as detailed under 'Quantitative Data' section below. Note that although all the behavioral data was coded in detail on Observer software for each observation, only significant events were reflected in the results chapter.

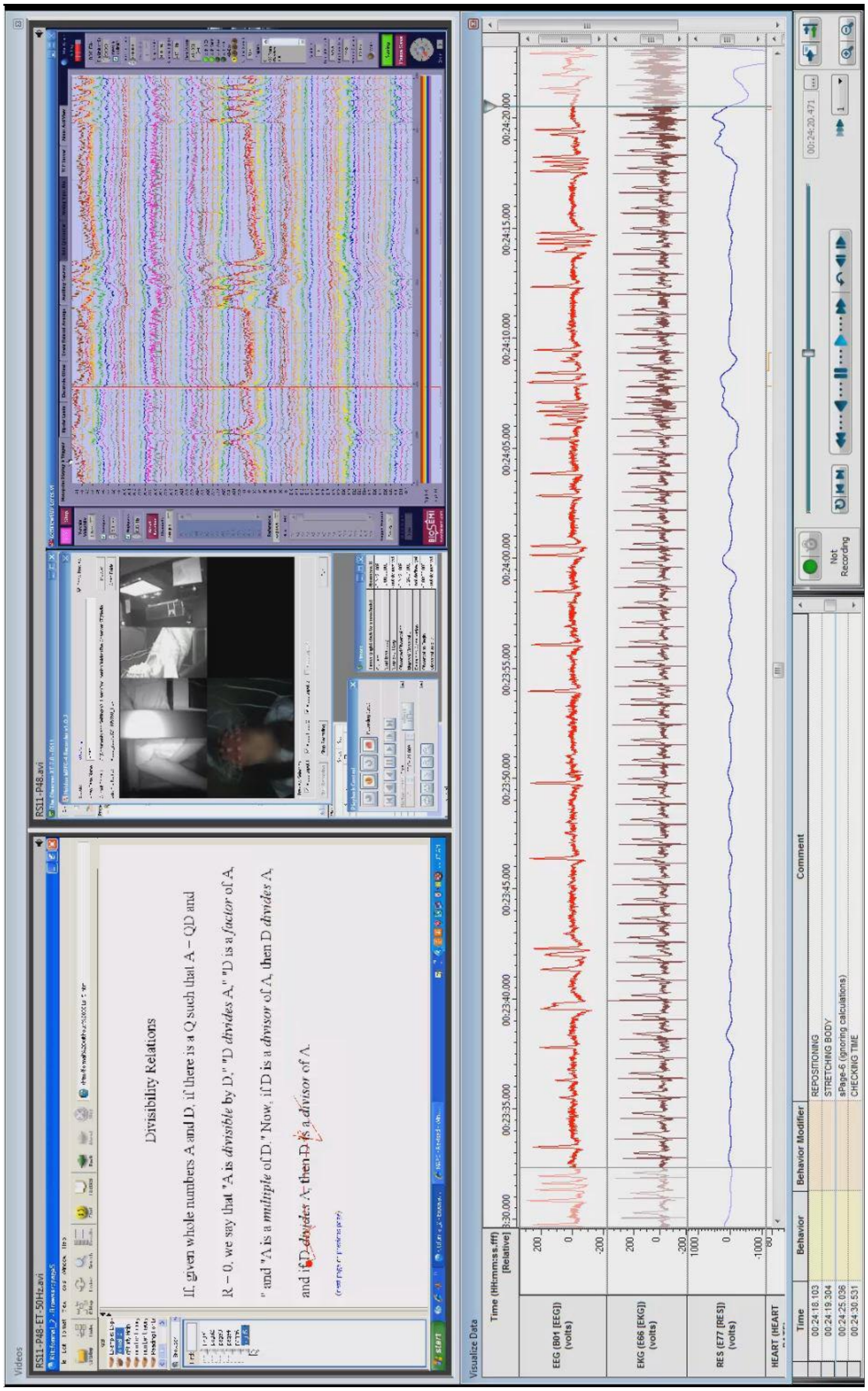


Figure 3.4. The analysis screen with events coded

Time	Behavior	Behavior Modifier	Comment
00:24:18.103			REPOSITIONING
00:24:19.304			STRETCHING BODY
00:24:25.036			sPage-6 (ignoring calculations)
00:24:30.531			CHECKING TIME

Figure 3.5. A section of the qualitative data coding

It is important to note that the Noldus Observer is capable of playing all components of the data simultaneously and at a desired speed (slow motion, fast motion, real time, or frame by frame). When a significant event occurs, the screen was captured using the Snagit program and imported into the results section of this document.

Heatmaps and gazeplots were exported in picture file format using Clearview. They were defined for a time interval where an important event is present. These pictures were than imported into the thesis.

Quantitative Data

After it is processed using AcqKnowledge, the physiology data were imported into Microsoft Office Excel as a spreadsheet. Each row on this document presented a sample of the data where columns are the amplitude (in volts) or rate (in bpm) values of each channel. Timestamps of the significant events and start/end times of different periods within each observation for all participants were coded on the Noldus Observer. These timestamps were integrated with the physiology data in Excel. Using the start and stop times of these periods, average blink, heart and respiration rates were calculated.

As another quantitative component, all survey and JOL answers were imported into Excel. The scores for surveys were rated based on the related literature for calculating scores for each survey and survey scales. JOLs were classified based on Calculation, Understanding and Reasoning. Self-reporting JOL scores were calculated by assigning ‘0’, ‘0.5’ and ‘1’ values for ‘Not Well’, ‘Well’, and ‘Very Well’ reports respectively. True answers within the tests were scored as 1, whereas false answers were scored as 0.

Clearview software was used to calculate precise total eye gaze times of each JOL. These data were then quantitatively exported to Excel. This approach provided an opportunity to precisely calculate total times spent on each page, total times spent on calculation, understanding or reasoning related content separately, and other periods (such as times spent on studying specific pages) throughout the observations. Gaze times, as well as number of fixations per any requested period of the data are present for analysis and for cross comparisons among these periods within a single participant data or across participants. Lastly all quantitative data were plotted in Excel for clearer interpretation.

3.3. Sections of the Data Screen

The data screen (viewed by researcher) combines different components of the data into one simple interface. Figure 3.7 demonstrates these components.

As the first component, the data screen involves the screen recording of the computer used by the participant (left upper corner of Figure 3.7). Eye tracking data is embedded on the screen that shows the eye gaze data of the participant starting from one second preceding the moment of screen capture. The data screen also involves the camera recording of the participant's body from three different angles (at the center of Figure 3.7), those are viewing his legs (to observe possible leg movements due to anxiousness), his face (to observe facial expressions), and the inside of the room from the side. The raw physiology data screen (right upper corner of Figure 3.7) is included to view body response in general (that is appears as fluctuations through all channels). The bottom of the data screen involves three physiology channels chosen and extracted for detailed analysis. First of these channels is the B01 channel of the EEG data. The B01 is the closest located EEG electrode to the eyes, thus, it easily captures eye related artifacts, such as eye blinks and eye fluttering. The sharp peaks on this channel demonstrate eye blinks while frequent peaks with small amplitude indicate eye flutters. The second channel is the EKG channel that demonstrates the heartbeats of participants. The third and last channel at the bottom displays the respiratory response of the participant's body.

Note that participant names in the narratives below are not their real names, and were assigned arbitrarily for confidentiality.

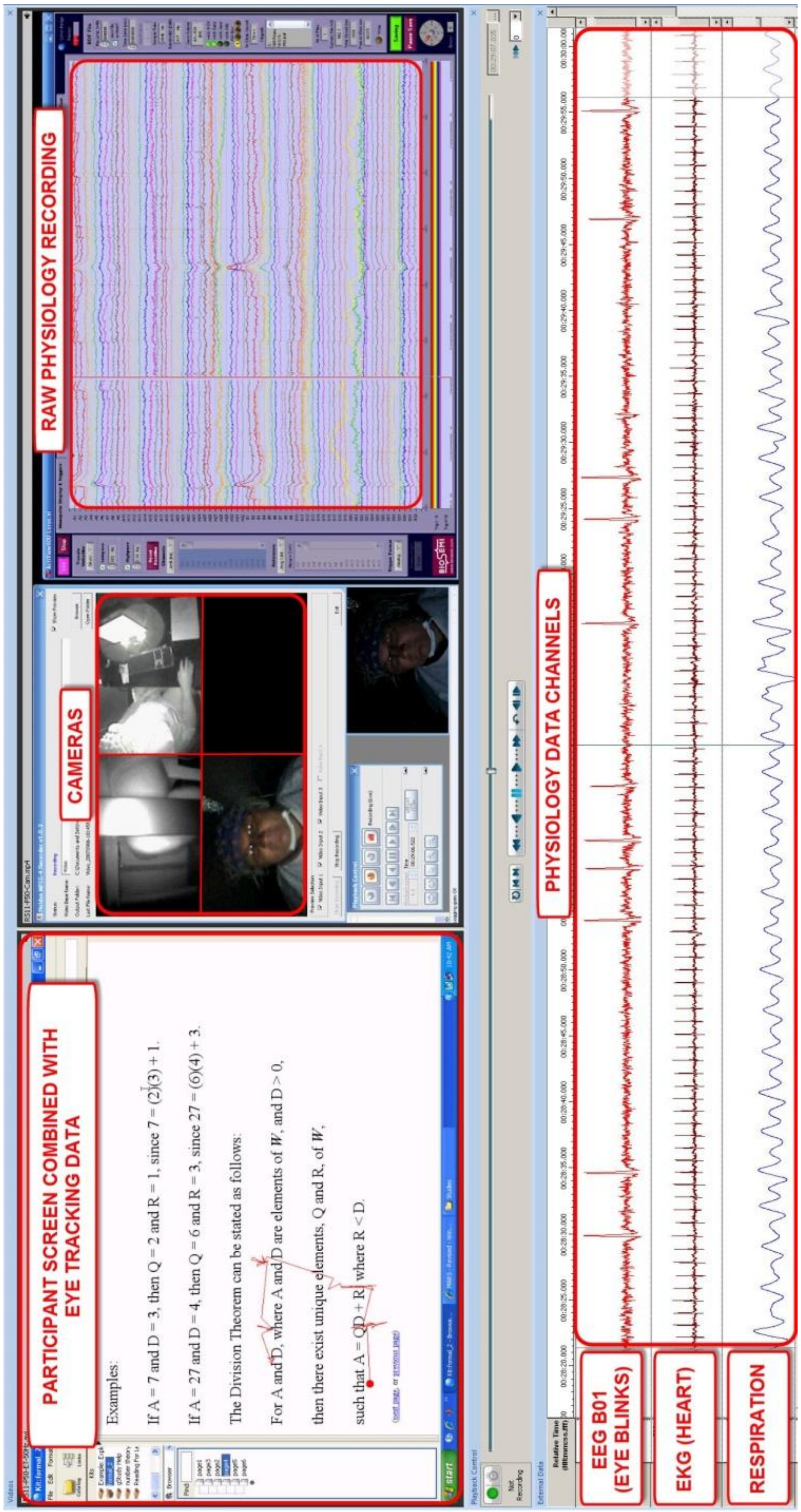


Figure 3.6. Sections of the sample data screen

Chapter 4.

Results

This chapter provides learner profiling (single case analyses) of four participants. Following this section, results chapter will include the learner profiles for each participant, which are extracted from learner profiling and provided as pedagogical guides for each participant so far as the data covered. Lastly the cross-case analyses will be detailed.

4.1. Participant-1 (Linda)

4.1.1. Demographics

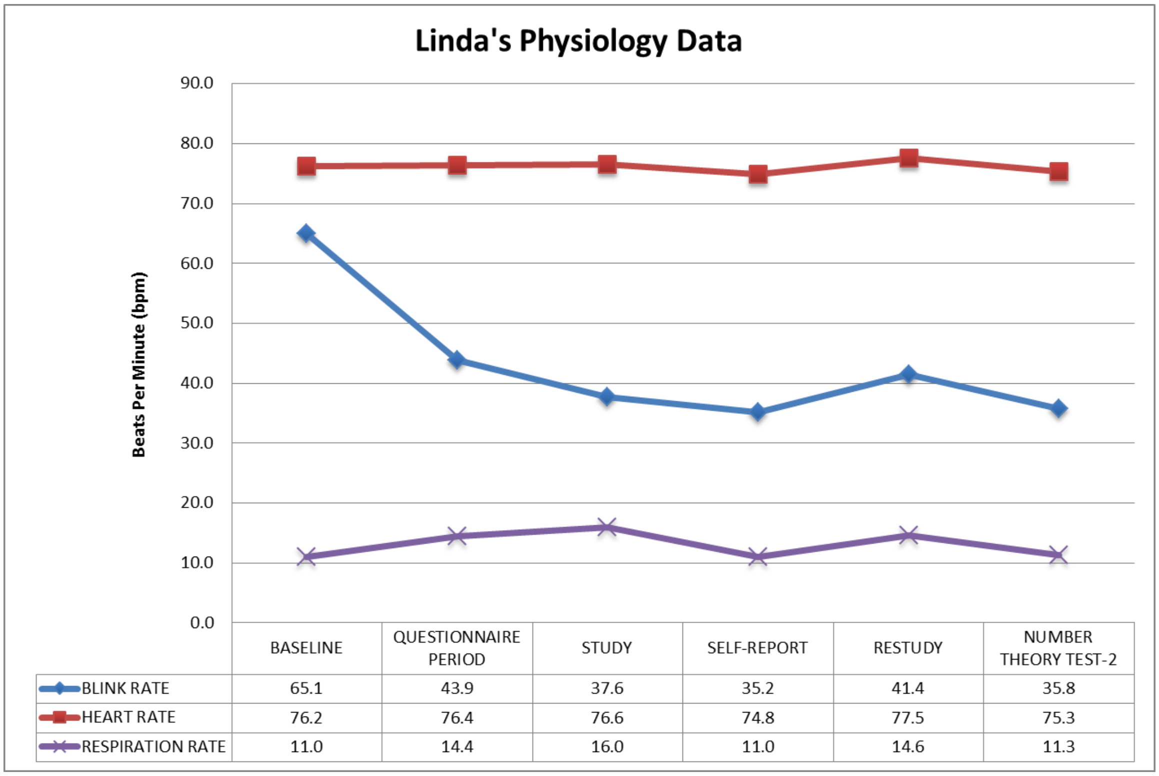
Linda is a 22 years old female undergraduate student in Molecular Biology & Biochemistry. She has Chinese parents and her mother tongue is Mandarin.

4.1.2. Psychological / Physiological State

Linda was observed being calm and relaxed before, during, and after the experiment. She self-reported feeling fine and calm before and after the experiment, whereas self-reporting herself as again fine and calm during the experiment, but then saying “first felt a bit worried about the math part, that I wouldn’t recall all of the information and it will be too hard for me to learn, but after reading the content, I felt fine”. Linda’s experiment took 47 minutes in total including all phases of the experiment.

4.1.3. Pre-Study Period

Linda’s eye blink rate was high during the relaxation period before the experiment commenced as a possible indication of anxiety. However, that effect diminished quickly once she started the experiment (Figure 4.1). Her heart and respiration signatures were consistent throughout the experiment. She was observed being calm and focused while answering the questionnaires. Her EBI scores on Quick Learning (4/7) and Simple Knowledge (6.4/7) scales were highest among all participants. These scores indicate her strategy driven personality while learning, and her belief in the simplicity and fast learnability of knowledge. They also align with the strategies she uses in her studying the material, as detailed below.



	BASELINE	QUESTIONNAIRE PERIOD	STUDY	SELF-REPORT	RESTUDY	NUMBER THEORY TEST-2
BLINK RATE	65.1	43.9	37.6	35.2	41.4	35.8
HEART RATE	76.2	76.4	76.6	74.8	77.5	75.3
RESPIRATION RATE	11.0	14.4	16.0	11.0	14.6	11.3

Figure 4.1. Linda's physiological data of throughout the experiment

4.1.4. Study Period[2]

For the first time through (2:20 in total), Linda skimmed the verbal pages (that involve expressions and verbal representations, such as definitions) sequentially and quickly (15-20 seconds each) by scanning these pages just once from the beginning to the end without focusing on any particular sections. For other pages those involve calculations and numbers, she approached them very intensely spending 30-40 seconds per page (i.e. P3, P6) and tried building connections between them (Figure 4.2).

[2] See Appendix H for the study material.

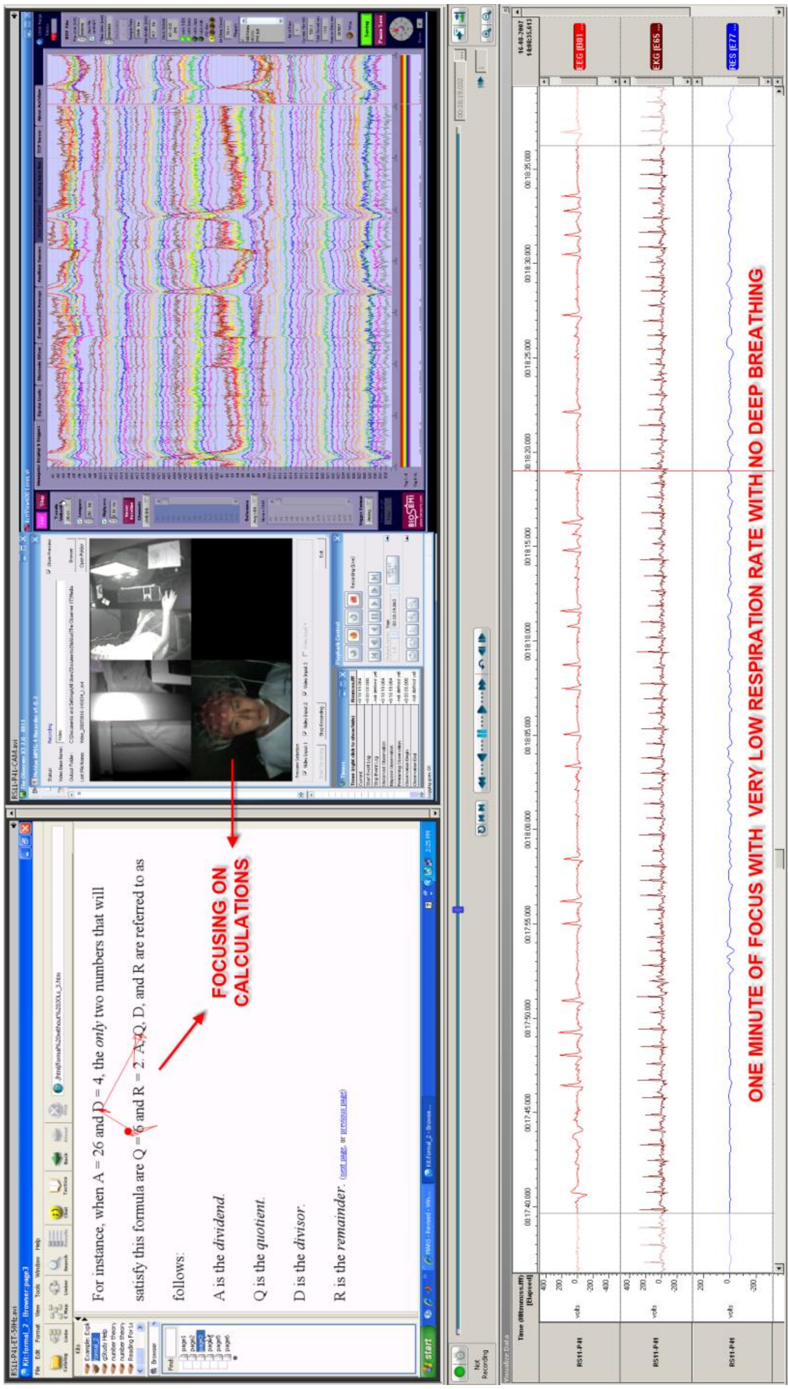


Figure 4.2. Relating calculations and numbers with deep focus

For the second time through (1:30 in total), unlike her first time through, Linda spend most of her time on pages involving understanding and reasoning (such as the second half of P3 and P5). These sections involve definitions of the concepts related to TDT (i.e. divisor, multiple, factor, dividend, quotient, divisor, remainder). For P6 (in contrast to what she did first time studying the page) she only focused on the parts involving reasoning (such as, '*If D divides A, then D is a factor of A*') and skipped the calculation parts much quicker. For the second time through, she reviewed P3 and P5 twice, and P6 once.

After checking the time (that was limited to 10 minutes) by looking at the right bottom section of the screen to see how much was left, Linda started her third time through studying the material with P1. She quickly reviewed each page sequentially (6-10 seconds each) until P5. P5 and the first half of P6 involve divisibility relations, which require understanding and reasoning skills. She spent most of her third time through, just focusing on P5 and P6 (~2 minutes). During this period, she took multiple deep breaths, changed her body position more often than usual, and did more frequent eye blinks (Figure 4.3). She also was observed checking the time very often during this stage (Figure 4.4). These behavioral indicators are interpreted as hardship confronted on these pages, and having time pressure close to the end of study phase.

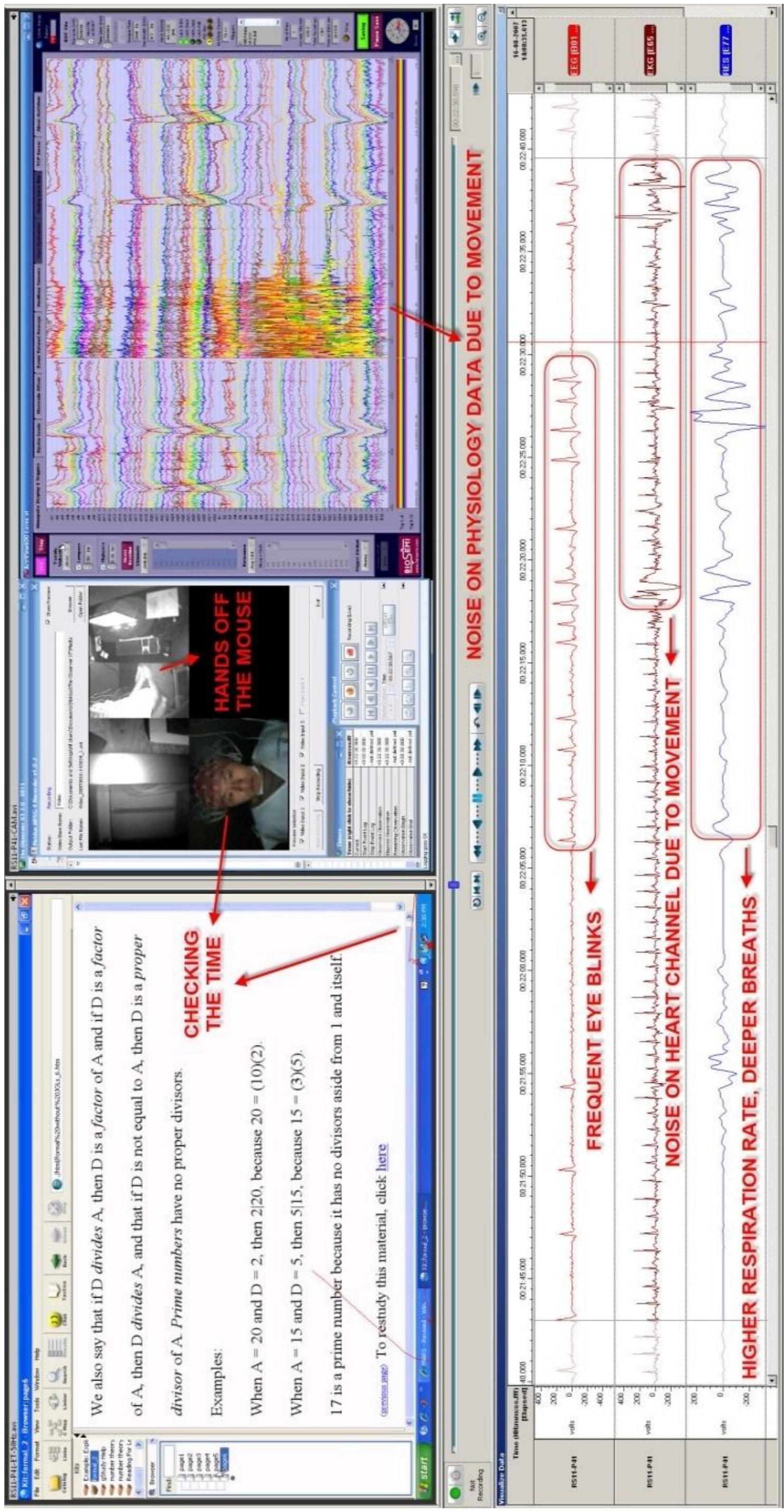
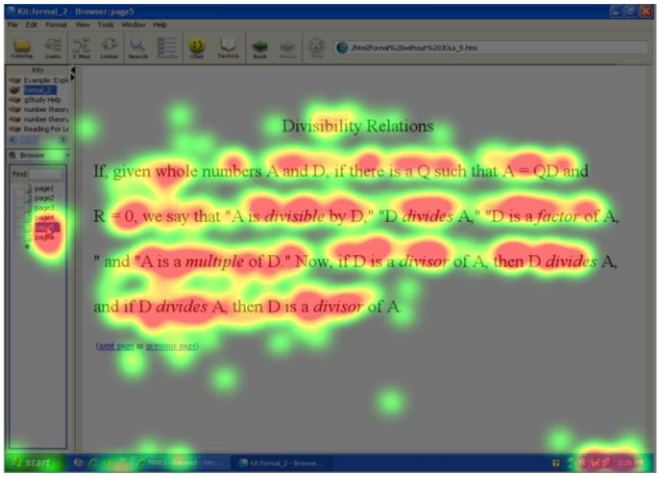


Figure 4.3. Hardship confronted, time pressure



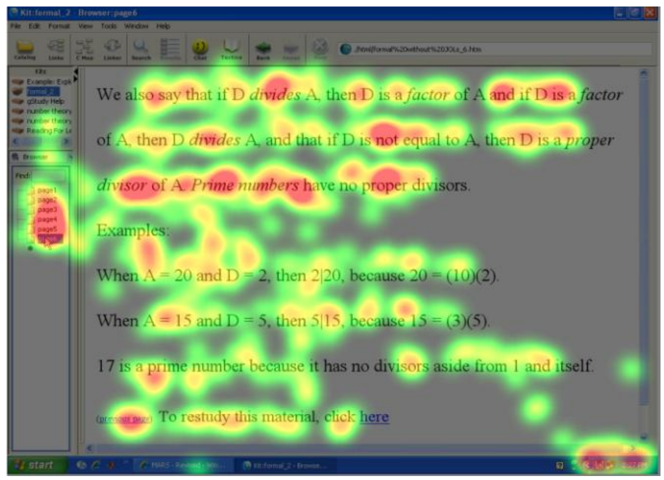


Figure 4.4. Checking time often: hotspots on P5 and P6 (Time is located at the right bottom)

After this period, she started showing signs of weariness. She made a few quick switches among pages without putting any particular attention to them, and then spent another 25 seconds reading P5. After that, she entered another “no attention” period where she checked the time often, looked at the sections on the screen those are unrelated to the content, or did not look at the screen at all.

4.1.5. Self-Reporting JOLs

Linda went through the pages sequentially during this phase, and promptly self-reported her understanding of the sections of the material. She was quicker in self-reporting Calculation JOLs, and tagged all of them as “very well” understood.

4.1.6. Restudy Period

Linda was the only participant who used her full time (10 minutes) for restudying the material, while the other participants spent less than 2:30 minutes in this phase. For her first time through reviewing the material within the restudy phase (1:50 in total), she quickly reviewed pages sequentially. She spent 10-20 seconds each page, with the exception of P5 where she spent around 35 seconds mostly attending to the relations among the terms *divisible, divisor,* and *divides*.

For the second time through (1:45 in total), just like her first time, she reviewed the pages in sequence starting from P1. This time she spent slightly longer time per age (15-25 seconds). She checked the time more often for this period (5 times), in comparison with her first time through restudying the material (where she had checked the time only once that is close to the end of this period). She spent less time reading the pages where she checks the time (i.e. P4 where she spends 12 seconds), therefore time pressure can be evaluated as a limiting factor for her restudying the material. She usually switched to a different page after checking the time.

Linda started her third time through (2:00 in total) again from P1 but after a second of memory recall regarding the content of that page, she preferred not to attend to it and switched back to studying understanding and reasoning related parts on P5 and P6 for another half a minute. After that, she quickly reviewed all pages from P1 to P3 for around 5-10s per page, then spent more time on restudying pages 4, 5 and 6 (20-25s each). She checked the time three times this round.

She started her fourth time through restudying the material from P1. As an interesting gesture, at the times she restudied the understanding and reasoning related content on pages 5 and 6, Linda often scratched her nose or touched her mouth with her fingers (Figure 4.5). Considering she had manifested similar behavioral cues when first studying this section, she apparently faced continuous hardship understanding this part of the content. According to eye tracking analysis of the study and restudy phases, she spent an average of ~11.8s looking at these understanding and reasoning related JOLs on P5 and P6, while spending an average of ~6.3s for all other JOLs within the study material (in other words she spent more time on these parts). In addition, the number of her eye fixations on these JOLs (939) was almost half of the total number of eye fixations she made throughout the study and restudy periods (1941). In addition, her average self-reporting JOL score was 29% for these JOLs compared to 67% for other JOLs (in other words, she had much less confidence). These results from eye-tracking analysis and self-report data are in parallel with the behavioral and physiological indicators that Linda had hard time studying the understanding and reasoning related content on pages 5 and 6.

After that period, Linda showed some indications of fatigue and boredom for ~2 minutes close to the end of the restudy. In addition, she checked the time more frequently during this last 2 minutes of the restudy phase. She started taking deep breaths more frequently, blinking her eyes more often, and repositioning her body more often. Her eyelids became half-closed and her face started looking tired. All of these indicators could be clearly monitored on the respiration channel and front facing camera recording on the analysis screen. Figure 4.6 demonstrates how these changes in the behavioral and physiological data are identifiable when compared to the other sections of the experiment.

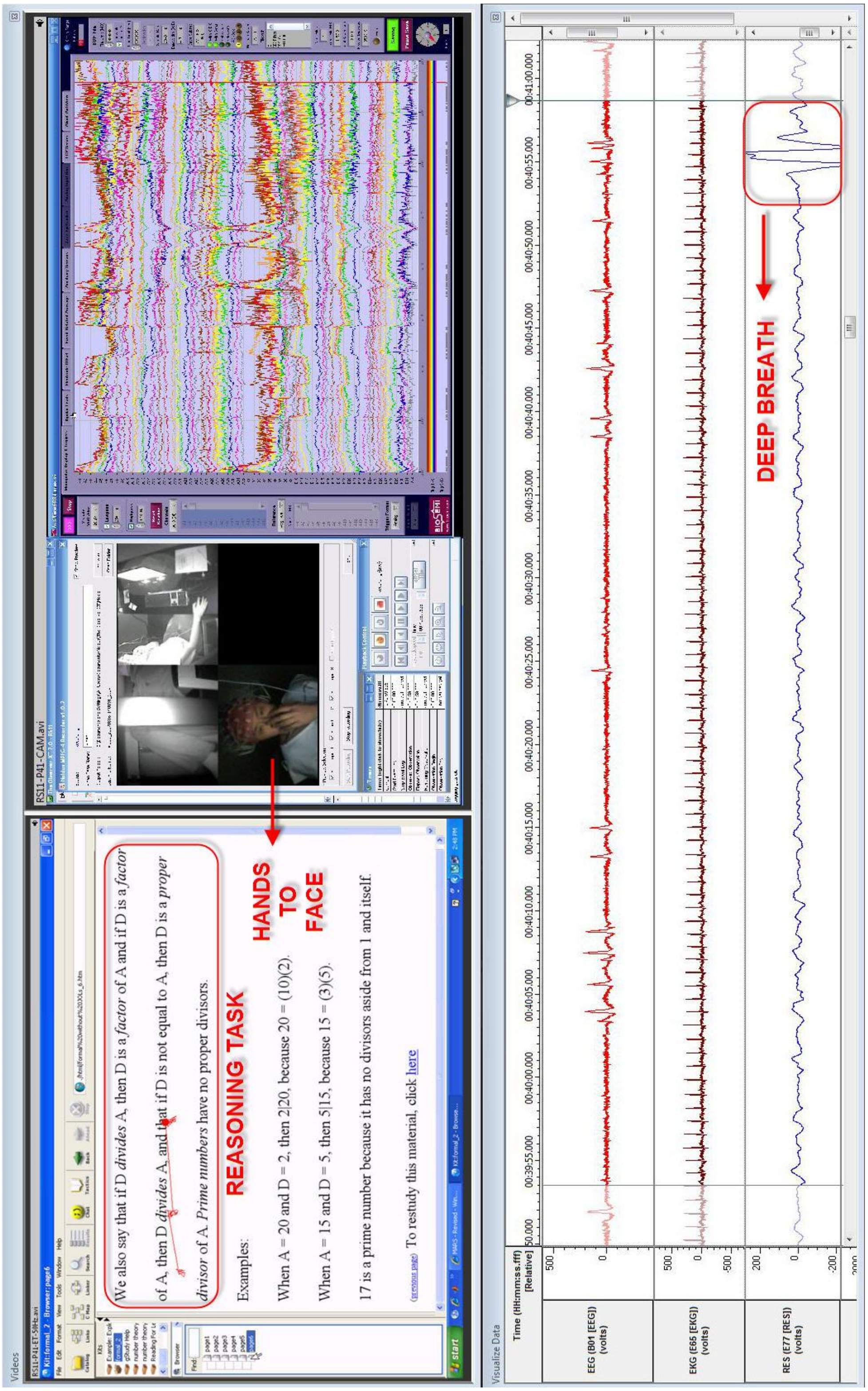


Figure 4.5. Restudying P5 and P6

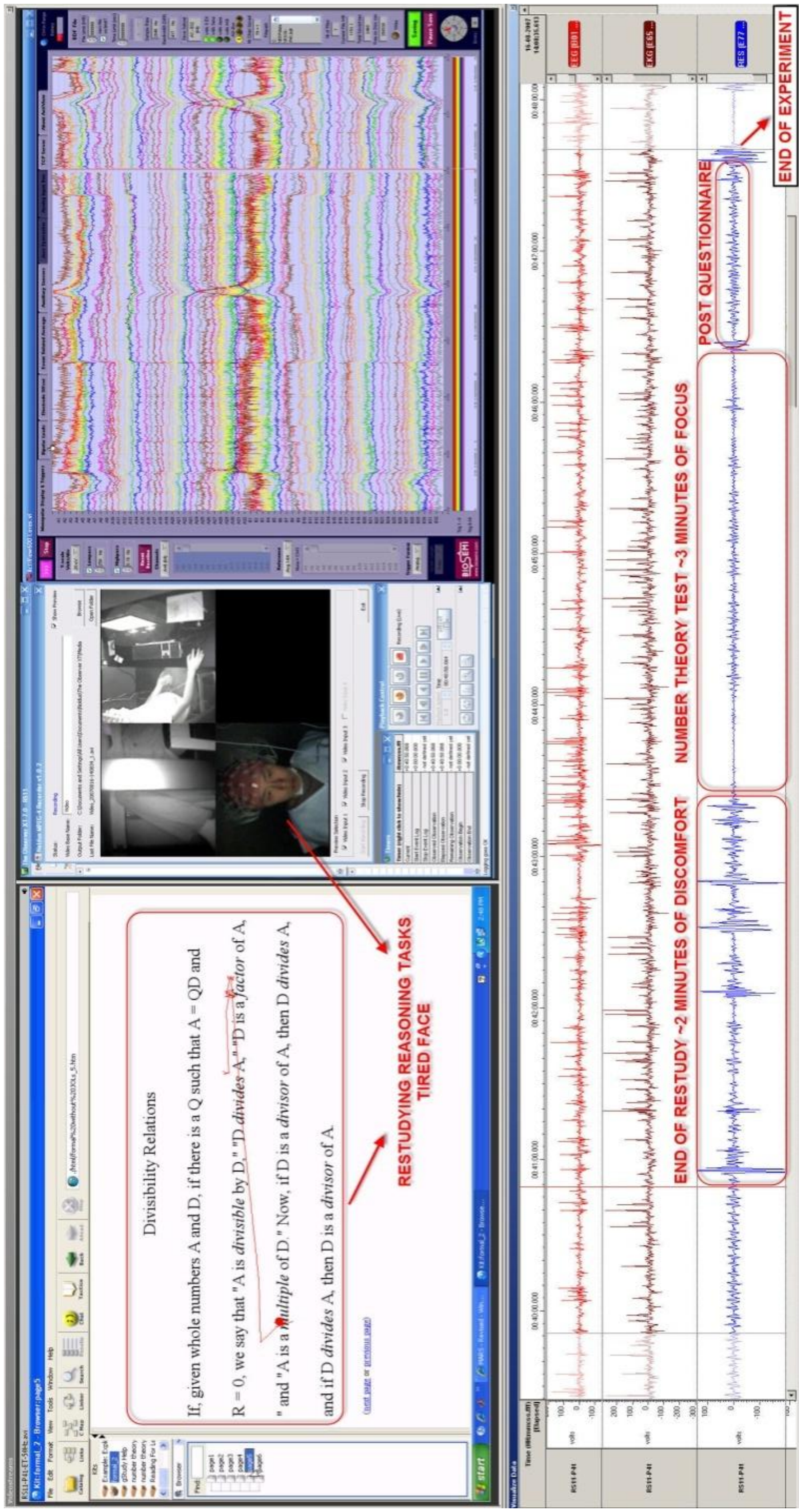


Figure 4.6. End of restudy: ~2 minutes of discomfort/boredom

4.1.7. Test

Linda was well focused and calm during the test period. She spent most time on the first question (almost a minute) where she checked all given numbers to see if they are factors of 42. For other questions, she spent an average of 2-15s per question. No other significant events were observed.

4.1.8. Post Questionnaire

For her answers to the post questionnaire, Linda indicated that the task was somewhat interesting to her (rated 4/7), and it was not challenging to her at all (rated 0/7).

At the end of the experiment, Linda was asked to provide a brief narrative of her experience. Her answers are as follows:

Researcher: Please go through the material and give us a verbal narrative of your experience as you were studying and restudying it."

Linda: When I started studying this material, I felt fine because learning the division was okay. But I found P5 a little confusing because there were a lot of words…I am one of those people who prefer skim reading. I try to read the point where I want to focus…I am also much more like a visual learner, so I prefer figures, diagrams in a text, so that I can put everything in contact…In general, the material was pretty easy to understand, and I was comfortable studying it.

4.1.9. Analysis

Based on the interpretations of her eye-tracking data, and as also her self-narrative indicated, Linda was using a speed study/skimming technique, and she can be evaluated as a quick learner. This interpretation is also cross-validated with her scores on two EBI scales: Quick Learning (4/7) and Simple Knowledge (6.4/7). Both scale scores were highest among all participants. Linda followed a strategy where she first focused on the content that she was most comfortable with the calculation tasks. Her JOL confidence scores for the calculation tasks were 100% and she received 100% success rate with calculation related questions on the test. Both scores are highest among all participants. Succeeding that, Linda did not spend much time on the calculation tasks, she spent the rest of her time focusing mainly on understanding and reasoning related content. Therefore, she can be evaluated having mastery-approach motivation for the calculation parts at the beginning of the experiment, whereas having mastery-avoidance orientation for calculation parts and mastery-approach orientation for understanding and reasoning parts for the rest of the experiment. Her average JOL scores for understanding and reasoning were 54% and 37% (the lowest among all participants) respectively. In other words, she self-reported her confidence with these sections of the study material lowest among all participants. This indicates that she was somewhat precautious or modest with her learning (i.e. ‘I could do better’). Nevertheless, her test scores for this content were 50% for understanding, and 67% for reasoning, which were highest among all participants. Finally, she was the only participant self-reporting that the experiment was not challenging to her at all (0/7). She was also the only participant taking her full time to restudy the material for 10 minutes. In summary, spending a long time on the tasks with intense focus on them, her unique and intense strategies studying the material, and in spite of her modest self-reporting confidence ratings, having the highest test scores, Linda was observed demonstrating the characteristics of a mastery learner throughout the experiment that prevails for calculation, understanding and reasoning sections.

4.2. Participant-2 (Betty)

4.2.1. Demographics

Betty is a 21 years old female undergraduate student in Molecular Biology & Biochemistry. She has Chinese parents and her mother tongue is Cantonese.

4.2.2. Psychological / Physiological State

Betty was observed being relaxed before the experiment, while seen as tired and anxious during and after the experiment. She self-reported feeling “curious and a bit anxious” before the experiment, “anxious” during the experiment, and “tired” after it. Betty also indicated that her eyes got tired during the experiment due to the bright monitor screen. She has myopia and uses contact lenses. This might be a reason why her eyes are more sensitive to the screen light than usual, and for her needing to rest her eyes more often than usual. Betty's experiment took 38 minutes.

4.2.3. Pre-Study Period

Betty looked calm and relaxed while answering the demographics questionnaire. After the DQ, just like all other participants, she took the pre-questionnaire. There she showed some indications of anxiety when faced with the question *"How comfortable are you at this time when you are informed that you are going to study a topic regarding the Division Theorem in this experiment?"* answering as *"Very Little"*. At this moment she reacted negatively by shaking her head horizontally while quietly chortling with frustration and hitting the mouse button harder than usual (Figure 4.7).

With the average score of 2.8 out of 5, Betty received the highest score from the MARS-R among all four participants (viz. likely to have highest math anxiety). She also received the highest test anxiety score from the MSLQ among all other participants (6 out of 7). Therefore her response to this question, along with her facial expressions and gestures, is in parallel with these results showing that Betty is a highly math anxious participant.

Betty was calm during the MSLQ and the EBI. She took deep breaths between the EBI and the MAI, and scratched her eyelids. As reported earlier, screen light might be a factor causing her to rest eyes frequently.

According to her EBI score, Betty's belief in her innate ability was the strongest among all four participants (5.3/7). Her extrinsic motivation scale score from the MSLQ is the highest (6.8/7) among all participants, and she is the only participant with a higher extrinsic motivation score in comparison to intrinsic motivation score. In addition, her MAI score for Procedural Knowledge was the highest among all four participants (6/7). These scores evoke that in addition to having high math anxiety, Betty believes that most knowledge is innate, in other words, knowledge depends on ability. This belief of her can be evaluated as a factor causing her lacking confidence learning mathematics because her belief of mathematical inability about herself. The scores also indicate that she is more of an extrinsically motivated learner than intrinsic, and prioritizes procedural knowledge to conceptual.

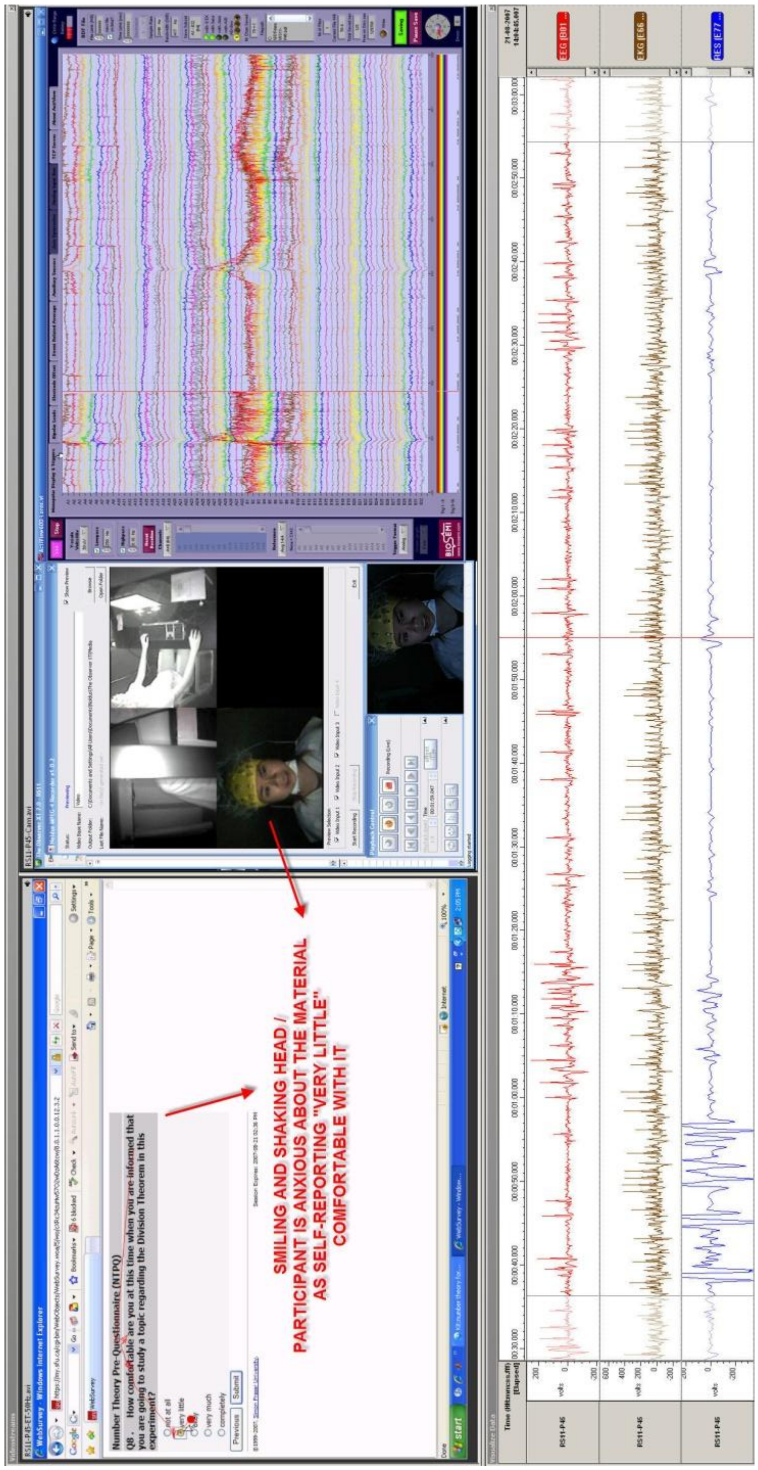


Figure 4.7. Reacting negatively when self-reporting discomfort with the content

4.2.4. Study Period

Betty started her first time through (5:30 in total) studying the material by reviewing pages in order, except for P2 that she reviewed twice. P2 is where the expression ‘A=QD+R’ is first introduced and defined. She spent most of her time reading the page trying to understand this part. She spent one minute in her first study of P2, then spend another 10s on it for the second time. Interestingly, she spent a total of 22 seconds focusing on the expression ‘A=QD+R’ in her first P2 review, and then spent all of her 10s for her second review of the page attending to the same expression (Figure 4.8 and 4.9). After her second time reviewing it, she hit the mouse button harder than usual to switch from the page, which might be interpreted as a sign of anxiety.

Betty spent 30-60 seconds on other pages, with the exception of P5 where she spent 80s. She often lost her attention when studying the tasks. For example at the end of P3, where she sees the definitions for the terms dividend, quotient, divisor and remainder, she lost her attention for ~15s with eyes closed/looking elsewhere. This might be caused by her disinterest to the task, anxiety, or because her eyes got tired.

One other time Betty confronted with the expression ‘A=QD+R’ was on P5 where she showed clear indication of discomfort and anxiety with this specific expression, such as negative changes in her facial expression, taking a deep breath and swallowing, and doing frequent rapid eye blinks (Figure 4.10). On this page, she spent nine seconds, again only focusing on the same expression, closed her eyes, took a deep breath, repositioned her body, swallowed and switched her attention to the rest of the page. She kept moving her attention back to this specific expression six more times on this page.

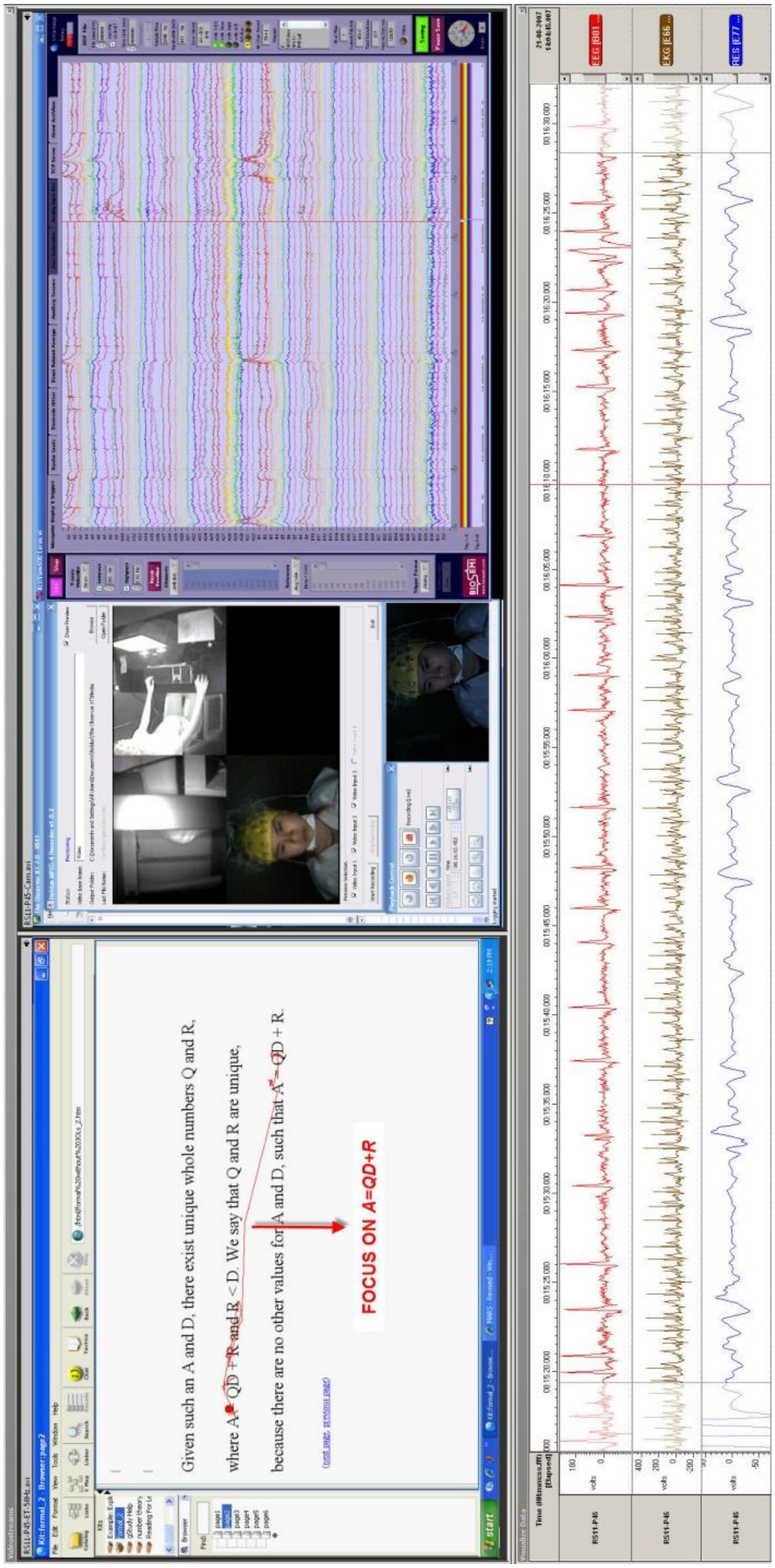


Figure 4.8. Focusing on A=QD+R on P2 (researcher screen)

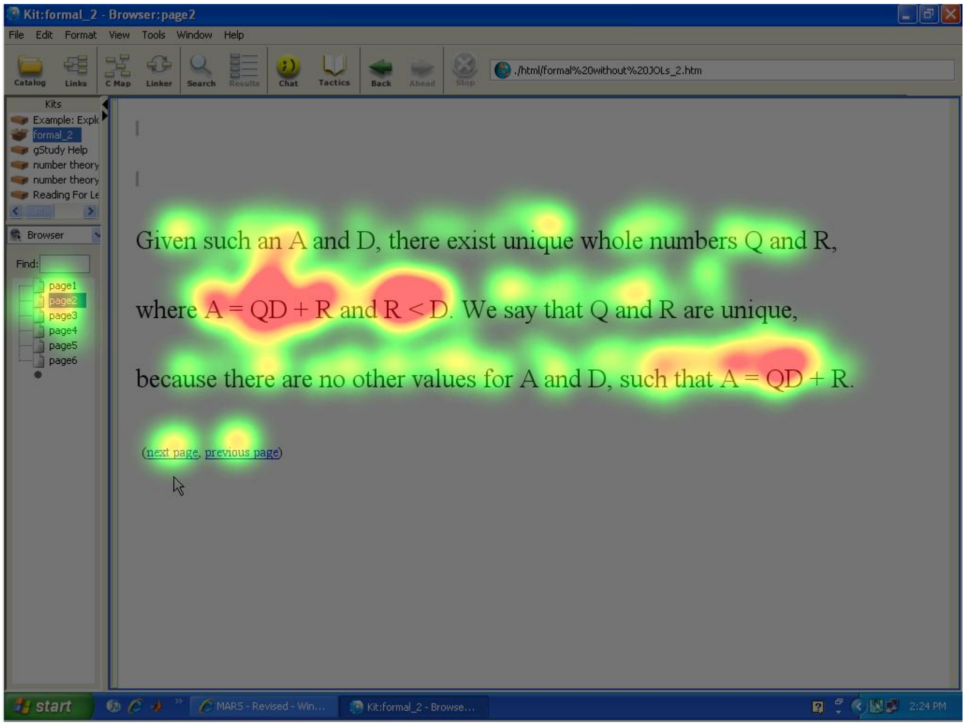


Figure 4.9. Focusing on A=QD+R on P2 (hotspot analysis)

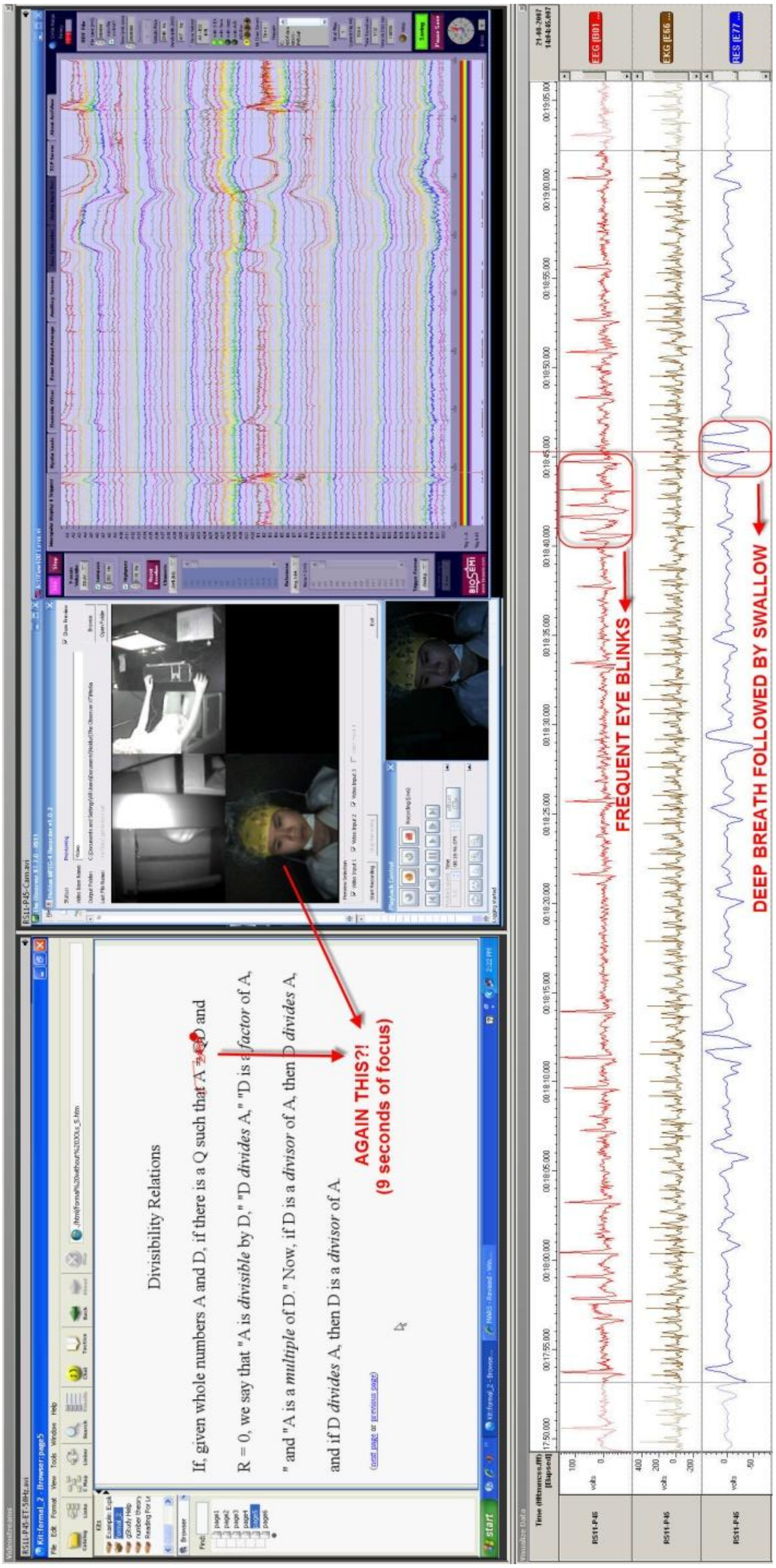


Figure 4.10. Discomfort with A=QD+R

She showed a brief moment of interruption at the end of P6 (a Calculation task) with two seconds with her eyes closed, and then looked elsewhere for a moment (Figure 4.11). This might be due to anxiety, or tired eyes.

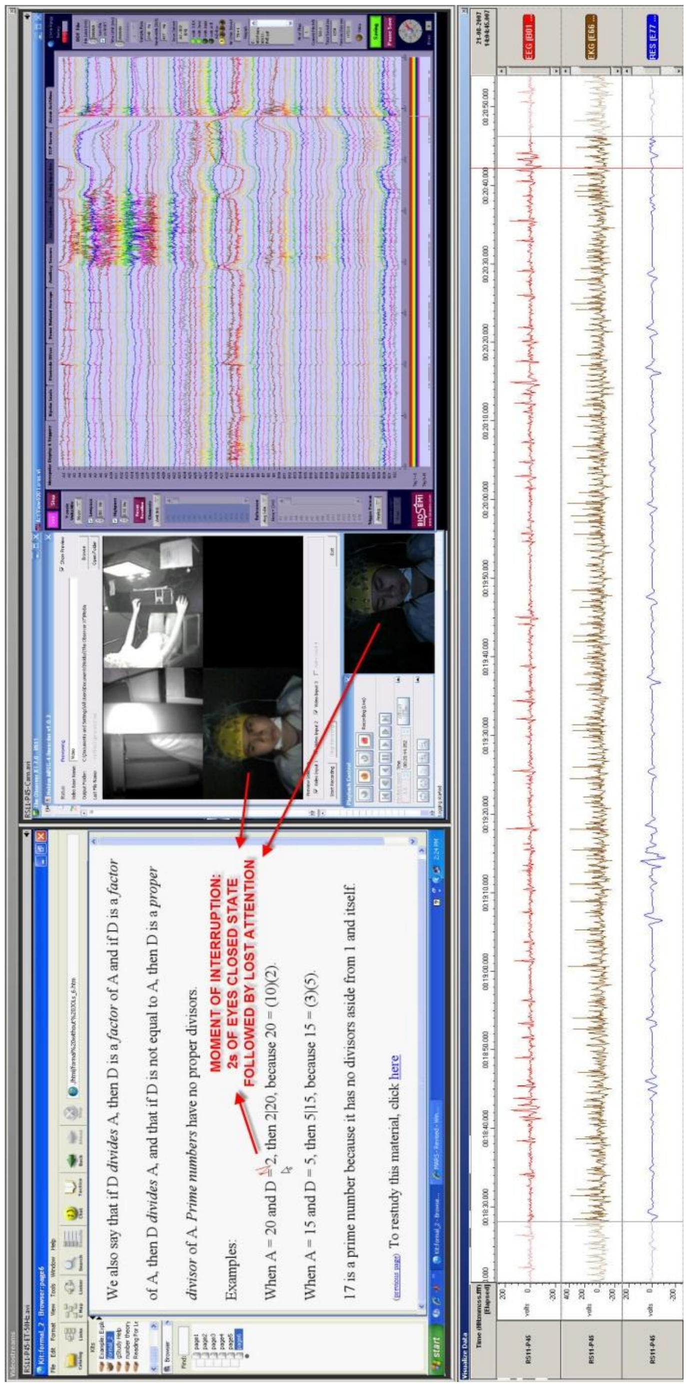


Figure 4.11. Moment of interruption

After that moment of interruption, she seemed to be starting her second time through studying the material; however, she was observed losing her motivation studying the task. She started her second time through studying the material by quickly navigating among pages with no particular attention to any. During this period, she did not look at the screen for 107 seconds by whether closing her eyes, looking elsewhere, or not paying attention to the content. During this stage, she hit the keyboard keys with her fingers in a fast and pointless manner; even though she did not need to use keyboard for this part of the experiment (just mouse was needed for navigating between pages). This no-attention period might be due to psychologically loss of motivation, or physically tired eyes (Figure 4.12). However, as detailed above and as the behavioral cues indicate, it is more likely that loss of motivation was caused by anxiety.

Close to the end of this period, after swallowing once and checking the time, Betty interestingly moved her gaze among the letters of A, Q, D and R on P3, one by one (Figure 4.13).

Betty continued her second time through by reviewing the second half of the P4, where there is some reasoning related content. Here she spent 22s of her time just looking at the expression ‘A=QD+R’. During that time her eye blink rate was considerably increased. At the end of this stage, she changed her body position, took a deep breath, swallowed, and then switched to P5 (Figure 4.14). These repeated instances of focus on the same expression followed by immediate interruption clearly indicate anxiety caused by this expression. This conclusion is also in parallel with Betty’s high scores on the anxiety scales of the related questionnaires, and her self-report having very little comfortable with the content. It is more likely that she tried hard to understand that specific expression because it is the heart of the definition of TDT. Therefore, she might have evaluated this expression as the key section of the content that is a prerequisite to understand the rest. In spite of spending a long time trying to understand this expression, failing to do so might have led to peak in her anxiety and loss of motivation to study the material.

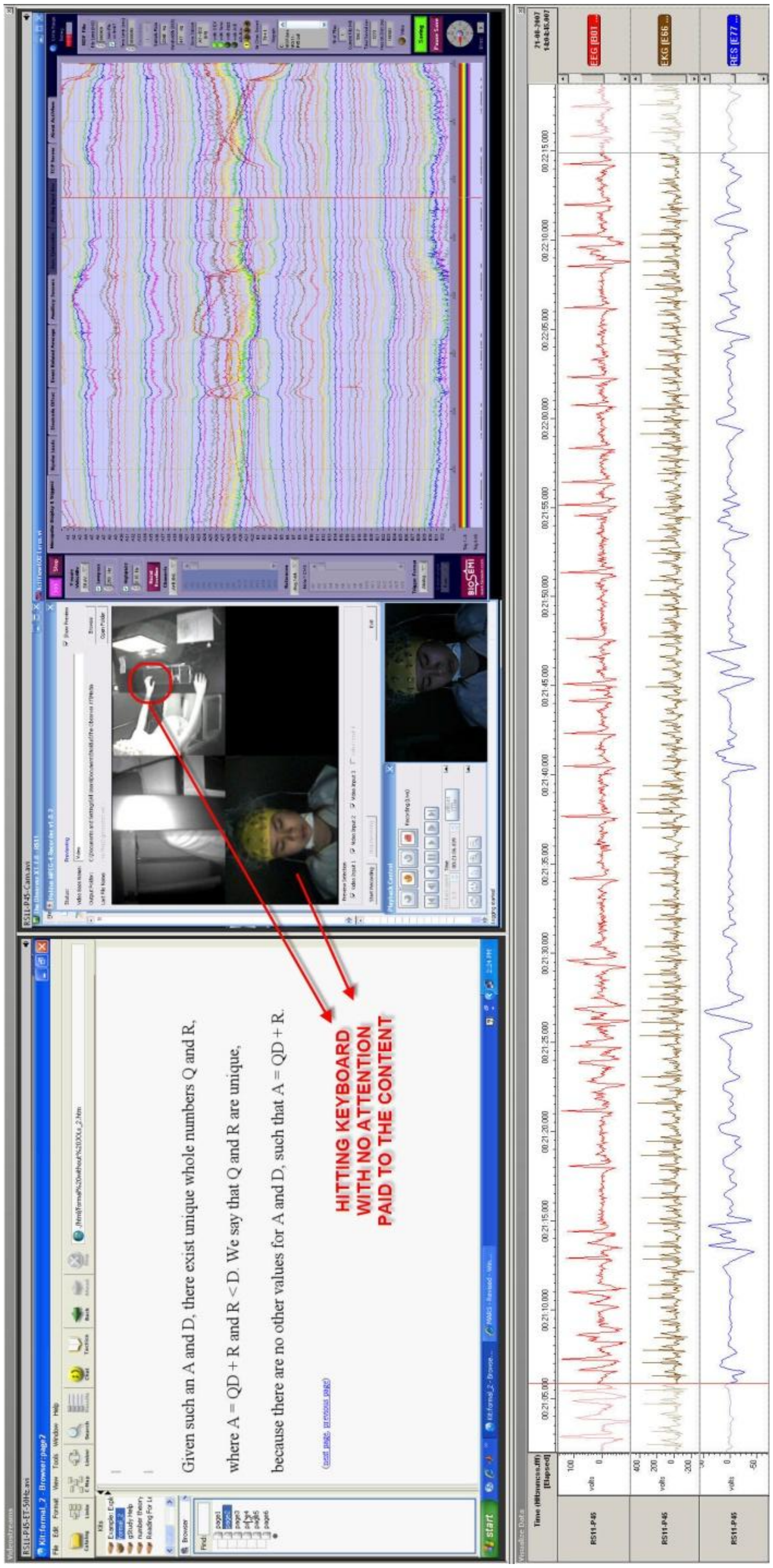


Figure 4.12. Loss of attention period

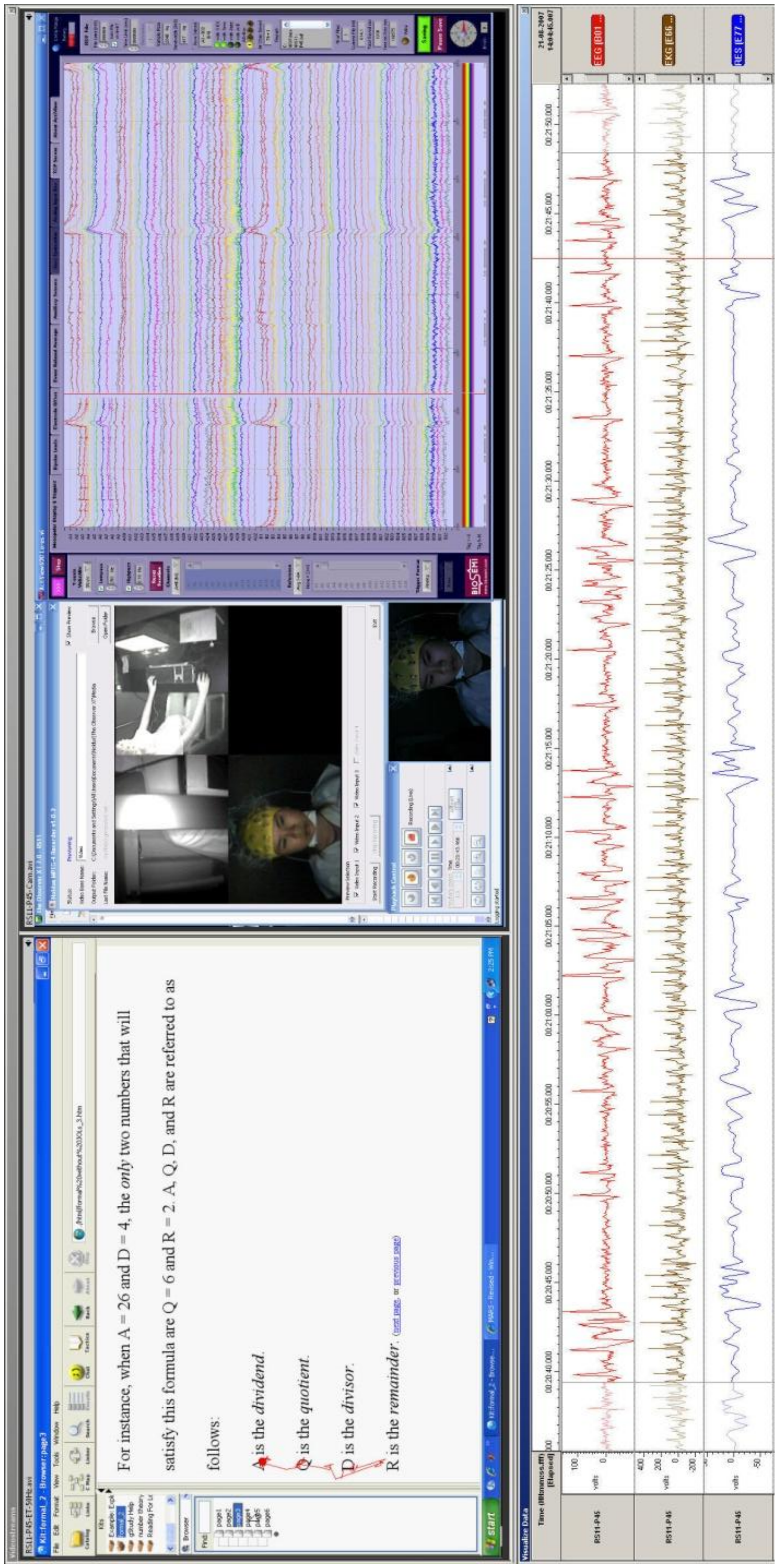


Figure 4.13. Looking at letters

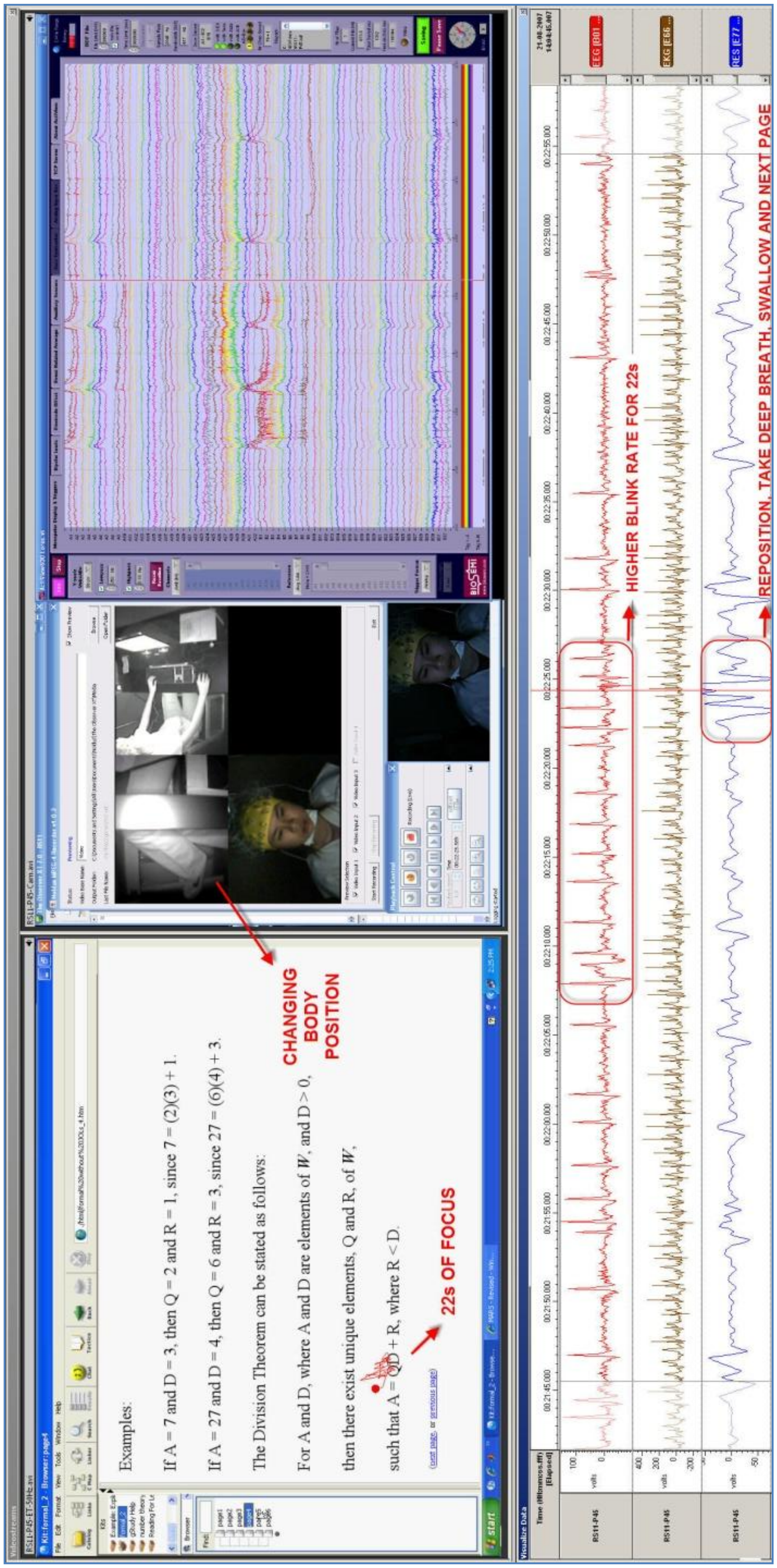


Figure 4.14. Discomfort with A=QD+R

On P5, while checking a version of the same expression (A=QD), Betty started to shake her legs as a possible indication of anxiety and boredom. Although still looking a little anxious, she was observed being relatively more focused on this page. She spent 2:30 minutes on this page where she tried building connections among divisibility concepts (i.e. divide, factor, multiple and divisor), and the expression ‘A=QD’ (the version of A=QD+R where R=0). Note that no other participant showed a similar behavior for more than half a minute. This was a very special timeframe indicating her paying extraordinary attention and struggle to identify their relations, thus I found it as a necessity to extract and honor this phenomenon by naming it.

Here I offer a new term, *The Spider’s Web,* to express the sophistication of the phenomenon manifested where learner’s eye motion and cognition (see the section of Chapter 2 explaining how the two relate each other) gradually and in parallel draw and construct connections among related concepts (Figure 4.15).

Betty lastly spent ~30s on P6 before the study period ended. At the end of study period, she was observed generally scowling, which might be whether because she was nervous or tired.

Figure 4.15. The Spider's Web trajectory

4.2.5. Self-Reporting JOLs

Betty went through the pages sequentially and promptly, and self-reported her understanding of the material. She tagged all of the JOLs involving the expression 'A=QD+R' as "well" understood.

4.2.6. Restudy Period

Betty looked unwilling to restudy the material. She displayed her frustration with facial expressions, and hitting the mouse button harshly and shaking her legs rapidly. During this phase, her blink and respiration rates significantly increased (Figure 4.16).

In her restudying material, Betty spent most of her time on P5 where she spent around one minute. She spent 10-30 seconds on the other pages. On P5, her respiration rate was significantly increased, she moved her body more frequently, and swallowed (Figure 4.17). These can be interpreted as indications of boredom and displeasure. (Later Betty self-reported that she restudied items that were difficult to understand for her).

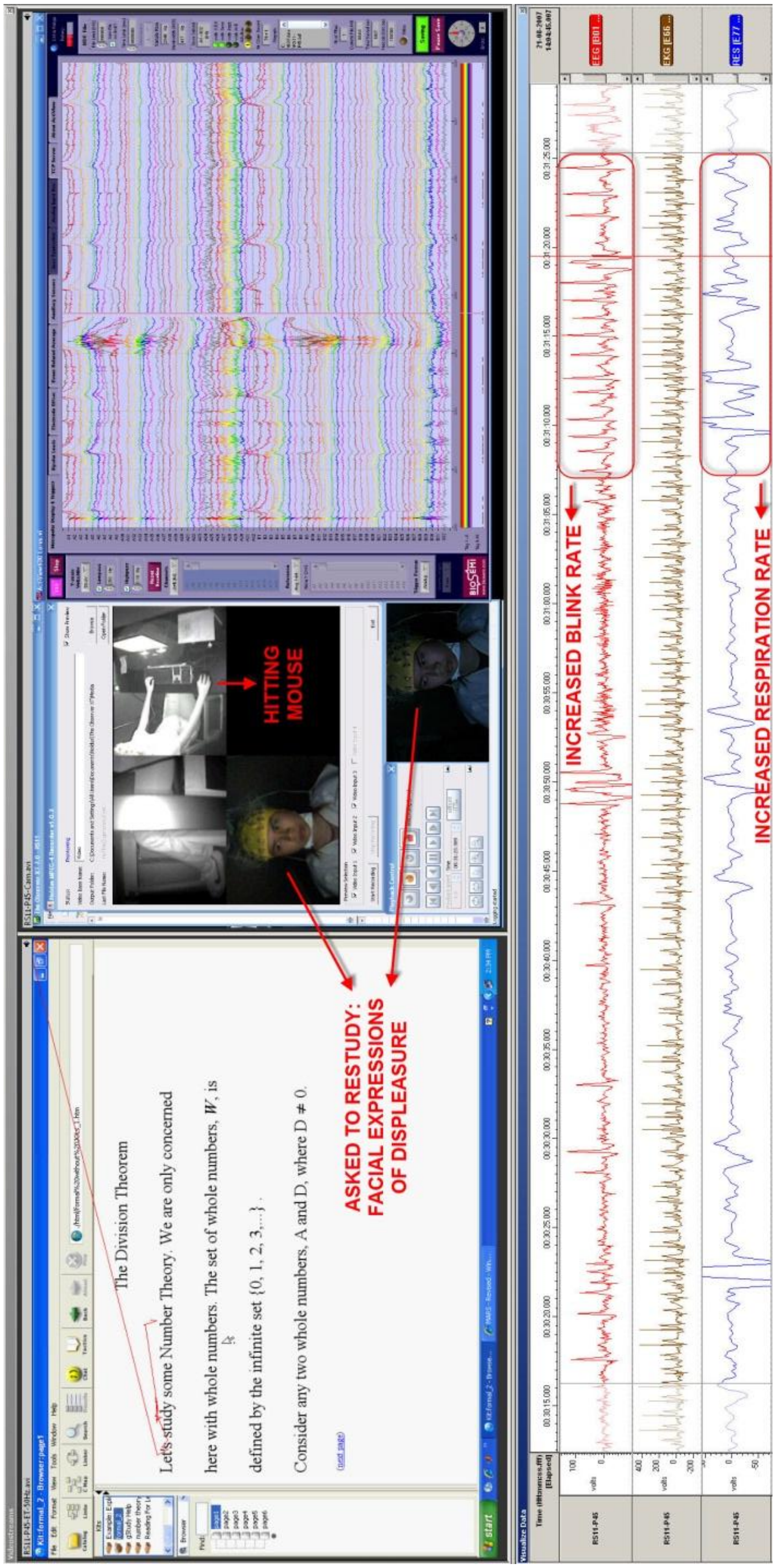


Figure 4.16. Reluctance to restudy

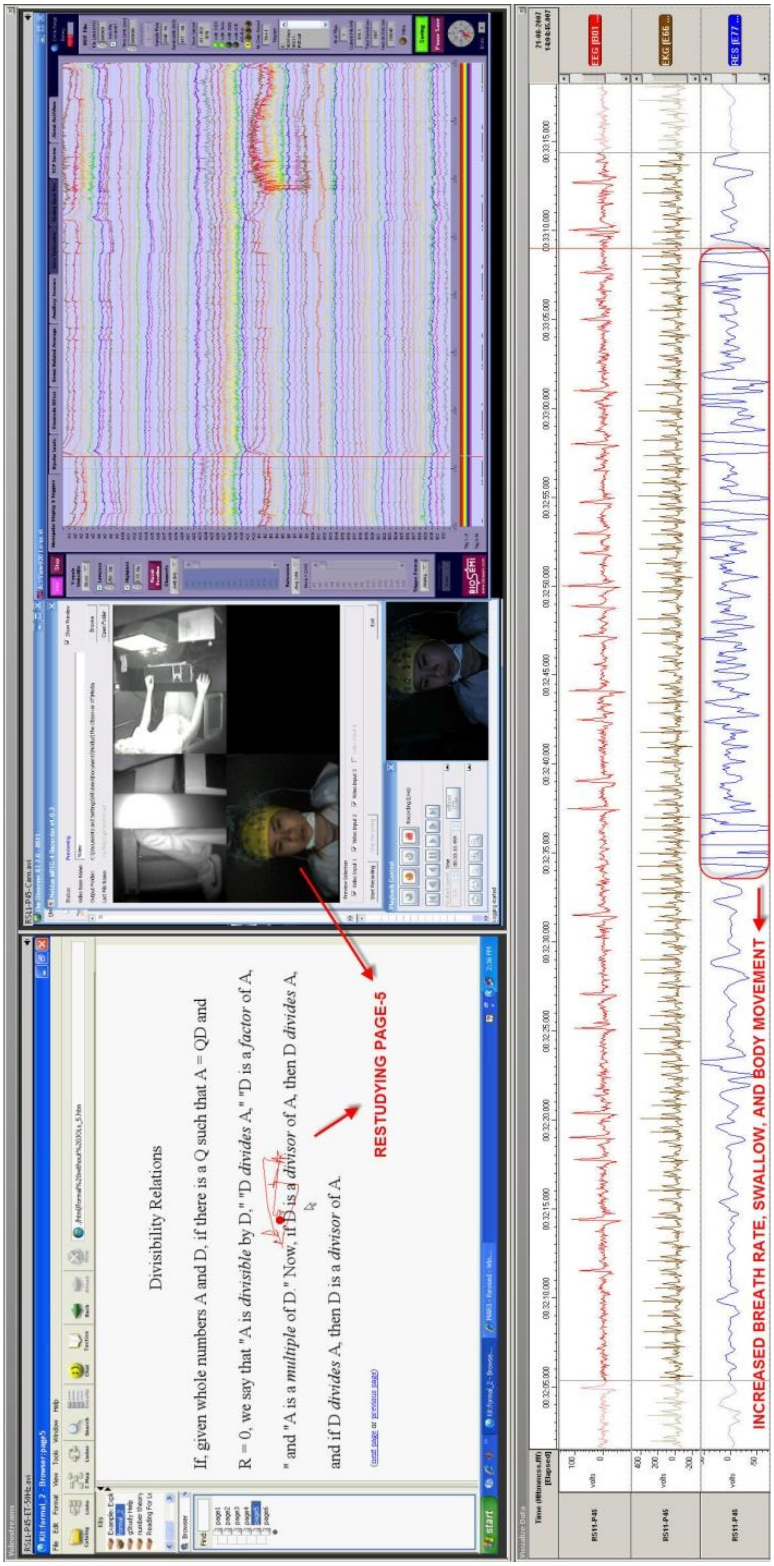
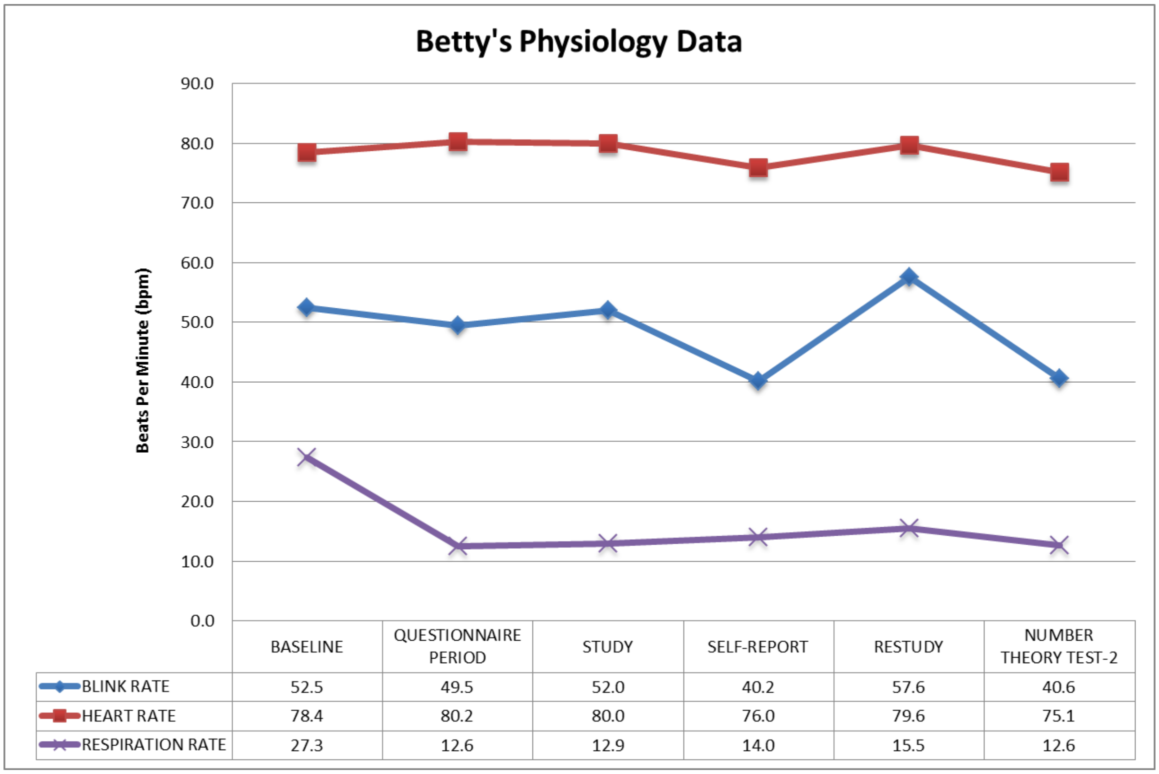


Figure 4.17. Restudying P5

Betty looked nervous and uninterested throughout the restudy phase, and showed some indicators of disengagement and anxiety, such as frequent swallowing and swinging legs. The only period where she was relatively more focused was when she restudied divisibility concepts (where she previously built the Spider's Web). Just like all participants, Betty was given 10 minutes to restudy the material. However after 2:30 minutes from the beginning of the restudy period, she opened her arms, saying "I am done?!" in a frustrated manner (Figure 4.18).

Figure 4.18. I am done?!

Another indication of Betty's frustration was an increased rate of eye blinks, which had a peak during the restudy period as seen in her physiological data (Figure 4.19).



	BASELINE	QUESTIONNAIRE PERIOD	STUDY	SELF-REPORT	RESTUDY	NUMBER THEORY TEST-2
BLINK RATE	52.5	49.5	52.0	40.2	57.6	40.6
HEART RATE	78.4	80.2	80.0	76.0	79.6	75.1
RESPIRATION RATE	27.3	12.6	12.9	14.0	15.5	12.6

Figure 4.19. Betty's physiological data of throughout the experiment

4.2.7. Test

Betty was very focused during the test phase. While self-rating her confidence for the first two questions, she often clenched her eyes, whispering to herself and self-validating her answers/calculations, then sometimes nodding before reporting her rating (Figure 4.20). She sometimes swallowed when self-rating. These are interpreted as evidences that she took the test and self-confidence ratings seriously with some test anxiety involved.

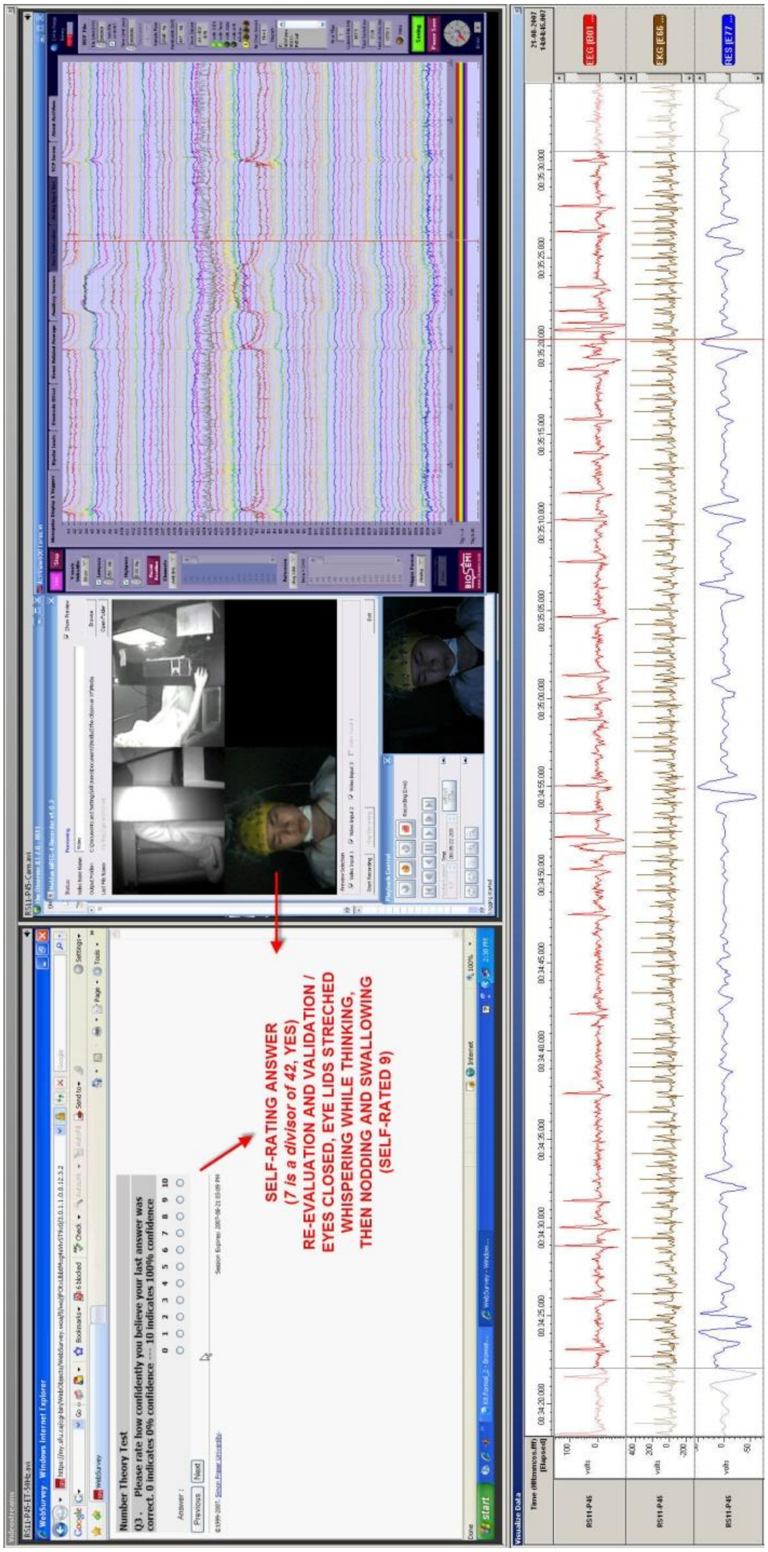


Figure 4.20. Self-rating confidence, validating calculation

4.2.8. Post Questionnaire

In the post questionnaire, Betty indicated that the task was not interesting to her at all (rated 1/7), and it was challenging to her (rated 5/7).

4.2.9. Analysis

The material was new and unexpected for Betty. She was anxious from the beginning, where she reported having very little comfortable with the material. As her self-report data along with physiological and behavioral cues indicates, she did not attempt to cover her disinterest and anxiety. Her frustration with the material and reluctance for studying it demonstrate that Betty had avoidance motivation for the most part. The MSLQ scores indicate that she is likely to be a performance learner. Although that was the case for the most part, there were times (such as building the Spider's Web) Betty showed qualities of a mastery-approach learner. In addition, she was interrupted by the bright screen monitor, and she had to rest her eyes frequently for that reason. Therefore, she could not be categorized to solely having mastery or performance / approach or avoidance motivation. For some reasoning parts, especially the parts involving 'A=QD+R', she had a very hard time making sense of the material. This usually caused anxiety, and losing her motivation to keep up with studying the material. She received 33% success rate from the test relating such reasoning content, which is considered somewhat low. She spent the least amount of total time on the experiment among all other participants. Lastly, her overall discomfort with the material is evaluated persisting throughout all CUR content. Therefore her behaviors and motivations were not classified based on CUR classification.

4.3. Participant-3 (Susan)

4.3.1. Demographics

Susan is a 22 years old female undergraduate student in Molecular Biochemistry. She has Vietnamese parents and her mother tongue is Vietnamese.

4.3.2. Psychological / Physiological State

Susan was observed being calm and relaxed before, during, and after the experiment. She self-reported feeling “a little worried that it was going to be hard-core math theory that was being tested on the exam part” before the experiment, feeling “pretty relaxed and trying best to answer all the questions” during the experiment, and feeling “same as before, but a little relieved” after the experiment. Susan’s experiment took 44 minute.

Susan meditates once a week in her daily life. After the experiment (referring to the relaxation periods), she commented that “the screen thing was the weirdest because I felt like I was being watched when I meditate; I usually don't have anyone watching while I am meditating”.

4.3.3. Pre-Study Period

Susan’s test anxiety scores from the MSLQ (1.6/7) and the MARS-R (1.6/5) were the lowest among all other participants. She was indeed observed as the most relaxed and calm participant among all four, with no major behavioral indications of anxiety. Her respiration rate was considerably lower (12.2bpm) than the others as well, that might be an indication for an overall calm psychological state and/or a result of her regular meditations. Her epistemological belief in innate ability (1.5/7) was the lowest among all other participants, while her belief in Omniscient Authority was 0.8/7. Those indicate her being a skeptical yet confident character in learning (i.e. I can do if I want to).

4.3.4. Study Period

For her first time through studying the material, Susan reviewed all pages one by one sequentially. On P2, she particularly focussed on the expression ‘A=QD+R’ for 10s, while holding her breath (Figure 4.21). When she first saw it, she kept her eyes closed for 2-3s. This might be a skill of Susan developed through her meditations that she can deeply focus on things she wants to, accompanied with holding breath.

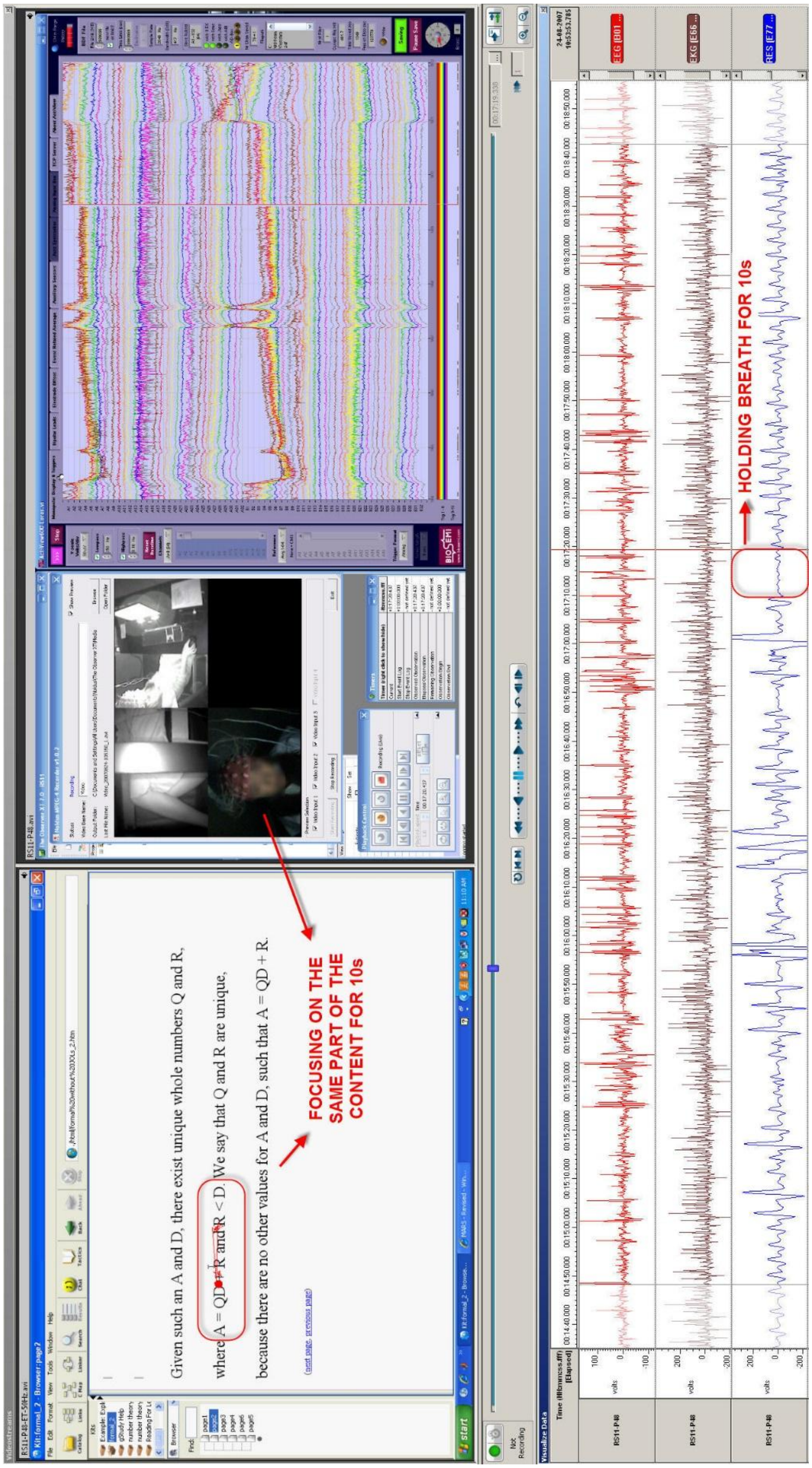


Figure 4.21. Focusing on A=QD+R

When Susan came to P4, she did not review the calculation parts, instead skipped them and started reading the page from the middle where there is no calculations involved (Figure 4.22).

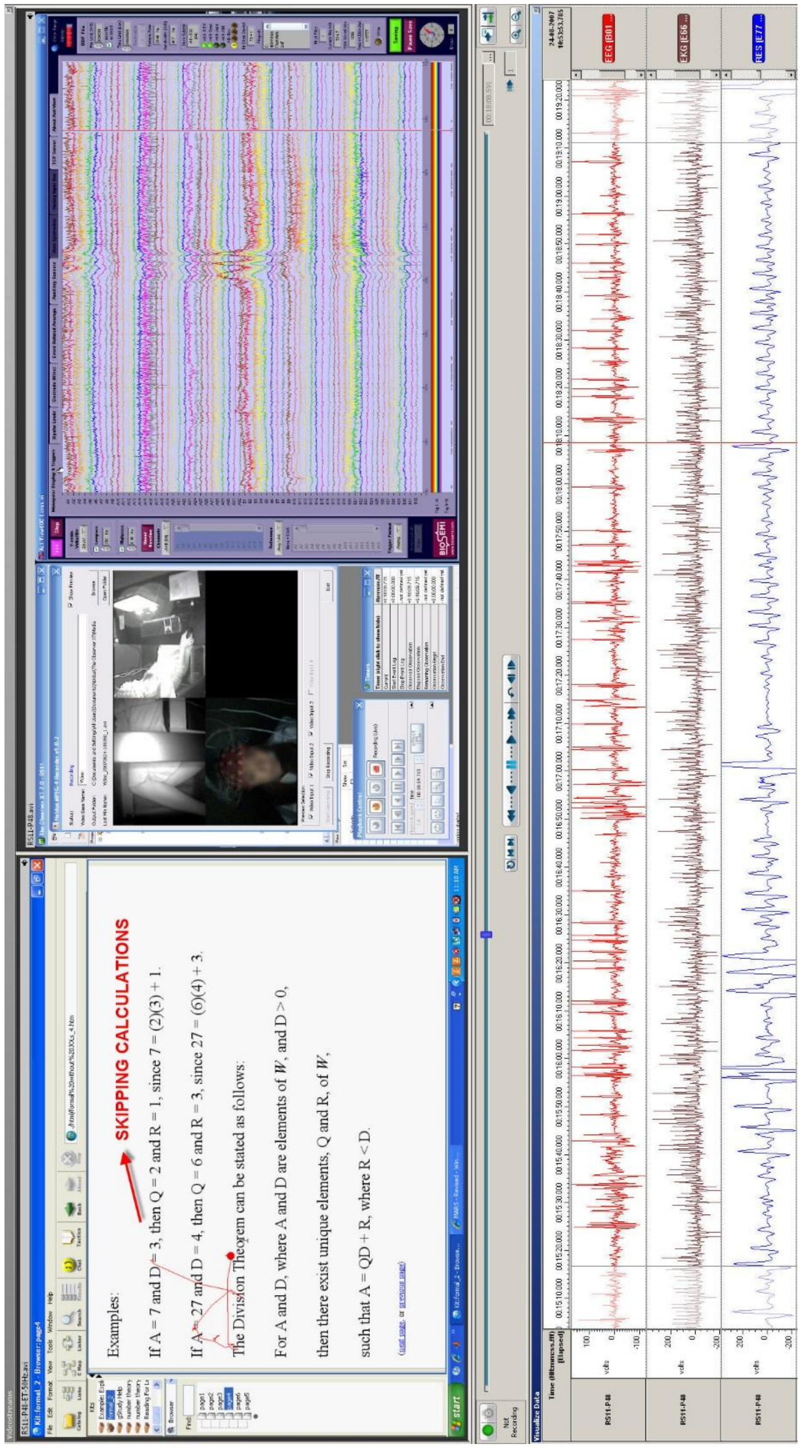


Figure 4.22. Skipping calculations

During her studying through the material second time, she skipped the parts involving calculations on P3 as well (Figure 4.23).

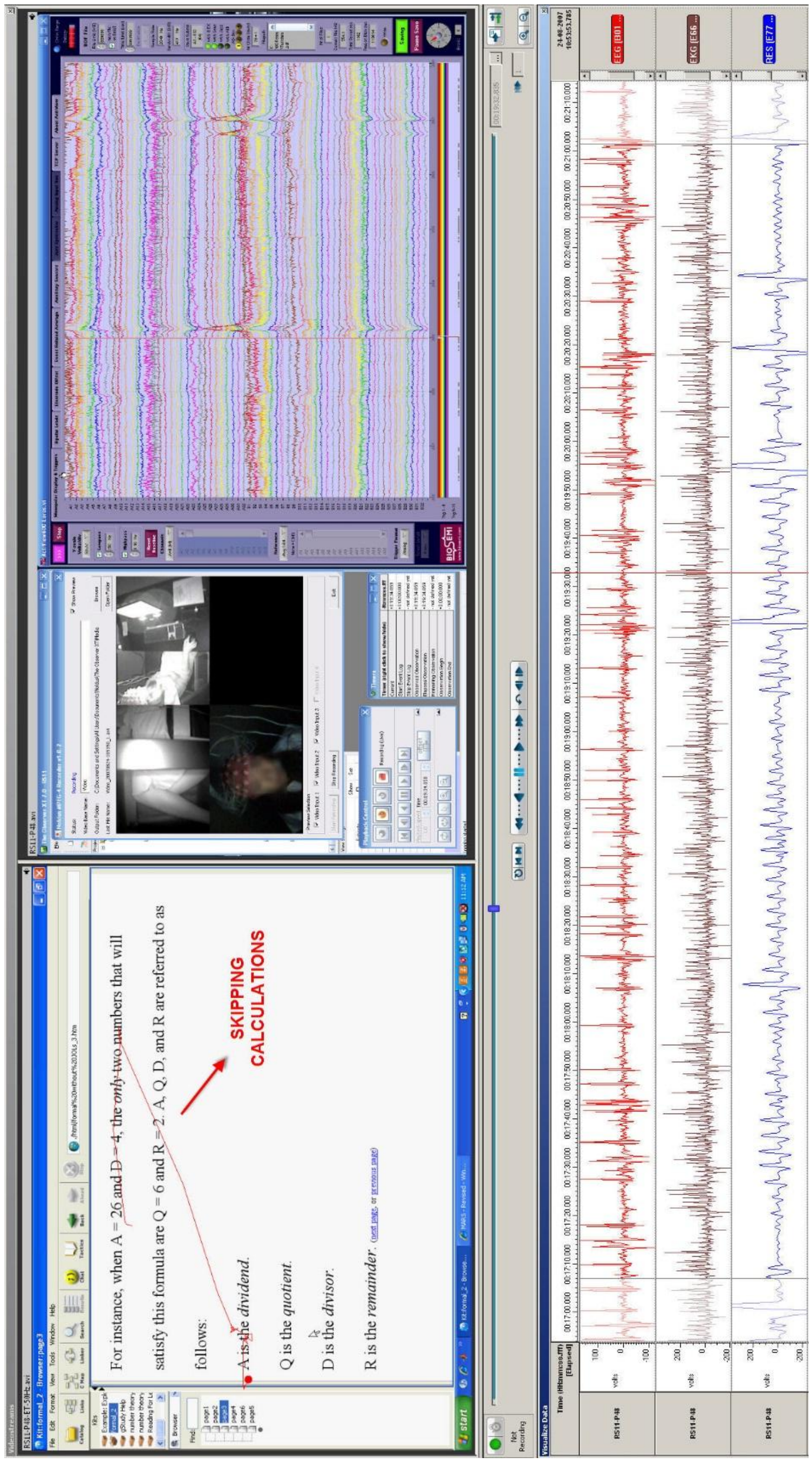


Figure 4.23. Skipping calculations

In her second time studying P4, Susan continued persistently avoiding calculations (Figure 4.24).

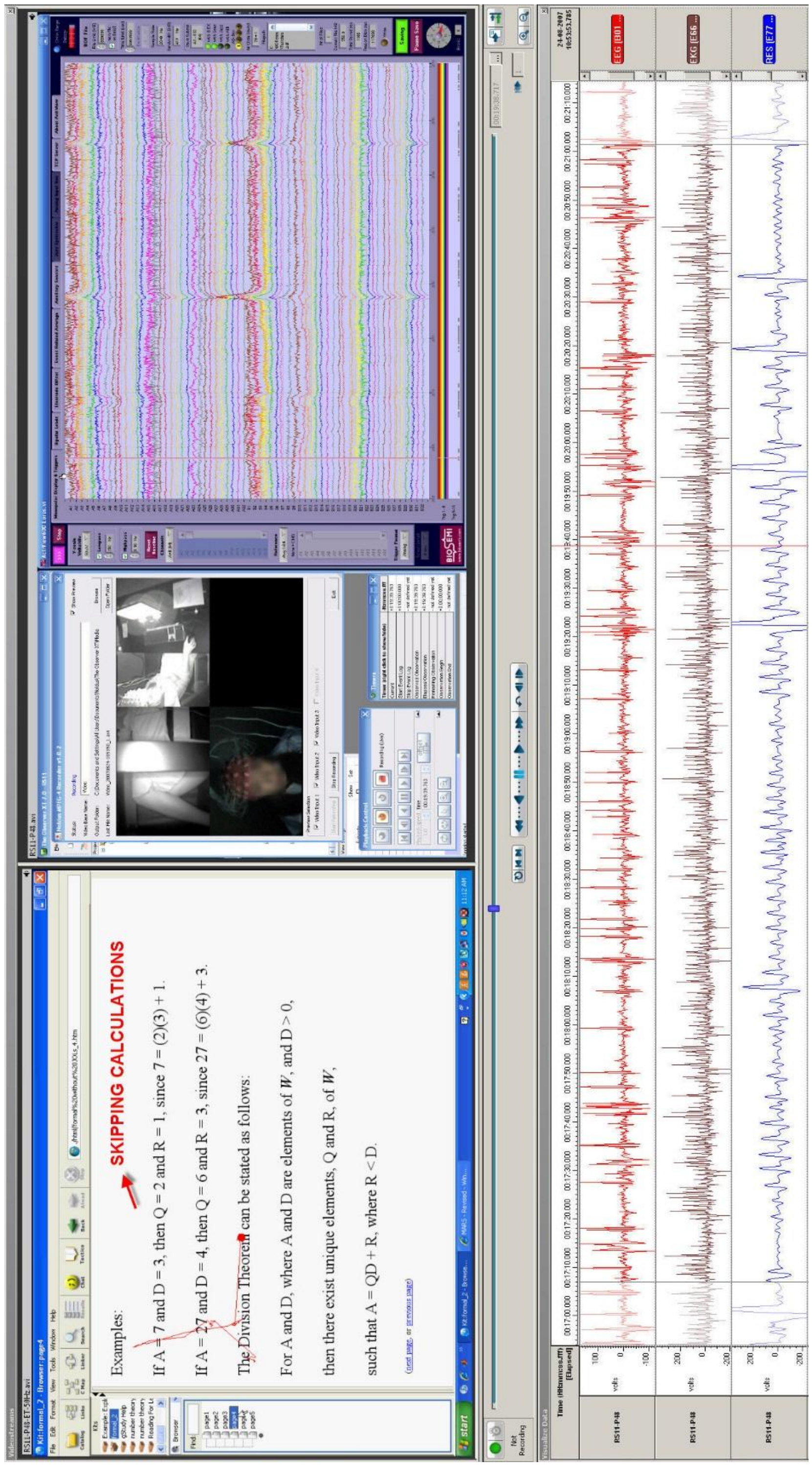


Figure 4.24. *Skipping calculations*

On her studying P6, just as on the other pages, she did not pay attention to the calculations (Figure 4.25).

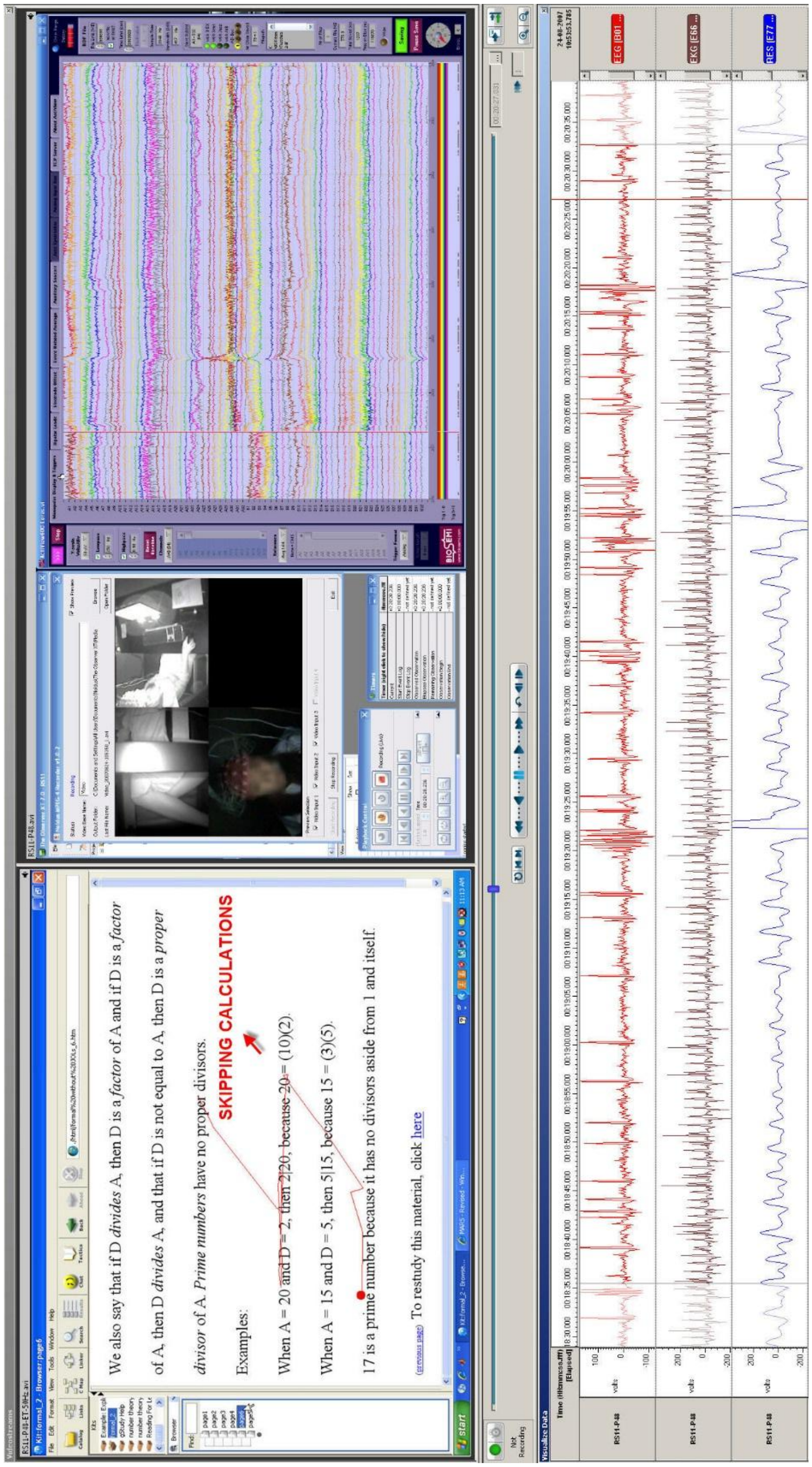


Figure 4.25. Skipping calculations

For her third time through, Susan started studying the material from P1 by sequentially (and this time more quickly) navigating among pages. Very interestingly, when she saw the same expression on P2 (A=QD+R), just like she did during her first time through, she closed her eyes and kept them closed for 2-3s. During these eyes-closed periods, her eyelids fluttered that indicates intense cognitive activity (Ehrlichman & Micic, 2012). (High frequency peaks with lower amplitude on the first wave extracted from EEG channel indicates eye fluttering. It could also be observed on the video recording of the front facing camera) (Figure 4.26). She later on tagged this JOL as “very well understood” on her self-report. This reaction of Susan to seeing ‘A=QD+R’ is worthwhile to compare to Betty’s reaction to highlight the difference between the behavioral manifestations of anxiety and cognitive activity of two different individuals interacting with the same mathematical expression.

For her third time studying P4, Susan kept ignoring the calculation parts. When it comes to reviewing reasoning related parts on this page, she was focused and tried to bridge them. After reviewing these parts, she took a few moments raising her head and looking up while using her reasoning. This happened twice with a few seconds of gap in between. After the second look up, Susan shook her head vertically with a long eye blink, which gave the impression that she completed the process of conceptualizing the information (Figure 4.27).

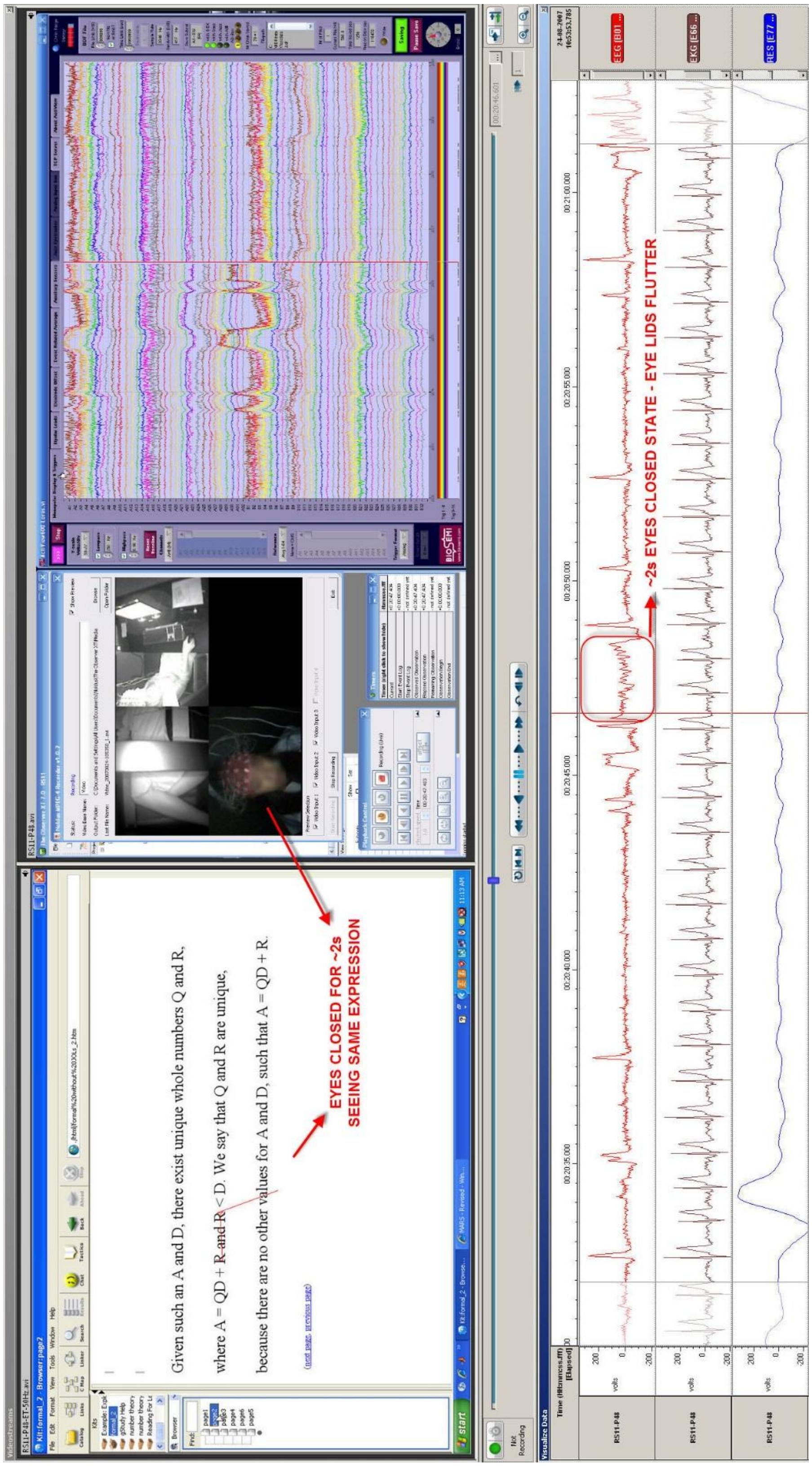


Figure 4.26. Eyes-closed state on A=QD+R

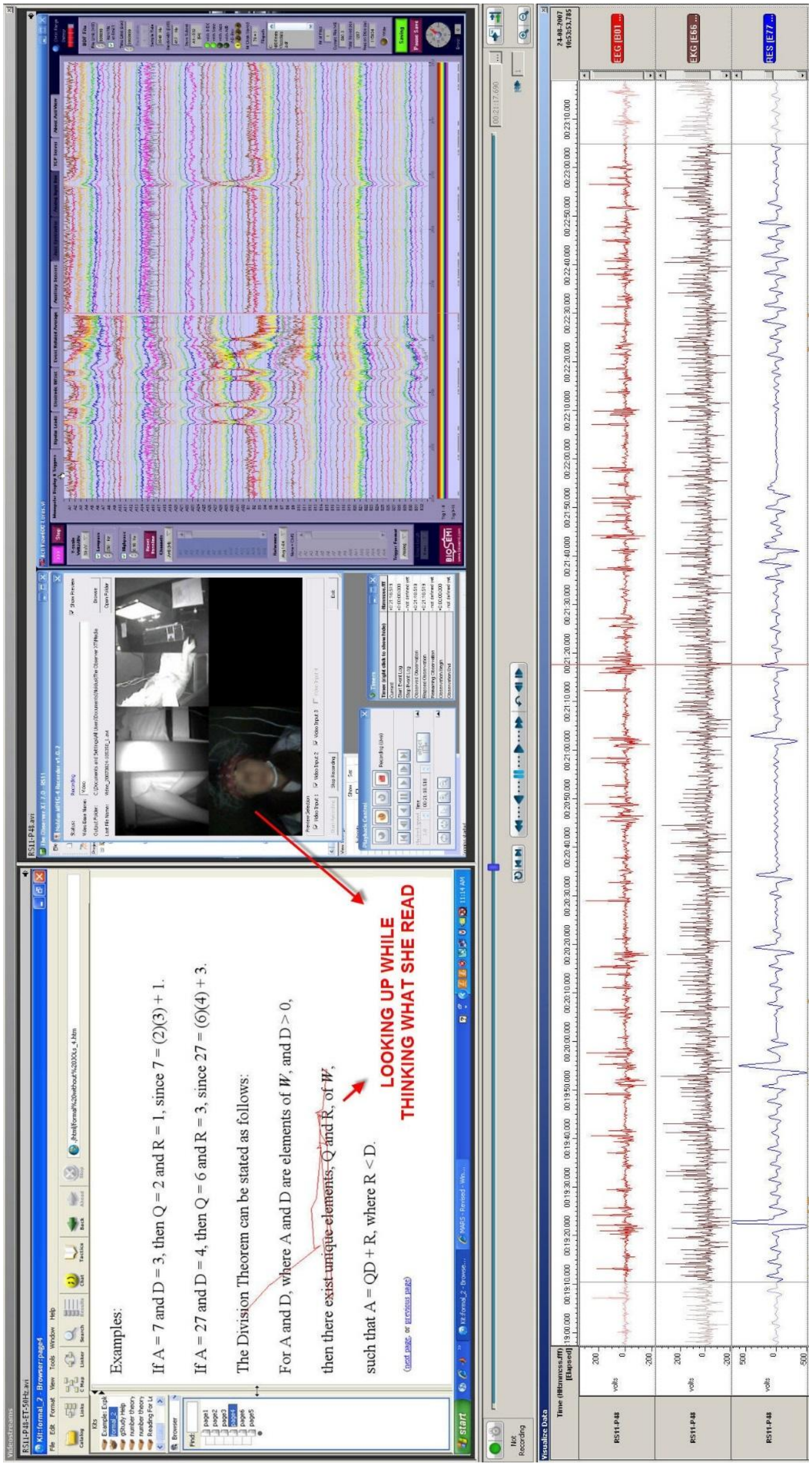


Figure 4.27. Facial expressions and gestures of conceptualizing

Susan started her fourth time through studying the material in reverse from P6 to P1. On P6, she skipped the calculation parts just like her previous times studying this page (Figure 4.28).

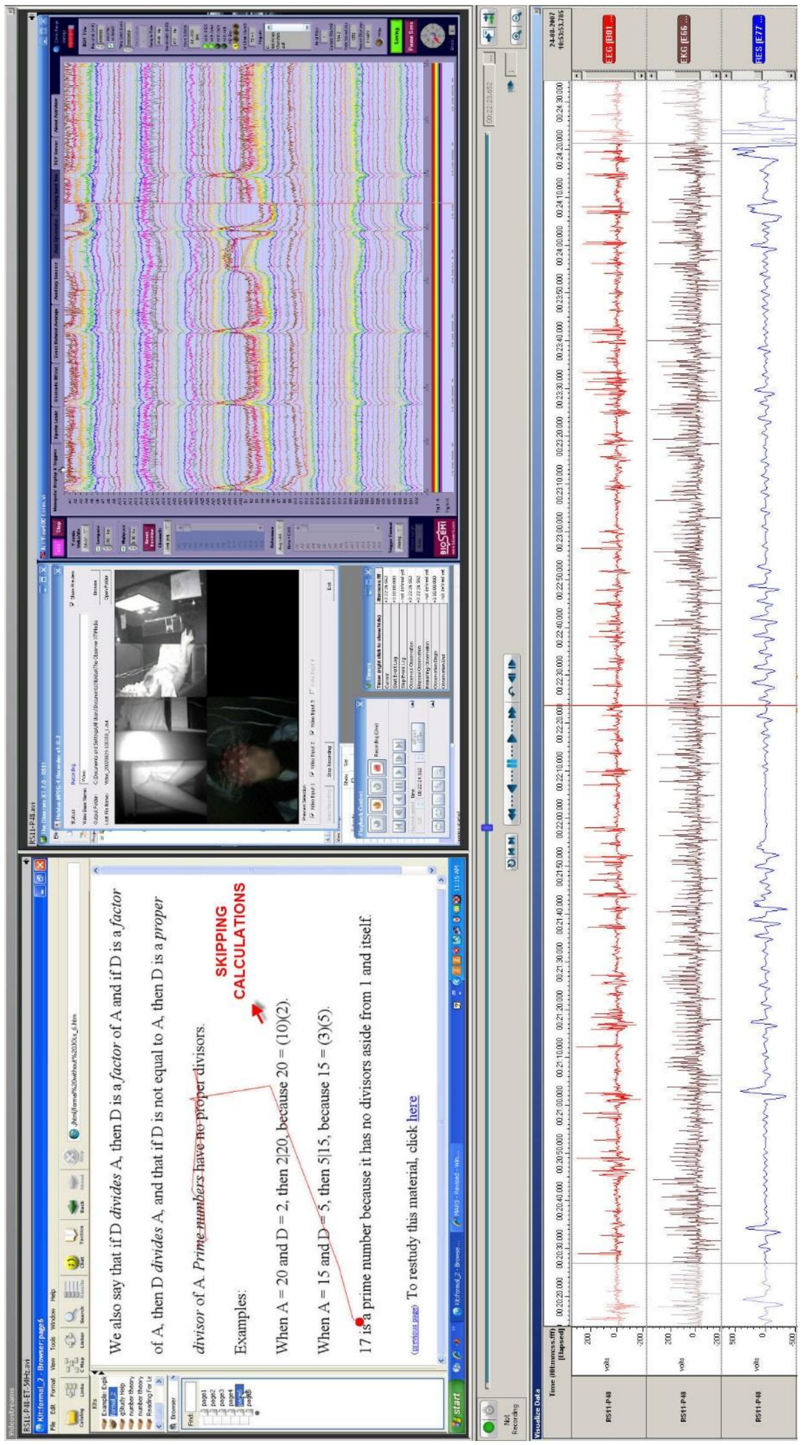


Figure 4.28. Skipping calculations on P6

She skipped calculations on P4 as well (Figure 4.29).

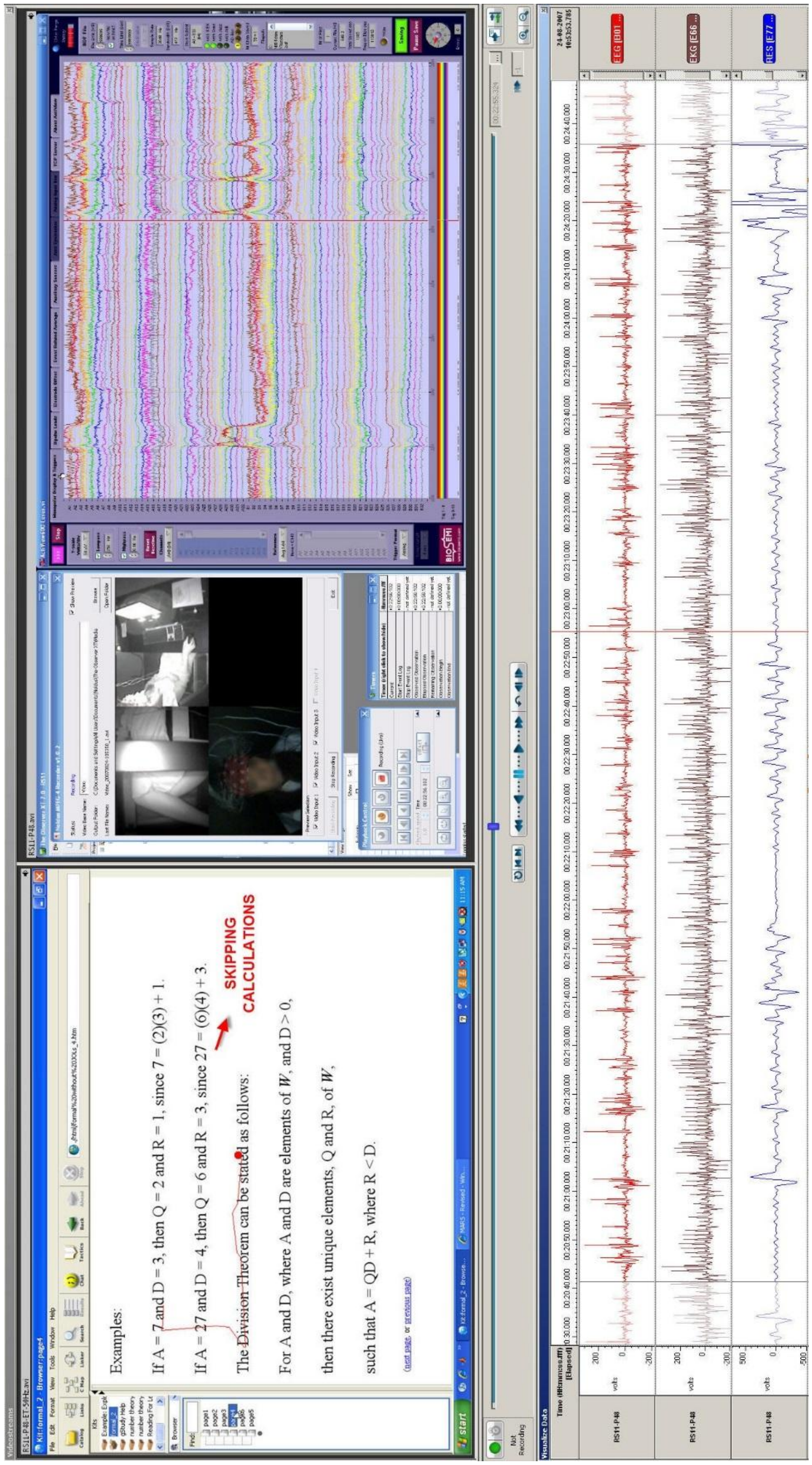


Figure 4.29. Skipping calculations on P4

For her fifth time through in the study period, Susan very quickly reviewed each page sequentially from P1 to P6, and again skipped all calculation parts (Figure 4.30).

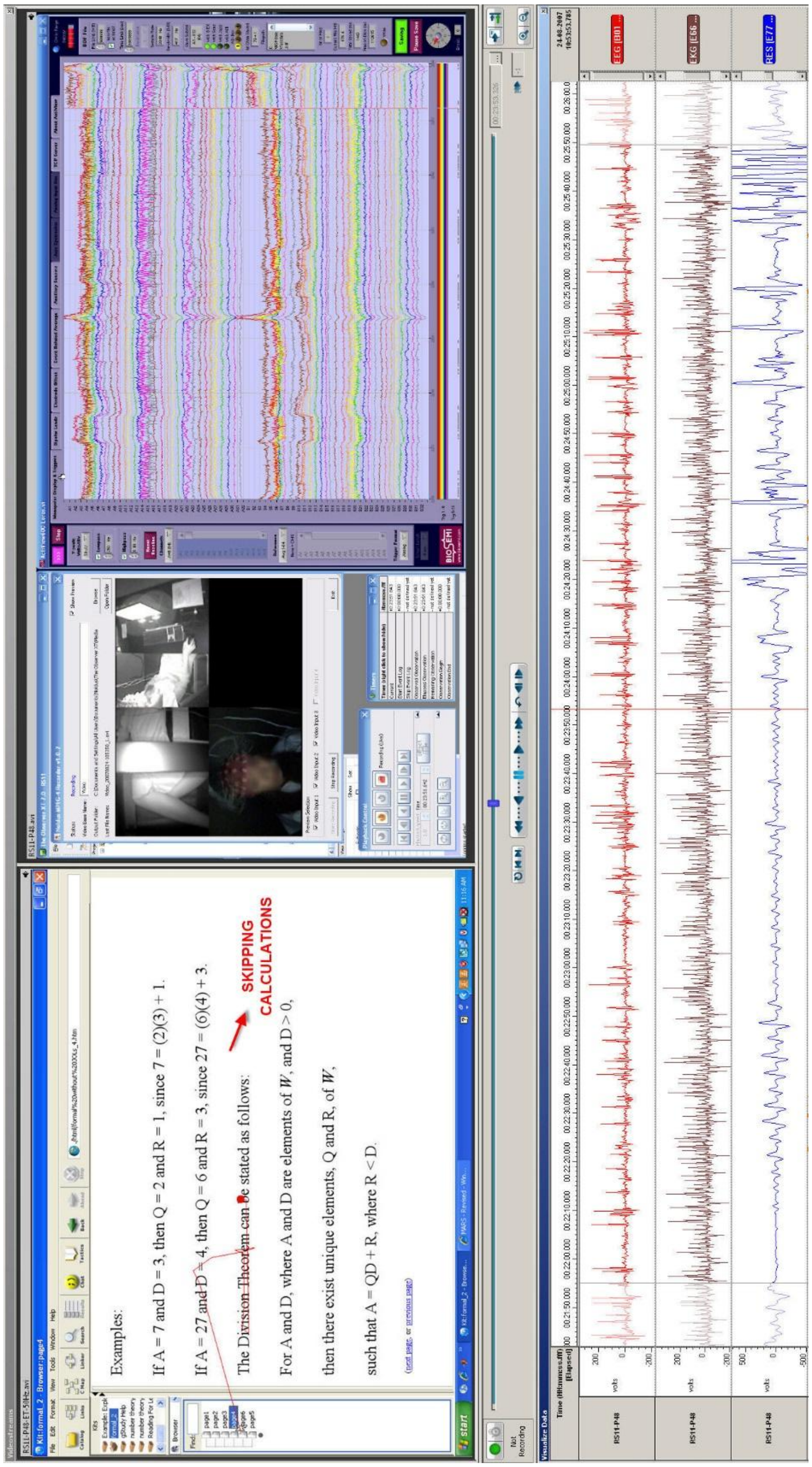


Figure 4.30. Skipping calculations on P4

While studying P5, she took a deep breath, repositioned her body and stretched her arms, as possible indicators of getting tired / bored. On P6, where she continued to ignore calculations, she checked the time once, and scratched her legs. Figure 4.31 shows a major escape from calculations on this page.

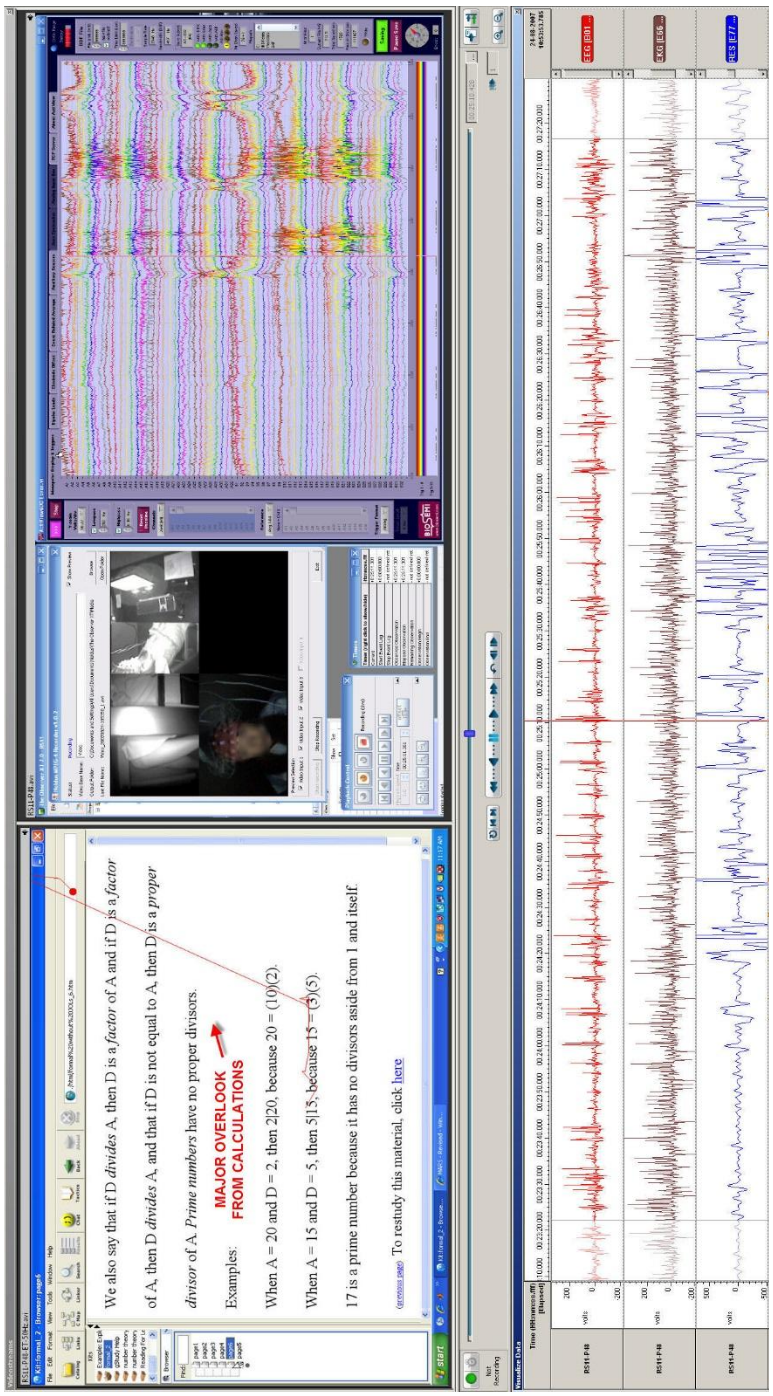


Figure 4.31. Major overlook from calculations

During the last two minutes of the study period, Susan showed indications of getting tired / bored by checking the time more often, taking deep breaths more often, repositioning and stretching her body, or playing with her cloths. This distraction period can be clearly identified on the respiration channel. Figure 4.32 provides an overview for the first six minutes of the study period where she was focussed and the last two minutes of the study period where she was distracted.

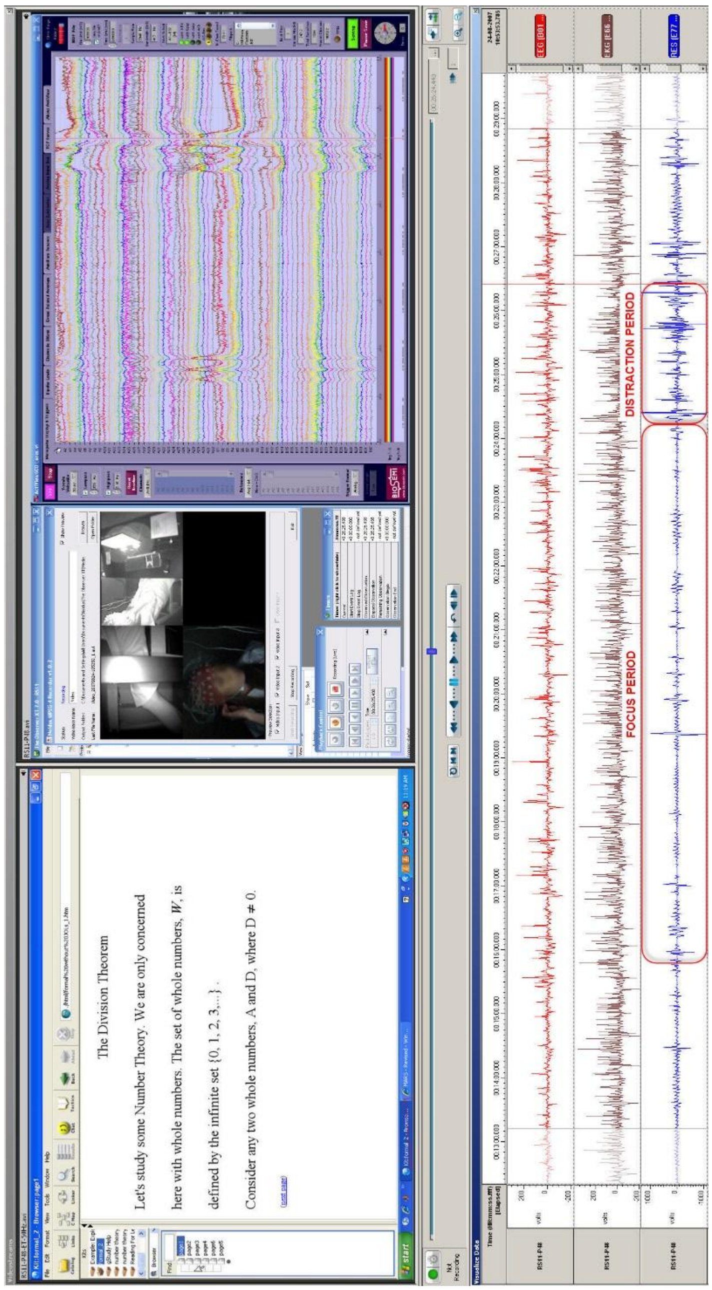


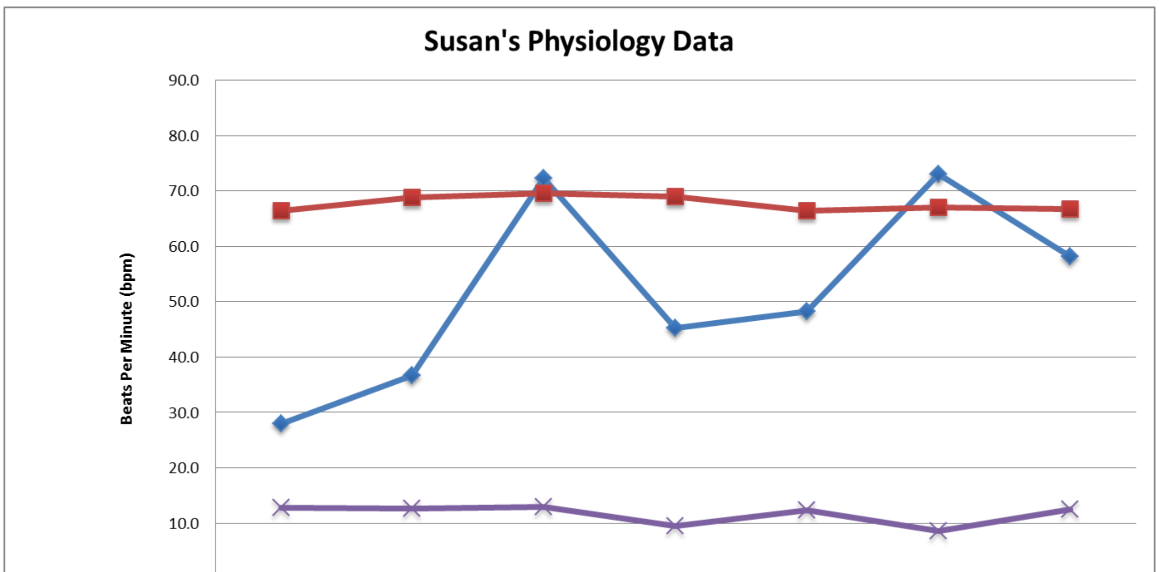
Figure 4.32. Study period: Focus vs. distraction on respiration channel

4.3.5. *Self-Reporting JOLs*

According to the self-reporting results, Susan's average confidence score for calculation is the lowest among all other participants (1.5/6) whereas her average scores for understanding (11/14) and reasoning (10/15) are the highest among all four participants.

4.3.6. *Restudy Period*

For the restudy period, which took less than 100 seconds (she ended by saying "Okay, I'm done"), Susan quickly went through the pages with no specific attention to any particular section or page. She looked calm and relaxed; there were no significant physiological or behavioral events observed. Her eye blink data indicated a significant rise during the study and restudy phases (Figure 4.33). Because it is not in parallel with her heart and respiration signature, this is not evaluated as an indication of anxiety. This effect is considered partially due to her eye fluttering behavior during the studying and restudying of the material.



	BASELINE	QUESTIONNAIRE PERIOD	STUDY	SELF-REPORT	NUMBER THEORY TEST-1	RESTUDY	NUMBER THEORY TEST-2
BLINK RATE	28.0	36.7	72.2	45.2	48.2	73.0	58.1
HEART RATE	66.5	68.9	69.6	69.0	66.5	67.0	66.8
RESPIRATION RATE	12.9	12.7	12.9	9.4	12.3	8.6	12.5

Figure 4.33. Susan's physiological data of throughout the experiment

4.3.7. Test

Susan took the test twice, before and after the restudy periods. According to the test scores, she is among the least successful in understanding (50% for the first trial and 0% for the second trial) and reasoning (17% for the first trial and 0% for the second trial) related questions. She answered all questions sequentially and promptly. She was observed being calm and relaxed during her test periods, no other significant events were observed.

4.3.8. Post Questionnaire

Answering the question if how much interesting was the learning task for her; Susan's rating was 0/7(not at all). When asked if how challenging the task was, her rating was 2/7. She reported that she restudied items that were difficult for her to understand.

4.3.9. Analysis

At the end of the observation and among different parts of it, just like all other participants, Susan was asked to close her eyes and relax. While all other participants entered a sleepy state during such periods with their eyes closed, Susan kept her eyes wide open and frequently repositioned her body. This resistance to enter the relaxation period of her was manifested throughout all relaxation periods of Susan. Later on during her post-experiment self-report, she explained that is because she was uncomfortable of being watched while relaxing.

An interesting physiological note is that when she closes her eyes, her eyelids immediately start fluttering when processing information (same effect also continuously happened at the additional sessions after the actual observation, where she was asked some questions she did mental calculations, such as counting backwards from 100 by sevens). Multiple times throughout the observation, she closed her eyes for a few seconds when she wanted to focus on something. Considering Susan is regularly meditating, this might be her intrinsic strategy to close her eyes, try to relax, tolerate, and digest information when she faces a challenging task.

Even though she was the only participant who had no relaxation state (which significantly diminishes respiration rate), Susan had the lowest average respiration rate (12.2bpm) among all participants. This data are in parallel with her observable calm state during the experiment.

Susan self-reports most calculations as not well understood, in spite of the fact that she did not attempt studying them, instead ignored as explained above. Therefore, she is considered having performance-avoidance motivation when it comes to calculations. Although it cannot be generalized, she was sometimes observed having mastery-approach motivation when it comes to understanding and reasoning related tasks.

4.4. Participant-4 (John)

4.4.1. Demographics

John is a 45 years old male graduate student in Education. He has British father and German mother, and his mother tongue is English.

4.4.2. Psychological / Physiological State

John is hyperopic and using glasses (he was the only participant using glasses). He has asthma; therefore, his respiration signal was indicating weak yet frequent breaths throughout the experiment. For this reason, his respiration rate is 19.8bpm on average, which is significantly higher than all other participants. He did not have enough sleep the night before the experiment that showed little or no observable effect on his performance.

After the experiment, John self-reported feeling somewhat nervous before the observation, feeling worried about some of his answers during the observation, and feeling relaxed after it. His experiment took 54 minutes.

4.4.3. *Pre-Study Period*

John had significant characteristics during the pre-study period that are different from the others. For example, interestingly for his DQ, he indicated that he is a high school graduate, although he has bachelor's degree and pursuing Master's degree. He was tending to skip similar personal questions. That was evaluated as an indication of his concern for privacy, although he was explicitly informed that his identity would be kept anonymous.

Otherwise, John looked relaxed and confident throughout the pre-study period. Although he was generally easygoing on the tasks cheered and smiled often, John tended to question and comment aloud on things that were unclear to him. As an example, he mentioned that the room was a little dark to type. He made similar comments, complaints, or asked questions in case he needed clarification. These behaviors are in parallel with his EBI Omniscient Authority score where he gets -0.8/7 that is the lowest score among all participants, that is to say he is likely to have a character where he does not hesitate questioning rules, conditions or authority.

For his pre questionnaire, John indicated that he is very much comfortable with thinking mathematically/logically, and very much comfortable with his thinking/reasoning skills. He also self-reported that he had average comfort with the content material (TDT) he was about to study.

According to the MSLQ results, he got the highest score on the Intrinsic Motivation Scale (6/7) and lowest on the Extrinsic Motivation Scale (3/7) among all participants. These results indicate that he is likely to be a mastery learner.

4.4.4. *Study Period*

John's first time through studying the material took only 1:50 minutes. He quickly scanned through some pages for less than 10s each. Studying some pages (such as P2), he often went back to the previous page (P1, in this case) to briefly review it, and bridge the two conceptually.

It was common throughout John's study of the material to skip verbal parts (those he most probably consider as less important), and to spend more time focussing on the key concepts, studying definitions of such concepts, and dealing with numerical examples of them. John's studying of P4 is an example of this strategy (Figure 4.34).

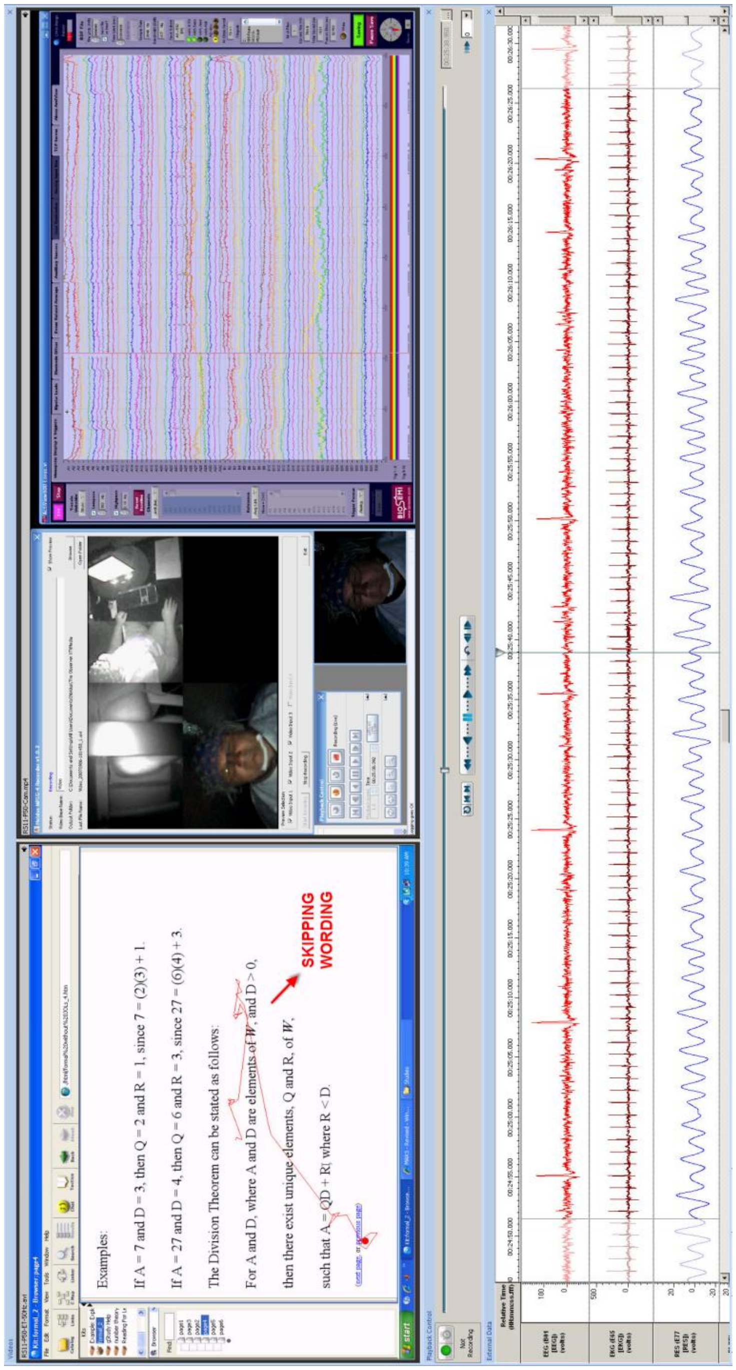


Figure 4.34. Skipping verbal parts on P4

It was observed that for his first time through, as a strategy, John mainly aimed to familiarize himself with the material and to spot key sections he considers most important, with the intention of focussing on these in more detail afterwards. The hotspot analysis of his eye tracking data on P4 is an example how he skipped wording and preferred to focus on key parts on that page (Figure 4.35).

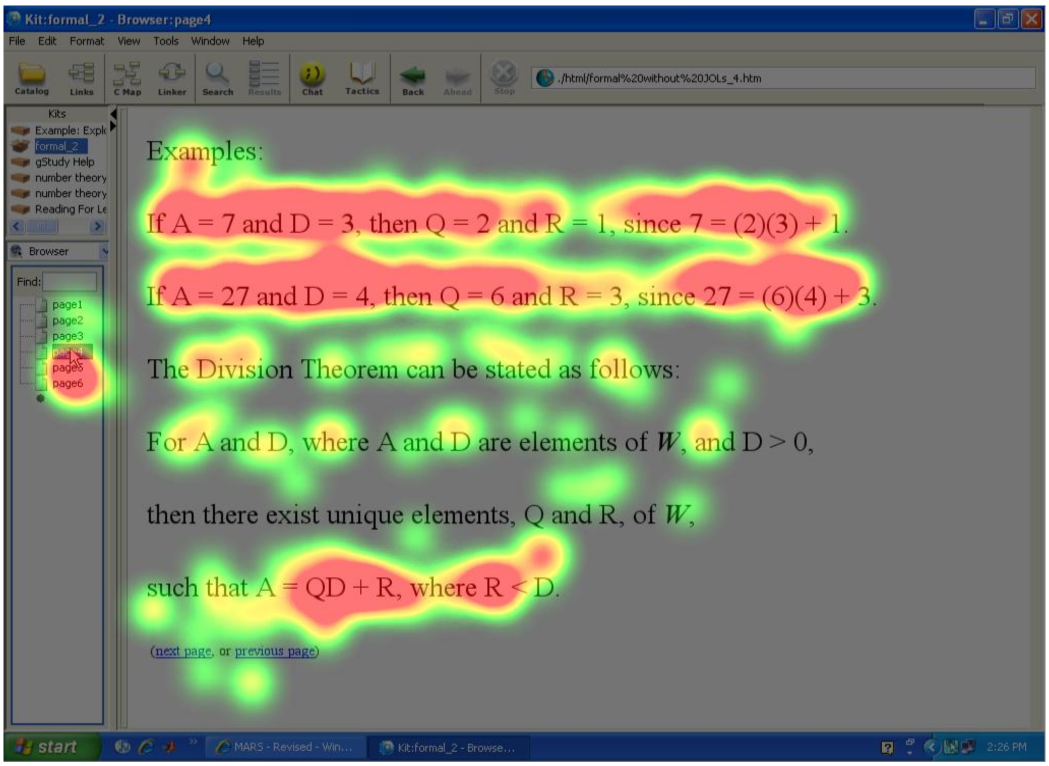


Figure 4.35. Hotspot analysis of P4 indicates skipping verbal parts and focusing on key content

Another evidence how John disregarded verbal sections showed itself on the quantitative eye-tracking analysis. According to this analysis, he spent least time on P1, where the content is verbal, and there are no definitions or examples. On this page, he spent only 5% of his total gaze time during the study period, and 2% of his total gaze time during the restudy period. He tagged almost all JOLs (except one, which he tagged as 'well' understood) as 'very well' understood on this page.

John was evaluated spotting key parts of the content consciously by determining which parts to spend more time, and which to spend less time. In other words, he did not ignore these verbal parts because they were hard to understand; rather he aimed to make better use of his time by focussing on parts he considered more important. Therefore, his behavior on the first page is evaluated as an outcome of mastery-avoidance motivation orientation. Note that he was the only participant following this strategy; therefore, he can be evaluated as a more expert reader and learner of mathematics content in comparison to the other participants. This selective characteristic of John in studying also shows itself in his Information Management Strategies scale score of MAI, which involves self-reporting items such as “I consciously focus my attention on important information”. His score for this scale was 6.3/7 that is highest among all participants.

Even though it was effective for calculation parts, this strategy became perpetual and later caused John to miss some important content that is important to gain for understanding the material. Some examples of such content are divisibility relations on P5 and division concepts on P3. Disregarding verbal content is thought as one of the reasons why John received 0% success rate for the understanding tasks in the Test-1. This score is the lowest among all participants.

John preferred to spend more time studying calculations. Figure 4.36 shows the number of average fixations and average gaze time per JOL for calculation, understanding and reasoning.

John’s second time through studying the material took 7 minutes and 13 seconds. He skipped the first page after spending just a few seconds on it. After that, he carefully reviewed P2. He was the only participant marking all JOLs on this page as very well understood. He spent 45s on P3. He apparently wanted take his time to focus more on this page, because he left the mouse, leaned back and took a deep breath before beginning to study it.

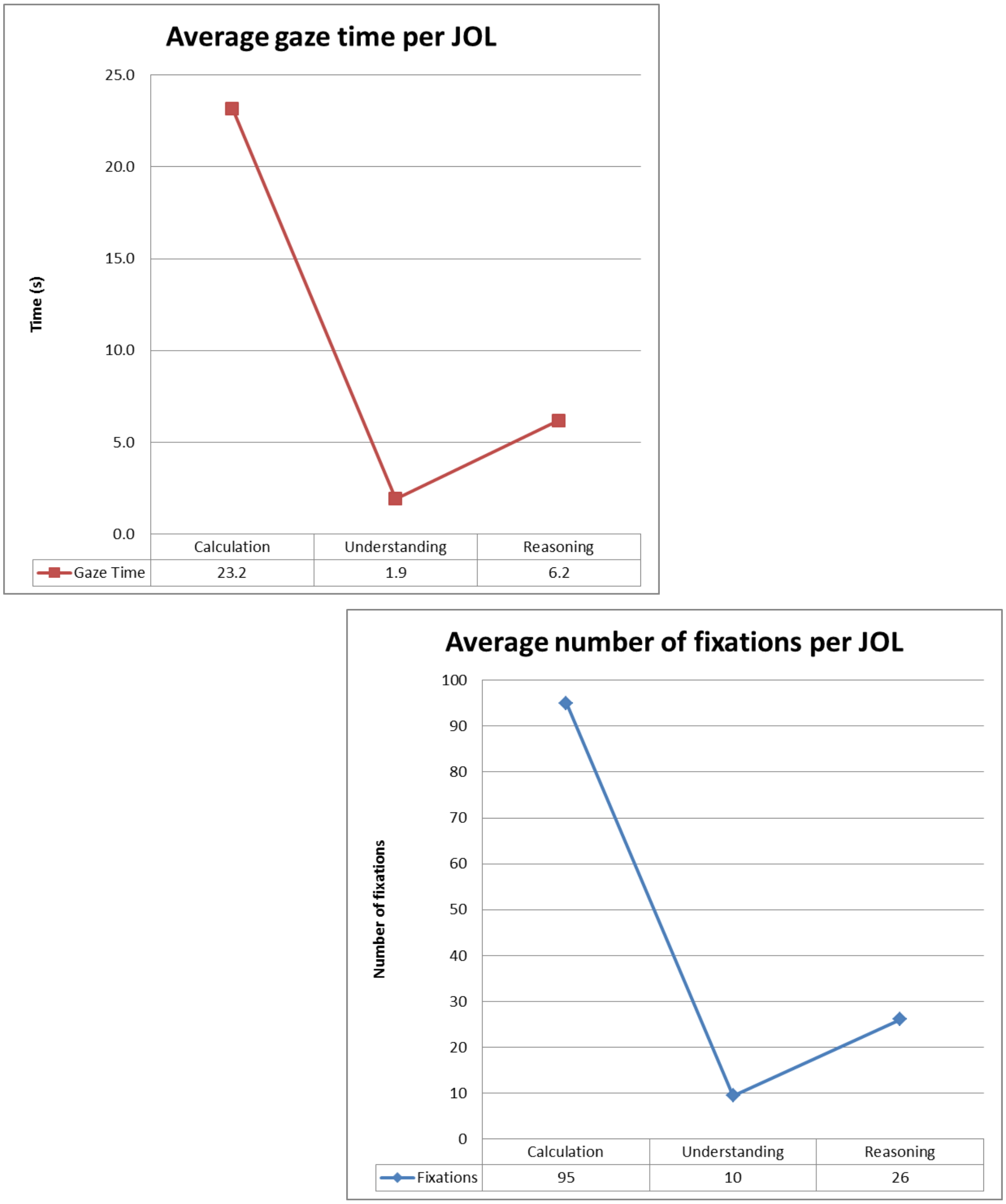


Figure 4.36. John's eye tracking data per JOL during study period

On P4, on his second time through studying the material, he once more spent most of his time trying to understand the calculation parts, and again skipping the wording, then just focusing on the general expression ‘A=QD+R’ (Figure 4.37).

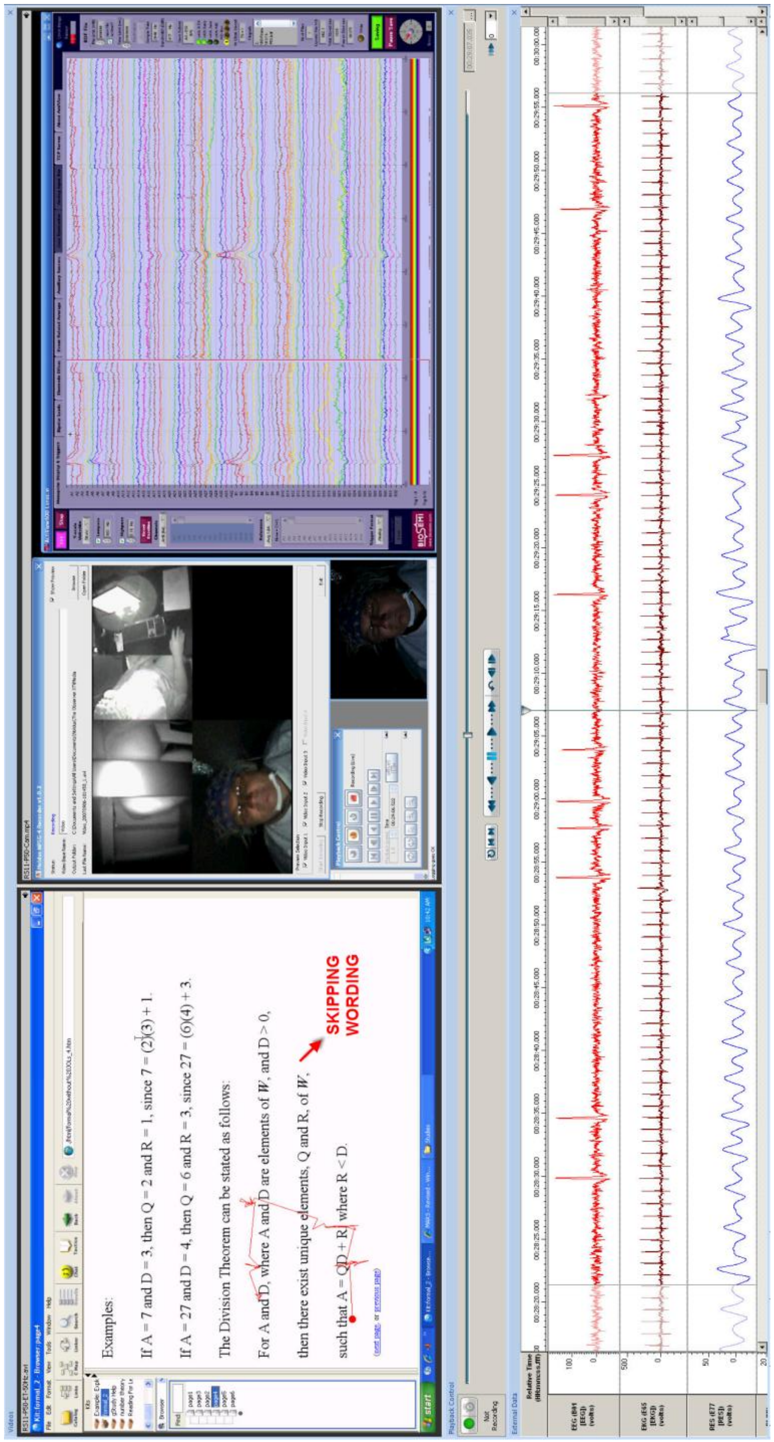


Figure 4.37. Skipping verbal parts again on P4

During the observation, John often highlighted the sections of the material that he wanted to spend more time focusing on. In this case, he highlighted expression ‘A=QD+R’ on P4 (Figure 4.38).

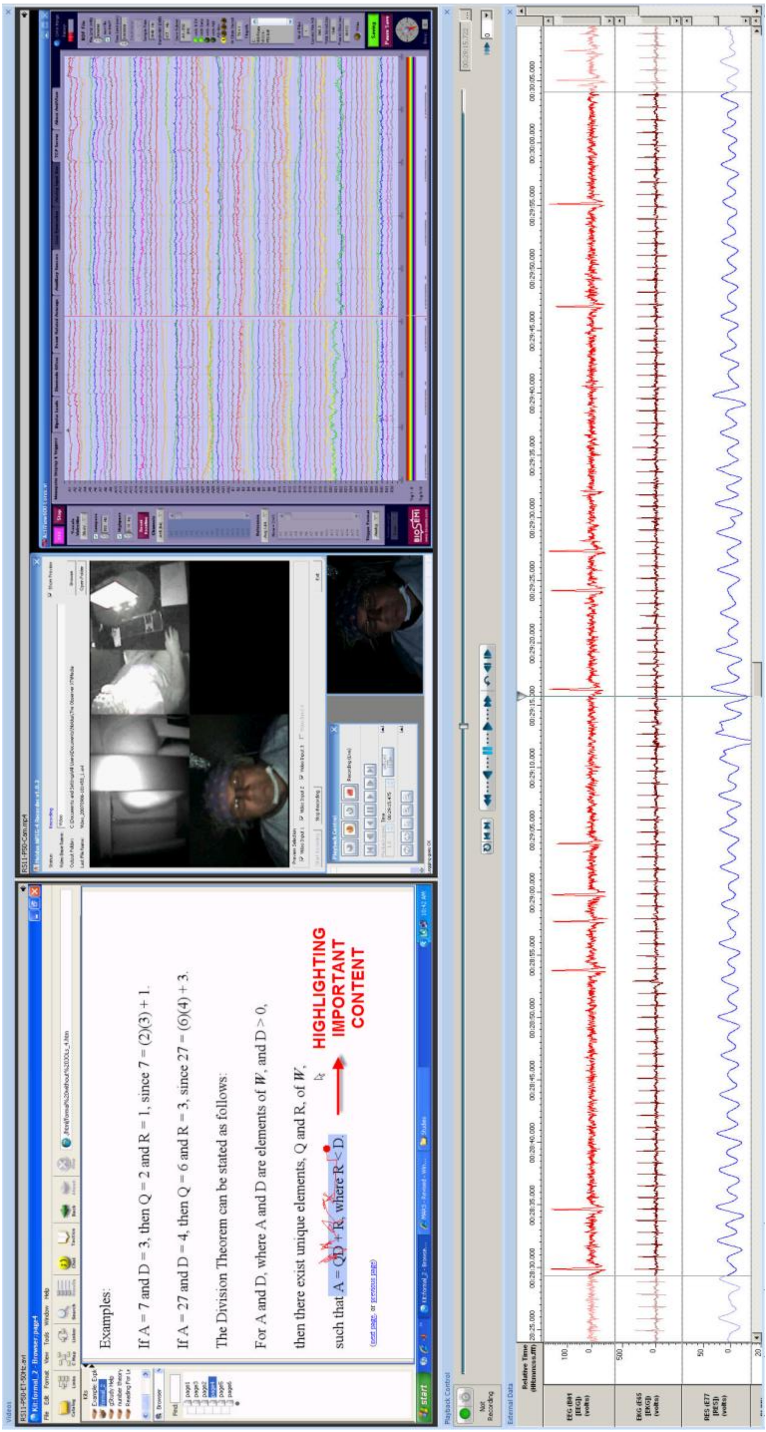


Figure 4.38. Highlighting important content

Figure 4.39 demonstrates another example of his highlighting technique and how he relates the general expression ‘A=QD+R’ to the numerical example.

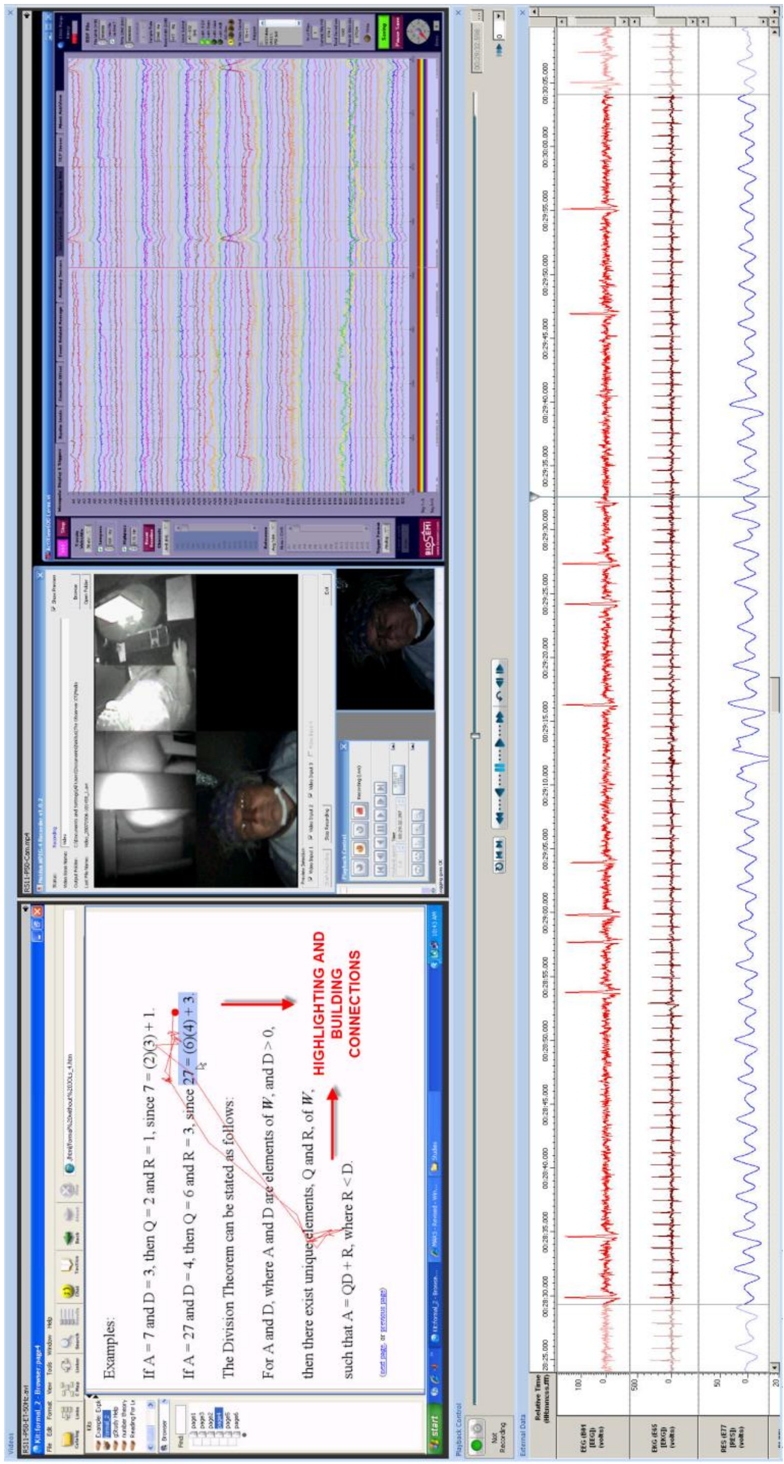


Figure 4.39. Highlighting and building connections

Then John reviewed P5. Considering P5 mainly involves verbal content, just as he did for P1, he preferred to skip this page quickly with no particular attention. P5 is where he spent second least time (9% of total study time) after P1 (where he used only 5% of study time).

Differently from other participants, John followed a conscious cross-comparison strategy *among* pages when studying the material and answering test questions, where he makes quick comparisons, and build connections *across* pages.

As an example (see Figure 4.40), he compared the different examples (with and without remainder) on P4 and P6, and then rechecked the definitions of the related concepts on P3. After re-reviewing the definition of uniqueness within the division theorem on P2, he went back to P4 where he once again reviewed the examples and their relations with the formal expression of TDT. He finally went back to P6, where he started, and reviewed the example one more time.

While all other participants focused on connecting information within a single page, what is unique with John's technique is his swiftness in switching among pages, spotting and reading related parts quickly, as though all of this information were on a single page. Note that in this case, it differs from Betty's Spider's Web that was an intense connection building between concepts within a single page, whereas John's Cross-Comparison Trajectory is among different pages that involve related content. This phenomenon also evidences that he could attribute each page to its content as he went through them, and consequently was able to use his short-term memory effectively to build connections across multiple pages. It is important to remember that John received 6.3/7 from the Information Management Strategies scale of the MAI that is highest among all participants. This high score is in parallel with his frequent use of strategies while studying the material.

Figure 4.40. Cross-comparison trajectory among pages, building connections

John spent his last (3rd) time through (which took only 20s) in quickly navigating among pages with no particular attention. He started yawning at this period, as a possible indication of boredom (Figure 4.41). Although lack of sleep as reported in his pre-screening questionnaire could be considered as another possible reason, boredom is considered as a more reasonable explanation because he did not show similar behaviors to this point.

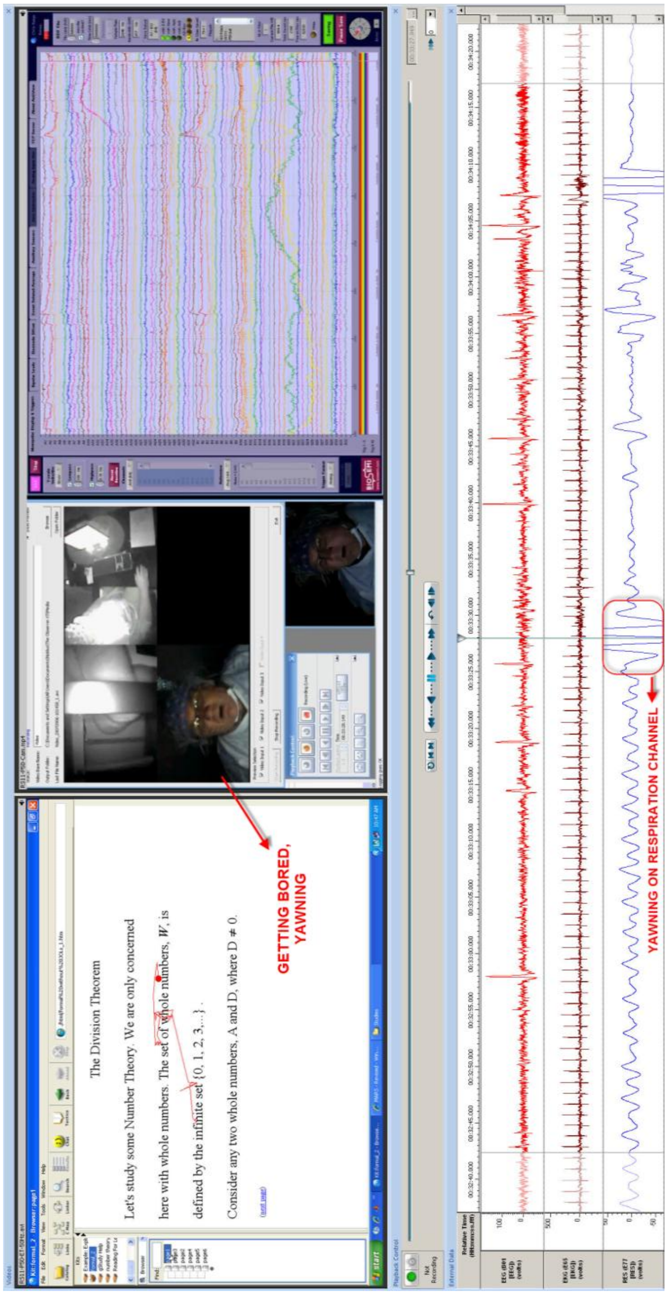


Figure 4.41. Getting bored, yawning

At the end of study period, without waiting for any instructions, John expressed he was done with studying, by saying “Okay, do I have to spend any more time on this? Because I think I got it”. After being told that he has some more time, he was observed being somewhat nervous and replied “Pooh, Okay! Review (!)”. Then he was told that he is welcome to switch to the next phase that is self-reporting JOLs.

4.4.5. Self-Reporting JOLs

John went through the pages sequentially and promptly, self-reporting his understanding of the material. Among all participants, he spent most time on self-reporting JOLs (350s). He was observed being diligent and taking his time during the self-report period.

4.4.6. Test

Whereas other participants were prompt and quick, John took his time, followed different strategies, and drew a unique physiological and psychological profile in the test phase.

While deeply thinking about something, doing calculations, having hardship or making his final decision, he often did multiple (double or triple) rapid blinks. In addition, after selecting an answer John often pressed and held the mouse button on the “Next” box (that links to the next question) for a few seconds. Prior to releasing the mouse button for switching to the next question, he used these short periods to look back to the sections of the current page that involve the question and selections. Figure 4.42 demonstrates an example where John held the mouse button and did multiple rapid blinks while double-checking his answer.

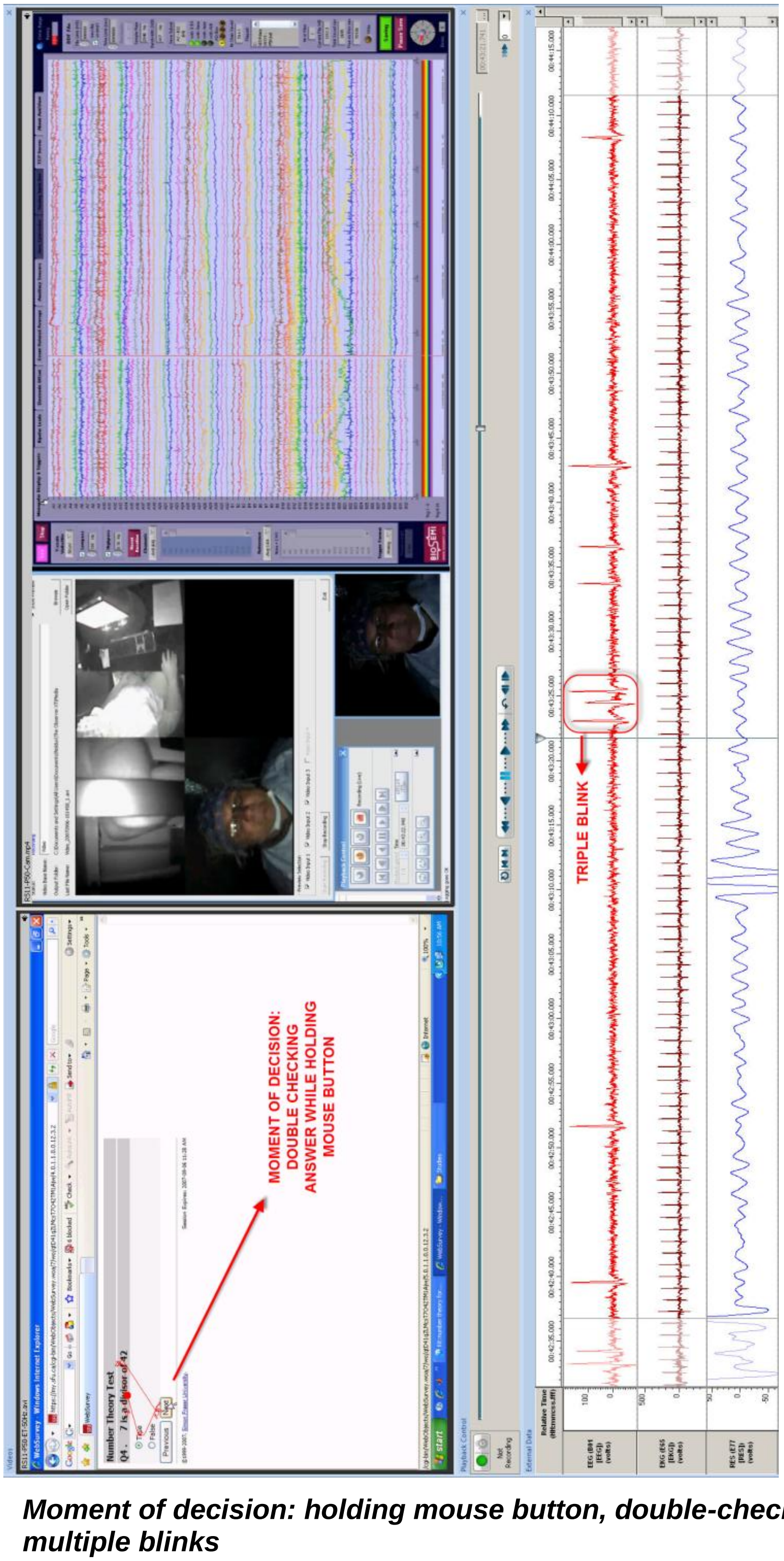


Figure 4.42. ***Moment of decision: holding mouse button, double-checking, doing multiple blinks***

Just as he used cross-validation and highlighting strategies in the study period, John used the same strategies in the test period as well. As an example on Q12, he first used his highlighting strategy for the section that he wanted to focus on (the number of *18*). He seemed to hesitate as to whether *18* is a divisor of *42*. He first selected “true” for his answer then put his mouse on “next”. However at this moment, he stopped, became so hesitant as to whether change his answer or not. Then he recalled a similar question he came across earlier (Q6) and started to build triangles between the selection, *next* and *previous* sections (Figure 4.43). This moment of decision took a few seconds. At the end, rather than releasing the mouse button to continue to the next question, or changing his answer, John decided to go back to Q6 that was six pages earlier.

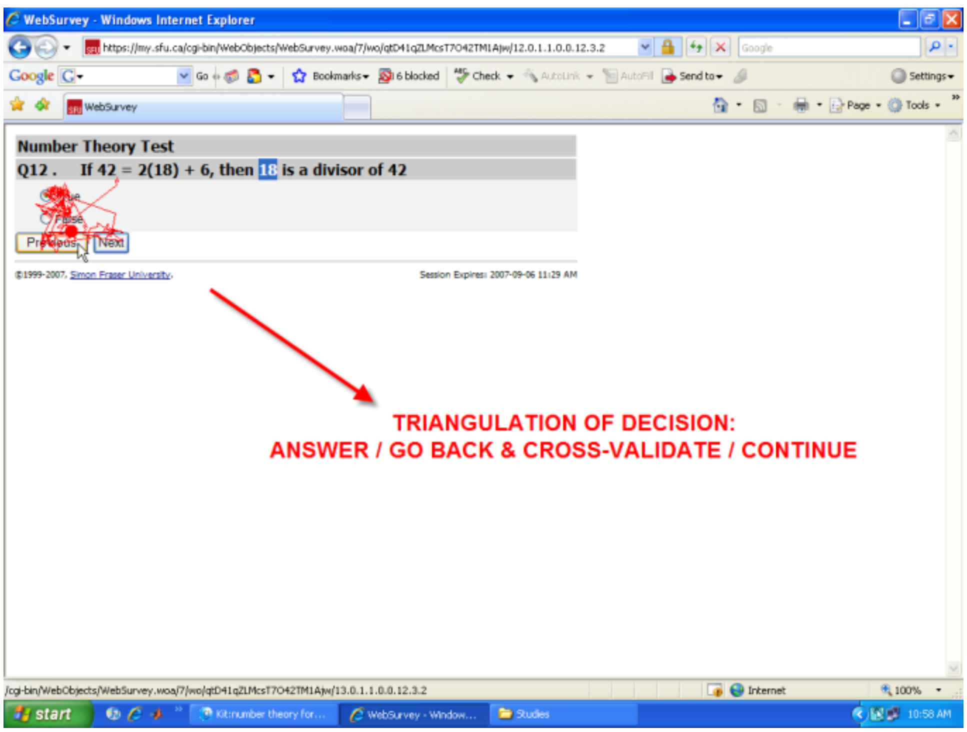


Figure 4.43. Triangulation of decision on Q12

Q6 and Q12 are similar questions those aimed at measuring participants’ understanding of the basic concepts with TDT (i.e. divisor and dividend). In his self-report, John did not report “very well” for any of the definitions (JOL scores for: dividend →[0.5], quotient →[0], divisor →[0.5], and reminder →[0.5]). Figure 4.44 shows his trajectory between Q12 and Q6. Note that John spent considerable amount of time for *triangulation* *of decision* each time he is about the answer a question. No other participant showed a similar behavior.

Figure 4.44. Cross-validation trajectory in the test phase

In finalizing his answer for Q12, John did double blinks twice, as he faced with hardship on that question (Figure 4.45). His self-confidence rating for his answer to Q12 was 10/10 after this cross-validation.

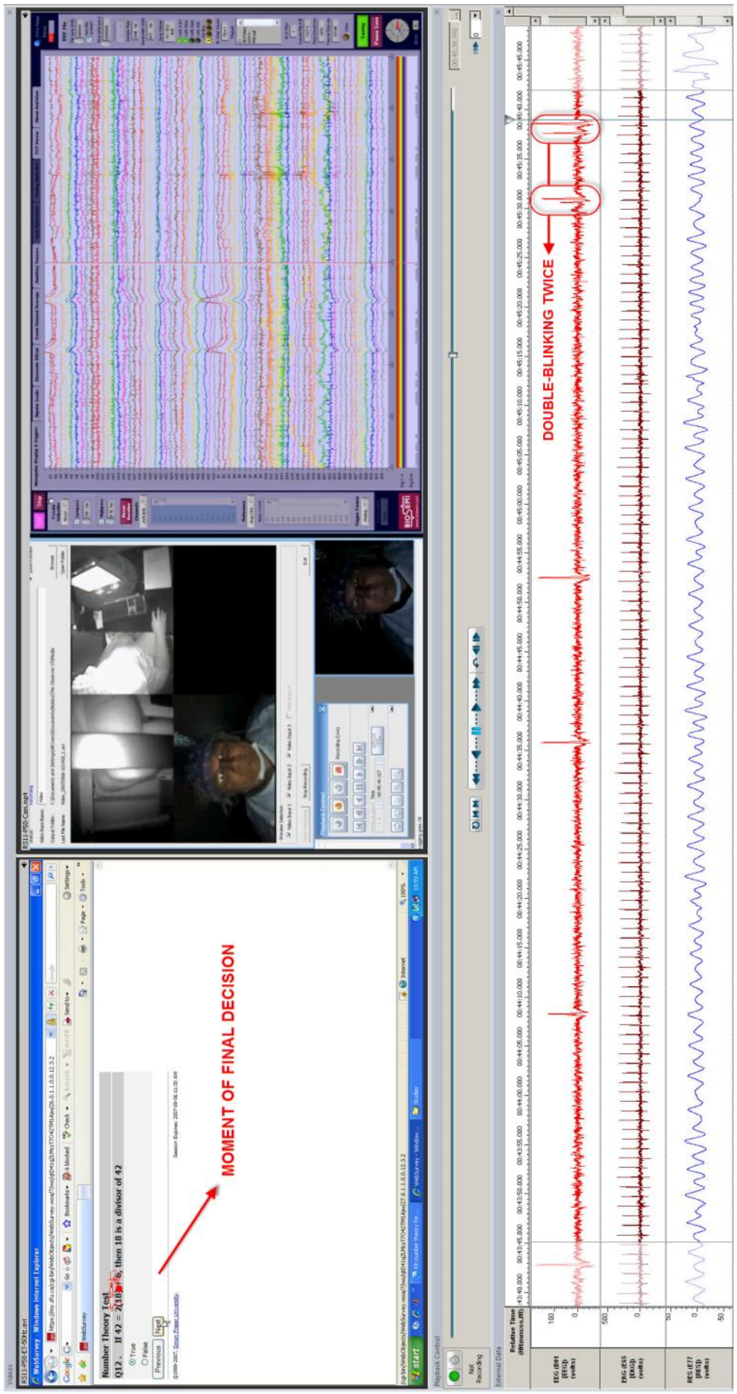


Figure 4.45. Moment of final decision

As mentioned before, John generally skipped all verbal parts in his study phase, and preferred to focus on formulas and numbers those he considers important. However, as he experienced trouble on Q12 and Q6, skipping the definitions for dividend, divisor, quotient and remainder on P3 caused him to face hardship and pressure in the test phase. He was given a second chance to restudy the material, and to take the test once again after restudy. According to the eye-tracking analysis, his gaze time on these four verbal JOLs on P3 during the study period was 5% of his total gaze time on all JOLs. However, after experiencing difficulty on Q6 and Q12, he spent 23% of his restudy time focussing exclusively on these JOLs. As a result, his second review of these JOLs during the restudy phase led him to learn the definitions of divisibility concepts, and to correct his answer for Q6. Figure 4.46 visually demonstrates his ignorance of these concepts during the study period, and his attention to them after the test, during the restudy period.

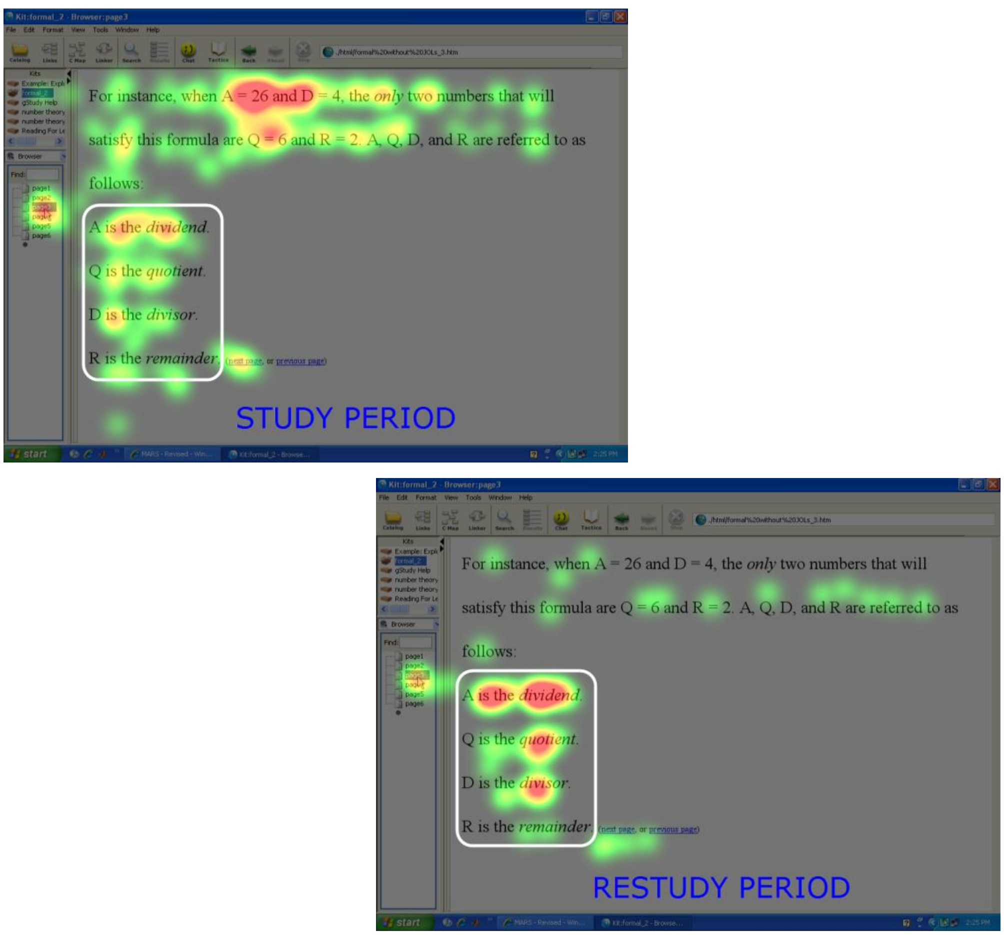


Figure 4.46. Ignorance in study vs. focus in restudy on P3 concepts

Another question relating the same content of divisibility concepts was Q14, where John again confronted with hardship answering. As soon as he read the question once his facial expressions changed from smile to serious, then he lowered his eyebrows and head angle. All of these behavioral changes occurred in five seconds (Figure 4.47).

Figure 4.47. Reactions to hardship on Q14: change of facial expressions in a few seconds

He did frequent multiple blinks during this question. He also held the mouse button at the moment of decision (Figure 4.48). Those are considered indicators of him having hardship with this question.

The next question John faced hardship with was Q20. He repositioned his body at the beginning when he read the question once. After double blinking, he clicked on “True” then “False” and then again “True”. Apparently, he was confused at this moment of decision, and he smiled (Figure 4.49). This was a nervous smile (aka non-Duchenne Smile) as he slightly raised the edges of his mouth and while not moving his eyebrows (Ekman et al., 1990). After that, John reviewed the question a few more times, and whispered to himself for a while during his review. Lastly, he changed his answer to “False” and self-rated his confidence for his answer as 4/10 (his lowest rate of confidence among all questions). This was the last question of the test.

John took the test twice, before and after the restudy period. For both trials, he spent the longest amount of time on the tests among all participants (328s for the first, and 225s for the second trial). He was observed taking the tasks very seriously, and took his time for answering the questions. While the other three participants answered questions one-by-one consequently and promptly, John did not hesitate to stop, double-check his answers, crosscheck with previous answers to other questions, and revise them if necessary. Also before continuing to next question, he had longer periods between selecting his answers and clicking next. He used these times generally for reviewing and revaluating the questions and double-checking his answers. Also just as he did for the study-restudy periods, John continued to use his highlighting technique to mark important parts of test questions. For these reasons, he is considered to have mastery-approach orientation for the test periods.

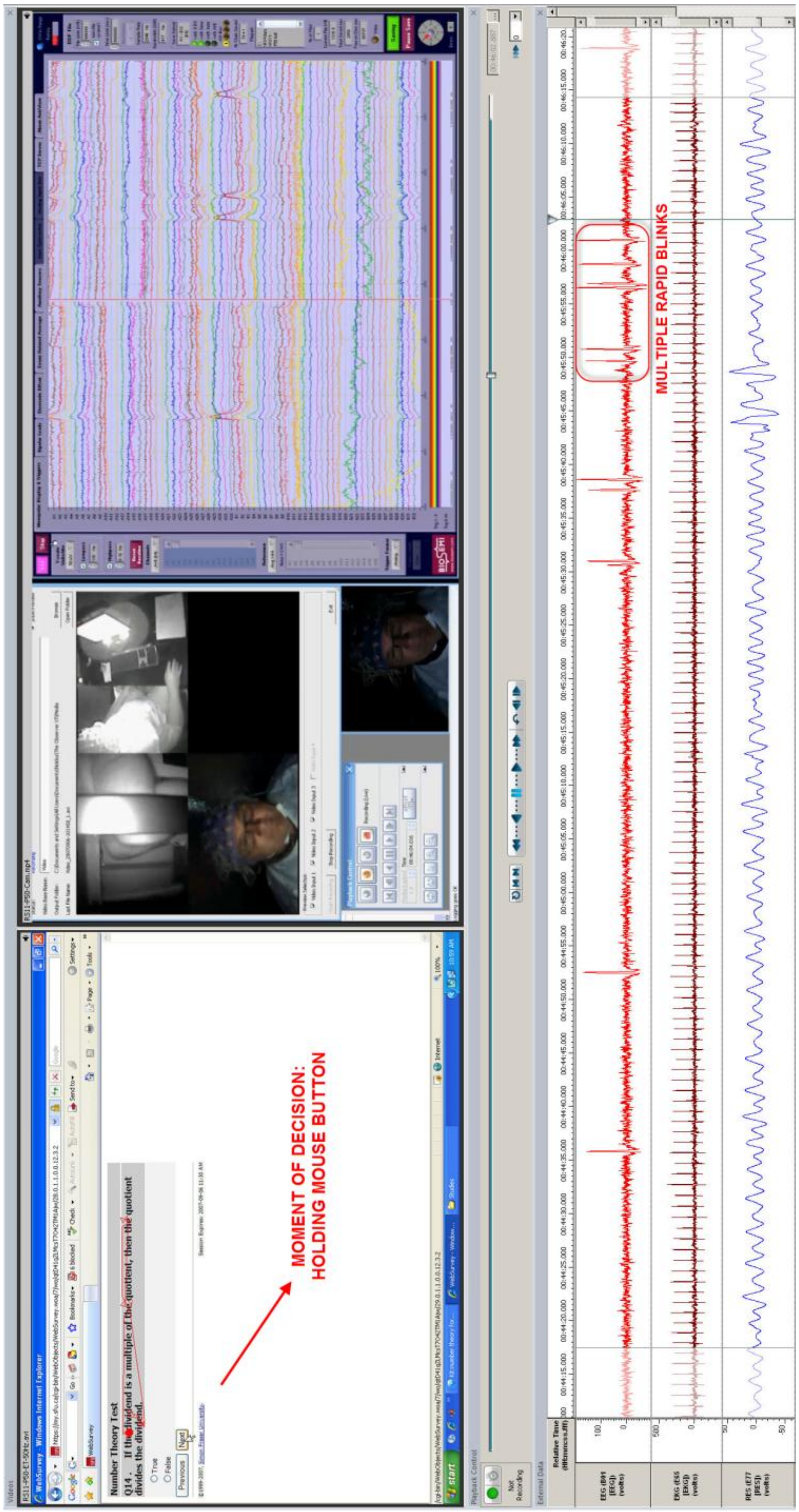


Figure 4.48. Indications of hardship: holding mouse, facial changes, multiple blinks

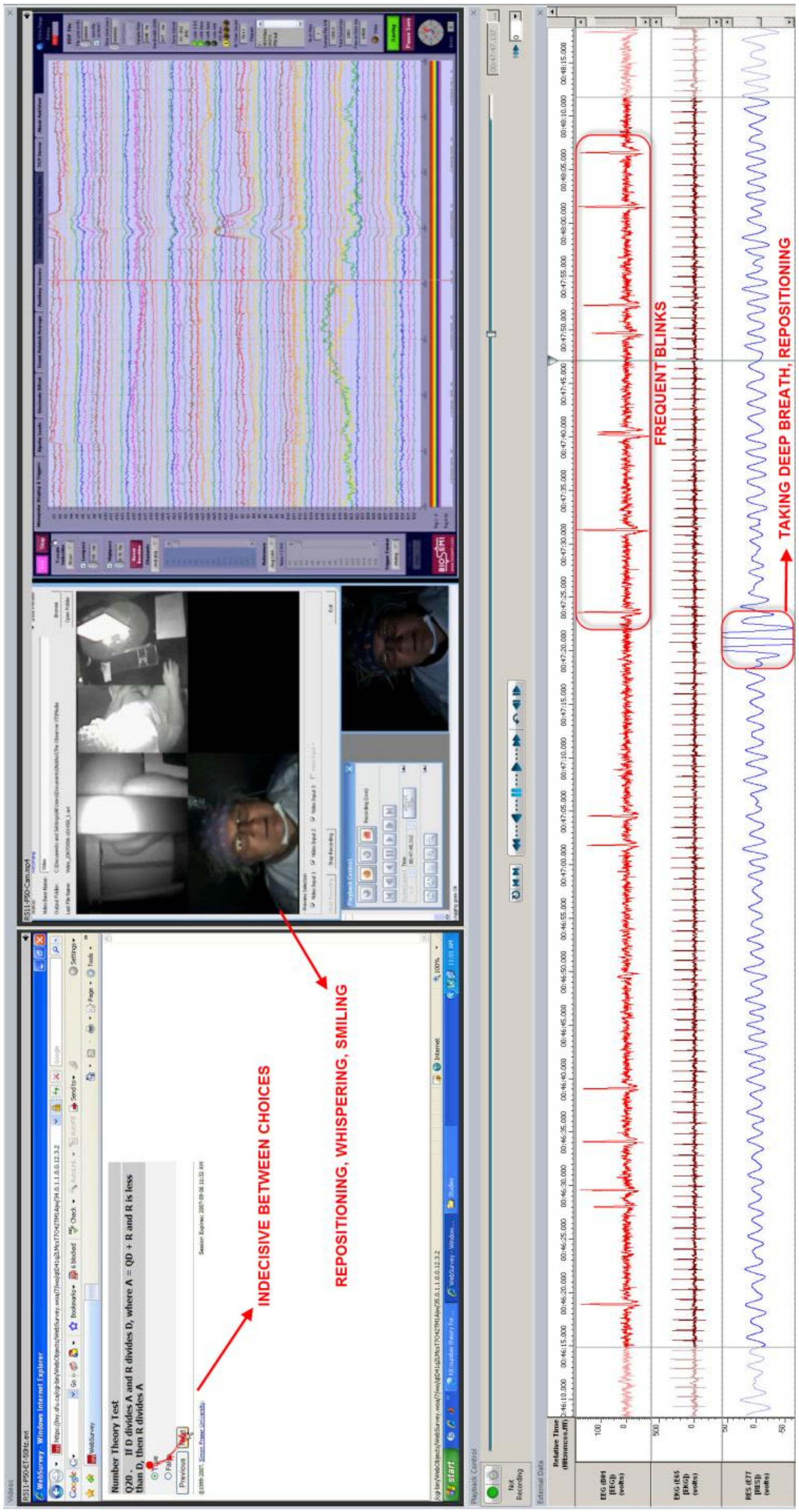


Figure 4.49. Behavioral and physiological indicators for facing hardship on Q20

John was generally relaxed and confident until the test period. Having more serious hardship on the first test trial was unexpected, thus this put some pressure on him. Figure 4.50 demonstrates the physiological signature of his heart and respiratory responses throughout the experiment. His respiration rate had a steep peak in the test period (21.5bpm). His anxiousness during the test period even continued after the test, and resulted as unrest during the relaxation period. As seen from the respiration data, he was significantly more anxious during the relaxation period just after the test, compared to the one before the test (respiration rates 18.3bpm and 9.8bpm respectively). Restudying the material brought him more confidence, and his respiration rate started to diminish gradually. (Note the parallel graphs for heart and respiration rates on Figure 4.50. These graphs were parallel for all participants. This is in alignment with the previous literature and recognizes positive correlation between these rates, thus also validates measurements of the current study).

John's unrest during the relaxation period after the test also showed in his eye related behavior. Although his eyes were closed, his eyelids fluttered with very high frequency. This was specific only to this relaxation period as his eyelids were steady for previous ones. Figure 4.51 shows the change in eye blink rate during that period on the raw data. Figure 4.52 demonstrates the peak in blink rate caused by the fluttering of the eye lids (It is important to remind that eye-fluttering effect was not considered as an artifact in data analysis rather is considered as a valuable physiological cue, thus was not corrected. Notice similar but a much weaker effect on Figure 4.52 after the study period that happened when John was exposed the content for the first time). More active processing of short-term working memory and mental calculation are evaluated having role in this behaviour (Remember similar phenomenon occurred with Susan). However, anxiety of confronting hardship on the test also might have partial effect in the case of John.

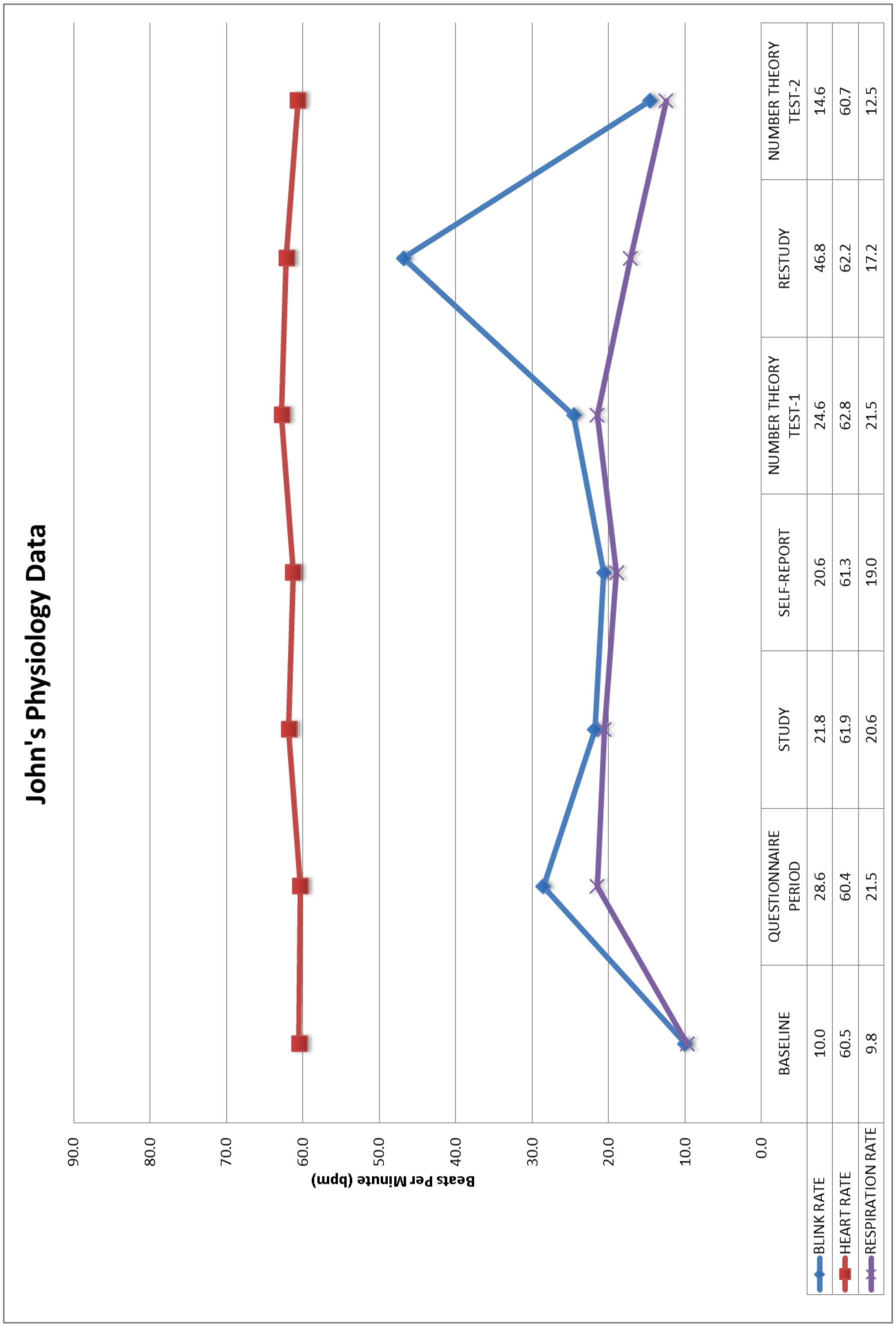


Figure 4.50. John's physiological data of throughout the experiment

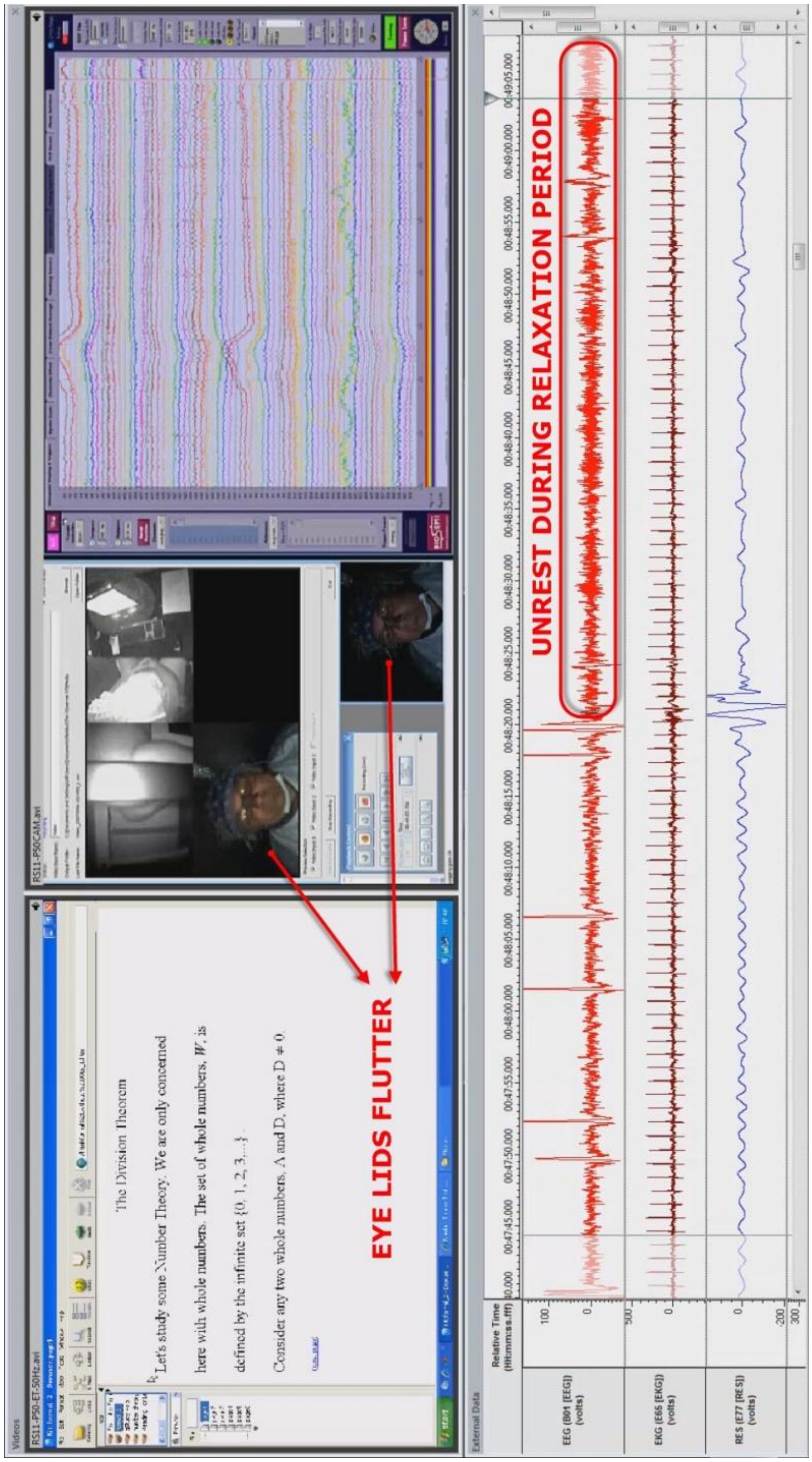


Figure 4.51. Unrest during relaxation period after the first test phase

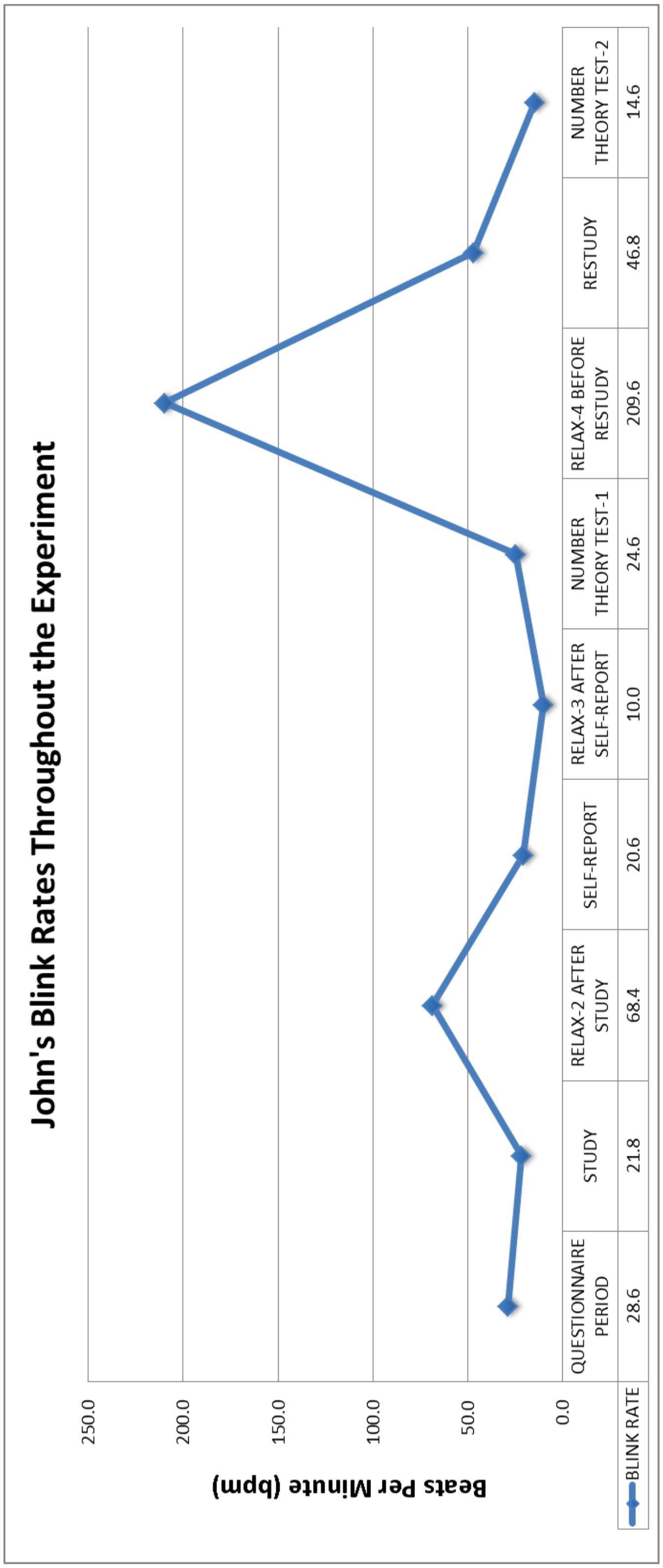


Figure 4.52. Peak on the blink rate caused by unrest during relaxation period after the first test

4.4.7. Restudy

After the eyes-closed relaxation period, John was allowed to restudy the material, where he spent ~2:30 minutes. He was very quick in reviewing pages and searched specifically for the definitions for Dividend, Quotient, Divisor and Remainder (P3). That was because he faced hardship on answering questions measuring his understanding of dividend (Q6) and divisor (Q12). On this page, as his usual technique for studying the important information, he highlighted the part that explains dividend. After highlighting this part, just before switching to the next page, he took a deep breath and repositioned his body (Figure 4.53). This is evaluated as the relief of understanding content that was previously confusing for him. That was the only time he took deep breath and repositioned his body throughout the restudy period. He quickly reviewed other pages and said “Okay” to express he was done with restudying.

As for the next step, John was exposed the test for a second time. During this second trial, he corrected his answer for Q6 (that is asking for the dividend in the expression $42 = 2\ (18) + 6$), and changed his confidence rating from 8 to 10 for this question. Another significant change John made in the second test trial was to change his answer for Q20 (which he had hard time answering on his first trial that was aimed at his understanding and reasoning skills), and his confidence rating for his answer to Q20 from 4 to 10. This increase in his confidence was thanks to his restudying such content.

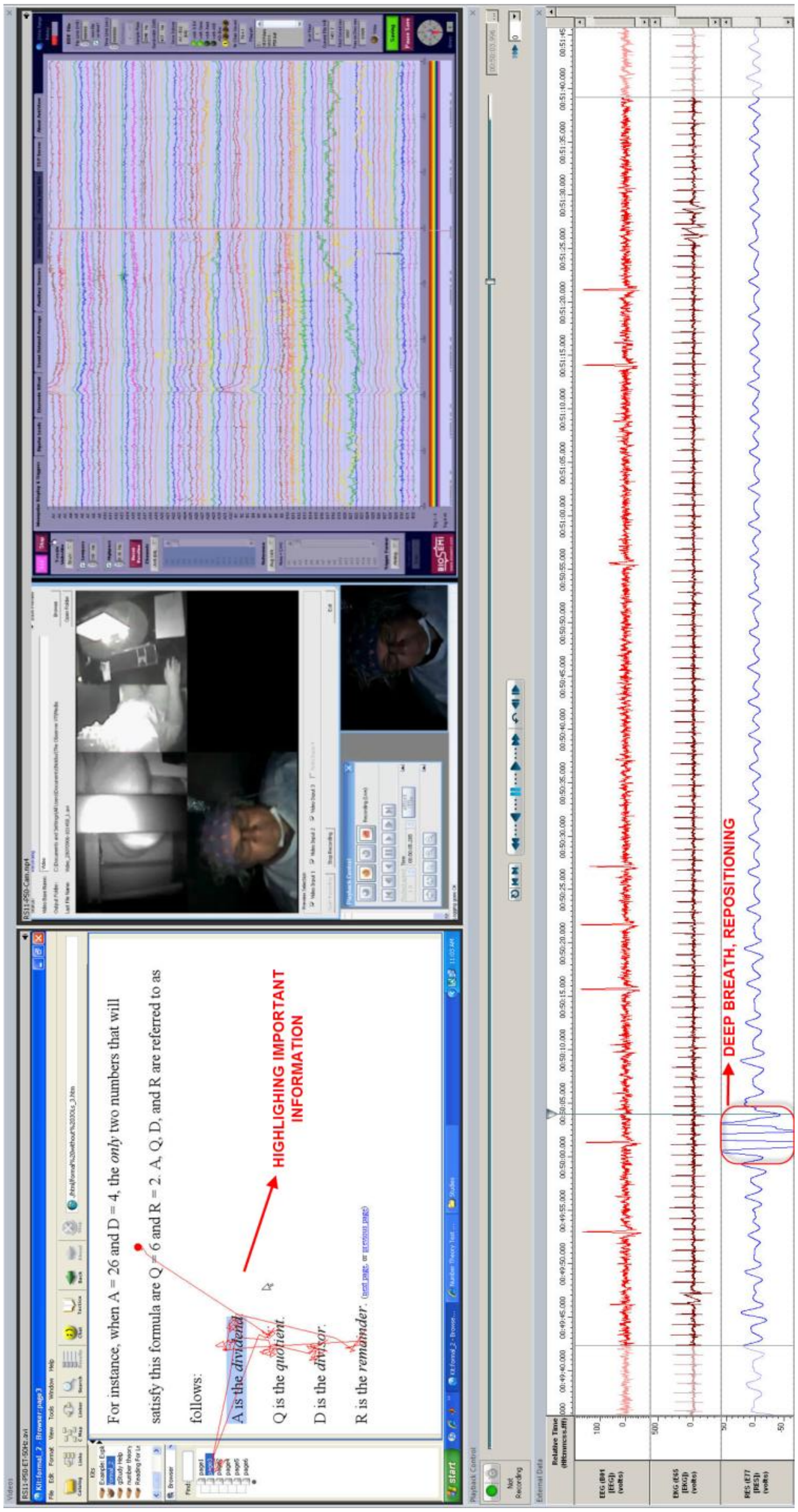


Figure 4.53. Relief of restudying P3

4.4.8. Analysis

John was observed focusing on most tasks diligently with the intention of learning them well throughout the observation. He clearly used the information he learned on answering test questions. He spent the highest amount of time on most tasks among all participants. He did not hesitate to express that he was done with the tasks as soon as he completed them, that is to say he was not wasting his time. For these reasons, he is considered having mastery approach orientation most of the time. John was the only participant making cross-validations and cross-comparisons among pages. He reviewed and connected different pages one after the other, even when those pages took minutes of study time and had different content between them. This is another indication that he was focusing on tasks with mastery motivation, that way he was able to compare, bridge, and integrate information. This interpretation supports the hypothesis of Barnett and Seefeldt (1989), it is in parallel with the findings of Epelboim and Suppes (2001) and Andrà, et al. (2009), and alings with the definition of high working memory capacity by Tse and Pu (2012); that expert learners tend to focus on the key parts of content, target for important cues, and establish intense attentional shifts between them while spending less time on the rest of the content they consider less important (e.g. verbal parts in a mathematics text).

John took his time studying the parts he considered important (mostly calculation tasks). He was the only participant spending more time on calculations with compare to understanding and reasoning related content. He often disregarded verbal parts, most probably because of considering them less important. That caused him having trouble understanding important information and being able to use it later on during the test phase. This behavior of him is not considered as having performance-avoidance motivation on the understanding tasks, rather a studying technique of him where he tends to ignore parts he considers less important. The evidence for this interpretation showed itself in the restudy phase where he focused on these previously ignored tasks more seriously with mastery-approach motivation, and used this information in his second trial of testing. His body responded both behaviorally and physiologically (such as his respiration, eye related behavior, facial expressions or gestures) whenever he faced hardship. These results are also in parallel with the qualitative hypothesis of R. E. Mayer (1983) that restudying increases what is learned, it is helpful for refocusing attention on understanding the relations between key concepts, as well as organizing, rewording and reacquiring them. John’s switch of mediators to understanding related content after the first test trial is also in parallel with the results of Pyc and Rawson (2012), which points out similar effects (such as attention to previously missed parts, better learning of the content, and improved success in retest) manifested when test + restudy + retest protocol is used.

4.5. Learner Profiles

The data above provide a detailed insight into the four learners in an expanded form. The learner profiles below summarize what can be gained from these detailed analyses, discussing the approaches, motivations and strategies of the learners and provide some pedagogical recommendations from a teacher’s perspective.

4.5.1. Linda

Linda is a well-motivated student. She is calm, relaxed and showing honest effort for studying. She follows a strategy where she first focuses on content she is the most comfortable with, that is calculations. When it comes to understanding and reasoning tasks, she is relatively more anxious. She often checks the time when she faces with challenging content, therefore she needs to do more practice with such content and train herself to stay calmer and focused at times she faces hardship. As she indicates on her self-report, she is more successful with visual content, therefore she should be offered more visualized, real life experience driven content and examples rather than theoretical expressions and definitions.

4.5.2. Betty

Betty is normally a well-motivated student. However due to her pre-judgments about the content and previous negative experiences with mathematics, she gets distracted from the content and become anxious with it easily. These biases demotivate her, they are causing her lack of self-esteem for the most part, and preventing her showing more effort to grasp the content. She is resistant and reacts negatively especially to theoretical parts. She requires special care, more dialogue and new ways of introduction to the material, such as real life examples and simplified definitions. She is relatively less successful reader, and reading long pages is often distracting and boring for her. Rather than deductive, she should be offered inductive instruction where she will be offered examples first, and then she can discover the definitions and rules based on these examples with the guidance of the teacher. Offering of a restudy session for the same material that is not well understood causes her getting more anxious, rather than helping her better understand the material. This picture could be different if she is first offered inductive teaching where she can understand the material, followed by the formal content exposed to her where she can conceptualize the material more systematically.

4.5.3. Susan

Generally speaking Susan is a calm student; however it is surprising that she could not keep her eyes closed and relaxed during the relaxation periods. As she self-reported, this was due to the fact that she feels uncomfortable of being watched while her eyes are closed. She does not like numbers and calculations; she generally switches her attention somewhere else whenever she faces with such content. She reports this type of content as well understood although she does not attend to it, therefore teacher should pay extra attention to her behaviors rather than her self-reports to evaluate her understanding of the lessons. It is difficult for her to understand mathematics without overcoming her anxiety with calculations. Therefore she should be provided extra care where she will be guided to discover fun ways of doing calculations and how these are helpful both in school and her daily life.

4.5.4. John

John is a well-motivated and experienced student. He shows extra effort in not only trying to fully understand the material, but also building connections among different parts of the material and applying what he learned once he is tested. When he needs to double check prior information, he can easily spot it. He has some other studying techniques such as ignoring less important content and highlighting important content. He does not need extra care and alternative teaching techniques for similar materials. He has the potential to be offered more practice with the content he learnt (such as more tests), and to be exposed more advanced content in a shorter period of time.

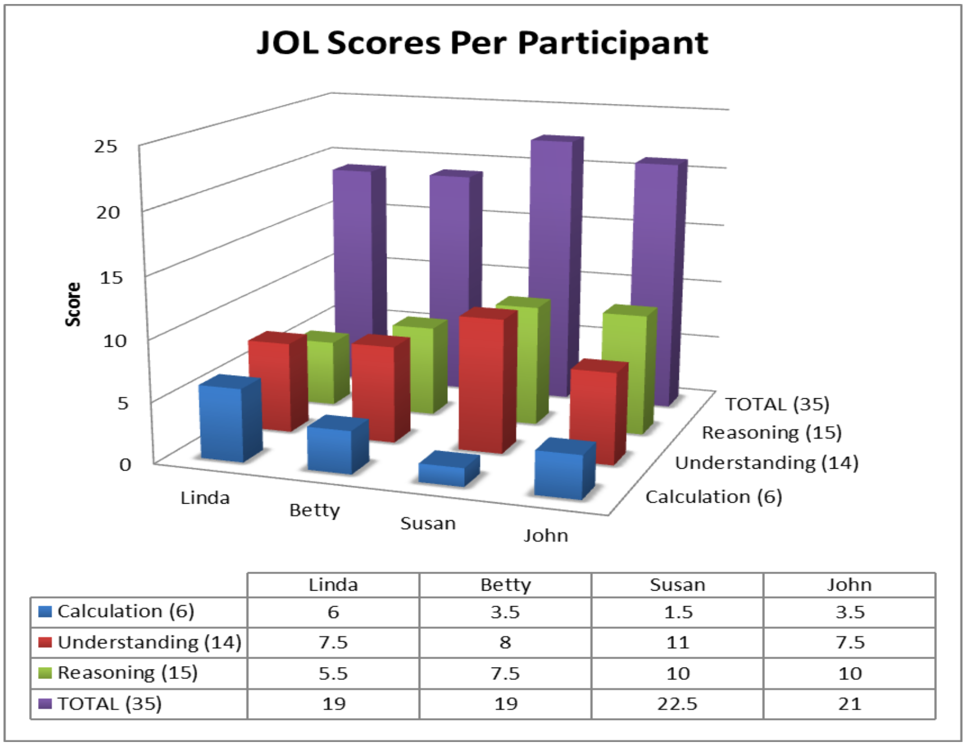
4.6. Cross-Case Analysis

All four participants demonstrate different personalities, motivational orientations, cognitive orientations, and different signatures of behavioral and physiological data. However there are some indicators that stand across multiple participants. Below quantitative data are analyzed across cases.

4.6.1. JOL Scores per Participant

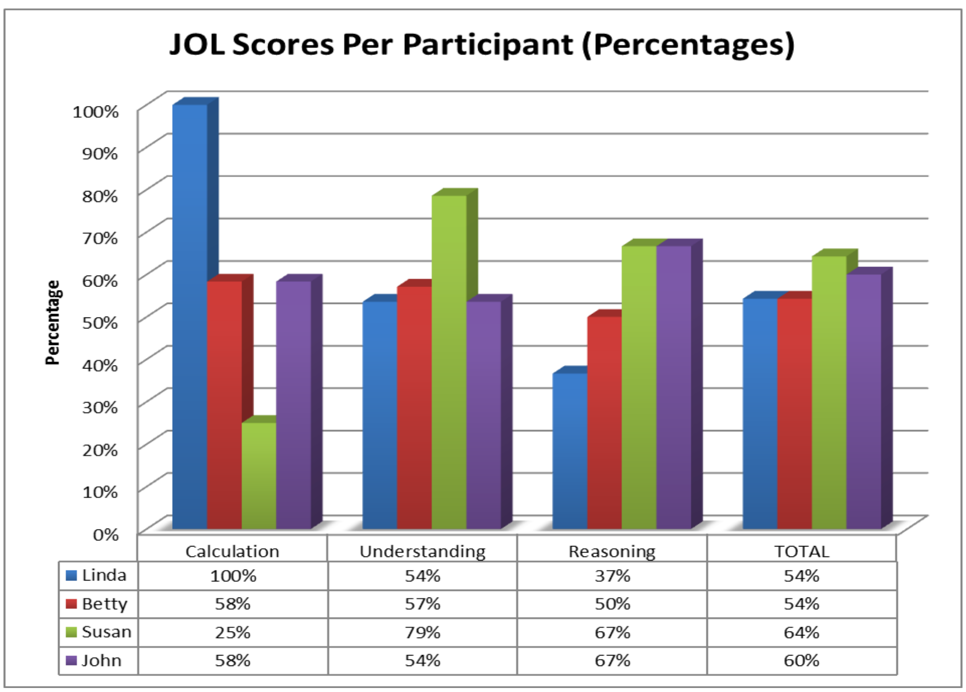
The JOL scores indicate an overall confidence of participants' understanding of the material. Susan shows a lack of confidence with the calculations those she did not study and self-rated her understanding as the lowest among all four (Figure 4.54).

When the same data are calculated based on percentages of overall JOL scores, it can be seen that only Linda had 100% JOL rating for calculations (Figure 4.55). Remember that Linda spent her time first on such content, than she spent most of her studying and restudying on the rest of the content. This indicates her high confidence of calculation capabilities with TDT. Another point is Susan's confidence with understanding related content. Apart from these two points, the JOL scores are in moderate levels within and across participants.



	Linda	Betty	Susan	John
Calculation (6)	6	3.5	1.5	3.5
Understanding (14)	7.5	8	11	7.5
Reasoning (15)	5.5	7.5	10	10
TOTAL (35)	19	19	22.5	21

Figure 4.54. JOL scores per participant



	Calculation	Understanding	Reasoning	TOTAL
Linda	100%	54%	37%	54%
Betty	58%	57%	50%	54%
Susan	25%	79%	67%	64%
John	58%	54%	67%	60%

Figure 4.55. JOL scores per participant (percentages)

4.6.2. *Time Spent On Each Section per Participant*

Linda and John spent more time in total than the other two participants. As it was evaluated that they showed indications of mastery motivation overall, this is seen as an outcome of their motivational orientations where they took their time on the material to learn it better (Figure 4.56).

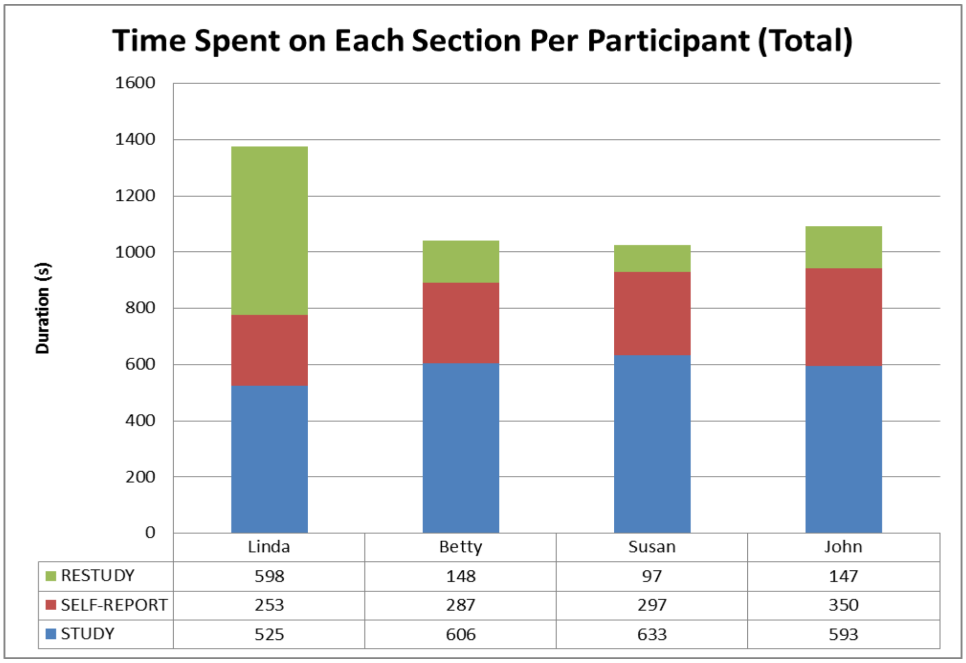
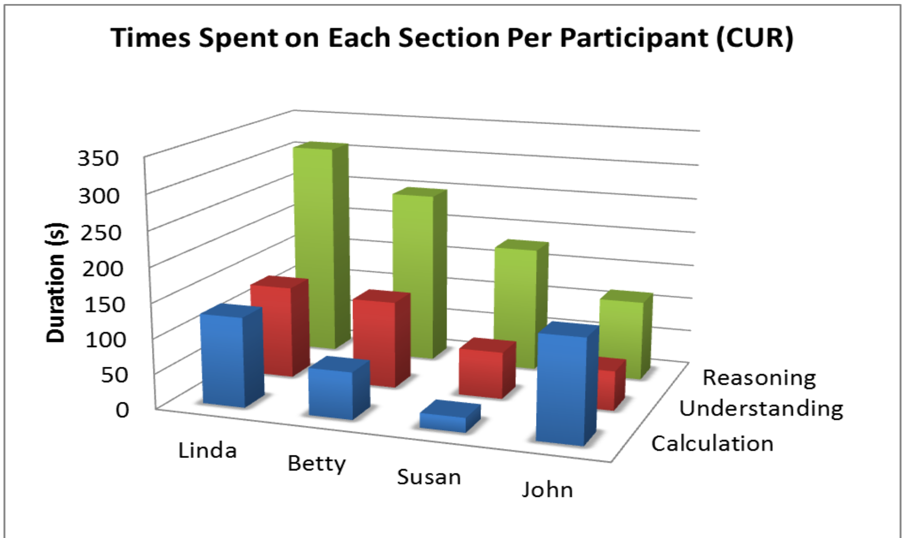


Figure 4.56. Time spent on each section per participant (total)

It also can be seen, although both having mastery motivation, Linda spent a considerably longer time restudying the material compared to John. This was because of the differences between their studying strategies. Linda is a learner who tends to aim at gaining the most from the time she is given. Although she understands the material, she is always willing to use her time to increase her knowledge and experience with the material. On the other hand, John only focusses on the material he wants to learn, after he feels comfortable with it, he does not spend time on it. In that respect, Linda can be evaluated having mastery-approach dominant motivation whereas John has a mastery-avoidance dominant motivation.

When we compare participants' levels of attention on CUR content, we notice that Susan spent significantly less time on the calculation content with respect to others (Figure

4.57). This is a clear picture of her performance-avoidance motivation with such content. It also can be seen that the mastery dominant learners (Linda and John) spend more time on calculation related content compared to the performance dominant learners (Betty and Susan). This is an indication that Linda and John could understand the definition of TDT quicker and spent more time on the numerical examples of the theorem whereas others had to spent most of their times trying to understand the theorem, thus could not understand the numerical examples.



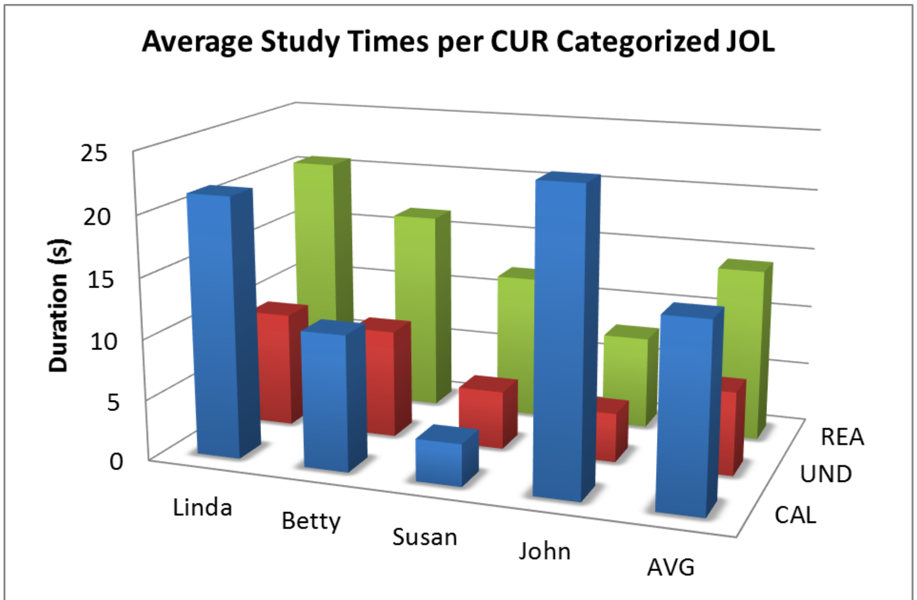
	Linda	Betty	Susan	John
Calculation	128	67	21	146
Understanding	133	125	67	56
Reasoning	311	249	180	115

Figure 4.57. Time spent on each section per participant (CUR)

4.6.3. Average Study Times for Calculation, Understanding and Reasoning

Figure 4.58 shows the average times spent on each type of JOL by each participant, and all participants on total. This was calculated based on the precise numerical output from the eye-tracking measurements showing the total times spent on each JOL for each CUR category. These measurements were then divided to the total number of JOLs for each category to eliminate this effect. The results indicate that Linda and John spent most time on calculations, whereas Susan spent least time on such content. This validates the result that Susan was the most performance-avoidance motivated learner when it comes to calculation tasks. The results also indicate that John spent least time on understanding and reasoning related content. As it was mentioned previously, his behavioral data are in parallel with this result.

When we look at the overall average times spend for all participants classified based on CUR, it is clear that participants more than twice total average times on calculation and reasoning related content with respect to the understanding related content (Figure 4.58). This can be though as a result that participants required more time to understand/verify calculations, and build logical connections, compared to understanding simple expressions and definitions. These results also indicate that calculation, understanding and reasoning are separate categories of mathematical cognition requiring different amounts of times to process knowledge.

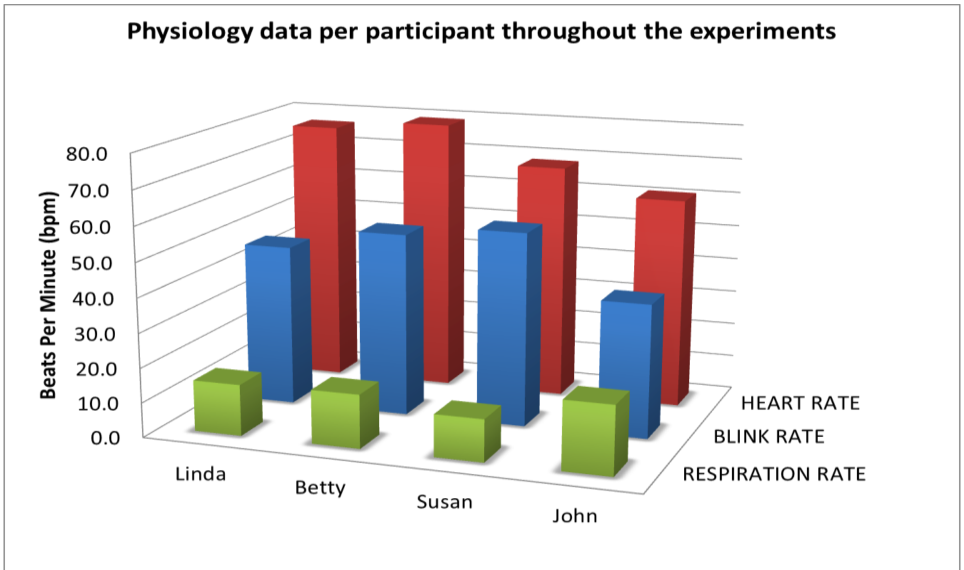


	Linda	Betty	Susan	John	AVG
CAL	21	11	3	24	15.1
UND	10	9	5	4	6.8
REA	21	17	12	8	14.2

Figure 4.58. Average times spent per JOL classified based on CUR

4.6.4. Physiology Data per Participant throughout the Experiments

When physiology data are compared across participants, it can be noticed that John's respiration rate is significantly higher than other participants. This is due to the fact that John has asthma. That was also seen in his visual respiration signature throughout the experiment as frequent breaths with low amplitude, as mentioned earlier. Susan's blink rate is higher than others (Figure 4.59). This is considered mainly because her eyes flutter periods each took a few seconds, during the study and restudy periods (that was discussed in more detail in her learner profiling above). Apart from these, physiological signatures across participants look steady.



	Linda	Betty	Susan	John
RESPIRATION RATE	14.8	15.5	12.2	19.8
BLINK RATE	46.7	53.0	56.0	38.6
HEART RATE	76.6	79.1	68.3	61.0

Figure 4.59. Physiology data per participant throughout the experiments

4.6.5. Questionnaire Scores

Number Theory Pre-Questionnaire Scores

Prior to the experiment for this questionnaire the participants rated their attitudes towards their CUR skills based on a 5-point confidence scale (not at all, very little, okay, very much, completely) (Table 4.1).

__Table 4.1.__ ***Result of the number theory pre-questionnaire***

	Question	**CUR Type**	**Linda**	**Betty**	**Susan**	**John**
1	In general, how comfortable are you with mental calculations with 1 digit whole numbers? (5)	Calculation	5	4	3	3
2	In general, how comfortable are you with mental calculations with 2 digit whole numbers? (5)	Calculation	3	4	3	2
3	In general, how comfortable are you with reading and recall of information? (5)	Understanding	3	3	4	3
4	In general, how comfortable are you with reading and comprehension of information? (5)	Understanding	4	3	4	3
5	In general, how comfortable are you with thinking mathematically/logically? (5)	Reasoning	3	4	3	4
6	Overall, how comfortable are you with thinking/reasoning skills? (5)	Reasoning	5	4	4	4
7	Are you comfortable with the study material content? (TDT) (5)	n/a	3	2	4	3

According to the results of the pre-questionnaire (Table 4.1), Linda felt more comfortable with her calculation and reasoning skills, and less comfortable with her understanding skills. As explained in her learner profiling, the data indicated that Linda indeed felt most comfortable with her calculation skills, therefore she followed a strategy where she first focused on such tasks, and after feeling comfortable with them, switched her attention to the rest of the content. Her verbal post-experiment self-report validated this result. Also as seen on Figure 4.55, her self-reporting JOLs indicated that her confidence of calculations remained very high (100%) after studying the material.

Her pre-questionnaire data indicated that Betty felt more comfortable with her Calculation and Reasoning skills, and less comfortable with her Understanding skills (Table 4.1). Betty is a special case. She was very anxious with the material overall, and self-reported her displeasure by rating herself very little comfortable with the content material (see last question on Table 4.1). As detailed in her learner profiling, her high mathematics anxiety score and her negative behavioral cues throughout her experience with the material further validate this result. Her anxiety and overall discomfort with the material is considered as a diminishing factor to distinguish her attitude towards CUR separately, and use her pre-questionnaire scores to cross-validate rest of the data based on this. It can also be concluded that her overall anxiety might have negatively affected her positive attitude towards calculations during the experiment, and she could not succeed well due to her loss of motivation caused by this anxiety.

It is important to highlight that although Susan indicated moderate level of comfort with calculations here (Table 4.1), as detailed in her analysis, her behavioral data indicated she was not comfortable at all with such content. This points out how self-report of a learner can be different from her actual learning behavior. This is a significant finding showing how contemporary methods such as eye-tracking can be helpful to test the validity of self-report data.

John felt more comfortable with his reasoning skills and least comfortable with his calculation skills (Table 4.1). However his analysis indicated that he succeeded well in calculations, and he had difficulties in understanding and reasoning tasks. As mentioned above, use of in depth learner profiling analysis evidences that self-report data from participants could be different than their actual attitudes and behaviors.

Motivation and Anxiety Survey Scores

Intrinsic and extrinsic motivation scales should be evaluated together. When we take the case of Betty although she has a relatively high intrinsic motivation score, her extrinsic motivation score is considerably higher than all other participants (Figure 4.60). She indeed showed more characteristics of a performance motivated learner with respect to all other participants, as detailed in her learner profiling above. On the other hand when we take the case of John, his intrinsic motivation score is significantly higher than his extrinsic motivation score. Also when we compare these scores of him with other participants, he has the highest from the former and the lowest from the latter. This is in parallel with his explicit mastery orientation throughout the experiment.

Scores of test anxiety and math anxiety are in parallel. The scores from these scales indicate that Betty was the most anxious participant and Susan was the least anxious participant. As explained in detail in their learner profiling above, their behavioral data indicates this is actually the fact. It is also worthwhile to point out that moderate level of anxiety is visible for the participants with mastery dominant motivation: Linda and John. This can be considered as an indication that such participants took the tasks seriously, which results in some emotional arousal that helps them keep motivated on the tasks, thus it is beneficial. This also indicates that having too much anxiety (the case of Betty) and too low anxiety (the case of Linda) might be associated with performance motivation. Future research is needed to verify this interpretation.

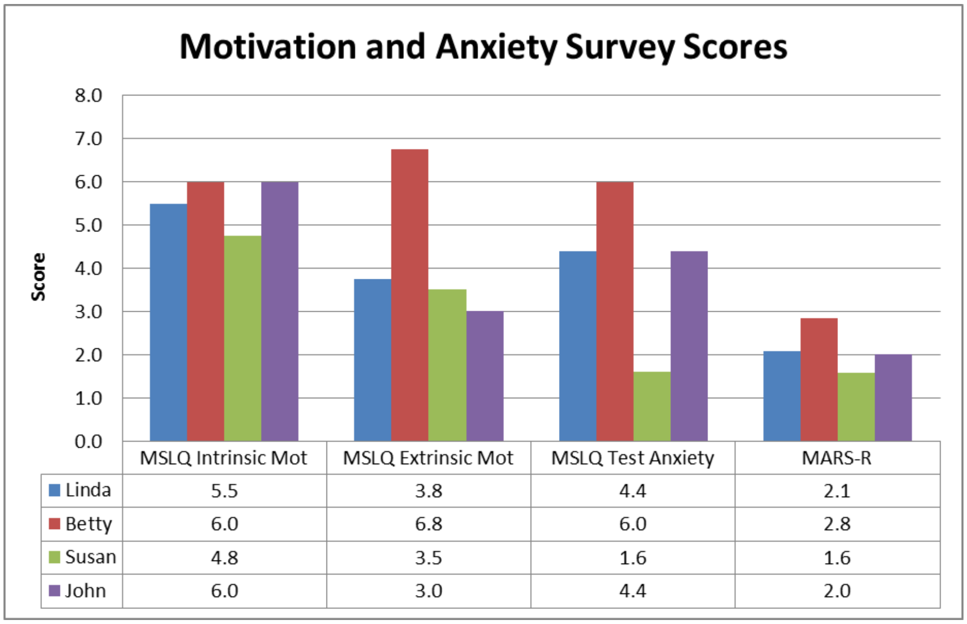


Figure 4.60. Motivation and anxiety survey scores

Epistemological Belief Inventory Scores

Betty's belief of innate ability is highest among all participants (Figure 4.61). This might be associated with her high math anxiety score and supporting behavioral evidence. In that respect, she might have low confidence with her mathematical abilities those she considers she lacks from the birth, which might be causing her anxiety and preventing her to be motivated to understand the material. John receives the lowest score from the omniscient authority score. This is in parallel with frequent comments questioning experimental settings and the content, as detailed above in the results section. As behavioral data confirms, this indicates his skeptical character where he does not hesitate questioning rules, conditions or authority. Linda received highest scores for quick learning and simple knowledge. As detailed in her learner profiling above, this scores are in parallel with her behavioral and self-report data indicating a strategy driven personality, belief in the simplicity and fast learnability of knowledge.

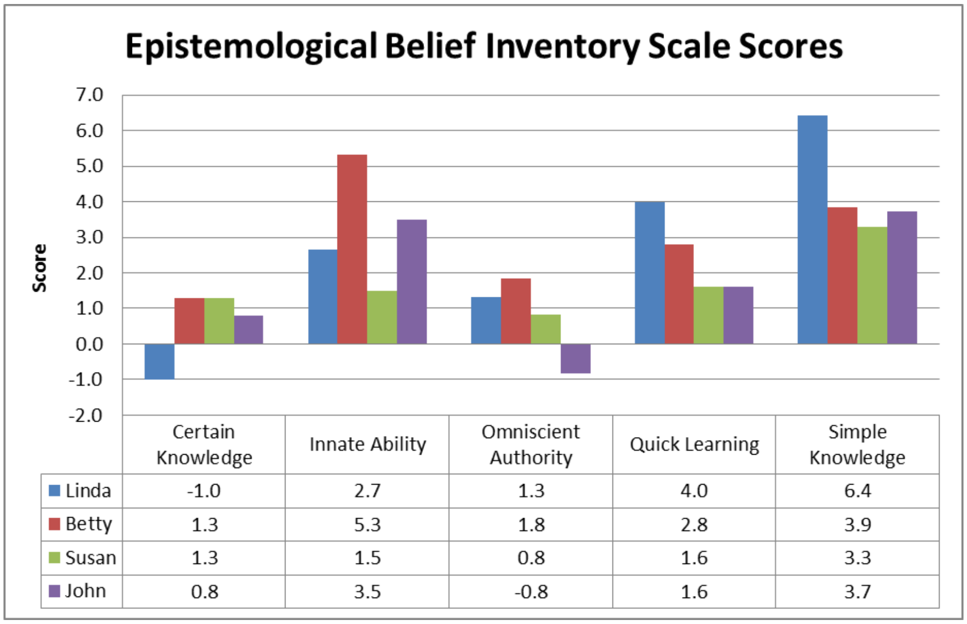
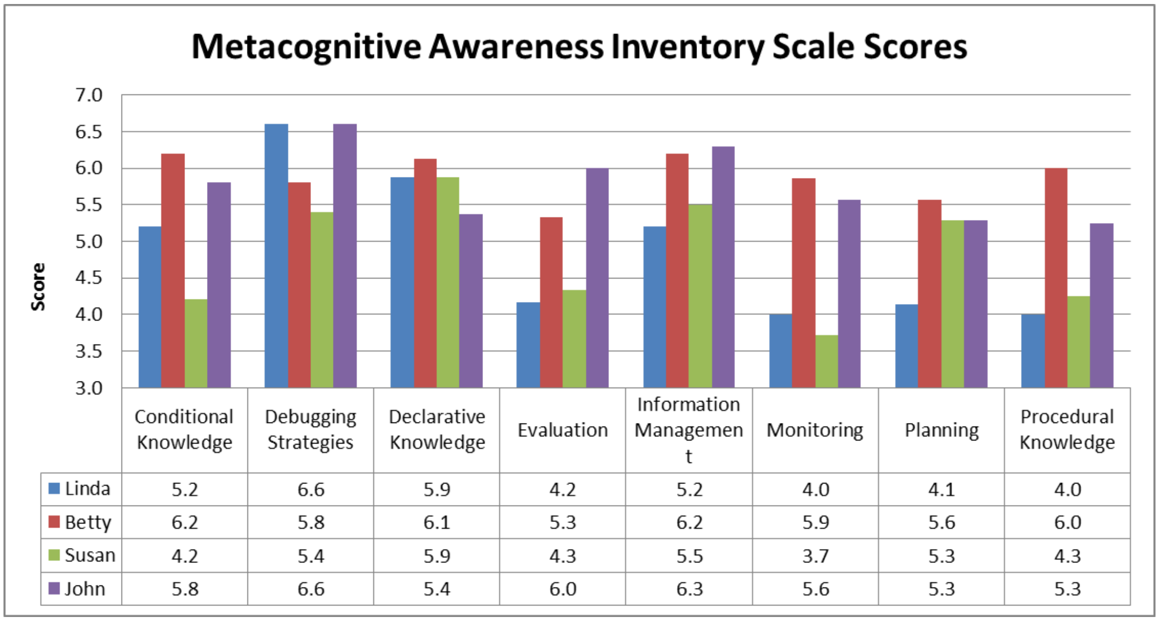


Figure 4.61. Epistemological belief inventory scores

Metacognitive Awareness Inventory Scale Scores

The MAI scores were evaluated for a few occasions during the single case analyses in the results section above. As for cross case analysis, interestingly it is observed that the MAI scores from five scales (conditional knowledge, evaluation, information management, monitoring, procedural knowledge) were higher for Betty and John in comparison to other participants (Figure 4.62). Considering Betty was observed as a performance motivation oriented learner whereas John was the opposite, these parallel scores indicates that the metacognitive awareness of learners are related to their motivational orientations. This might also indicate that the metacognitive awareness capabilities of learners show benefits when combined with mastery motivation (i.e. the case of John) whereas its positive effects diminish with performance motivation or/and high level of anxiety (i.e. the case of Betty). Future research studies with larger sample sizes are needed to further test this hypothesis.



	Conditional Knowledge	Debugging Strategies	Declarative Knowledge	Evaluation	Information Managemen t	Monitoring	Planning	Procedural Knowledge
Linda	5.2	6.6	5.9	4.2	5.2	4.0	4.1	4.0
Betty	6.2	5.8	6.1	5.3	6.2	5.9	5.6	6.0
Susan	4.2	5.4	5.9	4.3	5.5	3.7	5.3	4.3
John	5.8	6.6	5.4	6.0	6.3	5.6	5.3	5.3

Figure 4.62. Metacognitive awareness inventory scale scores

Chapter 5.

Discussion

After highlighting the importance of the learner profiling framework, this chapter will revisit the research questions expressed at the end of Chapter 2. In that regard, it covers the contributions of the current research to the literature supported by significant findings.

5.1. Education Research Methodology: Learner Profiling as a Framework

Before revisiting the research questions, first and foremost, this study offers a unique research framework to educational research, *learner profiling*. Learner profiling introduces a new way of studying learners where their cognitive and affective actions are monitored and recorded both behaviorally and physiologically, and combined with self-report data. As detailed before, behavioral methods involve audio-visual recordings, screen capture and eye-tracking, whereas physiological methods involve eye blink, heart and respiration responses. Self-reporting methods involve multiple surveys those aimed at measuring participants' metacognitive, epistemological, and emotional states as well as their self-judgments of learning of the material they study. Another angle is applying the restudy period to see its effects on other parameters mentioned above, as well as participants' learning. Test items relating to the content material and participants' experiences with the experiments were used as validation tools to detect their motivation. Future research using learner profiling methodology might customize number and type of methods used depending on the characteristics of the research. Offering such framework for combining multiple methods is important and has strong potential to be used by educational researchers in the future. In spite of the challenges brought with the new approaches and methods, the results are encouraging. I believe this research provides solid contributions to the literature and has great potential for improving educational research methodologies, especially with regards to the techniques involved and its offerings as a new compact form of in depth learner analysis.

It is also significant that my thesis offers a new term that is a *learner profile*. The literature provides no similar introduction of learner profiling or a learner profile. Building on my definition of learner profile, future research is expected to expand its content coverage as to include more topics from multiple disciplines (i.e. STEM fields, reading, music), in addition to the methodology as to include more efficient usage of traditional techniques, such as verbal self-reporting. That way it will be possible to determine motivational and cognitive orientations, abilities as well as inabilities and anxieties of learners, and to offer appropriate customized pedagogical solutions and strategies to learners for improving their knowledge spaces and versatility, as well as their motivational and affective strengths, to be better learners.

5.2. Revisiting Research Questions and Related Contributions to the Literature

The following subsections will address research questions proposed at the beginning and summarize how the findings contribute to the literature.

5.2.1. Revisiting Research Question 1: CUR Framework and Its Significance

The first research question of this study was "How beneficial is the calculation/understanding/reasoning (CUR) framework for classifying learners' cognitive orientations and corresponding mathematical content?" This study identifies mathematical content and cognition in three categories those are *calculation*, *understanding* and *reasoning.* Based on behavioral and physiological tools, the results provide evidence that abilities, motivations, interests, and achievements of learners can be distinctively classified based on CUR framework. For example Susan's performance avoidance pertains only to calculations, whereas John showed indications of mastery avoidance on understanding and reasoning tasks for the most part, while showing mastery approach on calculation related content, and Linda showed first mastery-approach and then mastery avoidance motivation on such tasks (Elliot & McGregor, 2001). Also pre-questionnaire and cross analysis results indicated that self-reports (Table 4.1) and study times (Figure 4.58) of learners vary among these three categories. It is necessary to note that while the findings of this study indicates CUR is a successful classification, future research with larger sample sizes is needed to further test its validity.

5.2.2. Revisiting Research Question 2: Effectiveness of Restudying Same Material

The second research question of this study was "How beneficial is restudying same material for learners cognitively or affectively?" It was observed that Betty and Susan, who consciously did not attend to some content and had less overall motivation during the study phase, spent the least amount of time with less attention on restudying in comparison with the other participants. Therefore there was no observable positive effect of restudy for these participants. On the other hand Linda and John, who showed indications of superior attention to the material and tasks with higher overall mastery motivation during the study phase, spent more time restudying the material, evaluated this period to better understand the material, and used it to revaluate their previous answers. Therefore based on the data from these four participants, restudying had positive effects on learners with higher overall mastery motivation, whereas it was not used as a helpful tool and had no positive effect on learners with more overall performance motivation.

5.2.3. Revisiting Research Question 3: A New Approach to Motivational Orientations

The third research question was "How beneficial is *learner profiling* for determining factors associated with learners' motivational orientations?" Using the learner profiling framework, the findings of the current study provide significant contributions to the psychology literature on motivation in that regard. Elliot and McGregor (2001) define four possible profiles for participant motivation; those are mastery-approach, mastery-avoidance, performance-approach and performance-avoidance. However detailed analyses of each participant in this study show that learners do not solely fit one of these categories; and motivational constructs of learners are much more organic and nuanced. Motivational orientations are heavily depended on the type of content as well as momentarily affective and cognitive states of learners, thus drawing conclusions with a single point of view can be misleading. As an example, Linda spent most of her time on calculation tasks whereas she did not spent much time on understanding and reasoning tasks. Later on she behaved exactly opposite way where she spent most of her time on understanding and reasoning tasks. This behavior was due to her studying strategy where she first spends her time studying the parts she is most comfortable with, to ensure she understands them very well and put them aside. After making sure that she understands these parts, she switches her attention to the rest of the content.

The first outcome from this example is that motivational orientations of learners depend on the type of content, even within the same discipline, subject or topic. The material used for this study is a mathematical text that involves calculation, understanding and reasoning content. It is very clear that Linda's motivation was oriented towards calculation tasks at the beginning, then shifted towards understanding and reasoning tasks. The second outcome of this example is how motivational orientation is dependent on time within a single session of study. Her ignorance of understanding and reasoning tasks at the beginning does not indicate that she had performance motivation, because she approached these tasks with mastery motivation for the rest of the experiment. Similarly her mastery-approach orientation shifts towards mastery-avoidance after she felt comfortable with calculation tasks.

Similar examples of having different motivational orientations relative to time and type of content are visible for other participants. Although generally having performance motivation, it was observed that Betty shifted towards mastery-approach motivation for some parts during the experiment, such as during the period of building the Spider's Web. During this period, in contrast to her anxious and nervous mood throughout the experiment, she did not show any behavioral or physiological symptoms of anxiety (Further research can test the relation between such symptoms and motivational orientations of learners and see how these indicators can be used to evaluate motivational orientations). In addition Susan's specific avoidance of calculation related content was easily distinguishable from the rest of her studying the material. John's ignorance of verbal content (which would be considered as performance-avoidance at first look) later turned to be mastery approach after he recognized this parts of the content was important for him to be successful on the test. As a result, his ignorance is not considered caused by his performance motivation, rather it was because he is an expert learner and preferred to spend less time on the content he judged less important. Therefore his motivation can be assessed changing from mastery-avoidance to mastery approach for verbal content. All these examples provide solid evidence that motivational orientations of the learners are much more nuanced than previously thought in the literature, and future research is recommended taking these results into consideration. In that respect, it is recommended using the terms of *mastery dominant learner* and *performance dominant learner* in lieu of 'mastery learner' and 'performance learner' to express a learner's overall motivational orientation.

5.2.4. Revisiting Research Question 4: Emotional and Cognitive Cues

The fourth research question was if "How beneficial is *learner profiling* for determining kinds of behaviors associated with different types of learners?" Unlike the psychophysiology literature that uses quantitative methodology, while benefiting from the findings of previous psychophysiology literature, this educational research study uses learner profiling with a qualitative dominant multiple case approach by focusing on each individual's behavioral and physiological measures to obtain deeper insight into their cognitive and affective states. Therefore the current study provides evidence that psychophysiological methods have potential use for spotting emotional states of learners based on single case observations. Behavioral cues such as facial expressions and eye tracking were cross validated with eye blink, heart and respiration rates of the participants. The results are significant and encouraging in terms of connecting such cues to underlying cognitive and affective behaviors.

As an example just focusing on facial expressions, John's non-enjoyment smiles particularly during the first trial of the test were clearly identifiable where he had a clearly observable pressure for not being able to answer some test items due to his lack of focus on the understanding and reasoning related parts of the study material. Having the highest scores on the anxiety tests and showing discomfort with the task, Betty's facial expressions often indicated displeasure. The periods of anxiety were identifiable on their facial expressions of Betty and John, where they demonstrated immediate facial reactions to what they did not like, whereas Susan and Linda's facial reaction were more stable throughout the observations. Therefore it is considered that not all learners provide the same amount of facial cues, this is heavily depended on their personalities (for example Susan often meditates and she was observed usually having calm and easygoing character throughout the experiment, thus her face did not manifest immediate emotional arousal). It is also interesting that some participants looked above (Betty), or closed their eyes (Susan) while calculating. These findings are important for educational research to determine behavioral cues pertaining specific types of learners with specific emotional responses to specific types of content. Collecting more systematic facial expression and gesture analyses of learners with larger sample size has strong potential for future research. In the future, results gained from a larger project supported by more data and number of samples can be used in teacher education to teach students' common facial expressions, their properties, their links to different subjects, and the underlying cognitive and affective states.

Monitoring blink, heart and respiration waves and rates provided meaningful data that are in parallel with behavioral measures of emotional states. While having seconds of deep focus on a task or content, participants often held their breath, or had shallow breaths at very low rates. At the moments of interruption and nonattendance to the tasks, it was common that participants changed their body position more frequently and did more frequent eye blinks. Whether changing body position, facial expression or taking a deep breath; moments of surprise or facing hardship almost always came along with some behavioral and physiological cues. Moments of anxiety were also identifiable in the data in the form of multiple frequent blinks, swallowing or deep breaths. The eye flutter effect during eyes closed states was observed for Susan, which was either due to memory recall (might be short or long term), or mental calculation. While Susan had this effect for shorter periods of time, John's eyes fluttered almost all through his relaxation period after the first test trial, therefore anxiety is considered having an important role in this case. Validating their answers, Betty often closed her eyes, while John did multiple frequent eye blinks keeping the mouse button pressed while double checking his answer. Future research with larger sample size needed to further study if these behaviors are common among similar types of learners, and if they have significance pointing out higher motivation or higher level of thinking. Highlighting text is another interesting behavior that was exposed by John while studying important content. Further research can also test if this is a common behavior of expert learners who have more experience with both reading content on a computer screen and mastery motivation on the task they study. It was also common that participants often left mouse and keyboard, and then changed body position at the moments they wanted to have an increased concentration on a new task or content they were exposed to.

In addition to the common behavioral and physiological indicators for emotional states that are visible across participants, there were some other individual-specific indicators. Linda often needed to check the time, which was either because she had time pressure, or she got tired / bored of the task. Betty used body language more often to express her negative feelings. She sometimes seemed to be very frustrated; she started hitting the mouse or keyboard. She placed particular attention on the expression 'A=QD+R' and spend considerable amount of time focusing on it with no indication of distraction. Confronting with that specific expression sometimes caused her discomfort where she took a deep breath, or repositioned her body immediately followed by switching her attention to a different content. It was very interesting that Susan almost always ignored calculation content; apparently she does not enjoy studying such content. John is a good case to observe common behaviors of an expert learner with mastery motivation. He used multiple strategies such as highlighting important content, making cross comparisons among different pages, or consciously ignoring verbal parts those he consider unimportant for understanding the material. His physiological data provided clear picture how his anxiety affected his respiration and heart rate that was caused by unexpected hardship he confronted during the first test trial, and how restudying the material lead to a big relief on him.

5.2.5. Revisiting Research Question 5: Eye Tracking as a Method for Revealing Connections

The fifth research question addressed if "How beneficial is eye tracking technology for determining connections built by learners among different concepts/sections within the study material?" Addressing this question, the results of the current study provides contribution to Mathematics Education literature with regards to the connections that participants built between related parts of the content. NCTM documents introduce Connection Standards such as "recognize and use connections among mathematical ideas" and "understand how mathematical ideas interconnect and build on one another to produce a coherent whole" (NCTM, 2000; NCTM, 2006). Thanks to eye-tracking data in this study, some connections that participants built could be spotted. The Spider's Web phenomenon in Betty's observation is an example to this (Figure 5.1) where she built a Spider's Web among divisibility concepts (divide, factor, multiple and divisor), and 'A=QD'. (Remember the literature under subsection titled 'Use of Eye-Tracking in Mathematics Education Research' providing evidence that our eyes focus on where our attention is focused and our eye gaze shifts to where our attention shifts). While all participants similarly built connections between related concepts on a single page, Spider's Web phenomenon is considered special and named differently because of its intensity of connection building for 2:30 minutes with no gaps, while other participant did not spend more than 30 seconds for connections.

These connections Betty builds within seconds are normally invisible to the teacher/researcher. Eye-tracking data along with the behavioral and physiological cues are telling us that she is very well focused during this period of bridging the concepts. These conceptual bridges Betty builds between divisibility concepts also inform us what specific concepts on such a definition are important for a learner to grasp the conceptual understanding of the material. This is an important cue for developing pedagogical approaches for teaching similar materials. Further research can focus what type of study techniques learners develop, and which are providing better performance on conceptual understanding later on. Similar approach can also be used to develop custom pedagogies for teaching different topics and subjects. Additionally, this methodology can be used to measure the effectiveness of the current and alternative ways of teaching similar subjects considering the extent to which they meet NCTM Standards pertaining to connections. Another example of developing connections can be seen in John's cross-comparison strategy (Figure 5.2). John was the only participant effectively building connections among different pages, and different phases within the experiment. John's Cross-Comparison Trajectory differs from Betty's Spider's Web in the sense that it involves connection building among pages, whereas Spider's Web happens on a single page. Considering he was evaluated having highest overall mastery orientation to the material, future research can further test if cross-comparison is a standard behavior of expert learners, if yes, then if this can be used to as a tool to measure learners' motivation.

Figure 5.1. The Spider's Web trajectory

Figure 5.2. Cross-comparison trajectory among pages, building connections

5.2.6. Revisiting Research Question 6: Learner Profiling as a Validation Tool for Self-Report Data

Learner profiling framework displays affective use of its tools for potentially testing the validity of self-reports by the participants. Although this study did not aim at directly and systematically test the validity of self-report data, some results indicate that the actual attitudes and beliefs of learners can be very different from what they report. For example, although Susan self-reported her attitude towards calculations 'okay' during the pre-questionnaire, and self-reported her understanding of most calculation related content as '*not well understood'* during her self-reporting JOLs, she did not even attempt studying them, instead ignored all such tasks. For this reason, it is concluded that Susan is to be a performance-avoidance learner when it comes to calculations. It would not be possible to know that she does not attend to calculations at all, based on traditional methods. That factual learning behavior of her could be determined based on the analysis of eye-tracking data. Another case how learners' self-report can be different from the reality is John's. During his pre-questionnaire, John self-reported that he feels relatively more comfortable with his reasoning skills and least comfortable with his calculation skills. However his analysis indicated that he succeeded well in calculations, and he had more difficulties in understanding and reasoning tasks. Also in the case of Betty, she self-reported all JOLs involving the expression 'A=QD+R' as well understood in spite of the fact that these expressions caused her anxiety most of the time, and she could not respond successfully to the questions involving that expression. These three examples indicate that self-report data may not always reflect the factual, and using learner profiling framework can potentially be used to test the validity of self-report data.

Chapter 6.

Conclusion

This last chapter will provide a section where I will discuss the limitations of the study and its potential for future research. The concluding remarks will summarize the study with a specific focus on its significance.

6.1. Limitations, Potential and Recommendations for Future Work

To my knowledge this research is its first example for single trial learner profiling combining behavioral and physiological methods. As a result of the triangulation approach, the validity was strengthened using numerous self-reporting tools to overcome the insufficient previous evidence from limited number of educational literature using physiological monitoring of single trials (Denzin, 1978).

Complexity of the tools and protocols of the analysis are considered as a limitation of this study. To overcome these difficulties for future research, establishment of interdisciplinary teams with a variety of backgrounds (such as education, psychology, medicine, computer science) is recommended. Also development of more practical, user friendly and compatible sets of hardware and software is necessary. Without more simplified use of technical equipment and software; recording, synchronizing, processing and combining multiple data sets require too much time, energy, experience and knowledge that limits the number of researchers capable of effectively and efficiently using these techniques.

Another limitation of the current study is the limited coverage of the learner profiles. This is mostly because the content was limited to the division theorem and related concepts. In addition the lack of post experiment interviews for all participants (except for Linda) is another reason for this limited coverage. Interviews during the experiment were considered being potentially distracting for the participants, however post-experiment interviews for all participants could be helpful and recommended for future research.

Expanding learner profiles will be possible if multiple sessions covering multiple topics and subjects with a single participant are done. At this point, this study defined learner profiling, and a learner profile, and aimed to reflect what data can be uncovered using non-traditional methods, and what practical outcomes can be achieved in educational context in that sense. I hope that future research will benefit from my experiences, and take it forward.

Improving ecological validity is another difficulty to overcome. The methods used in this study require participants to use a computer screen in a closed room with sensors attached to their body. Although this study has multiple tools to ensure observational control, such as questionnaires, behavioral and physiological tools to measure participants' level of anxiety, and fine adjustments in the room, such as comfortable seating and darker environment to improve concentration on the tasks, it is an important next step for future research to further improve ecological validity. It is anticipated that along with the future developments of professional grade portable and wireless sensors those can measure and record behavioral and physiological data (such as eye-tracking lenses, wireless EEG, EKG and respiration sensors), similar studies can be conducted in classroom environments in a couple of years. It is also important to mention that in spite of some disadvantages, these methods provide researchers an opportunity to see and record things that are normally unobservable in traditional qualitative research settings (e.g. as mentioned in Hannula, 2006). In a usual case study, a learner is often distracted by the researcher who accompanies to him, that affects ecological validity negatively. However the tools used in the current study provide learners chances to interact with materials without interruption, and provide researchers opportunities for spotting and deeply analyzing significant recorded events rather than having to rely solely on the verbal / written explanations of the participants. As an example, the Spider's Web phenomenon is normally very hard to spot with conventional qualitative methods such as interviews; such phenomena are hidden from researchers and even from the learners themselves who experience them. Therefore in terms of ecological validity learner profiling methodology -although still having a lot of space for improvement- can easily be considered having more advantages than disadvantages in comparison with the traditional qualitative methodology.

The data used in the current study is a part of larger research that involves data from more participants. Similar learner profiling can be applied to other participants in different topics, different subjects or disciplines. These data sets also involve EEG recordings. Although analyzing EEG data is beyond the scope of this study, future research based on the same or similar data sets can focus on EEG analysis. I see the CUR framework having strong potential for future research. An effective CUR classification of learner behavior during a study session requires overall motivation to the tasks. It is not considered as a high possibility to classify learners' motivational or cognitive orientations based on the CUR framework for learners with resistance to studying material, having overall high anxiety or discomfort with it (such as Betty).

In addition, the data indicates that understanding and reasoning tasks, content and cognition, as well as responding learner behaviors can sometimes overlap. Most of the data indicated that learner behaviors are similar for understanding and reasoning tasks. On the other hand, in theory, understanding and reasoning are two different levels of cognition, former pertains to reading, recall and comprehension of information such as understanding of simple definitions, while latter pertains to thinking logically, such as the ability to build if/then relations. In that parallel, results of pre-questionnaire indicates that learners self-report their attitudes and skills towards calculation, understanding and reasoning differently. Moreover as seen on Figure 4.58, results indicate that participants spend more than twice as much time on the reasoning related content compared to the understanding related content. These results evidences that understanding and reasoning are two separate categories, and future research is needed to further study how and in what extend learner behavior differs based on CUR classification.

I also consider that a potential forth category might be added to the CUR in geometrical contexts. In her post-experiment self-report Linda stated, "I am a visual learner, so I prefer figures, diagrams in a text, so that I can put everything in contact". This indicates that some learners are likely to be more successful when visual components are involved. Therefore I propose that visualization likely be used as the forth category (where the framework would be named as the CURV) attached to reasoning where spatial content is incorporated and spatial cognition is also investigated. This aspect is not covered in the current study due to the nature of the content that does not involve spatial representations of TDT. Future research is required to explore visualization as a fourth category of this CUR framework.

6.2. Concluding Remarks

This study focusing on four cases with qualitative dominant perspective evidences that learner profiling methodology has strong potential to be used as a framework in educational research. As a subset of mixed methods profiling, it has aspects from traditional qualitative methodology combined with effective methods of psychophysiology.

This exploratory multiple case study provides four important contributions to the literature. First and foremost, using learner profiling framework with a perspective of maximizing the variability of methods used for profiling each case enables augmented insight into understanding learners as well as strengthening cross-validation capabilities of the research. Using eye tracking technique provides the researcher invaluable information about learners' attendance, what content they focus on, what content they ignore, and how their bodies react to such different scenarios. For example ignorance of calculations by Susan indicated negative previous experience and her performance-avoidance motivation orientation on such content. Audio-visual recordings are important to gain insight into the behavioral responses of learners in the process of learning, such as the changes in their facial expressions and gestures. Recordings of non-enjoyment smiles of Betty and John when hardships confronted are some examples of such phenomena. The physiological recordings such as eye blink, heart and respiration rates are also important to understand how learners' bodies respond to learning process physiologically. Multiple physiological signatures those are visible across all participants (such as increase in heart and respiration rates due to anxiousness), as well as participant specific signatures (John's multiple frequent eye blinks at the moments of decision) were recorded as important contributions to the literature.

Secondly, the data from four participants indicated that CUR framework provides a successful classification for categorizing mathematical cognition and corresponding content. The analysis with the CUR framework perspective indicates that learners can be more interested, motivated, skilled, or successful studying one or more of these categories, while having no interest or skills for some others. For example, Linda had better performance and self-confidence on calculation tasks, whereas she was more anxious and less successful on understanding and reasoning tasks where she spent more time. Susan consciously avoided all calculation related content. While Betty's performance and motivation for all three types of content was below average, she focused more on some specific reasoning tasks. John focused well on all types, with the exception of verbal parts he considered not of much importance. These results provide evidence that learners are not fully attentive or inattentive when faced with mathematics content, they behave differently based on type of the content: calculation, understanding and reasoning.

Thirdly, results indicated that motivational constructs defined by Elliot and McGregor (2001) are time and content depended. The qualitative approach in this study combined with the CUR framework provides evidence that participants can have totally different motivational orientations depending on the CUR categories even while studying the same mathematics content (TDT in this case). Learner motivations can also change within minutes depending on their studying strategies or psychological states. As an example, Linda first spent her time mostly on calculation tasks and not much on understanding and reasoning tasks. She than showed an opposite study behavior, where she spent most of her time on understanding and reasoning tasks. This was because she preferred to first start and finish studying the content she is most comfortable with, which was calculations. Then she focused on to the other content which required more attention.

Lastly this study offers a qualitative dominant single case approach for monitoring learners' behavioral and physiological responses, and attempts to link them to emotional and cognitive states. The observations are validated using other measures and survey results and were built on the findings from previous literature. Facial cues are one example how behavioral data is linked to emotions. John's non-enjoyment smiles particularly during the first trial of the test are examples of such facial cues. The eye related behaviors of Betty looking above when thinking, and of Susan closing eyes while calculating (where her eyes lids flutter) are some examples pointing out the links between cognitive processes and behaviors. Physiology data that involve blink, heart and respiration waves and rates also indicates links with changes in emotional arousal of the participants. For example some participants often held their breath with lower frequency when they were focused. In other cases where they were interrupted, it was common that participants tended to move their bodies more often or blink more often. In some cases emotional arousal showed itself as frequent blinks, swallowing or deep breaths, for example in cases where participants confronted with hardship. The eye flutter effect was another behavioral cue that is linked to some underlying cognitive and affective processes such as memory processing (Susan) or anxiety (John).

The learner profiling research offers a new approach and tools in educational research; therefore it required a new theoretical and empirical framework. While there is sufficient literature separately for multiple methods used and theories reflected in this study, it is necessary to develop new lenses those cover these theories and methodologies. Working with such data requires skills, experience and expertise in using a variety of different software, as well as flexibility, knowledge, and experience in both qualitative and quantitative research methodologies. While it is challenging to overcome these difficulties, it is an enjoyable and valuable experience to gain the ability to apply and combine these methods, and to open up a new way for studying learners.

Also in acquiring such private data sets, extra care for ethical considerations, and perhaps even an entirely new *ethical* framework may be called for. Although it is the nature of research to gather data from individuals to study and present them in academic avenues, and the participants give consent for the use of their behavioural, physiological, and self-report data for research purposes, such data set should be considered highly confidential while their subjectivity is being unveiled.

Clearly, data sets conducted for research purposes must be kept secure and confidential, and used just for research purposes only, and not for practical applications in daily life. That is to say, misdistribution of such data sets in combination with real identities of the participants exposed could make private information available for the access of public (such as medical history, ethnicity and socioeconomic status) that will be potentially harmful or disturbing for participants. These issues are bound to become even more relevant in the future, as further technological innovations become available for monitoring and recording physiological manifestations of subjective states.

Looking to the future, consider for instance brain imaging technologies, such as EEG and fMRI. It is possible in a few decades that analysis of brain activity could uncover excessive amounts of information about the individuals, such as their identity, sexual orientation, memories, dreams, fears, desires and potentially even revealing their thoughts and visual experiences (e.g., Nishimoto, Vu, Naselaris, Benjamini, Yu, & Gallant, 2011). In that case, it might be possible to access and decode much private information about the individuals using previously recorded brain activity datasets.

Eventually expanding upon and adapting new technological innovations in support of learner profiling into practical situations, for example, to classroom environments might lead to further ethical considerations. As an example, using wireless signals, or use of networks, carry potential for undesired and unauthorised access (or "hacking") into these datasets. In another vein, drawing conclusions from monitoring and analysing embodied behavior of individuals might raise concerns from humanistic points of view (e.g. evaluations based on involuntary physiological cues might not be desired to be used by students). Extending and applying such research methods into daily life situations (e.g. classrooms) requires very sensitive planning and prior consultations and open discussions among researchers and other concerned stakeholders.

On the other hand it is important to highlight the fact that almost all available behavioral and physiological methods are already being widely used in many fields such as marketing research, human-computer interaction, cognitive neuroscience and psychophysiology, and they are becoming more and more accessible for public use (such as google glasses, inexpensive wireless EEG systems, smartphone applications that can measure and record heart signals, and facial expression analysis software, just like it was the same case for video recordings, which is now being widely used in educational research).

Ethical implications and issues of learner profiling in educational research and practice can be informed by the medical sciences. Medicine can be seen as the most sensitive field of study requiring extraordinary attention to ethical considerations, where much information is gathered from humans, such as fluids or tissue to obtain cortisol samples to measure anxiety, or DNA, which carries enormous amounts of information about an individual.

As complex, sensitive, and far-reaching as these ethical considerations are, they should not necessarily be considered as barriers to further research. Data gathered from human bodies (living and deceased), have led to much valuable knowledge and beneficial advances, thanks to which millions of lives have been saved, and overall human life quality improved. As a result, in terms of expanding methodological scope of educational research into the inner recesses of human embodiment, educational researchers are recommended to be very alert and attentive to potential ethical considerations, and yet not to be overly apprehensive or conservative in exploring potential benefits of learner profiling using contemporary and future equipment, methods, and techniques already widely used or yet to be developed in other disciplines.

It is no surprise that trying new approaches are exploratory and challenging. With the absence of similar previous research and having large quantities of data in hand require a highly careful attention to detail, revise and redesign approaches frequently, and many weeks of study time for observing the experiments. Beyond the technical side, the most crucial part of learner profiling is to observe, feel and breathe all data from participants, putting oneself into the learner's shoes to understand what cognitive and affective processes are at work underlying their reactions, and to gain instincts and reflexes for spotting and extracting key events from the data. After spotting significant events (behavioral and physiological cues providing links underlying cognitive and affective processes, such as changes in facial expressions) and key parts of the data (such as start/end of phases), an additional process is required to code and interpret different components of the data. These interpretations should be significant, meaningful, and valid throughout all components of the data sets, as well as valuable contributions to the literature.

I believe that the learner profiling research; which was defined, theoretically structured, and empirically studied here provides some promising theoretical and methodological advances to educational research. It has strong potential for future development and further important contributions to the literature for studying learners.

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Appendices

Appendix A. Consent to Participate in Research

CONSENT TO PARTICIPATE IN (MINIMAL RISK) RESEARCH
Simon Fraser University

Principal Investigator: Stephen R. Campbell, PhD **Phone:** [redacted]
Application Number: 37778 Email: [redacted]
Study Title: Mathematics Educational Neuroscience Laboratory
First REB Review: 2006/08/09 **Latest Amendment:** 2007/01/17

Informed consent involves:

- **Disclosure** of relevant information to prospective participants about the research;
- **Comprehension** of the information provided; and,
- **Voluntary agreement** to participate, free of any coercion or undue influence.

Disclosure
You are invited to participate in research conducted by Dr. Stephen R. Campbell, Ph.D. **Your participation in this study is entirely voluntary.** Data you provide is assigned an anonymous Participant Code. **You can discontinue your participation at any time.** Furthermore, your decision not to participate will have no adverse effects on your grades, your evaluations in any class, or your status in any program.
Please read all the information below and ask questions about anything you do not understand before deciding whether to participate. When all of your questions have been answered to your satisfaction, you may then, if you so choose, sign the attached consent forms to volunteer and participate in various aspects of this research study.

PURPOSE OF STUDY
This research study investigates the nature of mathematics related anxieties and understandings using computer learning environments and stimuli. It also investigates interdependencies between mathematics related anxieties and understandings. These data sets will be maintained securely and anonymously for subsequent analyses, and may be submitted for presentation to academic venues and in academic publications.

THREE DATA SETS (COMPONENTS) THAT MAY BE GATHERED
(1) The first kind of data that may be gathered is mainly psychometric, starting with various questionnaires pertaining to demographics, mathematics related anxieties and understandings, with optional follow up interviews. For those volunteering for the minimal risk components of this study (see below), questionnaires pertaining to medical history and present states for screening and ensuring safety must be obtained as well.

(2) The second kind of data that may be gathered is audiovisual data that will record research participants' vocal and facial expressions and body movements, as well as capturing computer activities, such as recording the computer screen, key strokes, and mouse movements and clicks. Eye gaze, eye movement, pupil dilation (**ET**) data may be acquired using photodetectors and low-level infrared light emitters.

(3) The third kind of data that may be gathered is called psychophysiological, including electroencephalograms (**EEG**) to measure 'brain waves', electromyograms (**EMG**) to measure muscle movement, electrocardiograms (**EKG**) to measure heart rate, skin conductance (**SC**), temperature (**T**), blood volume flow (**BF**), and respiration (**R**). These data sets are acquired using transducers and conductors (electrodes).

PROCEDURES USED IN COLLECTING COMPONENT DATA SETS

(1) Psychometric data. These data are acquired in the standard form of questionnaires and semi-structured interviews. Some initial screening questionnaires will help determine whether any further data will be collected. Time commitment: approx. ½ - 1 hour.

(2) Audiovisual data. Cameras and microphones will be used to acquire audiovisual data during computer tasks and/or clinical interviews. Keystroke and mouse movement data will also be recorded during computer tasks. Very low levels of infrared light (**LLIL**) are reflected off of the retina for ET data. Time commitment: approx. ½ - 1 hour.

(3) Psychophysiological data (see above). EEG, EMG, EKG, SC, T, BF, and R data are acquired through electrodes placed on the scalp and other select areas of the skin using small amounts of gel to detect extremely small differences in voltage potential that are naturally generated through biophysical means. SC involves a negligible current through the skin, and BF also involves LLIL. Time commitment: ½ - 1 hour in addition to (2).

POTENTIAL RISKS AND DISCOMFORTS

Component 1 involves no risk, and your participation in Component 1 places no obligation on you to volunteer to participate in Components 2 and 3 (see above).

Components 2 and 3 are minimal risk research. All instrumentation complies with the relevant requirements of the applicable Canadian Standards Association and SFU Research Ethics Board standards and guidelines pertinent to the use of such equipment.

Consent forms identifying participants will be kept separately and securely from all data acquired, and will be assigned an anonymous Participant Code. It is remotely possible that participants may be identifiable through the anonymous presentation or publication of audiovisual data (Component 2). If you do not wish for your likeness to be presented or published, you can indicate your preference(s) in this regard on Page 3 below. Your participation in Component 2 places no obligation on you to participate in Component 3.

Some discomfort from the use of conductive gel and/or tape for attaching electrodes may be incurred. Volunteers will have completed a prescreening questionnaire to mitigate risks based on their health history. For some participants, mathematics related anxieties may evoke physical reactions and/or emotional discomfort.

RIGHTS OF RESEARCH PARTICIPANTS

You will receive a copy of your signed consent form(s), and at any time you may revoke your permission to use non-aggregated data acquired from you at any time preceding submission for academic presentations and/or publications by notifying the principal investigator in writing. Page 4 is provided for your convenience in this regard.

At your option, you may also submit a "participant's feedback" form to the Director, Office of Research Ethics if you have served as a participant in this project and would care to comment on the procedures involved. All information submitted to the Director will be strictly anonymous, unless you indicate your name to be known to the Principal Investigator of this study. Page 5 is provided for your convenience in this regard.

Participant Code:

CONSENT TO PARTICIPATE IN (MINIMAL RISK) RESEARCH
Simon Fraser University

Principal Investigator: Stephen R. Campbell, PhD **Phone:** [redacted]
Application Number: 37778 **Email:** [redacted]
Study Title: Mathematics Educational Neuroscience Laboratory Study

Statement of Comprehension and Voluntary Agreement

I am being asked to participate in an educational neuroscience research study. By initialing the components I am agreeing to participate in, and by signing accordingly, I acknowledge that I have read and understand the information provided above, that I have been given an opportunity to ask questions, and all of my questions have been answered to my satisfaction. I attest that the demographic and pre-screening information that I may choose to provide for volunteering my participation in components 2 and 3 is, to the best of my knowledge, both accurate and complete. I further attest that am at least 19 years of age or older.

Understanding that all of the data I provide will be anonymously maintained, analyzed, presented and published, by checking and initialing, I am providing my consent to participate in the following components of this (minimal risk) research study:

☐ please check and initial — **Component 1** (no risk - see 1 above)

☐ please check and initial — **Component 2** (minimal risk - see 2 above)

☐ please check and initial — **Component 3** (minimal risk - see 3 above)

☐ I have seen the participant videos for Components 2 and 3

☐ **do not** present my anonymous likeness at academic venues

☐ **do not** publish my anonymous likeness in academic publications

By signing this informed consent form, I willingly agree, free of coercion and undue influence, to participate in this (minimal risk) research study, subject to my specific permissions, as indicated above:

Name of Participant Date

I have explained the research to the participant, and answered all of his/her questions. I believe that he/she understands the information described in this document and, with that understanding, freely and willingly consents to participate:

Name of Investigator Date

Participant Code:

Stephen R. Campbell, Ph.D.
Assistant Professor
Faculty of Education
Simon Fraser University
8888 University Drive
Burnaby, BC, V5A 1S6
Canada

Dear Dr. Campbell

RE: Withdrawal of Consent

Study Title: Mathematics Educational Neuroscience Laboratory Study

This is to inform you that I am withdrawing my previously given consent to (please check appropriate boxes):

☐ participate in the minimal risk components of this study (if applicable);
☐ allow my likeness to be presented and/or published (if applicable);
☐ allow any data acquired from me to be presented and/or presented

I understand that my right to withdraw this consent pertains only to data I have provided that has not already been used in an unidentifiable manner as part of a greater aggregate of data. I also understand this right will be waived for any permissions that I have provided for the anonymous presentation or publication of my likeness that have already been submitted to an academic venue or that have already been submitted to an academic publication.

Name of Participant   Date

Participant Feedback Form

Participant Code:

Participant Feedback Form

Completion of this form is OPTIONAL, and is not a requirement of participation in the study. However, if you have served as a participant and would care to comment on the procedures involved, you may complete the following form and send it to the Director, Office of Research Ethics, Strand Hall, 8888 University Drive, Burnaby, B.C., V5A 1S6. All information received will be strictly anonymous, unless you wish your name to be made known to the researcher, as shown below.

Name of Research Study:	Mathematics Educational Neuroscience Laboratory Study (# 37778)
Investigator Name:	Dr. Stephen R. Campbell, Ph.D.
Investigator Department:	Education

Did you sign an Informed Consent Form before participating in the project?

☐ Yes ☐ No

Where there significant deviations from the originally stated procedures?

☐ No ☐ Yes

If Yes, please describe the nature of the deviation, the date, place and time:

Please make any comments you may have:

Completion of the Information Below is Optional

Participant Last Name: First Name:

Participant Contact Information/Address:

Home/Cell Telephone Work Telephone:

Email:

Do you wish your feedback to be anonymous? ☐ Yes ☐ No

Appendix B. Demographic Questionnaire

Participant Code:

Demographic Questionnaire (Component 1)

This information is strictly confidential and will be used for research purposes only.
All data collected through this questionnaire is secured independently of your identity, which you may provide only at your option, and need not divulge.

Personal Information:

Email address:

Sex:

Age (years/months):

Language:

First Language (first learned at home):

Second Language:

Other(s) Language:

Educational Background (Levels and degrees completed and/or in progress):

- ☐ Elementary School
- ☐ Secondary School
- ☐ Undergraduate Student (Indicate Majors)
- ☐ Bachelor (Indicate Majors):
- ☐ Master (Specialization):
- ☐ Doctoral (Specialization):
- ☐ Other...

Specialization

Ethnicity: (Please Specify)

What are the ethnic and/or cultural origins of your mother and father? Please choose natural or adopted, and provide %, e.g., Scottish (1/2), French (1/4), Chinese (1/4)

Mother:

Father:

Socioeconomic Status (SES): (Please indicate the appropriate level)

Estimate *your immediate family's average SES* while you were growing up:

Estimate the *average SES of the schools you attended* while growing up:

Estimate the *overall value your family placed on education* while you were growing up:

Appendix C. Number Theory Pre-questionnaire

Number Theory Pre-Questionnaire (NTPQ)

1. In general, how comfortable are you with mental calculations with 1 digit whole numbers?
   - not at all
   - very little
   - okay
   - very much
   - completely
2. In general, how comfortable are you with mental calculations with 2 digit whole numbers?
   - not at all
   - very little
   - okay
   - very much
   - completely
3. In general, how comfortable are you with reading and recall of information?
   - not at all
   - very little
   - okay
   - very much
   - completely
4. In general, how comfortable are you with reading and comprehension of information?
   - not at all
   - very little
   - okay
   - very much
   - completely
5. In general, how comfortable are you with thinking mathematically/logically?
   - not at all
   - very little
   - okay
   - very much
   - completely
6. Overall, how comfortable are you with your thinking/reasoning skills?
   - not at all
   - very little
   - okay
   - very much
   - completely

7. How comfortable are you at this time when you are informed that you are going to study a topic regarding the Division Theorem in this experiment?

- not at all
- very little
- okay
- very much
- completely

Appendix D. Motivated Strategies for Learning Questionnaire

Motivation Questionnaire

The following questions ask about your motivation for and attitudes about this class. Remember there are no right or wrong answers; just answer as accurately as possible. Use the scale below to answer the questions. If you think the statement is very true of you, circle 7. If a statement is not at all true of you, circle 1. If the statement is more or less true of you, circle the number between 1 and 7 that best describes you.

1	2	3	4	5	6	7
not at all true of me						very true of me

1.	In a learning task like this one, I prefer the material that really challenges me so I can learn new things.	1	2	3	4	5	6	7
2.	If I study in appropriate ways, then I will be able to learn the material in this task.	1	2	3	4	5	6	7
3.	When I take a test I think about how poorly I am doing compared with other participants.	1	2	3	4	5	6	7
4.	I think I will be able to use what I learn in this task in others.	1	2	3	4	5	6	7
5.	I believe I will receive an excellent grade in this task.	1	2	3	4	5	6	7
6.	I'm certain I can understand the most difficult material presented in the readings for this task.	1	2	3	4	5	6	7
7.	Getting a good impression in this task is the most satisfying thing for me right now.	1	2	3	4	5	6	7
8.	When I take a test I think about items on other parts of the test I can't answer.	1	2	3	4	5	6	7
9.	It is my own fault if I don't learn the material in this task.	1	2	3	4	5	6	7
10.	It is important for me to learn the reading material in this task.	1	2	3	4	5	6	7
11.	The most important thing for me right now is obtaining a good impression so my main concern in this task is getting a good grade in the test.	1	2	3	4	5	6	7
12.	I'm confident I can learn the basic concepts presented in the reading material.	1	2	3	4	5	6	7
13.	If I can, I want to get better grades in this task than most of the other participants.	1	2	3	4	5	6	7
14.	When I take tests I think of the consequences of failing.	1	2	3	4	5	6	7

15.	I'm confident I can understand the most complex material presented in this task.	1	2	3	4	5	6	7
16.	In a learning task like this, I prefer learning material that arouses my curiosity, even if it is difficult to learn.	1	2	3	4	5	6	7
17.	I am very interested in the content area of this task.	1	2	3	4	5	6	7
18.	If I try hard enough, then I will understand the learning material.	1	2	3	4	5	6	7
19.	I have an uneasy, upset feeling when I take a test.	1	2	3	4	5	6	7
20.	I'm confident I can do an excellent job on this task.	1	2	3	4	5	6	7
21.	I expect to do well in this activity.	1	2	3	4	5	6	7
22.	The most satisfying thing for me in this task is trying to understand the content as thoroughly as possible.	1	2	3	4	5	6	7
23.	I think the learning material in this task is useful for me to learn.	1	2	3	4	5	6	7
24.	When I have the opportunity in this task, I choose the contents that I can learn from even if they don't guarantee a good grade in the test.	1	2	3	4	5	6	7
25.	If I don't understand the learning material, it is because I didn't try hard enough.	1	2	3	4	5	6	7
26.	I like the subject matter of this task.	1	2	3	4	5	6	7
27.	Understanding the subject matter of this task is very important to me.	1	2	3	4	5	6	7
28.	I feel my heart beating fast when I take a test.	1	2	3	4	5	6	7
29.	I'm certain I can master the knowledge in this task.	1	2	3	4	5	6	7
30.	I want to do well in this task because it is important to show my ability to my family, friends, employer, or others.	1	2	3	4	5	6	7
31.	Considering the difficulty of this task, and my skills, I think I will do well in this task.	1	2	3	4	5	6	7

Appendix E. Epistemic Beliefs Inventory

Epistemological Belief

The following questions ask about your general epistemological belief. **Remember there are no right or wrong answers; just answer as accurately as possible.** Use the scale below to answer the questions. If you strongly disagree the statement, circle 1. If you strongly agree the statement, circle 7. If you more or less agree or disagree, circle the number between 1 and 7 that best describes you.

1	2	3	4	5	6	7
strongly disagree						very strongly agree

1.	Most things worth knowing are easy to understand.	1	2	3	4	5	6	7
2.	What is true is a matter of opinion.	1	2	3	4	5	6	7
3.	Students who learn things quickly are the most successful.	1	2	3	4	5	6	7
4.	People should always obey the law.	1	2	3	4	5	6	7
5.	People's intellectual potential is fixed at birth.	1	2	3	4	5	6	7
6. r	Absolute moral truth does not exist.	1	2	3	4	5	6	7
7.	Parents should teach their children all there is to know about life.	1	2	3	4	5	6	7
8.	Really smart students don't have to work as hard to do well in school.	1	2	3	4	5	6	7
9.	If a person tries too hard to understand a problem, they will most likely end up being confused.	1	2	3	4	5	6	7
10.	Too many theories just complicate things.	1	2	3	4	5	6	7
11.	The best ideas are often the most simple.	1	2	3	4	5	6	7
12.	Instructors should focus on facts instead of theories.	1	2	3	4	5	6	7
13.	Some people are born with special gifts and talents.	1	2	3	4	5	6	7
14.	How well you do in school depends on how smart you are.	1	2	3	4	5	6	7
15.	If you don't learn something quickly, you won't ever learn it.	1	2	3	4	5	6	7
16.	Some people just have a knack for learning and others don't.	1	2	3	4	5	6	7
17.	Things are simpler than most professors would have you believe.	1	2	3	4	5	6	7
18.	If two people are arguing about something, at least one of them must be wrong.	1	2	3	4	5	6	7
19. r	Children should be allowed to question their parents' authority.	1	2	3	4	5	6	7
20.	If you haven't understood a chapter the first time through, going back over it won't help.	1	2	3	4	5	6	7
21.	Science is easy to understand because it contains so many facts.	1	2	3	4	5	6	7
22.	The more you know about a topic, the more there is to know.	1	2	3	4	5	6	7
23.	What is true today will be true tomorrow.	1	2	3	4	5	6	7
24.	Smart people are born that way.	1	2	3	4	5	6	7
25.	When someone in authority tells me what to do, I usually do it.	1	2	3	4	5	6	7
26.	People shouldn't question authority.	1	2	3	4	5	6	7
27.	Working on a problem with no quick solution is a waste of time.	1	2	3	4	5	6	7
28.	Sometimes there are no right answers to life's big problems.	1	2	3	4	5	6	7

Appendix F. Metacognitive Awareness Inventory

Metacognitive Awareness Inventory

This part of the inventory surveys your views of study strategies and how you use strategies. Using numbers between 1 and 7, write a number on the line that best corresponds to whether the statement is. **Remember there are no right or wrong answers; just answer as accurately as possible**.

0 = not true at all of you … … up to … … 7 = true of you

1. _____ I ask myself periodically if I am meeting my goals.
2. _____ I consider several alternatives to a problem before I answer.
3. _____ I try to use strategies that have worked in the past.
4. _____ I pace myself while learning in order to have enough time.
5. _____ I understand my intellectual strengths and weaknesses.
6. _____ I think about what I really need to learn before I begin a task.
7. _____ I know how well I did once I finish a test.
8. _____ I set specific goals before I begin a task.
9. _____ I slow down when I encounter important information.
10. _____ I know what kind of information is most important to learn.
11. _____ I ask myself if I have considered all options when solving a problem.
12. _____ I am good at organizing information.
13. _____ I consciously focus my attention on important information.
14. _____ I have a specific purpose for each strategy I use.
15. _____ I learn best when I know something about the topic.
16. _____ I know what the teacher expects me to learn.
17. _____ I am good at remembering information.
18. _____ I use different learning strategies depending on the situation.
19. _____ I ask myself if there was an easier way to do things after I finish a task.
20. _____ I have control over how well I learn.
21. _____ I periodically review to help me understand important relationships.
22.._____ I ask myself questions about the material before I begin.
23. _____ I think of several ways to solve a problem and choose the best one.
24. _____ I summarize what I've learned after I finish.
25. _____ I ask others for help when I don't understand something.
26. _____ I can motivate myself to learn when I need to.

0 = not true at all of you … … up to … … 100 = true of you

27. _____ I am aware of what strategies I use when I study.
28. _____ I find myself analyzing the usefulness of strategies while I study.
29. _____ I use my intellectual strengths to compensate for my weaknesses.
30. _____ I focus on the meaning and significance of new information.
31. _____ I create my own examples to make information more meaningful.
32. _____ I am a good judge of how well I understand something.
33. _____ I find myself using helpful learning strategies automatically.
34. _____ I find myself pausing regularly to check my comprehension.
35. _____ I know when each strategy I use will be most effective.
36. _____ I ask myself how well I accomplished my goals once I'm finished.
37. _____ I draw pictures or diagrams to help me understand while learning.
38. _____ I ask myself if I have considered all options after I solve a problem.
39. _____ I try to translate new information into my own words.
40. _____ I change strategies when I fail to understand.
41. _____ I use the organizational structure of the text to help me learn.
42. _____ I read instructions carefully before I begin a task.
43. _____ I ask myself if what I'm reading is related to what I already know.
44. _____ I reevaluate my assumptions when I get confused.
45. _____ I organize my time to best accomplish my goals.
46. _____ I learn more when I am interested in the topic.
47. _____ I try to break studying down into smaller steps.
48. _____ I focus on overall meaning rather than specifics.
49. _____ I ask myself questions about how well I am doing while I am learning something new.
50. _____ I ask myself if I learned as much as I could have once I finish a task.
51. _____ I stop and go back over new information that is not clear.
52. _____ I stop and reread when I get confused.

Appendix G. Math Anxiety Rating Scale—Revised

Math Anxiety Rating Scale

Please rate your anxiety when you do the following things related to math.

1. Looking through the pages in a math text.
0 (no anxiety) 1 2 3 4 (high anxiety).

2. Having to use the tables in the back of a math book.
0 (no anxiety) 1 2 3 4 (high anxiety).

3. Thinking about an upcoming math test one day before
0 (no anxiety) 1 2 3 4 (high anxiety).

4. Watching a teacher work an algebraic equation on the blackboard.
0 (no anxiety) 1 2 3 4 (high anxiety)

5. Being told how to interpret probability statements
0 (no anxiety) 1 2 3 4 (high anxiety).

6. Picking up a math textbook to begin working on a homework assignment
0 (no anxiety) 1 2 3 4 (high anxiety)

7. Taking an examination (quiz) in a math course
0 (no anxiety) 1 2 3 4 (high anxiety).

8. Reading and interpreting graphs or charts
0 (no anxiety) 1 2 3 4 (high anxiety).

9. Signing up for a course in statistics
0 (no anxiety) 1 2 3 4 (high anxiety).

10. Waiting to get a math test returned in which you expected to do well
0 (no anxiety) 1 2 3 4 (high anxiety).

11. Being given a "pop" quiz in math class
0 (no anxiety) 1 2 3 4 (high anxiety).

12. Walking on campus and thinking about a math course
0 (no anxiety) 1 2 3 4 (high anxiety).

Appendix H. Study Material

Page 1

The Division Theorem

Let's study some **Number Theory**. We are only concerned here with **whole numbers**. The set of whole numbers, *W*, is defined by the **infinite set {0, 1, 2, 3,...} .**

Consider any two whole numbers, A and D, where **D ≠ 0.**

(next page)

Page 2

Given such an A and D, there exist unique whole numbers Q and R,

where **A = QD + R** and **R < D**. We say that **Q and R are unique,**

because there are no other values for A and D, such that A = QD + R.

(next page, previous page)

Page 3

For instance, **when A = 26 and D = 4, the *only* two numbers that will satisfy this formula are Q = 6 and R = 2**. A, Q, D, and R are referred to as follows:

A is the *dividend*.

Q is the *quotient*.

D is the *divisor*.

R is the *remainder*. (next page, or previous page)

Page 4

Examples:

If A = 7 and D = 3, then Q = 2 and R = 1, since 7 = (2)(3) + 1.

If A = 27 and D = 4, then Q = 6 and R = 3, since 27 = (6)(4) + 3.

The Division Theorem can be stated as follows:

For **A and D, where A and D are elements of W, and D > 0**, then **there exist unique elements, Q and R, of W**, such that **A = QD + R, where R < D**.

(next page, or previous page)

Page 5

Divisibility Relations

If, **given whole numbers A and D, if there is a Q such that A = QD and R = 0**, we say that "**A is *divisible* by D**," "**D *divides* A**," "**D is a *factor* of A**, " and "**A is a *multiple* of D**." Now, **if D is a *divisor* of A, then D *divides* A**, and **if D *divides* A, then D is a *divisor* of A**.

(next page or previous page)

We also say that **if D *divides* A, then D is a *factor* of A** and **if D is a *factor* of A, then D *divides* A**, and that **if D is not equal to A, then D is a *proper divisor* of A**. ***Prime numbers* have no proper divisors**.

Examples:

When A = 20 and D = 2, then 2|20, because 20 = (10)(2).

When A = 15 and D = 5, then 5|15, because 15 = (3)(5).

17 is a prime number because it has no divisors aside from 1 and itself.

(previous page) To restudy this material, click here

Appendix I. Instructions to Participants

Instruction to the participants

You will notice there are several **shaded regions** in the text. Please label **each** shaded region with **your judgment of how well you think you have learned the content of each shaded region**: *very well*, *well*, or *not well*. To label a shaded region:

1. Move your cursor over the first word of the **entire shaded region**.

2. Left-double-click where your cursor is.

3. Right-click to show the pop-up menu.

4. Move your cursor over the top "Label As …"

5. Move your cursor over the label you choose:

Appendix J. List of JOLs and Corresponding CUR Categories

JOL#	CUR Type	JOL Content
P1-JOL1	Understanding	The Division Theorem
P1-JOL2	Understanding	Number Theory
P1-JOL3	Understanding	whole numbers
P1-JOL4	Understanding	Infinite set {0, 1, 2, 3, …}
P1-JOL5	Reasoning	Consider any two numbers, A and D
P1-JOL6	Reasoning	$D \neq 0$
P2-JOL1	Reasoning	Given such A and D, there exist unique whole numbers Q and R
P2-JOL2	Reasoning	$A = QD + R$
P2-JOL3	Reasoning	$R < D$
P2-JOL4	Reasoning	Q and R are unique, because are no other values for A and D
P3-JOL1	Calculation	When $A = 26$ and $D = 4$, the only two numbers that will satisfy this formula are $Q = 6$ and $R = 2$
P3-JOL2	Understanding	A is the dividend
P3-JOL3	Understanding	Q is the quotient
P3-JOL4	Understanding	D is the divisor
P3-JOL5	Understanding	R is the remainder
P4-JOL1	Calculation	If $A = 7$ and $D = 3$, then $Q = 2$ and $R = 1$, since $7 = (2)(3) + 1$
P4-JOL2	Calculation	If $A = 27$ and $D = 4$, then $Q = 6$ and $R = 3$, since $27 = (6)(4) + 3$
P4-JOL3	Reasoning	A and D, where A and D are elements of W, and $D > 0$
P4-JOL4	Reasoning	There exist unique elements, Q and R, of W
P4-JOL5	Reasoning	$A = QD + R$, where $R < D$
P5-JOL1	Understanding	Divisibility Relations
P5-JOL2	Reasoning	Given whole numbers A and D, if there is a Q such that $A = QD$ and $R = 0$
P5-JOL3	Understanding	A is divisible by D
P5-JOL4	Understanding	D divides A
P5-JOL5	Understanding	D is a factor of A
P5-JOL6	Understanding	A is a multiple of D
P5-JOL7	Reasoning	If D is a divisor of A, then D divides A
P5-JOL8	Reasoning	If D divides A, then D is a divisor of A
P6-JOL1	Reasoning	If D divides A, then D is a factor of A
P6-JOL2	Reasoning	If D is a factor of A, then D divides A
P6-JOL3	Reasoning	If D is not equal to A, then D is a proper divisor of A
P6-JOL4	Understanding	Prime numbers have no proper divisors
P6-JOL5	Calculation	When $A = 20$ and $D = 2$, then $2 \mid 20$, because $20 = (10)(2)$
P6-JOL6	Calculation	When $A = 15$ and $D = 5$, then $5 \mid 15$, because $15 = (3)(5)$
P6-JOL7	Calculation	17 is a prime number because it has no divisors aside from 1 and itself

Appendix K. Number Theory Test

Post-Test

1. 1, 2, 3, 6, 7, 14, 18, 42, are all divisors of 42
   - True
   - False

2. Please rate how confidently you believe your last answer was correct. 0 indicates 0% confidence --- 10 indicates 100% confidence
   0 1 2 3 4 5 6 7 8 9 10
   Answer :

3. 7 is a divisor of 42
   - True
   - False

4. Please rate how confidently you believe your last answer was correct. 0 indicates 0% confidence --- 10 indicates 100% confidence
   0 1 2 3 4 5 6 7 8 9 10
   Answer :

5. In the equation 42 = 2(18) + 6, the dividend is
   - 6
   - 2
   - 42
   - 18

6. Please rate how confidently you believe your last answer was correct. 0 indicates 0% confidence --- 10 indicates 100% confidence
   0 1 2 3 4 5 6 7 8 9 10
   Answer :

7. For any two whole numbers A and D, A is divisible by D, and D divides A, if and only if there exists unique whole numbers Q and R such that A = QD + R, and R is less than D.
   - True
   - False

8. Please rate how confidently you believe your last answer was correct. 0 indicates 0% confidence --- 10 indicates 100% confidence
   0 1 2 3 4 5 6 7 8 9 10
   Answer :

9. A is a prime number because A is divisible by both 1 and A
   - True
   - False

10. Please rate how confidently you believe your last answer was correct. 0 indicates 0% confidence --- 10 indicates 100% confidence
    0 1 2 3 4 5 6 7 8 9 10
    Answer :

11. If 42 = 2(18) + 6, then 18 is a divisor of 42

- True
- False

12. Please rate how confidently you believe your last answer was correct. 0 indicates 0% confidence --- 10 indicates 100% confidence

    0 1 2 3 4 5 6 7 8 9 10

    Answer :

13. If the dividend is a multiple of the quotient, then the quotient divides the dividend.
    - True
    - False

14. Please rate how confidently you believe your last answer was correct. 0 indicates 0% confidence --- 10 indicates 100% confidence

    0 1 2 3 4 5 6 7 8 9 10

    Answer :

15. If A is a prime number and D divides A, where D is not equal to 1, then A divides D.
    - True
    - False

16. Please rate how confidently you believe your last answer was correct. 0 indicates 0% confidence --- 10 indicates 100% confidence

    0 1 2 3 4 5 6 7 8 9 10

    Answer :

17. A is a divisor of D, if D = QA + R, where R is less than A
    - True
    - False

18. Please rate how confidently you believe your last answer was correct. 0 indicates 0% confidence --- 10 indicates 100% confidence

    0 1 2 3 4 5 6 7 8 9 10

    Answer :

19. If D divides A and R divides D, where A = QD + R and R is less than D, then R divides A
    - True
    - False

20. Please rate how confidently you believe your last answer was correct. 0 indicates 0% confidence --- 10 indicates 100% confidence

    0 1 2 3 4 5 6 7 8 9 10

    Answer :

Appendix L. Number Theory Post-questionnaire

Post-Questionnaire

The following 6 questions ask about your thoughts having come to your mind during the task. Remember there are no right or wrong answers; just answer as accurately as possible.

1. Was this learning task interesting to you? 0 indicates not interesting at all --- 7 indicates very interesting

    0 – 1 – 2 – 3 – 4 – 5 – 6 – 7

2. Was this learning task challenging for you? 0 indicates not challenging at all --- 7 indicates very challenging

    0 – 1 – 2 – 3 – 4 – 5 – 6 – 7

3. In general I chose the items for restudy
    - that were interesting to me
    - that were difficult to understand
    - that were easy to understand
    - that were possible test questions
    - for no particular reason

4. Did the shading and highlighting take your attention away from studying the material at hand?

    0-not at all – 1 – 2 – 3 – 4 – 5 – 6 – 7-very much

5. To what extent did you review the item that you just labeled during your first 10-minute study?
    - Always
    - sometimes
    - Never

Appendix M. List of Test Items and Corresponding CUR Categories

Item#	CUR Type	Test Item
1	Calculation	1, 2, 3, 4, 7, 14, 18, 42 are all divisors of 42
2	Calculation	7 is a divisor of 42
3	Understanding	In the equation 42=2(18)+6, the dividend is
4	Understanding	For any whole numbers A and D, A is divisible by D, and D divides A, if and only if there exists unique whole numbers Q and R such that A=QD+R, and R is less than D.
5	Reasoning	A is prime number because A is divisible by both 1 and A
6	Reasoning	If 42=2(18)+6, then 18 is a divisor of 42
7	Reasoning	If the dividend is a multiple of the quotient, then the quotient divides the dividend.
8	Reasoning	If A is a prime number and D divides A, where D is not qual to 1, then A divides D.
9	Reasoning	A is a divisor of D, if D=QA+R, where R is less than A
10	Reasoning	If D divides A and R divides D, where A=QD+R and R is less than D, then R divides A

Appendix N. Post-experiment Questionnaire

Post-Experiment Questionnaire

This information is strictly confidential and will be used for research purposes only. All data collected through this questionnaire is secured independently of your identity, which you may provide at your option, and need not divulge.

Participant Code:

1. How did you feel before the experiments?

2. How did you feel during the experiments?

3. How do you feel after the experiments (now)?

4. Do you have any suggestions that would have helped to improve your experience?

Appendix O. Qualification Notice

Qualification Notice
(For Psychophysiology)

If you think for any reason(s), for example:

- being pregnant;
- being exceptionally gifted (IQ > 1000);
- user of nicotine (more than 30 cigarettes/day), marijuana (more than 3 joints/week), alcohol (more than 15 drinks/week), or any other substance (ab)use that you would rather not divulge;
- being HIV positive;
- being unavailable to take part in any subsequent research sessions;
- history of epilepsy;
- head lice;

-- that you should not participate in our experiments, or that your participation in our research would unduly bias our results, please return this questionnaire blank. Thank you.

DISCLOSURE STATEMENT

IF you are invited to participate AND if you also agree to participate in the minimal risk components of research that this questionnaire is pre-screening for, THEN you will be asked to review this information and attest to the fact that it is both accurate and complete to the best of your knowledge. You should also understand that this information would then be securely maintained as part and parcel of your informed consent to participate in this minimal risk research. IF, however, upon review of this information by the investigator, you are NOT invited to participate in this research, OR, IF upon further consideration, you choose to withdraw consent and NOT to participate in this research, THEN these questionnaires will be promptly shredded and securely disposed of.